Typical stages in digital signal processing

Aliasing prefiltering

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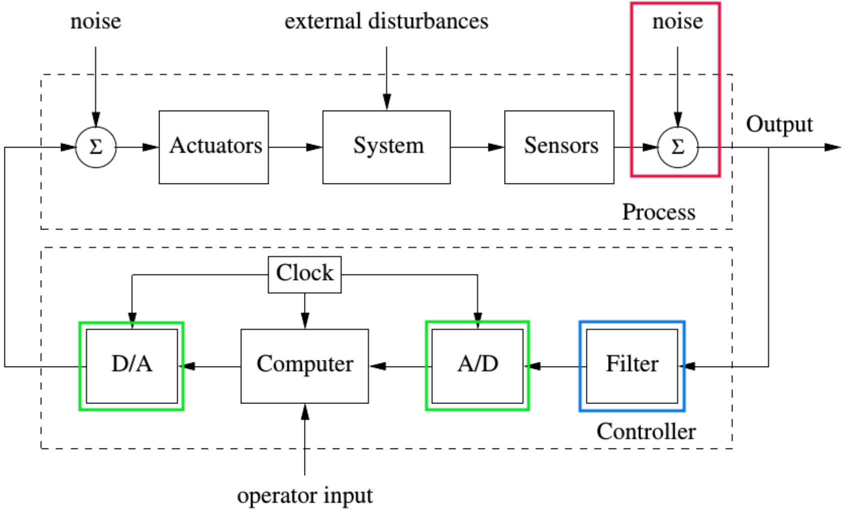
Summary

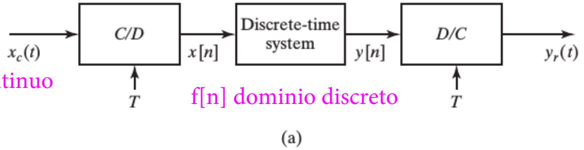
1. DSP in a the context of control systems
2. Digital processing of analog signals
3. Sampling signals in the frequency domain
   - Periodic sampling
   - Frequency-domain representation of sampling
4. Aliasing prefiltering

DSP in a the context of control systems



Conversor A/D ideal a frecuencia 1/T

D/A ideal a f=1/T



f(t) t continuo

f[n] dominio discreto

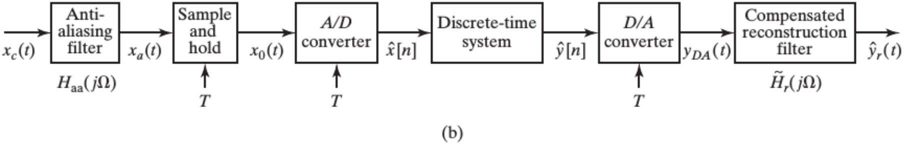


Figure 4.47 (a) Discrete-time filtering of continuous-time signals. (b) Digital processing of analog signals.

Periodic sampling

The discrete-time representation of a continuous-time signal is obtained through periodic sampling from a continuous-time signal $x_c(t)$ according to,

$$x[n] = x_c(nT), \qquad -\infty < n < \infty \,, \tag{1}$$

where T is the sampling period, and $f_s = 1/T$ is the sampling frequency, or $\Omega_s = 2\pi/T$ in radians/s

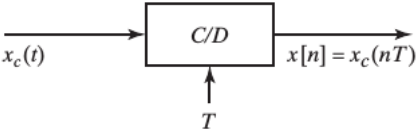


Figure 4.1 Block diagram representation of an ideal continuous-to-discrete-time (C/D) converter.

It is convenient to represent the sampling process mathematically in the two stages.

