1. [20 points] TRUE/FALSE. *No need for justification.*

(a) TRUE/FALSE

G

tree of G G

(b) TRUE/FALSE
(Let $G = (V, E)$
For every vertex $v \in V$ v
in the minimum spanning tree of G.

(c) TRUE/FALSE

$G = (V, E)$
G

G
The second smallest spanning tree of G

(d) TRUE/FALSE
If all edge capacities in a flow network are integer multiples of 35, then the value of the maximum flow must be a multiple of both 5 and 7.

(e) TRUE/FALSE
(Let $G = (V, E)$ be a directed graph with nonnegative weights on edges, and $\gamma(p, q)$ denote the length of the *longest simple path* between p and q.)
The triangle inequality $\gamma(p, q) + \gamma(q, r) \leq \gamma(p, r)$ holds for every p, q, and r in V.

2. [30 points] MINIMUM SPANNING TREE UPDATE

Consider an undirected, connected graph $G = (V, E)$ with edge weights $w : E \to \mathbb{Z}^+$, and a minimum spanning tree $T = (V, E')$ of G, both given as adjacency lists. Consider the following updates on G. For each case, decide whether an update might be necessary, and if

(a) $e \in E - E'$ $\hat{w}(e) > w(e)$

(b) The weight of a particular edge $e \in E - E'$ is decreased to $\hat{w}(e) < w(e)$.

(c) The weight of a particular edge $e \in E'$ is decreased to $\hat{w}(e) < w(e)$.

(d) The weight of a particular edge $e \in E'$ is increased to $\hat{w}(e) > w(e)$.

(e) A new edge $e = (u, v) \notin E$ is added to E $\hat{w}(e)$

3. [30 points] An Alternative Algorithm for All Pairs Shortest Path Problem
Let $G = (V, E)$ be a directed graph with n vertices and weighted (-, 0, or +) edges.

(a) How could we delete an arbitrary vertex v from this graph, without changing the

$V' = V - \{v\}$ $G' = (V', E')$ G' is equal to

G $O(n^2)$ time.

(b) Suppose we have already computed all pairs shortest-path distances in G'. Describe and analyze an algorithm to compute the shortest-path distances from v to every other vertex, and from every other vertex to v, in the original graph G, in $O(n^2)$ time.

(c) $O(n^3)$ time

by combining parts (a) and (b).

4. [30 points] FLOW NETWORKS

Consider the flow network $G = (V, E)$, where $V = \{s, a, b, c, d, e, f, g, t\}$, s is the source, t is the sink, and the edge set with capacities is $E = \{ ((s,a),3),\ ((s,b),6),\ ((a,c),4),\ ((a,d),2),\ ((b,d),3),\ ((b,e),5),\ ((c,f),1),\ ((d,f),6),\ ((d,g),7),\ ((e,g),2),\ ((f,t),8),\ ((g,t),5)\ \}$.

(a)

(b) $f : E \to \mathbb{R}$ G

(c) Is the maximum flow function f on G unique? Justify.

(d) Prove or disprove the claim: The maximum flow function on a flow network is unique if and only if the minimum cut on it is unique.

(e) Draw the residual graph for flow f

(f) has a unique maximum flow. [ ]