COMP9334
Capacity Planning for Computer Systems and Networks

Week 2: is and

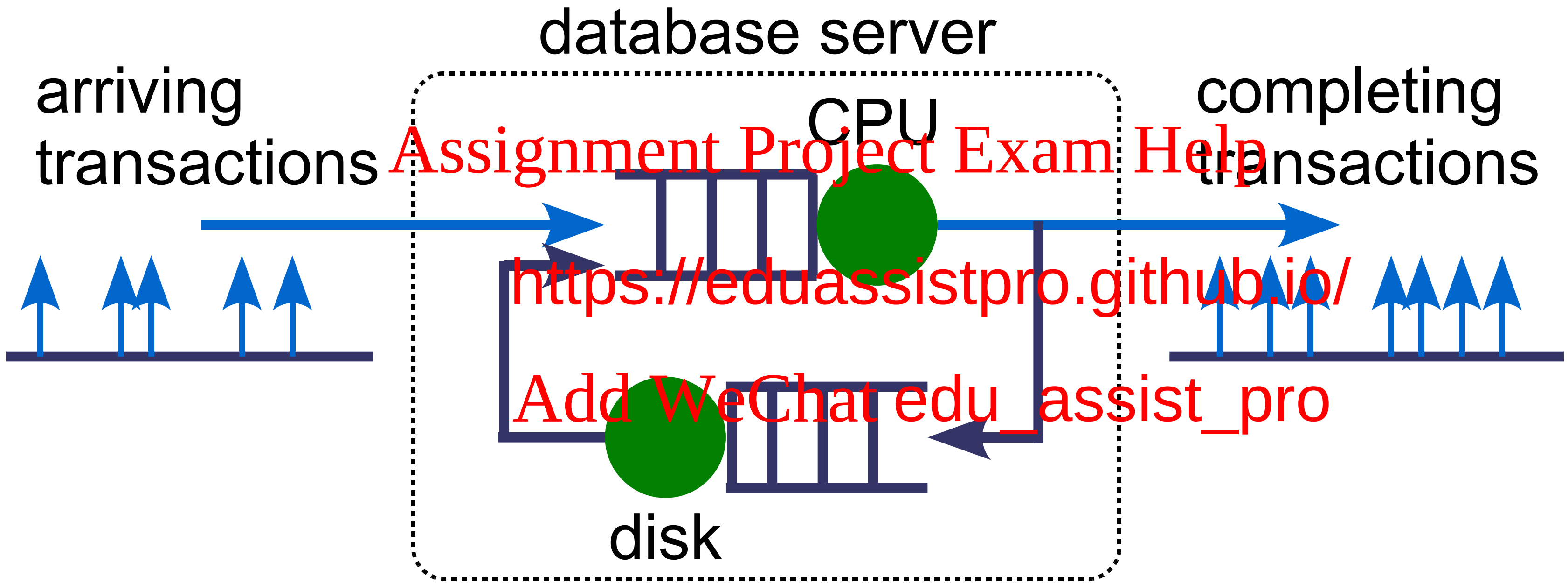
Workload Characteri

Last lecture

- Modelling of computer systems using Queueing Networks
  - Open networks
  - Closed networks

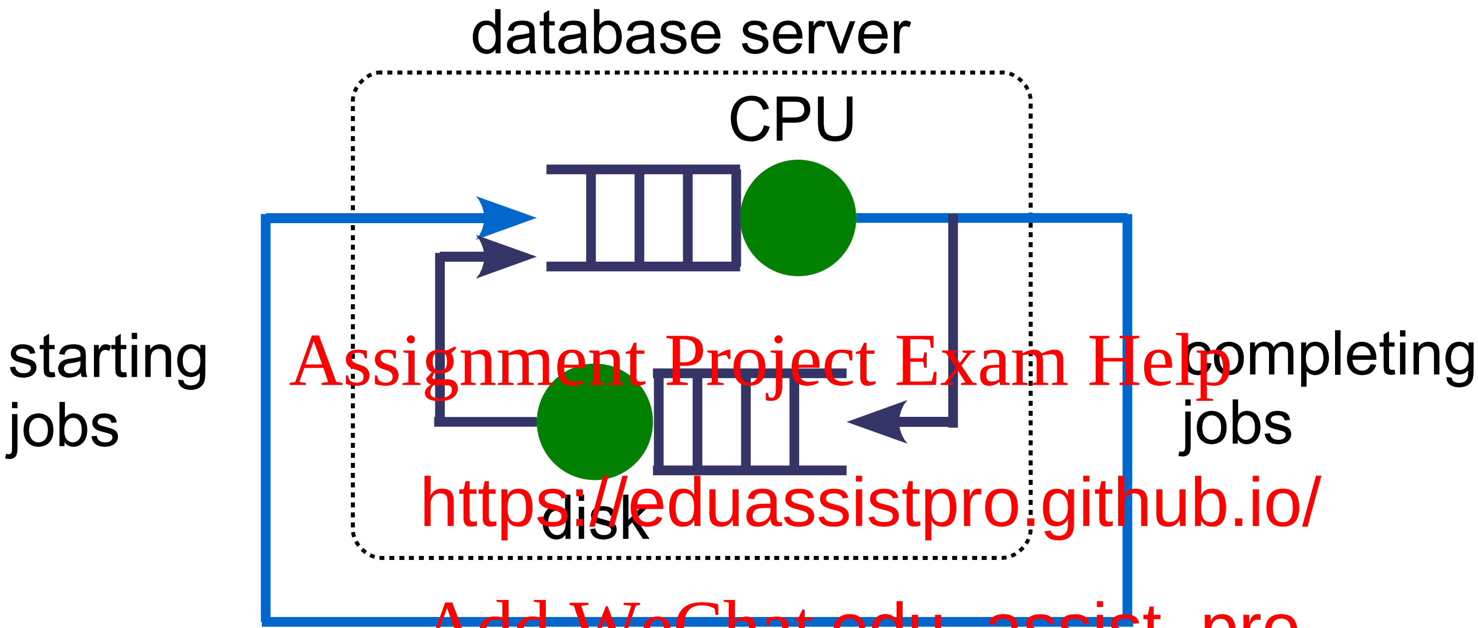
Open networks

Example: The server has a CPU and a disk.



A transaction may visit the CPU and disk multiple times.
An open network is characterised by external transactions.

Closed queuing networks



Closed queueing networks model

- Running batch jobs overnight
- Once a job has completed, a new job starts.

Good performance means high throughput.

#jobs in the system = multi-programming level

This lecture

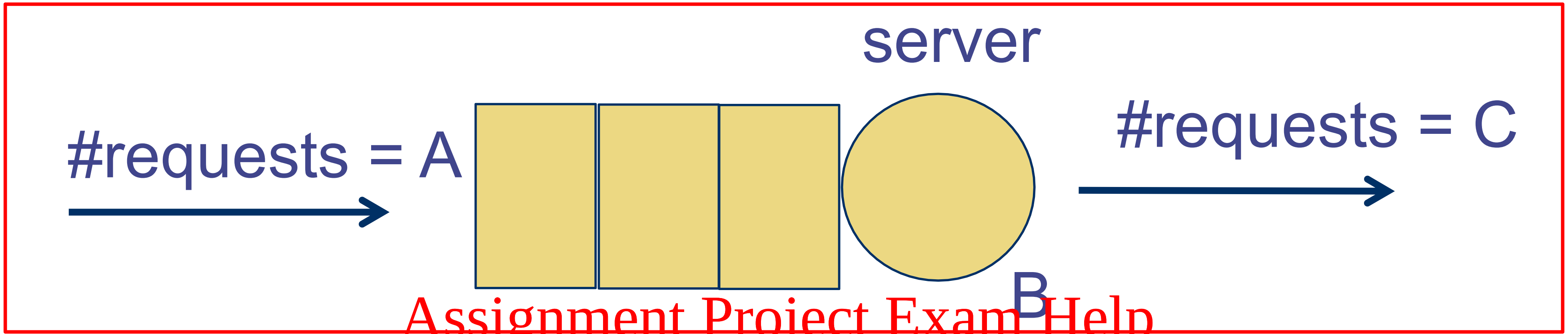
- The basic performance metrics
  - Response time, Throughput, Utilisation etc.
- Operational analysis
  - Fundamental Laws relating the basic performance metrics
  - Bottleneck and performance analysis
- Workload chara
  - Poisson proces

Operational analysis (OA)

- "Operational"
  - Collect performance data during day-to-day operation
- Operation laws
- Applications:
  - Use the data for building queueing network models
  - Perform bottle
  - Perform modifi

- iostat

Single-queue example (1)



In an observational y for time B
A requests arrived,

A, B and C are basic measurements

Deductions: Arrival rate λ = A/T
Output rate X = C/T
Utilisation U = B/T
Mean service time per completed request = B/C

Motivating example

- Given
  - Observation period = 1 minute
  - CPU
    - Busy for 36s.
    - 1790 requests arrived
    - 1800 request
  - Find
    - Mean service time per completi
    - Utilisation =
    - Arrival rate =
    - Output rate =

Utilisation law

- The operational quantities are inter-related
- Consider
  - Utilisation U = B / T
  - Mean service time per completion S = B / C
  - Output rate X = C / T

- Utilisation law – d X?
  -

- Utilisation law is an example of operational law.

