1. [20 points] TRUE/FALSE OR PICK ONE. *No need for justification.*

(a) TRUE/FALSE

Let $G = (V, E)$ be a directed graph with weights on edges, and $\gamma(p, q)$ denote the length of the *longest* s p q ngle inequality, i.e., $\gamma(p, q) + \gamma(q, r) \leq \gamma(p, r)$ p, q r V

(b) TRUE/FALSE

Let $G = (V, E)$ be a flow network with source $s \in V$ and sink $t \in V$, and non-negative edge capacities. If the maximum flow assignme the minimum cut for this network is also *unique*.

(c) TRUE/FALSE

Given two graphs G and G' with the same sets of vertices V and edges E, however different edge weight functions (w and w' respectively). Both weight functions are non-negative a ation: $w'(e) = w(e)^3$ for every edge e a b V st path between them in G is also a G'

(d) TRUE/FALSE

Suppose we are given a weighted, directed graph $G = (V, E)$ in which edges that leave the source vertex s may have negative wieghts, all other edge weights are non-negative, and there are no negative-weight cycles. Then, Dijkstra's algorithms correctly finds shortest paths from s in this graph.

(e) TRUE/FALSE

Let $G = (V, E)$ be a connected, undirected graph with a weight function $w : E \to \mathbb{N}$ defined on its edges. The shortest path between any two vertices in V is always part of some minimum spanning tree of G.

(f) TRUE/FALSE

Let $f_1, f_2 : V \times V \to \mathbb{R}$ be two different flows on a flow network $G = (V, E)$ with a capacity $c : V \times V \to \mathbb{R}^+ \cup \{0\}$. Then, $f_1 + f_2$, which is defined as $(f_1 + f_2)(e) = f_1(e) + f_2(e)$ for every $e \in V \times V$, is also a flow in G.

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(g) True/False

Consider the following pseudocode which describes a greedy algorithm that takes a graph $G = (V, E)$ and a weight function w on its edges as input and returns a set of edges T. The output T of the algorithm is a minimum spanning tree of G.

Maybe-MST$(G, w$

Sort the edges of G [illegible] w

$T \leftarrow E$

for each edge e, taken in non-increasing order by weight

  do if $T - \{e\}$ is a connected graph

    then $T \leftarrow T - e$

return T

(h) Pick One

Let $G = (V, E)$ be a directed graph with edge-weight function $w : E \to R$. Consider an adjacency matrix $A = (a_{ij})$ where $a_{ij} = w(i, j)$ or ∞. Let $d_{ij}^{(m)}$ denote the weight of a shortest path from i to j that uses at most m edges. Which of the following recurrences correctly formulate a dynamic programming solution for the all-pairs shortest path problem?

(i) $d_{ij}^{(m)} = \min_{i \leq l \leq}\{d_{ik}^{(m-1)}\} + a_{kj}$ [illegible] $m = 1, 2, \ldots, n-1$

(ii) $d_{ij}^{(m)} = \min\{d_{ij}^{(m-1)} + a_{ij}\}$ for $m = 1, 2, \ldots, n$

(iii) $d_{ij}^{(m)} = \min_m\{d_{ik}^{(m-1)} + a_{kj}\}$ for $k = 1, 2, \ldots, n$

(iv) $d_{ij}^{(m)} = \min_{1 \leq k \leq n}\{d_{ik}^{(m-1)} + a_{kj}\}$ for $m = 1, 2, \ldots, n-1$

(i) Pick One

Let $f_1, f_2 : V \times V \to R$ be two different flows on a flow network $G = (V, E)$ with a capacity $c : V \times V \to R^+ \cup \{0\}$. Then, $f_1 + f_2$, which is defined as $(f_1 + f_2)(e) = f_1(e) + f_2(e)$ for every $e \in V \times V$, is NOT necessarily a flow in G because it can violate

(i) the conservation law

(ii) the skew symmetry property

(iii) the capacity constraint

(iv) all of the above

(j) Pick One

Consider a sequence of n operations performed on a data structure, where c_i and $\hat{c}_i$ denote the actual and the amortized costs of operation i, respectively. Which ONE of the following inequalities is essential for amortized complexity analysis?

(i) $c_i \leq \hat{c}_i$

(ii) $c_i \geq \hat{c}_i$

(iii) $\sum_{i=1}^{n} c_i \leq \sum_{i=1}^{n} \hat{c}_i$

(iv) $\sum_{i=1}^{n} c_i \geq \sum_{i=1}^{n} \hat{c}_i$

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2. [10 points] SUPPORTING YOUR CLAIM
Pick any TWO of the statements in Question 1 (a)-(g) that you decided to be TRUE or FALSE. Give a complete proof of your decision.

