

Viewing Transform

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Intended Learning Outcomes

- Able to set up a camera coordinate system
- Understand the properties of different projection methods
- Able to set up t atrices and use appropriate Op https://eduassistpro.github.io/ lize the projection
- Describe the operation and f Add WeChat edu_assist_pro lipping

Image generation process

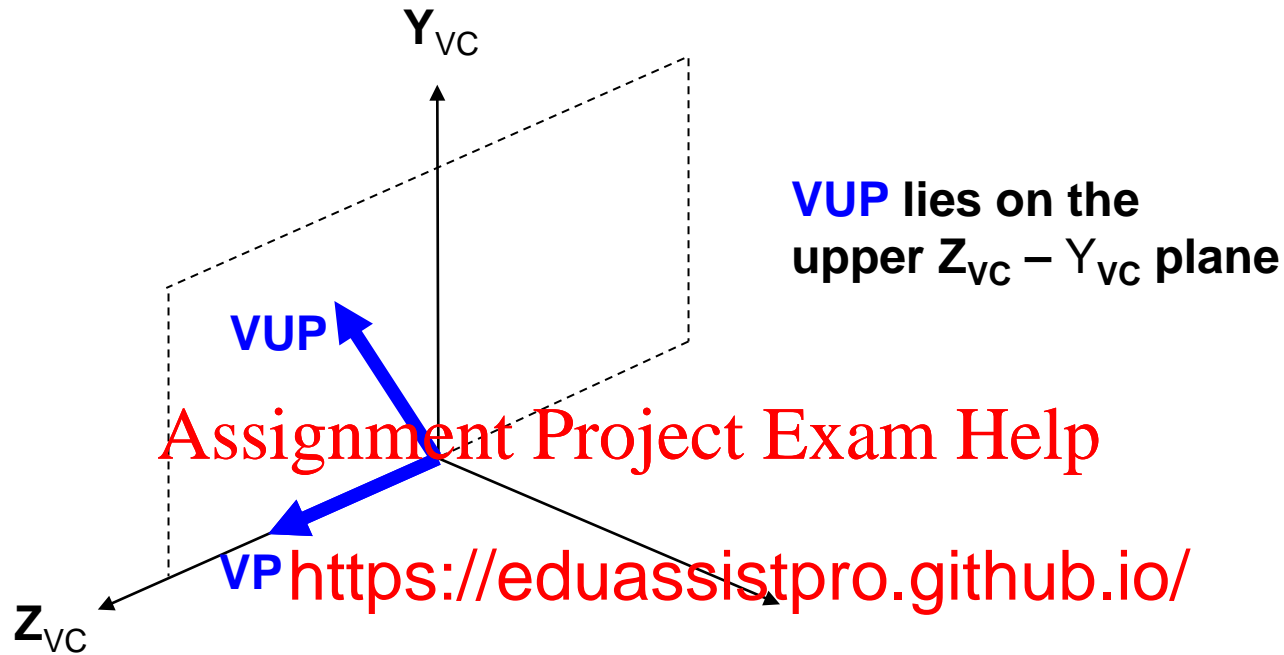
- A camera has its own coordinate system $\mathbf{X}^{(VC)}-\mathbf{Y}^{(VC)}-\mathbf{Z}^{(VC)}$, called the viewer coordinate system
- viewer coordinate system is alternatively called camera coordinate system
- To generate a viewing transformation, we first define a camera, then transform the world coordinate system (WC) to viewing coordinate system (VC)

$$\mathbf{X}^{(WC)}-\mathbf{Y}^{(WC)}-\mathbf{Z}^{(WC)} \rightarrow \mathbf{X}^{(VC)}-\mathbf{Y}^{(VC)}-\mathbf{Z}^{(VC)}$$

- Then projecting each point to a view plane

To specify a viewer coordinate system, we need to specify three vectors

- View Reference Point (**VRP**): origin of the viewing coordinate system (i.e. physical location of the camera)
 - View Plane Normal (**VPN**): direction of the viewing axis of the camera
 $X^{(VC)}-Y^{(VC)}-Z^{(VC)}$
 - View UP Vector (**VUP**): upward direction for the film (i.e. direction of the film plane)
- Note 1: These vectors do not need to be unit vector
- Note 2: These vectors are in WC