Application of OA

- Don't have to measure every operational quantities
  - Measure B to deduce U - don't have to measure U
- Consistency checks
  - If $U \neq S X$, something is wrong

- Operational laws rmance analysis
  - Bottleneck anal

  - Mean value analysis (Later in th

Equilibrium assumption

- OA makes the assumption that
  - C = A
  - Or at least C ≈ A
- This means that
  - The devices and system are in equilibrium
    - Arrival rate ut rate of requests for that device = Thr
    - The above statement also app tem, i.e. replace the word “device” by “system”

OA for Queueing Networks (QNs)

The computer system has K devices, labelled as 1,…,K.

The convention is to add an additional device 0 to represent the outside world.

OA for QNs (cont’d)

- We measure the basic operational quantities for each device (or other equivalent quantities) over a time of T
  - A(j) = Number of arrivals at device j
  - B(j) = Busy time for device j
  - C(j) = Number of completed jobs for device j

- In addition, we h

  - A(0) = Number o
  - C(0) = Number of completions fo

- Question: What is the relationship between A(0) and C(0) for a closed QNs?

Visit ratios

- A job arriving at the system may require multiple visits to a device in the system
  - Example: If every job arriving at the system will require 3 visits to the disk (= device j), what is the ratio of C(j) to C(0)?

    - We expect C(j)/C(0) = $\boxed{}$

  - V(j) = Visit ratio of device j
    - = Number of times a job visits

    - We have V(j) = C(j) / C(0)

Forced Flow Law

Since $$V(j) = \frac{C(j)}{C(0)}$$

The forced flow law is

$$V(j) = \frac{X(j)}{X(0)}$$

Service time versus service demand

- Ex: A job requires two disk accesses to be completed. One disk access takes 20ms and the other takes 30ms.

- Service time = the amount of processing time required *per visit* to the device
  - The quantities "2 ndividual service times.

- D(j) = Service demand of a job is the total service time required by that job
  - The service demand for this job = 20ms + 30 ms = 50ms

Service demand

- Service demand can be expressed in two different ways
  - Ex: A job requires two disk accesses to be completed. One disk access takes 20ms and the other takes 30ms.
    - D(j) = 50ms.
    - What are V(j) and S(j)?
      - Recall that e j
    -
  - Service demand D(j) = V(j) S(j)

Service demand law (1)

Given D(j) = V(j) S(j)

Since $$V(j) = \frac{X(j)}{X(0)}$$

t is X(j) S(j)?

Service demand law $$D(j) = \frac{U(j)}{X(0)}$$

Service demand law (2)

- Service demand law D(j) = U(j) / X(0)
  - You can determine service demand without knowing the visit ratio
  - Over measurement period T, if you find
    - B(j) = Busy time of device j
    - C(0) = Number of requests completed
  - You' ve enough information to find D(j)

- The importance of service de
  - You will see that service dema mental quantity you need to determine the performance of a queueing network
  - You will use service demand to determine system bottleneck today

Server example exercise

Measurement time = 1 hr		
	# I/O/s	Utilisation
Disk 1	32	0.30
Disk 2	36	0.41
	50	0.54
C		0.35
Total # jobs=13680		

What is the service time of Disk 2?
What is the service demand of Disk 2?
What is its visit ratio?

Server example solution

Measurement time = 1 hr		
	# I/O/s	Utilisation
Disk 1	32	0.30
Disk 2	36	0.41
	50	0.54
C		0.35
Total # jobs=13680		

Service time = U2/X2

System throughput

Service demand

Visit ratio

Little's law (1)

- Due to J.C. Little in 1961
  - A few different forms
    - The original form is based on stochastic models
  - An important result which is non-trivial
    - All the other operational laws are easy to derive, but Little's Law's derivation is more elaborate.

- Consider a single-
  - Navg = Average number of jobs i
    - When we count the number of jobs in a device, we include the one being served and those in the queue waiting for service

Little's Law (2)

- X = Throughput of the device
- Ravg = Average response time of the jobs
- Navg = Average number of jobs in the device
- Little's Law (for OA) says that

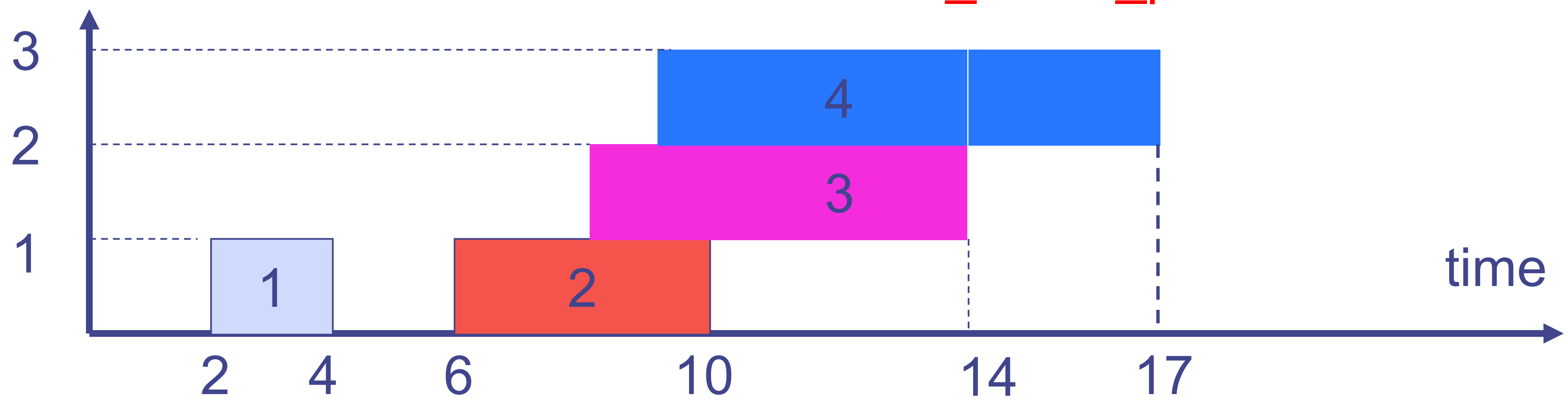
Navg

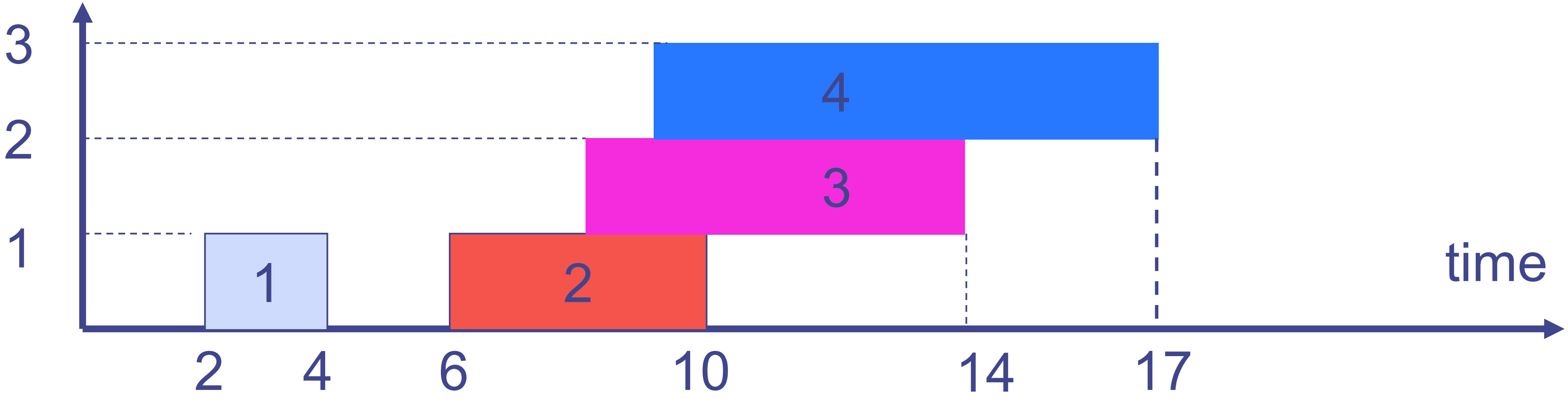
We will argue the validity of Littl ing a simple example.

