Language modeling

- Language modeling is the task of assigning a pr sequence (for evaluating translation cand
- It is also the task o sequence of w
- Perfect performance in language mod le to predict the next word in the sentence wit guesses that is fewer or less than that requir participant.

Language Modeling

Formally, the language model can be formulated a

$$P(w_{1:M}) = P(w_1)P(w_2\ w_1)P(w_3\ \boldsymbol{w}_{1:2}) \quad (w_M|\boldsymbol{w}_{1:M-1})$$

With Markov assu

$$P(\boldsymbol{w}_{1:M}) \approx \prod_{m=1}^{M} P(w$$

Text generated by GPT-2

The fact that I work at the univ
ability to find more people. T
My university is a very small community. Those are things that are going to have the effect, that go all these years and have the impact that it has on my life." That's one of the things that makes me really ne
being a student. If you're a computer programmer, there
somebody who you want to study. This is not how I work. Bec
learning, I have to be ver
want to get a career. I don
Now, you know, I think if I don't get the job, then I just don'
time. Maybe I feel like I can't do much, and all of these thin
to be my own business and I want to be able to get a job. I'd cer
be looking at that. I'm at a different time. I'm going to write about my business, my business, my business and I want to talk to you about the life that I've been working for the last 30 years or so. So that's why I've been able to be here for the last 15 years through the university. My life has been extremely productive and I've had a lot of great things, so I love it."

Try it yourself: https://gpt2.ai-demo.xyz

Noisy Channel Model for MT

- The Noisy channel model is the general framework for Statistical Machine Translation, with ma

- Language model is a crucial component of the that is used to "select" the best translation:

$$P_{e|}$$

$$= P_{s|e}(\quad {}_e(\boldsymbol{w}^{(e)})$$

where $\boldsymbol{w}^{(e)}$ is generated from a language model, $\boldsymbol{w}^{(s)}$ is a Spanish sentence generated from a translation model $P_{s|e}(\boldsymbol{w}^{(s)}|\boldsymbol{w}^{(e)})$

Perplexity: a metric for evaluating language models

- Given a text corpus of M words (where M could be in the millions) $w_1, w_2, w_3, \cdots w_M$, a language model function LM assigns a probability to a word based on its histo

$$\ell(\quad , \cdots , w_1)$$

$$m=1$$

- The perplexity of the LM with re :

$$Perplex(\boldsymbol{w}) = 2^{-\frac{\ell(\boldsymbol{w})}{M}}$$

What counts as a good language model?

- Good language models will assign high proba
  events in the co
- Perplexities ar
  language models are only comparable w
  same evaluation corpus.

Extreme Cases of Perplexity

- In the limit of a perfect language model, probability 1 is assigned to the held-out corpus, with $Perplex(\boldsymbol{w}) = 2^{-\frac{1}{M}\log_2 1} = 2^0 = 1$

- In the opposite limit, probability zero is assign held-out corpus, which responds to an infinite perplexity: $Perplex(\boldsymbol{w})$ $-$

- Assume a unif $w_i) = 1/V$ for all words in the vocabulary. Then,

$$\log_2(\boldsymbol{w}) = \sum_{m=1}^{M} \log_2 \frac{1}{V} = -\sum_{m=1}^{M} \log_2 V = -M \log_2 V$$

$$Perplex(\boldsymbol{w}) = 2^{\frac{1}{M} M \log_2 V} = 2^{\log_2 V} = V$$

Traditional approaches to language modeling

- Based on a n-order markov property
  $P(w_{m+1}|\boldsymbol{w}_{1:m}) \approx P(w_{m+1}|\boldsymbol{w}_{m-n:m})$

- The estimates are usually derived from corp

- The role of the la
  of $\hat{P}(w_{m+1}$

- The maximu
  $\hat{P}(w_{m+1}|\boldsymbol{w}_{m-n:m})$ is then

$$\hat{P}_{MLE}(w_{m+1}|\boldsymbol{w}_{m-n:m}) = \frac{\#(\boldsymbol{w}_{m-n:m+1})}{\#(\boldsymbol{w}_{m-n:m})}$$

