Competing with Equivocal Information*

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Abstract

I study strategic disclosure between multiple senders and a single receiver. The senders are competing for prizes awarded by the receiver. They decide whether to disclose a piece of information that is both verifiable and equivocal: it can influence the receiver both ways. The standard unraveling argument fails as a single sender doesn't disclose if the commonly known probability that her information is favorable is sufficiently high. Competition restores full disclosure only if some of the senders are sufficiently unlikely to have favorable information. With asymmetric information between candidates, however, all symmetric equilibria approach full disclosure as competition increases.

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1 Introduction

In the standard persuasion game framework, where an informed sender tries to persuade an uninformed receiver to take a certain action by selectively communicating verifiable information, the sender's information is assumed to be unequivocal: she knows how her information would influence the receiver's choice. In many situations, however, an agent controls the access of others to information but is unable to predict their reactions to it. A climate expert may understand the environmental effects of a particular emission reduction policy, but lack the economic and political expertise to apprehend its electoral value to those in charge of approving it. A movie producer may find it impossible to predict how the information conveyed in a trailer

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will affect the willingness of any particular consumer to watch her movie[1]. An advertiser or an advertising platform may not know how the information contained in a sponsored link will influence any particular consumer.

In the standard framework, all the relevant information is revealed in equilibrium because any action of the sender can be outguessed by the receiver. With equivocal information, this classical unraveling argument breaks down. When the receiver is sufficiently inclined to act as the sender wishes without any information, the sender has no incentive to inform her. A job candidate with a good resume, for example, is unlikely to reveal additional information about herself in a statement of purpose. Since it is both difficult to appreciate how such information will be interpreted by the employer and easy to make the statement of purpose deliberately vague, a candidate who thinks that she will be hired on the basis of her resume alone will not communicate potentially detrimental information. Furthermore, common knowledge that information is equivocal to the candidate prevents the employer from drawing any unfavorable conclusion from her behavior. A candidate with a weaker resume, however, may have to provide as much additional information as possible in order to sway the employer's decision. Similarly, the best strategy to advertise a movie from a popular director is to keep the trailer elliptic and mysterious, while the trailer of a movie from an unknown director will feature all its best scenes in order to attract audiences.

Such reasoning by a job candidate or a movie producer should, however, be altered by the presence of competitors. A weak job candidate who has to provide more information about herself in order to stay in the race, may get ahead of an *ex ante* stronger candidate if her information turns out to be favorable. This should incentivize *ex ante* stronger candidates to disclose more information as well. Thus, the forces of competition may be expected to mitigate the *real authority*[2] of *ex ante* strong candidates over the employment decision and lead to more disclosure.

[1]That is true even if she knows the average consumer's reaction.

[2]As defined by Aghion and Tirole (1997), real authority is the effective control over decisions as determined by the information structure, rather than by the formal right to decide.

One of the main results of the literature on persuasion games, as most generally stated in Milgrom and Roberts (1986), is the identification of a set of assumptions that, by ensuring skepticism on the part of the receiver, lead to efficient provision of information by the sender. This occurs when the receiver is capable of strategic reasoning, informed about the interests of the sender and aware of the type of information that is available to her. Milgrom and Roberts (1986) and Milgrom (2008) show that, even when these assumptions fail, competition among senders can sometimes lead to efficient disclosure. The analysis of the single sender/single receiver case in this paper[3] points at another important assumption in standard persuasion games: that the sender is able to anticipate the impact of the information she owns on the receiver. Interestingly, the inability of the sender to interpret her information is an advantage. By effectively eliminating the asymmetry of (interpretable) information, it renders the actions of the sender completely uninformative for the receiver. This allows the sender to fully benefit from the control she exerts over the availability of information.

This paper investigates the effects of competition in a theoretical framework designed to fit several economic situations. Several candidates with heterogeneous prospects compete for a limited number of homogeneous prizes or slots which can be interpreted as positions, grants, the decision to implement the project or policy proposed by the candidate, the decision of a buyer to purchase an item offered by the candidate. The model incorporates the search strategy of the receiver who decides how to allocate the slots. A candidate's *prospect* is her probability of being a good fit. I analyze the case in which the prospects of all candidates are common knowledge among them, as well as the case in which candidates know only their own prospect but not others' (asymmetric information among candidates).