Consider the single sever queue example from Week 1

Job index	Arrival time	Service time	Departure time
1	2	2	4
2	6	4	10
3	8	4	14
4	9	3	17

Let us use blocks o time span of the jobs, i.e. width of each block = re e of the job



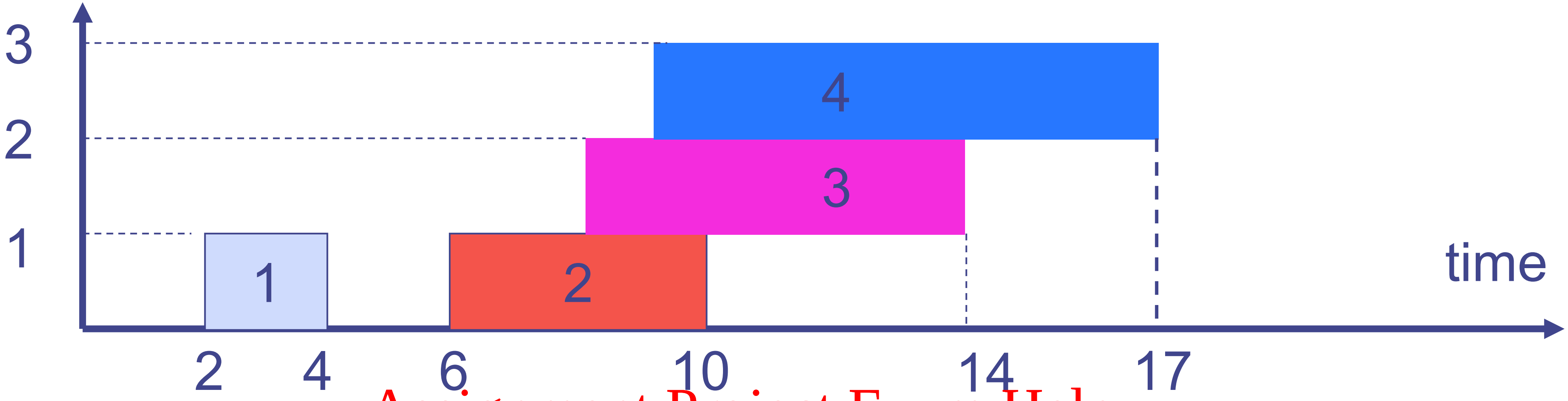


Assuming that in the measurement time interval [0,20]

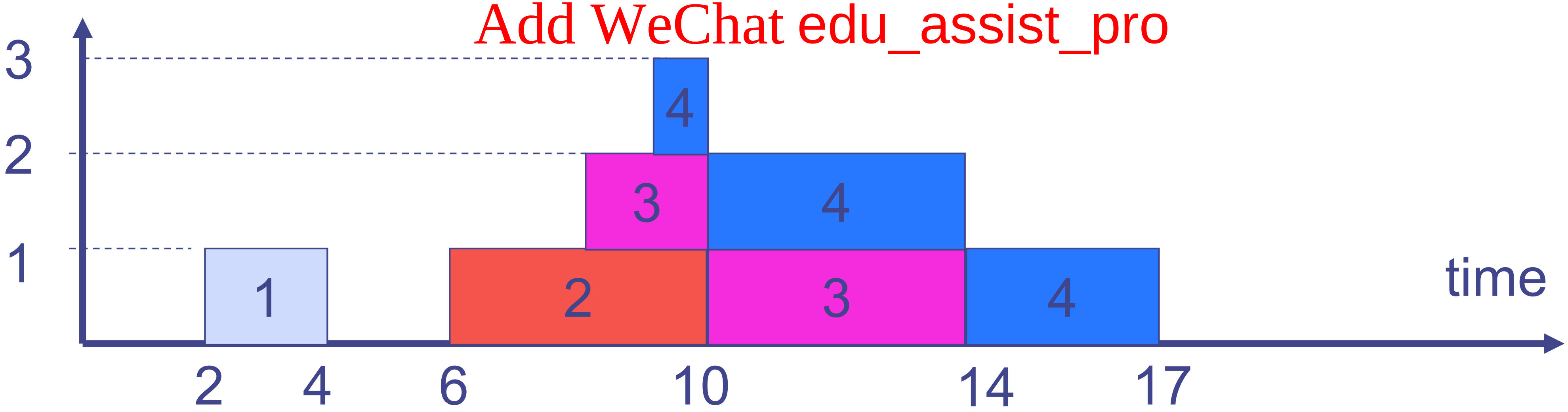
these 4 jobs arrive this device, i.e. the
device is in equilibri

Total area of the blocks
= Response time of job 1 + Response time of job 2 +
Response time of job 3 + Response time of job 4
= Average response time over the measurement interval *
Number of jobs departing over the measurement interval

This is one interpretation. Let us look at another.

