

Typical stages in digital signal processing

A/D Conversion

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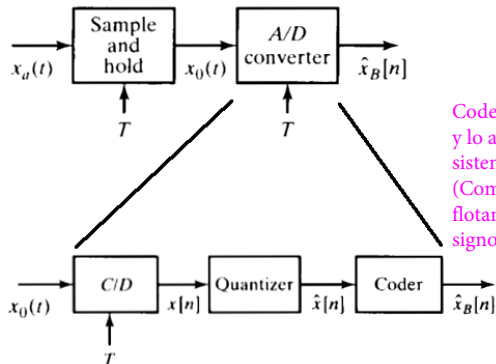
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A/D Conversion Stages

- The A/D converter is a physical device that converts a voltage or current amplitude at its input into a binary code representing a quantized amplitude value closest to the amplitude of the input.
- The sample-and-hold stage can be a **zero-order-hold**.

S&H mantiene un valor de tensión durante un tiempo determinado a la entrada del A/D, ya que la conversión de A/D toma un tiempo finito.



Coder: Toma string de bits y lo acomoda a algun sistema de numeracion (Complemento a 2, punto flotante, binario con o sin signo, etc)

Cuantizador da valor posible dentro de los binarios

Quantizer

- Uniformly spaced quantizer, function $Q(x)$.
- The number of quantization levels will be a power of two (2^B).
- Precision of quantizer, $\Delta = \frac{\text{full voltage range}}{2^{\text{word length}}} = \frac{2V_p}{2^B} \cdot [mV]$

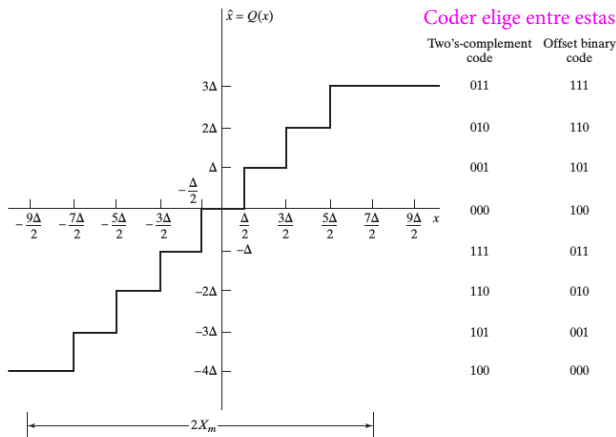


Figure 4.54 Typical quantizer for A/D conversion.

Quantizer Error example

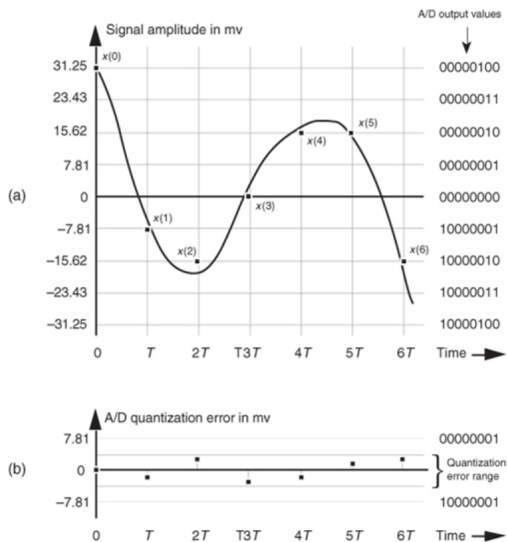
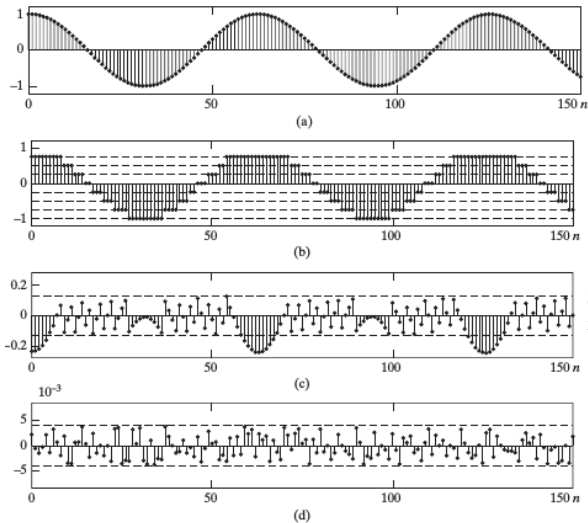