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$\mathbf{Z}_{VC} = \mathbf{VPN} $	(unit vector in WC)
$\mathbf{X}_{VC} = \mathbf{VUP} \times \mathbf{VPN} $	(unit vector in WC)
$\mathbf{Y}_{VC} = \mathbf{Z}_{VC} \times \mathbf{X}_{VC}$	(unit vector in WC)

Note: $| \quad |$ is used in the notes to denote normalization to unit vector

Transformation from WC to VC

- $\mathbf{P}^{(VC)} = \mathbf{M}_{VC \leftarrow WC} \mathbf{P}^{(WC)}$

- Applying coordinate system transformation method 1:

$$\mathbf{M}_{VC \leftarrow WC} = \begin{pmatrix} \mathbf{X}_{VC} & \mathbf{Y}_{VC} & \mathbf{Z}_{VC} & \mathbf{VP} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Note: \mathbf{X}_{VC} , \mathbf{Y}_{VC} , \mathbf{Z}_{VC} are unit column vector in WC

OpenGL commands

- *glMatrixMode (GL_MODELVIEW);*
- *gluLookAt (x0, y0, z0, xref, yref, zref, Vx, Vy, Vz);*

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- **VRP** = (x0, y0,
- **VPN** = (x0, y0, <https://eduassistpro.github.io/>
- **VUP** = (Vx, Vy, Vz) [Add WeChat edu_assist_pro](#)
- To remember this, it is convenient to remember (x0, y0, z0) as **where the camera is placed**, (xref, yref, zref) as **where the center of the scene is**, and (Vx, Vy, Vz) as a vector that tells **where it's up for the camera**

View Plane

- Also called projection plane or image plane
- It is usually a plane defined by $Z_{VC} = \text{constant}$, i.e., parallel to the $X_{VC} - Y_{VC}$ plane
- As the name indicates, the coordinates of the point (X, Y, Z) in viewer coordinates is projected onto the view plane
- i.e. 3D becomes 2D

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Projections : project $(X, Y, Z)^{(VC)}$ to $(x, y)^{(VC)}$

- Two general types: Parallel and Perspective Projections

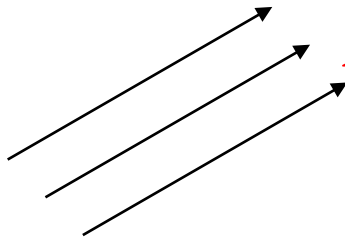
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Parallel project

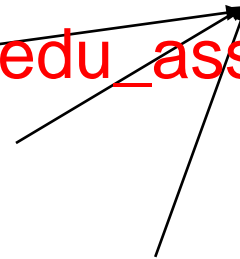
e projection

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all light rays are parallel



all light rays converge on a common point called projection reference point (PRP)

Parallel projection can be considered as the special case of perspective projection when $PRP = \infty$

Different properties

Parallel projection

coordinate positions are transformed to the
projection plane along
i.e. center of projection

preserves relative proportions

use in engineering drafting

Perspective Projection

coordinate positions are transformed to the
plane along lines that converge
towards the center of projection
(Reference Point)

does not preserve

use in realistic views

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Parallel Projections

- Specify a *projection vector* – a direction vector
- Two types:

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- *Orthographic* projection plane vector \perp projection plane
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- *Oblique* projection plane Add WeChat edu_assist_pro
projection - projection vector not \perp to projection plane

Orthographic Projection ($\alpha = 90^\circ$)

- Two types:
- *Front elevation, side elevation, rear elevation, plan view*
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Y-Z is shown
- *Isometric projection* - pr vector = $(\pm 1, \pm 1, \pm 1)$
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 - For a cube, each side will be displayed equally
 - The 8 possibilities corresponds to viewing in the 8 octants

Front elevation, side elevation, rear elevation, plan view

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front elevation view

Isometric projection

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Oblique Projection ($\alpha \neq 90^\circ$)

- Two types:
- *Cavalier projection* - projection vector makes an angle α of $\tan^{-1}1$ with p
 - for a cube, length of X axis, Y axis will remain the same; length of Z axis will be halved.
- *Cabinet projection* - projection vector makes an angle α of $\tan^{-1}2$ with projection plane
 - for a cube, length of X axis, Y axis will remain the same; length of Z axis will be halved.

