1. [20 points] TRUE/FALSE OR PICK ONE. *No need for justification.*

(a) TRUE/FALSE

Let $G = (V, E)$ be a directed graph with weights on edges, and $\gamma(p, q)$ denote the length of the p q

$\gamma(p, q) + \gamma(q, r) \leq \gamma(p, r)$ p q r V

(b) TRUE/FALSE

A *Bottleneck spanning tree* of an undirected graph G is defined as a spanning tree of G

(c) TRUE/FALSE

Given two graphs G and G' with the same sets of vertices V and edges E, however different edge weight functions (w and w' respectively). Both weight functions are non- $w'(e) = w(e)^3$ for every edge e E V

G G'.

(d) TRUE/FALSE

Suppose we are given a weighted, directed graph $G = (V, E)$ in which edges that leave the source vertex s may have negative weights, all other edge weights are non-negative, and there are no negative-weight cycles. Then, Dijkstra's algorithms correctly finds shortest paths from s in this graph.

(e) TRUE/FALSE

If an edge e is part of some minimum spanning tree of an undirected, connected graph G, then it must be a lightest edge across some cut of G.

(f) TRUE/FALSE

Let $f_1, f_2 : V \times V \to R$ be two different flows on a flow network $G = (V, E)$ with a capacity $c : V \times V \to R^+ \cup \{0\}$. Then, $f_1 + f_2$, which is defined as $(f_1 + f_2)(e) = f_1(e) + f_2(e)$ for every $e \in V \times V$, is also a flow in G.

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(g) TRUE/FALSE

If all edge capacities in a flow network are integer multiples of 9, then the maximum flow value is always a multiple of 9.

(h) TRUE/FALSE

graph $G = (V, E)$ and a weight function w on its edges as input and returns a set of edges T. The output T of the algorithm is a minimum spanning tree of G.

MAYBE-MST(G, w)

Sort the edges of G w

$T \leftarrow E$

for each edge e

  do if $T - \{e\}$ is a connected graph

    then $T \leftarrow T - e$

return T

(i) PICK ONE

Let $G = (V, E)$ $w : E \to \mathbb{R}$. Consider an adjacency matrix $A = (a_{ij})$ where $a_{ij} = w(i, j)$ or ∞. Let $d_{ij}^{(m)}$ denote the weight of a shortest path from i to j m

path problem?

(i) $d_{ij}^{(m)} = \min_{i \le k \le j}\{d_{ik}^{(m-1)}\} + a_{kj}$ for $m = 1, 2, \ldots, n-1$

(ii) $d_{ij}^{(m)} = \min\{d_{ij}^{(m-1)} + a_{ij}\}$ for $m = 1, 2, \ldots, n$

(iii) $d_{ij}^{(m)} = \min_{i \le j}\{d_{ik}^{(m-1)} + a_{kj}\}$ for $k = 1, 2, \ldots, n$

(iv) $d_{ij}^{(m)} = \min_{1 \le k \le n}\{d_{ik}^{(m-1)} + a_{kj}\}$ for $m = 1, 2, \ldots, n-1$

(j) PICK ONE

Consider a sequence of n operations performed on a data structure, where c_i and $\hat{c}_i$ denote the actual and the amortized costs of operation i, respectively. Which ONE of the following inequalities is essential for amortized complexity analysis?

(i) $c_i \le \hat{c}_i$ (ii) $c_i \ge \hat{c}_i$

(iii) $\sum_{i=1}^{n} c_i \le \sum_{i=1}^{n} \hat{c}_i$ (iv) $\sum_{i=1}^{n} c_i \ge \sum_{i=1}^{n} \hat{c}_i$

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2. [10 points] SUPPORTING YOUR CLAIM
Pick any TWO of the statements in Question 1 (a)-(h) that you decided to be TRUE or FALSE. Give a complete proof of your decision.

