Finite impulse response filtering

FIR windowed-sinc filters

Dr. Ing. Rodrigo Gonzalez

rodralez@frm.utn.edu.ar
rodrigo.gonzalez@ingenieria.uncuyo.edu.ar

UTN FACULTAD REGIONAL MENDOZA
UNIVERSIDAD TECNOLÓGICA NACIONAL

UNCUYO
UNIVERSIDAD NACIONAL DE CUYO

Classification of discrete filters

Table: Classification of discrete filters

	Finite impulse response (FIR)	Infinite impulse response (IIR)
Filtering in time domain	Moving average	Leaky Integrator
Filtering in frequency domain	Windowed-sinc Filters Equiripple Minimax	Bilinear z-transform

Information in frequency domain

- Information of a signal is contained in frequency response, phase and amplitude.
- **Many samples** in the signal are needed for frequency analysis.
  Necesitamos un vector de elementos de la señal
- The **frequency response** shows how information in frequency domain is being changed.
- Examples: telephone voice channel, equalizer...

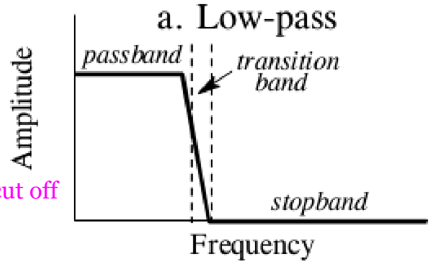
Con un ancho de banda menor no podemos saber quien nos habla por telefono. No distinguimos la voz. Perdemos información.

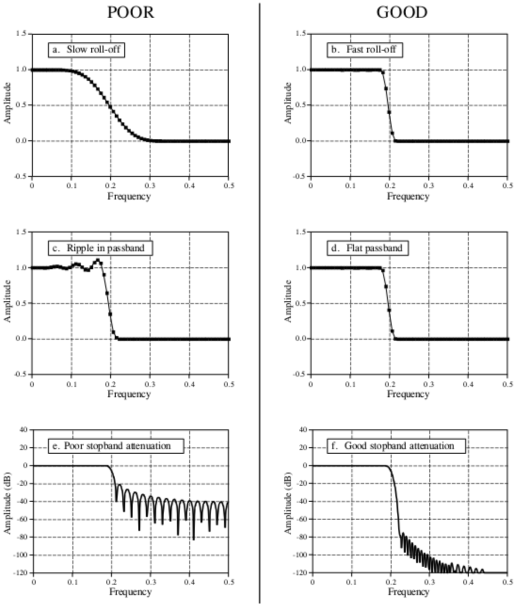
Frequency domain parameters

- Passband.
- Stopband.
- Cut-off frequency.
- Transition band (fast roll-off).

starts in f cut off

- Passband ripple.
- Stopband ripple.





Se desea:
-banda de transición corta

-Sin Ripple

-Atenuación está en dB. Se busca mayor atenuación

Strategy of filtering by windowed-sinc

- Taking the Inverse Fourier Transform of an ideal frequency response (1) produces an ideal sinc filter kernel (2, impulse response) with **infinite** length.

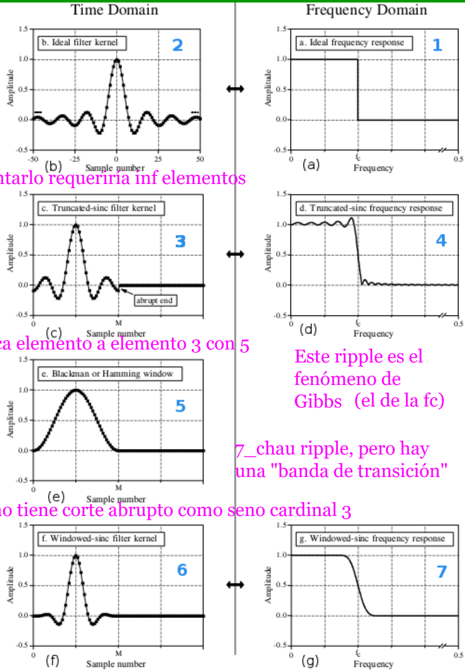
sinc=seno cardinal= sen(x)/x

$$h_s[n] = \frac{\sin(\pi f[n]/f_s)}{(\pi f[n]/f_s)}$$

- To get around this problem, ideal sinc filter is **truncated** to M+1 points, symmetrically chosen around the main lobe, where M is an even number.
- Truncated-sinc (3) produces the **Gibbs phenomenon** in frequency response (4), no matter how long M is made.
- Multiplying the truncated-sinc ($h_{st}[n]$, 3) by the Blackman window ($w[n]$, 5) results in a **windowed-sinc filter kernel** ($h_w[n]$, 6) with frequency response (7).

$$h_w[n] = h_{st}[n] \cdot w[n]$$

$$y[n] = h_w[n] * x[n]$$



Implementarlo requeriría inf elementos

Se multiplica elemento a elemento 3 con 5

Este ripple es el fenómeno de Gibbs (el de la fc)

7_chau ripple, pero hay una "banda de transición"

