

BOOLEAN

ALGEBRA

Assignment Project Exam Help

LOGIC

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

Bernhard Kainz (with thanks to
Edwards)

N. Dulay and **E.**

b.kainz@imperial.ac.uk

Learning Objectives

- At the end of this lecture you should:
 - Understand how logic relates to computing problems
 - Be able to represent
 - Truth tables
 - Logic circuits
 - Boolean algebra

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

What is Logic?

- Dictionary definitions (dictionary.com - reduced!)
 - reason or sound judgement
 - a system of principles governing correct or reliable inference
- Branch of philosophy
 - Principles of inference
- You use logic all the time in your everyday life

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

Propositional Logic

- The Ancient Greek philosophers created a system to formalise arguments called propositional logic

Assignment Project Exam Help

- A proposition is TRUE or FALSE
<https://eduassistpro.github.io/>
- Propositions can be compound of the operators AND, OR and NOT

Propositional Logic Example

- Propositions may be TRUE or FALSE, for example:
 - It is raining
 - The weather for
- A combined proposition exam
 - It is raining OR the weather forecast is bad

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

Propositional Logic Example

- Can assign values to propositions, for example: I will take an umbrella if it is raining OR the weather forecast is bad

Assignment Project Exam Help

- Means that the p
the Boolean com
forecast being b

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

- I will take an umbrella = it is raining her forecast is bad

Diagrammatic Representation

- Can think of the umbrella proposition as a result that we calculate from the weather forecast and the fact that it is raining by means of a logical OR

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

Truth Tables

- Since propositions can only take two values, we can express all possible outcomes of the umbrella proposition by a table
- Assignment Project Exam Help**

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

Complex Propositions

- Can make our propositions more complex, for example:
 - (Take Umbrella) = (NOT (Take Car)) AND ((Bad Forecast) OR (Raining))

Assignment Project Exam Help

<https://eduassistpro.github.io/>

- Diagrammatical representation

Add WeChat edu_assist_pro

Boolean Logic

- To perform calculations quickly and efficiently we need a more succinct notation than propositional logic
- Need to have well-defined semantics for all the “operators” that we intend to use
- Boolean Algebra satisfies the criterion above
- Named after George Boole
- Provides a system of logical operations
- Rules for combining operations
- Describes their application to binary numbers

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

George Boole: 1815-1864

Boolean Algebra – Fundamentals

- The truth values are replaced by 1 and 0
 - 1 = TRUE 0 = FALSE

Assignment Project Exam Help

- Propositions are <https://eduassistpro.github.io/>
 - R = it is raining W = The weather is good
- Operators are replaced by symbols
 - ' = NOT + = OR • = AND

Add WeChat edu_assist_pro

Boolean Algebra – Simplify Propositions

- Recall:

- (Take Umbrella) \equiv (NOT (Take Car) AND (Head Forecast) OR (Raining))

<https://eduassistpro.github.io/>

- Using notations notations, we

Add WeChat edu_assist_pro

- $U = (C') \cdot (W + R)$

Boolean Algebra – Precedence

- Operator Precedence
 - Highest precedence operator is evaluated first

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat (edu_assist_pro
(v)

math (logic) symbol

- Note that: $(C') \cdot (W + R)$ is not the same as $C' \cdot W + R$
- Logic operators in, e.g., C:
 - Logical: AND: $\&\&$ OR: $\|\|$ NOT: $!$
 - (Binary: AND: $\&$ OR: $|$ NOT: \sim)

Boolean Algebra – Truth Tables

- All possible outcomes of the operators can be written as truth tables

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

Boolean Algebra – Truth Tables

- Given any Boolean expression e.g.: $U = C' \cdot (W + R)$
- We can calculate a truth table for every possible value of the variables on
<https://eduassistpro.github.io/>
- For n variables there are 2^n possible combinations
[Add WeChat edu_assist_pro](#)

Boolean Algebra – Truth Tables

- Truth table for “Umbrella”
 - $U = C' \cdot (W + R)$

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

Boolean Algebra – Rules

- **Note: A and B can be any Boolean Expression**

Negation:

$$(A')' = A$$

$$A \cdot A' = 0$$

$$A + A' = 1$$

Associative:

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A + B) + C = A + (B + C)$$

Commutative:

$$A \cdot B = B \cdot A$$

$$A + B = B + A$$

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

Distributive:

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

$$A + (B \cdot C) = (A + B) \cdot (A + C)$$

Note the precedence

Boolean Algebra – Rules

Single variables (Idempotent law):

$$A \cdot A = A$$

$$A + A = A$$

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Simp

$$A \cdot 0 = 0$$

$$A \cdot 1 = A$$

$$A + 0 = A$$

$$A + 1 = 1$$

Add WeChat edu_assist_pro

Boolean Algebra – de Morgan's Rule

$$(A + B)' = A' \cdot B'$$

$$(A \cdot B)' = A' + B'$$

as before, A

ression

<https://eduassistpro.github.io/>

Can generalise to n Boolean var

Add WeChat [edu_assist_pro](#)

$$(A + B + C + D + \dots)' = A' \cdot B' \cdot C' \cdot D' \cdot \dots$$

$$(A \cdot B \cdot C \cdot D \cdot \dots \cdot X)' = A' + B' + C' + D' + \dots + X'$$

Boolean Functions – Schematic Representation

- A standard set of easy-to-recognise symbols is used to represent Boolean functions

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

- A circle is all that is required to indicate NOT. The triangle is just to indicate Input/Output direction

Inverting Functions

- A circle can be added to the AND and OR symbol outputs to create their inverted functions – NotAnd (NAND) and NotOr (NOR) gates

