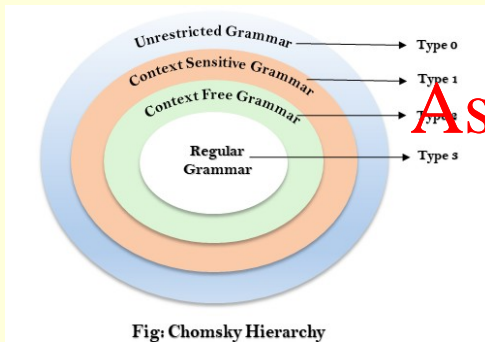


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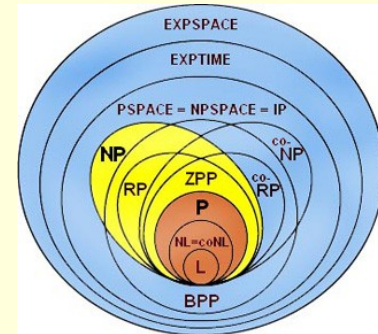
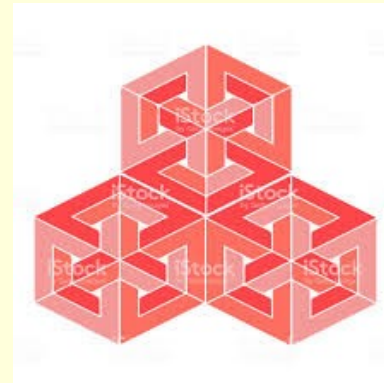
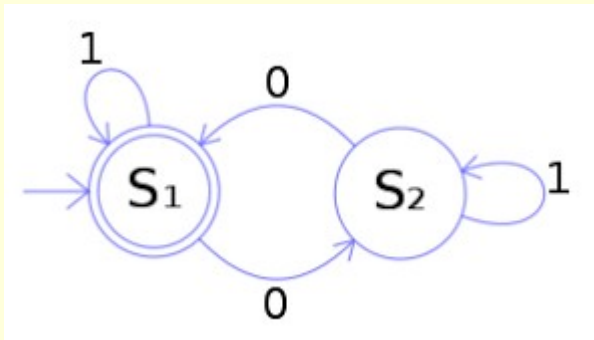
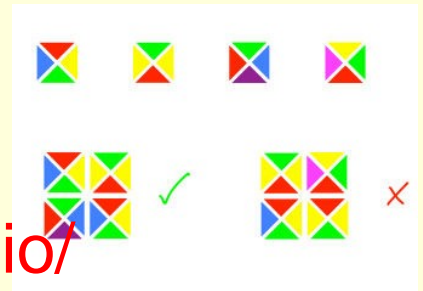
(We will commence soon. We are just allowing a few minutes for people to join and set up. *Please mute your microphone unless you are speaking.* You can raise your hand or use the chat at any time.)

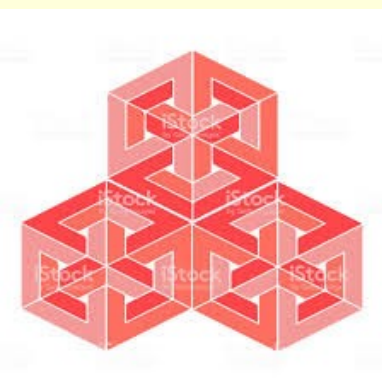
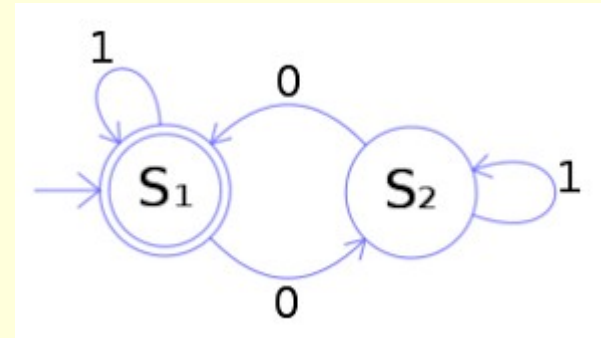
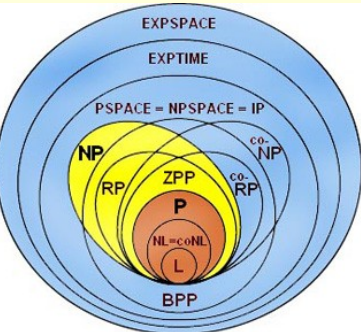


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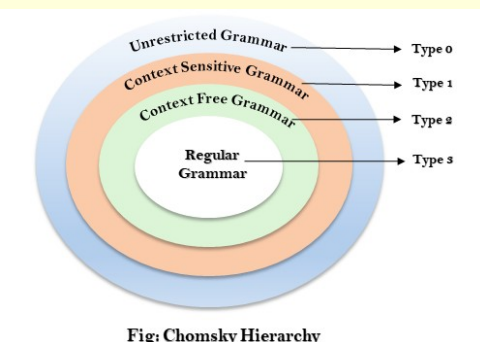
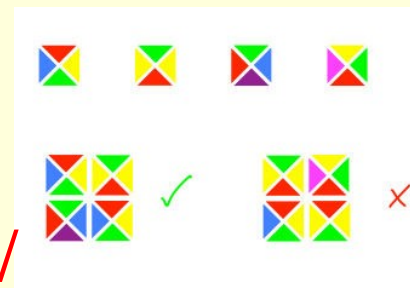


Fig: Chomsky Hierarchy

James Harland

james.harland@rmit.edu.au

* With thanks to Sebastian Sardina

Intro music 'Far Over' playing now ...



Week 7

Computing Theory

Acknowledgement



RMIT University acknowledges the people of the Woiwurrung and Boon wurrung language groups of the eastern Kulin Nations on whose unceded lands we conduct the business of RMIT University respectfully acknowledge Elders, past and present.

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RMIT also acknowledges the Traditional Custodians and their Ancestors of the lands and waters across Australia where we conduct our business.

(add your name here to volunteer for this or email me)

Overview

- Questions?
- Grammar normal forms

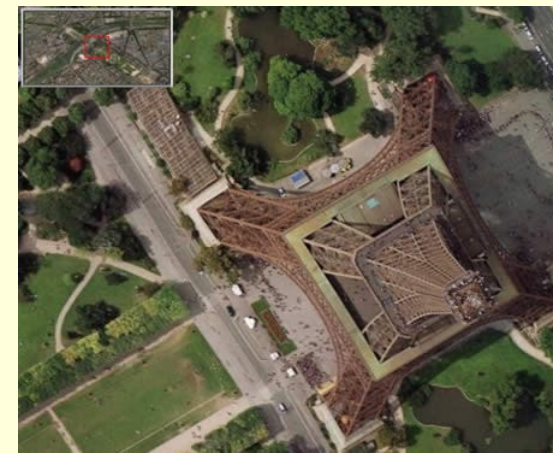
- Questions? Assignment Project Exam Help What can be done

- Pumping Lemm be done

- Questions? <https://eduassistpro.github.io/>

- Platypus Game Add WeChat edu_assist_pro

- Questions?



