COMS 4236: Introduction to Computational Complexity, Spring 2018

Problem Set 3, due Thursday March 8, 11:59pm on Courseworks

Please follow the homework submission guidelines posted on Courseworks.

Problem 1. [15 points]
a. Consider the following problem, called CIRCUIT-SAT-20.
Input: A Boolean circuit C with 20 input variables.
Question: Is C satisfiable, i.e. does there exist an assignment to the input variables of C for which the value of C is 1?

Show that CIRCUIT-SAT-20 is in P.

b. Suppose that A, B, C are three languages such that (1) $A \le_p B$, (2) $B \le_p C$, (3) A is NP-complete, and (4) C is NP-complete. Show that B is also NP-complete.

Problem 2. [20 p

a. Show that if a language L is complete for a cl r polynomial time reductions then its complement $\overline{L}$ is complete f the same type of reductions. (Recall that coC = { $\overline{L}$ | $L \in$ C }.)

b. We say that a class C is *closed* under a type of reductions (eg. log-space or polynomial time reductions) if, whenever L reduces to L' and L' is in C, then also L is in C.
Suppose that the class C is closed under a type of reductions and the language M is complete for C. Show that M is in coC if and only if C = coC.

Problem 3. [20 points]
a. Show that NP, coNP, and EXP=$\bigcup_{c>0} TIME(2^{n^c})$ are closed under polynomial time reductions.

b. Show that the class $E = \bigcup_{c>0} TIME(2^{cn})$ is not closed under polynomial time reductions.
That is, there are languages L, L' such that L $\le_p$ L' and L' $\in E$, but L $\notin E$.

(*Hint*: Use a padding function and the time hierarchy theorem.)

Problem 4. [25 points]

a. Consider the following transformation that maps a directed graph G=(V,E) with m nodes to another directed graph G'=(V',E') with m² nodes. The set of nodes of G' is V'={ [v,i] | v∈V, 1≤i≤m }, and the set of edges is E'= { ([u,i],[v,i+1]) | (u,v)∈E, 1≤i≤m-1 } ∪ { ([v,i],[v,i+1]) | v∈V, 1≤i≤m-1 }.

Use this transformation to show that the Graph Reachability problem is NL-complete (under log-space reductions) even when the input is restricted to acyclic graphs and the nodes are ordered topologically.

Specifically, show that the following *DAG Reachability* problem is NL-complete.

Input: A directed acyclic graph H=(N,A) with set of nodes N={1,…,n} in topological order, i.e. all edges (i,j) ∈A satisfy $i<j$; Two nodes s, t.

Question: Is there a path in H from s to t?

b. The *Graph Cyclicity* problem is as follows.

Input: A directed graph G.

Question: Does the graph contain a cycle?

Show that the Graph Cyclicity problem is NL-complete.

c. Is the *Graph Acyclicity* problem (Given a directed graph, is it acyclic?) NL-complete? Justify your answer.

Problem 5. [20 points]

a. Consider the following set of inequalities: $a \leq$ $0 \leq a$.

Show that for every assignment of values 0 or 1 to b and c, there is a unique value of a over the real numbers that satisfies the inequalities, namely the value $a = b \wedge c$, i.e. $a = 1$ if both b, c are 1, and $a = 0$ otherwise.

b. Give a set of inequalities in variables a, b, c that has the analogous property for $a = b \vee c$, i.e., for every assignment of values 0 or 1 to b and c, there is a unique real value of a that satisfies the inequalities, namely the value $a = b \vee c$.

c. Show that the following "*Linear Inequalities*" problem is P-hard under log-space reductions. (The problem is also in P but you do not have to show this.)

Linear Inequalities:

Input: A system of linear inequalities in a set of variables.

Question: Does the system have a solution over the reals, i.e. is there an assignment of real values to the variables that satisfies all the inequalities?

(*Hint:* Reduce from the Circuit Value problem with fan-in 2. Introduce variables for the gates and the inputs of the circuit and include appropriate inequalities for the gates and the given input assignment to the circuit.)