

# Typical stages in digital signal processing

## Aliasing prefiltering

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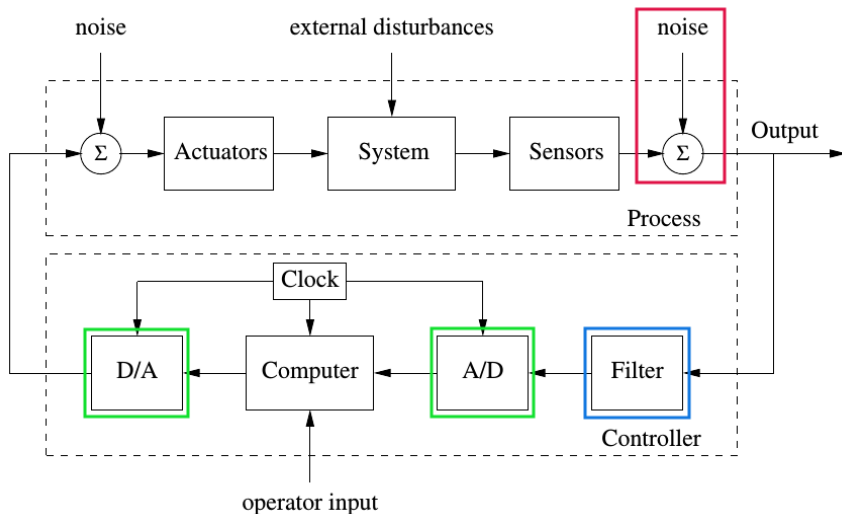
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- 1 DSP in a the context of control systems
- 2 Digital processing of analog signals
- 3 Sampling signals in the frequency domain
  - Periodic sampling
  - Frequency-domain representation of sampling
- 4 Aliasing prefiltering

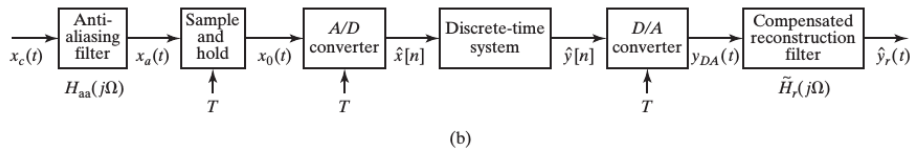
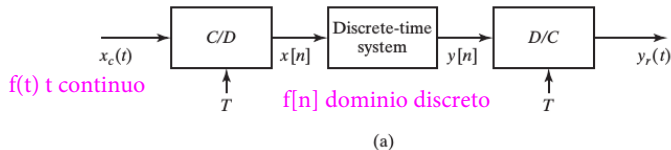
# DSP in the context of control systems



# Digital processing of analog signals

Conversor A/D ideal a frecuencia  $1/T$

D/A ideal a  $f=1/T$

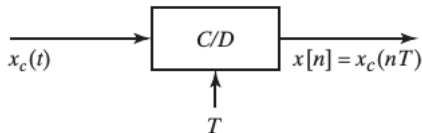


**Figure 4.47** (a) Discrete-time filtering of continuous-time signals. (b) Digital processing of analog signals.

The discrete-time representation of a continuous-time signal is obtained through periodic sampling from a continuous-time signal  $x_c(t)$  according to,

$$x[n] = x_c(nT), \quad -\infty < n < \infty, \quad (1)$$

where  $T$  is the sampling period, and  $f_s = 1/T$  is the sampling frequency, or  $\Omega_s = 2\pi/T$  in radians/s

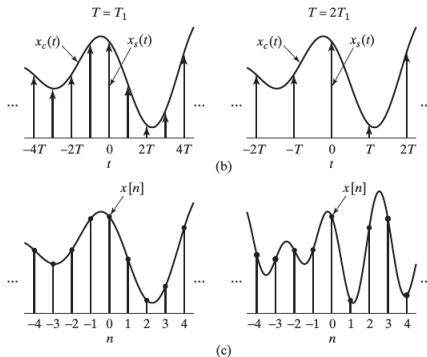
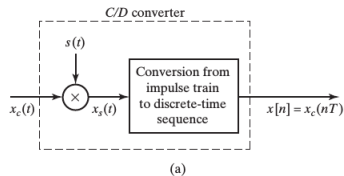


**Figure 4.1** Block diagram representation of an ideal continuous-to-discrete-time (C/D) converter.

# Sampling process

It is convenient to represent the sampling process mathematically in the two stages.