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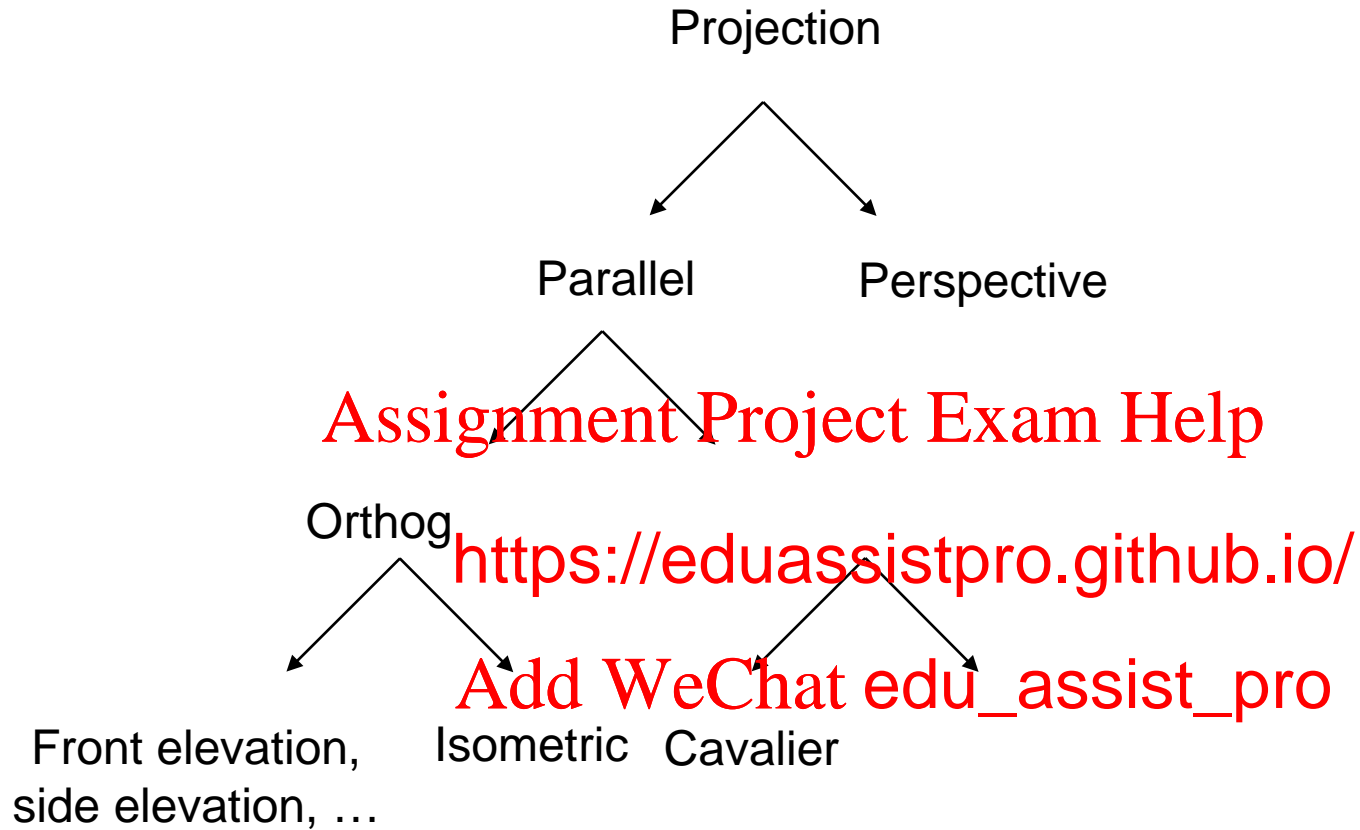
Cavalier Projection

