Competing with Equivocal Information*

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Abstract

This paper studies strategic disclosure by multiple senders competing for prizes awarded by a single receiver. They decide whether to disclose a piece of information that is both verifiable and equivocal (it can influence the receiver both ways). The standard unraveling argument breaks down: if the commonly known probability that her information is favorable is high, a single sender never discloses. Competition restores full disclosure only if some of the senders are sufficiently unlikely to have favorable information. When the senders are uncertain about each other's strength, however, all symmetric equilibria approach full disclosure as the number of candidates increases.
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1 Introduction

In the standard persuasion game framework where an informed sender tries to persuade an uninformed receiver to take the highest action by selectively communicating verifiable information, competition plays no role. Even with a single sender, all the information is revealed because the receiver understands the motives behind any action of the sender. For this argument to work, the sender must know how her information would influence the receiver's choice. In many instances, however, an agent controls the access of others to a piece of information but is unable to predict their reactions to it. When information is equivocal in such a way, the

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classical unravelling argument breaks down and competition may play a role. The contribution of this paper is to study competition in a model with equivocal information. The starting point is the benchmark model of Caillaud and Tirole (2007) in which equivocal information is captured in a simple way.

There are many situations where information may be equivocal. For example, a climate expert may understand the environmental effects of a particular emission reduction policy, but be unable to assess its electoral value to those in charge of approving it. A movie producer, or an online advertiser, may find it impossible to predict how the information conveyed in a trailer or an ad will affect the willingness to purchase of any particular consumer.[1]

When an uninformed receiver is sufficiently inclined to act as the sender wishes, the sender has no incentive to run the risk of informing her if she is uncertain about the impact of this information. A job candidate with a good resume, for example, is unlikely to reveal additional information about herself in a statement of purpose. Indeed, since it is both difficult to appreciate how such information will be interpreted by the employer and easy to make the statement of purpose deliberately vague, a candidate who thinks that she will be hired on the basis of her resume alone will not communicate potentially detrimental information. Furthermore, common knowledge that information is equivocal to the candidate prevents the employer from drawing any unfavorable conclusion from her behavior. A candidate with a weaker resume, however, may have to provide as much additional information as possible in order to sway the employer's decision. Similarly, the best strategy to advertise a movie from a popular director is to keep the trailer elliptic and mysterious, while the trailer of a movie from an unknown director will feature all its best scenes in order to attract audiences. But these examples ignore the effects of competition.

If there are multiple job candidates, for example, a weak one who has to provide more information about herself in order to stay in the race, may get ahead of an *ex ante* stronger candidate if her information turns out to be favorable. This should incentivize strong candidates

[1] That is true even if she knows the average consumer's reaction.

to disclose more information as well. Thus, the forces of competition may be expected to lead to more disclosure. This paper shows that competition is not sufficient for full disclosure in general, and that weak candidates play an important role.

In the model, several candidates with heterogeneous prospects compete for a limited number of homogeneous prizes or slots which can be interpreted as positions, grants, the decision to implement a project or a policy, or the decision to purchase an item. A candidate's *prospect* is her probability of being a good fit. I analyze the case in which the prospects of all candidates are common knowledge among them, as well as the case in which candidates ignore the prospects of their competitors.

The search policy of the decision maker plays an important role in setting the incentives of the candidates in the disclosure game. I choose to focus on a sequential policy that prioritizes learning and works as follows. First, the decision maker first examines all documented projects one by one (the one she has received information about), starting with the most promising one and moving downwards. If she has not found enough good projects among the documented ones, she fills the remaining slots with undocumented projects that have a positive expected value. I define as strong the projects that have a positive expected value and as weak the other ones. The sequential policy is salient because it is uniquely selected if the game is perturbed by adding small processing costs for the decision maker.[2] This choice of policy is important for the results and I show how other optimal policies that are not selected by the perturbation argument may lead to different results.

Under the sequential policy, a candidate who withholds information loses her priority in the decision process of the decision maker. It might, however, still be beneficial to do so in equilibrium if the probability that there are sufficiently many good projects in the set of documented projects is low. For the decision maker, it is clearly optimal that all information be made available to her since it expands her choice set.

The first important finding is that sufficient competition leads to full disclosure only if some

[2]More accurately, any policy that is optimal under this perturbation gives the same incentives to candidates as the sequential policy.

of the candidates are *weak*,[3] that is their prospects are sufficiently low that approving them without further investigation would be wasteful in expectation. This result holds both under symmetric and asymmetric information. It emphasizes the importance of weak candidates in this type of contests. Additional results show how the presence of weak candidates improves information disclosure. The second important finding limits the scope of the first one since, when candidates do not know about the strength of their competitors, all symmetric equilibria approach full disclosure asymptotically as the number of candidates increases. Hence, in this case, the role of weak candidates is muted and competition may suffice.

Related Literature. There is a large literature on strategic communication that distinguishes soft information (Crawford and Sobel, 1982), from hard information (Grossman, 1981; Grossman and Hart, 1980; Milgrom, 1981). The literature on *persuasion games*[4] studies the case of hard (certifiable) information in problems with a single sender trying to persuade a single receiver to take a certain action. Milgrom and Roberts (1986) and subsequent contributions identified conditions under which the unraveling argument holds. The receiver must be capable of strategic reasoning, informed about the interests of the sender and aware of the type of information that is available to her. Shin (2003) shows that it may fail if there is uncertainty about the precision of the sender's information and Wolinsky (2003) introduces a particular form of uncertainty about the preferences of the sender. In Dziuda (2011), the unraveling argument fails because of the structure of provability: every argument disclosed is verifiable, but the receiver cannot know whether every argument has been disclosed. Another reason that unraveling may fail is if the direction in which the sender tries to influence the receiver changes with respect to the sender's type (see Giovannoni and Seidmann, 2007; Hagenbach, Koessler and Perez-Richet, 2012; Seidmann and Winter, 1997).

I also focus on hard information. The single sender/single receiver case of this paper, which

[3]In the case of asymmetric information, if the type distribution puts a positive weight on weak types.

[4]For a review of this literature see Milgrom (2008); recent contributions also include Glazer and Rubinstein (2001, 2004). Sobel (2007) summarizes the literature on information transmission. Kartik (2009) builds a bridge between the hard and soft information approaches.

was first analyzed by Caillaud and Tirole (2007), points at another important assumption: the sender must be able to anticipate the impact of the information she owns on the receiver. Interestingly, the inability of the sender to interpret her information is an advantage. By effectively eliminating the asymmetry of information, it renders the actions of the sender uninterpretable by the receiver and allows the sender to fully benefit from the control she exerts over the availability of information.

Caillaud and Tirole (2007) extend their analysis to a multiple-receivers framework and use a mechanism design perspective to understand optimal persuasion strategies when decisions affecting the sender are made by a committee under a qualified majority rule. I analyze a multiple-senders/single-receiver version of the same benchmark model to explore the effects of competition. There are few studies of the effects of competition in the literature on hard information. Milgrom and Roberts (1986) and Milgrom (2008) show that, even when the conditions for the unraveling argument fail, competition among senders can sometimes lead to efficient disclosure.

The paper is also related to Kamenica and Gentzkow (2011). They model an information transmission game in which the sender has to commit to a signaling technology before knowing her type, which has the same implications as the equivocal information assumption. As in this paper, the game is effectively a symmetric information game. They make no restrictions on the signaling technology that can be chosen by the sender. By contrast, the model in this paper and Caillaud and Tirole (2007) restrains the alternatives to perfect signaling or no signaling. The same authors consider the effects of competition for their model in Gentzkow and Kamenica (2011). Their framework is broader in a way but does not encompass the model in this paper. A first difference is the restriction on signaling technologies, a second difference is that information is costless to process in their model. The results are very different since, with two players or more, full revelation is always an equilibrium outcome in their model. Bhattacharya and Mukherjee (2011) also consider competition in a situation where information is unequivocal and the senders can conceal information by pretending to be uninformed. Finally, in their conclusion, Che, Dessein and Kartik (2010) suggest that competition may even harm the receiver in an extension of their framework.

The assumption that an economic agent can control access to information that she cannot process plays an important role in Eso and Szentes (2003). They propose an agency model where the principal can release, but not observe, information that would allow the agent to refine her knowledge of her own type. They show that when the full mechanism design problem is considered altogether, the optimal mechanism calls for full disclosure and allows the principal to appropriate the rents of the information she controls exactly as if it were observable to her. Eso and Szentes (2007) develop an auction model with similar conclusions.

This paper is also connected to the literature on obfuscation which studies the incentive for firms to manipulate the search cost of consumers. Ellison and Ellison (2009) provide evidence of such practices. Carlin (2009); Ellison and Wolitzky (2008); Wilson (2010) develop static models of obfuscation and Carlin and Manso (2010) provide a dynamic model. In the literature on marketing and advertising, Bar-Isaac, Caruana and Cuñat (2010) study the incentive of a monopoly to manipulate the cost of consumers to learn their true valuation. In a similar spirit, Lewis and Sappington (1994) and Johnson and Myatt (2006) study the optimal information structure of the consumer about her valuation from the point of view of a monopolist.

Finally, the advertising literature makes predictions about the relationship between product quality and the informativeness of advertising. This question is connected to the analysis of the relationship between product quality and levels of advertising in the literature. As summarized in Bagwell (2007), the empirical literature does not strongly support a systematic positive relationship. Bagwell and Overgaard (2006) and Bar-Isaac, Caruana and Cuñat (2010) offer possible theoretical explanations for a negative relationship. To the extent that the quantity of advertising and informativeness are related, the benchmark model of Caillaud and Tirole (2007) and of this paper offers an alternative simple theoretical explanation in the case of a monopolist.

2 The Model

2.1 Setup

For clarity, the model is described in the language of project adoption, although it fits other situations as well. Finitely many candidates with a single project each are indexed by the set $\mathcal{N} = \{1, \ldots, N\}$. They seek to maximize the probability that their project be adopted by a decision maker who can implement only $M \leq N$ of them. From now on, and for most of the paper I assume $M = 1$, but I explain how to extend some of the results to $M > 1$ in SECREF. A project is either good or bad for the decision maker. A good project yields an expected gain $G > 0$, whereas a bad project yields an expected loss $L > 0$.

All the players share the belief that the projects are independent from one another, and assign probability $\rho_n \in (0, 1)$ to the event that project n is good. With minimal loss of generality,[5] $\rho_1 > \cdots > \rho_N$. Each candidate n controls information that would allow the decision maker to figure out the value of project n but is irrelevant to other projects. The candidates, however, are unable to process this information and can only decide whether to communicate it to the decision maker. Alternatively, I could assume that the candidates must commit to a communication decision before observing the value of their project. When a candidate discloses her information, I say that her project is *documented*. The communication decisions are taken simultaneously. The decision maker can then decide whether to *investigate* each documented project, and which projects to approve. Her decisions are not contractible and she cannot commit to a policy at the outset.

Approving a project with prospect ρ without inquiring provides the decision maker with an expected payoff $\rho(G + L) - L$. Let $\hat{\rho} \equiv L/(L + G)$ denote the cutoff prospect at which the expected value of a project becomes positive. Let $\mathcal{N}_W \equiv \left\{n \in \mathcal{N}; \rho_n < \hat{\rho}\right\}$ be the set of *weak candidates* or *weak set*, and $\mathcal{N}_S \equiv \left\{n \in \mathcal{N}; \hat{\rho} < \rho_n\right\}$ be the set of *strong candidates* or *strong*

[5]There is a small loss of generality since ties are ruled out, but this is a measure 0 event if the probability profile is drawn from an atomless joint distribution on $[0, 1]^N$. However, some results are sensitive to this assumption as indicated below.

set. Rubberstamping a weak candidates yields a negative payoff, whereas it is positive for a strong candidate. To simplify the discussion, I assume that all the candidates are either weak or strong, that is none of them has a prospect exactly equal to $\hat{\rho}$.

At the end of the disclosure game, the decision maker may face documented and undocumented projects. Let $\mathcal{D}$ denote the set of documented projects, or *documented set*, and $\mathcal{H} = \mathcal{N} \smallsetminus \mathcal{D}$ the set of undocumented projects, or *hidden set*. Let $\mathcal{H}_W \equiv \mathcal{H} \cap \mathcal{N}_W$, $\mathcal{H}_S \equiv \mathcal{H} \cap \mathcal{N}_S$, $\mathcal{D}_W \equiv \mathcal{D} \cap \mathcal{N}_W$ and $\mathcal{D}_S \equiv \mathcal{D} \cap \mathcal{N}_S$ denote the weak and strong subsets of these sets.

One goal of the paper is to study the effect of weak candidates. More precisely, I study the effects of *strengthening the weak set*, which can mean either one of two operations: (i) adding more weak candidates; (ii) raising the prospects of current weak players (while keeping them weak).

Finally, the following notations will be useful. For any subset $\mathcal{J} \subseteq \mathcal{N}$, denote its cardinality by J, and let $j(1) < \cdots < j(J)$ be the ordered elements of this subset, so that $\rho_{j(1)} > \cdots > \rho_{j(J)}$.

Definition 1 (Truncated Subsets)**.** *For any subset $\mathcal{J} = \{j(1), \ldots, j(J)\} \subseteq \mathcal{N}$ and any $k < J$, let $\mathcal{J}^-(k) \equiv \{j(1), \ldots, j(k)\}$ and $\mathcal{J}^+(k) \equiv \{j(k+1), \ldots, j(J)\}$ be the left and right truncations of $\mathcal{J}$ at k. By convention, $\mathcal{J}^-(0) = \mathcal{J}^+(J) = \emptyset$.*

For a project $n \in \mathcal{N}$, and a subset of projects $\mathcal{J} \subseteq \mathcal{N}$, let $r_{\mathcal{J}}(n)$ be the *rank* of n in $\mathcal{J}$. This does not require n to be an element of $\mathcal{J}$: if $n \notin \mathcal{J}$ then $r_{\mathcal{J}}(n)$ is the rank that n would have in $\mathcal{J} \cup \{n\}$. For example, if $\mathcal{N}$ consists of three projects 1, 2 and 3 such that $\rho_1 > \rho_2 > \rho_3$ and $\mathcal{J} = \{1, 3\}$, then $r_{\mathcal{J}}(3) = r_{\mathcal{J}}(2) = 2$ as project 3 is the second strongest project in $\mathcal{J}$ and project 2 would be the second strongest project in $\mathcal{J} \cup \{2\}$.

2.2 The Sequential Policy Assumption

In the absence of a processing cost, it is clearly optimal for the decision maker to investigate all the documented projects before considering any of the undocumented ones. I will call this principle the learning priority principle. However, there are multiple ways to do it. The decision maker could for example investigate all the documented projects, and then select one project randomly among those tat turned out to be good. The particular way she randomizes is irrelevant. She could also search sequentially among the documented projects in any order, and validate he first good project that she finds. If in any of these procedures no good project is found among the documented one, then the decision maker should validate the strongest of the projects in $\mathcal{H}_S$.

This multiplicity of optimal procedures for the decision maker is unsatisfying. More importantly, different procedures may give different incentives to the candidates in the disclosure game. Finally, they are not all equally tractable. For these reasons, I select the following optimal policy of the decision maker, that I call the *sequential policy*.

Assumption 1 (Sequential Policy)**.** *Given any triple* $(\mathcal{D}, \mathcal{H}, M)$*, the decision maker uses the following two-step procedure:*

Step 1. *Investigate and conditionally approve all projects in* $\mathcal{D}$ *sequentially in the order* $d(1) \rightarrow d(2) \rightarrow \ldots$ *as long as there are some empty slots.*

Step 2. *Fill the* $m \geq 0$ *remaining slots with the* $\min\{m, H_S\}$ *strongest projects in* $\mathcal{H}_S$*.*

This assumption is important for the results, and I will illustrate how they may fail when the decision maker investigates all documented projects and chooses uniformly among the ones that turn out to be good. It is also quite tractable. But the main reason to select the sequential policy is that it remains optimal when a small processing cost is introduced. This is not the case, for example, of the uniform policy. In fact, every policy that remains optimal gives the same incentives as the sequential policy in the disclosure game. See Section 6 for a formal treatment of this result.

3 Sequential Policy and Incentives

Probability of Implementation. As a consequence of the sequential policy assumption, the probability that a project is approved depends on the probabilities of finding good projects in subsets of $\mathcal{D}$. Hence it is useful to introduce the following notations. For a subset $\mathcal{K}$ of $\mathcal{D}$, let $f(p,\mathcal{K})$ denote the probability of finding exactly p good projects in $\mathcal{K}$, and let

$$F(p,\mathcal{K}) \equiv \sum_{q=0}^{p} f(q,\mathcal{K})$$

be the probability that there are no more than p good projects in $\mathcal{K}$.

$$f(p,\mathcal{K}) = \sum_{\substack{\mathcal{J}\subset\mathcal{K}\\ J=p}} \prod_{j\in\mathcal{J}} \prod_{l\in\mathcal{K}\smallsetminus\mathcal{J}} \rho_j(1-\rho_l), \tag{1}$$

and

$$F(p,\mathcal{K}) = \sum_{\substack{\mathcal{J}\subset\mathcal{K}\\ J\leq p}} \prod_{j\in\mathcal{J}} \prod_{l\in\mathcal{K}\smallsetminus\mathcal{J}} \rho_j(1-\rho_l). \tag{2}$$

$F(p,\mathcal{K})$ is clearly increasing in p. It is also decreasing in $\mathcal{K}$ for the set containment order and decreasing in ρ_k for any $k \in \mathcal{K}$ (see Lemma 9 in Appendix C). Intuitively, adding new candidates to a pool or increasing the prospect of any project already in the pool increases the probability that at least p projects in the pool are good.

With these notations, the probability that $d(k)$, the k-th best project in $\mathcal{D}$, is implemented by the decision maker is

$$\rho_{d(k)}F(M-1,\mathcal{D}^{-}(k-1)),$$

and the probability that $h(k)$, the k-th project in $\mathcal{H}$, is implemented is

$$F(M-k,\mathcal{D}) \cdot \mathbb{1}_{h(k)\in\mathcal{H}_S}.$$

When $M = 1$, these expressions become

$$\rho_{d(k)} F(0, \mathcal{D}^-(k-1)) = \rho_{d(k)} \prod_{\ell=1}^{k-1} \left(1 - \rho_{d(\ell)}\right)$$

for the first one, and the only undocumented project that stands a chance of being rubber-stamped is the strongest one, with probability

$$F(0, \mathcal{D}) \cdot \mathbb{1}_{h(1) \in \mathcal{H}_S} = \prod_{\ell=1}^{D} \left(1 - \rho_{d(\ell)}\right) \cdot \mathbb{1}_{h(1) \in \mathcal{H}_S}.$$

Implied Preferences of the Decision Maker. My focus is on the amount of disclosure, and the rank of withholding candidates. It is important, however, to keep in mind the effect of changes in disclosure on the payoff of the decision maker. Assumption 1 implies the following expression for the expected payoff of the decision maker

$$\begin{aligned} V(\mathcal{D}, \mathcal{H}, M) = {} & (1 - F(M-1, \mathcal{D}))MG + G \sum_{p=1}^{M-1} p f(p, \mathcal{D}) \\ & + \sum_{r=1}^{\min(M, H_S)} F(M-r, \mathcal{D}) \left(\rho_{h(r)}(G+L) - L\right). \end{aligned} \tag{3}$$

The first term measures the payoff from implementing M good projects, weighted by the probability $1 - F(M-1, \mathcal{D})$ of finding them. The second term measures the expected payoff obtained when less than $M - 1$ good projects are found in $\mathcal{D}$. Finally, the last term measures the payoff from filling the remaining slots with projects in $\mathcal{H}_S$. Then I can show the following about the preferences of the decision maker: first, she obviously prefers larger documented sets in the set containment order as it enlarges her choice set; second, when $M = 1$ she prefers to get information from stronger candidates.

Lemma 1 (Preferences of the Decision Maker)**.**

(i) The decision maker prefers a larger documented set: $\mathcal{D}_0 \subseteq \mathcal{D}_1 \Rightarrow V(\mathcal{D}_1, \mathcal{H}_1, M) \geq V(\mathcal{D}_0, \mathcal{H}_0, M)$.

(ii) Suppose $M = 1$ and consider two documented sets $\mathcal{D}^0 = \hat{D} \cup \{m\}$ and $\mathcal{D}^1 = \hat{D} \cup \{n\}$, with $\rho_n > \rho_m$. Then $\mathcal{D}_0 \subseteq \mathcal{D}_1 \Rightarrow V(\mathcal{D}_1, \mathcal{H}_1, 1) \geq V(\mathcal{D}_0, \mathcal{H}_0, 1)$.

Proof. See Appendix C $\square$

To get an intuition for the second point, consider the two partitions $[\mathcal{D}^0, \mathcal{H}^0]$ and $[\mathcal{D}^1, \mathcal{H}^1]$ of the set of projects. In words, the only difference between the two partitions is that in $\mathcal{D}^1$ the stronger project n is documented whereas the weaker one m is undocumented, while the opposite is true in $\mathcal{D}^0$. Because this swap between the roles of n and m can be decomposed in a series of swaps between adjacent projects, we might as well assume that $m = n + 1$. If there exists a project $h \in \mathcal{H}^0_S \cap \mathcal{H}^1_S$, so that it is strong and undocumented in both partitions, and $\rho_h > \rho_n$, then when n or $n + 1$ are undocumented, then neither is ever implemented when the search among documented projects fails. Then the only effect of the swap is to change the probability that the search among documented projects is successful, and then $[\mathcal{D}^1, \mathcal{H}^1]$ is clearly better than $[\mathcal{D}^0, \mathcal{H}^0]$ for the decision maker. However, if n is strong and dominates all the projects in $\mathcal{H}^0 \cap \mathcal{H}^1$, the effect of the swap is ambiguous: when n is documented, the search among documented projects is more likely to be successful, but the fall back option is less promising. The proof shows that the first effect dominates when there is a single slot, so that $[\mathcal{D}^1, \mathcal{H}^1]$ remains better than $[\mathcal{D}^0, \mathcal{H}^0]$ in this case. This result needs no longer be true when $M > 1$, as the following example shows.

