Search Fundamentals

12 Aug 2020

1. Distributed DFS and BFS

2. ClassicalDFS

3. Ci

4. CidonDFS §2

5. Bellman-Ford algorithm

6. Maximal Independent Set

Distributed DFS

- Despite its sequential appearance, DFS can work faster on distributed networks!
-
- edge twice (needed), but avoids all or most fr
  time complexity = $2|V| - 2$, but at th
  increased message complexity = 3
  - Small error in Tel's text: message complexity is not $4|E|$...
- Cidon DFS was designed for async networks, thus also works for sync networks (even better).

Distributed BFS

- For sync networks, Echo (aka SyncBFS) finds a BFS spanning tree: time complexity $= 2D + 1$, message complexity $= 2|E|$

- Despite its parallel appearance, BFS is harder to implement

- of Bellman-Ford: shows similar issues, bu l

- AsyncBFS [Lynch]: messages = $O(D)$; time+pileups $= O(D|V|)$
- LayeredBFS [Lynch]: messages $= O(D|V| + |E|)$; time$-$pileups $= O(D^2)$
- We do not further follow these issues here...

Cidon DFS

- Cidon DFS uses two types of tokens:
  - One single classical **tok** token, sent on tree edges and, *occasionally*, on fronds

- [illegible]

  - ahead of **tok**
  - together with **tok** (can be combined)
  - alone (on fronds only)

Read more: Tel §6.4, or Cidon's original paper:
Yet Another Distributed Depth-First-Search Algorithm

http://cidon.eew.technion.ac.il/files/var/448324-cidon_dfs_87.pdf

Classical vs Cidon DFS – Examples

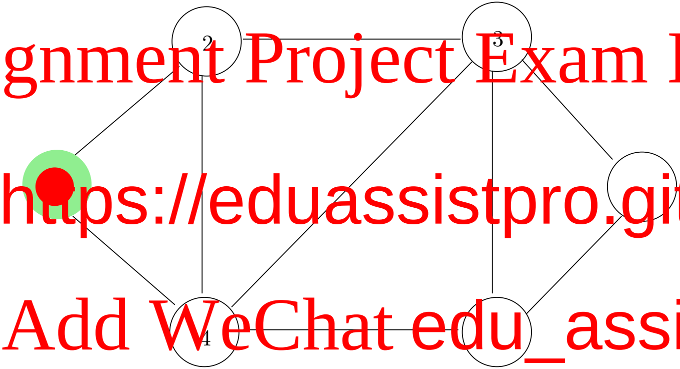
- In all cases, there is one single **tok** token, thus the time complexity sums the delays in this token's delivery

- In classical DFS, frond edge $\{2, 4\}$ is sequentially twice

- **vis** hich now saves 2 time intervals (while keeping m

- In Cidon DFS §2, frond edge $\{2,$
  - in one direction, by the **vis** toke of the **tok** token,
  - in the reverse direction, by a pair of **tok+vis** tokens (**vis** is not strictly necessary)
  - this still saves 1 time interval (but increases messages count)