Addressing the zero count problem

- Zero count for any o…
  for the entire corpus, meaning infinite perplexity!
- Add-α smoothing:

$$\hat{p}_{ad} \ldots \frac{\ldots \alpha}{\ldots) + \alpha|V|}$$

- Another technique is to back off to a lower n-… there is a count. The Jelinek Mercer inter… g:

$$\hat{P}_{int}(w_{m+1}|\boldsymbol{w}_{m-n:m})$$
$$= \lambda_{m-n:m}\frac{\#(\boldsymbol{w}_{m-n:m+1})}{\#(\boldsymbol{w}_{m-n:m})} + (1-\lambda_{m-n:m})\hat{P}_{int}(\boldsymbol{w}_{m+1}|\boldsymbol{w}_{m-(n-1):m})$$

Notice this is a recursive formulation.

Limitations of smoothed MLE based models

- Smoothing based on backoff to lower-orde sequential na rd large n-gram
- MLE-based language models suffer fro tion across contexts

Neural language models

- Treat word predi
  the goal of computing the probability $P(w|u)$, where $w \in V$ is a word, and u is the context that depends on *previous* words
- Parametrize the probability $P(w|u)$
  K-dimensional dense vectors, $\beta_w \in$ ^{K}:

  $u))$

  The vector of probabilities can be compu
  SoftMax transformation to the vector o

  $$P(\cdot|u) = \text{SoftMax}([\boldsymbol{\beta}_{w_1} \cdot \boldsymbol{v}_u, \boldsymbol{\beta}_{w_2} \cdot \boldsymbol{v}_u, \cdots, \boldsymbol{\beta}_{w_V} \cdot \boldsymbol{v}_u])$$

- The word vectors β_w are parameters of the model and can be estimated directly, e.g., using the negative log likelihood of the training corpus as the objective

Computing the context vector

- There are different ways to compute the context vector $\boldsymbol{v}$, and one effective way is to use a **Recurrent Neural Network** or RNN. The basic idea is to recurrently update th vector while moving through a sequence

- Let $\boldsymbol{h}_m$ represent the contextual inform m in the sequence

$$\boldsymbol{x}_m \triangleq \phi_{w_m}$$

$$\boldsymbol{h}_m = \text{RNN}(\boldsymbol{x}_m, \boldsymbol{h}_{m-1})$$

$$P(w_{m+1}|w_1, w_2, \cdots, w_m) = \frac{\exp(\boldsymbol{\beta}_{m+1} \cdot \boldsymbol{h}_m)}{\sum_{w' \in V} \exp(\boldsymbol{\beta}_{w'} \cdot \boldsymbol{h}_m)}$$

where ϕ is a matrix of word embeddings, and $\boldsymbol{x}_m$ is the word embedding for w_m

S s

Sequence-to-sequence models

- Sequence goes in, sequence comes out
- Sequence-to-sequence models are a powe framework that have found success in a wide ra applications
  - Autom goes in, text com
  - Machin ce goes in, target language sentence comes out
  - Image captioning: Image goes in, capt
  - text summarization: whole text goes in, summary comes out
  - Automatic email responses: Generating automatic responses to incoming emails
  - etc. etc.

The encoder decoder architecture

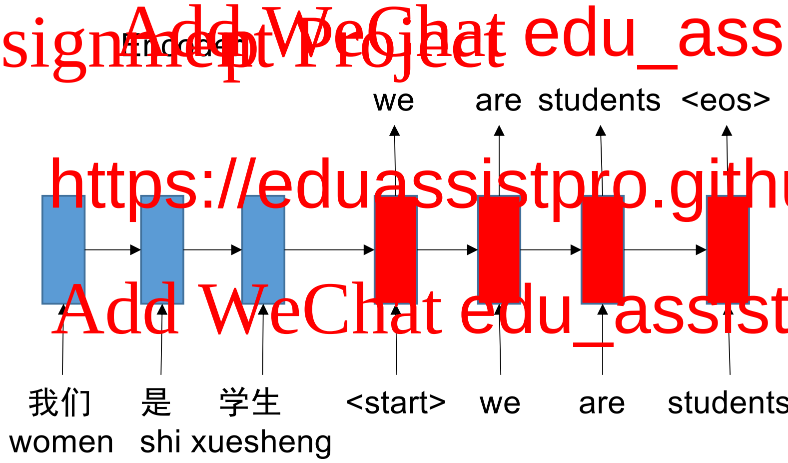
- The encoder netw
vector or a matrix representation; the decoder network then converts the encoding into a sentence in the target language

$$\boldsymbol{z} = \text{Encode}(\quad)$$

where the second line means the decoder d
conditional probability $P(\boldsymbol{w}^{(t)}|$

- The decoder is typically a recurrent neural network (e.g., LSTM) that generates one word at a time, while recurrently updating a hidden state.
- The encoder decoder networks are trained end-to-end from parallel sentences.