- 1 An impulse train  $s(t)$  is multiplied by a continuous-time signal  $x_c(t)$ .
- 2 The continuous-time signal  $x_s(t)$  is transformed to a discrete-time sequence  $x[n]$ .



**Figure 4.2** Sampling with a periodic impulse train, followed by conversion to a discrete-time sequence. (a) Overall system. (b)  $x_s(t)$  for two sampling rates. (c) The output sequence for the two different sampling rates.

$x_s(t)$  es tren de impulsos modulado

Bloque convierte tren a valores binarios

(c) son bastante distintos porque abarcan rangos de  $x_c(t)$  distintos, ojo

Si tomás la der entre -2 a 2 coincide con la otra. Creo que es un mal ejemplo al problema de usar  $T$  muy largo

# Nyquist-Shannon Sampling Theorem

Let  $x_c(t)$  be a bandlimited signal with,

$$X_c(j\Omega) = 0 \text{ para } |\Omega| \geq \Omega_N. \quad (2)$$

johm implica que ya estamos en dominio de la frecuencia  
ohmN es máxima frecuencia  
que tiene la señal

Then  $x_c(t)$  is uniquely determined by its samples  $x[n] = x_c(nT)$ ,  $n = 0, \pm 1, \pm 2, \dots$  if,

$$\Omega_s = \frac{2\pi}{T} \geq 2\Omega_N. \quad (3)$$

The frequency  $\Omega_N$  is commonly referred to as the **Nyquist frequency**, and the frequency  $2\Omega_N$  as the **Nyquist rate**.

The Nyquist rate is the minimum sampling frequency in order to be able of reconstructing  $x_c(t)$ .

# Frequency-domain representation of sampling

$x_s(t)$  is obtained multiplying  $x_c(t)$  by a periodic impulse train  $s(t)$ ,

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT), \quad \begin{array}{l} \text{Vale 1 en } t=nT \text{ o vale infinito?} \\ \text{Dice profe que 1, y tiene sentido} \end{array} \quad (4)$$

$$x_s(t) = x_c(t) s(t), \quad (5)$$

$$= x_c(t) \sum_{n=-\infty}^{\infty} \delta(t - nT), \quad (6)$$

$$= \sum_{n=-\infty}^{\infty} x_c(nT) \delta(t - nT) \quad \text{by sifting property.} \quad (7)$$

The Fourier transform of the periodic impulse train  $s(t)$  is the periodic impulse train,

$$S(j\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_s), \quad \text{where } \Omega_s = \frac{2\pi}{T}. \quad (8)$$

The Fourier transform of  $x_s(t)$  is the continuous-variable convolution of  $X_c(j\Omega)$  and  $S(j\Omega)$ ,

$$\text{El producto de las señales en el dominio del tiempo (7) es una convolución en el dom de la frecuencia(9)} \quad X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega), \quad (9)$$

$$X_s(j\Omega) = \frac{1}{T} X_c[j(\Omega - k\Omega_s)]. \quad \text{Reemplacé (8) en (9)} \quad (10)$$

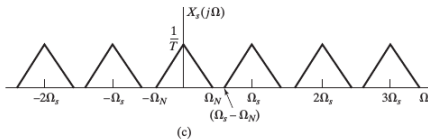
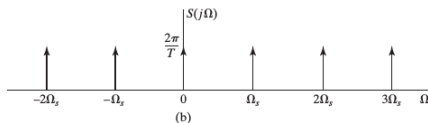
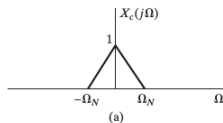