Figure: 12.1 [2]

Quantizer Error example, II



Cuantizada en 3 bits

Se comio la parte negativa

Cuantizar una señal es como meterle ruido

Figure 4.57 (continued) (b) Quantized samples of the cosine waveform in part (a) with a 3-bit quantizer. (c) Quantization error sequence for 3-bit quantization of the signal in (a). (d) Quantization error sequence for 8-bit quantization of the signal in (a).

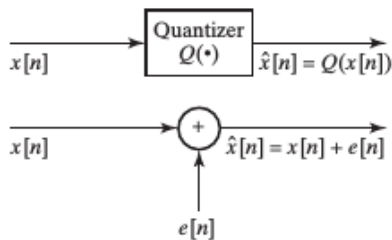


Figure 4.56 Additive noise model for quantizer.

Cuantizador "empeora" señal original al agregarle ruido

The precision of the quantizer is given by [2]:

$$\Delta = q = \frac{\text{full voltage range}}{2^{\text{word length}}} = \frac{2V_p}{2^B} \cdot [mV] \quad (1)$$

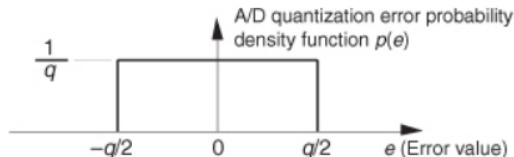
- $SNR = (P_{\text{signal}})/(P_{\text{noise}})$ relates two powers. Relacion señal a ruido En valores de potencia
- Since q is defined as a random variable, its power cannot be represented explicitly.
- A statistical version of SNR is used,

$$SNR_{ADC} = 10 \cdot \log_{10} \left(\frac{\text{input signal variance}}{\text{A/D quantization noise variance}} \right), \quad [\text{dB}] \quad (2)$$

$$= 10 \cdot \log_{10} \left(\frac{\sigma_{\text{signal}}^2}{\sigma_{ADC}^2} \right). \quad (3)$$

Relaciona varianzas en vez de potencias

ADC Signal-to-Noise relationship, II



$$\sigma_{ADC}^2 = \int_{-q/2}^{q/2} (e - \mu)^2 p(e) de = \int_{-q/2}^{q/2} e^2 p(e) de = \frac{1}{q} \int_{-q/2}^{q/2} e^2 de = \frac{q^2}{12}, \quad (4)$$

$$\sigma_{ADC}^2 = \left(\frac{2V_p}{2^B} \right)^2 \cdot \frac{1}{12} = \boxed{\frac{V_p^2}{3 \cdot 2^{2B}}}, \quad (5)$$

$$\text{Load Factor, } LF = \frac{rms_{signal}}{V_p} = \frac{\sigma_{signal}}{V_p} \Rightarrow \sigma_{signal}^2 = \boxed{LF^2 \cdot V_p^2}, \quad (6)$$

$$\begin{aligned} SNR_{ADC} &= 10 \cdot \log_{10} \left(\frac{\sigma_{signal}^2}{\sigma_{ADC}^2} \right), \\ &= 10 \cdot \log_{10} \left[\left(LF^2 \cdot V_p^2 \right) \cdot \frac{3 \cdot 2^{2B}}{V_p^2} \right] = 10 \cdot \log_{10} \left[\left(LF^2 \cdot 3 \cdot 2^{2B} \right) \right], \end{aligned} \quad (7)$$

$$= 10 \cdot \left[\log_{10}(LF^2) + \log_{10}(3) + 2 \log_{10}(2) \cdot B \right], \quad (8)$$

$$= 20 \cdot \log_{10}(LF) + 4.77 + 6.02 \cdot B. \quad [\text{dB}] \quad (9)$$

ADC Signal-to-Noise relationship considerations

$$\begin{aligned} SNR_{ADC} &= 20 \cdot \log_{10}(LF) + 4.77 + 6.02 \cdot B, \quad [\text{dB}] \\ &= 20 \cdot \log_{10}\left(\frac{rms_{signal}}{V_p}\right) + 4.77 + 6.02 \cdot B. \quad [\text{dB}] \end{aligned}$$

Se busca SNR lo más grande posible->Aumentar cantidad de Bits, o el LF.