The main finding is that in either case sufficient competition leads to full disclosure (*i.e.* disclosure by all candidates) only if some of the candidates are *weak*[4], that is their prospects are sufficiently low that approving them without further investigation would be wasteful in expectation. This result emphasizes the importance of weak candidates in this type of contests.

[3] Caillaud and Tirole (2007) were the first to analyze this case.

[4] In the case of asymmetric information, if the type distribution puts a positive weight on weak types.

It may have policy implications for the preselection of pools of candidates in procurement contests or when hiring. The results can also be applied to the disclosure of information to buyers in a market. A market with strong competitors[5] only may harm the consumer by limiting disclosure. For instance, a horizontal merger between two weak competitors is often considered to be pro-competitive if it creates a stronger player able to compete more aggressively with other strong players. The model suggests that such a merger may harm the consumer by reducing her information. The model is too limited to be used as a policy guide for mergers, but the consequences of horizontal mergers on information provision may be worth considering. Yet this aspect of mergers is not mentioned in the 1992 Horizontal Merger Guidelines[6]. The second finding is that when candidates do not know about the strength of their competitors, equilibria approach full disclosure asymptotically as the number of candidates increases. Hence, in this case, the role of weak candidates is more limited.

The cost of processing or acquiring information can have important consequences in many environments. In the examples above, it can be costly to process additional information about candidates, even when they make this information available. If an engineer's project is sufficiently promising, her CEO may be willing to implement it without further assessment because learning about the details would be too costly, or she may be constrained to *rubberstamp* the project in the absence of more detailed information. In the presence of many candidates with different prospects, about which additional information may be available or not, and is costly to process when available, the decision maker faces a complex search problem. For example, the presence of a second candidate with a sufficiently strong prospect, by changing the outside option of the decision maker, may make it valuable to conduct a further assessment that would otherwise have been wasteful. In the absence of processing costs, a decision maker would optimally start by processing all available information, before rubberstamping any project. A

[5]The model will make the meaning of strong clearer. For this discussion, it means competitors whose products are sufficiently likely to be satisfactory to the consumer that she would make the purchase in the absence of additional information, even when she would prefer to consult this additional information.

[6]This document, jointly issued by the Federal Trade Commission and the Department of Justice, is vailable at http://www.ftc.gov/bc/docs/horizmer.htm.

theoretical contribution of this paper is to provide conditions on the prospects under which she adopts a similar behavior in the presence of processing costs, and to characterize an optimal search algorithm for the decision maker in such cases. The presence of small processing costs also induces an optimal order in the treatment of available information: in order to save on processing costs, it is optimal to process projects with better prospects first. Therefore the set of optimal policies in the presence of these costs, even as they go to zero, is a subset of the policies that are optimal when processing is costless.

In the model, a project is either good or bad for the decision maker. Each candidate has information that would allow the decision maker to perfectly figure out the value of her project, but is unable to process this information and to anticipate its effect on the decision maker. A candidate decides whether to provide documentation about her project. The decision maker can process each piece of information at a cost. If the processing cost is sufficiently low, I show that it is an optimal policy for the decision maker to first learn sequentially about all the documented projects, and allocate a slot to each project that is found to be good. In this processing phase, she examines the projects in the order of decreasing prospects as long as there are slots to allocate. She starts allocating slots to undocumented projects only after having examined all the documented projects, even those less promising than the non-documented ones. Hence, by withholding information, a candidate loses her priority in the decision process of the decision maker. It might, however, still be beneficial to do so in equilibrium if the probability that there are sufficiently many good projects in the set of documented projects is low. For the decision maker, it is clearly optimal that all information be made available to her since it expands her choice set.

When candidates are perfectly informed about their opponents, I show that there is no equilibrium with full disclosure in the absence of weak candidates. With one slot, increasing the number of weak candidates and improving their initial prospects both make full disclosure more likely. Furthermore, in the game with a single slot to fill, when a pure strategy equilibrium exists, it is unique. In such an equilibrium, at most one candidate withholds information. I show that increasing the number of weak candidates or improving their initial prospects always leads to a weaker candidate withholding information, to the benefit of the decision maker.

I also analyze the bayesian game with imperfect information of the candidates about the prospects of their peers. Full disclosure never obtains in the absence of weak types, but is the unique equilibrium in their presence whenever there are sufficiently many candidates. The importance of weak candidates, however, is mitigated by the presence of uncertainty since I show that, even in the absence of weak types, all symmetric equilibria approach full disclosure as the number of candidate becomes large.