1. An impulse train $s(t)$ is multiplied by a continuous-time signal $x_c(t)$.
2. The continuous-time signal $x_s(t)$ is transformed to a discrete-time sequence $x[n]$.

xs(t) es tren de impulsos modulado

Bloque convierte tren a valores binarios

(c) son bastante distintos porque abarcan rangos de xc(t) distintos, ojo

Si tomás la der entre -2 a 2 coincide con la otra. Creo que es un mal ejemplo al problema de usar T muy largo

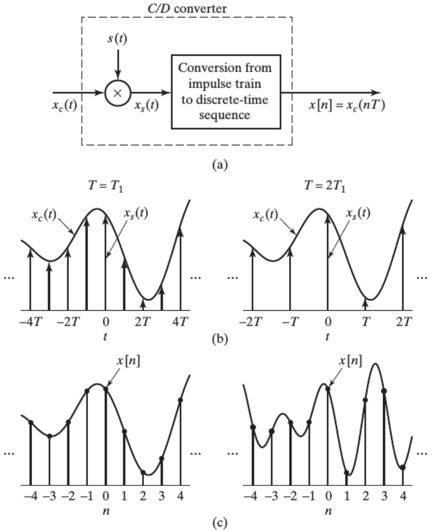


Figure 4.2 Sampling with a periodic impulse train, followed by conversion to a discrete-time sequence. (a) Overall system. (b) $x_s(t)$ for two sampling rates. (c) The output sequence for the two different sampling rates.

Let $x_c(t)$ be a bandlimited signal with,

johm implica que ya estamos en dominio de la frecuencia

$$X_c(j\Omega) = 0 \ \text{ para } \ |\Omega| \geq \Omega_N \, . \tag{2}$$

ohmN es máx frecuencia que tiene la señal

Then $x_c(t)$ is uniquely determined by its samples $x[n] = x_c(nT)$, $n = 0, \pm 1, \pm 2, \ldots$ if,

$$\Omega_s = \frac{2\pi}{T} \geq 2\Omega_N \, . \tag{3}$$

N de Nyquist?

The frequency Ω_N is commonly referred to as the **Nyquist frequency**, and the frequency $2\Omega_N$ as the **Nyquist rate**.

frecuencia de Nyquist

tasa de Nyquist

The Nyquist rate is the minimum sampling frequency in order to be able of reconstructing $x_c(t)$.

Frequency-domain representation of sampling

$x_s(t)$ is obtained multiplying $x_c(t)$ by a periodic impulse train $s(t)$,

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \,, \tag{4}$$

Vale 1 en t=nT o vale infinito?
Dice profe que 1, y tiene sentido
Pero me suena que delta de dirac es de área 1

$$x_s(t) = x_c(t)\, s(t) \,, \tag{5}$$

$$= x_c(t) \sum_{n=-\infty}^{\infty} \delta(t - nT) \,, \tag{6}$$

$$= \sum_{n=-\infty}^{\infty} x_c(nT)\, \delta(t - nT) \qquad \text{by sifting property.} \tag{7}$$

The Fourier transform of the periodic impulse train $s(t)$ is the periodic impulse train,

$$S(j\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_s), \qquad \text{where } \Omega_s = \frac{2\pi}{T} \,. \tag{8}$$

The Fourier transform of $x_s(t)$ is the continuous-variable convolution of $X_c(j\Omega)$ and $S(j\Omega)$,

El producto de las señales en el dominio del tiempo (7) es una convolución en el dom de la frecuencia(9)

$$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega) \,, \tag{9}$$

$$X_s(j\Omega) = \frac{1}{T} X_c[j(\Omega - k\Omega_s)] \,. \tag{10}$$

Reemplacé (8) en (9)

Frequency-domain representation of sampling, 2

- Fourier transform of $x_s(t)$ consists of periodically repeated copies of $X_c(j\Omega)$
- These copies are shifted by integer multiples of the sampling frequency.
- It is evident that