Encoder decoder



Training objective

If the output layer of the decoder is a logistic function, then the entire network can be trained to maximize the conditional log-likelihood (or minimize the negative log-likelihood):

$$\log \ldots \sum^{M^{(t)}} \ldots {}_{-1}, \boldsymbol{z})$$

$$w_m^{(t)} \mid \boldsymbol{w}_{1:m-1}^{(t)}, \boldsymbol{w}^{(s)} \sim \mathrm{Sof}\ldots\Big)$$

where $\boldsymbol{h}_{m-1}^{(t)}$ is a recurrent function of the previously generated text $\boldsymbol{w}_{1:m-1}^{(t)}$ and the ecoding $\boldsymbol{z}$, and $\boldsymbol{\beta} \in \mathbb{R}^{(V^{(t)} \times K)}$ is the matrix of output word vectors for the $V^{(t)}$ words in the target language vocabulary

The LSTM variant

- In the LSTM varia[illegible] set to the final hidde[illegible] sentence:

$$\boldsymbol{h}^{(s)} = \text{LSTM}(\boldsymbol{x}^{(} \text{[illegible]}$$

where $\boldsymbol{x}^{(s)}$ is the embedding of the sour[illegible] $w_m^{(s)}$.

- The encoding then provides the initial hi[illegible] decoder LSTM:

$$\boldsymbol{h}_0^{(t)} = \boldsymbol{z}$$
$$\boldsymbol{h}_m^{(t)} = \text{LSTM}(\boldsymbol{x}_m^{(t)}, \boldsymbol{h}_{m-1}^{(t)})$$

where $\boldsymbol{x}_m^{(t)}$ is the embedding of the target language word $w_m^{(t)}$

Tweaking the encoder decoder network

- Adding layers: The encoder and decoder net implemented as **deep LSTMs** wit den state
- Adding **att** ord or words in the source language when gener target language