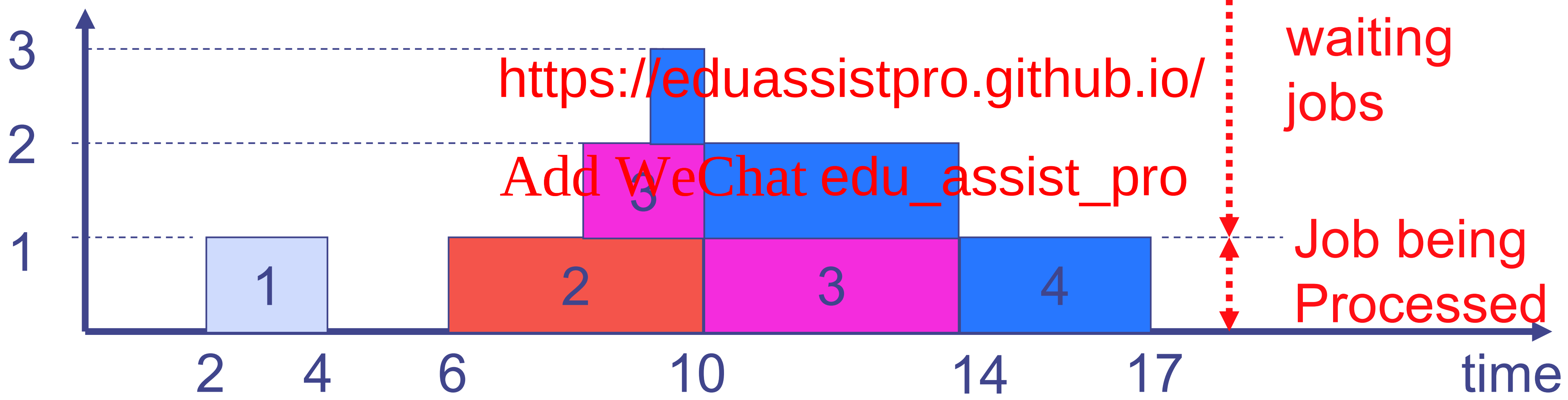


Let us assume thes and let them fall to the ground. Like t

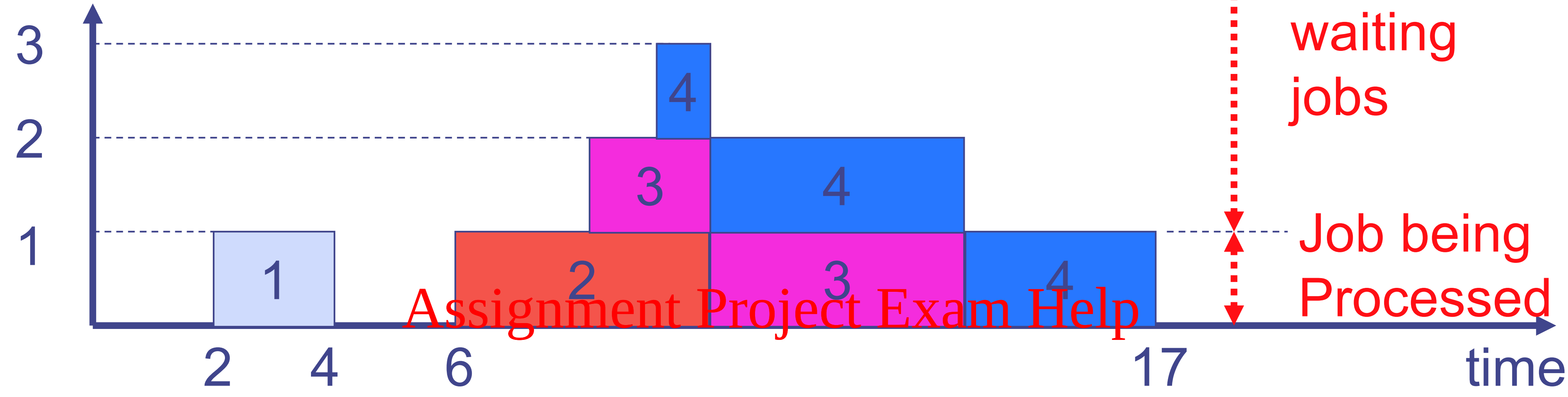


There is an interpretation of the height of the graph.

Job index	Arrival time	Service time
1	2	2
2	6	4
3	8	4
4	9	3

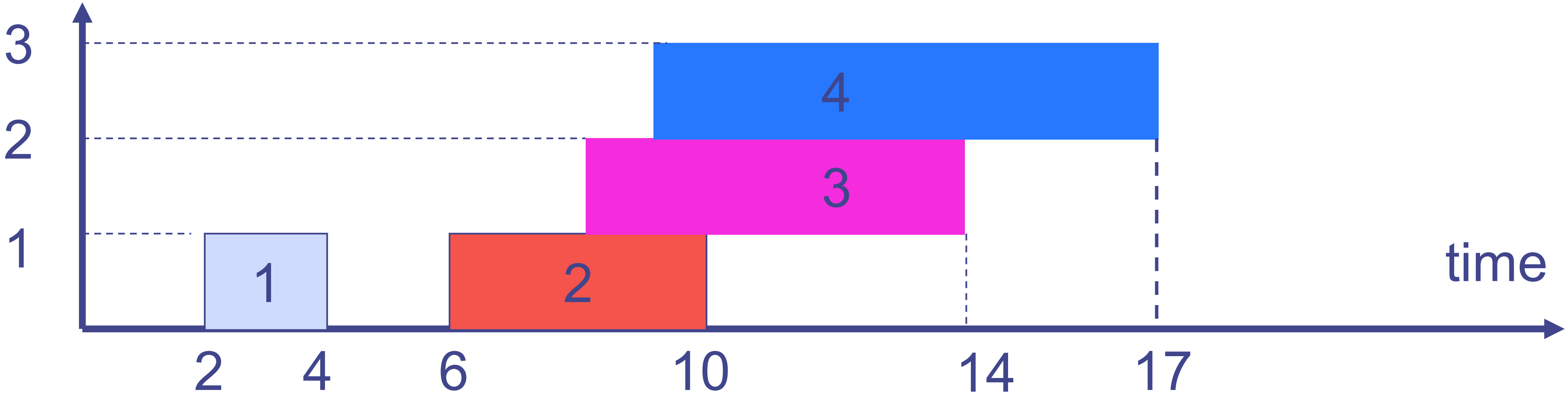


Interpretation: Height of the graph = number of jobs in the device
E.g. Number of jobs in [9,10] = 3
E.g. Number of jobs in [11,12] = 2 etc.

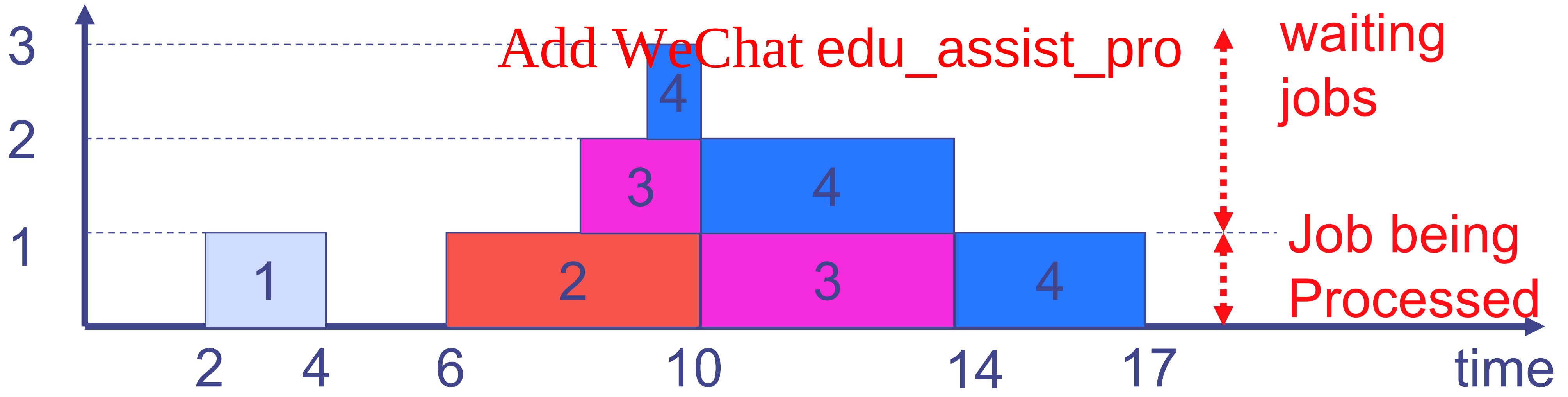


Again, consider the measurement interval of [0,20].

Area under the graph in [0,20]
= Height of the graph in [0,1] + Height of the graph in [1,2] + …
Height of the graph in [19,20]
= #jobs in [0,1] + #jobs in [1,2] + … + #jobs in [19,20]
= Average number of jobs in [0,20] * 20



Area = Average response time over [0,T] * Number of



Area = Average number of jobs in [0,T] * T

Deriving Little's Law

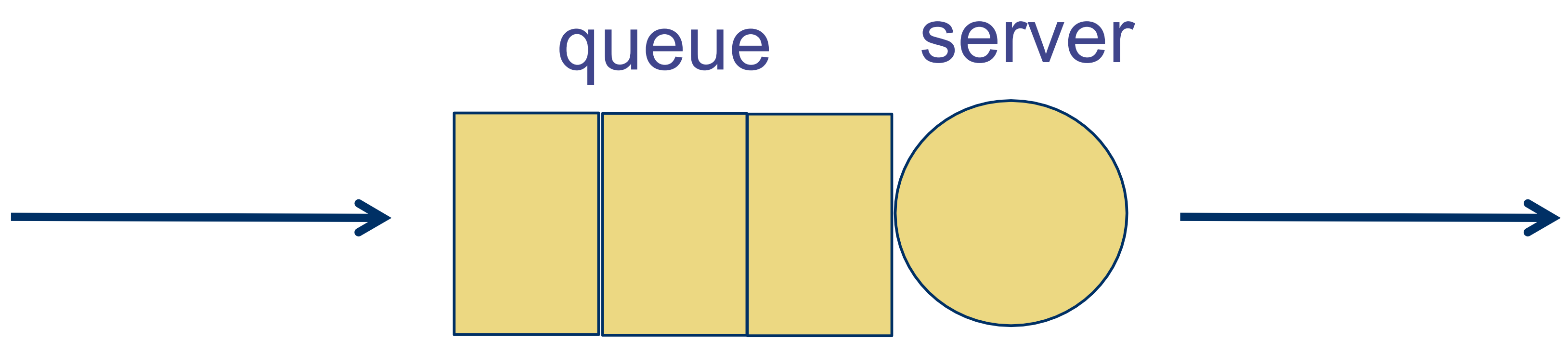
Area = Average response time of all jobs *
Number of jobs leaving in [0,T] (Interpretation #1)
= Average number of jobs in [0,T] * T (Interpretation #2)

Since Number of job
= Device throughput in [0,T]

We have Little's Law.