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Cabinet Projection

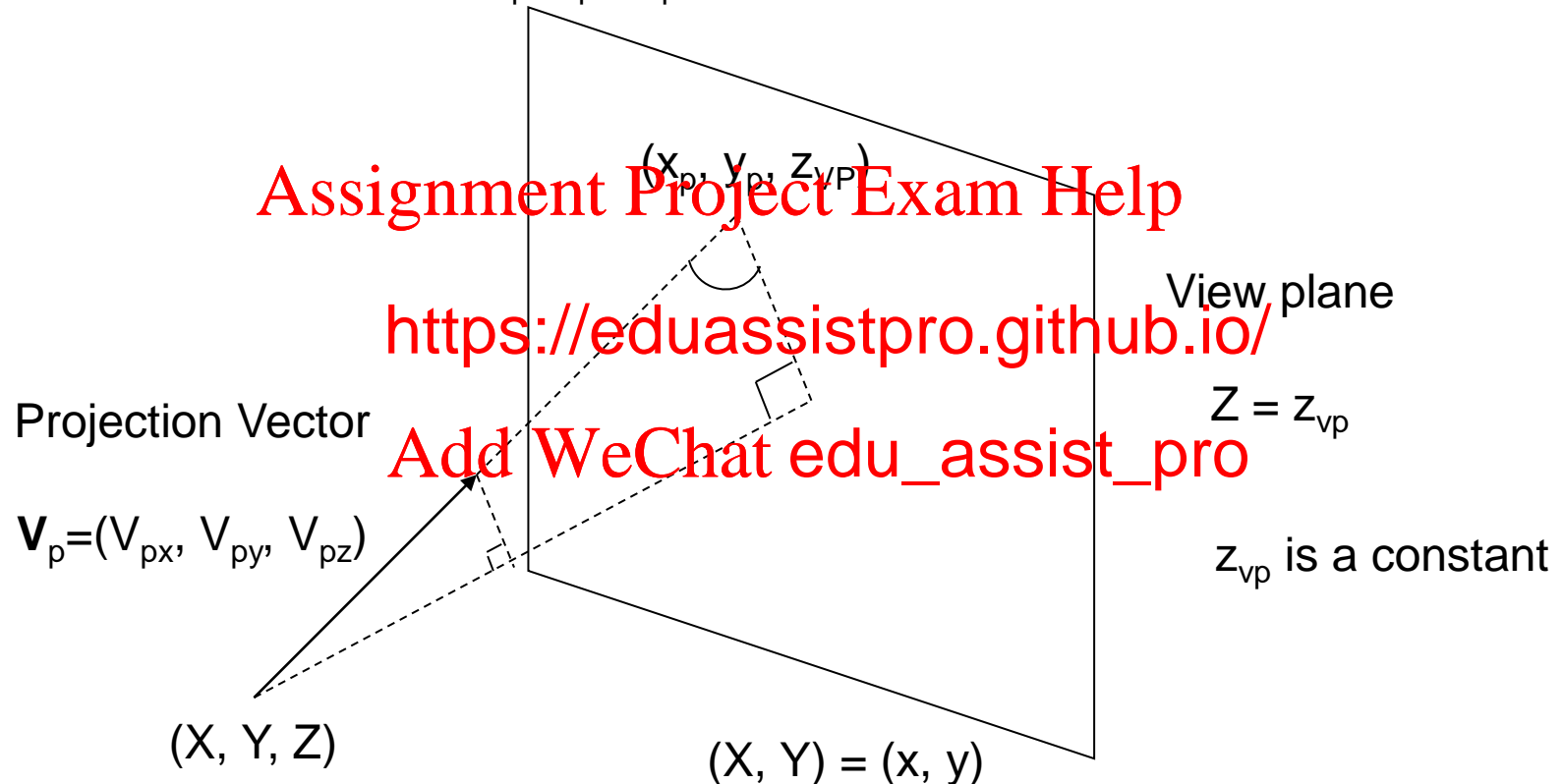


4 x 4 Transform for Parallel Projection

All quantities in this slide are already in VC

$$\mathbf{P}^{(VC)} = (X, Y, Z)$$

$$\mathbf{P}^{(im)} = (x_p, y_p, z_{vp})$$



$\alpha = 90^\circ$ Orthographic projection

$\alpha \neq 90^\circ$ Oblique projection

By similar triangles,

$$\frac{x_p - X}{z_{vp} - Z} = \frac{V_{px}}{V_{pz}}$$

$$\frac{y_p - Y}{z_{vp} - Z} = \frac{V_{py}}{V_{pz}}$$

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Rearranging, Add WeChat edu_assist_pro

$$x_p = X + (z_{vp} - Z) \frac{V_{px}}{V_{pz}} \quad (1)$$
$$y_p = Y + (z_{vp} - Z) \frac{V_{py}}{V_{pz}}$$

- $\mathbf{P}^{(im)} = (x_p, y_p, z_{vp}, 1)$ $\mathbf{P}^{(VC)} = (X, Y, Z, 1)$
- $\mathbf{P}^{(im)} = \mathbf{M}_{parallel} \mathbf{P}^{(VC)}$

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$$\mathbf{M}_{parallel} = \begin{pmatrix} 0 & 1 & -\frac{V_{px}}{V_{pz}} & \frac{V_{py}}{V_{pz}} \\ 0 & 0 & 0 & z_{vp} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Verify that the first two rows implement eqn (1) above
- The third row is such that the projected point is at $Z = z_{vp}$. However, this is not maintained; in OpenGL, $\mathbf{p}^{(im)} = (x_p, y_p, Z)$, i.e. the original Z is kept for depth tests

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- Third row of eqn (10-13), pg. 350 of text is set to $0 \ 0 \ 1 \ 0$, which has maintaining the original Z .

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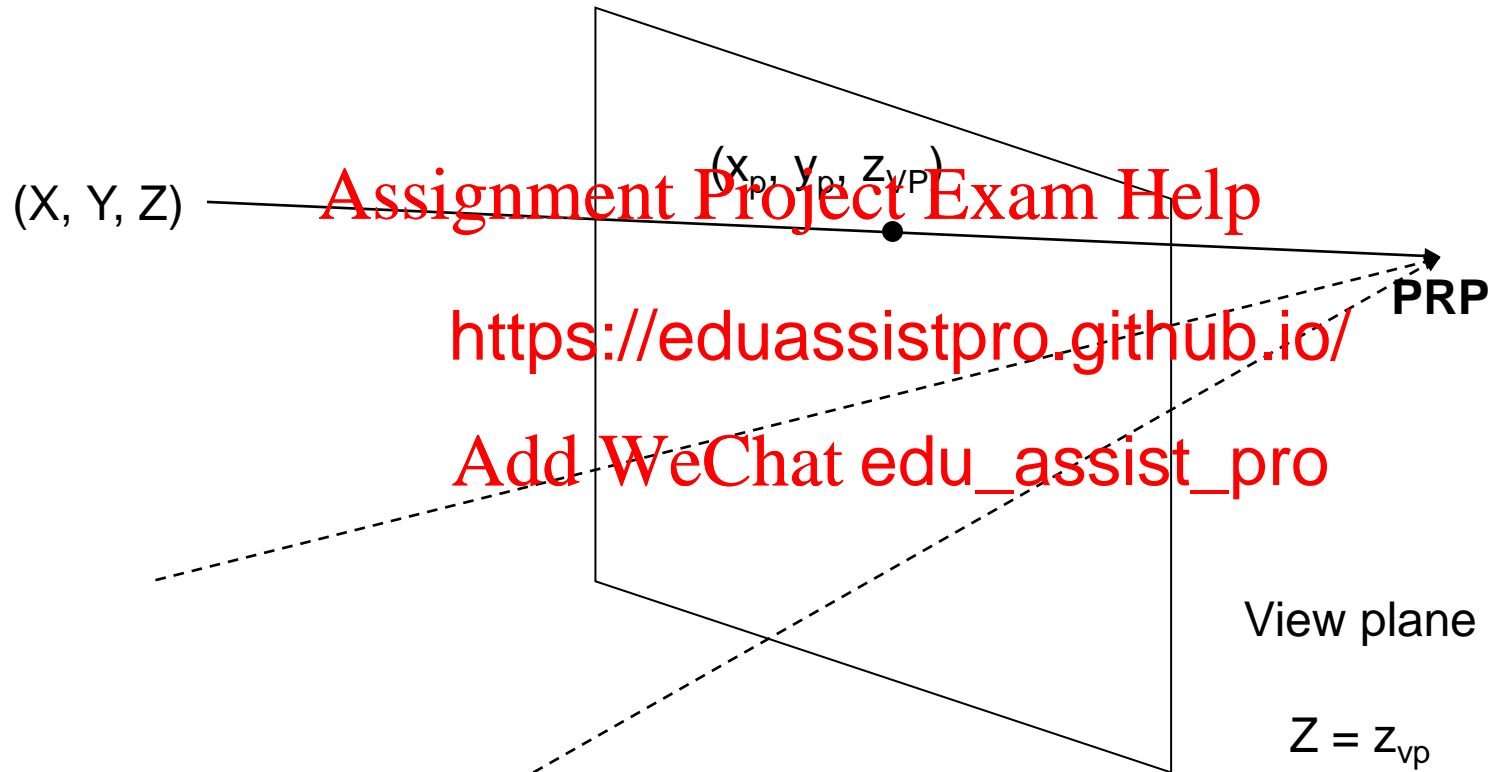
Perspective Projection