Example 1. *Consider the case of three strong candidates for two slots. If the two most promising candidates disclose their information while the third one does not, the payoff of the decision maker is $V_1 = 2\rho_1\rho_2 G + G(\rho_1(1 - \rho_2) + \rho_2(1 - \rho_1)) + (1 - \rho_1\rho_2)(\rho_3(G + L) - L)$. The first term gives her payoff if the search in her documented set is fully successful weighted by the probability of such a success; the second term is the weighted payoff of the search when it is only partially successful, the third term is the cost of the search; the last term is the weighted payoff from rubberstamping the third project when the search is not fully successful. If the first and the third candidate disclose their information while the second does not* *her payoff is* $V_2 = 2\rho_1\rho_3 G + G(\rho_1(1-\rho_3) + \rho_3(1-\rho_1)) + (1-\rho_1\rho_3)(\rho_2(G+L) - L)$. *Then* $V_2 - V_1 = (1-\rho_1)(\rho_2 - \rho_3)L > 0$. *Hence the decision maker prefers to get information from candidate 3 than from candidate 2 when candidate 1 is disclosing. In fact, it is easy to show that with 3 strong candidates and two slots, the decision maker always prefers to obtain her information from weaker candidates.*

4 The Disclosure game

To read this section, recall that in the disclosure game, the candidates simultaneously decide whether to disclose their information to the decision maker. I start with the benchmark case of a single candidate. Then I provide some remarks on the general case, and a general result about the impossibility of full disclosure without weak candidates. Finally I analyze the single slot case.

4.1 Benchmark: One Candidate

This case is also the benchmark case of Caillaud and Tirole (2007) who first analyzed it. Without processing cost, the result is extremely simple. A weak candidate cannot be rubberstamped so she will disclose her information. A strong candidate with prospect ρ is certain to be rubberstamped if she does not disclose, while if she does, her project is approved with probability $\rho < 1$, and hence it is clearly optimal for her to withhold.

Proposition 1 (Caillaud and Tirole 2007)**.** *If the candidate is weak, she discloses and her project is implemented if it turns out to be good. If the candidate is strong, she does not disclose and her project is implemented with probability 1.*

Hence when a candidate is strong, she has *real authority* over the final decision in the sense of Aghion and Tirole (1997) since she effectively controls the decision. This generates a non-monotonicity in the expected payoff of the decision maker as a function of ρ since it is equal to ρG for $\rho < \hat{\rho}$, but then falls back to 0 and is equal to $\rho(G+L) - L$ for $\rho > \hat{\rho}$.

4.2 Multiple Candidates

Full Disclosure. Since full disclosure is the optimal outcome of the disclosure game for the decision maker, it is important to characterize the conditions under which it obtains. To start this section, I provide a general negative result in the absence of weak candidates. The intuition for this result is very simple. If all projects are strong and all candidates disclose, the weakest project (which is strong nevertheless) has the same probability of being reached by the search whether it is documented or not. But, conditionally on being reached, the project is accepted with probability one if it is undocumented, while if it is documented the probability that it is implemented is equal to its prospect. Therefore, the weakest candidate would deviate from full disclosure.

Proposition 2 (Full Disclosure−Impossibility)**.** *Full disclosure is impossible in the absence of weak candidates.*

While the intuition for this result is quite compelling, it is important to note that it is sensitive to my assumptions: if prospects can be tied, or if the policy of the decision maker is not the sequential policy, the result may not hold. To see this consider the following examples.

Example 2 (Ties)**.** *Suppose that there are two strong projects with equal prospect ρ. Then the payoff of a candidate in the case of full disclosure is given by $\rho/2 + \rho(1-\rho)/2$, if we assume that the decision maker randomizes uniformly to choose which project she investigates first. If a candidate withholds while the other one discloses, her payoff is $1-\rho$. Hence full disclosure is an equilibrium when ρ is sufficiently close to 1.*

Example 3 (Uniform Policy)**.** *Consider the following policy for the decision maker: first, check all documented projects, and pick uniformly among the ones that turn out to be good; second, if there is no good project among the documented ones pick uniformly among the strong non-documented projects. Suppose that there are two strong candidates with prospects $\rho_1 > \rho_2$. Then the payoff of candidate 1 under full disclosure is $\rho_1\rho_2/2+(1-\rho_2)\rho_1$, and symmetrically for candidate 2. If candidate 1 is the only one who withholds, her payoff is $1-\rho_2$, and symmetrically for* *candidate 2. It is clear that full disclosure is an equilibrium when both prospects are sufficiently close to 1. In fact disclosing is then a strictly dominant strategy for both players.*

General Remarks. Since any weak and undocumented project is discarded by the decision maker, a weak candidate is certain that her project stands no chance if she refuses to disclose. If she discloses, she faces the probability of adoption $F\big(M-1, \mathcal{D}^-\big(r_{\mathcal{D}}(n)-1\big)\big)\rho_n > 0$, where $r_{\mathcal{D}}(n)$ denotes the rank of project n in the documented set, and $\mathcal{D}^-\big(r_{\mathcal{D}}(n)-1\big)$ is therefore the set of documented projects which are stronger than n. This leads to the following remark.

Remark 1. It is a dominant strategy for weak candidates to disclose.

Therefore, in any equilibrium $\mathcal{H} = \mathcal{H}_S$, and $\mathcal{N}_W \subseteq \mathcal{D}$. Note that disclosing always yields a positive probability of approval. Withholding, on the other hand, yields a null probability of approval for all but the first M projects in $\mathcal{H}_S$. Consequently, in any equilibrium, $H \leq M$, for otherwise candidate $h(M+1)$ would gain by disclosing.

Remark 2. Any equilibrium action profile $[\mathcal{D}, \mathcal{H}]$ satisfies $H \leq M$.

Interestingly, this remark bounds the amount of withholding in equilibrium. For the same reason, when $H = M$, no candidate weaker than $h(M)$, the M-th candidate in $\mathcal{H}$, has any incentive to withhold.

Remark 3. Given any action profile $[\mathcal{D}, \mathcal{H}]$ such that $H = M$, a candidate $n \in \mathcal{D}$ such that $r_{\mathcal{N}}(n) > r_{\mathcal{N}}(h(M))$ has no incentive to deviate.

4.3 The Single Slot Case

From now on, I assume that $M = 1$. In this case, it is clear from the remarks above that there is at most one candidate who withholds, and that if there is such a candidate, her project is strong.

Now consider an action profile $[\mathcal{D}, \mathcal{H}]$ such two consecutive strong candidates, n and $n-1$, both disclose. Then if n deviates by withholding information, she is certain that her project is implemented if the search among documented projects fails. Then her payoff if she deviates is given by the probability that the search fails, $F(0, \mathcal{D} \setminus \{n\})$. If she decides to stick to her original choice and disclose, then her project is implemented with probability ρ_n if the search reaches her, that is if none of the candidates stronger than n turns out to be good . Hence her payoff from disclosing is given by $\rho_n F(0, \mathcal{D}^{-}\{n-1\})$, the probability that the search among documented projects that are stronger than n fails times the probability that n is good. Then I may characterize the incentive to deviate of candidate n by her relative gain from deviating which is given by the following ratio

$$\delta(n) = \frac{F\big(0, \mathcal{D} \setminus \{n\}\big)}{\rho_n F\big(0, \mathcal{D}^{-}\{n-1\}\big)}$$

But the same formula applies to the next stronger candidate $n-1$, and a simple calculation yields

$$\frac{\delta(n)}{\delta(n-1)} = \frac{\rho_{n-1}}{\rho_n} > 1.$$

Hence n has a stronger incentive to deviate than $n-1$. The next remark follows from this observation.

Remark 4. Consider a strategy profile such that candidates $n-k, n-k+1, \cdots, n-1, n$ are strong and disclose. Then, if any of these candidates is better of by deviating, candidate n must be better of by deviating.

This remark considerably reduces the number of incentives constraint that need to be checked to verify that a profile is an equilibrium. For full disclosure we need only check that the weakest of the strong candidates, N_S, has no incentive to deviate. The following proposition gives this condition and makes the additional point that when full disclosure is an equilibrium it is generically the unique equilibrium, and in fact the game is even dominance solvable.[6]

[6] The result in Proposition 3 is a direct consequence of the property in Remark 4. In a former version of the paper available upon request, I show that this property holds for any M whenever $\rho_1 \leq 1/2$, so that Proposition 3 holds as well.

Proposition 3 (Full Disclosure – Characterization). *Full disclosure is an equilibrium of the disclosure game with a single slot if and only if the weakest of the strong candidates has no incentive to deviate, that is*

$$\rho_{N_S} \geq \frac{F\big(0, \mathcal{N} \setminus \{N_S\}\big)}{F\big(0, \mathcal{N}^-(N_S - 1)\big)} = \prod_{n \in \mathcal{N}_W} (1 - \rho_n). \tag{4}$$

Furthermore, the disclosure game is dominance solvable whenever the inequality in (4) holds strictly. In particular, full disclosure is then the unique equilibrium.

Proof. It is easy to see that (4) ensures that the weakest of the strong candidate has no incentive to deviate from full disclosure, and Remark 4 implies that it is a sufficient condition for full disclosure equilibrium. When it holds strictly, the game is dominance solvable for the following reason. First withholding is a strictly dominated strategy for weak candidates by Remark 1 so theses strategies can be eliminated. If (4) holds strictly, it is strictly better for the weakest of the strong candidate, N_S, to withhold when the other strong candidates withhold. If some of the other strong candidates withhold, then candidate N_S is also strictly better of by disclosing because if she does not it is one of the stronger withholding candidate that is chosen when the search among documented projects is unsuccessful (see Remark 2). Hence withholding is a strictly dominated strategy for N_S and can be eliminated. By Remark 4, when (4) holds strictly, it also holds strictly when replacing N_S by $N_S - 1$. But then I can repeat the argument I just made for candidate $N_S - 1$ and show that withholding is strictly dominated. Doing that iteratively until reaching the strongest of the strong candidate solves the game. □

Proposition 2 and 3 both show the importance of weak candidates. It is competition from weak candidates, who cannot afford secrecy, that puts pressure on stronger candidates to reveal their information. More generally, the right-hand side of (4) is decreasing in $\mathcal{N}_W$ both for the set order and for the strength order, implying that a better pool of weak candidates makes condition (4) easier to satisfy. The only strong candidate that has an effect on full disclosure is the weakest of the strong candidates. Adding candidates stronger than N_S or improving the prospect of existing ones cannot help to get full disclosure.

Beyond full disclosure, I can use Remark 4 to characterize the pure strategy equilibria. In fact, to verify that an action profile where the only withholding candidate is $n \in \mathcal{N}_S$ is an equilibrium I need to check two things:

(i) n would not be better of by disclosing:

$$\rho_n \leq (1-\rho_{n+1}) \cdots (1-\rho_N).$$

(ii) $n-1$ would not be better of by withholding:

$$(1-\rho_{n+1}) \cdots (1-\rho_N) \leq \rho_{n-1}.$$

Indeed, the candidates who are weaker than n have no incentive to withhold by Remark 3, and if any candidate who is stronger than n has an incentive to withhold, then so does $n-1$ by Remark 4. Of course, (ii) is void when $n=1$.

Then I can characterize equilibria as follows. Let $n^* = \min\{n \in \mathcal{N}_S : \rho_n \leq (1-\rho_{n+1}) \ldots (1-\rho_N)\}$, and $n^* = 0$ when this set is empty. n^* is the strongest candidate of the strong set whose prospect is less than the probability that none of the projects with lower prospects is good. It is also the strongest candidate of the strong set who prefers to withhold when everyone else discloses. If there is no such candidate, that is $n^* = 0$, then full disclosure is the equilibrium outcome. If $n^* = 1$, then candidate 1 is the only one withholding information in equilibrium. Otherwise, either n^* satisfies (i) and (ii) and then the unique pure strategy equilibrium is such that n^* withholds and all other candidates disclose.