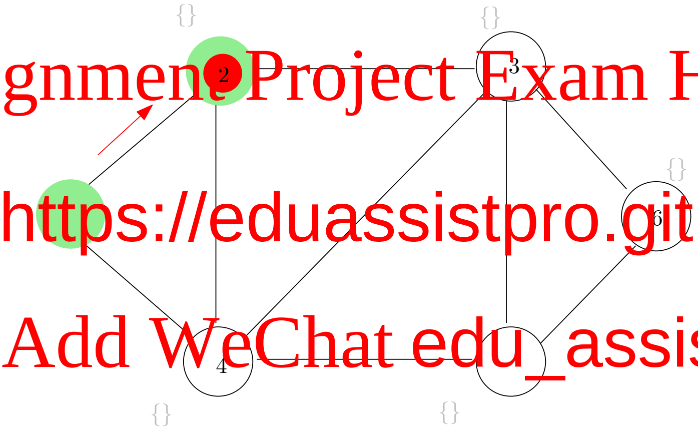
Classical DFS



$\text{Time Units} = 0$

$\text{Messages} = 0$

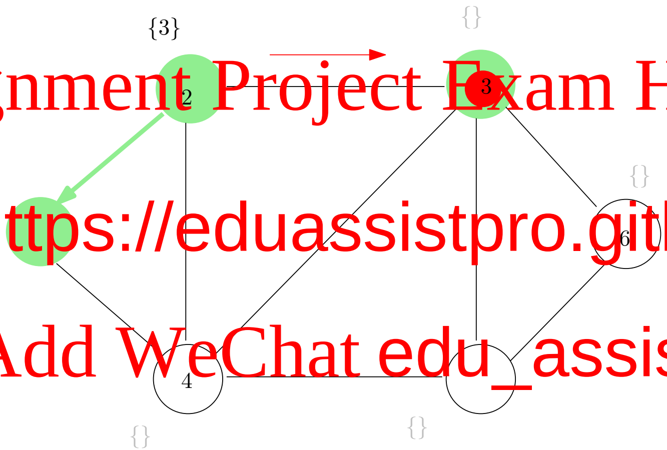
Classical DFS



Time Units = 1

Messages = 1

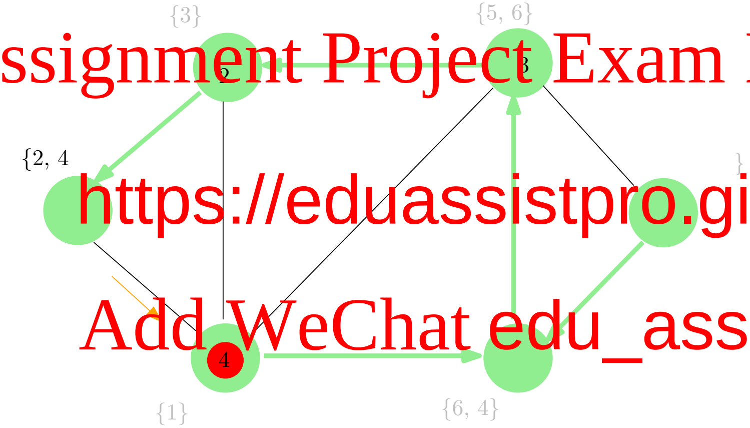
Classical DFS



Time Units = 2

Message = 2

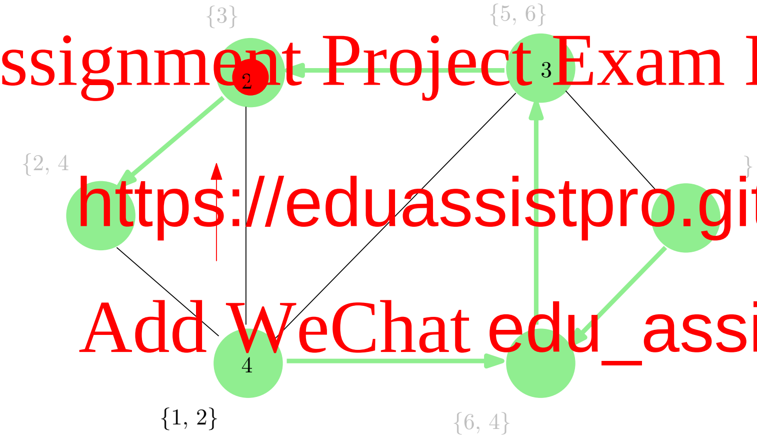
Classical DFS



Time Units $= 10$