Weekly Schedule



	Lecture/Lectorial	Tutorial	Assessment
1	Formal languages, grammars	Motivations & Mathematical preliminaries	
2	Finite State Machines	Grammars	Quiz 1
3	Pushdown Automata, nondeterminism	NFAs and DFAs	Quiz 2
4	Turing machines	Pushdown automata	Quiz 3
5	Computability, universality	Computability	Quiz 4
6	Pumping Lemma, NFA \rightarrow DFA conversion	Complexity	Assignment 1, Quiz 5
7	Chomsky Hierarchy	Nondeterministic Pumping Lemma	Quiz 6
8	Unrestricted grammars		Quiz 7
9	Complexity and intractability	Unrestricted grammars	Quiz 8
10	NP-completeness	Complexity and intractability	Quiz 9
11	Zero-knowledge proofs	NP-completeness	Quiz 10
12	Research and requests	Sample exercise	Assignment 2
14-16	--	--	Final exercise

Foundations

relationships

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Analysis

Assessment

Week 7

Computing Theory

Questions?

Questions?



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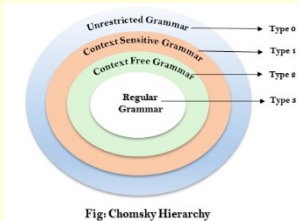
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Questions?



Chomsky Hierarchy



Automata	Languages	Grammars
	Undecidable languages	---
Turing Machines	↔ Recursively enumerable languages	↔ Unrestricted grammars
Linear Bounded Automata	↔	↔ Context-sensitive grammars
(Nondeterministic) Pushdown Automata	↔ Context-free languages	↔ Context-free grammars
Deterministic Pushdown Automata	?? (Deterministic CF?)	???
Nondeterministic Finite Automata & Deterministic Finite Automata	↔ Regular languages	↔ Regular grammars & regular expressions

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Week 7

Week 6

Week 7

Computing Theory

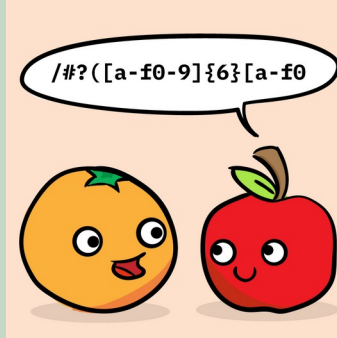
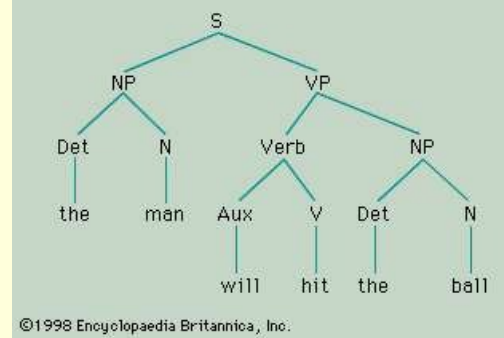
Grammars

What are grammars again?

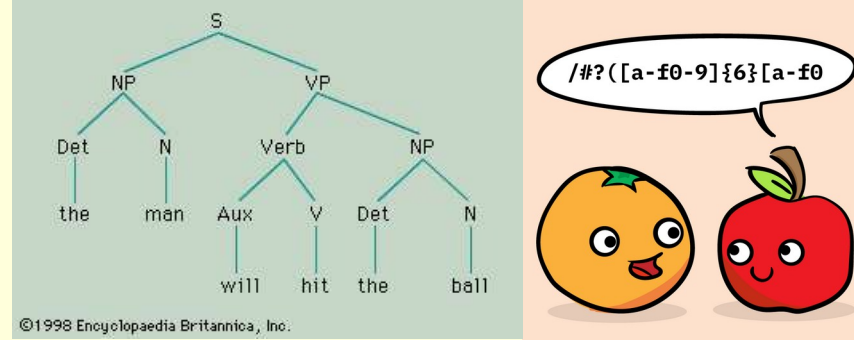
"Around the survivors a perimeter create!"

Much studied by Noam Chomsky in the 1950's

- Similar to rules use <https://eduassistpro.github.io/>
- Special start symbol S
- Set of rules of the form $X \rightarrow Y$ ("whenever you see X , you can replace it with Y ")
- For any string $w_1 X w_2$, can obtain string $w_1 Y w_2$
- Stop when no more rules apply



Grammars



Inherently nondeterministic!

S zN@E.mad
N 1 | 1E
E DD | DDE
D 0 | 1 | 2

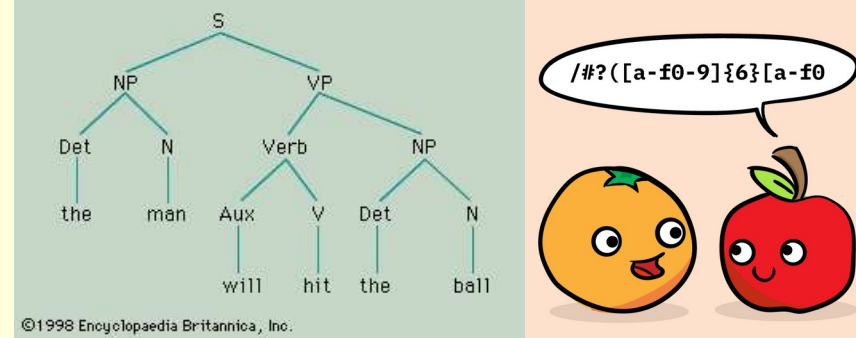
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S zN@E.mad z1E@E.mad z1D@E.mad z1D@E.mad
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... z10D@DD.mad z101@DD.mad z101@0D.mad z101@01.mad
... z11D@DD.mad z110@DD.mad z110@1D.mad z110@10.mad
... z10D@DD.mad z100@DD.mad z100@0D.mad z100@00.mad
... z12D@DD.mad z122@DD.mad z122@2D.mad z122@22.mad
...

Grammars



Question: What **exactly** is a grammar?

Answer: A set of replacement rules for strings

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V: set of **non-terminal** S, A, B, C, ...)

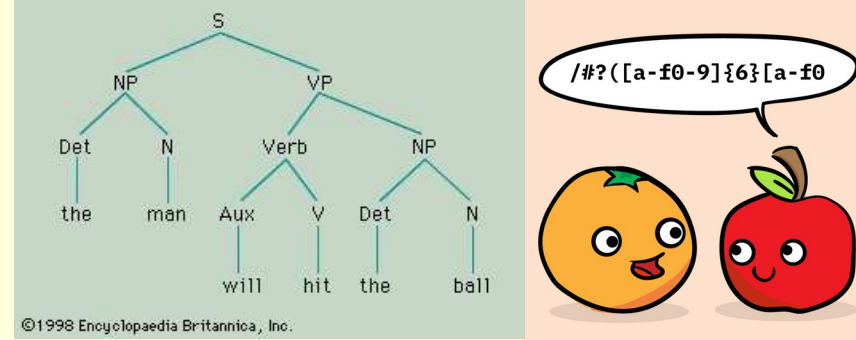
T: set of **terminal sym** <https://eduassistpro.github.io/>

A rule is of the form **$\underbrace{A \rightarrow W}_{\text{non-empty string over } V \cup T} C$** (Add WeChat edu_assist_pro)

Derivation: Given rule $L \mid R_1 \mid R_2 \mid \dots \mid R_n$ and string xLy , a permitted step is $xLy \rightarrow xR_i y$ for any $i = 1 \dots n$

Take transitive closure of steps from **S** ...

