Finite impulse response filtering
Moving average filter

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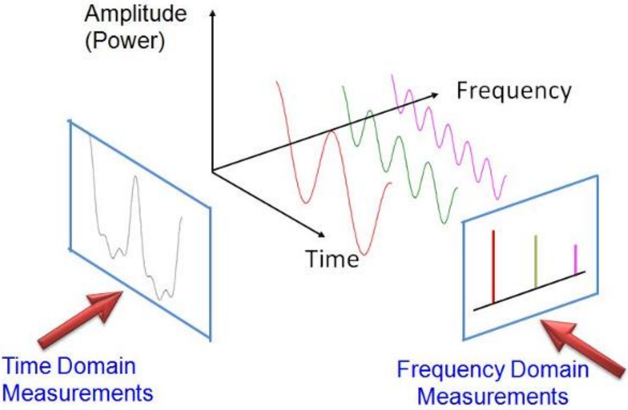
Filtering in different domains

- Filtering in **time domain** (signal restoration, smoothing, denoising).
- Filtering in **frequency domain** (signal separation).
- Choosing filtering in time or frequency domain depends on **where the information is**.



Classification of discrete filters

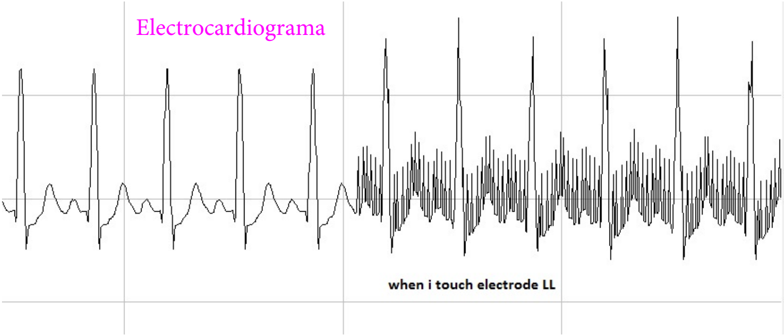
Table: Classification of discrete filters

	Finite impulse response (FIR)	Infinite impulse response (IIR)
Filtering in time domain	Moving average	Leaky Integrator
Filtering in frequency domain	Windowed Filters Equiripple Minimax	Bilinear z-transform

Information in time domain

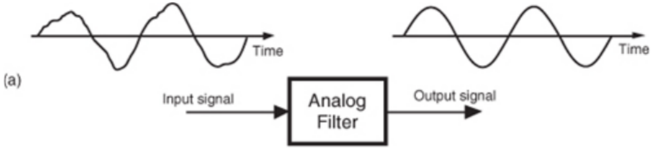
- Information is contained in amplitude and time of the signal.
- **Each sample** contains information that is interpretable without reference to any other sample.En el dom de la frecuencia en cambio para filtrar necesitamos varias muestras
- The **step response** describes how information in time domain is being modified by the system. Cuán bien o mal trabaja el sistema
- Examples: electrocardiography (ECG) signal, accelerometer, temperature...

Electrocardiograma

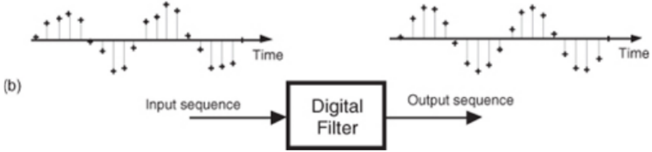


FIR filtering structure

Figure 5-1 Filters: (a) an analog filter with a noisy tone input and a reduced-noise tone output; (b) the digital equivalent of the analog filter.