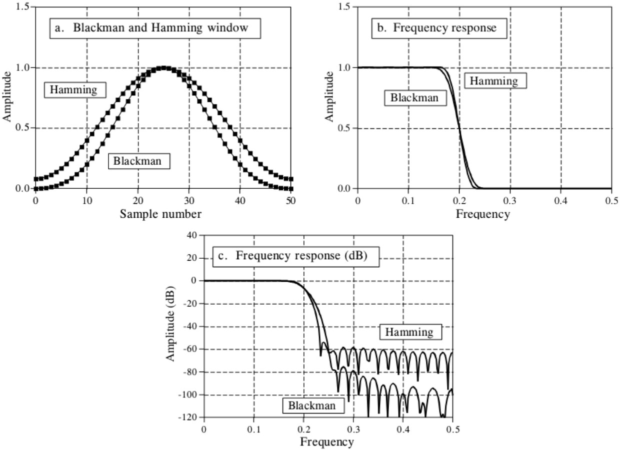
6 no tiene corte abrupto como seno cardinal 3

Differences between Blackman and Hamming

- The two windows have M = 50 (51 points)
- Which of these two windows should you use? It's a trade-off between parameters.
- The Hamming window has about a 20% faster roll-off than the Blackman.

banda de transición 20% más angosta

- However, the Blackman has a better stopband attenuation, -74dB (-0.02%) vs. -53dB (-0.2%).
- The Blackman has a passband ripple of only about 0.02%, while the Hamming is typically 0.2%.
- In general, the Blackman should be your first choice; a slow roll-off is easier to handle than poor stopband attenuation.

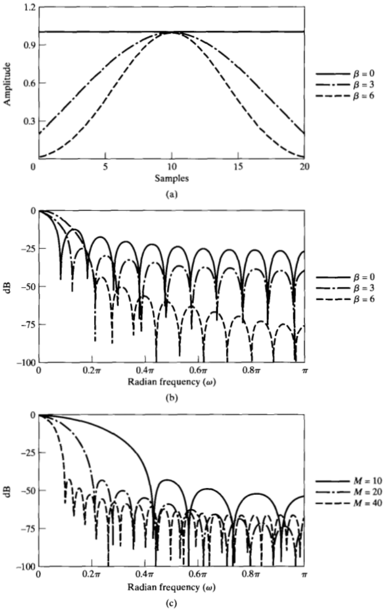


Blackman ofrece mucho más atenuación en la banda atenuada, pero con banda de transición más ancha respecto a Hamming. Se parte eligiendo Blackman ya que el ancho se puede achicar aumentando el orden del filtro.

Es una ventana de ventanas

- The Kaiser window has two parameters:
  - Lenght, M+1.
  - Shape parameter, β.
- Trade-off between side-lobe amplitude and main-lobe width.

Cambiando Beta pasamos por varias ventanas estándares.
Con B=0 ventana rectangular
Pasamos de hamming a blackman o una intermedia.
Varía fco respecto a fs
Relación de compromiso entre atenuación y ancho de banda de la banda pasante



(a)
(b)
(c)

Normalized performances of windowed-sinc filters

Name of window function $w[n]$	Transition width ΔF in (Hz), (normalised)	Pass-band ripple A_p in (dB)	Ripple δ_p, δ_s	Side-lobe level in (dB)	Stop-band attenuation A_s in (dB)
Rectangular	0.9/N	0.741	0.089	-13	21
Hanning	3.1/N	0.0546	0.063	-31	44
Hamming	3.3/N	0.0194	0.0022	-41	53
Blackman	5.5/N	0.0017	0.000196	-57	74
Kaiser β=4.54	2.93/N	0.0274			50
β=5.65	3.63/N	0.00867			60
β=6.76	4.32/N	0.00275			70
β=8.96	5.71/N	0.000275			90

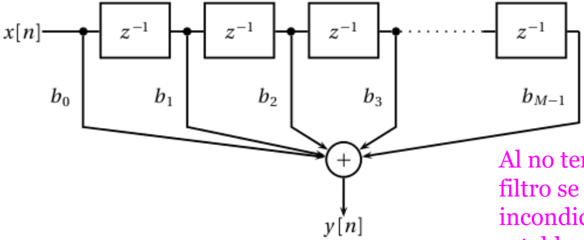
Atenuación del
primer lóbulo lateral

FIR structures

$$H(z) = b_0 + b_1 z^{-1} + \dots + b_{M-1} z^{M-1}$$

Los b son coeficientes del filtro FIR. z^(-n) es retraso n

No tiene denominador. Entonces no hay realimentación.



Al no tener polos el filtro se dice que es incondicionalmente estable

Figure 7.22 Direct FIR implementation.

Forma directa filtro FIR

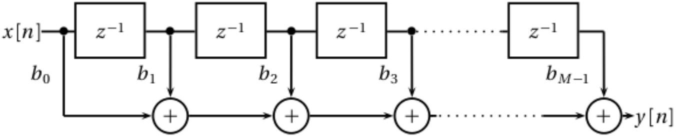


Figure 7.23 Transversal FIR implementation.

Idéntico matemáticamente al anterior. Forma transversal del filtro FIR. Es la convolución de una señal con los coeficientes del filtro. función mac

Bibliography

1 Steven W. Smith, The Scientist and Engineer's Guide to Digital Signal Processing. Chapter 16. www.dspguide.com.

2 Paolo Prandoni and Martin Vetterli. Signal processing for communications. Taylor and Francis Group, LLC. 2008. Sections 5.4 and 5.5 https://www.sp4comm.org/.