PID controllers

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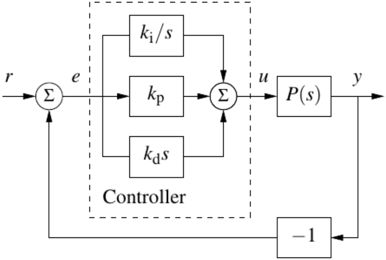
A survey of controllers for more than 100 boiler-turbine units in the Guangdong Province in China [1]:

- 94.4% of all controllers were PI,
- 3.7% PID,
- 1.9% used advanced control.



$$u(t) = k_p \cdot e(t) + k_i \int_0^t e(\tau)d\tau + k_d \frac{de(t)}{dt} = k_p \left(e(t) + \frac{1}{T_i} \int_0^t e(\tau)d\tau + T_d \frac{de(t)}{dt} \right) , \quad (1)$$

$$k_i = \frac{k_p}{T_i} , \quad (2)$$

$$k_d = k_p \cdot T_d . \quad (3)$$

[1] Li Sun, Donghai Li, and Kwang Y. Lee. Optimal disturbance rejection for pi controllerwith constraints on relative delay margin.ISA Transactions,2016.

Al aumentar Kp se achica la brecha pero se hace más oscilatorio

Desaparece la brecha, a costa de más oscilación

Pico disminuye y el sistema responde más rápido

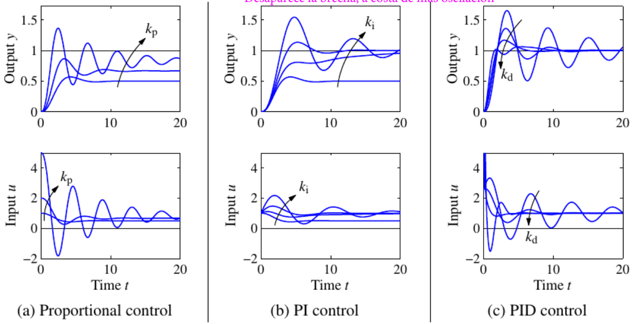


Figure 11.2: Responses to step changes in the reference value for a system with a proportional controller (a), PI controller (b) and PID controller (c). The process has the transfer function $P(s) = 1/(s+1)^3$, the proportional controller has parameters $k_p = 1$, 2 and 5, the PI controller has parameters $k_p = 1$, $k_i = 0$, 0.2, 0.5, and 1, and the PID controller has parameters $k_p = 2.5$, $k_i = 1.5$ and $k_d = 0$, 1, 2, and 4.

$$u = k_p \cdot e + k_i \int_0^t e(\tau)d\tau + k_d \frac{de}{dt} = k_p \left(e + \frac{1}{T_i} \int_0^t e(\tau)d\tau + T_d \frac{de}{dt} \right) \tag{4}$$

- If the plant mathematical model cannot be obtained at all, then an analytical or computational approach to the design of a PID controller is not possible.
- Then we must resort to experimental approaches to the tuning of PID controllers.
- Ziegler and Nichols suggested rules for tuning PID controllers (values K_p , T_i, and T_d) based on:
  - Experimental step responses (Method 1).
  - Based on the value of K_p that results in marginal stability when only proportional control action is used (Method 2).

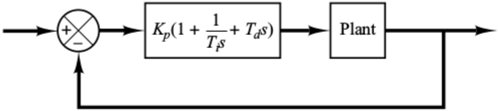


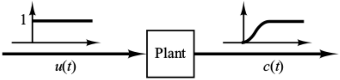
Figure 8–1 PID control of a plant.

PID Control Tuning

Method 1

- We obtain experimentally the response of the plant to a unit-step input.
- This method applies if the response to a step input exhibits an S-shaped curve.
- If the plant involves neither integrator(s) nor dominant complex-conjugate poles, then such a unit-step response curve may look S-shaped.

Figure 8–2
Unit-step response of a plant.



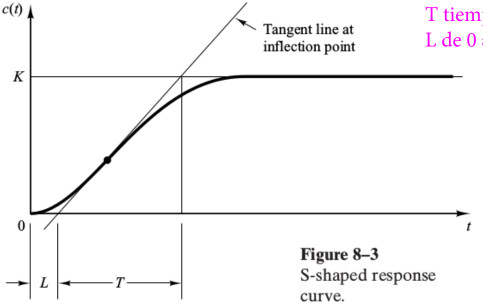


Figure 8–3
S-shaped response curve.

T tiempo de 10% a 90%
L de 0 a 10%

- The S-shaped curve may be characterized by two constants, delay time L and time constant T.

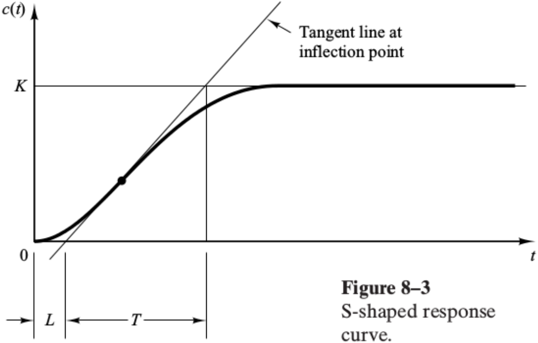


Figure 8–3
S-shaped response curve.

