3D Object Representation

Intended Learning Outcomes

- Understand the concept of standard graphics object
- Able to mathematically manipulate and program in OpenGL two types of planar representation: tables and mesh
- Distinguish the and non-parametric equ the advantage of using the former in computer
- Able to mathematically manip rogram in OpenGL quadrics and super-quadrics

Standard Graphics Object

- **standard graphics object = a set of (planar) polygons**

- Complicated obj ing many polygons

- Dedicated hardware are designed rendering of standard graphics objects.

Two methods for storing standard graphics objects

- Method 1: use table (vertex, edge, polygon, attribute)

Method 2: Quadrilateral Mesh

- A n x m array of vertex positions (X, Y, Z)
- Represent a surface of (n-1) x (m-1) quadrilaterals
- Each quadrilateral may be further subdivided into two triangle
- Two ways t esh
  - Way 1: By specifying an e
  - Way 2: By 3D digitizer

3-D scanner

3D d y triangulation

3D scanner is available in CityU Library:

http://www.cityu.edu.hk/lib/about/facility/3d/index.htm

Glut functions

- *glutWire* as wireframe
- *glutSolid* as fill area polygon patches

glutSolidCu

- Tetrahedron, Cube, Octahedron, Dodecahedron, Icosahedron

Mathematical Concepts for Plane

- Plane

$$aX + bY + cZ + d = 0$$

- Only 3 parameters define t the fourth can be set to 1 or 0
- d = 1 does not pass through (0, 0, 0)
- d = 0 pass through (0, 0, 0)

Normal

- Important concept in lighting and shading
- Normal vector
  - vector ⊥ to t
  - "Unit vector"
- Solving for Normal
  - Normal **n** = (a, b, c)
  - Select 3 vertices on the plane **V1**, **V2**, **V3**

$$\mathbf{n} = (V2 - V1) \times (V3 - V1)$$

Distinguishing "Inside" from "Outside"

- Useful for "collision detection"

