

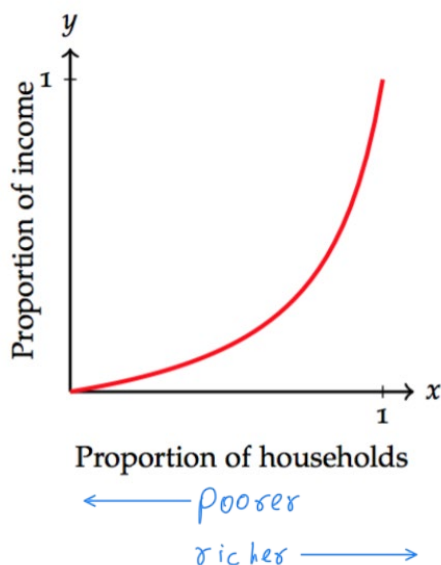
6.1 b – Income Inequality & the Gini Index

* Corrado Gini was an Italian economist who devised this index in 1912

* The Gini index is a measure of income inequality within a country

* First, let's discuss something called a Lorenz curve...

Let P designate the set of households in a country during a given period of time. The **Lorenz curve**, $L_P(x)$, for that country and year contains the points (x, y) such that the poorest $(100x)\%$ of households received $(100y)\%$ of the country's total income.



Ex) Suppose $L_P(x) = 0.3x + 0.7x^5$ defines a Lorenz curve for a population set P .

a) Write a sentence that explains what $L_P(0.45) = 0.148$ means.

The poorest 45% of the households earn 14.8% of the total income of Population P

b) How much income was earned by the poorest 20% of households in P ?

$$\begin{aligned} L_P(0.2) &= 0.3(0.2) + 0.7(0.2)^2 \\ &= 0.060048 \approx 0.06 = 6\% \end{aligned}$$

c) How much income was earned by the wealthiest 10% of households in P ?

* First calculate the income of the poorest 90%

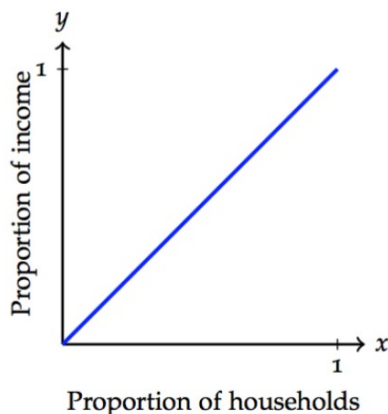
$$\begin{aligned} L_P(0.9) &= 0.3(0.9) + 0.7(0.9)^2 \\ &= 0.683343 \\ &= 68.3\% \end{aligned}$$

* The wealthiest earn $1 - L_P$ of the total income

$$\begin{aligned} &= 1 - 0.683343 \\ &= 0.316657 \\ &= 31.7\% \end{aligned}$$

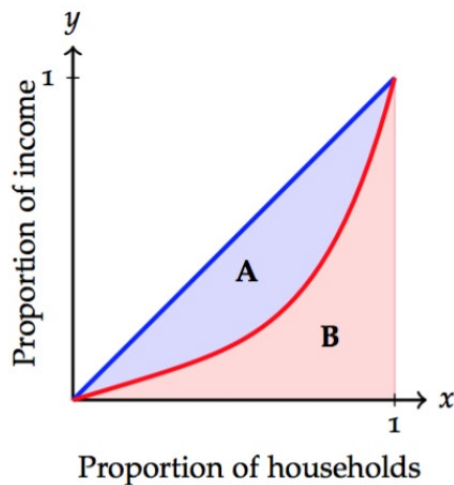
Perfectly Equal Income Distribution

* Let Id (for identity) represent a hypothetical egalitarian country in which every household earns an equal share of income. $L_{Id}(x) = x$ is the Lorenz curve for this population. The poorest $a\%$ of the population received $a\%$ of the income (... everyone receives the same income).



Gini Index

The Gini index is the area between the Lorenz curve and the line $y = x$ divided by the area under $y = x$.



$$G_2 = \frac{\int_0^1 (L_{x_1}(x) - L_p(x)) dx}{\int_0^1 (L_{x_1}(x)) dx}$$

$$G_2 = \frac{A}{A+B}$$

So the Gini index can be calculated using:

$$G = 1 - 2 \int_0^1 L_P(x) dx$$

Note: a) This index can range from 0, meaning every household receives the same income, to 1, meaning a single household receives all the income in the entire country.

b) A higher index indicates that the country's income distribution is more unequal.

c) $G = 0.4$ is often reported as a percentage (40%)

d) World Bank estimates for the Gini indices of countries range from 0.255 to 0.630

e) Taxation can change a country's Gini index (estimates indicate the U.S.'s pre-tax Gini index at 0.486 and post-tax at 0.378)

Ex) Compute the Gini index for the Lorenz curve $L_p(x) = 0.5x^2 + 0.5x^4$.

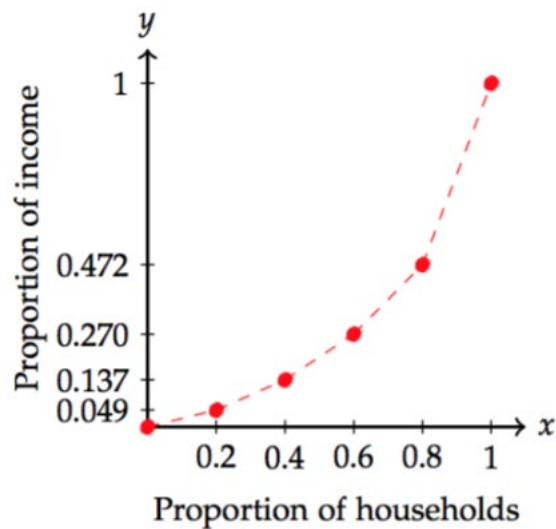
Ex) The Gini index for the Lorenz curve $L_p(x) = x^a$ is $G = 0.8$. Find the value of a .

Approximating the Gini Index

The table below shows Lorenz curve data for several countries as estimated by the World Bank. Each estimate was based on that country's most recent census.

	Proportion of households					
	0	0.2	0.4	0.6	0.8	1
Brazil	0	0.029	0.100	0.224	0.414	1
China	0	0.047	0.144	0.297	0.529	1
India	0	0.085	0.206	0.363	0.571	1
Mexico	0	0.049	0.137	0.270	0.472	1
Nigeria	0	0.044	0.127	0.257	0.460	1
Russia	0	0.061	0.165	0.313	0.526	1

Ex) a) Use the Trapezoid Rule to approximate the Gini index for Mexico



b) Why couldn't we use Simpson's rule to approximate the Gini index for any country in the given table?

c) Use the Trapezoid Rule to approximate the Gini index for Russia

d) Using the Gini indices calculated for Mexico and Russia, in which country is the income more equally distributed?

Properties of the Lorenz Curve

For any population set P :

- a) the domain of the Lorenz curve is the closed interval $[0, 1]$
- b) the range of the Lorenz curve is also the closed interval $[0, 1]$
- c) $L_P(0) = 0$ means 0% of households receive 0% of the income
- d) $L_P(1) = 1$ means 100% of households receive 100% of the income