Average number of jobs in [0,T]
= Average response time of all jobs * Device throughput in [0,T]

Using Little's Law (1)



- A device consist ue

- The device comp uests per second

- On average, there are 3.2 req device
- What is the response time of the device?

Intuition of Little's Law

- Little's Law
  - Mean #jobs = Mean response time * Mean throughput
- If # jobs in the device ⬆, then response time ⬆
  - And vice versa

Applicability of Little's Law

- Little's Law can be applied at many different levels
- Little's law can be applied to a device
  - Navg(j) = Ravg(j) * X(j)
- A system with K devices
  - Navg(j) = #jobs in device j
  - Average numb vg = Navg(1) + …. + Navg(K)
  - Average response time of devic
  - Average response time of the system = Ravg
- We can also apply it to an entire system
  - Navg = Ravg * X(0)

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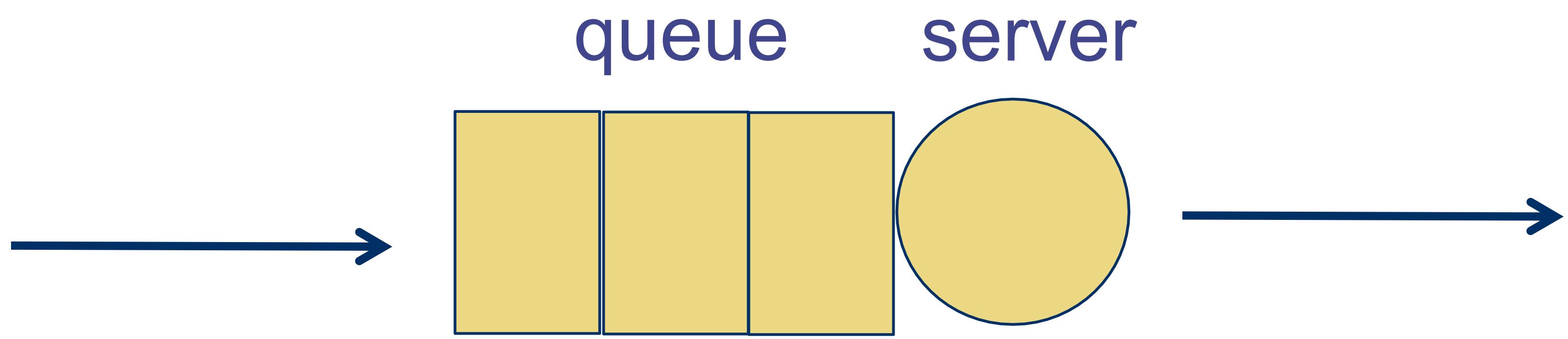
Using Little's Law (2)

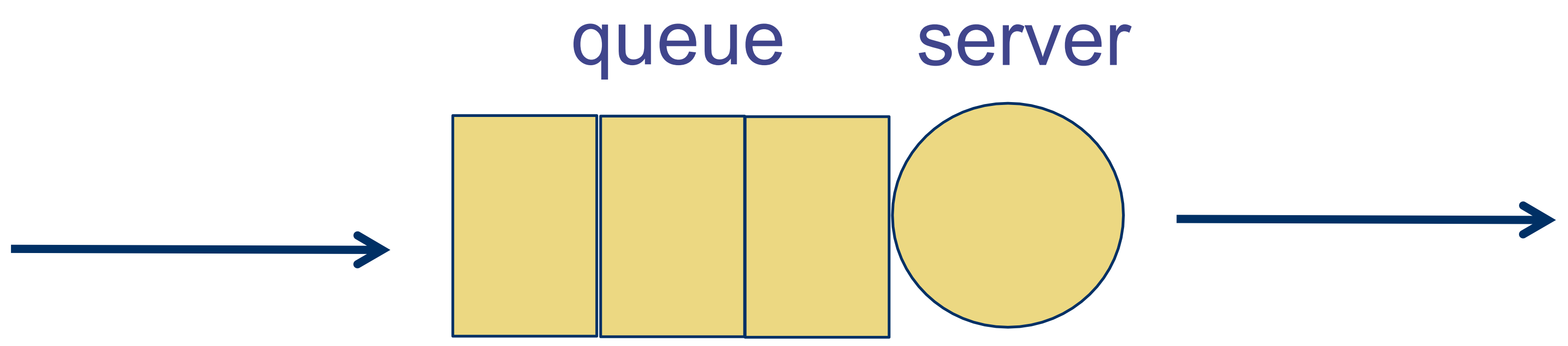


- The device compl uests per second
- On average, ther

  - 3.2 requests in the device

  - 2.4 requests in the queue
  - 0.8 requests in the server
- What is the mean waiting time and mean service time?
- Hint: You need to draw "boxes" around certain parts of the device and interpret the meaning of response time for that box.

Using Little's Law (2)



- The device compl uests per second
- On average, ther

  - 3.2 requests in the device

  - 2.4 requests in the queue
  - 0.8 requests in the server
- What is the mean waiting time and mean service time?

Interactive systems

M users

results

jobs

Each user sends a job to the system

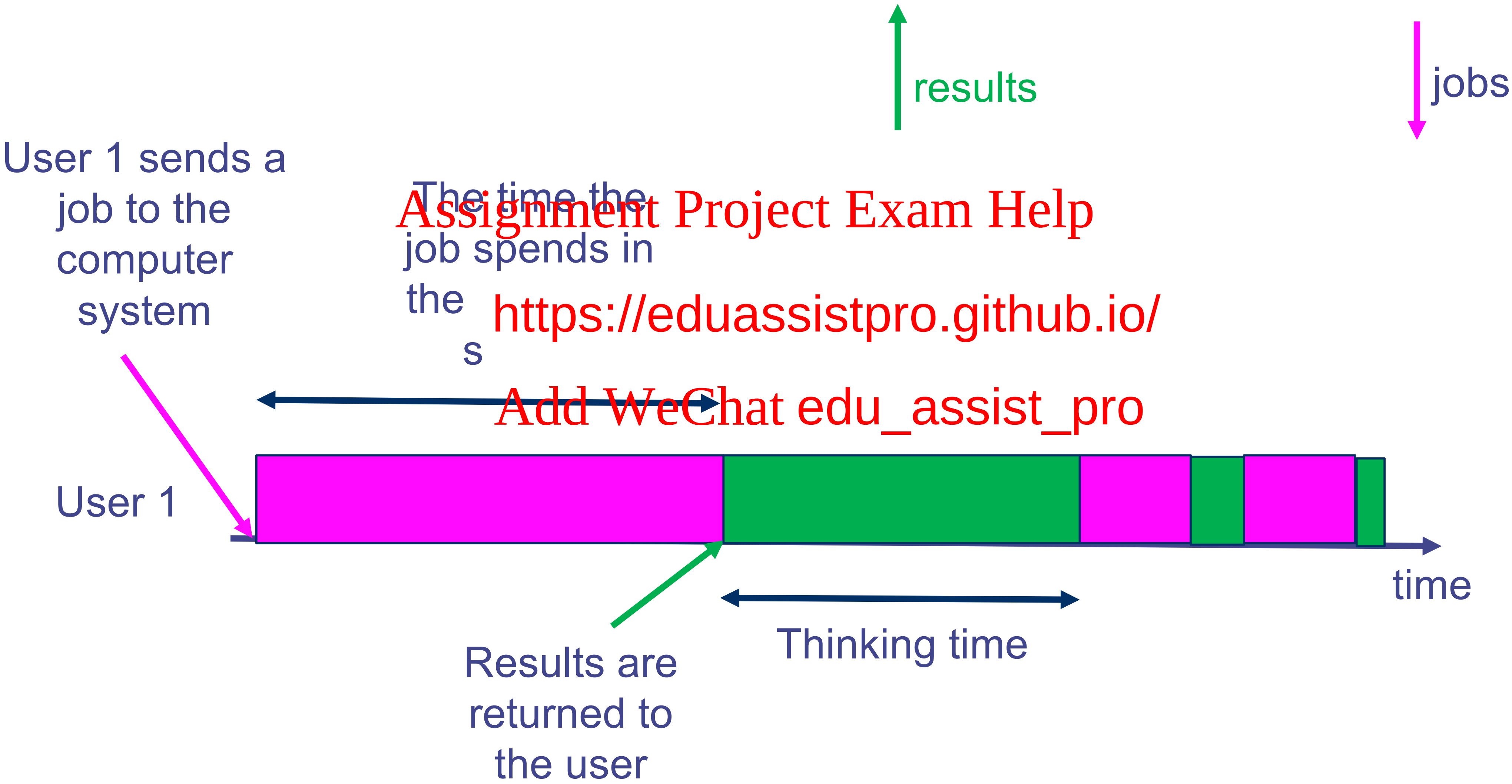
The system sends the results to the user.