Related Literature. There is a large economic literature on the strategic communication of information that distinguishes between soft information (Crawford and Sobel, 1982), and hard information (Grossman, 1981; Grossman and Hart, 1980; Milgrom, 1981). The literature on *persuasion games*[7] studies the case of hard (certifiable) information in problems with a single sender trying to persuade a single receiver to take a certain action. For example, a seller tries to influence the decision of a buyer with verifiable information. Milgrom and Roberts (1986) and subsequent contributions identified conditions under which the unraveling argument holds. Shin (2003) shows that it may fail if there is uncertainty about the precision of the sender's information, Wolinsky (2003) introduces a particular form of uncertainty about the preferences of the sender. In Dziuda (2008), the unraveling argument fails because of the structure of provability: every argument disclosed is verifiable, but the receiver cannot know whether every argument has been disclosed. I also focus on hard information[8]. The unraveling argument breaks down because the sender cannot interpret her own information. The model allows me to study competition in the provision of information, which is absent from most of this literature except Milgrom and Roberts (1986).

Caillaud and Tirole (2007) analyze a single-sender/multiple-receivers model from a mech-

[7]For a review of this literature see Milgrom (2008); recent contributions also include Glazer and Rubinstein (2001), Glazer and Rubinstein (2004). Sobel (2007) summarizes the literature on information transmission. Kartik (2009) builds a bridge between the hard and soft information approaches.

[8]The distinction between soft and hard information is less meaningful in the context of this paper, since it is not clear how and why the sender would falsify information that she cannot interpret. In this light, it seems natural to assume non-falsifiable information, as implicitly done in this paper.

anism design perspective in order to understand optimal persuasion strategies when decisions affecting the sender are made by a committee under a qualified majority rule, with obvious political economy applications to lobbying situations. I analyze a multiple-senders/single-receiver version of the same benchmark model from a game-theoretic perspective to explore the effects of competition. Competition is an important feature of lobbying, and as such this paper is a contribution to the literature on lobbying.

The assumption that an economic agent can control access to information that she cannot process plays an important role in other recent papers. Eso and Szentes (2003) propose an agency model where the principal can release, but not observe, information that would allow the agent to refine her knowledge of her own type. They show that when the full mechanism design problem is considered altogether, the optimal mechanism calls for full disclosure and allows the principal to appropriate the rents of the information she controls exactly as if it were observable to her. Eso and Szentes (2007) develop an auction model with similar conclusions.

This paper is also connected to the literature on obfuscation which studies the incentive for firms to manipulate the search cost of consumers. The term was coined by Ellison and Ellison (2009) who provide evidence of such practices. Carlin (2009), Ellison and Wolitzky (2008), Wilson (2009) develop static models of obfuscation and Carlin and Manso (2009) provide a dynamic model. In the literature on marketing and advertising, Bar-Isaac, Caruana and Cuñat (2010) study the incentive of a monopoly to manipulate the cost of consumers to learn their true valuation. In a similar spirit, Lewis and Sappington (1994) and Johnson and Myatt (2006) study the optimal information structure of the consumer about her valuation from a monopolist's perspective.

Finally, the advertising literature makes predictions about the relationship between product quality and the informativeness of advertising. This question is connected to the analysis of the relationship between product quality and levels of advertising in the literature. As summarized in Bagwell (2007), the empirical literature on the topic does not strongly support a systematic positive relationship. Bagwell and Overgaard (2006) and Bar-Isaac, Caruana and

Cuñat (2010) offer possible theoretical explanations for a negative relationship. To the extent that the quantity of advertising is an acceptable measure of its informativeness, this paper offers an alternative and simple theoretical explanation, in the case of a monopolist.

2 The Model

2.1 Setup

For clarity, the model is described in the language of project adoption, although it fits other situations as well. Finitely many candidates with a single project each are indexed by the set $\mathcal{N} = \{1, \ldots, N\}$. They seek to maximize the probability that their project be adopted by a *decision maker* who can implement only $M \leq N$ of them. A project is either good or bad for the decision maker. A good project yields an expected gain $G > 0$, whereas a bad project yields an expected loss $L > 0$[9].

