Lecture22_NetworkFlows1

Wednesday, October 21, 2020 3:31 PM

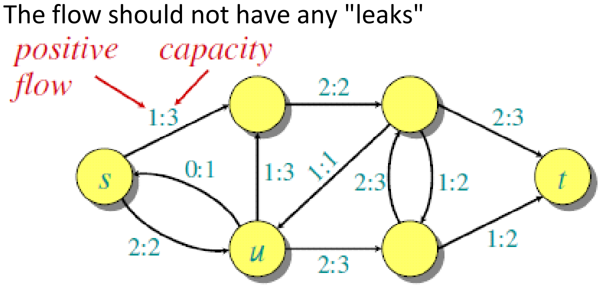
Matching problem

Fl

De E) with two distinguished vertices: a source s and a sink t. Each edge $(u, v) \in E$ has a nonnegative capacity $c(u, v)$. If $(u, v) \notin E$, then $c(u, v) = 0$.

The flow goes into the vertex should equal to the flow goes out from that vertex

- The flow should not have any "leaks"



Flow conservation (like Kirchoff's current law):
- Flow into u is $2 + 1 = 3$.
- Flow out of u is $0 + 1 + 2 = 3$.

The value of this flow is $1 - 0 + 2 = 3$.

- The positive flow is not greater than the capacity
- Flow conservation: in == out
- Sink vertex doesn't need to have flow conservation
- The nature of the problem: at some edges we are wasting our capacity
- ➢ Find the maximum assignment

Maximum-flow problem: Given a flow network G, find a flow of maximum value on G.

➢ The value of the maximum flow of the above graph is 4.

Flow cancellation

Notational simplification

Equivalence of definitions

Theorem. The two definitions are equivalent.

Proof. ($\Rightarrow$) Let $f(u, v) = p(u, v) - p(v, u)$.

- ***Capacity constraint:*** Since $p(u, v) \le c(u, v)$ and $p(v, u) \ge 0$, we have $f(u, v) \le c(u, v)$.
- ***Flow conservation:***

$$\sum_{v \in V} f(u,v) = \sum_{v \in V} (p(u,v) - p(v,u)) = \sum_{v \in V} p(u,v) - \sum_{v \in V} p(v,u)$$

- ***Skew symmetry:***

$$f(u, v) = p(u, v) - p(v, u) = -(p(v, u) - p(u, v)) = -f(v, u).$$

- If given a network flow, can we create a flow that satisfies its two properties

➢ From summation notation to **set notation**

set = sumation

all vertices except source and sink

Simple properties of flow

lemma 1

lemma2

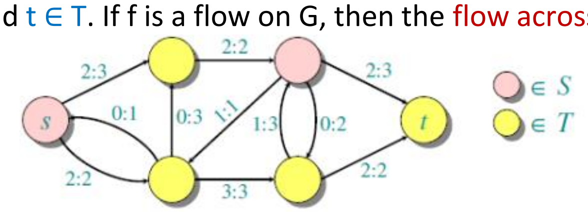
lemma 3

conservation of flow

- Whatever we send to the network should end up in the sink

Cuts

Definition. A cut (S, T) of a flow network G = (V, E) is a partition of V such that s ∈ S and t ∈ T. If f is a flow on G, then the flow across the cut is f (S, T).



$$f(S, T) = (2 + 2) + (-2 + 1 - 1 + 2)$$
$$= 4$$

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- Split the graph to two side: source to one side, and sink to another side

Another characterization of flow value

Lemma. For any flow f and any cut (S, T), we have | f | = f (S, T).

Proof.
$$\begin{aligned} f(S,T) &= f(S,V) - f(S,S) \\ &= f(S,V) \\ &= f(s,V) + f(S-s,V) \\ &= f(s,V) \\ &= |f|. \quad \square \end{aligned}$$

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Upper bound on the maximum flow value

he capacity of any cut.

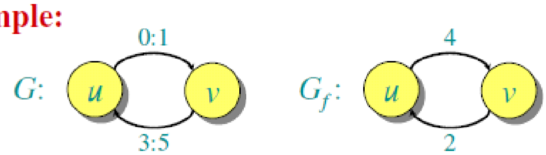
Residual network

Definition. Let f be a flow on G = (V, E). The residual network G_f (V, E_f) is the graph with strictly positive residual capacities

c_f (u, v) = c(u, v) – f (u, v) > 0.

Edges in E_f admit more flow.

Example:



Lemma. $|E_f| \le 2|E|$. $\square$

Augmenting paths

n.

- The graph above is not entire graph, therefore there are some node without conservation int the network flow

Max-flow, min-cut theorem

Theorem. The following are equivalent

1. f is a maximum flow.
2. f admits no augmenting paths.
3. | f | = c(S, T) for some cut (S, T).