Proposition 4 (Equilibrium Characterization)**.** *There exists an equilibrium in pure strategies if $n^* \in \{0,1\}$ or if n^* satisfies $\rho_{n^*-1} \geq (1-\rho_{n^*+1}) \ldots (1-\rho_N)$. When it exists, this equilibrium is the unique pure strategy equilibrium. There is full disclosure if $n^* = 0$, and otherwise the only withholding candidate in equilibrium is n^*.*

Proof. See Appendix A. □

The next proposition shows that improving the set of weak candidates $\mathcal{N}_W$ can only lead to a better pure strategy equilibrium of the disclosure game from the point of view of the decision maker. Improving the prospect of the strongest candidates, however, does not change the identity of the withholding candidate.

Proposition 5 (Single Slot – Comparative Statics)**.** *Improving the set of weak candidates, either by adding more weak candidates or by increasing the prospects of the current ones, increases n^*. Making candidates such that $\rho > \rho_{n^*}$ stronger cannot change the identity of the withholding candidate in equilibrium.*

Proof. See Appendix A. □

To conclude this section, I consider the example of two strong candidates, and possibly many weak candidates. In general, mixed strategy equilibria are difficult to characterize and may involve mixing by more than two candidates, but they can be analyzed in a simple way in this example.

Example 4. *When $N_S = 2$, a mixed strategy equilibrium obtains whenever $(1-\rho_2)f(0,\mathcal{N}_W) < \rho_1 < f(0,\mathcal{N}_W)$ (from Proposition 4). In this case, the mixed strategy equilibrium is unique and the two strong candidates play as follows. The stronger one discloses her information with probability $\lambda_1 = \frac{\rho_2}{\rho_1\rho_2+(1-\rho_1)f(0,\mathcal{N}_W)}$, while the weaker one discloses her information with probability $\lambda_2 = \frac{f(0,\mathcal{N}_W)-\rho_1}{\rho_2 f(0,\mathcal{N}_W)}$. For the details of the calculation, see Appendix A. The following table describes the evolution of the equilibria as the weak set becomes stronger, that is as $f(0,\mathcal{N}_W)$ decreases. The equilibrium is generically unique. As the weak set becomes stronger, the equilibrium switches from candidate 2 disclosing, to a mixed strategy equilibrium such that 1 becomes increasingly likely to disclose whereas 2 becomes less so, to an equilibrium in which only 1 discloses, and finally to full disclosure. The arrows in the table indicate the sense of variation of λ_1 and λ_2 with respect to $f(0,\mathcal{N}_W)$.*

$f(0,\mathcal{N}_W)$	*candidate 1*	*candidate 2*
$1 \geq \cdot \geq \rho_1/(1-\rho_2)$	*0*	*1*
$\rho_1/(1-\rho_2)$	$\frac{\rho_2(1-\rho_2)}{\rho_1(\rho_2(1-\rho_2)+\rho_1(1-\rho_1))} \geq \lambda_1 \geq 0$	*1*
$\rho_1/(1-\rho_2) \geq \cdot \geq \rho_1$	$\lambda_1 \downarrow$	$\lambda_2 \uparrow$
ρ_1	$1 \geq \lambda_1 \geq \frac{\rho_2}{\rho_1(\rho_2+1-\rho_1)}$	*0*
$\rho_1 \geq \cdot \geq \rho_2$	*1*	*0*
$\rho_2 \geq \cdot \geq 0$	*1*	*1*

5 Incomplete Information

In some applications, it may be unreasonable to assume that candidates know each other's prospect, especially when their number is large. Then it is still true that weak types are needed for full disclosure, and more generally that they favor information revelation. However, their role is less important since, even in the absence of weak types, competition leads to full disclosure in the limit as the number of candidates goes to infinity.

I assume that the prospects of the projects are drawn independently from an atomless distribution with cumulative density function Ψ and full support $\mathcal{S} = [\underline{x}, \overline{x}] \subseteq (0,1)$.The corresponding probability density function ψ is assumed to be bounded away from 0 by some $\underline{\psi} > 0$.[7] All prospects are observed by the decision maker and I maintain Assumption 1. N is common knowledge. The *type* of candidate n is her realized prospect $\rho_n \in \mathcal{S}$. Types lying in $\mathcal{S} \cap (0, \hat{\rho})$ are weak, and types in $\mathcal{S} \cap (\hat{\rho}, \overline{x})$ are strong. If $\hat{\rho} \leq \underline{x}$, weak types are *absent*, and otherwise they are *present*.

Let $\lambda_n(\rho)$ denote the strategy of candidate n that consists in disclosing with probability $\lambda_n(\rho)$ when her type is ρ. I consider Bayesian Nash equilibria. In general, I consider only symmetric equilibria but I show in Proposition 6 that under certain conditions the game is dominance solvable and then full disclosure is the unique equilibrium.[8]

[7]The only result that relies on this assumption is Proposition 8

[8]For a rigorous treatment, I would need to define distributional strategies as in Milgrom and Weber (1985), and derive $\lambda_n(\rho$ from a distributional strategy. The exact notion of equilibrium is that of Bayesian Nash

Full disclosure denotes the strategy profile such that all the candidates disclose with probability 1 regardless of their type. Finally, if $(\lambda_1, \ldots, \lambda_N)$ is an equilibrium, then so is any strategy profile $(\lambda'_1, \ldots, \lambda'_N)$ such that λ'_n and λ_n differ on a subset of measure 0 of the set on which n is indifferent between disclosing or not. The characterization results in what follows are up to this known issue.

Let $\boldsymbol{\lambda} = (\lambda_1, \cdots, \lambda_N)$ be a strategy profile. Supposing that all other candidates are playing according to $\boldsymbol{\lambda}$, the payoffs of candidate n are given by

$$\begin{aligned} V^{\boldsymbol{\lambda}}_{D,n}(\rho) &\equiv \rho E\left[\prod_{m\neq n}(1-\lambda_m(\rho_m)\rho_m \mathbb{1}_{\rho_m>\rho})\right] \\ &= \rho \prod_{m\neq n}\left(1-\int_{\rho}^{\overline{x}} x\lambda_m(x)d\Psi(x)\right), \end{aligned} \tag{5}$$

if she discloses, and

$$\begin{aligned} V^{\boldsymbol{\lambda}}_{H,n}(\rho) &\equiv E\left[\prod_{m\neq n}\lambda_m(\rho_m)\left(1-\rho_m\right)+\left(1-\lambda_m(\rho_m)\right)\mathbb{1}_{\rho_m<\rho}\right]\mathbb{1}_{\rho\geq\hat{\rho}} \\ &= \mathbb{1}_{\rho\geq\hat{\rho}}\prod_{m\neq n}\left(\int_{\underline{x}}^{\overline{x}}\lambda_m(x)(1-x)d\Psi(x)+\int_{\underline{x}}^{\rho}\left(1-\lambda_m(x)\right)d\Psi(x)\right), \end{aligned} \tag{6}$$

if she withholds.[9] $V^{\boldsymbol{\lambda}}_{D,n}$ is continuous in type[10] while $V^{\boldsymbol{\lambda}}_{H,n}$ has a single discontinuity at $\hat{\rho}$ if weak types are present. $V^{\boldsymbol{\lambda}}_{D,n}$ is also strictly increasing in ρ, while $V^{\boldsymbol{\lambda}}_{H,n}$ is only weakly increasing in ρ. In particular, it is constant on any interval of types on which all other players disclose with probability 1. The reason is that the probability of being considered by the decision maker when withholding depends on a candidate's own type ρ only through the implied probability that a candidate with type higher than ρ also withholds, which is invariant while ρ stays within

equilibrium in distributional strategies. To simplify the exposition, I start directly from $\lambda_n(.)$ in the main body of the paper, and relegate the rigorous treatment to Appendix B.

[9] In both equations, the equality is a consequence of the independence of the prospects.

[10] The only non-obvious part of the argument consists in showing that $\int_{\rho}^{\overline{x}} x\lambda_m(x)d\Psi(x)$ is continuous in ρ. Because λ_m is bounded between 0 and 1, $\left|\int_{\rho}^{\overline{x}} x\lambda_m(x)d\Psi(x) - \int_{\rho'}^{\overline{x}} x\lambda(x)d\Psi(x)\right| \leq \left|\int_{\rho}^{\rho'} d\Psi(x)\right| = |\Psi(\rho)-\Psi(\rho')|$ for any pair (ρ, ρ'). Hence, Ψ being atomless, this difference goes to 0 when $\rho' \to \rho$, and that concludes the argument. A similar argument works for $V^{\boldsymbol{\lambda}}_{H}$.

an interval on which other candidates disclose.

If $\boldsymbol{\lambda} = (\lambda, \dots, \lambda)$ is a symmetric strategy profile, dropping the n index for the payoff functions,

$$V_D^\lambda(\rho) = \rho \left(1 - \int_\rho^{\overline{x}} x\lambda(x) d\Psi(x) \right)^{N-1}, \tag{7}$$

and,

$$V_H^\lambda(\rho) = \left(\int_{\underline{x}}^{\overline{x}} \lambda(x)(1-x)d\Psi(x) + \int_{\underline{x}}^{\rho} (1-\lambda(x))\, d\Psi(x) \right)^{N-1} \mathbb{1}_{\rho \geq \hat{\rho}}. \tag{8}$$

A profile $\boldsymbol{\lambda}$ is an equilibrium if n is willing to play according to λ_n when other candidates follow $\boldsymbol{\lambda}$. Let $\Lambda_n \equiv \{\rho \in \mathcal{S} : \lambda_n(\rho) \in (0,1)\}$, $\Lambda_n^0 \equiv \{\rho \in \mathcal{S} : \lambda_n(\rho) = 0\}$ and $\Lambda_n^1 \equiv \{\rho \in \mathcal{S} : \lambda_n(\rho) = 1\}$. Then λ is an equilibrium strategy if and only if

(i) $\forall \rho \in \Lambda_n^0, \quad V_{D,n}^{\boldsymbol{\lambda}}(\rho) \leq V_{H,n}^{\lambda}(\rho)$,

(ii) $\forall \rho \in \Lambda_n, \quad V_{D,n}^{\boldsymbol{\lambda}}(\rho) = V_{H,n}^{\lambda}(\rho)$,

(iii) $\forall \rho \in \Lambda_n^1, \quad V_{D,n}^{\boldsymbol{\lambda}}(\rho) \geq V_{H,n}^{\lambda}(\rho)$.

Before going further, note that there exists a symmetric equilibrium (see Appendix B for a proof). As in the case with perfect information, it is clear that any strategy that prescribes to disclose with probability less than 1 for weak types is strictly dominated.

Lemma 2. *Any strategy λ_n such that $\lambda_n(\rho) < 1$ for some $\rho < \hat{\rho}$ is strictly dominated.*

Proof. See Appendix B. □

Therefore in equilibrium $[\underline{x}, \hat{\rho}) \subseteq \Lambda_n^1$. It is easy to prove that when weak types are present, sufficient competition yields full disclosure.

Proposition 6 (Full Disclosure)**.** *If weak types are present, full disclosure is an equilibrium if and only if*

$$N \geq \hat{N} \equiv 1 + \frac{\log(1/\hat{\rho})}{\log\left(1 - \int_{\hat{\rho}}^{\overline{x}} x d\Psi(x)\right) - \log\left(1 - \int_{\underline{x}}^{\overline{x}} x d\Psi(x)\right)}. \tag{9}$$

Furthermore the game is dominance solvable whenever this inequality holds strictly. In particular, full disclosure is then the unique equilibrium. In the absence of weak types, full disclosure is never an equilibrium.

