COT5405/CIS4930: ANALYSIS OF ALGORITHMS

Date: April 18, 2017, Tuesday
Time: 8:20pm – 10:10pm (110 minutes)
Professor: Alper Ungor (Office CSE 534)

This is a closed book exam. No collaborations are allowed. Your solutio
concise, but complete, and handwritten clearly. Use only the space provided in this booklet, including the even numbered pages. Feel free to refer to algorithms, definitions and concepts discussed in class rather than describing them in detail unless of course you are asked for it specifically.

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First Name:

Last Name:

UF ID:

	Credit	Max
Problem 3		
Problem 4		
TOTAL		110

1. [20 points] PICK ONE *No justification is needed for this problem.*

(a) Which one of the following is incorrect for Two Dimensional Range Trees?

i. Counting range queries take $O(\log^2(n))$ time.

ii. Reporting rang

iii. Construction ti

iv. Space complexi

(b) CLIQUE-DEGREE-4 problem: Given a graph $G = (V, E)$, where all vertices have degree at most 4, and an integer k, determine whether V has a subset S of size at least k that forms a clique in G. Which of the following statement(s

(a) CLIQUE-DEGREE-4 is NP-Hard; (b) CLIQUE xinating CLIQUE-DEGREE-4 is NP-Hard; (d) There exists a polynom CLIQUE-DEGREE-4 to VERTEXCOVER problem.

i. (a) and (d)

ii. (b) and (d)

iii. (a) and (c)

iv. (b) and (c)

v. All of them

(c) MAJORITY-CHECK problem: Given a set S ine whether more than half the numbers in S are exactly the sam e compared in constant time, which of the following stateme

(a) MAJORITY-CHECK is in NP; (b) There exists no comparison-based algorithm solving MAJORITYCHECK in $\Theta(n)$ time; (c) MAJORITY-CHECK is proven to be NP-hard via a reduction from SUBSETSUM problem.

i. only (a)

ii. (a) and (b)

iii. (a) and (c)

iv. All of them

v. None of them

(d) MAX-SAT: Given a CNF (Conjunctive Normal Form) Boolean formula and a positive integer k, does there exist a truth assignment that satisfies at least k clauses.
Which one of the following is correct about this problem?

i. This problem can be solved in polynomial time since k is a small number.

ii. This problem is NP-hard since it is a special case of the SAT problem.

iii. This problem is NP-hard since it is a generalization of the SAT problem.

iv. This problem can be shown to be NP-hard by giving a reduction from MAX-SAT to CLIQUE problem.

v. None of the above

2. [30 points] GEOMETRIC ALGORITHMS - CO-CIRCULARITY CHECK
Following procedures each with planar input points and constant running time are provided:
$\underline{\text{CCW}(p, q, r)}$ decides if r is LEFTOF, ON, or RIGHTOF the oriented line going through p, q.
$\underline{\text{INCIRCLE}(p, q, r, s)}$ decides whether s is INSIDE, ON, or OUTSIDE the unique circle going through p, q, r.

(a) **Draw** five points s of points (a, b, c), (a, b, d), and (a, b, e) are all the same, a, b, c, d are co-circular, and yet a, b, c, e are not co-circular, i.e., $\text{circumradius}(a, b, c) = \text{circumradius}(a, b, d) = \text{circumradius}(a, b, e)$, $\text{INCIRCLE}(a, b, c, d) = \text{ON}$ and $\text{INCIRCLE}(a, b, c, e) \neq \text{ON}$.

(b) Given n p
whether any f $O(n^4)$ time.
Design an $O(n^3 \log n)$ time.
(*Hints: Sorting takes* $O(n \log n)$ *time. Rec* *g algorithm.*)

3. [30 points] NP-Completeness - Strongly Independent Sets

Consider a simple graph $G = (V, E)$. A subset $S_1 \subseteq V$ is called an *independent set*, if no two vertices in S_1 have an edge (path of length one) between them. A subset $S_2 \subseteq V$ is called a *strongly independent set*, if no two vertices in S_2 have a path of length one or two between them, i.e., $\forall\, u, v \in S$ $\in E$ and $(w, v) \in E$.

IndependentSet Problem: Given a simple graph G and an integer k, determine if G contains an independen

StronglyIndependentSet Problem: Given a simple graph G and an integer k, determine if G contains a strongly independent set of size at least k.

(a) **Draw** a connected graph that has an independent set of size 4. **Draw** another connected graph that has a strongly independent set of size 4.

(b) **Prove** that lete. (*Hints:* IndependentSet P *nal vertices positioned strategically and forming a clique amo*)

4. [30 points] Approximation - Pack your Books, You Got a Job at Google
Imagine that you just graduated UF and got a job at Google. For your move to California, you need to pack all your books to boxes each of which has capacity a real number between 1 and 2. For simplicity, assume that each book has weight a real number between 0 and 1. You want to minimize the n

PackBooks Proble — e $\{w_1, \ldots, w_n\}$, $0 < w_i < 1$, m boxes with capacities 1 m j k, is it possible to pack all books in at most k boxes such that total weight in each box is within its capacity.

(a) **Prove** that PackBooks Problem is NP-hard. (*Hints: Consider a special case of this problem that you are familiar with. No reduction necessary.*)

(b) **Design** and **analyze** an approximation algorit n of the PackBooks Problem with an approximatio