The user after a thinking time, sends another job to the system.

Thinking time = time spent by the user

An interactive system is an example of closed system.

Interactive systems (Time line)



Interactive system (1)

- M interactive clients
- Z = mean thinking time
- R = mean response time of the computer system

throughput

Interactive system (2)

- Mavg = mean # interactive clients
- Z = mean thinking time
- X0 = throughput

ply Little's Law to
ive part, we
Mavg = Z * X0

Interactive system (3)

- Navg = average # clients in the computer system
- R = mean response time at the computer system

throughput

Little's Law to the

er system, we

avg = R * X0

Interactive system (4)

- Mavg = X0 * Z
- Navg = X0 * R
- The system is closed, the total number of users M is stant, we have

  avg + Navg

  re,

  * (Z+R)

The operational laws

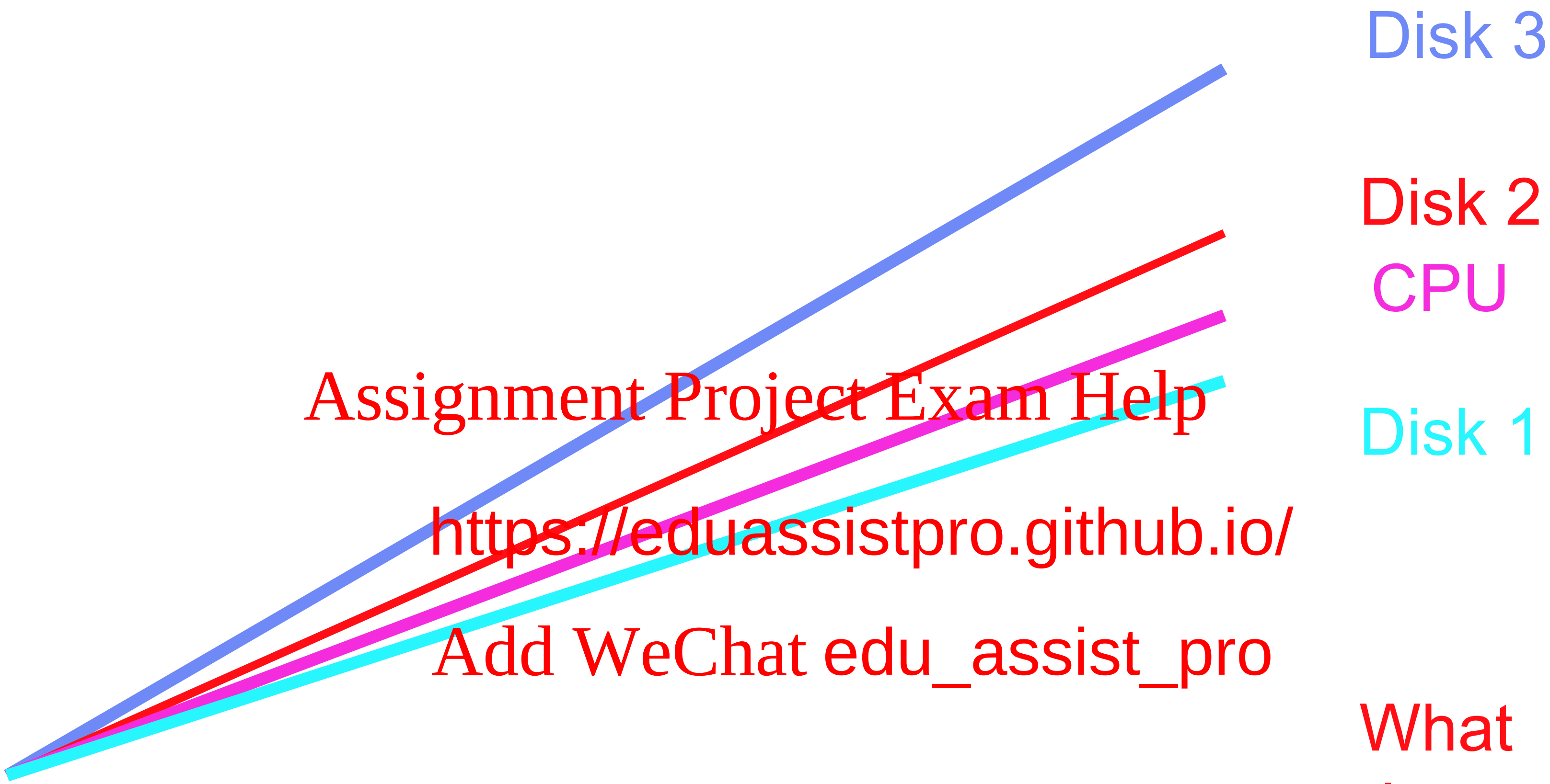
- These are the operational laws
  - Utilisation law U(j) = X(j) S(j)
  - Forced flow law X(j) = V(j) X(0)
  - Service demand law D(j) = V(j) S(j) = U(j) / X(0)
  - Little's law N = X R
  - Interactive response time M = X(0) (R+Z)
- Applications
  - Mean value anal
  - Bottleneck analysis
  - Modification analysis

Bottleneck analysis - motivation

	D(j)	Utilisation
Disk 1	79ms	0.30
Disk 2	108ms	0.41
Disk 3	142ms	0.54
	92ms	0.35

Service demand law: D(j) = U(j) / X(0)
==> U(j) = D(j) X(0)
Utilisation increases with increasing throughput and service demand

Utilisation vs. throughput plot U(j) = D(j) X(0)



What determines this order?

Observation: For all system throughput:
Utilisation of Disk 3 > Utilisation of Disk 2 > Utilisation of CPU > Utilisation of Disk 1

Bottleneck analysis

- Recall that utilisation is the busy time of a device divided by measurement time
  - What is the maximum value of utilisation?
- Based on the example on the previous slide, which device will reach the maximum utilisation first?

Bottleneck (1)

- Disk 3 has the highest service demand
- It is the bottleneck of the whole system

Operational law: $$X(0) \leq \frac{1}{D(j)}$$

Utilisation limit: $$U(j) \leq 1$$

Bottleneck (2)

$$X(0) \leq \frac{1}{D(j)}$$ Should hold for all K devices in the system

$$\Rightarrow X(0) \leq \min \frac{1}{D(j)}$$

$$\Rightarrow X(0) \leq \frac{1}{\max D(j)}$$

Bottleneck throughput is limited by the maximum service demand

Bottleneck exercise

	D(j)	Utilisation
Disk 1	79ms	0.30
Disk 2	108ms	0.41
Disk 3	142ms	0.54
	92ms	0.35

The maximum system throughput is 1 / 0.142 = 7.04 jobs/s. What if we upgrade Disk 3 by a new disk that is 2 times faster, which device will be the bottleneck after the upgrade? You can assume that service time is inversely proportional to disk speed.

