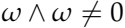


# 1. Ejercicio 2.2

W E A R A 2 3



1999



1999

AI1R4

A12 IR





$w \in A_1^2 R_1^4$   $w \in A_1^4 R_1^4$



$$\omega = \lambda_1 \varepsilon_1 \wedge \varepsilon_2 + \cdots + \lambda_6 \varepsilon_3 \wedge \varepsilon_4$$

$$\omega = \lambda_1 \varepsilon_1 \wedge \varepsilon_2 + \cdots + \lambda_6 \varepsilon_3 \wedge \varepsilon_4 \tag{1}$$

$$\text{si } \lambda_1 = \lambda_6 = 1, \quad \omega \wedge \omega = 2\varepsilon_1 \wedge \varepsilon_2 \wedge \varepsilon_3 \wedge \varepsilon_4$$

# 2. Ejercicio 2.3

$$\mathbb{R}^3 \overset{i}{\rightarrow} Alt^1(\mathbb{R}^3), \quad \mathbb{R}^3 \overset{j}{\rightarrow} Alt^2 \mathbb{R}^3$$

$$i(v)(v) = (v, v), \quad j(v)(v_1, v_2) = \det(v, v_1, v_2),$$





Q1 Q2 E I 2

$$i(v_1) \wedge i(v_2) = j(v_1 \times v_2).$$





$$i\hbar\frac{\partial}{\partial t}\psi_1 + \hat{H}\psi_1 = i\hbar\frac{\partial}{\partial t}\psi_2 + \hat{H}\psi_2$$

$$i(\lambda v_1 + v_2) = i\lambda v_1 + i v_2 = \lambda i(v_1) + i(v_2)$$



$$i(e_1)(v) = e_1(v) \quad \wedge_1 i(e_j)(v) = e_j(v)$$





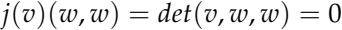


$$v \in \mathbb{R}^3 \Rightarrow j(v) \in \mathcal{A}l^2(\mathbb{R}^3)$$



$$\det(v, v_1 + v_2, v_3) = j(v)(v_1 + v_2, v_3) = j(v, v_1, v_3) + j(v, v_2, v_3) = \det(v, v_1, v_3) + \det(v, v_2, v_3)$$









$$i(\lambda v_1 + v_2)(v_1, v_2) = \det(\lambda v_1 + v_2, v_1, v_2) = \lambda \det(v_1, v_1, v_2) + \det(v_2, v_1, v_2) = \lambda i(v_1) + i(v_2)(v_1, v_2)$$



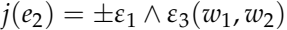
1e1 A2e1 A2e1

AI2R3

1. 123456789

$$(w_1, w_2) \mapsto \det \begin{pmatrix} 1 & 0 & 0 \\ w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{pmatrix} = w_{12}w_{23} - w_{13}w_{22} \stackrel{?}{=} \varepsilon_2 \wedge \varepsilon_3(w_1, w_2).$$

$$e_2 \wedge e_3(v_1, v_2) = \sum_{\sigma} \text{sgn}(\sigma) e_2(v_{\sigma(1)}) e_3(v_{\sigma(2)}) = e_2(v_1) e_3(v_2) - e_2(v_2) e_3(v_1) = v_1 v_2 - v_2 v_1$$





परिभाषा

$\frac{1}{2} \ln \frac{1}{2} = -\frac{1}{2} \ln 2$

$$i(\sigma_1) \wedge i(\sigma_2) = \sum \sigma_3(\sigma_1) i(\sigma_2) \wedge i(\sigma_1) \wedge \sigma_2 = A_{03} \sigma_1 \wedge \sigma_2 + A_{02} \sigma_1 \wedge \sigma_3 + A_{01} \sigma_2 \wedge \sigma_3$$

$$i(v_1) \wedge i(v_2)(e_1 e_2) = i(v_1)(e_2) i(v_2)(e_1) = v_1 v_2 - v_1 v_2 = 0$$

# 3. Ejercicio 2.5

APR 20

$$\langle w_1 \wedge \dots \wedge w_p, \tau_1 \wedge \dots \wedge \tau_n \rangle,$$

WIP: AI1V1



$$(w, \tau) = (i^{-1}(w), i^{-1}(\tau)).$$

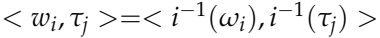
1911-11-11



2 = 1000

$$\{\beta_{\sigma(1)} \wedge \dots \wedge \beta_{\sigma(p)} \mid \sigma \in S(p, n-p)\}$$

