Infinite impulse response filters

Leakey Integrator filter

Dr. Ing. Rodrigo Gonzalez

rodralez@frm.utn.edu.ar
rodrigo.gonzalez@ingenieria.uncuyo.edu.ar

Table: Classification of discrete filters

	Finite impulse response (FIR)	Infinite impulse response (IIR)
Filtering in time domain	Moving average	Leaky Integrator
Filtering in frequency domain	Windowed Filters Equiripple Minimax	ZOH method Bilinear z-transform

The MA filter equation,

$$y[n] = x[n] * h[n] = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k]\,, \tag{1}$$

$$y[n] = \frac{1}{M}\left[\sum_{k=1}^{M-1} x[n-k] + x[n]\right] = \frac{1}{M}\left[\sum_{k=1}^{M-1} x[n-k]\right] + \frac{1}{M}x[n]\,. \tag{2}$$

Since,

$$y[n-1] = \frac{1}{M-1}\left[\sum_{k=1}^{M-1} x[n-k]\right] \implies y[n-1](M-1) = \left[\sum_{k=1}^{M-1} x[n-k]\right]\,. \tag{3}$$

Then,

$$y[n] = \frac{M-1}{M}y[n-1] + \frac{1}{M}x[n]\,. \tag{4}$$

Defining $\lambda = \frac{M-1}{M}$,

Unica variable de ajuste es lambda que es siempre<1

$$y[n] = \lambda\, y[n-1] + (1-\lambda)\, x[n]\,. \tag{5}$$

Esto NO pertenece a un Filtro FIR, porque esos solo dependen de valores pasados de la entrada.
Los filtros IIR dependen de valores pasados de la salida.

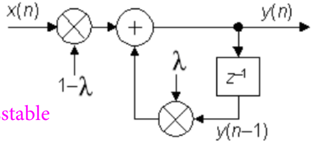
It can be seen that the leaky integrator filter is an IIR filter. Why?

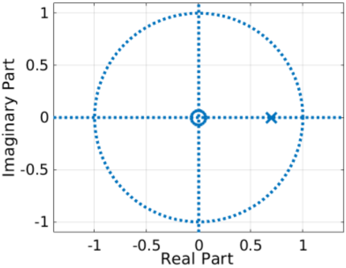
Leaky integrator filter

$$y[n] = \lambda\, y[n-1] + (1-\lambda)\, x[n]\,.$$

- No longer a convolution.
- Instead, a **constant coefficient difference equation**. Initial conditions must be set.
- LI is also known as **Single Pole Recursive filter** [2].
- The new system is LTI.
- LI is stable for $|\lambda| < 1$. Since $\lambda = \frac{M-1}{M}$, the filter is stable.
- The value of λ (which is the pole of the system) determines the smoothing power of the filter.

$$\frac{Y(z)}{X(z)} = \frac{1-\lambda}{1-\lambda z^{-1}}$$

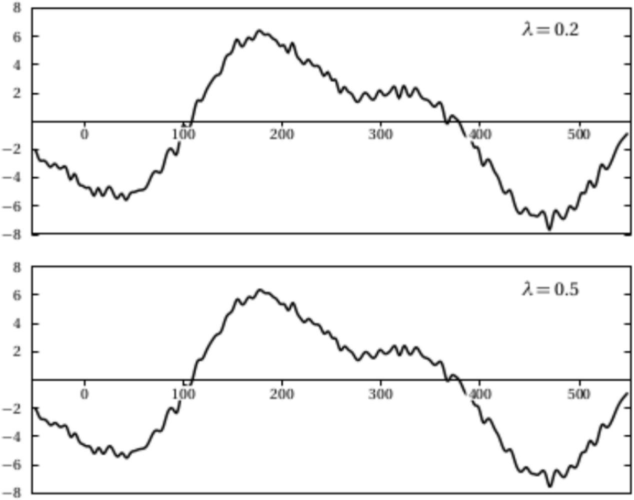




polo en lambda que es <1 -->Estable
cero en....