# Frequency-domain representation of sampling, 2

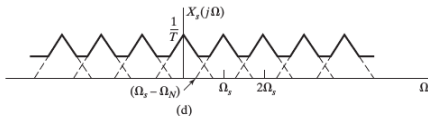
- Fourier transform of  $x_s(t)$  consists of periodically repeated copies of  $X_c(j\Omega)$
- These copies are shifted by integer multiples of the sampling frequency.
- It is evident that

$$\Omega_s - \Omega_N \geq \Omega_N, \text{ or,}$$

$$\Omega_s \geq 2\Omega_N$$

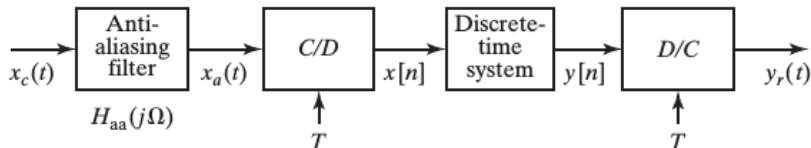


Aliasing



**Figure 4.3** Frequency-domain representation of sampling in the time domain. (a) Spectrum of the original signal. (b) Fourier transform of the sampling function. (c) Fourier transform of the sampled signal with  $\Omega_s \geq 2\Omega_N$ . (d) Fourier transform of the sampled signal with  $\Omega_s < 2\Omega_N$ .

The aliasing filter is an **analog** low-pass or band-pass filter.

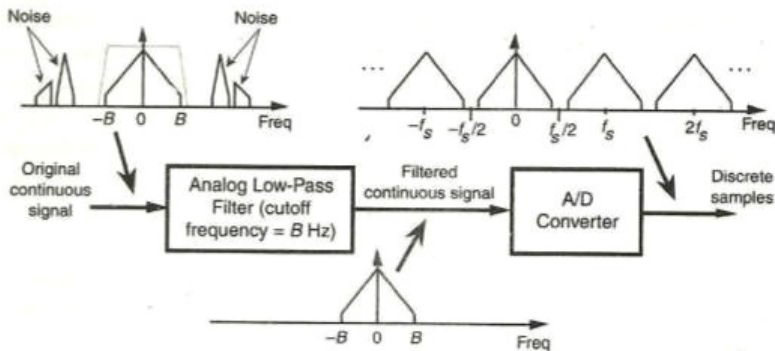


**Figure 4.48** Use of prefiltering to avoid aliasing.

El filtro garantiza que la máxima frecuencia que entre sea  $\omega_m$

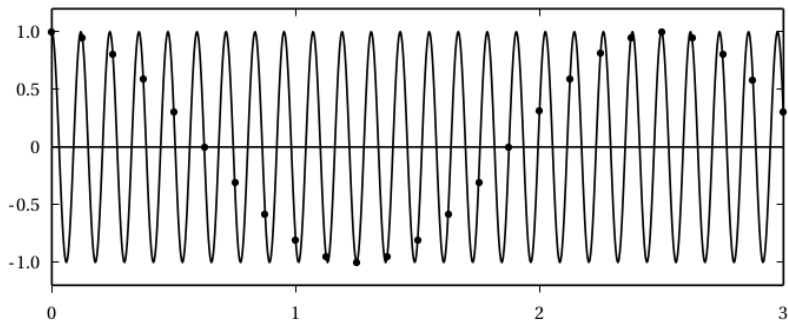
## Aliasing prefiltering, 2

Even if the signal is naturally bandlimited (as music), wideband additive noise may fill in the higher frequency range, and as a result of sampling, these noise components would be aliased into the low-frequency band.



El filtro anti Aliasing se come el ruido

## Aliasing prefiltering, example



**Figure 9.8** Example of aliasing: a sinusoid at 8400 Hz,  $x(t) = \cos(2\pi \cdot 8400t)$  (solid line) is sampled at  $F_s = 8000$  Hz. The sampled values (dots) are indistinguishable from those of at 400 Hz sinusoid sampled at  $F_s$ .

La señal la vas a ver, pero va a ser incorrecta

# ADC signal conditioning circuits (Ref. [4])

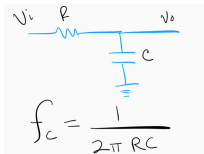


Figure: RC low-pass filter.