- **ALL** light rays goes through the Projection Reference Point (**PRP**), also called center of projection.

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Example:

i) **PRP** = **VRP**

ii) $Z = z_{vp}$ is the view plane

By similar triangles,

$$\frac{x_p}{z_{vp}} = \frac{X}{Z}$$

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Multiplying each side by z_{vp} yields

$$x_p = \frac{z_{vp} \cdot X}{Z} = \frac{X}{Z / z_{vp}} \quad (2)$$

$$y_p = \frac{z_{vp} \cdot Y}{Z} = \frac{Y}{Z / z_{vp}}$$

- $\mathbf{P}^{(VC)} = (X, Y, Z, 1)$

- $\mathbf{P}^{(im)} = \mathbf{M}_{\text{perspective}} \mathbf{P}^{(VC)}$

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$$\mathbf{M}_{\text{perspective}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/z_{vp} & 0 \end{bmatrix}$$

- Verify that the first two rows and the fourth row implements eqn (2) above using homogeneous coordinates operation
- Note that the f anymore
- The third row i point is at $Z =$
 z_{vp} . However, this is not mai penGL, $\mathbf{p}^{(im)} =$
 (x_p, y_p, Z) , i.e. the original Z is pth tests

Clipping

- Any object not within the clipping volume does not need to be processed – this eliminates most of the objects at one go
- For a convex c by planes, one can check whether the signs of the plane equations (2).

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OpenGL – first set matrix mode

- `glMatrixMode (GL_PROJECTION);`
- Note: `GL_PROJECTION` is used as it deals with projection
- There are two transformation matrices: `GL_MODELVIEW` and `GL_PROJECTION`
- A point is pre-multiplied by

