NUMERICAL OPTIMISATION
TUTORIAL 9: NONSMOOTH OPTIMISATION

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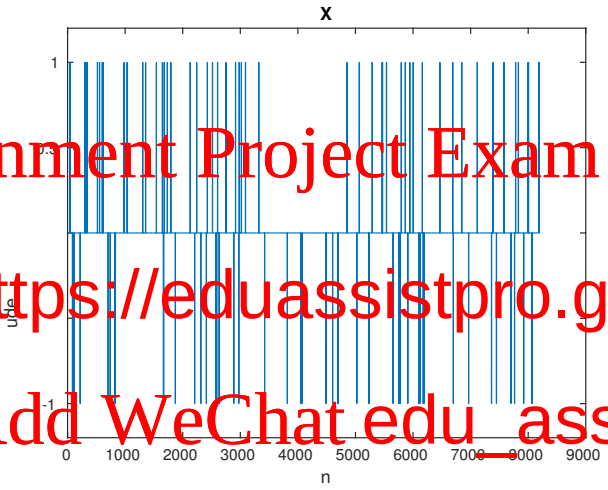
EXERCISE 1

Consider a *sparse* signal $x = \{x_n\}_{n=1\ldots N}$ of length $N = 2^{13}$ consisting of:

- $T = 100$ randomly distributed spikes with values $\{\pm 1\}$.
- the remaining, $N - T$, values equal to 0.

A possible realisation of the signal x is:



Following the compressed sensing paradigm we take $K = 2^{10}$ *compressed measurements* of the form

$$y_i = a_i^{\mathrm{T}} x,$$

where $a_i \in \mathcal{R}^N$ is an appropriately chosen sensing vector. Assuming that the measurement is corrupted with zero-mean Gaussian noise with standard deviation $\sigma = 0.005$, the measurements can be compactly written as

$$\tilde{y} = Ax + e,$$

where a_i^{T} is the ith row of the measurement matrix $A \in \mathcal{R}^{K \times N}$ and $e \in \mathcal{N}(0, \sigma)$ is the normally distributed noise vector.

We consider two different measurement types:

- A is a orthogonal random matrix (use `randn()` and `orth()` to construct it)
- A is a subsampled Welsh-Hadamard transform (the forward and inverse WH transform is available in Matlab: `fwht()`, `ifwht()`)

While the problem to recover x from $\tilde{y}$ is underdetermined, under assumption of *sparsity* it is still possible to recover sparse signals from such incomplete measurements (under certain assumptions on A and sparsity of the signal vs. number of measurements). While forcing sparsity would lead to a

combinatorial problem, the L_1 norm has been proven to provide a good relaxation resulting in the following compressed sensing recovery problem

$$x_{CS} = \arg\min_x \frac{1}{2}\|Ax - y\|_2^2 + \lambda\|x\|_1, \tag{CS}$$

where λ is the regularisation parameter chosen depending on the properties of A and the standard deviation of the noise.

Implement and solve the compressed sensing recovery problem (CS) with

- ISTA
- FISTA
- ADMM

EXERCISE 2

Repeat the same experiment in 2D. This is to demonstrate how to apply the same methods in image settings.