Message $= 10$

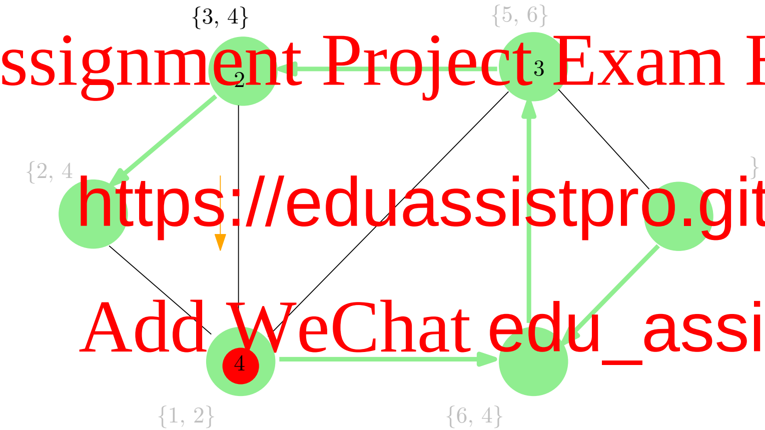
Classical DFS : 4 → **tok** → 2



Time Units = 11

Message = 11

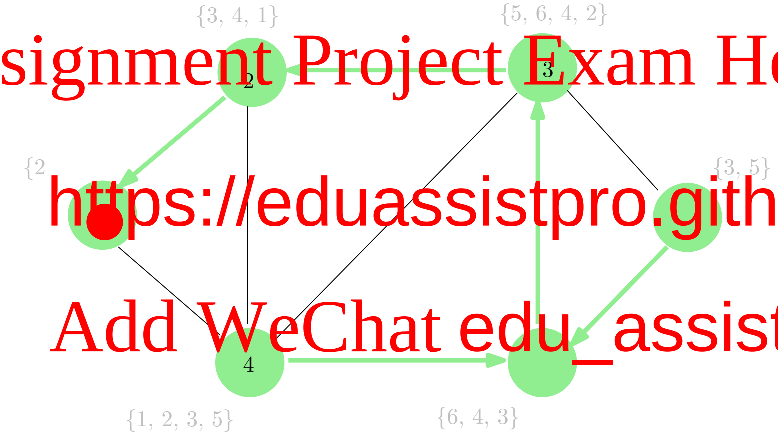
Classical DFS : 2 → **tok** → 4



Time Units = 12

Message = 12

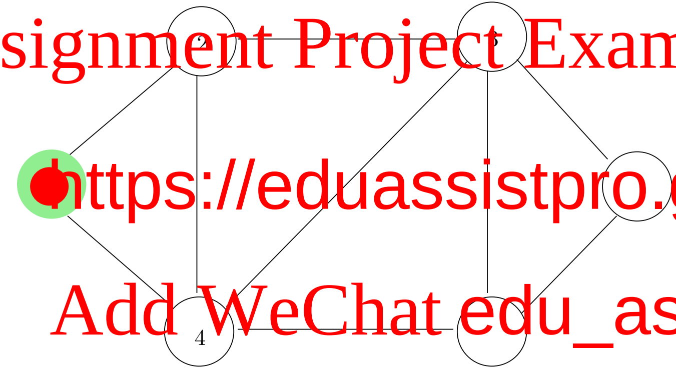
Classical DFS



Time Units = 18 = 2M

Message = 18 = 2M

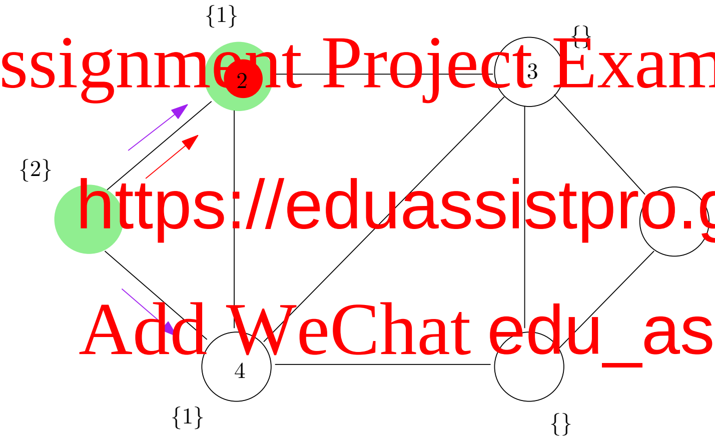