Proof. See Appendix B. □

In order to understand the role of weak and strong candidates, it is interesting to look at the effect of the distribution of types on $\hat{N}$. The next result shows that increasing the weight on stronger types in the weak set and decreasing the weight on stronger types in the strong set while keeping the relative weights of these sets constant, hence concentrating the distribution around $\hat{\rho}$, leads to a lower threshold $\hat{N}$. Obviously it is also better to have stronger candidates for the decision maker, therefore decreasing the weight on stronger types in the strong set has an additional detrimental effect. Increasing the weight on stronger types in the weak set, however, is unambiguously better for the decision maker.

Proposition 7 (Comparative Statics)**.** *Consider two distributions* Φ *and* Ψ *with the same support* $\mathcal{S}$ *that includes weak types,* $\hat{\rho} \in \mathcal{S}$*, and such that for every* $x \in \mathcal{S}$*,* $\Phi(x) \leq \Psi(x)$ *if* $x \leq \hat{\rho}$*, and* $\Phi(x) \geq \Psi(x)$ *if* $x \geq \hat{\rho}$*. Then* $\hat{N}_\Phi \leq \hat{N}_\Psi$*.*

Proof. See Appendix B. □

Knowing that sufficient competition leads to full disclosure as long as weak types are present, one may wonder about the effect of competition in the absence of weak types. The following proposition shows that, even though full disclosure is never an equilibrium, any sequence of symmetric equilibria approaches full disclosure as N goes to infinity.

Proposition 8 (Competition at the Limit)**.** *In the absence of weak types, if* $\{\lambda_N\}_{N=1}^{\infty}$ *is a sequence of symmetric equilibria, then for every* $\rho \in (\underline{x}, \overline{x}]$

$$\lim_{N\to\infty} \lambda_N(\rho) = 1.$$

Proof. See Appendix B. □

6 Selecting the Sequential Policy

In this section, I assume that acquiring information about a documented project has a cost $c > 0$, and there is no restriction on the number of slots M. I show that all the results carry through as long as the cost c is sufficiently small. In fact, introducing the cost c eliminates all possible policies of the decision maker that do not give the same incentives in the disclosure game as the sequential policy. More precisely, consider the following assumption which is satisfied whenever c is sufficiently small:

Assumption 2. *The processing cost* $c > 0$ *satisfies:*

(AL) *Affordable Learning:* $c < LG/(L+G)$.

(NE) *No Extremes:* $c < \min\Big(G\rho_N, L(1-\rho_1)\Big)$.

(LP) *Learning Priority: if* $N \geq 2$, *then* $c < (L+G)\rho_N(1-\rho_1)$.

The *Affordable Learning* assumption ensures that some projects are worth investigating for the decision maker; the *No Extremes* assumption ensures that no project has such a good prospect that the decision maker would be better of by rubberstamping it than by investigating, and that no project has such a bad prospect that she would not investigate it. The learning priority assumption implies that a decision maker with one slot to fill and a pool of projects reduced to the best and the worst projects from the original pool would always choose to process all available information before rubberstamping a project. Indeed, suppose the decision maker has only received documentation about N, the worst project, and is therefore contemplating two choices: (i) rubberstamp the best project, with expected payoff $\rho_1(G + L) - L$, or (ii) first investigate project N and then rubberstamp project 1 if N is a bad project, with expected payoff $\rho_N G + (1 - \rho_N)(\rho_1(G + L) - L) - c$. The latter dominates the former if and only if (LP) is satisfied. This learning priority idea can be extended to show the following.

Proposition 9 (Optimal Policy)**.** *Suppose Assumption 2 is satisfied. Then the sequential policy is optimal for the decision maker. Furthermore, the probability that a certain project is approved is invariant across all optimal policies of the decision maker.*

Proof. See Appendix C □

A corollary of Proposition 9 is that all the result of the paper carry through if the processing cost is sufficiently small. It also means that the sequential policy of the decision maker, and therefore the disclosure game it gives rise to, is essentially the only one that survives the introduction of a small processing cost.

7 Conclusion

The results of this paper highlight the importance of *ex ante* weaker candidates to elicit information transmission in certain types of contests. They are less important, however, when candidates have imperfect information about their competitors. Then increasing the number of player ensures full disclosure asymptotically.

Appendix A The disclosure game

***Proof of Proposition 4* (Single Slot – Characterization).** As argued in the main body of the paper, there is an equilibrium such that n is the only candidate withholding information if and only if $\rho_n \leq (1-\rho_{n+1}) \dots (1-\rho_N)$ and $\rho_{n-1} \geq (1-\rho_{n+1}) \dots (1-\rho_N)$, which is by definition the case for n^*. The only point to prove is therefore uniqueness. Let $F_n = (1-\rho_{n+1}) \dots (1-\rho_N)$.

$\{F_n\}$ is an increasing sequence whereas $\{\rho_n\}$ is a decreasing sequence. Then, by definition of n^*, $\rho_n \leq F_n$ if and only if $n \geq n^*$. But since for there to be an equilibrium in which n is the only withholding candidate $\rho_{n-1} \geq F_n$ must also hold, n^* is the only possibility. Indeed if $n \geq n^*$, $\rho_{n-1} \leq F_{n-1} < F_n$ so that the second condition for an equilibrium cannot hold. $\square$

Proof of Proposition 5 **(Single Slot – Comparative Statics)**. We can rewrite the definition of n^* as the smallest strong candidate n such that $\rho_n \leq (1-\rho_{n+1})\cdots(1-\rho_{N_S})f(0,\mathcal{N}_W)$. Clearly, strengthening the weak set by adding weak candidates or by making the existing weak candidates stronger decreases $f(0,\mathcal{N}_W)$. But then it also reduces the set of strong candidates n that satisfy the inequality above, and therefore it raises n^*, the smallest n that satisfies it. Increasing ρ_k for candidates who are stronger than n^* has no effect on the expression that defines n^*, and therefore leaves n^* unchanged. $\square$

Characterization of the Mixed Strategy Equilibrium in Example 4. In order to make 1 indifferent between disclosing and withholding, λ_2 must satisfy $(\lambda_2(1-\rho_2)+(1-\lambda_2))f(0,\mathcal{N}_W) = \rho_1$ where the left-hand side is 1's payoff when withholding and the right-hand side is her payoff when she discloses. The same indifference condition for candidate 2 gives $\lambda_1(1-\rho_1)\rho_2 + (1-\lambda_1)\rho_2 = \lambda_1(1-\rho_1)f(0,\mathcal{N}_W)$. $\square$

Appendix B Incomplete Information

A distributional strategy of candidate n is a probability measure λ_n on $\mathcal{S} \times \{0,1\}$ for which the marginal distribution of $\mathcal{S}$ is ψ, where $\{0,1\}$ is a description of the action set and 1 corresponds to disclosing. This formalism introduced by Milgrom and Weber (1985) allows one to describe mixing behaviors by the players while avoiding the measurability issue noted in Aumann (1964). The probability that player n discloses information given that her type is ρ is then $\lambda_n(1|\rho)$. To simplify the notations, I denote this probability by $\lambda_n(\rho)$. The equilibrium notion for the disclosure game is Bayesian Nash equilibrium in distributional strategies. In general, I consider only symmetric equilibria but I show in Proposition 6 that under certain conditions the game is dominance solvable and then full disclosure is the unique equilibrium.

I start by showing the existence of a symmetric Bayesian Nash equilibrium in distributional strategies. This result needs a minor adaptation of Milgrom and Weber (1985) because of the symmetry requirement.

Lemma 3 (BNE existence)**.** *There exists a symmetric Bayesian Nash Equilibrium in distributional strategies for the disclosure game.*

Proof. Propositions 1 and 3 and Theorem 1 of Milgrom and Weber (1985) imply that when the set of distributional strategies is topologized by weak convergence, the players' strategy sets are compact, convex metric spaces and the payoff functions are continuous and linear in each single player's strategy, then the best response function $\beta : \Sigma \to 2^{\Sigma}$ that maps a strategy σ into the set of best responses of any player (the game is symmetric) to σ is a Kakutani map (that is upper-semicontinuous, non-empty valued and convex valued), where Σ is the set of distributional strategies of each player. Then the Kakutani-Glicksberg-Fan fixed point theorem implies that there exists a symmetric equilibrium of the disclosure game in distributional strategies . □

***Proof of Proposition 6* (Full Disclosure).** First note that (9) is equivalent to

$$\hat{\rho}\left(\frac{1-\int_{\hat{\rho}}^{\overline{x}} x d\Psi(x)}{1-\int_{\underline{x}}^{\overline{x}} x d\Psi(x)}\right)^{N-1} \geq 1,$$

and the left-hand side is equal to the ratio $V_D^1(\hat{\rho})/V_H^1(\hat{\rho})$ of the payoffs of a player with type $\hat{\rho}$ when all other players disclose with probability 1. Therefore, (9) implies that there is no incentive of a player with type $\hat{\rho}$ to deviate from full disclosure. It is therefore clearly a necessary condition for equilibrium.

To show that it is also sufficient, note that for a candidate with type $\rho > \hat{\rho}$, when all the other candidates disclose with probability 1,

$$\frac{V_D^1(\rho)}{V_H^1(\rho)} = \rho\left(\frac{1-\int_\rho^{\overline{x}} x d\Psi(x)}{1-\int_{\underline{x}}^{\overline{x}} x d\Psi(x)}\right)^{N-1} > \hat{\rho}\left(\frac{1-\int_{\hat{\rho}}^{\overline{x}} x d\Psi(x)}{1-\int_{\underline{x}}^{\overline{x}} x d\Psi(x)}\right)^{N-1} \geq 1,$$

implying that there is no incentive to deviate from the full disclosure profile for such a candidate. Since it is a dominant strategy for the types below $\hat{\rho}$ to disclose, this proves that (9) is also a sufficient condition. In the absence of weak types, $\hat{N}$ is infinite and full disclosure cannot be an equilibrium.