Se monta la entrada sobre una continua para que el conversor vea picos positivos y negativos

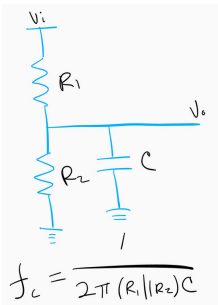


Figure: RC low-pass filter with voltage divider.

## Offset, amplificación y alta Z

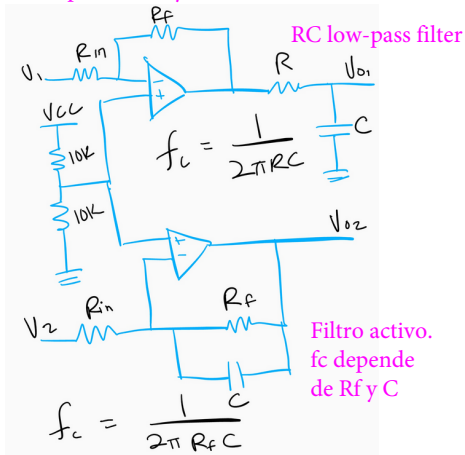
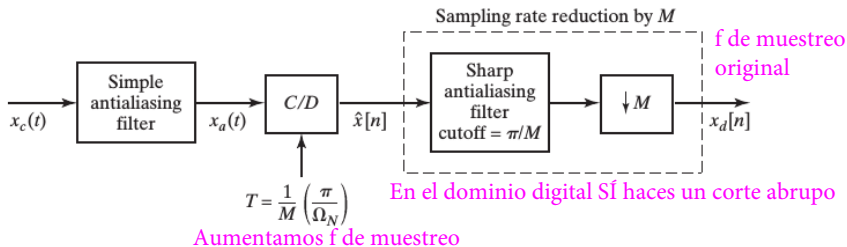


Figure: The top circuit is filtered using a passive filter while the bottom part uses an active filter.

Para V2 de alta  $f$  la ganancia del operacional se vuelve 0, porque el C hace de cortocircuito

Un filtro que corte tan abrupto es difícil de conseguir. Se usa esta técnica



**Figure 4.49** Using oversampled A/D conversion to simplify a continuous-time antialiasing filter.

# Oversampling frequency response

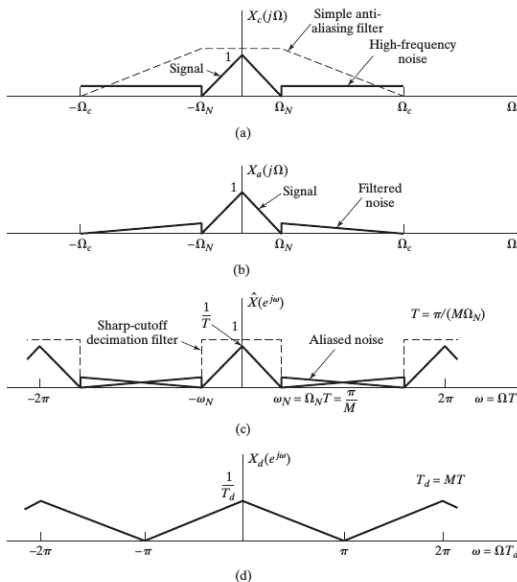


Figure 4.50 Use of oversampling followed by decimation in C/D conversion.

Este paso de bajar f de muestreo es opcional, te quedás con grafica (c)

- 1 Alan V. Oppenheim and Ronald W. Schaffer. *Discrete-time signal processing, 3rd Ed.* Prentice Hall. 2010. Sections 4.1, 4.2 and 4.3.
- 2 Richard G. Lyons. *Understanding Digital Signal Processing, 3rd Ed.* Prentice Hill. 2010. Section 12.3.1.
- 3 Paolo Prandoni and Martin Vetterli. *Signal processing for communications.* Taylor and Francis Group, LLC. 2008. Section 9.6.
- 4 Stratify Labs. ADC Signal Conditioning. [Link](#).