- Use (a, b, c

aX+bY+cZ = 0 : Outside on the plane

< 0 : side

- Use **V1**, **V2**, **V3**

V1, **V2**, **V3** selected CCW => Outside

CW => Inside

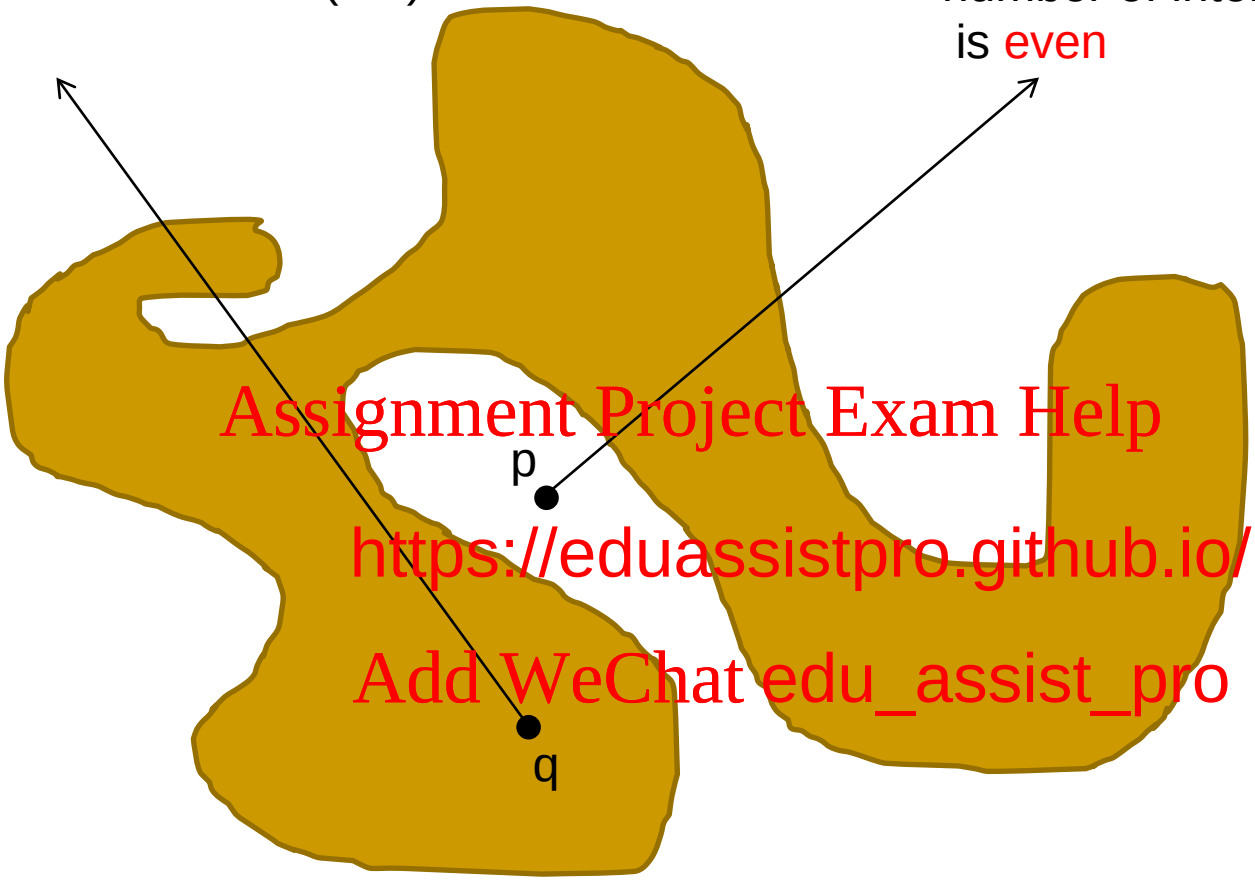
Inside-Outside Test

- To determine whether a pixel ***p*** is inside or outside an object ***S***
- Send a ray $p + t\,v$ which starts at the pixel, t is a scalar, ***v*** is an arbitrary direction vector
- Find all non-degenerate† intersections between the ray and ***S***
- *If the number o ven), **p** is inside (outside) **S***
- It is not easy to check non-degenerate intersections. One can solve this problem by sending out n rays in random directions and then use majority voting

† a degenerate intersection is one which the ray grazes the surface

Point q is inside as the number of intersections (= 3) is odd

Point p is outside as the number of intersections (= 2) is even



The yellow object is depicted as a 2D object but the technique can be applied to any n-dimensional object (n > 2)

Superquadrics

- 2D QUADRICS (conic section)

$$aX^2 + bY^2 + cXY + dX + eY + f = 0$$

- 3D QUADRICS

$$aX^2 + bY^2 + cZ^2 + dXY + eXZ + fYZ + gX + hY + iZ + k = 0$$

In 2D,

- Circle $X^2 + Y^2 = r^2$
- Ellipse $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$
- Parabola $Y^2 = 4aX$
- Hyperbola $X^2 - Y^2 = r^2$

In 3D

- Sphere $X^2 + Y^2 + Z^2 = r^2$
- Ellipsoid $\left(\frac{X}{r}\right)^2 + \left(\frac{Y}{r}\right)^2 + \left(\frac{Z}{r}\right)^2 = 1$
- Paraboloid
- Hyperboloid ?

(ans. to be discussed in tut.)

“Super”-quadrics

- Introduce to additional parameters s1 and s2
- Allow continuous transformation from “circle” to “square” (1 align)
- Example (2D)

$$\left(\frac{X}{a}\right)^{\frac{2}{s}} + \left(\frac{Y}{b}\right)^{\frac{2}{s}} = 1$$

Super-ellipsoid

$$\left[\left(\frac{X}{r_x}\right)^{2/s_2} + \left(\frac{Y}{r_y}\right)^{2/s_2}\right]^{s_2/s_1} + \left(\frac{Z}{r_z}\right)^{2/s_1} = 1$$

Non-parametric and Parametric forms

- Non-parametric form
  - Z = f(X, Y) or f(X, Y, Z) = 0
  - Used in mathematics
- Parametric form
  - Introduced two additional parameters u, v
  - X = f1 (u, v) Y = f2 (u, v) Z = f3 (u, v)
  - Used in CG

Parametric form of the super-ellipsoid

$$\left[\left(\frac{X}{r_x}\right)^{2/s_2} + \left(\frac{Y}{r_y}\right)^{2/s_2}\right]^{s_2/s_1} + \left(\frac{Z}{r_z}\right)^{2/s_1} = 1$$

Non-parametric

$$X = r_x \cos^{s_1}\phi \cos^{s_2}\theta$$

$$Y = r_y \cos^{s_1}\phi \sin^{s_2}\theta$$

$$Z = r_z \sin^{s_1}\phi$$

rametric

OpenGL functions

- Does not have superquadrics function
- Can display sphere, cone, cylinder
- Quadrilater