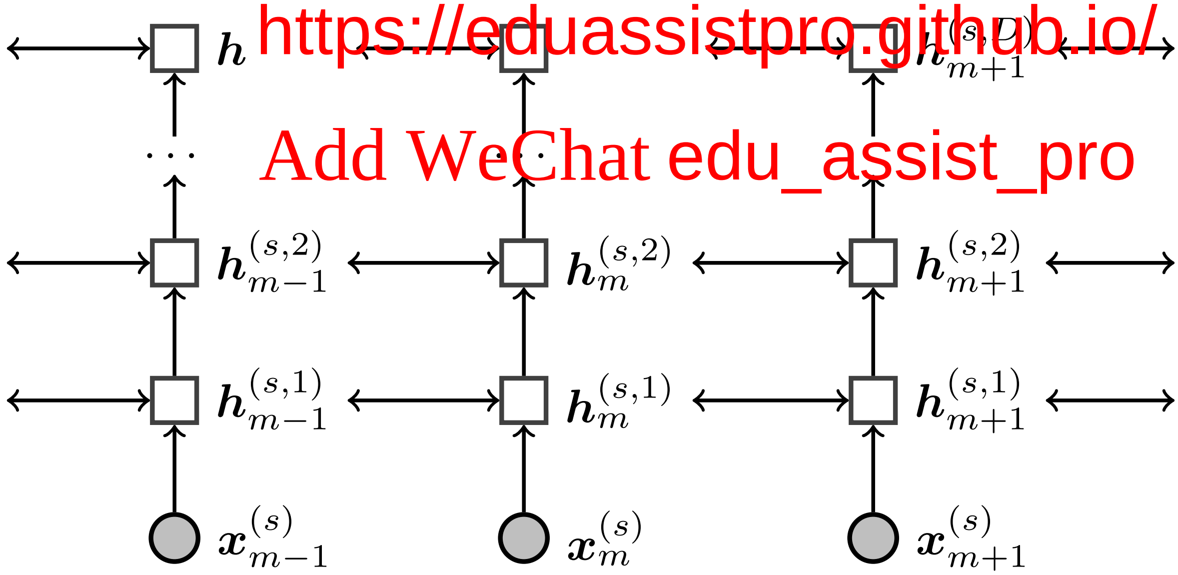
Multi-layered LSTMs

Each hidden state

LSTM at layer $i+1$

$$h_m^{(s,1)} = \text{LSTM}(x_m^{(s)}, h_{m-1}^{(s)})$$

$$h_m^{(s,i+1)} = \text{LSTM}(h_m^{(s,i)}, h^{(s,i+1)}$$



Neural attention

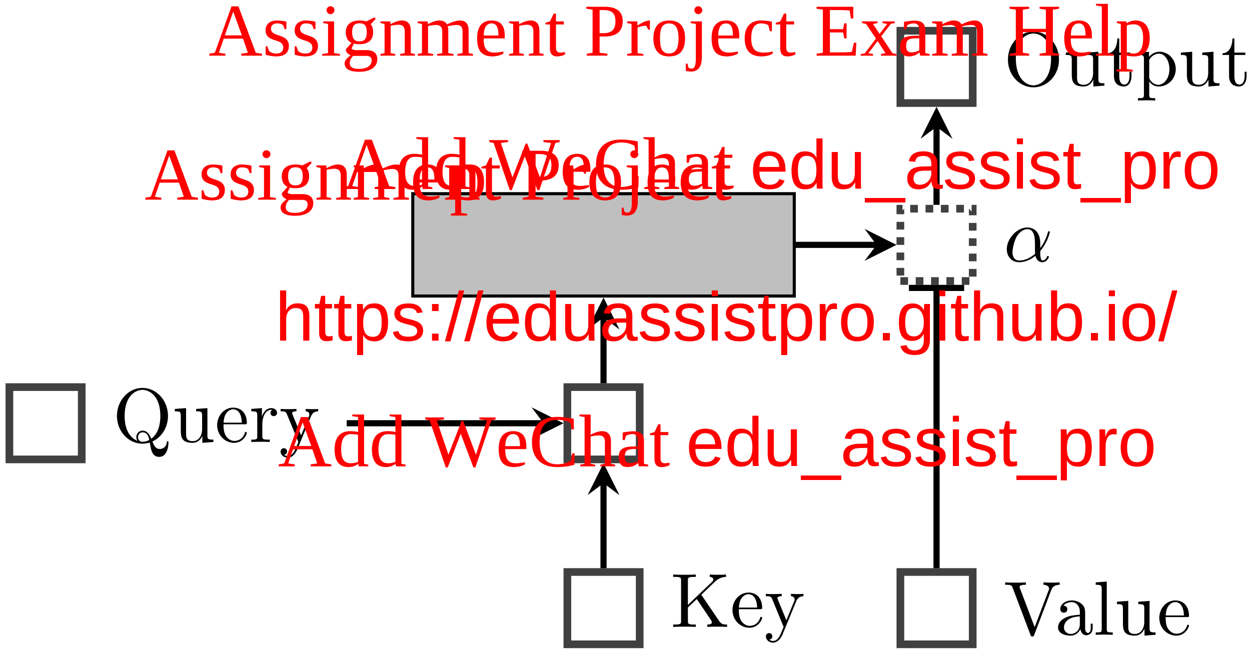
- Attention can be t
memory of key-value pairs, with the keys, values and queries all being vectors

- For each key n in the memory, we comp
respect to the query m, which measur y"
between th

- The scores are pically
softmax, which results in a vector of non-n f
length N, which equal to the size of the me
$[\alpha_{m \to 1}, \alpha_{m \to 2}, \cdots, \alpha_{m \to N}]$

- Multiply each value in the memory v_n by the attention $\alpha_{m \to n}$, and sum them up, we get the output of the attention.
- The attention is typical concatenated with the decoding hidden state to output the target word

“Querying”



https://eduassistpro.github.io/
Add WeChat edu_assist_pro

Step by step computation of attention

- Computing cometwork:

$$\psi_\alpha(m,n) = \boldsymbol{v}_\alpha \cdot \tanh(\boldsymbol{\Theta}_\alpha[\boldsymbol{h}_m ; \boldsymbol{h}_n])$$