Now suppose that (9) holds with a strict inequality, and that there exists $\rho_k \geq \hat{\rho}$ such that all strategies prescribing to disclose with probability less than 1 for some $\rho < \rho_k$ have been eliminated and call $\mathcal{L}_k$ the set of remaining strategies. Because the function

$$\rho\left(\frac{1-\int_\rho^{\overline{x}} x d\Psi(x)}{1-\int_{\underline{x}}^{\overline{x}} x d\Psi(x)}\right)^{N-1}$$

is increasing in ρ, it must be strictly greater than 1 when evaluated at ρ_k. Now given a strategy profile $\lambda \in \mathcal{L}_k$, the incentive to disclose for a type $\rho > \rho_k$ is given by

$$\begin{aligned}
\frac{V_{D,n}^\lambda(\rho)}{V_{H,n}^\lambda(\rho)} &= \rho \prod_{m\neq n} \frac{1-\int_\rho^{\overline{x}} x\lambda_m(x)d\Psi(x)}{\int_{\underline{x}}^{\overline{x}} \lambda_m(x)(1-x)d\Psi(x) + \int_{\rho_k}^{\rho}(1-\lambda_m(x))\,d\Psi(x)} \\
&= \rho \prod_{m\neq n} \frac{1-\int_\rho^{\overline{x}} x\lambda_m(x)d\Psi(x)}{\int_{\underline{x}}^{\rho_k} \lambda_m(x)(1-x)d\Psi(x) + \int_{\rho_k}^{\rho}(1-\lambda_m(x)x)\,d\Psi(x) + \int_\rho^{\overline{x}}(1-x)\lambda_m(x)d\Psi(x)} \\
&\geq \rho\left(\frac{1-\int_\rho^{\overline{x}} x d\Psi(x)}{1-\int_{\underline{x}}^{\overline{x}} d\Psi(x) + \int_{\rho_k}^{\rho} x d\Psi(x)}\right)^{N-1} \equiv L(\rho_k,\rho),
\end{aligned}$$

where the lower bound is attained on $\mathcal{L}_k$ by the strategy $\lambda(x) = 1 - \mathbb{1}_{x\in(\rho_k,\rho)}$. $L(.,.)$ is clearly continuous and the limit of $L(\rho_k, \rho)$ as $\rho \to \rho_k$ is

$$\rho_k \left(\frac{1 - \int_{\rho_k}^{\overline{x}} x d\Psi(x)}{1 - \int_{\underline{x}}^{\overline{x}} x d\Psi(x)} \right)^{N-1} > 1.$$

By continuity, it must also be strictly greater than 1 on a neighborhood to the right of ρ_k. Thus we can define

$$\rho_{k+1} = \sup \{\rho \in (\rho_k, \overline{x}) : L(\rho_k, \rho) > 1\}.$$

If $\rho_{k+1} < \overline{x}$, it must be true that $L(\rho_k, \rho_{k+1}) = 1$. We can define $\mathcal{L}_{k+1}$ to be the set of strategies that prescribe to disclose with probability 1 whenever $\rho < \rho_{k+1}$. The construction of ρ_{k+1} implies that, provided that players are restricted to use strategies in $\mathcal{L}_k$, strategies in $\mathcal{L}_k \setminus \mathcal{L}_{k+1}$ are strictly dominated.

Let $\rho_0 = \hat{\rho}$ and define $\mathcal{L}_0$ accordingly. I have already proved in Lemma 2 that strategies that are not in $\mathcal{L}_0$ are strictly dominated. Therefore I can apply the construction above to find an increasing sequence $\{\rho_0, \rho_1, \dots\}$ and the corresponding shrinking sequence $\{\mathcal{L}_0, \mathcal{L}_1, \dots\}$ stopping whenever $\rho_k = \overline{x}$. If this happens, the construction implies that the only strategy that survives the iterated elimination of strictly dominated strategies is to disclose with probability 1 everywhere except perhaps at $\rho = \overline{x}$.

Suppose that it is not the case so that for every k, $\rho_k < \overline{x}$. Because the sequence $(\rho_k)_{k \geq 0}$ is increasing and bounded, it admits a limit $\rho_\infty \leq \overline{x}$. By construction, the relationship $L(\rho_k, \rho_{k+1}) = 1$ must hold for every k. Then, taking k to infinity and using the continuity of $L(.,.)$

$$L(\rho_\infty, \rho_\infty) = \rho_\infty \left(\frac{1 - \int_{\rho_\infty}^{\overline{x}} x d\Psi(x)}{1 - \int_{\underline{x}}^{\overline{x}} d\Psi(x)} \right)^{N-1} = 1.$$

But the function

$$\rho \left(\frac{1 - \int_{\rho}^{\overline{x}} x d\Psi(x)}{1 - \int_{\underline{x}}^{\overline{x}} x d\Psi(x)} \right)^{N-1}$$

is increasing in ρ and strictly greater than 1 at $\hat{\rho}$ and therefore also at $\rho_\infty > \hat{\rho}$. A contradiction.

Therefore all strategies that disclose with probability less than 1 anywhere except at $\rho = \overline{x}$ are eliminated in a finite number of steps. But then the incentive to disclose for type $\overline{x}$ is given by

$$\overline{x}\left(\frac{1}{1-\int_{\underline{x}}^{\overline{x}} x d\Psi(x)}\right)^{N-1} \geq \hat{\rho}\left(\frac{1-\int_{\hat{\rho}}^{\overline{x}} x d\Psi(x)}{1-\int_{\underline{x}}^{\overline{x}} x d\Psi(x)}\right)^{N-1} > 1.$$

Hence strategies that prescribe to disclose with probability less than 1 for $\overline{x}$ can be eliminated as well. □

***Proof of Proposition 7* (Comparative Statics).** To show that $\hat{N}_\Phi \leq \hat{N}_\Psi$, note that I can rewrite

$$\hat{N}_\Psi = 1 + \frac{\log(1/\hat{\rho})}{-\log R_\Psi},$$

where

$$R_\Psi = \frac{1-\int_{\underline{x}}^{\overline{x}} x d\Psi(x)}{1-\int_{\hat{\rho}}^{\overline{x}} x d\Psi(x)} = 1 - \frac{\int_{\underline{x}}^{\hat{\rho}} x d\Psi(x)}{1-\int_{\hat{\rho}}^{\overline{x}} x d\Psi(x)},$$

and $\hat{N}_\Psi$ is increasing in R_Ψ. Integrating by parts, I obtain

$$R_\Psi = 1 + \frac{\int_{\underline{x}}^{\hat{\rho}} \Psi(x)dx - \hat{\rho}\Psi(\hat{\rho})}{1-\overline{x}+\hat{\rho}\Psi(\hat{\rho})+\int_{\hat{\rho}}^{\overline{x}} \Psi(x)dx}.$$

Clearly, $R_\Phi \leq R_\Psi$ since $\Phi(x) \leq \Psi(x)$ on $[\underline{x}, \hat{\rho}]$ and $\Phi(x) \geq \Psi(x)$ on $[\hat{\rho}, \overline{x}]$. Hence $\hat{N}_\Phi \leq \hat{N}_\Psi$. □

Asymptotic analysis. I start by proving five useful lemmas. The first lemma shows that if a strong type ρ discloses information with probability 1 in a symmetric equilibrium, then all the types above ρ also disclose with probability 1. The intuition is the following. If there exists an interval Ω on which λ is equal to 1, then as already noted V_H^λ is constant on Ω while V_D^λ is strictly increasing. If λ is an equilibrium strategy, $V_D^\lambda > V_H^\lambda$ on Ω. But then, the continuity of the two payoff functions implies that V_H^λ can never catch up with V_D^λ as ρ increases, so that disclosing must be strictly better than withholding.

Lemma 4. *If λ is a symmetric equilibrium such that there exists a strong type $\rho > \hat{\rho}$ satisfying*

$\rho \in Int(\Lambda^1)$, *then* $[\rho, \overline{x}] \subseteq \Lambda^1$.

Proof. Let $\Omega \subseteq (\hat{\rho}, \overline{x})$ be an open interval such that $\lambda = 1$ on Ω and $x \in \Omega$. Let $y = \sup\{\rho : \forall \rho' \in [x, \rho],\ \lambda(\rho') = 1\}$. Suppose $y < \overline{x}$. By continuity of the payoff functions, it must be true that $V_D^\lambda(y) = V_H^\lambda(y)$. However, $V_D^\lambda(.)$ is strictly increasing on (x, y) while $V_H^\lambda(.)$ is constant on the same interval. Furthermore, since λ is an equilibrium strategy, $V_D^\lambda(x) > V_H^\lambda(x)$, but then by continuity $V_D^\lambda(y) > V_D^\lambda(x) > V_H^\lambda(x) = V_H^\lambda(y)$, a contradiction. □

Therefore, in any equilibrium $Int(\Lambda^1) = (\underline{x}, \hat{\rho}) \cup (\rho^*, \overline{x})$ for some $\rho^* \in [\hat{\rho}, \overline{x}]$, or $Int(\Lambda_1) = (\underline{x}, \overline{x})$. In the absence of weak types, $Int(\Lambda^1) = (\rho^*, \overline{x})$. The next lemma shows that no disclosure cannot be an equilibrium.

Lemma 5. $\lambda = 0$ *is never a symmetric equilibrium.*

Proof. No disclosure cannot be an equilibrium as the lowest type would then have an incentive to disclose $V_D^0(\underline{x}) = \underline{x} > 0 = V_H^0(\underline{x})$. □

The next lemma shows that, if weak types are absent, and there exists a symmetric equilibrium with an open interval of type who withhold, then all the types higher than the types in this open interval must be withholding as well..

Lemma 6. *Suppose weak types are absent. There exists some* N_0 *such that for* $N > N_0$, *if the strategy* λ *defines a symmetric Bayesian Nash equilibrium of the disclosure game such that there exists* $\rho \in Int(\Lambda^0)$, *then* $[\rho, \overline{x}] \subseteq \Lambda^0$.

Proof. There must be some $\eta > 0$ such that $\Omega = [\rho, \rho + \eta) \subseteq \Lambda^0$. I can differentiate $V_D^\lambda(.)$ and $V_H^\lambda(.)$ on Ω to obtain

$$\begin{aligned}\left(V_D^\lambda\right)'(\rho) = \left(1 - \int_\rho^{\overline{x}} x\lambda(x)d\Psi(x)\right)^{N-1} &= \frac{V_D^\lambda(\rho)}{\rho} \\ &< \frac{V_H^\lambda(\rho)}{\underline{x}},\end{aligned}$$

where the inequality is due to the fact that $\rho \in \Lambda^0$ and that λ is an equilibrium. Also,

$$\begin{aligned}
\left(V_H^\lambda\right)'(\rho) &= (N-1)\psi(\rho)\left(\int_{\underline{x}}^{\rho}(1-x\lambda(x))d\Psi(x)+\int_{\rho}^{\overline{x}}\lambda(x)(1-x)d\Psi(x)\right)^{N-2} \\
&= (N-1)\psi(\rho)V_H^\lambda(\rho)^{1-\frac{1}{N-1}} \\
&\geq (N-1)\underline{\psi}V_H^\lambda(\rho),
\end{aligned}$$

where $\underline{\psi}$ is the lower bound of ψ and the last inequality obtains because V_H is always bounded above by 1.

It is clear then that for $N > 1+\frac{1}{\underline{x}\underline{\psi}}$, $V_H^\lambda(.)$ grows faster than $V_D^\lambda(.)$ on Ω. But since $V_H^\lambda(\rho) > V_D^\lambda(\rho)$, it means that V_H^λ must remain above V_D^λ on Ω so that the payoff from disclosing can never catch up with the payoff from withholding. This implies that no type above ρ would want to disclose with positive probability, hence $[\rho, \overline{x}] \subseteq \Lambda^0$. □

The payoff from withholding and the payoff from disclosing must equate each other on any interval of types that mix. The next lemma uses this fact, and the differentiability of the payoff function to derive an asymptotic expression of the equilibrium probability of disclosing on that interval.

Lemma 7. *For every $\varepsilon > 0$, there exists some N_1 such that if λ defines a symmetric Bayesian Nash equilibrium of the disclosure game for some $N > N_1$ such that $Int(\Lambda) \neq \emptyset$, then for every $\rho \in Int(\Lambda)$,*

$$\left|\lambda(\rho) - \frac{1}{1+\rho}\right| < \varepsilon.$$

Proof. For every $\rho \in Int(\Lambda)$, $V_H^\lambda(\rho) = V_D^\lambda(\rho)$. Since the two functions are differentiable on $Int(\Lambda)$, their derivatives must be equal as well, implying

$$(N-1)(1-\lambda(\rho))\psi(\rho)V_H^\lambda(\rho)^{1-\frac{1}{N-1}} = \frac{V_D^\lambda(\rho)}{\rho} + \rho^2(N-1)\lambda(\rho)\psi(\rho)\left(\frac{V_D^\lambda(\rho)}{\rho}\right)^{1-\frac{1}{N-1}}.$$

Noting that $V_H^\lambda(\rho) = V_D^\lambda(\rho)$, and after some algebra, I obtain

$$\frac{1}{1+\rho} - \lambda(\rho) = \frac{V_D^\lambda(\rho)^{\frac{1}{N-1}}}{(N-1)\rho(1+\rho)\psi(\rho)} + \frac{\rho\lambda(\rho)}{1+\rho}\left(\rho^{\frac{1}{N-1}} - 1\right).$$

Hence

$$\left|\frac{1}{1+\rho} - \lambda(\rho)\right| \leq \frac{1}{(N-1)\underline{x}(1+\underline{x})\underline{\psi}} + \frac{\overline{x}}{1+\overline{x}}\left(1 - \underline{x}^{\frac{1}{N-1}}\right).$$

Since both terms on the right-hand side go to 0 and are independent of ρ, this proves the lemma. $\square$

The last lemma uses Lemma 7 to bound the mixing probability for high values of N.