Grammars



A rule is of the form $(V \ T)^+ (V \ T)^*$

non-empty string over $V \ T$ string over $V \ T$

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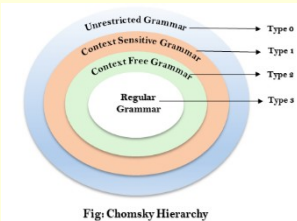
$L \ R_1 \mid R_2 \mid \dots \mid R$

grammars result from

<https://eduassistpro.github.io/>

Name	Type	Constraint	Example
Unrestricted	0	--	AbC AC
Context-sensitive	1	$ L \leq R_i $ or S	AbC abc
Context-free	2	$ L = 1$	A AC
Regular	3	$ L = 1$ (ie $L \ V$) and $R_i \ T \ \{\}$ TV	A a A bB A

Chomsky Hierarchy



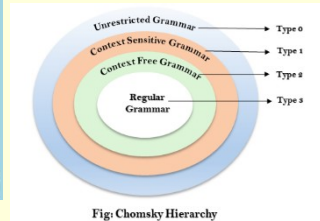
Automata	Languages	Grammars
---	<i>Undecidable languages</i>	---
Turing Machines	Recursively enumerable languages	Unrestricted grammars Type 0 A B C
Linear Bounded Automata	Context-sensitive languages	Context-sensitive grammars Type 1 C abc
(Nondeterministic) Pushdown Automata	Context-free languages	Context-free grammars Type 2 A AC
Nondeterministic Finite Automata & Deterministic Finite Automata	Regular languages	Regular grammars & regular expressions Type 3 A a bB

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Chomsky Hierarchy



- Grammars generate languages
- Automata accept languages

Relationships Assignment Project Exam Help

- Given a grammar G , such that $L(G) = L(M)$?
(so M accepts w iff G generates w)
- Given an automaton M , such that $L(M) = L(G)$?
(so G generates w iff M accepts w)

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Types

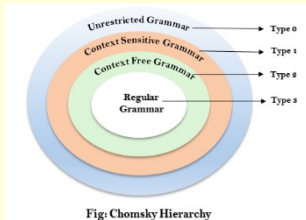
What of grammar is G ?

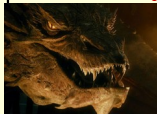
Regular? Context-free? Context-sensitive? Unrestricted?

What of automaton is M ?

DFA? NFA? PDA? LBA? DTM? NDTM?

Chomsky Hierarchy



Automata	Languages	Grammars
	Undecidable languages	---
Turing Machines	Recursively enumerable languages	Unrestricted grammars
Linear Bounded Automata	Context-sensitive languages	Context-sensitive grammars
(Nondeterministic) Pushdown Automata	Context-free languages	Context-free grammars
Deterministic Pushdown Automata	?? (Deterministic CF?)	???
Nondeterministic Finite Automata & Deterministic Finite Automata	Regular languages	Regular grammars & regular expressions

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Questions?

Questions?



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Questions?

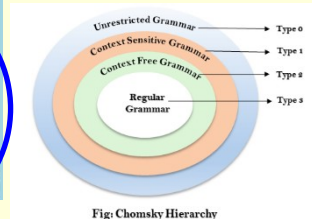


Normal Forms

Grammars are rather free form ...
The rule below is legal!



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S SUPERCALIFRAGILISTICEXPALIDOCIOUS

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For any context-free G , there are grammars G_1 and G_2 such that $L(G) = L(G_1)$ and $L(G) = L(G_2)$ and

G_1 is in Chomsky* normal form and

G_2 is in Greibach* normal form

*Noam Chomsky

*Sheila Greibach

Chomsky normal form

S
 $A \rightarrow BC$
 $A \rightarrow a$

Derived string only
grows by 1

Week 7

Greibach normal form

S
 $A \rightarrow aA_1A_2...A_n$ where $n \geq 0$
pop input push
Like PDA

Regular

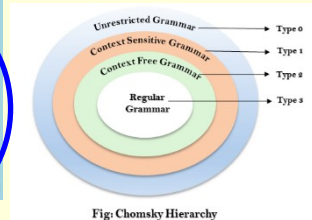
$A \rightarrow aB \mid a$

input state

Like FSA

Computing Theory

Chomsky Normal Form



Grammar is in Chomsky normal form if every rule is of the form

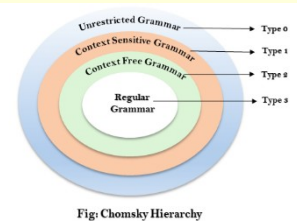
$S \rightarrow BC$
 $A \rightarrow a$

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where B, C are variables <https://eduassistpro.github.io/>

- $S \rightarrow \epsilon$ is the only rule with the empty right hand side
- Otherwise the right hand side has length 2
- All context-free grammars can be (efficiently) converted into an equivalent one in CNF
- String in derivation grows by 1 each step or stops
- Some variants exist

Chomsky Normal Form



$S \mid ASA \mid BSB \mid A \mid B$
 $A \mid 0$
 $B \mid 1$

(from week 1 slides)

$S \mid aB \mid bS \mid cS \mid$
 $B \mid aB \mid bC \mid cS \mid$
 $C \mid aB \mid bS \mid$

(from week 1 slides)

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$S \mid ASA \mid BSB \mid 0 \mid$
 $A \mid 0$
 $B \mid 1$

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$S \mid AC \mid BD \mid 0 \mid 1$
 $A \mid 0$
 $B \mid 1$
 $C \mid SA$
 $D \mid SB$



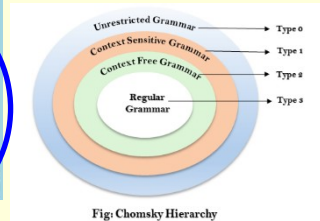
Week 7

$S \mid a \mid AB \mid DS \mid ES \mid$
 $B \mid a \mid AB \mid b \mid DC \mid ES$
 $C \mid a \mid AB \mid DS$
 $A \mid a$
 $D \mid b$
 $E \mid c$



Computing Theory

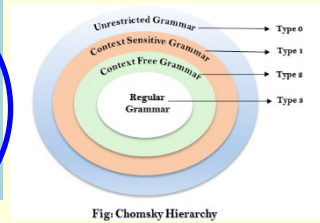
Chomsky Normal Form



Conversion Process:

1. Make start state non-recursive
2. Eliminate A rules (other than S)
3. Eliminate 'chain' rules (e.g., $A \rightarrow B$, $B \rightarrow C$ becomes $A \rightarrow C$)
4. Eliminate variables that do not derive any terminal string (e.g., $A \rightarrow B$, $B \rightarrow A$)
5. Eliminate variables not reachable from S
6. Replace terminals with variables (e.g., $A \rightarrow bCb$ to $A \rightarrow BCB$, $B \rightarrow b$)
7. Reduce variables in each rule (e.g., $A \rightarrow BCB$ to $A \rightarrow BD$, $D \rightarrow CB$)
(rule with n variables becomes $n-1$ rules with 2 variables)

Chomsky Normal Form



Derivations are much simpler to manage in with CNF

S : delete S from the current string
(only time derived string gets shorter)

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$A \rightarrow BC$: replace A with BC
(adds one variable t 1 in length)

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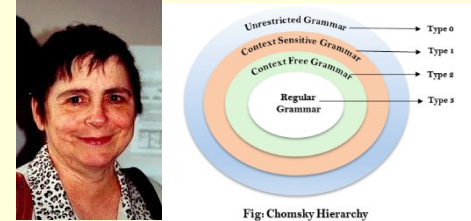
$A \rightarrow a$: replace A with a
(one less variable, one more terminal stays the same length)

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So if we have $S \rightarrow T$, then $|T|$ grows with every application of a rule like $A \rightarrow BC$

Precise relationship between depth of parse tree and $|T|$

Greibach Normal Form



Grammar is in Greibach normal form if every rule is of the form

S

$A \rightarrow a$

$A \rightarrow aA_1A_2 \dots A_n$

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- No rules like $A \rightarrow Ba$

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- Generalisation of regular grammar

$n \leq 1$

- All context-free grammars can be (efficiently) converted into an equivalent one in GNF
- Conversion is more complex than for Chomsky normal form
- Needs to eliminate 'left' recursions, so that terminals can be made to appear 'predictably from the left'

Computing Theory

Week 7

Questions?

Questions?



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Questions?



Quiz time!

Go to **Canvas** and find the quiz **Lectorial 7 Question set**

- Not worth any marks
- You can consult other students if you wish
- Time limit will be 10 minutes

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Are you ready?

Are you sure?

Go!

The pictures will take 10 minutes to disappear!

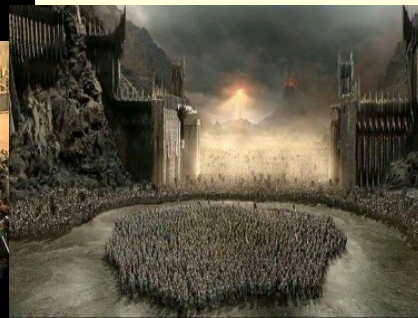
Thomas music means 1 minute left!



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Questions?

Questions?



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Questions?





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How did you go?

Question 1: Convert the grammar below to Chomsky normal form.

$S \rightarrow aaSB \mid$
 $B \rightarrow bB \mid b$

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<https://eduassistpro.github.io/form?>

Is the above gram

$S \rightarrow CD \mid$
 $B \rightarrow AB \mid b$
 $A \rightarrow b$
 $C \rightarrow EE$
 $E \rightarrow a$
 $D \rightarrow SB$

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$S \rightarrow aaTB \mid$
 $T \rightarrow aaTB \mid$
 $B \rightarrow AB \mid b$
 $A \rightarrow b$
 $C \rightarrow EE$
 $E \rightarrow a$
 $D \rightarrow SB$

How did you go?



twinkl.com



Question 2: Convert the grammar below to Chomsky normal form.

$S \rightarrow aSbb \mid A$
 $A \rightarrow cA \mid c$

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Is the above gram

[https://eduassistpro.github.io/
form?](https://eduassistpro.github.io/form?)

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$S \rightarrow EB \mid CA \mid c$

$E \rightarrow FS$

$B \rightarrow DD$

$D \rightarrow b$

$A \rightarrow CA \mid c$

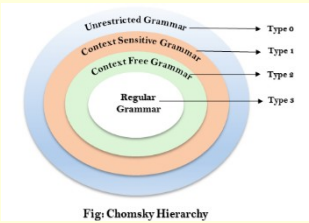
$C \rightarrow c$

$F \rightarrow a$

Week 7

Computing Theory

Grammars and PDAs



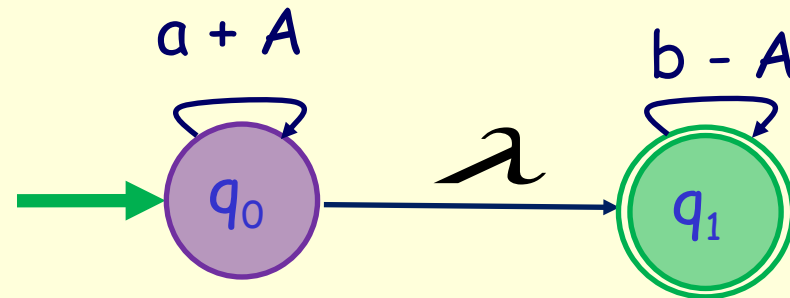
(Nondeterministic) Pushdown Automata	Context-free languages	Context-free grammars A AC
-----------------------------------------	------------------------	-------------------------------

- For every context-free grammar G , there is an PDA M such that $L(G) = L(M)$
- For every PDA M , there is a context-free grammar G such that $L(M) = L(G)$

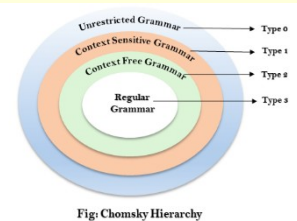
<https://eduassistpro.github.io/>

Consider $L = \{a^n b^n \mid n \geq 0\}$

$S \rightarrow aSb \mid \epsilon$



Grammars and PDAs



Given any grammar, construct an equivalent PDA as follows

- PDA has two states (say p and q)
- Initially push S onto the stack
- For each rule $L \rightarrow R_i$ in \mathcal{R} , add a transition $(q, L, L) \rightarrow (q, R_i)$ (replaces L on the stack)
- For each terminal x , add a transition $(q, x, x) \rightarrow (q, \epsilon)$ (remove non-terminal from top of the stack)

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Lots of

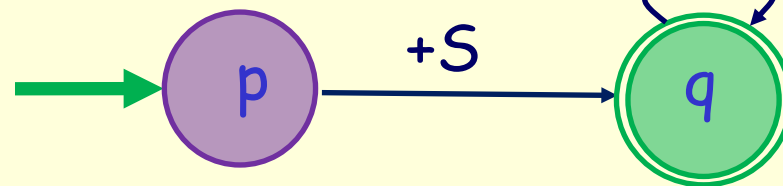
One for each rule case \longrightarrow

$-L+R_i$

One for each terminal \longrightarrow

$x-x$

(Also shows only ever "need" two states in a PDA)

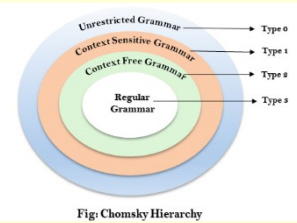


"Use extra states if need be ..."

Week 7

Computing Theory

Grammars and PDAs



$S \rightarrow aSb \mid$

$-S \rightarrow aSb$

$-S$

$a \rightarrow a$

$b \rightarrow b$

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$S \rightarrow aSb \mid aaSbb \mid aaaSbbb \mid aaabbbb$

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$(p, aaabbbb,) \rightarrow (q, aaabbbb, S) \rightarrow (q, aaabbbb, aSb) \rightarrow (q, aabbbb, Sb)$

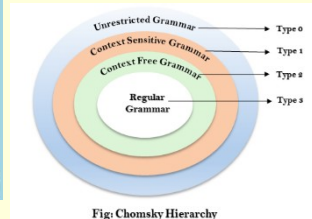
$(q, aabbbb, aSbb) \rightarrow (q, abbbb, Sbb) \rightarrow (q, abbbb, aSbbb)$

$(q, bbbb, Sb) \rightarrow (q, bbbb, b) \rightarrow (q, bbb, b) \rightarrow (q, bb, b) \rightarrow (q, b, b) \rightarrow (q, ,)$

(Greibach normal form simplifies this process, but is not required)



Grammars and PDAs



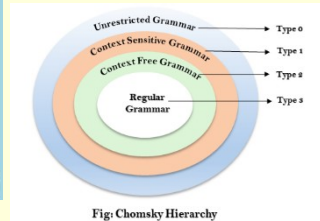
Given any PDA, construct an equivalent grammar as follows
(this is a little trickier than the other way around!) each step in a derivation deletes a symbol ...

- Transform the PDA so that every transition pops the stack (!!)
- $(p, x,) = (q,)$ Assignment Project Exam Help
- $(p, x, A) = (q, A)$ for each A ch A
- $(p, x,) = (q, B)$ execution
- Construct grammar
- Variable $(q_i A q_j)$ represents execution ending in q_j and popping A Add WeChat: edu_assist_pro
- $S (s Z f)$ where s, f are start and final states of the PDA
- For each $(q_1, x, B) = (q_2, A)$, add rule $(q_1 B p) \times (q_2 A p)$ for all states p
- For each $(q_1, x, A) = (q_2, AB)$, add rule $(q_1 A p) \times (q_2 A r) (r B p)$ for all states p, r same same
- $(q q)$ for all states q

Week 7 $(q_1 A q_2) * w$ iff $(q_1, w, A) (q_2,)$

Computing Theory

CFGs and PDAs



What can PDAs do?

- For every PDA there is an equivalent CFG
- Recognise regular languages
- Recognise context-free languages
- DPDAs are weaker than PDAs (!)

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What can't PDAs do?

- Recognise context-sensitive languages
- Recognise recursive languages

Questions?

Questions?



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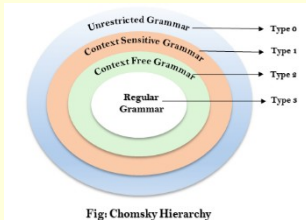
<https://eduassistpro.github.io/>

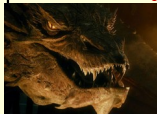
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Questions?



Chomsky Hierarchy



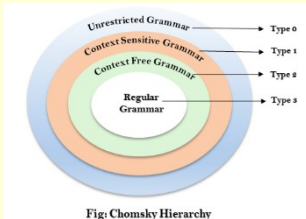
Automata	Languages	Grammars
	Undecidable languages	---
Turing Machines	Recursively enumerable languages	Unrestricted grammars
Linear Bounded Automata	Context-sensitive languages	Context-sensitive grammars
(Nondeterministic) Pushdown Automata	Context-free languages	Context-free grammars
Deterministic Pushdown Automata	?? (Deterministic CF?)	???
Nondeterministic Finite Automata & Deterministic Finite Automata	Regular languages	Regular grammars & regular expressions

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Limitation of CFGs



Type	Memory Size	Memory access
DFAs	Bounded*	Defined by machine
PDA's (Linear Bounded Automata)	Unbounded ?? (see Week 8)	Top of stack only ?? (see Week 8)
Turing Machines		Random (ie unlimited)

* bounded means th

n in advance

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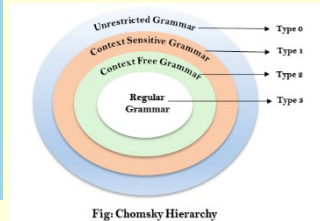
What happens when a CFG in normal form processes a string longer than the number of variables?



"Now you are getting somewhere ..."

Some variable must be repeated!

Limitation of CFGs



Let G be a context-free grammar in Chomsky normal form, and $A \stackrel{*}{\Rightarrow} w$ with (binary) derivation tree T .

If $\text{depth}(T) = n$, then $\text{length}(w) \leq 2^{n-1}$.

Proof: By induction on n .
Base case: $\text{depth}(T) = 1$
either $S \Rightarrow a$ or $A \Rightarrow a$

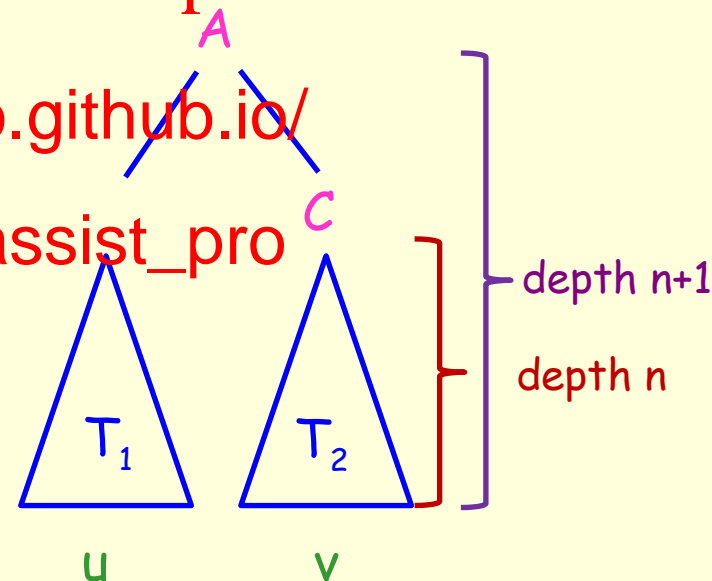
Inductive case: Assume it holds for all derivations of depth n . Let $A \stackrel{*}{\Rightarrow} w$ be a derivation of depth $n+1$. As G is in Chomsky normal form, we must have

$A \Rightarrow BC \stackrel{*}{\Rightarrow} uv$ where $B \stackrel{*}{\Rightarrow} u$ and $C \stackrel{*}{\Rightarrow} v$

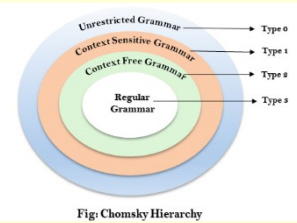
By the hypothesis $\text{length}(u) \leq 2^{n-1}$ and $\text{length}(v) \leq 2^{n-1}$ so $\text{length}(uv) \leq 2^{n-1} + 2^{n-1} = 2^n$

Chomsky normal form

S
 $A \Rightarrow BC$
 $A \Rightarrow a$



Limitation of CFGs



Let G be a context-free grammar in Chomsky normal form, and $S \xRightarrow{*} w$ a derivation of w . If $\text{length}(w) \geq 2^n$, then the derivation tree has depth at least $n+1$.