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3. [20 points] AMORTIZED ANALYSIS
A sequence of n operations is performed on a data structure. The kth operation has a cost of k when k is an exact power of 4, and otherwise it has a cost of 1. Analyze the amortized cost per operation using any one of these methods: (a) aggregate, (b) accounting, (c) potential.

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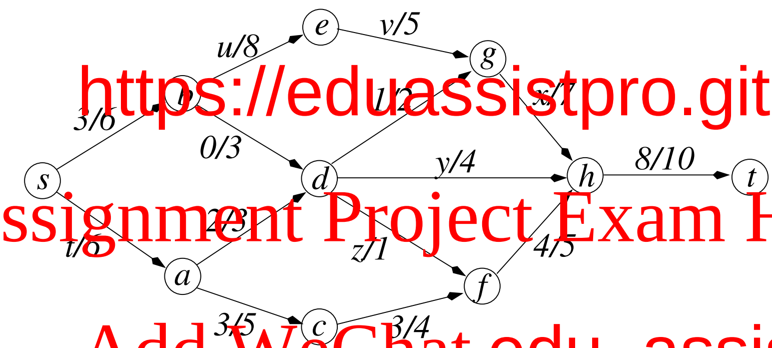
4. [25 points] MAXIMUM FLOW

Figure shows a flow network on which an $s-t$ flow has been assigned. The two numbers on each edge shows the flow and the capacity values, respectively.



(a) What are the values of t, u, v, x, y, and z

(b) What is the value of this flow? Is this a maximum $s-t$ flow in this network?

(c) Draw the residual graph for this flow.

(d) Find a minimum $s-t$ cut in this network. What is the capacity of this minimum cut?

zero flow $< s, a, c, f, h, t >$

with flow 4 $< s, a, d, f, h, t >$ $< s, b, d, h, t >$ with flow 3.

(f) Start with a **zero flow**

of augmentations with their flow values, one for each algorithm.)

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5. [15 points] MST of an Almost Tree Graph

A graph $G = (V, E)$ is called an *almost tree* if it is connected and has most $n + c$ edges where $n = |V|$ and c is small constant number. Design and analyze and algorithm for a given *almost tree* graph G with distinct costs on its edges, computes a minimum spanning tree of G in time $O(n)$.

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6. [10 points] Shortest Path Algorithm

For the directed weighted graph shown below, use Dijkstra's algorithm to compute the shortest paths from node A to all other nodes by filling in the table. At each step add a new vertex to M, the set of nodes whose shortest path length from A is correctly computed. The first two steps are al

$d(X)$: the cost of the A X

$p(X)$: the predecessor of node X along the current shortest path estimate from A.

Step	M		A	B	C	D	E	F	G
0		d		∞	∞	∞	∞	∞	∞
		p					-	-	-
1	A	d				∞	∞	∞	∞
		p		A	A			-	-
2		d							
		p							
3		d							
		p							
4		d							
		p							
5		d							
		p							
6		d							
		p							
7		d							
		p							

- Trace back on array p to output the shortest path from A to G:

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7. [10 points] MINIMUM SPANNING TREES

Kruskal's algorithm for computing a Minimum Spanning Tree of an input graph G (as described in your textbook) considers the edges of G in non-decreasing or selected MST edges the selected edges, re

Union-Find maintains a collection of sets of vertices, those who are connected to each other with the selected MST edges. Union-Find is initially a collection of one vertex sets (see the first row of the table below). find(X) operation returns the ID of a set a vertex X belongs to. union(X, Y) operation merges two sets with a Based on this description, complete the table below, showing iterati n on the given graph abow (first step is already given).

Step	Edge Considered	Sets in Union-Find Data Structure	Union-Find Operations	Select as MST Edge?
1	(D, F)	{A}, {B}, {C}, {D}, {E}, {F}, {G}	$\mathrm{find}(D) \neq \mathrm{find}(F)$ $\mathrm{union}(D, F)$	Yes
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				

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