All the players share the belief that the projects are independent from one another, and assign probability $\rho_n \in (0, 1)$ to the event that project n is good[10] . With minimal loss of generality[11], $\rho_1 > \cdots > \rho_N$. I refer to the order that underlies this ranking as the *strength order on projects*. Each candidate n controls information that would allow the decision maker to figure out the value of project n but is irrelevant to other projects. Candidates, however, are unable to process this information[12] and can only decide whether to communicate it to the decision maker, who can then process it (*investigate*) at a cost $c > 0$. A project whose information is made available to the decision maker is said to be *documented*.

The timing is as follows. First, the candidates simultaneously decide whether to disclose their information. Then the decision maker decides which information to process and which

[9]In the multi-seller/buyer interpretation of the model, projects are items for sale to a seller with demand for a fixed quantity M and these payoffs implicitly assume away any price heterogeneity across sellers.

[10]This assumption is relaxed in Section 5.

[11]There is a small loss of generality since ties are ruled out, but this is a measure 0 event if the probability profile is drawn from an atomless joint distribution on $[0, 1]^N$.

[12]Alternatively, I could assume that the candidates must commit to a communication decision before observing the value of their project.

projects to approve. Her decisions are not contractible and she cannot commit to a policy at the outset.

2.2 Assumptions and Notations

Assumptions. Approving a project with prospect ρ without inquiring provides the decision maker with an expected incremental payoff $\rho(G+L)-L$, whereas investigating and conditionally approving the project gives the expected incremental payoff $\rho G-c$. Let $\underline{\rho} \equiv c/G$, $\hat{\rho} \equiv L/(L+G)$ and $\overline{\rho} \equiv 1-c/L$.

Assumption 1 (Affordable Learning **(AL)**). *The processing cost is sufficiently low to ensure that investigating is profitable in some cases: $c < LG/(L+G)$.*

(AL) ensures that $\underline{\rho} < \hat{\rho} < \overline{\rho}$. The interval (0,1) can then be partitioned into four intervals (see Figure 1) such that:

(i) if $\rho \in (0, \underline{\rho})$, the project is not worth considering for either immediate (*i.e.* rubberstamping) or conditional approval (*i.e.* after investigation);

(ii) if $\rho \in (\underline{\rho}, \hat{\rho})$, the project is worth investigating, but rubberstamping is wasteful;

(iii) if $\rho \in (\hat{\rho}, \overline{\rho})$, the first-best option is to investigate, but rubberstamping beats mere rejection;

(iv) if $\rho > \overline{\rho}$, rubberstamping is the first-best option, investigating is wasteful.

Hence (AL) simply says that at least some projects are worth investigating.

Notations. For any subset $\mathcal{J} \subseteq \mathcal{N}$, denote its cardinality by J, and let $j(1) < \cdots < j(J)$ be the ordered elements of this subset, so that $\rho_{j(1)} > \cdots > \rho_{j(J)}$.

Definition 1 (Truncated Subsets). *For any subset $\mathcal{J} = \{j(1), \ldots, j(J)\} \subseteq \mathcal{N}$ and any $k < J$, let $\mathcal{J}^-(k) \equiv \{j(1), \ldots, j(k)\}$ and $\mathcal{J}^+(k) \equiv \{j(k+1), \ldots, j(J)\}$ be the left and right truncations of $\mathcal{J}$ at k. Also for $1 \leq k < k+r \leq J$, write $\mathcal{J}(k, k+r) = \{j(k+1), \ldots, j(k+r)\}$. By convention, $\mathcal{J}^-(0) = \mathcal{J}^+(J) = \emptyset$.*