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Pumping Lemma f

uages

For any context-free language L , if L is infinite, then there is a constant $n \geq 1$ such that for any $w \in L$ with $|w| \geq n$, there are strings x, y, z, u, v such that $w = xyzuv$ and

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1. $|yzu| \leq n$
2. y or u
3. $x y^i z v^i u \in L$ for all $i \geq 0$

Questions?

Questions?



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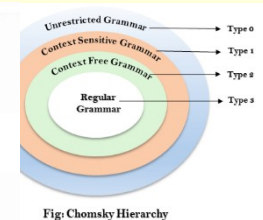
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Questions?



Pumping Lemma



Usual use is to show languages not *context-free* by contradiction

1. Assume L is *context-free*
2. Apply Pumping
3. Choose string w
4. Use $|yzu| \leq n$ to
5. Choose i such that
6. Contradiction!

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<https://eduassistpro.github.io/> and u

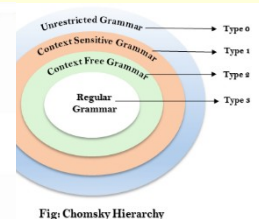
Add WeChat edu_assistpro (works)



Conclude that L is not *context-free*

All such proofs the same except steps 3 & 5

Pumping Lemma



The language $L = \{a^i b^i c^i \mid i \geq 0\}$ is not context-free

Proof: Assume L is context-free. Then the Pumping Lemma applies and so there is an $n \geq 1$ such that for all $w \in L$ such that $|w| \geq n$, $w = xyzuv$ where

1. $|yzu| \leq n$
2. y or u
3. $x y^i z u^i v \in L$ for all $i \geq 0$

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Choose $w = a^n b^n c^n$ and so $w \in L$ and $|w| \geq n$

Pumping Lemma

$w = xyzuv = a^n b^n c^n$ and $|yzu| \leq n$.

If y or u contains ab or bc , then $xyyzuuuv \notin L$

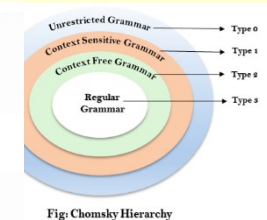
So both y and u contain only one of a or b or c .

But then also $xyyzuuuv \notin L$ as this will contain unequal numbers of a 's, b 's and c 's.

Hence L is not context-free.



Pumping Lemma



The language $L = \{a^i b^j a^i b^j \mid i, j \geq 0\}$ is not context-free

Proof: Assume L is context-free. Then the Pumping Lemma applies and so there is an $n \geq 1$ such that for all $w \in L$ such that $|w| \geq n$, $w = xyzuv$ where

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1. $|yzu| \leq n$
2. y or u
3. $x y^i z u^i v \in L$ for all $i \geq 0$

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Choose $w = a^n b^n a^n b^n$ and so $w \in L$ and $|w|$

e Pumping Len

$w = xyzuv = a^n b^n a^n b^n$ and $|yzu| \leq n$.

If y or u contains ab or ba , then $xyyzuuuv \notin L$

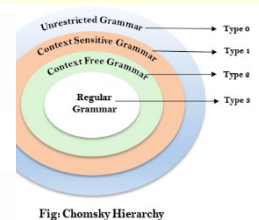
So both y and u contain only one of a or b .

But then also $xyyzuuuv \notin L$ as this will contain unequal numbers of a 's or b 's.

Hence L is not context-free.



Pumping Lemma



The language $L = \{ xx \mid x \in \{a,b\}^* \}$ is not context-free

Proof: Assume L is context-free. Then the Pumping Lemma applies and so there is an $n \geq 1$ such that for all $w \in L$ such that $|w| \geq n$, $w = xyzuv$ where

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1. $|yzu| \leq n$
2. y or u
3. $x y^i z u^i v \in L$ for all $i \geq 0$

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Choose $w = a^n b^n a^n b^n$ and so $w \in L$ and $|w|$

e Pumping Len

$w = xyzuv = a^n b^n a^n b^n$ and $|yzu| \leq n$.

If y or u contains ab or ba , then $xyyzuuv \notin L$

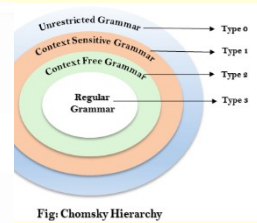
So both y and u contain only one of a or b .

But then also $xyyzuuv \notin L$ as this will contain unequal numbers of a 's or b 's.

Hence L is not context-free.



CFGs



Languages known to be context-free include

$$L = \{a^i b^i \mid i \geq 0\}$$

$$L = \{ww^R \mid w \in \{a,b\}^*\}$$

$$L = \{a^i b^j c^k \mid i=j \text{ or } j=k\}$$

$$L = \{a^i b^j a^i \mid i, j \geq 0\}$$

$$L = \{w \mid w \in \{a,b\}^*, n_a(w) = n_b(w)\}$$

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Languages known not to be context-free

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$$L = \{a^i b^i c^i \mid i \geq 0\}$$

$$L = \{a^i b^j a^i \mid i, j \geq 0\}$$

$$L = \{xx \mid x \in \{a,b\}^*\}$$

$$L = \{a^m \mid m \text{ is prime}\}$$

$$L = \{a^m \mid m = n^2 \text{ for some } n \geq 0\}$$

$$L = \{w \mid w \in \{a,b,c\}^*, n_a(w) = n_b(w) = n_c(w)\}$$

Questions?

Questions?



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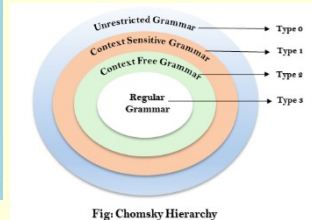
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Questions?



Properties of Languages



If L_1 and L_2 are regular languages, then so are

1. $L_1 \cup L_2$
2. $L_1 L_2$
3. L_1^* (and L_2^*)
4. L
5. $L_1 \cap L_2$
6. $L_1 \setminus L_2$

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1,2,3: Consider R_1, R_2 such that $L(R_1) = L_1$

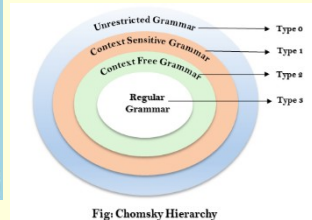
Then consider $R_1 \cup R_2, R_1 R_2, R_1^*$ and R_2^*

4: Get a DFA for L . Swap the final and non-final states to get a DFA for L

5: Follows from 1 and 4, as $L_1 \cup L_2 = L_1 \cup L_2$

6: Follows from 1 and 5, as $L_1 \setminus L_2 = L_1 \cap L_2^c$

Properties of Languages



If L_1 and L_2 are context-free languages, then so are

$L_1 L_2, L_1 L_2, L_1^*$ and L_2^*

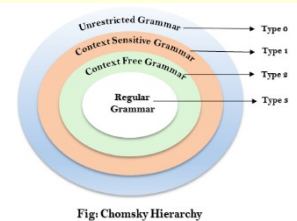
Let S_i be the start symbol for a context-free grammar for L_i .