`[GL_PROJECTION] [GL_MODELVIEW]`

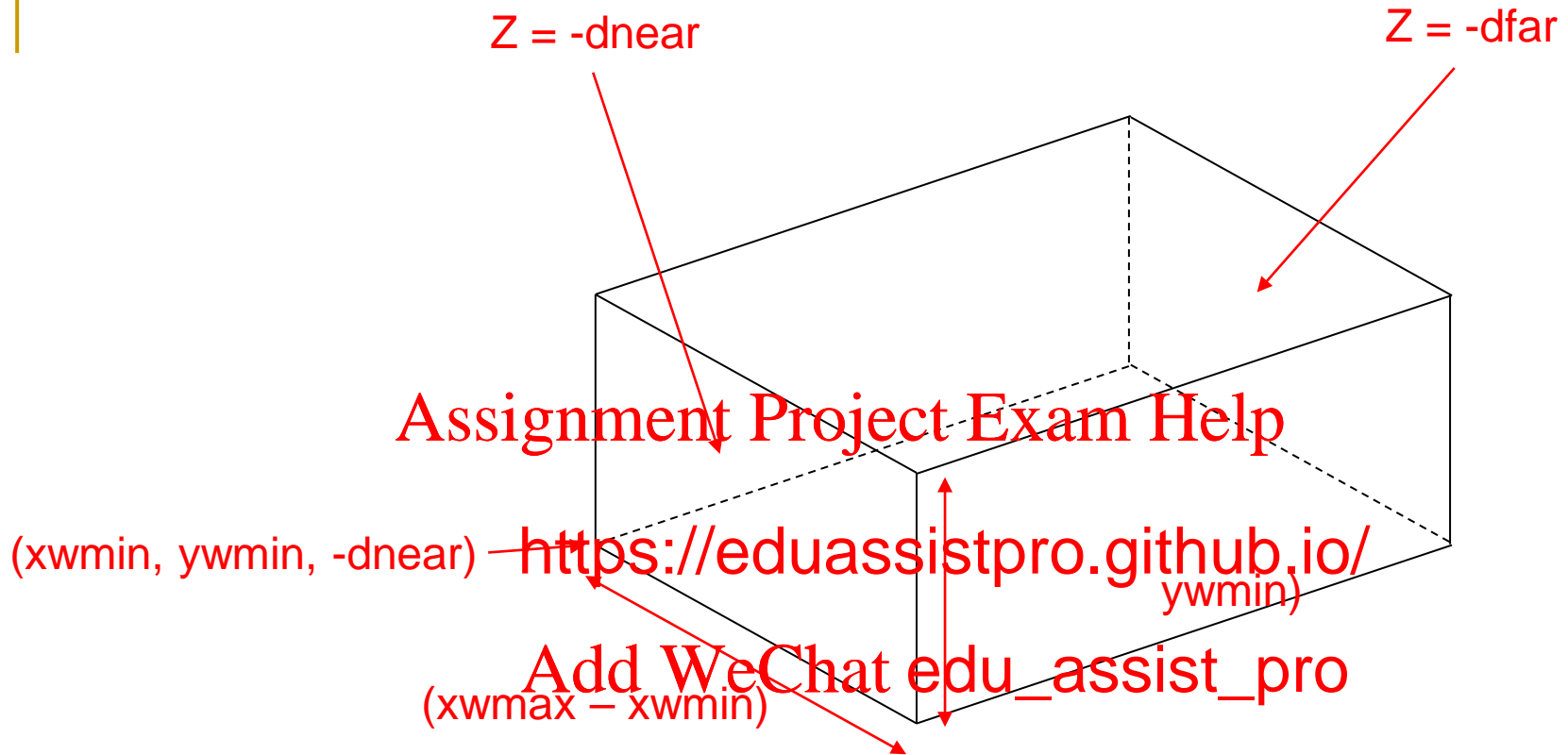
- `glOrtho` and `gluPerspective` commands may be used

OpenGL – Orthographic projection

- *glOrtho* (*xwmin*, *xwmax*, *ywmin*, *ywmax*, *dnear*, *dfar*)

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- Projection v
- Clipping plane <https://eduassistpro.github.io/>
- Near clipping plane $Z = -d$ serve as the view plane
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- Only points whose X and Y are in $|xwmin, xwmax|$ and $|ywmin, ywmax|$ respectively are displayed
- Clipping volume is a rectangular box



Only objects inside the rectangular shaped clipping volume is further processed

OpenGL – Perspective projection

- *gluPerspective* (*theta*, *aspect*, *dnear*, *dfar*)
 - ❑ **PRP = VRP**
 - ❑ $Z = -dnear$ (e –ve sign)
 - ❑ *dnear* and *dfar* are clipping planes
 - ❑ $Z = -dnear$ and $Z = -dfar$
 - ❑ *theta* is the angle of view
 - ❑ *aspect* = (width /height)
 - ❑ *theta* and *aspect* together determines size of image window
-
- ❑ clipping volume is a frustum

$Z = -d_{near}$

$Z = -d_{far}$

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VRP <https://eduassistpro.github.io/>

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aspect = width / height of the blue plane

Only objects inside the frustum shaped clipping volume is further processed

References

- Text : Ch. 10.2–10.7 discusses the viewing transform and the various types of projection
- Text : Ch. 10.8 discusses general perspective projection and then discusses the special case of orthographic projection. We only discuss the special case of orthographic projection.
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- Text : Ch. 10.9–10.10 discuss OpenGL commands