APR 20



Al1v



$$\langle w_i, \tau_j \rangle = \langle i^{-1}(w_i), i^{-1}(\tau_j) \rangle = i(i^{-1}(w_i))(i^{-1}(\tau_j)) = w_i(i^{-1}(\tau_j))$$

$$\langle \tau_j, \omega_i \rangle = \langle i^{-1}(\tau_j), i^{-1}(\omega_i) \rangle = i(i^{-1}(\tau_j))(i^{-1}(\omega_i)) = \tau_j(i^{-1}(\omega_i))$$

$$\begin{aligned}
\det(\langle \omega_i, \tau_j \rangle) &= \det(\omega_i(i^{-1}(\tau_j)))_{i,j \in \{1, \dots, p\}} = \\
&= \det \begin{pmatrix} \omega_1(i^{-1}(\tau_1)) & \cdots & \omega_1(i^{-1}(\tau_p)) \\ \vdots & & \vdots \\ \omega_p(i^{-1}(\tau_1)) & \cdots & \omega_p(i^{-1}(\tau_p)) \end{pmatrix} \\
&= \omega_1 \wedge \cdots \wedge \omega_p(i^{-1}(\tau_1), \dots, i^{-1}(\tau_p)) \\
&= (\tau_1 \wedge \cdots \wedge \tau_p)(i^{-1}(\omega_1, \dots, i^{-1}(\omega_p))).
\end{aligned} \tag{2}$$

$$(a_1 \wedge \cdots \wedge a_p, \tau_1 \wedge \cdots \wedge \tau_p) = a_1 \wedge \cdots \wedge a_p(i^{-1}(\tau_1), \cdots, i^{-1}(\tau_p)).$$



$$\begin{aligned}
\langle \omega_1 \wedge \cdots \wedge \omega_p, \tau_1 \wedge \cdots \wedge \tau_p \rangle &= \\
&= \omega_1 \wedge \cdots \wedge \omega_p(i^{-1}(\tau_1), \dots, i^{-1}(\tau_p)) \\
&= \langle \tau_1 \wedge \cdots \wedge \tau_p, \omega_1 \wedge \cdots \wedge \omega_p \rangle \\
&= (\tau_1 \wedge \cdots \wedge \tau_p)(i^{-1}(\omega_1), \dots, i^{-1}(\omega_p)).
\end{aligned} \tag{3}$$

Welp, AI is up to  
the task.

$$\omega = \sum_{\sigma \in S(p, n-p)} \omega(e_{\sigma(1)}, \dots, e_{\sigma(p)}) \varepsilon_{\sigma}$$

$$\tau = \sum_{\bar{\sigma} \in S(p, n-p)} \omega(e_{\bar{\sigma}(1)}, \dots, e_{\bar{\sigma}(p)}) \varepsilon_{\bar{\sigma}}$$

$$\langle \omega, \tau \rangle = \sum_{\sigma} \sum_{\bar{\sigma}} \omega_{\sigma} \tau_{\bar{\sigma}} \langle \varepsilon_{\sigma}, \varepsilon_{\bar{\sigma}} \rangle. \quad (4)$$

$$\langle \omega, \omega \rangle = \sum_{\sigma} (\omega_{\sigma})^2 \geq 0$$

$$\langle \varepsilon_{\sigma}, \varepsilon_{\bar{\sigma}} \rangle = \varepsilon_{\sigma(1)} \wedge \dots \wedge \varepsilon_{\sigma(p)}(e_{\bar{\sigma}(1)}, \dots, e_{\bar{\sigma}(p)}) = I_{\sigma}(\bar{\sigma})$$

$$\langle \omega, \omega \rangle = 0 \Leftrightarrow \omega_{\sigma} = 0 \forall \sigma \in S(p, n-p) \Rightarrow \omega = 0 \in \text{Alt}^p(V).$$



APR 20

$$\begin{aligned}
\langle \omega, i(\tilde{\zeta}_1) \wedge \cdots \wedge i(\tilde{\zeta}_p) \rangle &= \\
&= \langle \sum_{\sigma} \omega_{\sigma} \varepsilon_{\sigma}, i(\tilde{\zeta}_1 \wedge \cdots \wedge i(\tilde{\zeta}_p)) \rangle \\
&= \sum_{\sigma} \omega_{\sigma} \langle \varepsilon_{\sigma}, i(\tilde{\zeta}_1) \wedge \cdots \wedge i(\tilde{\zeta}_p) \rangle \\
&= \sum_{\sigma} \omega_{\sigma} \varepsilon_{\sigma}(\tilde{\zeta}_1, \dots, \tilde{\zeta}_p) \\
&= \omega(\tilde{\zeta}_1, \dots, \tilde{\zeta}_p).
\end{aligned} \tag{5}$$

$$(e_{\sigma(1)} \wedge \dots \wedge e_{\sigma(p)}) = (e_{\sigma(1)} \wedge \dots \wedge e_{\sigma(p)}) = e_{\sigma(1)} \wedge \dots \wedge e_{\sigma(p)}.$$

$$a = \Sigma (e_{\sigma(1)} \wedge \dots \wedge e_{\sigma(p)}) e_{\sigma(1)} \wedge \dots \wedge e_{\sigma(p)} = \Sigma (a \cdot e_{\sigma(1)} \wedge \dots \wedge e_{\sigma(p)}) e_{\sigma(1)} \wedge \dots \wedge e_{\sigma(p)}$$