  $\Omega_s - \Omega_N \geq \Omega_N$, or,

  $\Omega_s \geq 2\Omega_N$

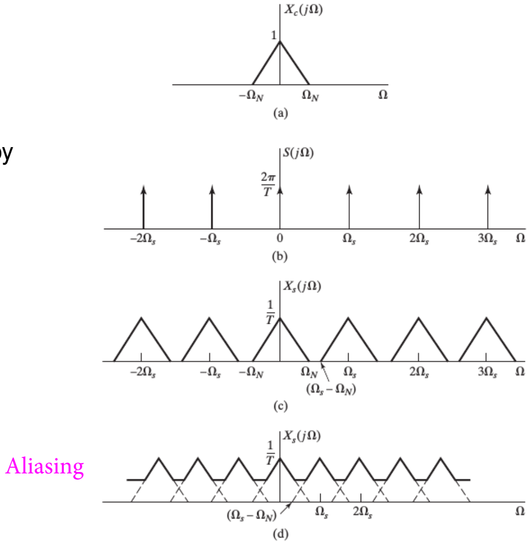


Figure 4.3 Frequency-domain representation of sampling in the time domain. (a) Spectrum of the original signal. (b) Fourier transform of the sampling function. (c) Fourier transform of the sampled signal with $\Omega_s > 2\Omega_N$. (d) Fourier transform of the sampled signal with $\Omega_s < 2\Omega_N$.

Aliasing prefiltering

The aliasing filter is an **analog** low-pass or band-pass filter.

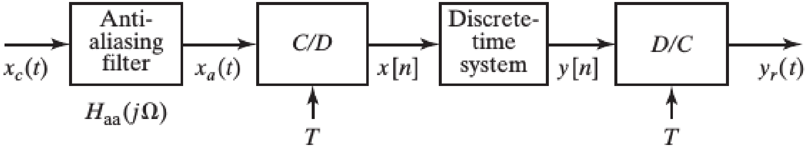
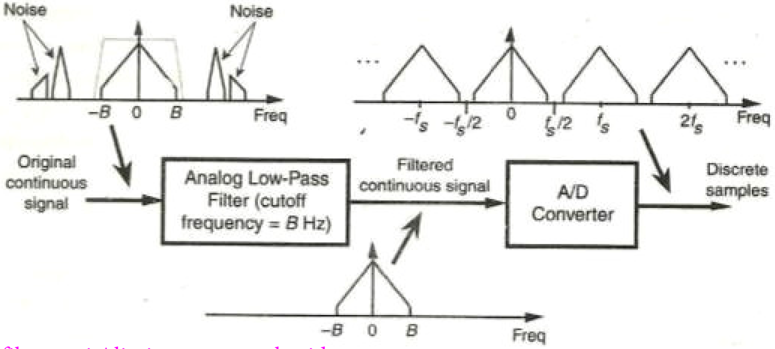


Figure 4.48 Use of prefiltering to avoid aliasing.

El filtro garantiza que la máx frecuencia que entre sea ohmN

Even if the signal is naturally bandlimited (as music), wideband additive noise may fill in the higher frequency range, and as a result of sampling, these noise components would be aliased into the low-frequency band.



El filtro anti Aliasing se come el ruido

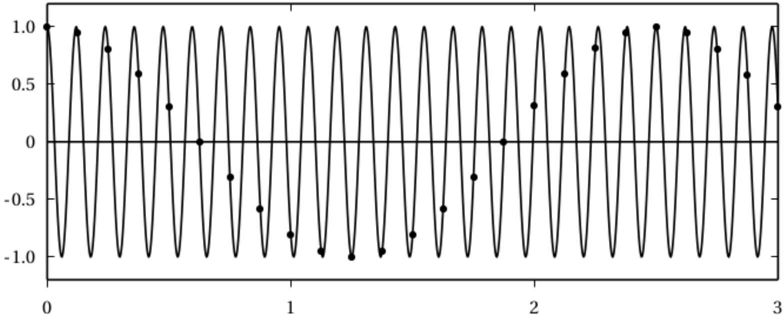


Figure 9.8 Example of aliasing: a sinusoid at 8400 Hz, $x(t) = \cos(2\pi \cdot 8400t)$ (solid line) is sampled at $F_s = 8000$ Hz. The sampled values (dots) are indistinguishable from those of at 400 Hz sinusoid sampled at F_s.