Cidon DFS §1



Time Units $= 0$

Messages $= 0$

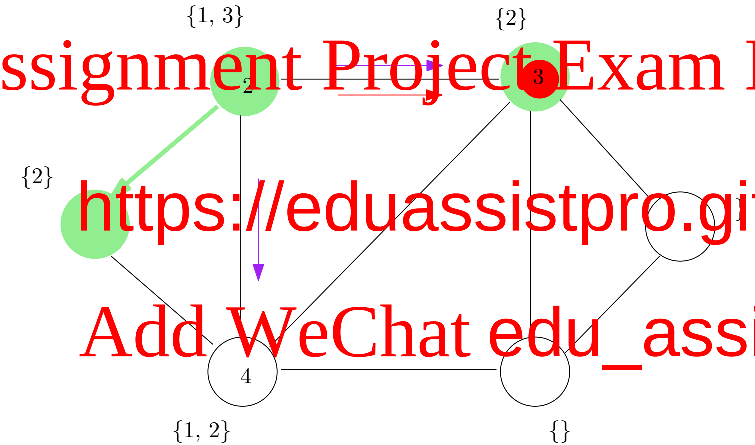
Cidon DFS §1



Time Units = 1

Messages = 3

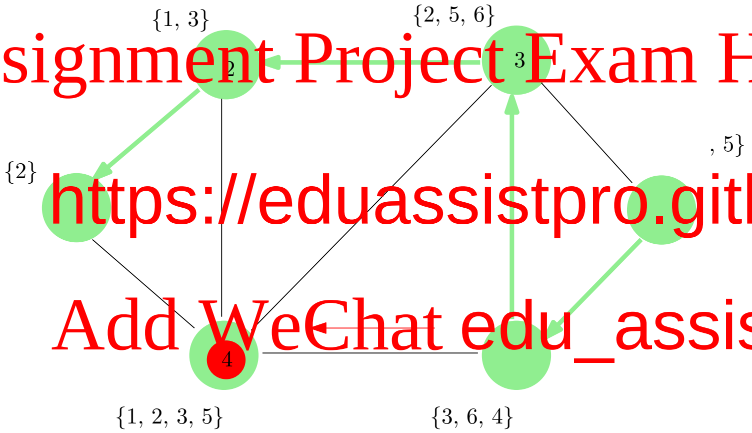
Cidon DFS §1 : 2 → **vis** → 4



Time Units = 2

Messages = 6

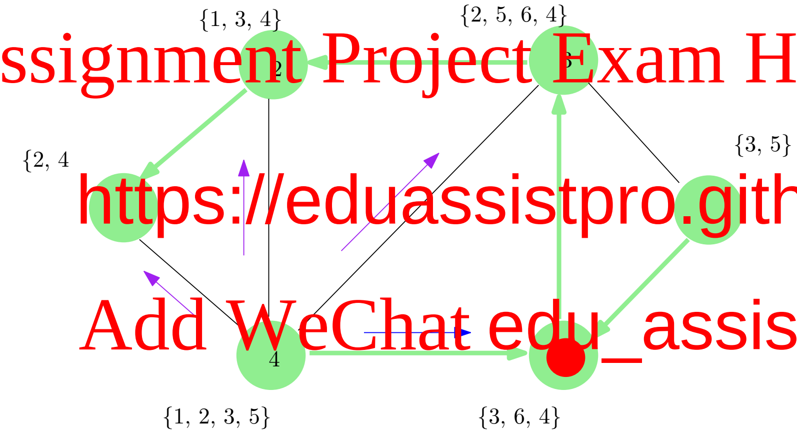
Cidon DFS §1



Time Units = 6

Messages = 16

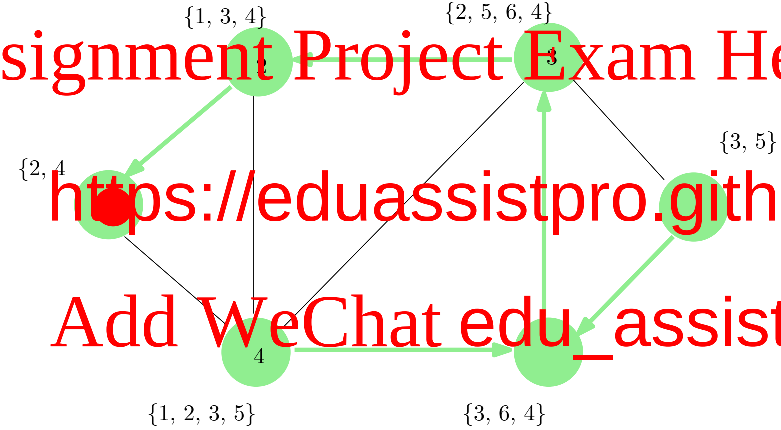