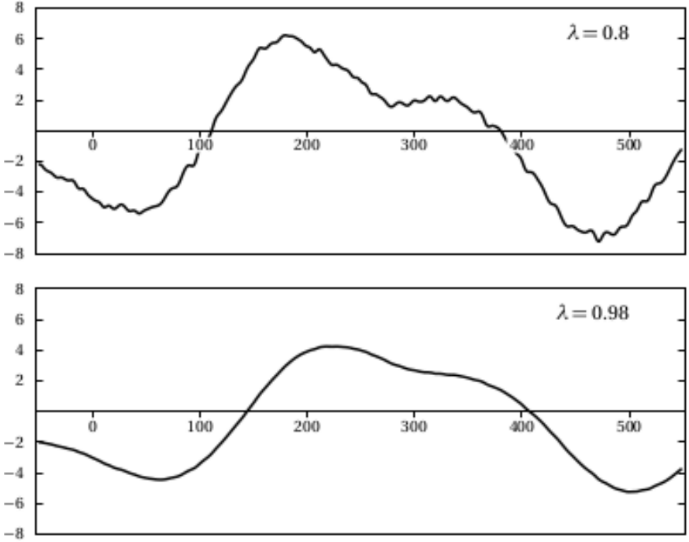
Por problemas de redondeo si lambda se acerca a la circunferencia de radio 1 podría resultar inestable. Se intenta estar lejos

Time domain response



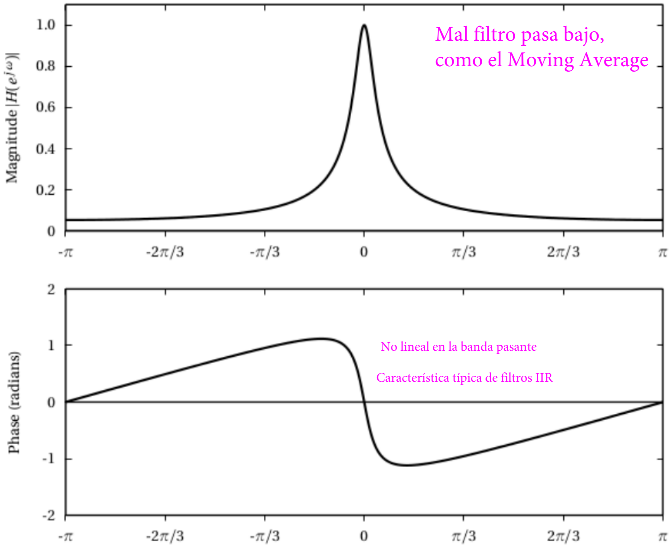
Time domain response, 2

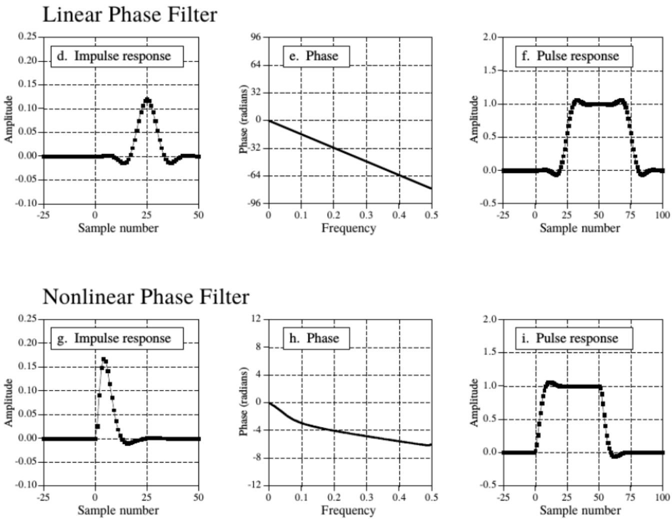
Note how the signal is delayed as λ grows.



Frequency domain response

Magnitude and phase response of the leaky integrator for $\lambda = 0.9$. Phase response is **nonlinear**.





Simétrico

Componentes de sen y cos se van retrasando y resulta en una deformación del pulso

Bibliography

1. Paolo Prandoni and Martin Vetterli. Signal processing for communications. Taylor and Francis Group, LLC. 2008. Section 5.3.2.
2. Steven W. Smith, The Scientist and Engineer's Guide to Digital Signal Processing. Chapter 19. www.dspguide.com