COT5405: ANALYSIS OF ALGORITHMS

Date: 2019
Time: 100 minutes
Professor: Alper Ungor (Office CSE 534)

This is a closed book exam. No collaborations are allowed. Your solutions s be concise, but complete, and handwritten clearly. Use only the space provided in this booklet, including the even numbered pages. When appropriate, feel free to give reference to the algorithms, definitions and concepts discussed in class rather than describing them in detail.

First name:

Last name:

	Credit	Max
Problem 3		
Problem 4		
Total		110

1. [20 points] TRUE/FALSE. *No need for justification.*

(a) TRUE/FALSE
For an undirected, connected graph G with distinct edge weights, the minimum spanning tree of G includ

(b) TRUE/FALSE
(Let $G = (V, E)$ be an undirected connected graph with d
For every vertex $v \in V$, the edge with the smallest weigh ust be an edge in the minimum spanning tree of G.

(c) TRUE/FALSE
(Consider a nct edge weights. The second smal e spanning tree of G with the s
The second smallest spanning tree of G is u

(d) TRUE/FALSE
If all edge capacities in a flow network are integer multiples of 35, then the value of the maximum flow must be a multiple of both 5 and 7.

(e) TRUE/FALSE
(Let $G = (V, E)$ be a directed graph with nonnegative weights on edges, and $\gamma(p, q)$ denote the length of the *longest simple path* between p and q.)
The triangle inequality $\gamma(p, q) + \gamma(q, r) \leq \gamma(p, r)$ holds for every p, q, and r in V.

2. [30 points] MINIMUM SPANNING TREE UPDATE

Consider an undirected, connected graph $G = (V, E)$ with edge weights $w : E \to \mathbb{Z}^+$, and a minimum spanning tree $T = (V, E')$ of G, both given as adjacency lists. Consider the following updates on G. For each case, decide whether an update might be necessary, and if so, describe and analyze an

(a) The weight of a particul e)

(b) The weight of a particular edge $e \in E - E$ is decreased to $\hat{w}(e) < w(e)$.

(c) The weight of a particular edge $e \in E'$ is decreased to $\hat{w}(e) < w(e)$.

(d) The weight of a particular edge $e \in E'$ is increased to $\hat{w}(e) > w(e)$.

(e) A new edge $e = (u, v) \notin E$ is added to E with wei

3. [30 points] AN ALTERNATIVE ALGORITHM FOR ALL PAIRS SHORTEST PATH PROBLEM
Let $G = (V, E)$ be a directed graph with n vertices and weighted (-, 0, or +) edges.

(a) How could we delete an arbitrary vertex v from this graph, without changing the shortest-path dist
algorithm that con hted edges, where
$V' = V - \{v\}$, an G' is equal to
the shortest-path d $n^2)$ time.

(b) Suppose we have already computed all pairs shortest-path distances in G'. Describe and analyze an algorithm to compute the shortest-path distances from v to every other vertex, and from every other vertex to v, in the original graph G, in $O(n^2)$ time.

(c) Describe and analyze a new all-pairs shortest path algorithm tha $O(n^3)$ time by combining parts (a) and (b).

4. [30 points] FLOW NETWORKS

Consider the flow network $G = (V, E)$, where $V = \{s, a, b, c, d, e, f, g, t\}$, s is the source, t is the sink, and the edge set with capacities is $E = \{ ((s, a), 3),\ ((s, b), 6),\ ((a, c), 4),\ ((a, d), 2),\ ((b, d), 3),\ ((b, e), 5),\ ((c, f), 1),\ ((d, f), 6),\ ((d, g), 7),\ ((e, g), 2),\ ((f, t), 8),\ ((g, t), 5) \}$.

(a) Draw this flow networ

(b) Give a maximum flow f cut

(c) Is the maximum flow function f on G unique? Justify.

(d) Prove or disprove the claim: The maximum flow function on a flow network is unique if and only if the minimum cut on it is unique.

(e) Draw the residual graph for flow f that you built in p

(f) Describe and analyze an efficient algorithm to determine whet has a unique maximum flow. [*Hint: First give a charac h.*]