Cidon DFS §1 : 4 → **vis** → 2



Time Units = 7

Messages = 20

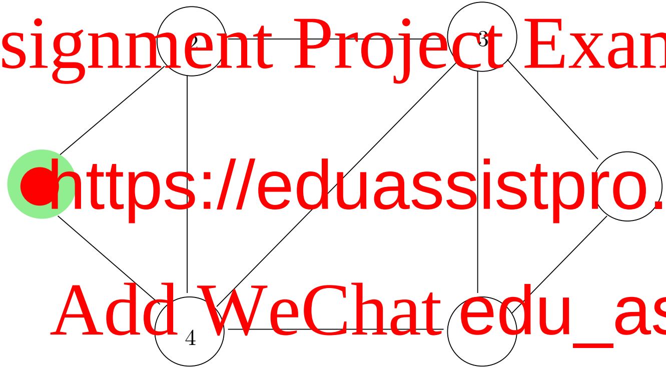
Cidon DFS §1



Time Units = 10 = 2N - 2

Messages = 23 ≤ 3M

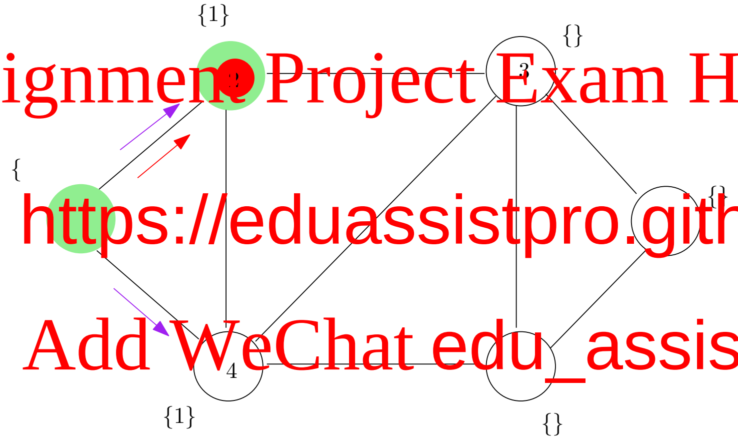
Cidon DFS §2



Time Units = 0

Messages = 0

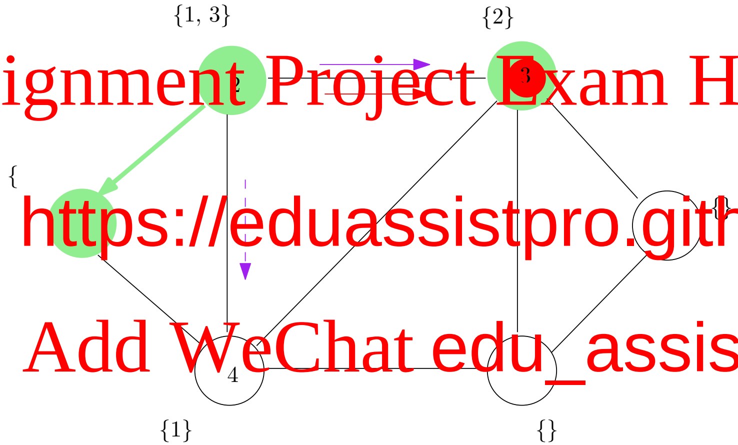
Cidon DFS §2



Time Units = 1

Messages = 3