No existen filtros analógicos en el dominio del tiempo



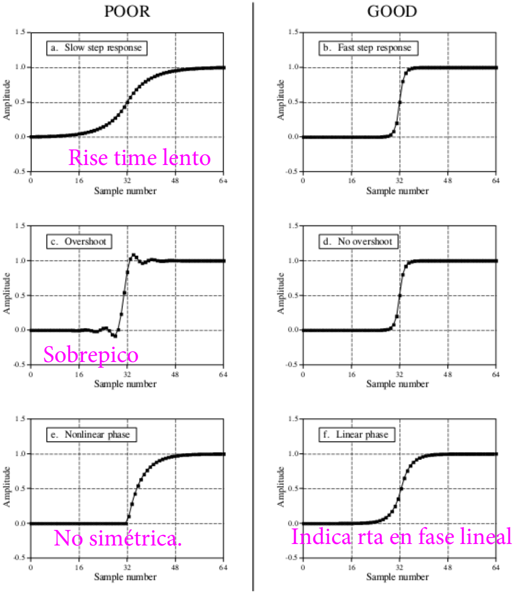
Time domain parameters, step response

- Risetime (between 10%∼90% amplitude).
- Overshoot.
- Linear phase.

It is not possible to optimize a filter for both domains.

Good performance in the time domain results in poor performance in the frequency domain, and vice versa.

Se deben usar 2 filtros para manejar bien ambos dominios



Moving average filter

- The moving average filter is a convolution of the input signal with a rectangular pulse having an area of one.
- *Local average*.
- There is a delay of N/2 samples between input and output.

Ahí empieza a dar salida. Pero son los N últimos que pasaron

$$h[n] = \frac{1}{N} \sum_{k=0}^{N-1} \delta[n-k] \,, \tag{1}$$

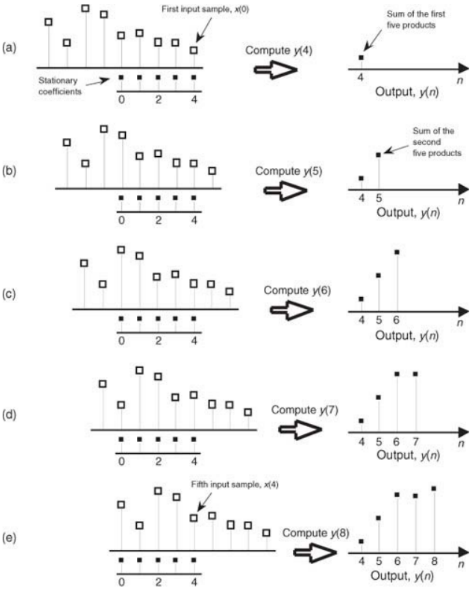
$$h[n] = \begin{cases} \frac{1}{N} & 0 \leq n < N \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$y[n] = x[n] * h[n] = \frac{1}{N} \sum_{k=0}^{N-1} x[n-k] \tag{3}$$

- It can be seen that the moving average filter is a FIR filter. Why?

Porque la salida actual no depende de salidas pasadas->No hay realimentación->Filtro FIR

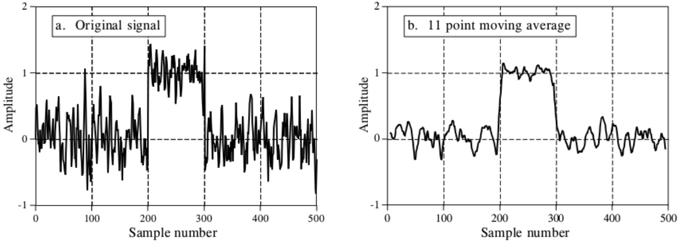
Moving average filter, example



Primer elemento sale en tiempo 4

Noise Reduction vs. Step Response

- MA reduces random white noise while trying to keep the sharpest step response.



