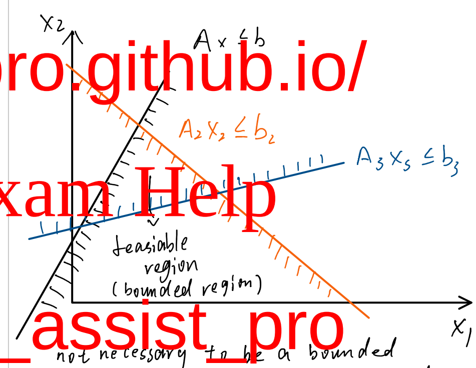
Lecture19_ShortestPaths3

Tuesday, October 20, 2020 9:20 PM

Linear Programming

Let A be an m×n matrix, b be an m-vector, and c be an n-vector. Find an n-vector x that maximizes c^Tx subject to Ax <= b, or determine that no such solution exists.

subject to $Ax \le b$
max $c^T x$



not necessary to be a bounded
unbounded
empty

Algorithms for the general problem

• Simplex methods—practical, but worst-case exponential time.
• Interior-point methods—polynomial time and competes with simplex.

Feasibility problem: No optimization criterion.
Just find x such that Ax <= b.

- In general, just as hard as ordinary LP.

Solving a system of difference constraints

Linear programming where each row of A contains exactly one 1, one –1, and the rest 0's.

optimal could be one of the corner of the converx region

- For each
- For each inequality / constrain put an edge from the one wit with positive sign. The weight of edge will be the right hand s

Unsatisfiable constraints

Theorem. If the constraint graph contains a negative-weight cycle, t differences is unsatisfiable.

Proof. Suppose that the negative-weight cycle is $v_1 \to v_2 \to \cdots \to v_k \to v_1$. Then, we have

$$\begin{aligned} x_2 - x_1 &\le w_{12} \\ x_3 - x_2 &\le w_{23} \\ &\vdots \\ x_k - x_{k-1} &\le w_{k-1,k} \\ x_1 - x_k &\le w_{k1} \\ \hline 0 &\le \text{weight of cycle} \\ &< 0 \end{aligned}$$

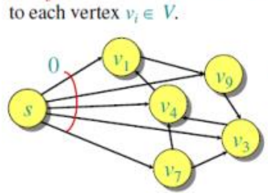
Therefore, no values for the x_i can satisfy the constraints. □

- Right side: negative number -> contradiction

Satisfying the constraints

Theorem. Suppose no negative-weight cycle exists in the constraint graph. Then, the constraints are satisfiable.

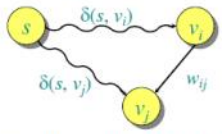
Proof. Add a new vertex s to V with a 0-weight edge to each vertex $v_i \in V$.



Note: No negative-weight cycles introduced ⇒ shortest paths exist.

- s: dummy variable, a vertex that connect to all vertices.

Claim: The assignment $x_i = \delta(s, v_i)$ solves the constraints. Consider any constraint $x_j - x_i \le w_{ij}$, and consider the shortest paths from s to v_j and v_i:

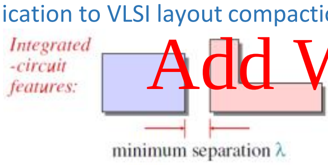


The triangle inequality gives us $\delta(s,v_j) \le \delta(s, v_i) + w_{ij}$. Since $x_i = \delta(s, v_i)$ and $x_j = \delta(s, v_j)$, the constraint $x_j - x_i \le w_{ij}$ is satisfied. □

Bellman-Ford and linear programming

Corollary. The Bellman-F
variables in **O(mn)** time.

Application to VLSI layout compaction



Problem: Compact (in one dimension) the space between the features of a VLSI layout without bringing any features too close together.

- Subject to the second optimalization function above