1911-12



$$\langle b_i, b_j \rangle = \begin{cases} 0 & \text{si } i \neq j \\ 1 & \text{si } i = j \end{cases}$$

1919-1919



Al1v

1991-1993  
1994-1996  
1997-1999

APR 20

$$\langle \beta_{\sigma(1)} \wedge \cdots \wedge \beta_{\sigma(p)}, \beta_{\bar{\sigma}(1)} \wedge \cdots \wedge \beta_{\bar{\sigma}(p)} \rangle = \beta_{\sigma(1)} \wedge \cdots \wedge \beta_{\sigma(p)} (\beta_{\bar{\sigma}(1)}, \dots, \beta_{\bar{\sigma}(p)}) = \begin{cases} 0 & \text{si } \sigma \neq \bar{\sigma} \\ 1 & \text{si } \sigma = \bar{\sigma} \end{cases}$$

4. Ejercicio 2.6

W E A I P W

A 10x10 grid of 100 grayscale images. Each image is a small, pixelated representation of a digit from 0 to 9. The digits are arranged in a regular grid, with each digit appearing once in each row and column. The digits are rendered in various styles, including different shades of gray, pixelation, and slight rotations, giving the grid a diverse and artistic appearance.

W E A I P W





APPROPRIATE

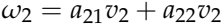
$$w_i = \sum_j w_{ij} x_j = 1 \quad \forall i \quad w_i = 1 \quad \forall i$$

$$\psi(v_1, \dots, v_p) = \psi(v_1, \dots, v_p, 0, \dots, 0)$$



A pixelated, black and white graphic of the text "A = 1/2 pi e 1/2". The text is rendered in a stylized, blocky font where each character is composed of many small squares. The equals sign is represented by two parallel horizontal bars. The pi symbol is a circle with a vertical line through its center. The entire graphic is set against a white background.

[illegible]





$$\begin{aligned}
 \omega(\omega_1, \omega_2) &= \omega(a_{11}v_1 + a_{12}v_2, a_{21}v_1 + a_{22}v_2) = \\
 &= a_{11}a_{22}\omega(v_1, v_2) + a_{12}a_{21}\omega(v_2, v_1) = \\
 &= a_{11}a_{22}\omega(v_1, v_2) - a_{12}a_{21}\omega(v_1, v_2) = \\
 &= (a_{11}a_{22} - a_{12}a_{21})\omega(v_1, v_2) = \det(A) \cdot \omega(v_1, v_2).
 \end{aligned} \tag{6}$$



$$\begin{aligned}
\omega(\omega_1, \dots, \omega_p) &= \omega\left(\sum_{j=1}^p a_{ij} v_j, \dots, \sum_{j=1}^p a_{pj} v_j\right) = \\
&= \sum_{\tau \in S(p)} \prod_{k=1}^p a_{k\tau(k)} \omega(v_{\tau(1)}, \dots, v_{\tau(p)}) = \\
&= \underbrace{\sum_{\tau \in S(p)} \operatorname{sgn}(\tau) \prod_{k=1}^p a_{k\tau(k)}}_{\det(A)} \cdot \omega(v_1, \dots, v_p).
\end{aligned} \tag{7}$$

# 5. Ejercicio 2.7

$$\begin{aligned}
Alt^{p+q}(f)(\omega_1 \wedge \omega_2) &= \\
&= \omega_1 \wedge \omega_2(f(\xi_1), \dots, f(\xi_{p+q})) \\
&= \sum_{\sigma \in S(p, n-p)} sgn(\sigma) \omega_1(f(\xi_{\sigma(1)}), \dots, f(\xi_{\sigma(p)})) \omega_2(f(\xi_{\sigma(p+1)}), \dots, f(\xi_{\sigma(p+q)})) \\
&= \sum_{\sigma \in S(p, n-p)} sgn(\sigma) Alt^p(f) \omega_1(\xi_{\sigma(1)}, \dots, \xi_{\sigma(p)}) Alt^q(f) \omega_2(\xi_{\sigma(p+1)}, \dots, \xi_{\sigma(p+q)}) \\
&= Alt^p(f)(\omega_1) \wedge Alt^q(f)(\omega_2)(\xi_1, \dots, \xi_{p+q}).
\end{aligned} \tag{8}$$