Considerations about LF :

- Ideally, if $rms_{signal} \gg V_p$, SNR_{ADC} **increases**, but this will produce a severe distortion in the sampling signal (saturation).
 - On the other hand, if $rms_{signal} \ll V_p$, SNR_{ADC} **decreases**.
- AD es muy mala idea alimentarlo con miliVolts.

Potencia efectiva de la señal se busca mucho mas grande que tension pico. Pero hay limites fisicos. A un ADC no puedes darle tanta potencia efectiva, porque son de baja

Considerations about B (numbers of bits):

- SNR_{ADC} increases 6 dB by each bit in ADC's quantizer.
- So, the more bits the better, isn't it?

Suelen ir de la mano rms y V_p

Other sources of error should be taken into account:

- It was considered that ADC's $V_{MAX} = V_p$.
- The proposed model for the probability density function of $e[n]$ may not be uniform.
- Therefore, SNR_{ADC} from Eq. 9 should be **decreased by 3 or 6 dB**.

Esto es una pérdida por consideraciones no tenidas en cuenta

ADC Signal-to-Noise relationship for a sinusoidal signal

For a sinusoidal signal, $rms_{signal} = V_p/\sqrt{2}$.

$$SNR_{ADC} = 20 \cdot \log_{10} \left(\frac{rms_{signal}}{V_p} \right) + 4.77 + 6.02 \cdot B, \quad (10)$$

$$= 20 \cdot \log_{10} \left(\frac{V_p/\sqrt{2}}{V_p} \right) + 4.77 + 6.02 \cdot B. \quad (11)$$

Thus, the maximum SNR_{ADC} is,

$$SNR_{ADC} = 20 \cdot \log_{10} \left(1/\sqrt{2} \right) + 4.77 + 6.02 \cdot B, \quad (12)$$

$$= -3.01 + 4.77 + 6.02 \cdot B, \quad (13)$$

$$= 1.76 + 6.02 \cdot B. \text{ [dB]} \quad (14)$$

LF hace aporte negativo.

ADC resolution for a particular signal

Consider the following example:

- The SNR for an audio output amplifier is 110 dB.
- A 24-bits ADC is chosen to sample the output amplifier (professional audio).
- The amplifier is excited by an input sinusoidal signal. *-3 es factor de seguridad por consideraciones*

$$SNR_{ADC} = 1.76 + 6.02 \cdot 24 - 3 = 143.24 \text{ dB}.$$

- How many bits are used to measure noise? $(143 - 110)/6 \simeq 5.5$ bits!.
- In a control loop, picking a bad ADC resolution could lead to a catastrophic scenario.
- What happens if $B = 10$ bits? $SNR_{ADC} = 61.96$ dB, $(110 - 61.96)/6 \simeq 8$ extra bits are needed!.

Summary: the number of bits B in an ADC must provide $SNR_{ADC} \geq SNR_{signal}$.

- Rule of thumb: the ADC resolution should be choose in order to provide 6 dB (1 bit) above the SNR of the signal to be sample.
- Additional bits (noisy bits) can be eliminated by right shifting.

In C: `adc_read >>= 5;`

- 1 Alan V. Oppenheim and Ronald W. Schaffer. *Discrete-time signal processing*, 3rd Ed. Prentice Hall. 2010. Section 4.8.
- 2 Richard G. Lyons. *Understanding Digital Signal Processing*, 3rd Ed. Prentice Hill. 2010. Section 12.3.1.
- 3 Paolo Prandoni and Martin Vetterli. *Signal processing for communications*. Taylor and Francis Group, LLC. 2008. Section 9.6.