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3. [16 points] Amortized Analysis

Recall the amortized analysis of a sequence of n insertions into a dynamic table whose capacity is doubled every time it becomes full. (As usual, inserting a new item into an empty slot takes unit time, as well as transfering each item into the expanded table.) Here, we

$T \quad 3T$

to give an upper bound on the amortized cost of each insertion operation.

(b) Table size is increased by 1000, i.e., it goes from T to $T + 1000$. Use *Accounting Method* to give an upper bound on the amortized cost of each insertion operation.

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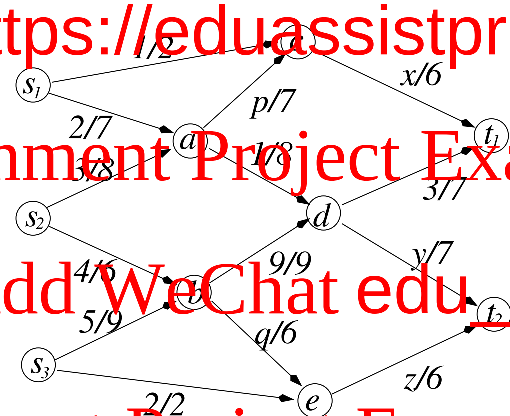
4. [28 points] Flow Network with Multiple Sources/Sinks

Consider a variant of the Flow Network problem where we have multiple sources and multiple sinks. Figure shows a flow network with three sources s_1, s_2 and s_3 and two sinks t_1 and t_2 on which a flow has been assigned. The two numbers on each edge shows the flow and the capacity values, resp



(a) Describe a simple transformation that turns a flow network with multiple sources/sinks into a flow ... can use the same exact algorithms

(b) What are t ... p q x y ... z ... d flow feasible?

(c) What is the value of the total flow out of ... aximum flow in this network?

(d) Draw the residual graph for this flow.

(e) Find a minimum $\{s_1, s_2, s_3\} - \{t_1, t_2\}$ cut in this network. What is the capacity of this minimum cut?

(f) Starting with a **zero flow** consider a sequence of three augmentations: (i) $< s, s_1, a, d, t_2, t >$ with flow 7, (ii) $< s, s_2, a, c, t_1, t >$ with flow 6, and (iii) $< s, s_3, b, d, t_1, t >$ with flow 7. Give a fourth augmentation that can follow these. Is there a fifth possible?

(g) Start with a **zero flow** in this flow network, and illustrate that the number of augmentations performed by the Edmonds-Karp algorithm can be more than the number of augmentations performed by the Ford-Fulkerson algorithm. (Simply list two sequences of augmentations with their flow values, one for each algorithm.)

5. [20 points] MST UPDATE

You are given an undirected connected graph $G = (V, E)$ with positive edge weights, and a minimum spanning tree $T = (V, E')$ of G with respect to those weights. You may assume G and T are given as adjacency lists. Now suppose the weight of a particular edge $e \in E$ is modified from $w(e)$ $\hat{w}(e)$ mum spanning tree to reflect this chang h. For each of the following cases, deci o describe a linear-time algorithm for updating the tree.

(a) $e \notin E'$ and $\hat{w}(e) > w(e)$

(b) $e \notin E'$ and $\hat{w}(e) < w(e)$

(c) $e \in E'$ and $\hat{w}(e) > w(e)$

(d) $e \in E'$ and $\hat{w}(e) < w(e)$

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6. [14 points] SHORTEST PATHS

For the directed weighted graph shown below, use Dijkstra's algorithm to compute the shortest paths from node A to all other nodes by filling in the table. At each step add a new vertex to M, the set of nodes whose shortest path length from A is correctly computed. The first two steps are al

$d(X)$: the cost of the A X

$p(X)$: the predecessor of node X along the current shortest path estimate from A.

Step	M		A	B	C	D	E	F	G
0		d		∞	∞	∞	∞	∞	∞
		p					-	-	-
1	A	d				∞	∞	∞	∞
		p		A	A			-	-
2		d							
		p							
3		d							
		p							
4		d							
		p							
5		d							
		p							
6		d							
		p							
7		d							
		p							

- Trace back on array p to output the shortest path from A to G:

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