Table 8–1 Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

function $C(s)/U(s)$ may then be approximated by a first-order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Ziegler and Nichols suggested to set the values of K_p, T_i, and T_d according to the formula shown in Table 8–1.

Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right) \\ &= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s} \end{aligned}$$

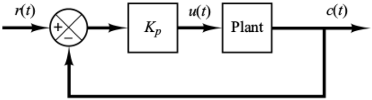
Thus, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

- We first set $T_i = \infty$, and $T_d = 0$.

Este método cuando la rta al escalón no fue tipo S
No es sistema de 1er orden, es más complejo

- Increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations.
- Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally determined.
- If the output does not exhibit sustained oscillations, then this method does not apply.

Figure 8–4
Closed-loop system with a proportional controller.



A veces NO querés que la planta oscile en la práctica. Puede ser peligroso

Figure 8–5
Sustained oscillation with period P_{cr}. (P_{cr} is measured in sec.)

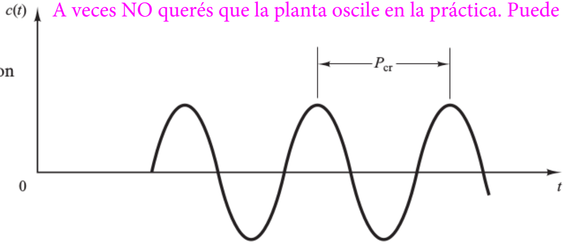


Table 8–2 Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right) \\ &= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s} \end{aligned}$$

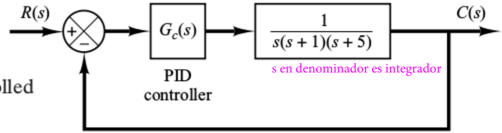
Thus, the PID controller has a pole at the origin and double zeros at $s = -4/P_{cr}$.

EXAMPLE 8–1 Consider the control system shown in Figure 8–6 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler–Nichols tuning rule for the determination of the values of parameters K_p, T_i, and T_d. Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25% or less.

Planta tiene un integrador, asi que hay que usar método 2

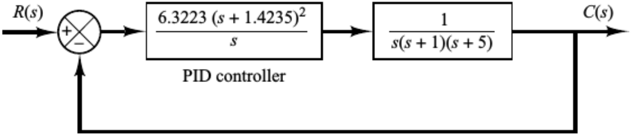


s en denominador es integrador

Figure 8–6 PID-controlled system.

En el libro resuelve detallado

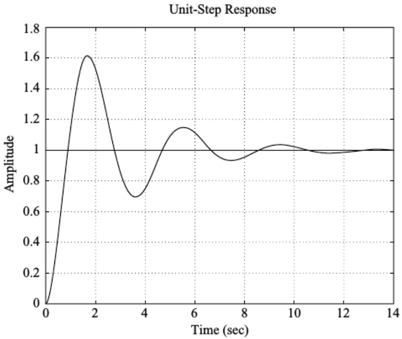
$$
\begin{aligned}
K_{cr} &= 30 \\
P_{cr} &= 2.8099 \\
K_p &= 0.6 \cdot K_{cr} = 18 \\
T_i &= 0.5 \cdot P_{cr} = 1.405 \\
T_d &= 0.125 \cdot P_{cr} = 0.35124
\end{aligned}
$$



Maximum overshoot is close to 62%.

MATLAB Program 8–1

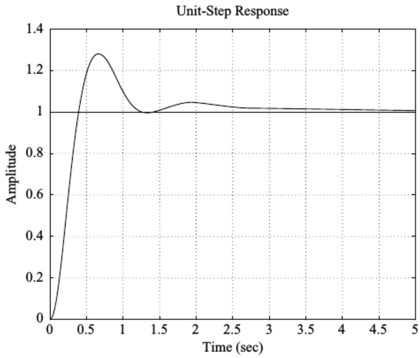
```
% ---------- Unit-step response ----------

num = [6.3223  18  12.811];
den = [1  6  11.3223  18  12.811];
step(num,den)
grid
title('Unit-Step Response')
```



The Ziegler–Nichols tuning rule has provided a starting point for fine tuning.

$$K_p = 39.42$$
$$T_i = 3.077$$
$$T_d = 0.7692$$

Maximum overshoot is fairly close to 25%.



- The value of Kp increases the speed of response.
- However, varying the location of the double zero has a significant effect on the maximum overshoot.

Bibliography

- Karl J. Astrom and Richard M. Murray *Feedback Systems*. Version v3.0i. Princeton University Press. September 2018. Chapter 11.
- Ogata, Katsuhiko. Modern Control Engineering. Fifth Edition. Prentice Hall. 2009. Chapter 8.