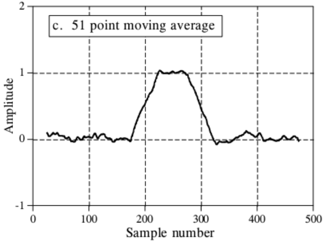
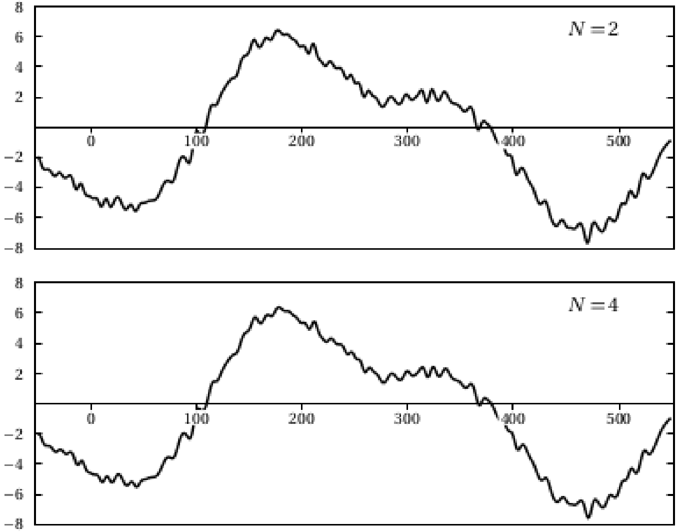


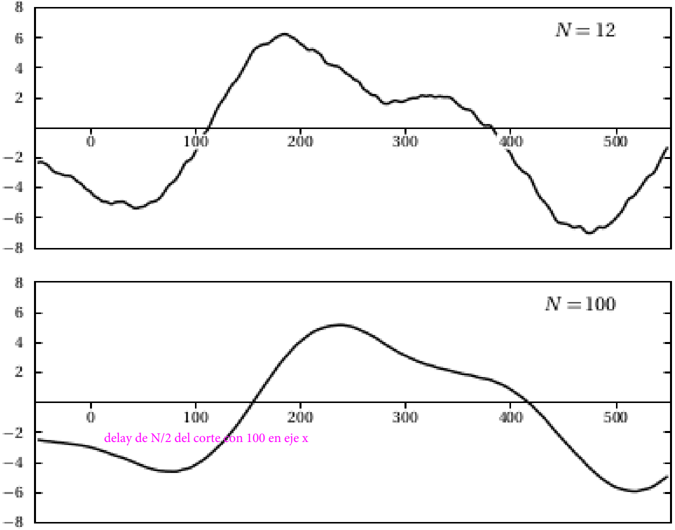
FIGURE 15-1
Example of a moving average filter. In (a), a rectangular pulse is buried in random noise. In (b) and (c), this signal is filtered with 11 and 51 point moving average filters, respectively. As the number of points in the filter increases, the noise becomes lower; however, the edges becoming less sharp. The moving average filter is the *optimal* solution for this problem, providing the lowest noise possible for a given edge sharpness.

Reducimos ruido a costa de perder componentes de alta frecuencia

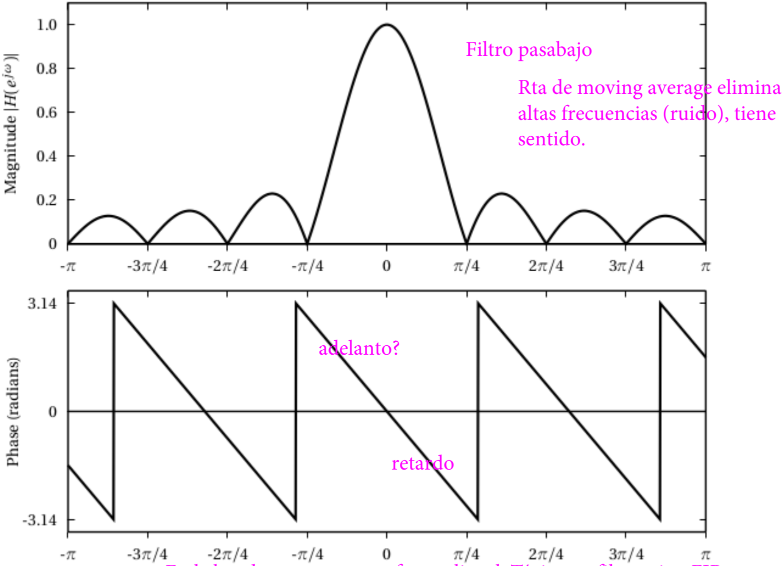

N = 2
N = 4
8
6
4
2
−2
−4
−6
−8
0
100
200
300
400
500

Noise Reduction, 2

- Note how the signal is delayed as N grows.