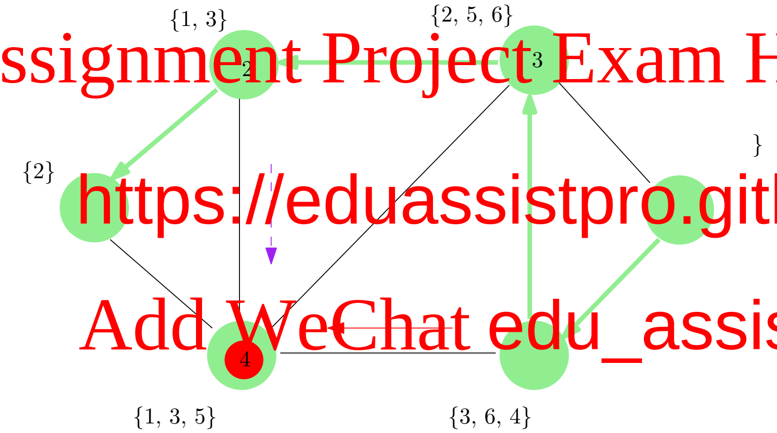
Cidon DFS §2 : 2 $\dashrightarrow$ **vis** $\dashrightarrow$ 4



Time Units $= 1 + \varepsilon$

Messages $= 6$

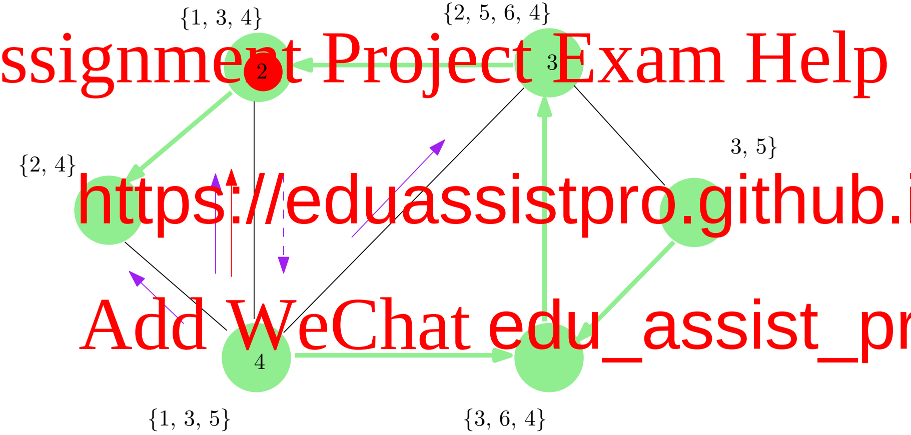
Cidon DFS §2



Time Units $= 1 + 5\varepsilon$

Messages $= 16$

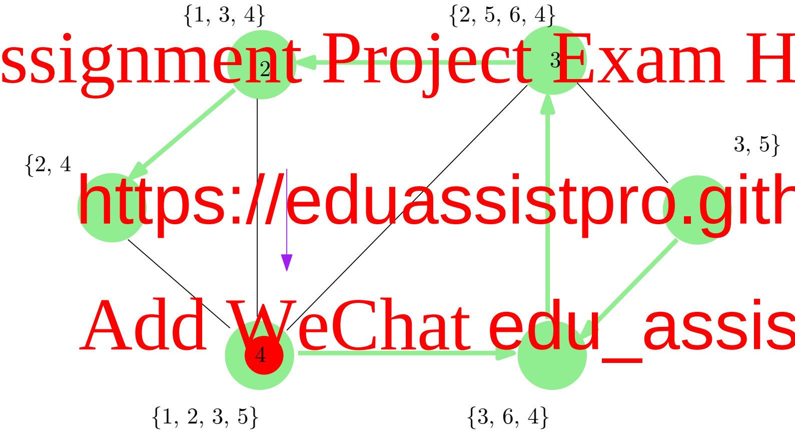
Cidon DFS §2 : 2 - - → **vis** - - → 4, 4 → **tok+vis** → 2