Another throughput bound

- Little's law

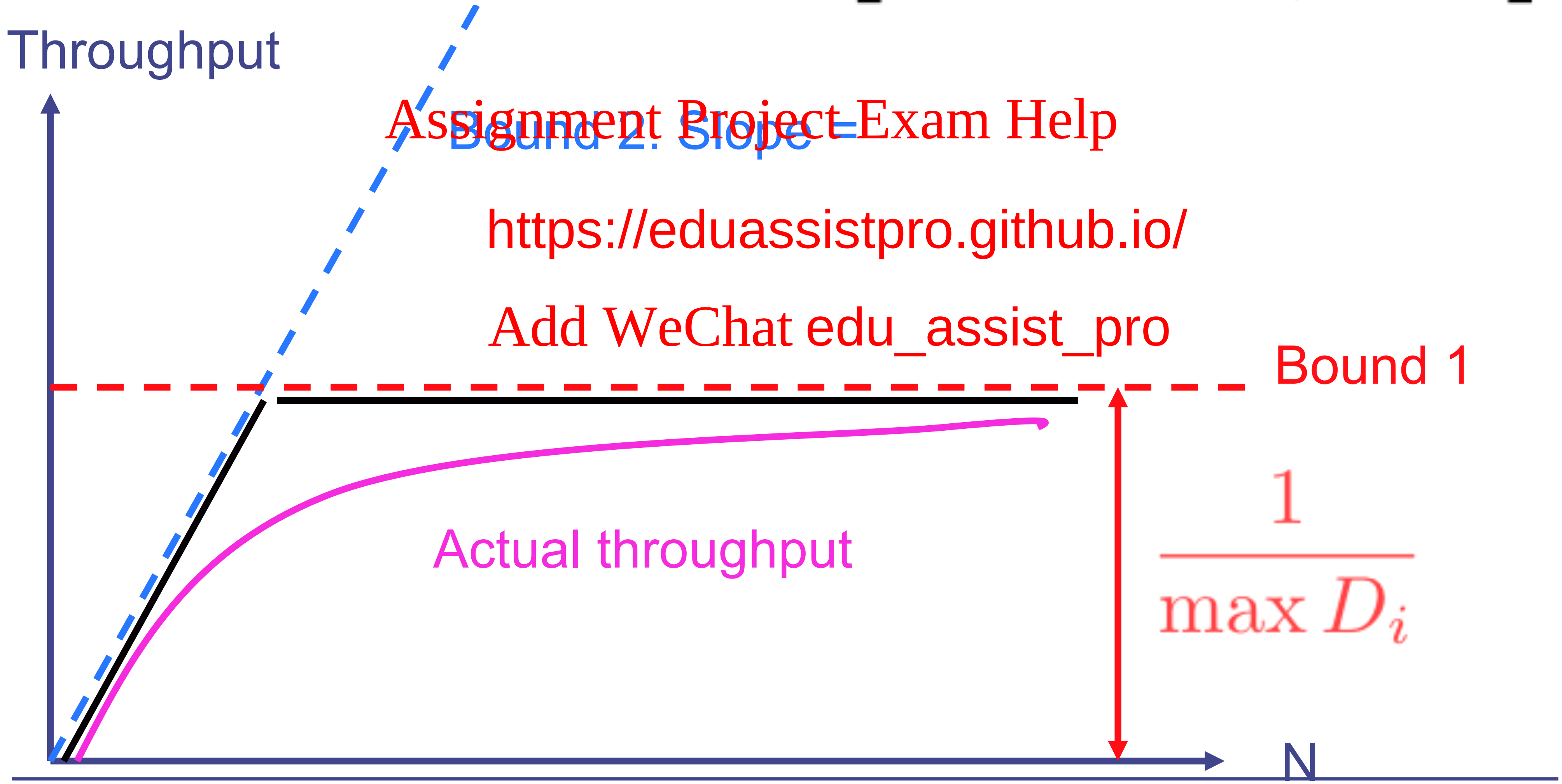
$$N = R \times X(0) \geq \left(\sum_{i=1}^{K} D_i\right) \times X(0)$$

Previously, we have $X(0) \leq \frac{1}{\max D(j)}$

Therefore: $X(0) \leq \min \left[\frac{1}{\max D_i}, \frac{N}{\sum_{i=1}^{K} D_i} \right]$

Throughput bounds

$$X(0) \leq \min \left[\frac{1}{\max D_i}, \frac{N}{\sum_{i=1}^{K} D_i} \right]$$



Bottleneck analysis

- Simple to use
  - Needs only utilisation of various components
- Assumes service demand is load independent

Modification analysis (1)

- (Reference: Lazowska Section 5.3.1)
- A company currently has a system (3790) and is considering switching to a new system (8130). The service demands for these two systems are given below:

System	Service demand (seconds)	
		sk
3790		0
8130	5.1	

- The company uses the system for interactive application with a think time of 60s.
- Given the same workload, should the company switch to the new system?
- Exercise: Answer this question by using bottleneck analysis. For each system, plot the upper bound of throughput as a function of the number of interactive users.

Modification analysis (2)

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Operational analysis

- These are the operational laws
  - Utilisation law U(j) = X(j) S
  - Forced flow law X(j) = V(j) X(0)
  - Service demand law D(j) = V(j) S(j) = U(j) / X(0)
  - Little's law N = X R
  - Interactive response time M = X(0) (R+Z)

- Operational anal nd the system performance but u to find the throughput and response time m

- To order to find the throughput and response time, we need to use queueing analysis
- To order to use queueing analysis, we need to specify the workload

Workload analysis

- Performance depends on workload
  - When we look at performance bound earlier, the bounds depend on number of users and service demand
  - Queue response time depends on the job arrival rate and job service time

- One way of spe use probability distribution.

- We will look at a well-known ess called Poisson process today.

- We will first begin by looking at exponential distribution.

Exponential distribution (1)

- A continuous random variable is exponentially distributed with rate λ if it has probability density function

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

P hat $x \leq X \leq x + \delta x$ is

$f(x)\,\delta x = \lambda \exp(-\lambda x)\,\delta x$

Exponential distribution - cumulative distribution

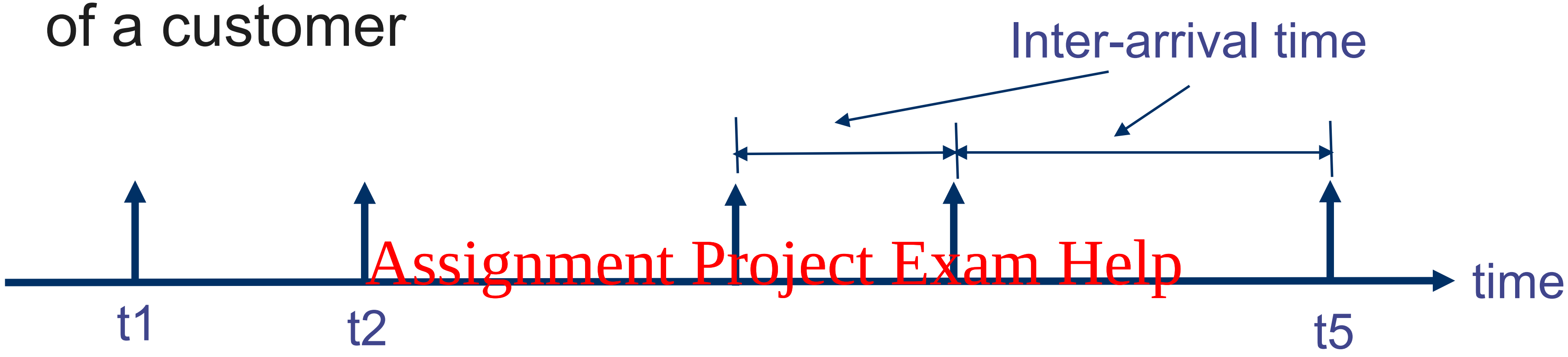
- The cumulative distribution function F(x) = Prob(X ≤ x) is:

$$F(x) = \int_0^x \lambda e^{-\lambda z} dz = 1 - e^{-\lambda x} \text{ for } x \geq 0$$

What is Prob(X ≥ x)?

