NUMERICAL OPTIMISATION - ASSIGNMENT 4

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EXERCISE 1

Consider the linear system $Ax = b$ with $A \in \mathbb{R}^{n\times n}$, $x \in \mathbb{R}^n$, $b \in \mathbb{R}^n$.

(a) Implement the linear preconditioned Conjugate Gradient method.
Submit your implementation via ***Cody Coursework***. **[20pt]**

Consider the starting point $x_0 = (0, \dots, 0)^T$, a tolerance `tol = 1e-12` and a dimension `n = 100`. Let b be the vector defined by $Ax^* = b$ for the following value for x^*:

```
xtrue = zeros(n,1);
xtrue(floor(n/4):floor(n/3)) = 1;
xtrue(floor(n/3)+1:floor(n/2)) = -2;
xtrue(floor(n/2)+1:floor(3/4*n)) = 0.5;
```

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(b) Solve the given linear system for the following A matrices:

- `A1 = diag(1:n`
- `A2 = diag(for`
- 1d negative L
  `A3 = -diag(ones(n-1, 1), -1) - diag(ones(n-1, 1), 1) + dia` ;

https://eduassistpro.github.io/

Compare the theoretical and actual convergence rates accordi ues of A_i, $i = 1, 2, 3$. Use the true solution[1] to evaluate th
Submit solution via ***TurnitIn***. **[20pt]**

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EXERCISE 2

(a) Implement the Fletcher-Reeves conjugate gradient method.
Submit your implementation via ***Cody Coursework***. **[20pt]**

(b) Implement the Polak-Ribière conjugate gradient method.
Submit your implementation via ***Cody Coursework***. **[20pt]**

(c) Minimise the function

$$f(x, y) = x^2 + 5x^4 + 10y^2$$

from the initial points $x_0 = (1.2, 1.2)^T$ and $x_0 = (-1, -1)^T$ with a tolerance `tol = 1e-12` using your implementation of the non-linear conjugate gradient methods. Explain your results highlighting any relevant feature in the minimisation process regarding the convergence of the methods. How would you modify them to ensure convergence?
Submit solution via ***TurnitIn***. **[20pt]**

Remark. The submission to `TurnitIn` should not be longer than 6 pages. Avoid submitting more code than needed (if any) and focus on explaining your results.

[1] In practice, the true solution is not available, so a common practice is to consider the norm of the residuals.