- Softmax attention

$$\alpha = \frac{\exp \psi}{\quad ')}$$

- Compute the Z

$$\boldsymbol{c}_m = \sum_{n=1}^{M^{(s)}} \quad {}_{m\to n} \quad {}_{n}$$

- incorporate the context vector into the decoding model:

$$\tilde{\boldsymbol{h}}_m^{(t)} = \tanh\left(\Theta_c[\boldsymbol{h}_m^{(t)}, \boldsymbol{c}_m]\right)$$

$$P(w_{m+1}^{(t)} | \boldsymbol{w}_{1:m}^{(t)}, \boldsymbol{w}^{(s)} \propto \exp\left(\boldsymbol{\beta}_{w_{m+1}^{(t)}} \cdot \tilde{\boldsymbol{h}}_m^{(t)}\right)$$

Seq2seq: Initialization

- Word embeddin

$$\boldsymbol{E} = \mathit{embed}\left(\begin{bmatrix} \mathit{women} \\ \mathit{shi} \\ \mathit{xuesheng} \\ \mathit{START} \\ \\ \\ \\ \mathit{EOS} \end{bmatrix}\right) = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \\ & .5 \\ 0 & \\ 1 & 0.2 \\ 2 & 0.1 \\ & 0.2 \\ & 1 \end{bmatrix}$$

- RNN parameters

$$\boldsymbol{W}^{(s)} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix} \boldsymbol{U}^{(s)} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \boldsymbol{b}^{(s)} = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$$

$$\boldsymbol{W}^{(t)} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix} \boldsymbol{U}^{(t)} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \boldsymbol{b}^{(t)} = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}$$

Encoder

$$z \triangleq \boldsymbol{h}^{(s)}$$

$$\boldsymbol{h}_n^{(s)} = \tanh(\boldsymbol{W}^{(s)} \times \boldsymbol{x} + \boldsymbol{U}^{(s)} \times \boldsymbol{h} + \boldsymbol{b})$$

$$\boldsymbol{h}_1^{(s)} = \tanh(\boldsymbol{W}^{(s)} \ldots \boldsymbol{E}[women] + \boldsymbol{U}^{(s)} \ldots \begin{bmatrix} \ldots.3364 \\ 0.5717 \end{bmatrix}$$

$$\boldsymbol{h}_2^{(s)} = \tanh \; \boldsymbol{W}^{(} \ldots = \begin{bmatrix} 0.3153 \\ 0.7289 \end{bmatrix}$$

$$\boldsymbol{h}_3^{(s)} = \tanh(\boldsymbol{W}^{(s)} \times \boldsymbol{E}[xuesheng] + \ldots_2^{(s)}) = \begin{bmatrix} 0.0086 \\ 0.5432 \end{bmatrix}$$

$$C = \boldsymbol{h}_3^{(s)} = \begin{bmatrix} 0.0086 \\ 0.5432 \end{bmatrix}$$

where C is a context vector

Decoder

$$\boldsymbol{h}_m^{(t)} = \tanh(\boldsymbol{W}^{(t)} \times \boldsymbol{x} + \boldsymbol{U}^{(t)} \times \boldsymbol{h} + \boldsymbol{b})$$

$$\boldsymbol{h}_1^{(t)} = \tanh(\boldsymbol{W}^{(t)} \times \boldsymbol{E}[START] + \boldsymbol{U}^{(t)} \quad \begin{bmatrix} 0.6929 \\ 0.2482 \end{bmatrix}$$

$$\boldsymbol{h}_2^{(t)} = \tanh \ \boldsymbol{W} \quad = \begin{bmatrix} 0.6203 \\ 0.2123 \end{bmatrix}$$

$$\boldsymbol{h}_3^{(t)} = \tanh(\boldsymbol{W}^{(t)} \times \boldsymbol{E}[are] + \boldsymbol{U}^{(t)} \quad \begin{bmatrix} 0.6039 \\ 0.1870 \end{bmatrix}$$

$$\boldsymbol{h}_4^{(t)} = \tanh(\boldsymbol{W}^{(t)} \times \boldsymbol{E}[students] + \boldsymbol{U}^{(t)} \times \boldsymbol{h}_3^{(t)} + \boldsymbol{b}^{(t)}) = \begin{bmatrix} 0.6220 \\ 0.0980 \end{bmatrix}$$