Arrival process

- Each vertical arrow in the time line below depicts the arrival of a customer



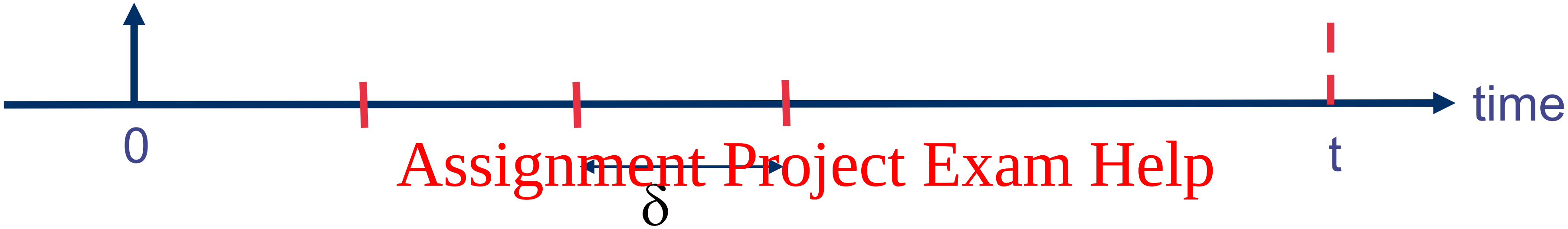
- An arrival can mean
  - A telephone call arriving at a call center
  - A transaction arriving at a computer system
  - A customer arriving at a checkout counter
  - An HTTP request arriving at a web server
- The inter-arrival time distribution will impact on the response time.
- We will study an inter-arrival distribution that results from a large number of **independent** customers.

Many independent arrivals (1)

- Assume there is a large pool of N customers
- Within a time period of δ (δ is a small time period), there is a probability of $p\delta$ that a customer will make a request (which gives rise to an arrival)
- Assuming the probability that each customer makes a request is independent, the probability that a customer arrives in time period δ is $Np\delta$
- If a customer arrive ability that the next customer does not

Many independent arrivals (2)

- Divide the time t into intervals of width δ



- No arrival in [0,t] ... ch interval δ
- Probability of no arrival in δ = 1
- There are t / δ intervals
- Probability of no arrival in [0,t] is

$$(1 - Np\delta)^{\frac{t}{\delta}} \rightarrow e^{-Npt} \text{ as } \delta \rightarrow 0$$

Exponential inter-arrival time

- We have showed that the probability that there is no arrival in [0,t] is exp(- N p t)
- Since we assume that there is an arrival at time 0, this means

  Probability(inter-arrival time > t) = exp(- N p t)

- This means

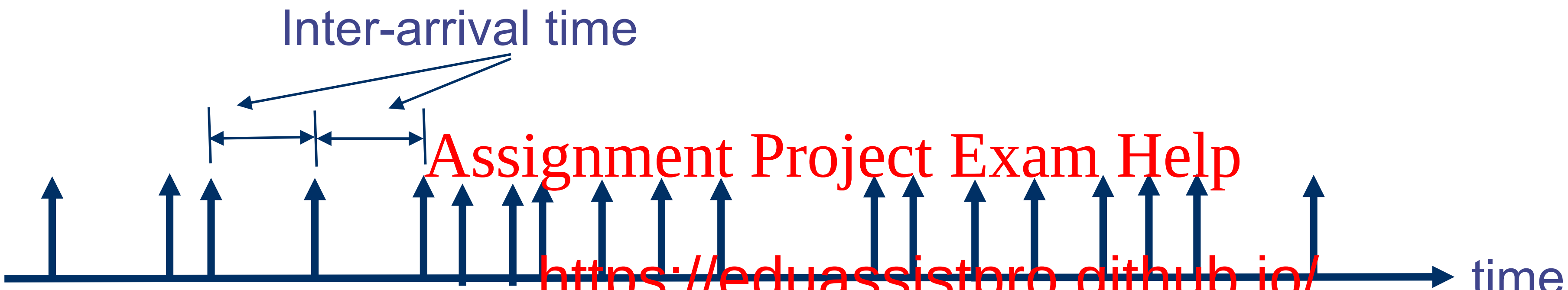
  Probability(inter-arrival ti - exp(- N p t)

- What this shows is the inter-arrival time distribution for independent arrival is exponentially distributed

- Define: λ = Np
  - λ is the mean arrival rate of customers

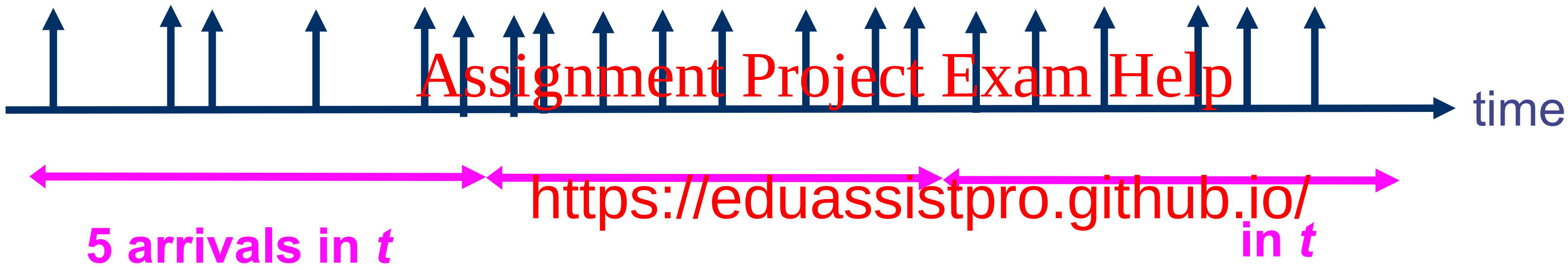
Two different methods to describe arrivals

Method 1: Continuous probability distribution of inter-arrival time



Two different methods to describe arrivals

Method 2: Use a fixed time interval (say *t*), and count the number of arrivals within *t*.



- The number of arrivals in *t* is ra
- The number of arrivals must be an non-negative integer
- We need a discrete probability distribution:
  - Prob[#arrivals in t = 0]
  - Prob[#arrivals in t = 1]
  - etc.

Poisson process (1)

- Definition: An arrival process is Poisson with parameter λ if the probability that *n* customer arrive in any time interval *t* is

$$\frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

mple:

$\lambda = 5$ and $t = 1$

Note: Poisson is a discrete probability distribution.

Poisson process (2)

- Theorem: An exponential inter-arrival time distribution with parameter λ gives rise to a Poisson arrival process with parameter λ

- How can you prove this theorem?
  - A possible method i e intervals of width δ. A fini stribution and with $\delta \rightarrow 0$, we get a Poisson distribution.

Customer arriving rate

- Given a Poisson process with parameter λ, we know that the probability of n customers arriving in a time interval of t is given by:

$$\frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

- What is the me rs arriving in a time interval of t?

- That's why λ is called the arrival rate.

Customer inter-arrival time

- You can also show that if the inter-arrival time distribution is exponential with parameter λ, then the mean inter-arrival time is $1/\lambda$
- Quite nicely, we have

Mean arrival rate = 1 / mean inter-arrival time

Application of Poisson process

- Poisson process has been used to model the arrival of telephone calls to a telephone exchange successfully
- Queueing networks with Poisson arrival is tractable
  - We will see that in the next few weeks.
- Beware that not all arrival processes are Poisson! Many arrival processe et today are not Poisson. We will

References

- Operational analysis
  - Lazowska et al, Quantitative System Performance, Prentice Hall, 1984. (Classic text on performance analysis. Now out of print but can be download from http://www.cs.washington.edu/homes/lazowska/qsp/
    - Chapters 3 and 5 (For Chapter 5, up to Section 5.3 only)
  - Alternative 1: You can read Menasce et al, "Performance by design", Chapter 3. Note that Menasce doesn't cover certain aspects of performance bounds. So, you will also ne Lazowska.
  - Alternative 2: You c rs 6 and 7. The treatment is more rigorous. You can gross over th entioning ergodicity.

- Little's Law (Optional)
  - I presented an intuitive "proof". A more formal proof of this well known Law is in Bertsekas and Gallager, "Data Networks", Section 3.2

- Tutorial exercises based on this week's lecture are available from course web site
  - We will discuss the questions in next week's tutorial time