Language	Grammar
$L_1 L_2$	$S \rightarrow S_1 S_2$
$L_1 L_2$	$S \rightarrow S_1 S_2$
L_1^*	$S \rightarrow S_1 S \mid$

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Example Language



$L = \{a^i b^j c^k \mid i j \text{ or } j k\}$ Is L context-free?

$L = L_1 L_2$ where $L_1 = \{a^i b^j c^k \mid i j\}$ and $L_2 = \{a^i b^j c^k \mid j k\}$

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So L is context-free

One of i and j must be the

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is context-free

L_1 :

maximum is i

↓

$S \rightarrow TC$

$T \rightarrow aTb \mid A \mid B$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

$C \rightarrow cC \mid$

↑

maximum is j

L_2 :

$S \rightarrow AT$

$T \rightarrow bTc \mid B \mid C$

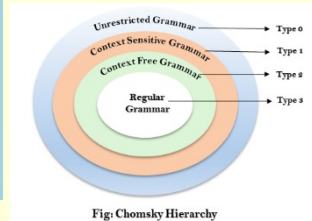
$A \rightarrow aA \mid$

$B \rightarrow bB \mid b$

$C \rightarrow cC \mid c$

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Properties of Languages



If R is a regular language and L is context-free, $L \cap R$ is context-free
(**Think:** still only have to count one thing in $L \cap R \dots$)

Build 'composite' PDA as follows.

Let the PDA that recognises L be $M_1 = (Q_1, \Sigma, \Gamma, \delta_1, q_{01}, F_1)$

Let the DFA that recognises R be $M_2 = (Q_2, \Sigma, \delta_2, q_{02}, F_2)$

Construct new PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$

So what is Q ??

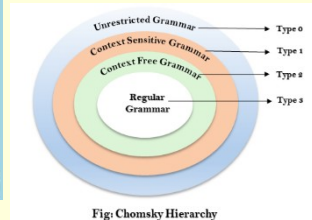
Add WeChat edu_assist_pro composite state of new PDA

$([p, q], a, A) \rightarrow ([p', q'], B)$ where $(p', B) \xrightarrow{\delta_1} (p, a, A)$ and $\delta_2(q, a) = q'$

$([p, q], \epsilon, A) \rightarrow ([p', q], B)$ where $(p', B) \xrightarrow{\delta_1} (p, \epsilon, A)$

Then show that $([p_0, q_0], w, \epsilon) \xrightarrow{*} ([p_i, q_i], u, \epsilon)$ iff $(p_0, w, \epsilon) \xrightarrow{*} (p_i, u, \epsilon)$ and $(q_0, w) \xrightarrow{*} (q_i, u)$

Properties of Languages



If L_1 and L_2 are context-free languages, then $L_1 L_2$, and \overline{L} may not be!

Consider $L_1 = \{ a^i b^j c^k \mid i = j \}$ and $L_2 = \{ a^i b^j c^k \mid j = k \}$

$L_1 L_2 = \{ a^i b^j c^k \mid i = j = k \}$ which is not context-free

It is true though that L is context-free, \overline{L} is not context-free.

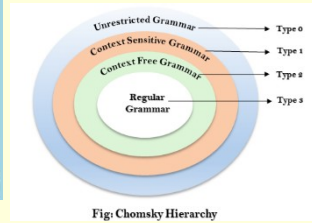
(Think: still only have to count one thing)

Consider $L = \{ a^i b^j c^k \mid i = j \text{ or } j = k \}$

Is \overline{L} context-free? $\overline{L} = L_3 \cup L_4$ where $L_3 = \{ w \mid w \text{ contains } ba, ca \text{ or } cb \}$ and $L_4 = \{ a^i b^j c^k \mid i = j = k \}$

Now $\overline{L} \cap a^* b^* c^* = L_4$ which is not context-free. So \overline{L} cannot be context-free!

PDAs vs DPDAs



If L can be recognised by a deterministic PDA, so can \bar{L}

- Take DPDA for L
- Swap accepting and non-accepting states
- New machine is a DPDA for \bar{L}

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This means that DPDA
Assume the opposite, i

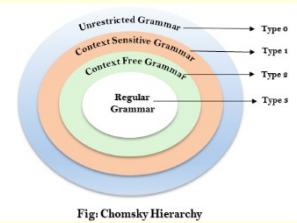
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ent DPDA


- Add WeChat edu_assist_pro
1. Let L be a context-free language
 2. Then there is a PDA for L
 3. By our assumption, there must be a DPDA for L
 4. This means that there is a DPDA for \bar{L}
 5. Then there is a PDA for \bar{L}
 6. Then \bar{L} is context-free for any language L



So our assumption is wrong, ie some PDAs have no equivalent DPDA (!!!)

Chomsky Hierarchy



Automata	Languages	Grammars
	Undecidable languages	---
Turing Machines	Recursively enumerable languages	Unrestricted grammars
Linear Bounded Automata	Context-sensitive languages	Context-sensitive grammars
(Nondeterministic) Pushdown Automata	Context-free languages	Context-free grammars
Deterministic Pushdown Automata	?? (Deterministic CF?)	???
Nondeterministic Finite Automata & Deterministic Finite Automata	Regular languages	Regular grammars & regular expressions

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Chomsky Hierarchy

Undecidable

Turing machines

Undecidable

$L(G) =$

Linear
bounded TM

ND Pushdown
automata

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Regu

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Context-free

$wc w^R$

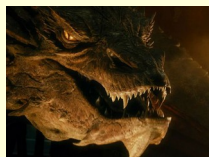
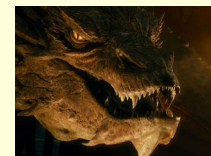
Context sensitive

$a^n b^n c^n$

Unrestricted

$\{(M, w) \mid M \text{ accepts } w\}$

$L(G_1) = L(G_2)$



Halting problem

Week 7

Computing Theory

Chomsky Hierarchy

Automata	Languages	Grammars
---	Undecidable languages	---
Turing Machines	Recursively enumerable languages	Unrestricted grammars
Linear Bounded Automata	Context-sensitive languages	Context-sensitive grammars
(Nondeterministic) Pushdown Automata	Context-free languages	Context-free grammars
Deterministic Pushdown Automata	?? (Deterministic CF?)	???
Nondeterministic Finite Automata & Deterministic Finite Automata	Regular languages	Regular grammars & regular expressions