Softmax over similarities between hidden layers and target embeddings

$$score_1\left(\begin{bmatrix} we \\ are \\ students \\ EOS \end{bmatrix}\right) = softmax\left(\begin{bmatrix} \boldsymbol{h}_1^{(t)} \times \boldsymbol{E}[we] \\ \boldsymbol{h}_1^{(t)} \times \boldsymbol{E}[are] \\ \boldsymbol{h}_1^{(t)} \times \boldsymbol{E}[students] \\ \boldsymbol{h}_1^{(t)} \times \boldsymbol{E}[EOS] \end{bmatrix}\right) = \begin{bmatrix} 0.1913 \\ 0.2000 \\ 0.1733 \\ 0.4354 \end{bmatrix}$$

$$score_2\left(\begin{bmatrix} we \\ are \\ students \\ EOS \end{bmatrix}\right) = softmax\left(\begin{bmatrix} \boldsymbol{h}_2^{(t)} \times \boldsymbol{E}[we] \\ \boldsymbol{h}_2^{(t)} \times \boldsymbol{E}[are] \\ \boldsymbol{h}_2^{(t)} \times \boldsymbol{E}[students] \\ \boldsymbol{h}_2^{(t)} \times \boldsymbol{E}[EOS] \end{bmatrix}\right) = \begin{bmatrix} 0.1999 \\ 0.2070 \\ 0.1826 \\ 0.4116 \end{bmatrix}$$

$$score_3\left(\begin{bmatrix} we \\ are \\ students \\ EOS \end{bmatrix}\right) = softmax\left(\begin{bmatrix} \boldsymbol{h}_3^{(t)} \times \boldsymbol{E}[we] \\ \boldsymbol{h}_3^{(t)} \times \boldsymbol{E}[are] \\ \boldsymbol{h}_3^{(t)} \times \boldsymbol{E}[students] \\ \boldsymbol{h}_3^{(t)} \times \boldsymbol{E}[EOS] \end{bmatrix}\right) = \begin{bmatrix} 0.2012 \\ 0.2098 \\ 0.1867 \\ 0.4023 \end{bmatrix}$$

$$score_4\left(\begin{bmatrix} we \\ are \\ students \\ EOS \end{bmatrix}\right) = softmax\left(\begin{bmatrix} \boldsymbol{h}_4^{(t)} \times \boldsymbol{E}[we] \\ \boldsymbol{h}_4^{(t)} \times \boldsymbol{E}[are] \\ \boldsymbol{h}_4^{(t)} \times \boldsymbol{E}[students] \\ \boldsymbol{h}_4^{(t)} \times \boldsymbol{E}[EOS] \end{bmatrix}\right) = \begin{bmatrix} 0.2037 \\ 0.2147 \\ 0.1959 \\ 0.3857 \end{bmatrix}$$

$$P(Y|X) = score_1 \times score_2 \times score_3 \times score_4$$

Attention

- The idea: Differen… sed when generating targ…

$$
\begin{aligned}
C_1 &= 0.98 \times \boldsymbol{h}_1^{(s)} + 0.01 \times \boldsymbol{h}_2^{(s)} + 0.01 \times \boldsymbol{h}_3^{(s)} \\
C_2 &= 0.01 \times \boldsymbol{h}_1^{(s)} + 0.98 \times \boldsymbol{h}_2^{(s)} + 0.01 \times \boldsymbol{h}_3^{(s)} \\
C_3 &= 0.01 \times \boldsymbol{h}_1^{(s)} + 0.01 \times \boldsymbol{h}_2^{(s)} + 0.98 \times \boldsymbol{h}_3^{(s)}
\end{aligned}
$$