La señal la vas a ver, pero va a ser incorrecta

ADC signal conditioning circuits (Ref. [4])

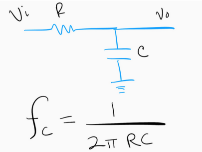


Figure: RC low-pass filter.

Se monta la entrada sobre una continua para que el conversor vea picos positivos y negativos

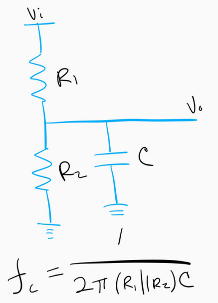
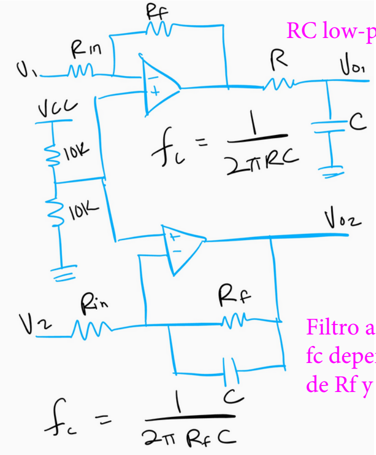


Figure: RC low-pass filter with voltage divider.

Offset, amplificacion y alta Z

RC low-pass filter



Filtro activo. fc depende de Rf y C

Figure: The top circuit is filtered using a passive filter while the bottom part uses an active filter.

Para V2 de alta f la ganancia del operacional se vuelve 0, porque el C hace de cortocircuito

Un filtro que corte tan abrupto es dificil de conseguir. Se usa esta técnica

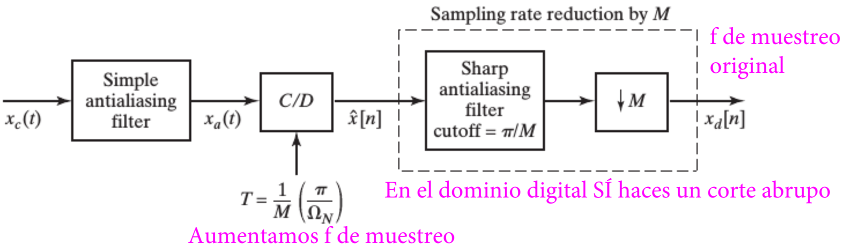
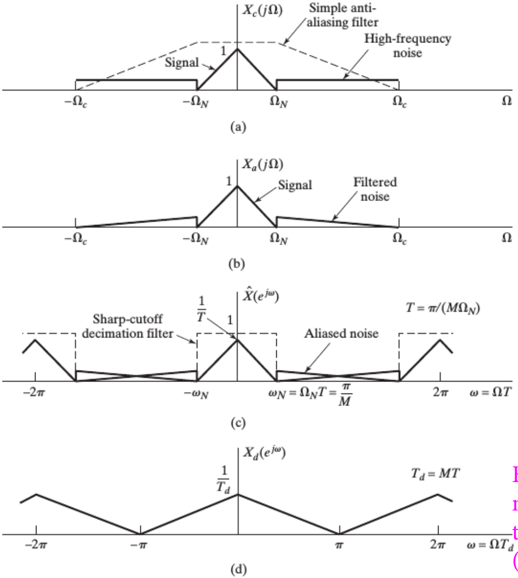


Figure 4.49 Using oversampled A/D conversion to simplify a continuous-time antialiasing filter.

Oversampling frequency response



Este paso de bajar f de muestreo es opcional, te quedás con grafica (c)

Figure 4.50 Use of oversampling followed by decimation in C/D conversion.

Bibliography

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