Problem Reduction

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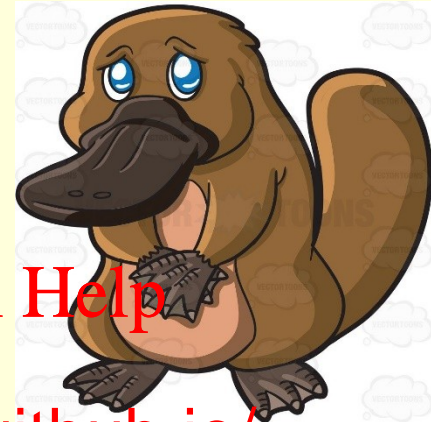
Closure properties

Pumping Lemma

The Platypus Game



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Week 7

<https://www.youtube.com/watch?reload=9&v=0gM5TjSOQ48>
<https://zoneringtones.com>

Computing Theory

The Platypus Game



Assignment 2

Variations

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- Standard (as previous)
- Variable length (50, 100, 200)
- Green - score 2 points
- Tree - score 5 points for reaching either
- Tiebreak - plays an extra match on a random seed up to 200 steps

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Report on your results with 2,500 machines (!!)

OneDrive folder will be shared with you (find the file matching your student number)

Top 10 from each of you will go into the 'knockout' phase

Week 7

Computing Theory



Still coming!

The Platypus Game



3 player tournament

1 vs 1 vs 1

1 vs 1 vs 2

1 vs 1 vs 3

1 vs 2 vs 2

1 vs 2 vs 3

...

1 vs n vs n

2 vs 2 vs 2

2 vs 2 vs 3

...

3 vs 3 vs 3

...

(n-1) vs (n-1) vs (n-1)

(n-1) vs (n-1) vs n

n vs n vs n

$$\sum_{i=1}^n i(i+1)/2 = \left(\sum_{i=1}^n i^2 + \sum_{i=1}^n i \right) / 2$$

$$\text{Assignment Project Exam Help} = \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4} + \frac{n}{6}$$

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When n=26
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this is 3,24



Around 100 times more than a 2-player tournament!

The Platypus Game

4 player tournament

1 vs 1 vs 1 vs 1

1 vs 1 vs 1 vs 2

...

1 vs 1 vs 1 vs n

1 vs 1 vs 2 vs 2

...

1 vs 2 vs 2 vs 2

...

1 vs n vs n vs n

2 vs 2 vs 2 vs 2

2 vs 2 vs 2 vs 3

...

2 vs n vs n vs n

...

3 vs 3 vs 3 vs n

...

(n-1) vs (n-1) vs (n-1) vs n

n vs n vs n vs n

Week 7

$$\sum_{i=1}^n i(i+1)(i+2)/6$$

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When n = 268,

this is 219,790,485

Around 10,000 times more than a 2-player tournament!

When n = 90, this is 2,919,735



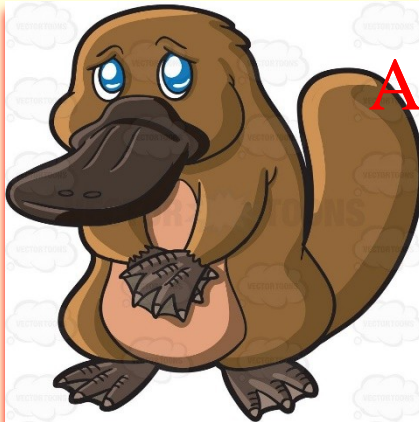
Computing Theory

Assignment 2



- Detailed specification will be released soon
- Platypus tournament for 2,500 machines
- 'Second version' of Universality task from Assignment 1
- Research on Assignment 2 Project Exam Problems

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That's it!



I am out of here!

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The lecture

ed.

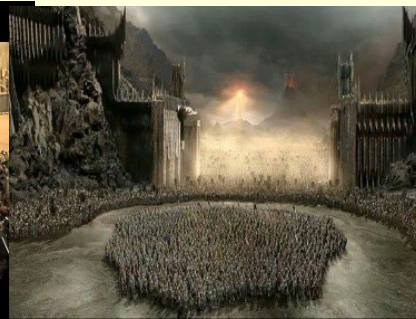
Break time! (We resume when all the pictures are gone! This will take 3 minutes!)



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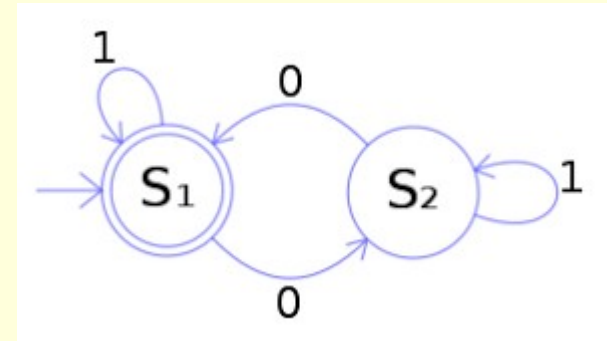
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I AM BACK!

Marking

Computing Theory

Marking



Why did I lose marks for ...??



Where did you last see them?
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At <https://eduassistpro.github.io/>

Perhaps you should add WeChat [edu_assist_pro](#) in at night ...

Never feed them after midnight!

...

Alternative Scheme?



Troll	Dreadful	Poor	Acceptable	Exceeds Expectations	Outstanding
-------	----------	------	------------	----------------------	-------------

Outstanding - CONGRATULATIONS! Your exemplary powers of deduction and a formidable knowledge of the inner workings of the magical world reveal you to be a witch or wizard of genuine skill and learning.

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Exceeds Expectations - rformance!

Acceptable - demonstrat <https://eduassistpro.github.io/>

Poor - Alas - we regret to inform you that you have failed. This may have been due to factors outside your control (eg: poltergeist attack, examination nerves or a malfunctioning quill.) Please do not disconsolate.

Dreadful - We are sorry to inform you that you have failed.

Troll - You would appear either to have abandoned the test due to factors outside your control (eg, earthquake, poltergeist attack), or else you are a troll, in which case you are to be congratulated on being able to use a computer and have achieved the grade of O.F.T. (Outstanding for Trolls).

Marking

Computing Theory

Alternative Scheme?



Troll	Dreadful	Poor	Acceptable	Exceeds Expectations	Outstanding
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Outstanding - CONGRATULATIONS! Your exemplary powers of deduction and a formidable knowledge of the inner workings of the magical world reveal you to be a witch or wizard of genuine skill and learning.

Assignment Project Exam Help

Exceeds Expectations - performance!

Acceptable - demonstrated <https://eduassistpro.github.io/>

Poor - Alas - we regret to inform you that you have failed. This may have been due to factors outside your control (eg: poltergeist attack, examination nerves or a malfunctioning quill.) Please do not disconsolate.

Dreadful - We are sorry to inform you that you have failed.

Troll - You would appear either to have abandoned the test due to factors outside your control (eg, earthquake, poltergeist attack), or else you are a troll, in which case you are to be congratulated on being able to use a computer and have achieved the grade of O.F.T. (Outstanding for Trolls).

Marking

Computing Theory