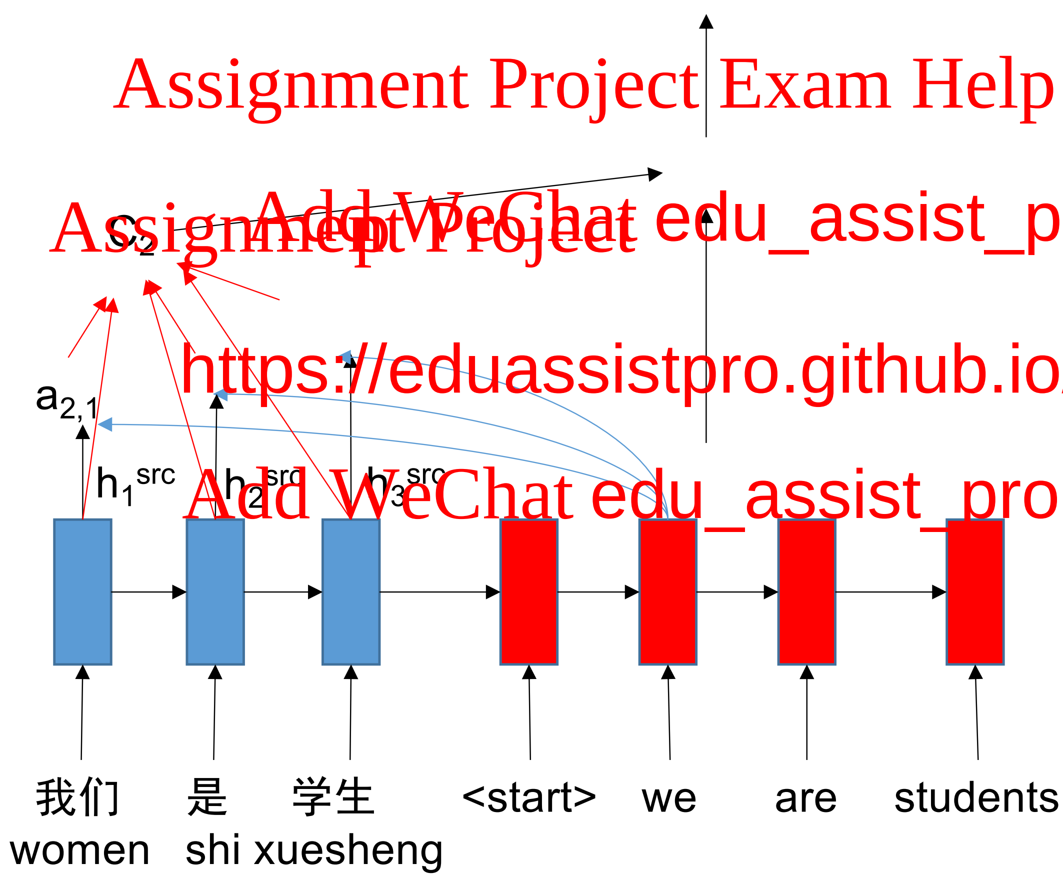
$$
C_m = \sum_n \alpha_{m \to n} \times \boldsymbol{h}_n^{(s)}
$$

$$
\alpha_{m \to n} = \frac{\exp(score(\boldsymbol{h}_m^{(t)}, \boldsymbol{h}_n^{(s)}))}{\sum_{n'=1} \exp(score(\boldsymbol{h}_m^{(t)}, \boldsymbol{h}_{n'}^{(s)}))}
$$

$$
score(\boldsymbol{h}_m^{(t)}, \boldsymbol{h}_n^{(s)}) = \boldsymbol{h}_m^{(t)\top} \boldsymbol{h}_n^{(s)}
$$

- Other scoring variants exist

Computing attention



Other attention variants

- Additive attention:

$$\psi_{\alpha}(m, n) = \boldsymbol{v}_{\alpha} \quad \tanh(\quad {}_{\alpha} \quad {}_{m} \qquad {}_{n}$$

- Multiplicative attention

$$\psi_{\alpha}(m, n) = \boldsymbol{h}_{m} \quad \boldsymbol{\Theta}_{\alpha} \boldsymbol{h}_{n}$$

Drawbacks of RNNs

- For RNNs, input is of each state ($\boldsymbol{h}_i$) depends on the previous state $\boldsymbol{h}_{i-1}$.
- This prevents parallel computation for all tokens in the input sequence simultaneously, making it diffic advantage of modern computation archit speed.
- We can imagi sequence int Conceptually, this can be viewed as a fully where each token is a node in the graph, and t of its hidden state depends on all other tokens in the graph.
- With this approach, the computation of the hidden state of a hidden state $\boldsymbol{h}_i$ does not depends on the computation of another hidden state. It only depends on the input sequence.
- With this approach, the order information would have to be captured separately, with position encoding.