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

Building Blocks for Circuits

- NAND/NOR are the commonly used building blocks for most circuits

Assignment Project Exam Help

- NAND / NOR can be implemented using transistors

<https://eduassistpro.github.io/>

- NAND is complete

Add WeChat edu_assist_pro

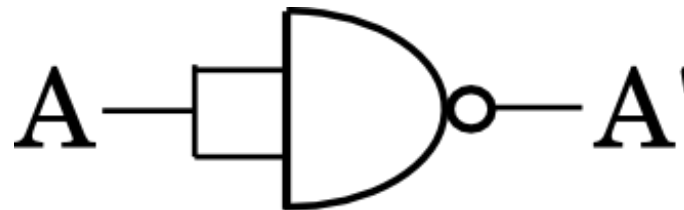
- A set of Boolean functions f_1, f_2, \dots is “complete” if and only if any Boolean function can be generated by a combination these functions
- Also called “universal gate”

NAND Gate – NOT

- It is possible to build all other gates out of NAND gates
- Create a NOT gate using the idempotent law:
 - $A \cdot A = A$ therefore

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro



NAND Gate – AND

- Create an AND gate using the Involution law:

- $(A')' = A$

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro



NAND Gate – OR / NOR

- To make an OR gate we need to apply de Morgan's theorem:

Assignment Project Exam Help

- $A + B = (A' \cdot B')'$

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

- Just invert output to get a **NOR** gate 😊

NAND – Complex Circuits

- Consider two cascading NAND Gates

Assignment Project Exam Help

<https://eduassistpro.github.io/>

- What circuit have we created?
 - Use Boolean Algebra to find out
 - $I = (B \cdot C)'$
 - $X = (A \cdot I)' = (A \cdot (B \cdot C)')'$
 - Apply de Morgan's law, we get
 - $X = A' + ((B \cdot C)')' = A' + (B \cdot C)$

NAND – Complex Circuits

- Truth table for $X = A' + (B \cdot C)$

A	B	C	$B \cdot C$	$X = A' + (B \cdot C)$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	1
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

XOR and XNOR Gates

- Very useful gates

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

A	B	XNOR
0	0	1
0	1	0
1	0	0
1	1	1

Behind the scenes

- Instantaneous on IC?

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro



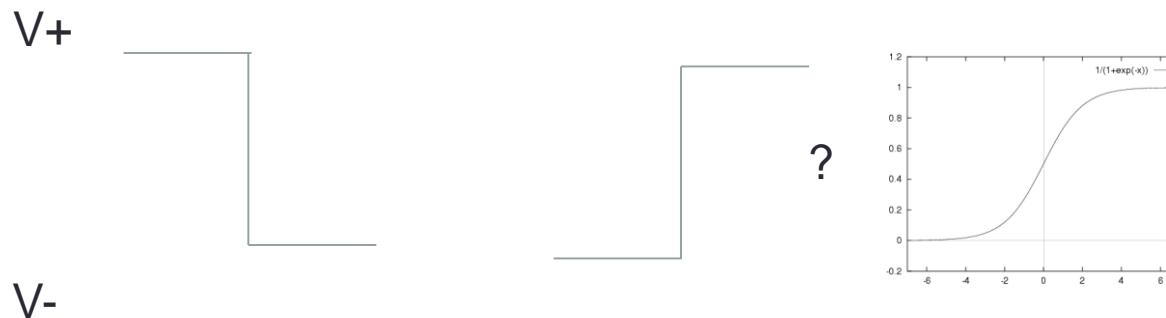
Behind the scenes

- Instantaneous on IC?

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro



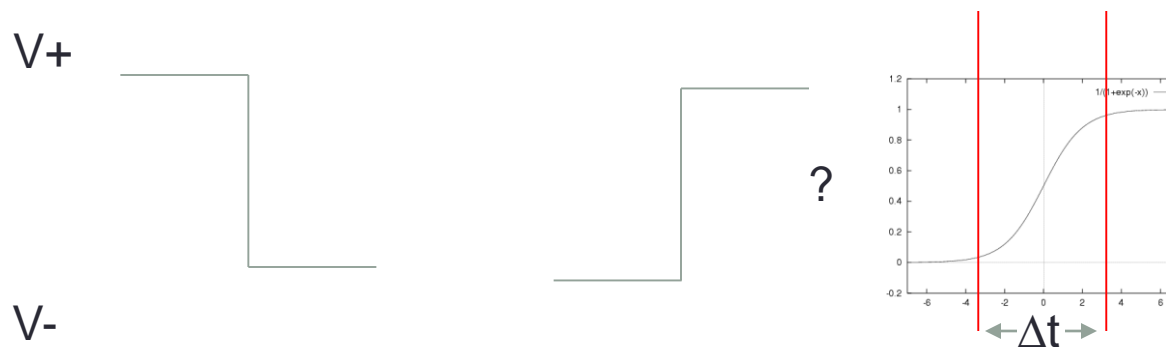
Behind the scenes

- Time delay and saturation limit state 'switching' speed of real in-silico circuits

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro



Behind the scenes

- Transistor-transistor logic TTL NAND gate

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro



Behind the scenes

- CMOS NAND in silico

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

Behind the scenes

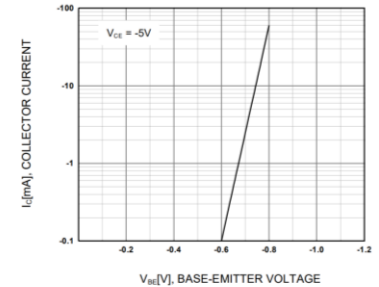


Figure 4. Base-Emitter On Voltage

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro