Modeling of physical systems

Dr. Ing. Rodrigo Gonzalez

rodrigo.gonzalez@ingenieria.uncuyo.edu.ar

Control y Sistemas

Ingeniería Mecatrónica,
Facultad de Ingeniería,
Universidad Nacional de Cuyo

May 2020

UNCUYO
UNIVERSIDAD
NACIONAL DE CUYO

Modeling of physical systems

System

Model

Analysis

Simulation

Prediction

Design

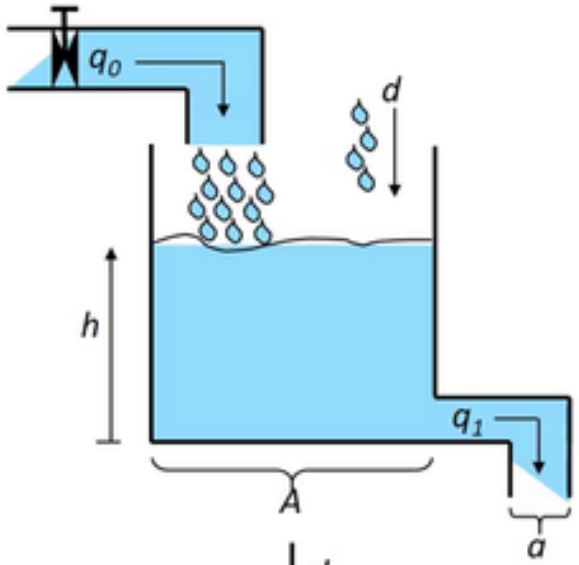
- *A system* is a piece of the reality
- A *model* is a simplified description of the reality

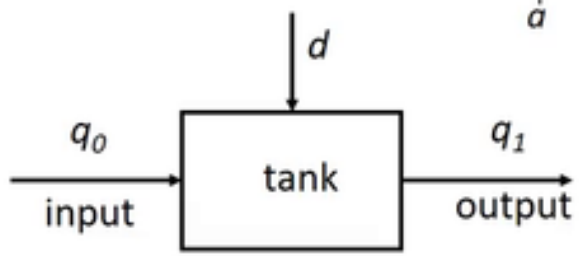
Modeling of physical systems

Models can be of different kinds:

- Drawings, schematics
- Block scheme, flow chart
- Mathematical models

We are interested in mathematical models of dynamical systems





$$A\frac{dh}{dt} = q_0 + d - a\sqrt{2gh}$$

$$q_1 = a\sqrt{2gh}$$

Modeling of dynamic systems

Our focus: mathematical models of physical (dynamic) systems, in particular linear, time invariant models (LTI) in continuous or discrete time:

- Differential- and difference equations

Ec diferenciales (t continuo)

$$\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_2y(t) = b_1u(t)$$

$$y(k) + a_1y(k-1) + a_2y(k-2) = b_1u(k)$$

Ec en diferencias (t discreto)

- Transfer functions

Típicamente sistemas SISO

$$Y(s) = \frac{b_1}{s^2 + a_1s + a_2}U(s)$$

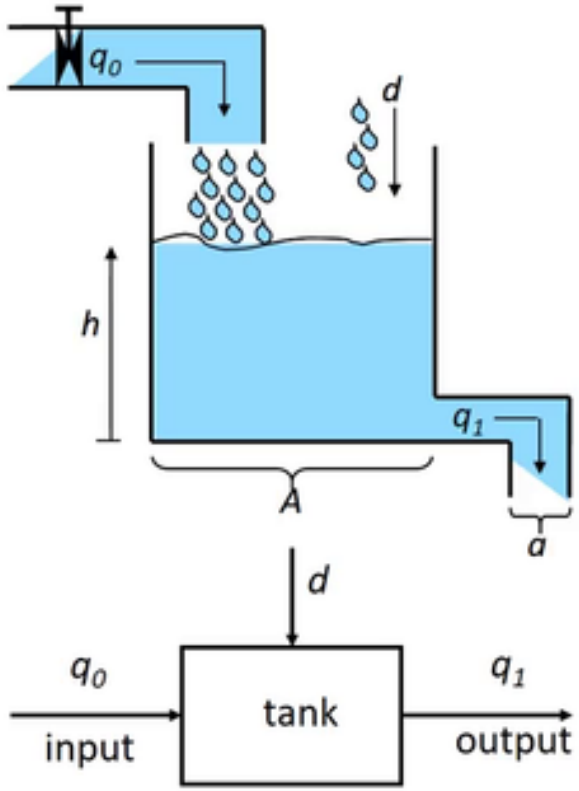
- State-space models

Sirve para MIMO

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

How do we build models?

- Physical relations
- Empirical knowledge
- Data

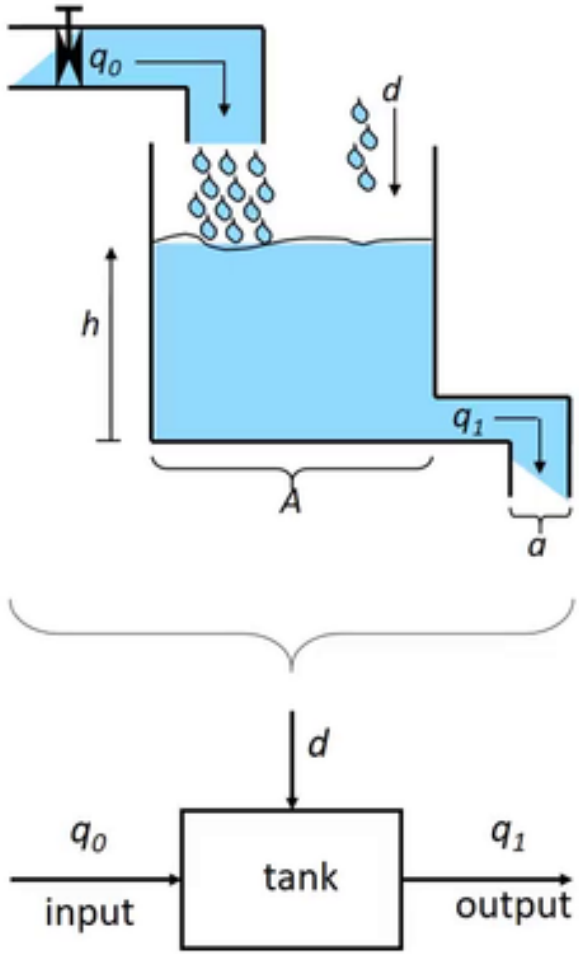


$$A\frac{dh}{dt} = q_0 + d - a\sqrt{2gh}$$

$$q_1 = a\sqrt{2gh}$$

Three phase method

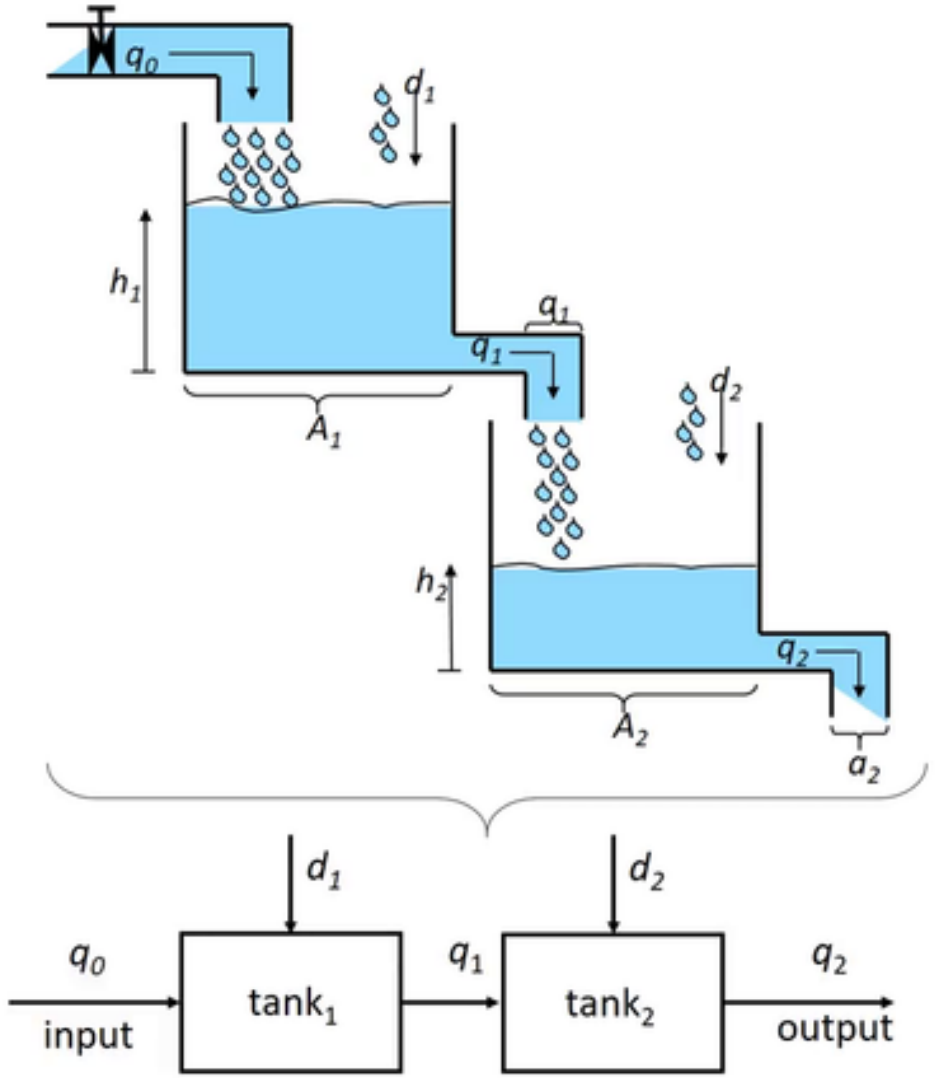
- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$



Podemos complejizar el modelo

Three phase method

- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$



Three phase method

- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations 2 grupos
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$

Conservation laws (balance equation):

- Mass balance [kg]
- Force balance [kgm/s^2 = N] (Newton's law)
- Torque balance [kgm^2/s^2 = Nm] (Newton)
- Voltage balance [V] (Kirchhoff's voltage law)
- Current flows [A] (Kirchhoff's current law)
- Volume flows [m^3/s]
- Energy flows [J/s = W]
- ...

Three phase method

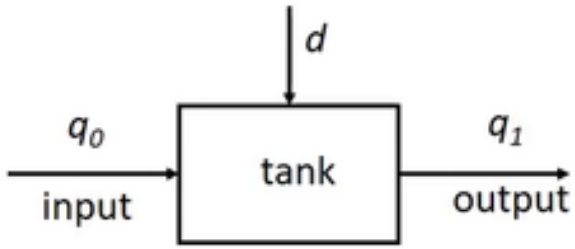
- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$

Constitutive relations (relate variables of different kind):

- Ohm's law: U=RI (voltage and current)
- Ideal gas law: pV=nRT (pres, vol and temp)
- Hooke's law: F=kx (force and distance)
- Air resistance: $F=bv^2$ (force and velocity)
- ...

Three phase method

- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$



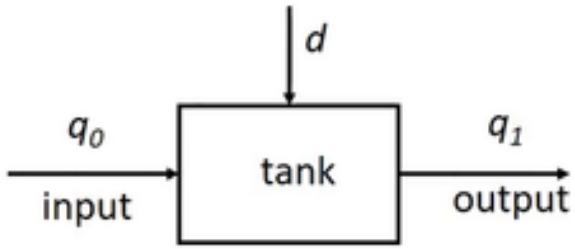
Conservation laws (balance equation):

What is it that changes? What is causing this change?

[change of volume per time instant, "accumulation"] = [supplied, input] - [consumed, output]

Three phase method

- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$



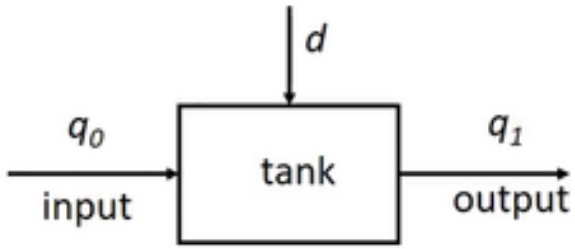
Conservation laws (balance equation):

- Volume flows [m^3/s]

$$\frac{dV}{dt} = q_0 + d - q_1$$

Three phase method

- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$



Conservation laws (balance equation):

- Volume flows [m^3/s]

$$\frac{dV}{dt} = q_0 + d - q_1$$

Constitutive relations:

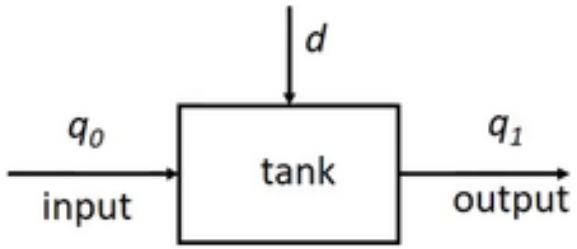
$$V = Ah$$

Volume and level

$$q_1 = a\sqrt{2gh}$$

Bernoullis equation

Three phase method



- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$

$$\frac{dV}{dt} = q_0 + d - q_1$$

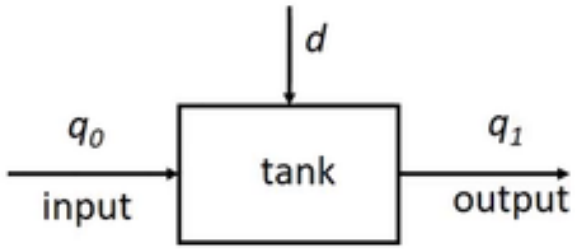
$$q_1 = a\sqrt{2gh}$$

$$V = Ah$$

$$\dot{x} = \frac{dx}{dt} = f(x, u, d)$$

$$y = h(x, u, d)$$

Three phase method



- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$

$$\frac{dV}{dt} = q_0 + d - q_1$$

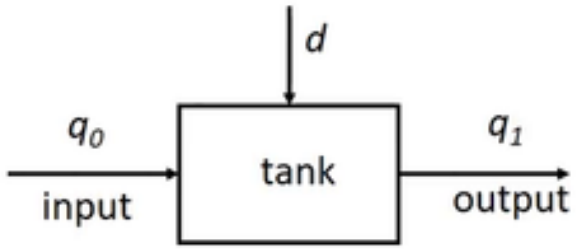
$$q_1 = a\sqrt{2gh}$$

$$V = Ah$$

Choose state variables:

- What is changing? V or h

Three phase method



- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$

$$\frac{dV}{dt} = q_0 + d - q_1$$

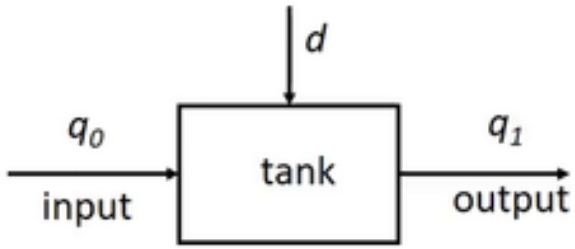
$$q_1 = a\sqrt{2gh}$$

$$V = Ah$$

Choose state variables: V

$$\frac{dV}{dt} =$$

Three phase method



- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$

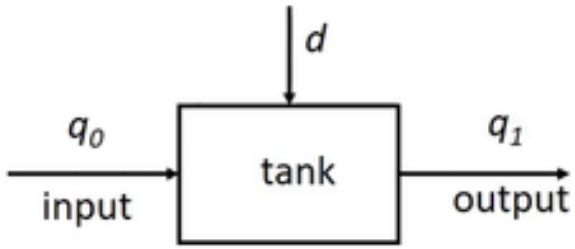
$$q_1 = a\sqrt{2gh}$$

$$V = Ah$$

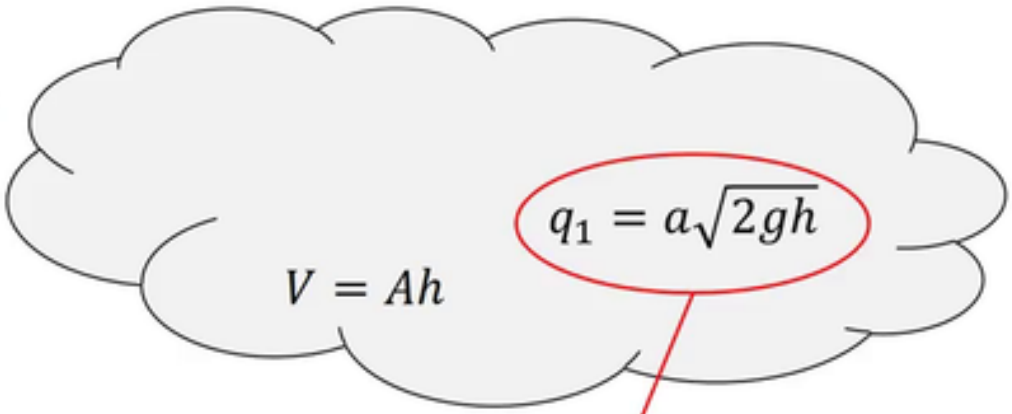
Choose state variables: V

$$\frac{dV}{dt} = q_0 + d - q_1$$

Three phase method



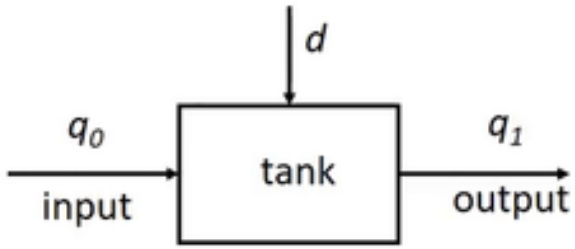
- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$



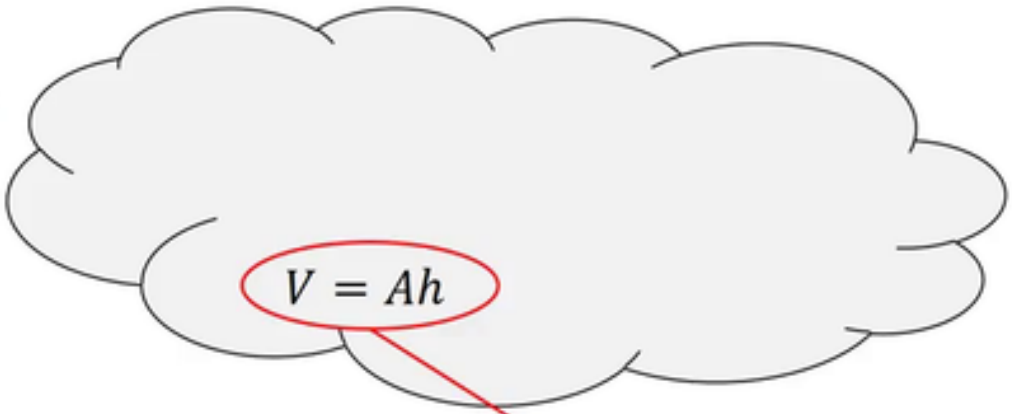
Choose state variables: V

$$\frac{dV}{dt} = q_0 + d - q_1$$

Three phase method



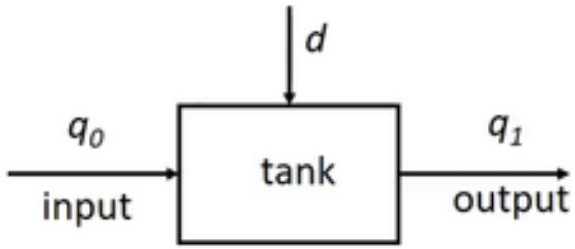
- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$



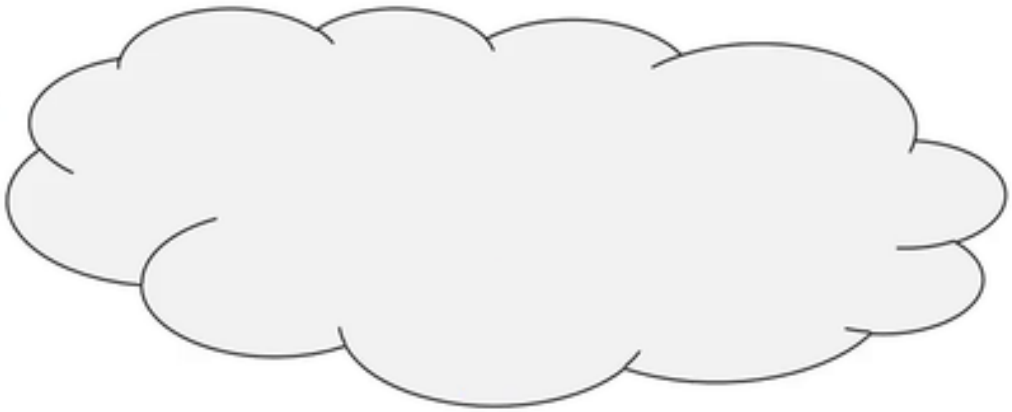
Choose state variables: V

$$\frac{dV}{dt} = q_0 + d - a\sqrt{2gh}$$

Three phase method



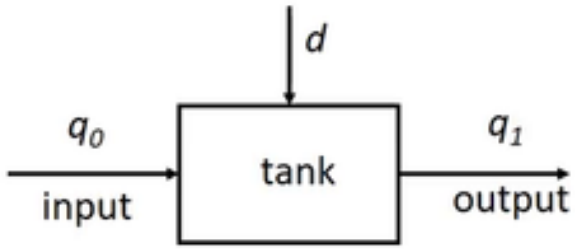
- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$



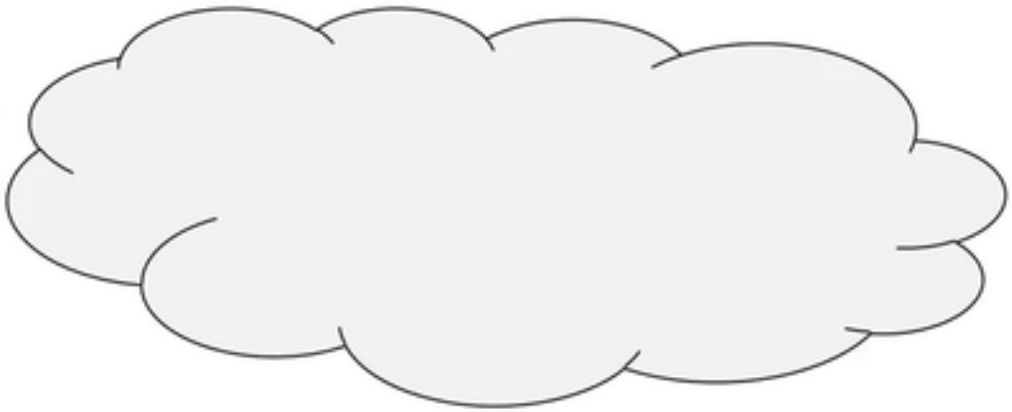
Choose state variables: V

$$\frac{dV}{dt} = q_0 + d - a\sqrt{2g\,V/A}$$

Three phase method



- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$



Choose state variables: V

$$\frac{dV}{dt} = q_0 + d - a\sqrt{2g\,V/A}$$

$$q_1 = a\sqrt{2gh}$$

Three phase method

- Structuring
  - Divide into subsystems
  - Inputs, outputs, internal variables?
- Basic equations
  - Conservation laws
  - Constitutive relations
- Form state-space model
  - Choose state variables
  - Form $\dot{x} = \cdots$

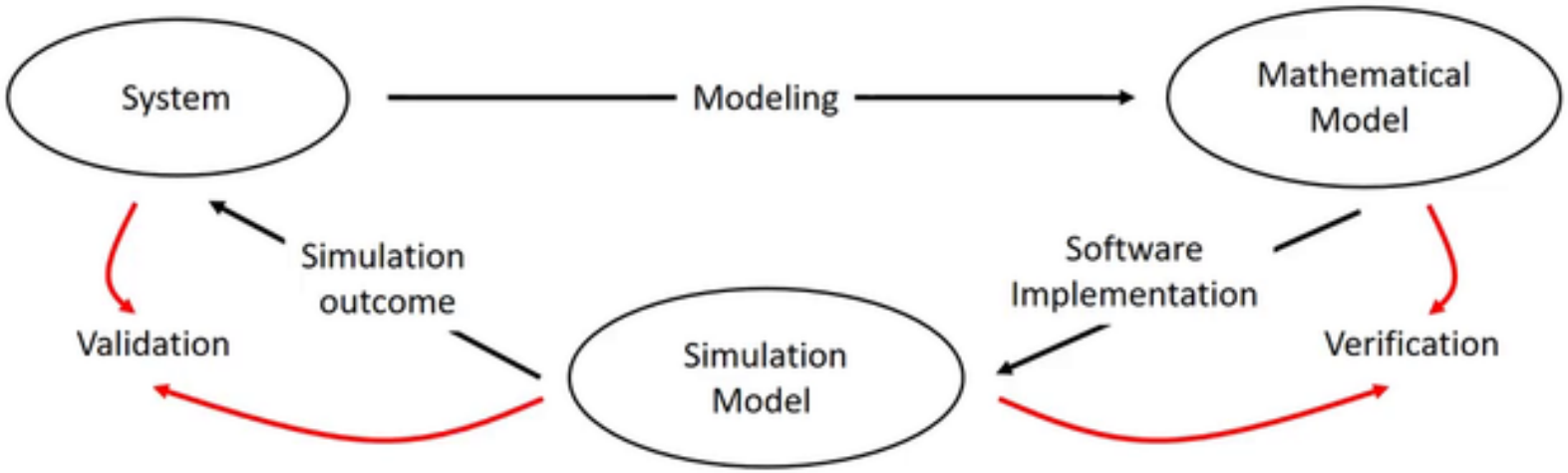
$$\begin{cases} \dfrac{dV}{dt} = q_0 + d - a\sqrt{2g\,V/A} \\ q_1 = a\sqrt{2g\,V/A} \end{cases}$$

Variable change: $x = V, u = q_0$ *and* $y = q_1$

$$\begin{cases} \dot{x} = \dfrac{dx}{dt} = u + d - a\sqrt{2g\,x/A} \\ y = a\sqrt{2g\,x/A} \end{cases}$$

Model verification and validation

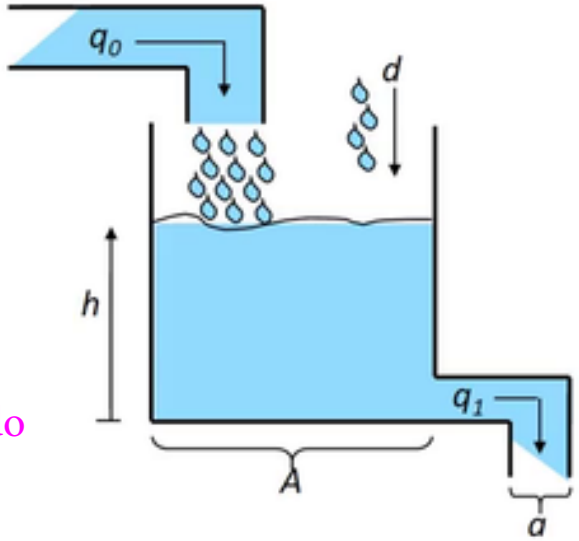
- Verification is the process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model.
- Validation is the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model.

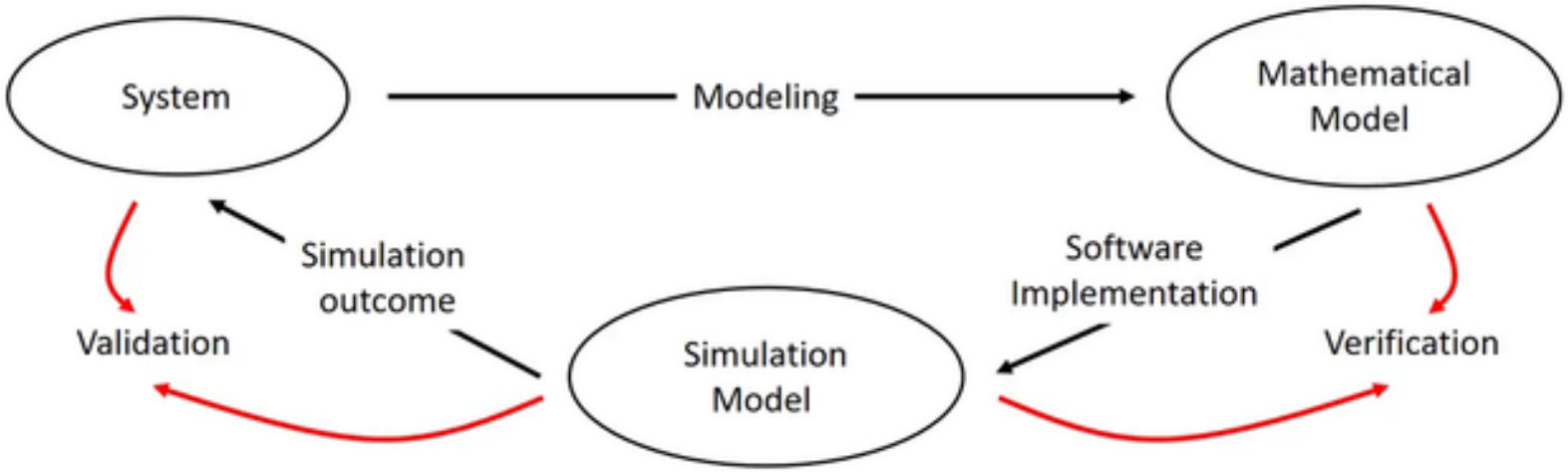


Model verification and validation

- All models have a limited domain of validity.
- Be aware of the model's (lack of) accuracy.

  Por ej zonas que se asumen lineales y funcionan en dominio acotado





Model Verification and Validation

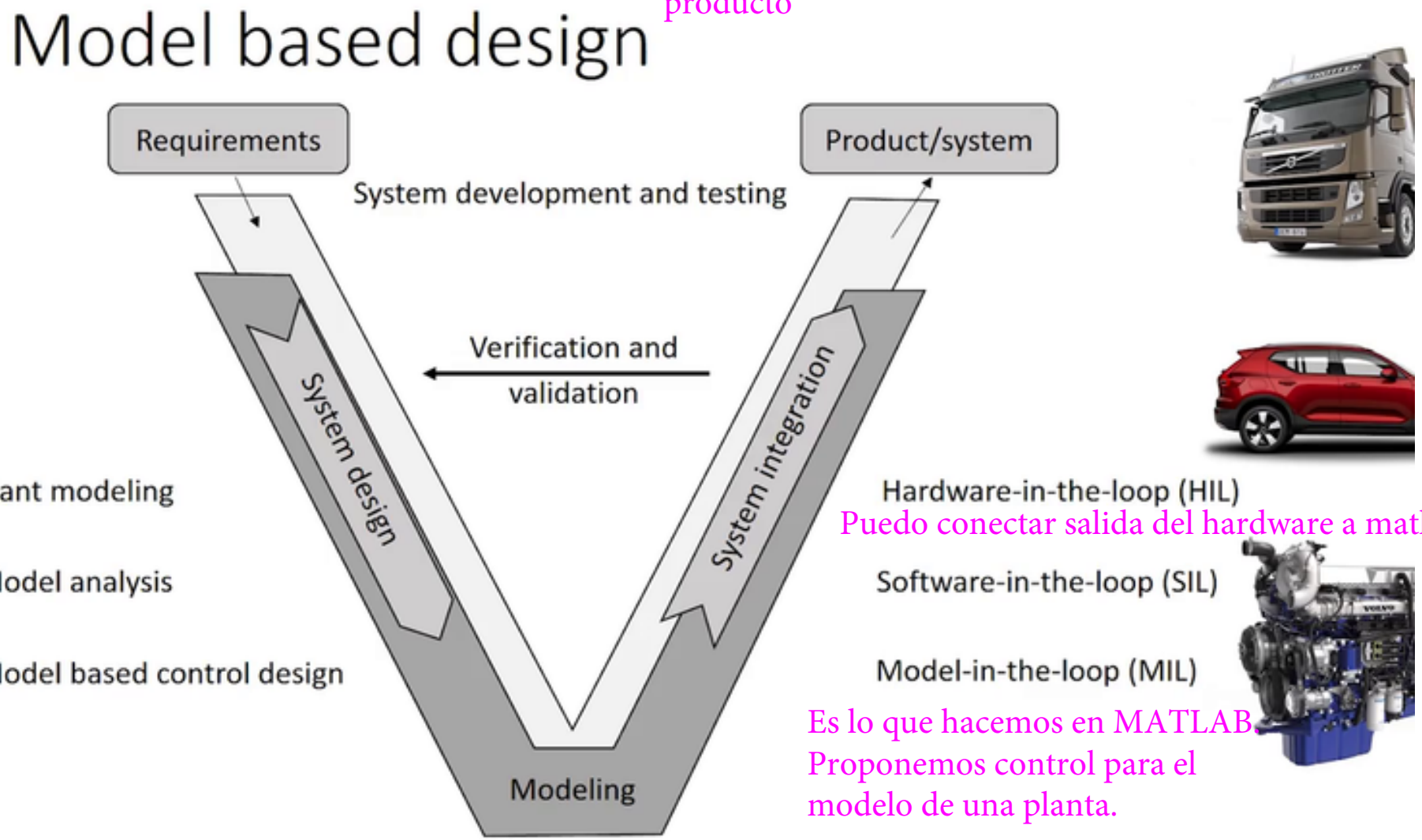
"All models are wrong but some are useful."

George Box, *Robustness in the strategy of scientific model building*, in Launer, R. L.; Wilkinson, G. N., Robustness in Statistics, Academic Press, pp. 201–236, 1979

"Models and simulations can never replace observations and experiments – but they constitute an important and useful complement."

Lennart Ljung and Torkel Glad, *Modeling and Identification of Dynamic Systems, Studentliteratur, 2016*

El modelo está presente en todo el ciclo de desarrollo del producto



Puedo conectar salida del hardware a matlab

Es lo que hacemos en MATLAB. Proponemos control para el modelo de una planta.

Bibliography

- Karl J. Astrom and Richard M. Murray *Feedback Systems*. Version v3.0i. Princeton University Press. September 2018. Chapter 3.
- Karmopp, Dean et al. *Systems Dynamics: Modeling, Simulation, and Control of Mechatronic Systems*. Fith Edition. John Wiley & Sons, Inc. 2012. Chapters 1 y 2.1.

Simulating a Simscape Model