Time Units = 1 + 6ε

Messages = 20

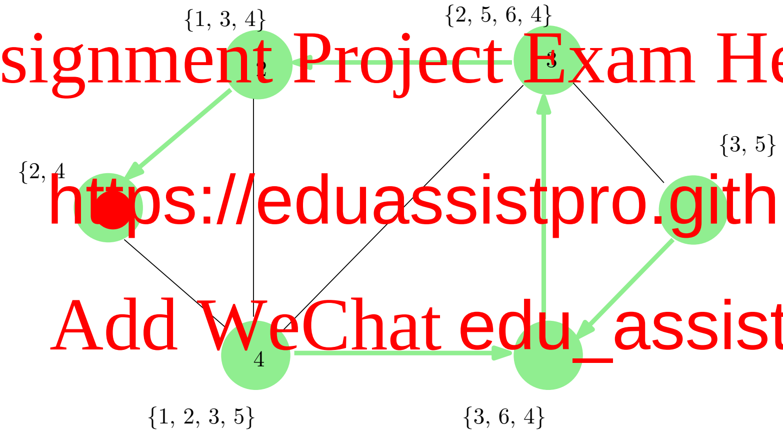
Cidon DFS §2 : 2 → **vis** → 4



Time Units = 2

Messages = 20

Cidon DFS §2



Time Units $= 6 \leq 2N - 2$

Messages $= 24 \leq 3M$

Distributed Bellman-Ford algorithm

- Classical Bellman-Ford algorithm finds all shortest paths from a single source – like Dijkstra
-
- g$|V|$)

- Classical Bellman-Ford: Time complexi
- Distributed Dijkstra: more difficult distri

- Distributed Bellman-Ford $\approx$ a simple extension of Echo

Distributed Bellman-Ford algorithm

- Sync Bellman-Ford ("Echo ++"):
  - Time complexity = $O(|V|)$

-
  - Message complexity: $O(|V|\quad)$ (terrible worst case, but often much lower in reality)
  - Time complexity = $O(|V|)$ – if any m most 1 time units – no FIFO [Tel]
  - Time complexity = $O(|V|^{|V|})$ – if we consider the congestion (pileups) on FIFO channels [Lynch]
  - Are these realistic? ...

Sync Bellman-Ford - Start

Problem: Find all shortest paths from node 3.

-

- blue = unvisited
- green = visited

Sync Bellman-Ford - Round 1

Update formula: *new distance* = minimum between *old distance* and all newly (*received distance* + *edge length*)

- Nodes 4 and 2:
  update distances

Sync Bellman-Ford - Round 2

- Nodes 5, 6, 1:
  update distances

Sync Bellman-Ford - Round 3

-
  update distance (recalculation!)

Sync Bellman-Ford - Round 4

- no recalculation (but could have been)

Distributed Bellman-Ford – termination?

All nodes have successfully terminated

We can see this, but the nodes don't know this yet

How to detect and, optionally, disseminate the termination info?

-
- For Sync Bellmann-Ford: by attaching a time-to-live (TTL) to the broadcast token

  - Initially equal to the number of nodes
  - After receiving it, each node decrements this by 1, at each round (thus sync mode required)
  - When TTL $= 0$, each node knows that the algorithm has terminated (guaranteed)

Async Bellman-Ford – worst case (sketch, cf. Lynch §15.4)