Lemma 8. *There exists some $\ell > 0$ and some N_2 such that if λ defines a symmetric Bayesian Nash equilibrium of the disclosure game for some $N > N_2$ such that $Int(\Lambda) \neq \emptyset$, then for every $\rho \in Int(\Lambda)$,*

$$1 - \ell > \lambda(\rho) > \ell.$$

Proof. This is a corollary of Lemma 7 obtained by choosing $\ell = \varepsilon = \min\left(\frac{1}{2(1+\overline{x})}, \frac{1}{2}\left(1 - \frac{1}{1+\underline{x}}\right)\right)$. $\square$

***Proof of Proposition 8* (Competition at the Limit).** By Lemma 4 and Lemma 6, there are five possible types of symmetric Bayesian Nash equilibria when N is sufficiently high: (i) $\lambda = 1$; (ii) $\lambda = 0$; (iii) $\Lambda = [\underline{x}, y)$ and $\Lambda^1 = (y, \overline{x}]$ for some $y \in (\underline{x}, \overline{x})$; (iv) $\Lambda = [\underline{x}, y)$ and $\Lambda^0 = (y, \overline{x}]$ for some $y \in (\underline{x}, \overline{x})$; (v) $\Lambda = [\underline{x}, \overline{x}]$. By Lemma 5 and Proposition 6, (i) and (ii) are impossible. In the next steps of the proof, I rule out (iv) and (v) as well. Then I work with equilibria of type (iii) and show that the cutoff y at which mixing starts must converge to $\underline{x}$ as N goes to infinity.

Suppose that there exists a sequence $(\lambda_k)_{k=1}^\infty$ of equilibria of type (iv) for the disclosure game with $N(k)$ candidates, where $N(k)$ increases strictly with k. To each λ_k is associated a $y_k \in (\underline{x}, \overline{x})$ and since $[\underline{x}, \overline{x}]$ is compact, I can assume, up to an extraction, that the sequence y_k

converges to some $y_\infty \in [\underline{x}, \overline{x}]$. Because in any equilibrium of type (iv), type $\overline{x}$ withholds, I can write

$$V_D^k(\overline{x}) = \overline{x} < \left(1 - \int_{\underline{x}}^{\overline{x}} x\lambda_k(x)d\Psi(x)\right)^{N(k)-1} = V_H^k(\overline{x}).$$

Using Lemma 8, for a sufficiently high k

$$\left(1 - \int_{\underline{x}}^{\overline{x}} x\lambda_k(x)d\Psi(x)\right)^{N(k)-1} \leq \left(1 - \ell\underline{x}\Psi(y_k)\right)^{N(k)-1},$$

and for the right-hand side to be greater than $\overline{x}$ for every k it must be true that $y_\infty = \underline{x}$. But then, looking at the incentives of type y_k, we have

$$V_D^k(y_k) = y_k \xrightarrow[k\to\infty]{} \underline{x},$$

and

$$V_H^k(y_k) = \left(\int_{\underline{x}}^{y_k} (1 - x\lambda(x))d\Psi(x)\right)^{N(k)-1} \leq ((1 - \ell\underline{x})\Psi(y_k))^{N(k)-1} \xrightarrow[k\to\infty]{} 0,$$

implying that for k sufficiently high $V_D^k(y_k) > V_H^k(y_k)$ which is a contradiction since these payoffs should be equal for λ_k to be an equilibrium.

Suppose now that there exists a similar sequence $(\lambda_k)_{k=1}^\infty$ of equilibria of type (v). Then for k sufficiently high,

$$V_H^k(\overline{x}) = \left(1 - \int_{\underline{x}}^{\overline{x}} x\lambda_k(x)d\Psi(x)\right)^{N(k)-1} < (1 - \ell\underline{x})^{N(k)-1} \xrightarrow[k\to\infty]{} 0.$$

Hence for k sufficiently high $V_H^k(\overline{x}) < \overline{x} = V_D^k(\overline{x})$, which contradicts the fact that λ_k is an equilibrium.

Hence, for N sufficiently large, the only possible equilibria are of type (iii). They exist by Proposition 3. Let $(\lambda_k)_{k=1}^\infty$ be a sequence of equilibria of this type, with corresponding number of players $N(k)$ that is strictly increasing in k. The payoffs of type y_k are given by

$$V_D^k(y_k) = y_k \left(1 - \int_{y_k}^{\overline{x}} x d\Psi(x)\right)^{N(k)-1},$$

and

$$V_H^k(y_k) = \left(1 - \int_{y_k}^{\overline{x}} x d\Psi(x) - \int_{\underline{x}}^{y_k} x\lambda_k(x) d\Psi(x)\right)^{N(k)-1},$$

and are equal since λ_k is an equilibrium. Hence, for every $k > 0$,

$$\left(1 - y_k^{\frac{1}{N(k)-1}}\right)\left(1 - \int_{y_k}^{\overline{x}} x d\Psi(x)\right) = \int_{\underline{x}}^{y_k} x\lambda_k(x) d\Psi(x).$$

Since $\left(1 - \int_{y_k}^{\overline{x}} x d\Psi(x)\right)$ is bounded and $0 < \underline{x} < y_k < \overline{x} < 1$, the left-hand side goes to 0 as $k \to \infty$. Because the right-hand side is bounded below by $\ell \underline{x} \Psi(y_k)$, it must be true that $y_\infty = \underline{x}$. □

Appendix C Decision Maker Policies

Lemma 9. *For any $\mathcal{K} \subseteq \mathcal{N}$, $k \in \mathcal{K}$, $p \in \{1, \cdots, K\}$ and $q \leq K$,*

$$f(p, \mathcal{K}) = \rho_k f(p-1, \mathcal{K} \setminus \{k\}) + (1-\rho_k) f(p, \mathcal{K} \setminus \{k\}), \tag{10}$$

and

$$F(q, \mathcal{K}) = F(q, \mathcal{K} \setminus \{k\}) - \rho_k f(q, \mathcal{K} \setminus \{k\}). \tag{11}$$

Proof. For $p \geq 1$ the probability of finding p good projects in $\mathcal{K}$ is equal to the probability of finding $p-1$ good projects in $\mathcal{K} \setminus \{k\}$ times the probability that k is a good project plus the probability of finding p good projects in $\mathcal{K} \setminus \{k\}$ times the probability that k is not a good project. This is exactly (10). (11) is obtained by summation of (10) for $p \leq q$. □

***Proof of Lemma 1* (Preferences of the Decision Maker)**. Point (i) is clear. For (ii), consider a swap of the following type. For a fixed set of projects $\mathcal{N}$ take two projects n and m, with $n < m$ ($\rho_n > \rho_m$). Let $[\hat{\mathcal{D}}, \hat{\mathcal{H}}]$ be a partition of $\mathcal{N} \setminus \{n, m\}$. Let $\mathcal{D}_0 = \hat{\mathcal{D}} \cup \{m\}$, $\mathcal{H}_0 = \hat{\mathcal{H}} \cup \{n\}$, $\mathcal{D}_1 = \hat{\mathcal{D}} \cup \{n\}$ and $\mathcal{H}_1 = \hat{\mathcal{H}} \cup \{m\}$. Then $[\mathcal{D}_0, \mathcal{H}_0]$ and $[\mathcal{D}_1, \mathcal{H}_1]$ are both partitions of $\mathcal{N}$ that are obtained from one another by swapping the roles of n and m.

I prove that $V(\mathcal{D}_1, \mathcal{H}_1, 1) \geq V(\mathcal{D}_0, \mathcal{H}_0, 1)$ for the case where $m = n + 1$. This proves the general result, as any other swap can be decomposed in a series of swaps between adjacent projects. When $M = 1$ and the learning partition of the decision maker is given by $[\mathcal{D}, \mathcal{H}]$, the expected payoff of the decision maker is

$$V(\mathcal{D}, \mathcal{H}, 1) = (1 - f(0, \mathcal{D}))G + f(0, \mathcal{D})\big(\rho_{h(1)}(G + L) - L\big).$$

Let Δ be the change in payoff due to the swap $\Delta = V(\mathcal{D}_1, \mathcal{H}_1, 1) - V(\mathcal{D}_0, \mathcal{H}_0, 1)$. Using Lemma 9, and supposing $r_{\mathcal{H}}(n) > 1$ or $n \notin \mathcal{N}_S$, it is equal to

$$\Delta = \big(\rho_n - \rho_{n+1}\big) f(0, \hat{\mathcal{D}})(1 - \rho_{h(1)})(G + L) \geq 0$$

If $\{n, n+1\} \subseteq \mathcal{N}_S$ and $r_{\mathcal{H}}(n) = r_{\mathcal{H}}(n+1) = 1$, then

$$\Delta = (\rho_n - \rho_{n+1}) f(0, \hat{\mathcal{D}})G - (\rho_n - \rho_{n+1}) f(0, \hat{\mathcal{D}})(G + L) + L f(0, \hat{\mathcal{D}})(\rho_n - \rho_{n+1}) = 0.$$

Finally, if $n \in \mathcal{N}_S$, $n + 1 \notin \mathcal{N}_S$ and $r_{\mathcal{H}}(n) = 1$, then

$$\Delta = (\rho_n - \rho_{n+1}) f(0, \hat{\mathcal{D}})G - f(0, \hat{\mathcal{D}})(1 - \rho_{n+1})\big(\rho_n(G + L) - L\big) = f(0, \hat{\mathcal{D}})\rho_{n+1}(1 - \rho_n)(G + L) \geq 0.$$

□

***Proof of Proposition 9* (Optimal Policy)**. The set of expected payoffs that a decision maker can reach is completely characterized by the triple $(\mathcal{D}, \mathcal{H}, M)$. Available policies can be described as finite sequences of investigation and approval decisions. Hence the decision maker's problem is to maximize a function over a finite set which implies the existence of an optimal policy. Because (NE) excludes projects for which it is not optimal to investigate on an individual basis, any policy is equivalent to a sequential policy where, at each stage, the decision maker can either rubberstamp a project from $\mathcal{H}$ or investigate and conditionally approve[11] a project from $\mathcal{D}$.

Let $V(\mathcal{D}, \mathcal{H}, M)$ denote the highest achievable payoff at $(\mathcal{D}, \mathcal{H}, M)$. It is intuitive that, at each stage, the decision maker is best off by choosing between the strongest project in $\mathcal{H}$ and the strongest project in $\mathcal{D}$. Hence in each state $(\mathcal{D}, \mathcal{H}, M)$, the choice of the decision maker can be described as a choice between (i) approving $h(1)$ and moving on to the state $(\mathcal{D}, \mathcal{H}^+(1), M-1)$; or (ii) investigating $d(1)$, approving it if it is good and moving on to the state $(\mathcal{D}^+(1), \mathcal{H}, M-1)$, or simply moving on to the state $(\mathcal{D}^+(1), \mathcal{H}, M)$ if $d(1)$ is bad. Of course there is also the option to rubberstamp a project in $\mathcal{D}$, but investigating is always better because of (NE). Then it is easy to see that any optimal policy should satisfy the following criteria:

(i) Projects in $\mathcal{H}_W$ are discarded and only the first M projects in $\mathcal{H}_S$ are ever considered.