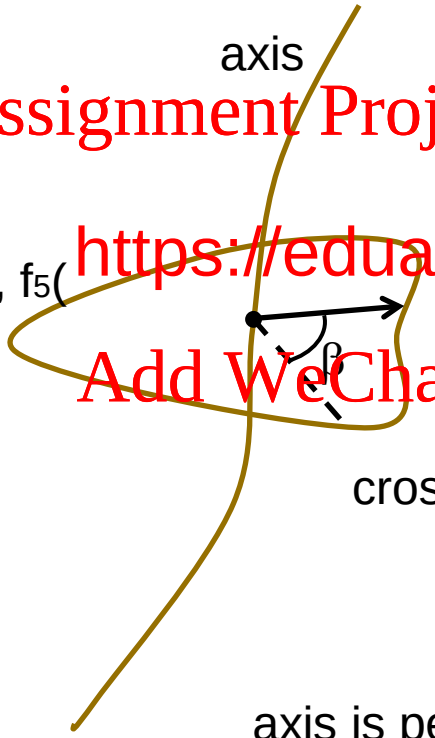
glutWireSphere (r, nLo _ nLatitudes)

Generation of complicated shapes

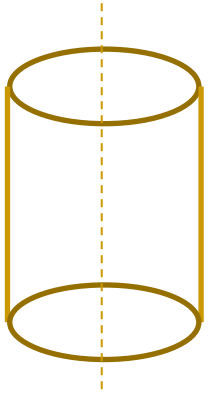
- Complicated shapes can be generated using quadrilateral mesh and parametric form
- Two examp
  - Generalize
  - Generalized Symmetry

Generalized Cylinder

real life example



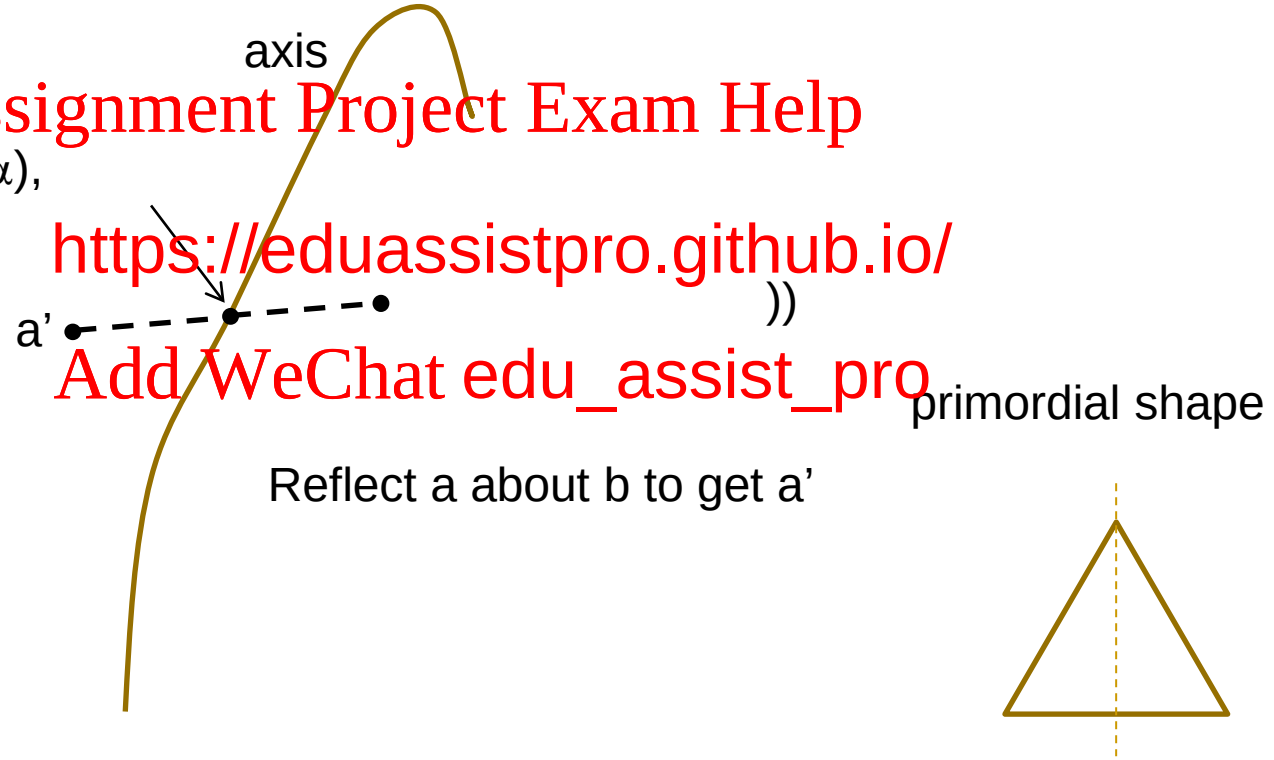
primordial shape



quadrilateral mesh parameterized by α and β

Generalized Reflectional Symmetry

real life example



quadrilateral mesh parameterized by α and β, with β varying linearly from a to a'

References

Ex: Practice using the index

For example, text

- OpenGL Line Functions Sec. 4-4
- Superquadr
- Parametric and non-paorms: A-8, A-9