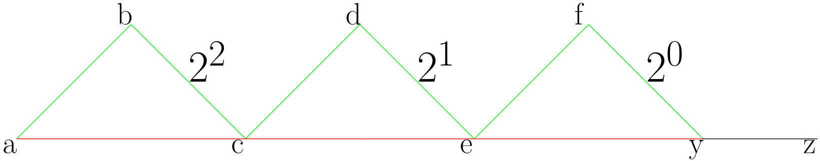
Sketch for

- $k = 3$ (can be any number)
- $N = 2k + 2 = 8$ nodes
- $M = 3k + 1 = 10$ edges

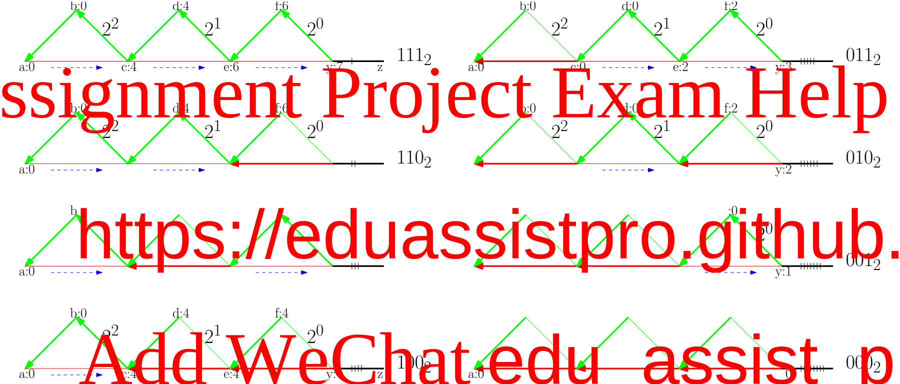
Cost

Initia

Green: fast link; Red: slow link; Black: slow & critical i



Async Bellman-Ford – worst case (cont)



Dotted blue arrows : messages still in transit

Shapes of the shortest-so-far cost paths $\xleftrightarrow{1:1}$ base 2 numbers

Exponential message complexity $\geq 2^k = 2^{(N-2)/2} = \Omega(\sqrt{2}^N)$

Exponential time complexity if FIFO – congestion on black edge

Async Bellman-Ford – worst case

- How to explain the time complexity?

- Time complexity without FIFO pileups $= O(|V|)$?

- Time complexity with FIFO pileups = exponential?

-

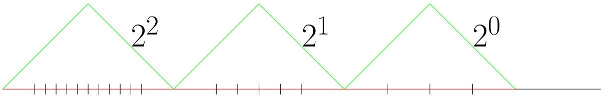
- If we consider congestion, then these piled-
  be successively delivered at t inte

- Total delivery time on the last edge: 2 ,
  i.e. exponential time complexity [Lynch §15.4]

- This argument fails, if we do NOT consider FIFO congestion, because then even the slowest message will not be affected by the others, and will only take a maximum 1 time unit.

Echo and Bellman-Ford – complexity highlights

- Sync Echo (aka Sync BFS) : BFS ST, no link changes, fast
- Async Echo : arbitrary ST, no link changes, but not so fast
- Sync BF : shortest paths ST, many link changes, not so fast
-
- Why the worst case argument does NOT ap
  - emulating slow links by extra edges expo V!
  - the formula is still exponential on $N = exp(k)$!



Maximal Independent Set

- Independent = no two neighbours
- Maximal = cannot be extended
-

- Impossible to solve with conventional mea symmetric case!

- Luby's algorithm can still break the ties with randomization techniques

Luby's algorithm

- Sync algorithm, which works in stages, each consisting of the following 3 rounds

1.

2. Winners notify their neighbours. Processes that receive such messages from their neighbours beco

3. Losers notify their neighbours. All proce conceptual reshaping of the graph. Bot conceptually disconnected from further participation. Remaining nodes are still competing and will regenerate new random values in their next stage!

Luby's algorithm

- Luby's algorithm will stop with probability 1, expected

- [illegible] $|V|^4$, will be likely distinct (but this is not necessary)
- Our diagrams will be sketched only, we won' rounds individually

MIS – Example

- Another solution could be $\{5, 1, 3\}$ (which is larger)

Luby – Stage #1, Round #1

- red = winner
- we only show most relevant messages

Luby – Stage #1, Rounds #2&#3

- one winner so far: 5
-
-

- winners and losers are conceptually disconnected
- still competing: 1, 2 and 3

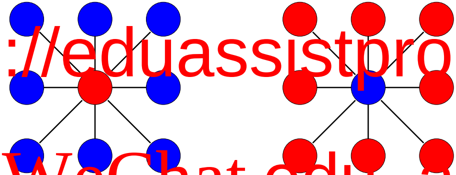
Luby – Stage #2

-
- winners' neighbours:
  1 and 3

- the end

More about MIS

- Minimum Maximal Independent Set vs. Maximum Maximal Independent Set



- Related readings (NOT required) – S. Bute `https://ufdc.ufl.edu/UFE0001011/00001/pdf`