(ii) If $M > D$, the first $K = \min(M - D, H_S)$ projects in $\mathcal{H}_S$ are rubberstamped.

(iii) If $M > H_S$, there is an optimal policy that consists of filling as many of the first $M - H_S$ slots as possible with documented projects that are found to be good after processing, and then solving for the continuation problem.

Indeed, the first part of (i) comes from the fact that projects in $\mathcal{H}_W$ cannot be investigated, and the expected payoff of rubberstamping them is negative. The second part of the statement is obvious given the existence of the cap M on the number of projects that can be implemented. (ii) is true because any project in $\mathcal{H}_S$ has a positive expected payoff, and therefore the first $\min(M-D, H_S)$ projects in $\mathcal{H}_S$ should be used to fill the slots that cannot be filled by projects

[11]The decision maker could delay approval after observing that a project is good, but there is no advantage in doing so since all good projects yield the same expected payoffs. She could also rubberstamp a project in $\mathcal{D}$, but (LP) implies that this option is always dominated.

in $\mathcal{D}$ since $D < M$. Finally (iii) holds because it never hurts to fill slots that cannot be filled by projects in $\mathcal{H}_S$ with projects in $\mathcal{D}$.

Having made these observation, I can prove Proposition 9 by a double induction on M and N that extends (LP) to more than two candidates and one slot.

If $\mathcal{D} = \mathcal{N}$ it is clearly optimal to investigate projects starting from the strongest one and then moving down in the strength order, approving a project each time it is found to be good, until all slots are filled. The policy of the proposition clearly does this. If $\mathcal{H} = \mathcal{N}$ it is also clear that the policy of the proposition is optimal: since projects in $\mathcal{N}_S$ have positive expected payoffs the strongest ones should be approved according to the availability of slots.

Consider states that satisfy $D \geq M = H$ and $\mathcal{H} \subseteq \mathcal{N}_S$. Below, I show, by a double induction on D and $M = H$, that the policy described in the proposition is optimal for all such states, and that it is the unique optimal policy up to some details in the order of learning explained below. Take this result as given for now. It implies that the policy of the proposition is optimal, although not uniquely, in any other state. This is a consequence of observations (i), (ii) and (iii) above. Indeed, by (i) projects in $\mathcal{H}_W$ are irrelevant. Hence I can assume that $M \leq N - H_W = D + H_S$ for additional slots would never be filled. By (ii), I can assume $D \geq M$ for otherwise the optimal policy consists in rubberstamping projects in $\mathcal{H}_S$ until a state where $D = M$ is reached and then continuing with the optimal policy. Furthermore, rubberstamping can occur at any place in the sequence describing the optimal policy. Hence it can be done at the end of the sequence so that the policy of the proposition is indeed optimal. This is the first source of non uniqueness of the optimal policy. Since the projects rubberstamped in this operation are the strongest in $\mathcal{H}$, and are certain to get approved, however, they are also irrelevant to the probability that any given project is approved across different optimal policies. Observations (i) and (iii) allow me to consider only the cases where $H = M$. As a consequence, an optimal policy can always be described as: always start by learning as much as possible, and then rubberstamp. But since the order of investigation never affects the probability of having to rubberstamp some projects in the end, it is always optimal to investigate stronger projects first as it minimizes the cost of the search. This is not uniquely optimal, however, for the following reason. If there are M slots available then the order in which the first M projects in $\mathcal{D}$ are investigated is irrelevant since at least M projects will be investigated regardless of what is found. Hence the proof that follows implies optimality, and uniqueness up to this subtlety. But the argument just made implies that the probability that a given project is approved is unaltered by which particular optimal policy is used.

Initiation $\mathcal{D} = \{d\}$, $\mathcal{H} = \{h\}$, $M = 1$. In this case the choice is between (i) rubberstamping h, and (ii) investigating d, approving d if it is of the good type, rejecting d and rubberstamping h if it is of the bad type. The first choice pays $\rho_h(G + L) - L$ while the second one pays $\rho_d G - c + (1 - \rho_d)(\rho_h(G + L) - L)$. Letting Δ be the gain from learning,

$$\Delta = \rho_d(1 - \rho_h)(G + L) - c > 0.$$

where the inequality holds because of (LP). Hence the unique optimal policy is to learn first.

Induction Step, $D > M = H = 1$. Suppose the result holds for any triple $(\mathcal{D}, \mathcal{H}, M)$ such that $D = K > H = M = 1$ and $\mathcal{H} \subseteq \mathcal{N}_S$, and consider a state $(\mathcal{D}, \mathcal{H}, M)$ with $D = K + 1$. The decision maker can either (a) rubberstamp h and end, or (b) investigate a project d in $\mathcal{D}$, approve d and end if it is of the good type, and move on to the state $(\mathcal{D} \setminus \{d\}, \{h\}, 1)$ otherwise. Hence we only need to compare the payoff of (b)

$$\rho_d G - c + (1 - \rho_d)V(\mathcal{D} \setminus \{d\}, \{h\}, 1),$$

to the payoff $\rho_h(G + L) - L$ of (a). Letting Δ denote the gain from learning

$$\Delta = \rho_d(1 - \rho_h)(G + L) - c + (1 - \rho_d)V(\mathcal{D} \setminus \{d\}, \{h\}, 1) > 0,$$

where the first two terms add up to something positive by (LP), and the last term is non-negative because the decision maker always has the option to discard all remaining projects and get 0. Hence, investigating first is optimal, and by the induction hypothesis it is also the best continuation policy. Because investigating in the order of decreasing strength minimizes the cost of search, it is optimal to do so. This proves the claim. It is unique up to the subtlety about the order of learning explained above.

Induction Step, $D > M = H > 1$. Suppose the result holds for all $(\mathcal{D}, \mathcal{H}, M)$ with $\mathcal{H} \subseteq \mathcal{N}_S$ such that $H = M \leq K$, or $H = M = K + 1$ and $D \leq J$. Consider a triple $(\mathcal{D}, \mathcal{H}, M)$ where $H = M = K + 1$ and $D = J + 1$. Consider the choice between (a) investigating (and conditionally approving) project $d(1)$ in $\mathcal{D}$, and (b) rubberstamping a project $h \in \mathcal{H}$. Then the decision can move on with an optimal continuation policy. I only consider the investigation of $d(1)$ since it is clearly the best option among policies that start with investigation. The first option yields

$$\rho_{d(1)}\Big(G + V(\mathcal{D} \setminus \{d(1)\}, \mathcal{H} \setminus \{h(H)\}, M - 1)\Big) + (1 - \rho_{d(1)})V(\mathcal{D} \setminus \{d(1)\}, \mathcal{H}, M) - c,$$

while the second option yields

$$\rho_h(G + L) - L + V(\mathcal{D}, \mathcal{H} \setminus \{h\}, M - 1),$$

which by induction can be rewritten as

$$\rho_h(G+L)-L+\rho_{d(1)}\Big(G+V(\mathcal{D}\setminus\{d(1), \mathcal{H}\setminus\{h, h(H)\}, M-2)\Big)+(1-\rho_{d(1)})V(\mathcal{D}\setminus\{d(1)\}, \mathcal{H}\setminus\{h\}, M-1)-c$$

The gain from learning is then

$$\Delta = \rho_{d(1)} \underbrace{\Big(V(\mathcal{D}\smallsetminus\{d(1)\}, \mathcal{H}, M-1) - \big(V(\mathcal{D}\smallsetminus\{d(1)\}, \mathcal{H}\smallsetminus\{h, h(H)\}, M-2) + \rho_h(G+L) - L\big)\Big)}_{A}$$

$$+(1-\rho_{d(1)}) \underbrace{\Big(V(\mathcal{D}\smallsetminus\{d(1)\}, \mathcal{H}, M) - \big(V(\mathcal{D}\smallsetminus\{d(1)\}, \mathcal{H}\smallsetminus\{h\}, M-1) + \rho_h(G+L) - L\big)\Big)}_{B}.$$

$A > 0$. Indeed in state $(\mathcal{D}\smallsetminus\{d(1)\}, \mathcal{H}\smallsetminus\{h(H)\}, M-1)$ an available policy is to rubberstamp h and then move on to state $(\mathcal{D}\smallsetminus\{d(1)\}, \mathcal{H}\smallsetminus\{h, h(H)\}, M-2)$ and continue with the optimal policy. Because of the induction hypothesis, this is not optimal at $(\mathcal{D}\smallsetminus\{d(1)\}, \mathcal{H}\smallsetminus\{h(H)\}, M-1)$, and A is exactly the difference of payoffs between the former policy and the optimal one. A similar argument shows that $B > 0$. Therefore $\Delta > 0$ implying that learning is optimal at $(\mathcal{D}, \mathcal{H}, M)$. Once again this implies that the policy of the proposition is uniquely optimal up to the order of learning.

Induction Step, $D = M = H$. Suppose now that the result holds for all $(\mathcal{D}, \mathcal{H}, M)$ with $\mathcal{H} \subseteq \mathcal{N}_S$ and $H = M \leq K$, and consider a triple $(\mathcal{D}, \mathcal{H}, M)$ such that $D = H = M = K + 1$. Then the payoff of learning about $d(1)$ is

$$\rho_{d(1)}\Big(G + V(\mathcal{D}\smallsetminus\{d(1)\}, \mathcal{H}\smallsetminus\{h(H)\}, M-1)\Big) + \big(1-\rho_{d(1)}\big)\Big(\rho_{h(1)}(G+L) - L + V(\mathcal{D}^+(1), \mathcal{H}^+(1), M-1)\Big) - c,$$

and the payoff of rubberstamping $h(1)$ ($h(1)$ is clearly better than any other h here) is

$$\rho_{h(1)}(G+L) - L + V(\mathcal{D}, \mathcal{H}\smallsetminus\{h(1)\}, M-1),$$

or, because of the induction hypothesis,

$$\rho_{h(1)}(G+L) - L + \rho_{d(1)}\Big(G + V(\mathcal{D}\smallsetminus\{d(1)\}, \mathcal{H}\smallsetminus\{h(1), h(H)\}, M-2)\Big)$$
$$+ \big(1-\rho_{d(1)}\big) V(\mathcal{D}\smallsetminus\{d(1)\}, \mathcal{H}\smallsetminus\{h(1)\}, M-1) - c.$$

Hence the gain from learning is

$$\Delta = \rho_{d(1)}\Big(V(\mathcal{D} \setminus \{d(1)\}, \mathcal{H} \setminus \{h(H)\}, M-1)$$
$$-\big(V(\mathcal{D} \setminus \{d(1)\}, \mathcal{H} \setminus \{h(1), h(H)\}, M-2) + \rho_{h(1)}(G+L) - L\big)\Big).$$

By the induction hypothesis, rubberstamping $h(1)$ is an available but non optimal policy at $(\mathcal{D} \setminus \{d(1)\}, \mathcal{H} \setminus \{h(H)\}, M-1)$, hence $\Delta > 0$. This concludes the proof. □

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