7.2 – Trigonometric Integrals

Evaluate the integrals.

* Note that we actually already know how to find this integral already using "u" substitution.

Ex) $\int \sin^4 2x \ \cos 2x \, dx$

$$u = \sin 2x$$
$$du = 2\cos 2x \, dx$$
$$\Rightarrow \frac{1}{2} du = \cos 2x \, dx$$

$$= \frac{1}{2}\int u^4 \, du$$

$$= \frac{1}{2} \frac{u^5}{5} + C$$

$$= \frac{u^5}{10} + C$$

$$= \frac{(\sin 2x)^5}{10} + C$$

* Note that in the following problem the power on sine is **odd**. The method used here would also work if the power on cosine were odd or if the power on sine and cosine were both odd.

Let's use the idea from the previous example but this time we'll let "u" be $\cos x$, peel off one of the sines below to use as our "du", and put the remaining sines in terms of cosine using the Pythagorean identity.

Recall a Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \cos^2 x$

$$u = \cos x$$
$$du = -\sin x \, dx$$
$$\Rightarrow -du = \sin x \, dx$$

Ex) $\int \sin^5 x \ \cos^2 x \, dx$

$$= \int (\sin^4 x \cos^2 x)(\sin x \, dx)$$

$$= \int [(1 - \cos^2 x)^2 \cos^2 x][\sin x \, dx$$

$$= \int (1-u^2)^2 u^2 (-du)$$

$$= -\int (1-2u^2+u^4)\, u^2\, du$$

$$= -\int (u^2 - 2u^4 + u^6)\, du$$

$$= -\frac{u^3}{3} + \frac{2u^5}{5} - \frac{u^7}{7} + C$$

$$= \frac{2}{5}(\cos x)^5 - \frac{1}{3}(\cos x)^3 - \frac{1}{7}(\cos x)^7 + C$$

Recall these identities which reduce the power: $\sin^2 x = \frac{1}{2}[1-\cos(2x)]$, $\cos^2 x = \frac{1}{2}[1+\cos(2x)]$

Note that in the following problem the power on sine and cosine are **both even**.

Ex) $\int 16 \sin^2 x \cos^2 x \, dx$

Summary: Evaluating $\int \sin^m(x)\cos^n(x)\,dx$

Case 1: m or n is odd

- Choose the trig function with the **odd power** (if both are odd, just choose one!).
- Save one factor to be used for du.
- Convert the remaining **even power** using the appropriate form of the identity:

$$\sin^2(x) + \cos^2(x) = 1 \quad \Rightarrow \quad \begin{cases} \sin^2(x) = 1 - \cos^2(x) \\ \cos^2(x) = 1 - \sin^2(x) \end{cases}$$

- Apply u-substitution.

Case 2: m and n are BOTH even

- Reduce **all even powers** using the identities

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$
$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

- You may need to expand and apply the previous step multiple times until you have eliminated all even powers.
- It is sometimes helpful to use the identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

Recall: $\tan^2 x = \sec^2 x - 1$

Ex) $\int \sec^3 x \tan^3 x\, dx$

Ex) $\int \sec^4 x \tan^2 x\, dx$

<u>Additional examples</u>

Ex) $\int_0^{\pi} \sqrt{1-\cos^2\theta}\, d\theta$

<u>Reminder of another identity that could be useful for WebAssign HW</u>:

$\cos 2x = \cos^2 x - \sin^2 x$