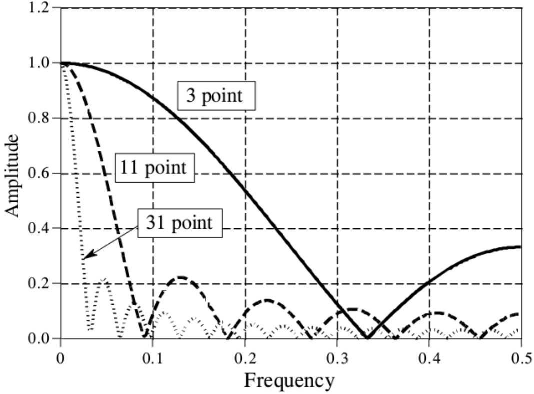
Un cuadrado en dom del tpo tiene rta en f de seno cardinal y vicewversa



En la banda pasante rta en fase es lineal. Típico en filtros tipo FIR

- The moving average filter is a bad low-pass filter.
- In short, the moving average is a good *smoothing filter* (the action in the time domain), but a bad low-pass filter (the action in the frequency domain).

$$|H[f]| = \frac{1}{N}\left|\frac{\sin(\pi \cdot f \cdot N)}{\sin(\pi \cdot f)}\right| \tag{4}$$



Eje de f normalizado segun f de muestreo. La máxima es 0.5 porque es la mitad de la fs.

MA cut-off frequency

Criterio para no eliminar componentes de alta frecuencia que son parte de la señal de interés

The cut-off frequency, f_{co}, can be defined as the frequency at which the magnitude ratio is 0.707 (-3 dB),

seno cardinal

$$|H[f]| = \frac{1}{N} \left| \frac{\sin(\pi \cdot f \cdot N)}{\sin(\pi \cdot f)} \right| \tag{5}$$

$$0.707 = \frac{1}{N} \left| \frac{\sin(\pi \cdot f_{co} \cdot N)}{\sin(\pi \cdot f_{co})} \right| \quad \text{=-3dB} \tag{6}$$

If N is the length of the moving average, then an approximate normalized cut-off frequency $F_{co} = f_{co}/f_s$ (valid for $N \geq 2$) is [3,4],

$$F_{co} = \frac{0.885894}{\sqrt{N^2 - 1}} \tag{7}$$

$$\implies N = \sqrt{\frac{0.885894^2}{F_{co}^2} - 1} \tag{8}$$

Maximum filter order

The following equation imposes a limit to the value of N from the maximum frequency of the signal to be smoothed.

round porque N es entero

$$N_{max} = \text{round}\left(\sqrt{\frac{0.885894^2 \cdot f_s^2}{f_{co}^2} - 1}\right) \tag{9}$$

Desnormalizar para que aparezca fs y fcorte.

Example:

- The dynamics of a robots will be studied using an accelerometer.
- The maximum dynamic frequency of the robot it is assumed to be around 10 Hz.
- If the accelerometer is sampled at 100 Hz, the maximum order of an MA filter is:

$$N_{max} = \text{round}\left(\sqrt{\frac{0.885894^2 \cdot 100^2}{10^2} - 1}\right) = 9 \tag{10}$$

What happens if the sampling frequency is increased?

Bibliography

1 Paolo Prandoni and Martin Vetterli. Signal processing for communications. Taylor and Francis Group, LLC. 2008. Sections 5.2 and 5.3.1. https://www.sp4comm.org/.

2 Steven W. Smith, The Scientist and Engineer's Guide to Digital Signal Processing. Chapters 14 and 15. www.dspguide.com.

3 What is the cut-off frequency of a moving average filter? Link.

4 3 dB cut-off frequency of moving average. Link.