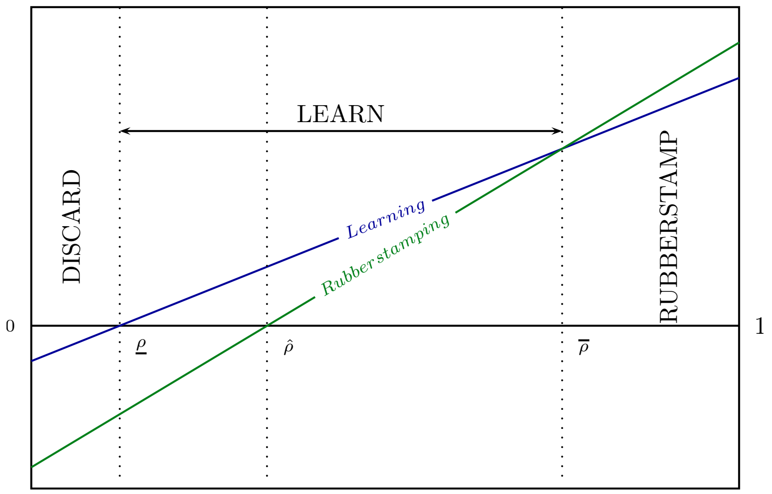

Figure 1: The Decision Maker's Payoffs with One Candidate

For a project $n \in \mathcal{N}$, and a subset of projects $\mathcal{J} \subseteq \mathcal{N}$, let $r_{\mathcal{J}}(n)$ be the *rank* of n in $\mathcal{J}$. This does not require n to be an element of $\mathcal{J}$: if $n \notin \mathcal{J}$ then $r_{\mathcal{J}}(n)$ is the rank that n would have in $\mathcal{J} \cup \{n\}$. For example, if $\mathcal{N}$ consists of three projects 1, 2 and 3 such that $\rho_1 > \rho_2 > \rho_3$ and $\mathcal{J} = \{1, 3\}$, then $r_{\mathcal{J}}(3) = r_{\mathcal{J}}(2) = 2$ as project 3 is the second strongest project in $\mathcal{J}$ and project 2 would be the second strongest project in $\mathcal{J} \cup \{2\}$.

For comparisons between sets of projects, I use the usual *set containment order* $\subset$, and the *strength order* for sets of the same cardinality defined below.

Definition 2 (Strength Order on Sets)**.** *For two sets of projects $\mathcal{K}, \mathcal{K}' \subseteq \mathcal{N}$ with the same cardinality $K = K'$, $\mathcal{K}$ is stronger than $\mathcal{K}'$, denoted $\mathcal{K} > \mathcal{K}'$ if for every $\kappa = 1, \ldots, K$, $\rho_{k(\kappa)} \geq \rho_{k'(\kappa)}$, with at least one of these inequalities holding strictly.*

3 The Decision Maker's Choice[13]

3.1 More Assumptions

As a preliminary, notice that projects with $\rho < \underline{\rho}$ will always be rejected without investigation. Indeed $\rho < c/G < L/(L+G)$ implies that the expected incremental payoff of investigating,

[13]See Appendix A for proofs that are not in the text.

$\rho G - c$, is negative, as is the expected incremental payoff of rubberstamping $\rho(G+L)-L$. The presence of these projects is also irrelevant to the disclosure game, since they have no effects on the payoffs of other candidates. In the remainder of the paper, I therefore assume without loss of generality that there are no such projects that is $\rho_n > \underline{\rho}$ for all $n \in \mathcal{N}$. I also assume, although not without loss of generality, that no candidate is sufficiently strong to make the incremental payoff of investigating smaller than the incremental payoff of rubberstamping,

Assumption 2 (No Extremes **(NE)**). *For every $n \in \mathcal{N}$, $\underline{\rho} < \rho_n < \overline{\rho}$.*

Let $\mathcal{N}_W \equiv \left\{n \in \mathcal{N}; \underline{\rho} < \rho_n < \hat{\rho}\right\}$ be the set of *weak candidates* or *weak set*, and $\mathcal{N}_S \equiv \left\{n \in \mathcal{N}; \hat{\rho} < \rho_n < \overline{\rho}\right\}$ be the set of *strong candidates* or *strong set*. Weak candidates are those for which the incremental payoff of investigation is greater than that from rubberstamping, whereas strong candidates are those for which this inequality is reversed. In the remainder of the paper, I also make the following assumption.

Assumption 3 (Learning Priority **(LP)**). *Whenever $N \geq 2$, $\rho_N(1-\rho_1) > c/(L+G)$.*

This assumption is satisfied if $\mathcal{N}_S = \emptyset$ or if $\mathcal{N}_W = \emptyset$, but it is not satisfied in general. If, for example, $\rho_1 \simeq \overline{\rho} = 1 - c/L$ and $\rho_N \simeq \underline{\rho} = c/G$, then $\rho_N(1-\rho_1) \simeq c^2/(LG) < c/(L+G)$, where the last inequality is a consequence of (AL). (LP) is always satisfied when learning is costless for the decision maker ($c = 0$). It can be interpreted as a bound on the processing cost, or as ruling out excessive heterogeneity in prospects.

It is called a learning priority assumption because it implies that a decision maker with one slot to fill and a pool of projects reduced to the best and the worst projects from the original pool would always choose to process all available information before rubberstamping a project. Indeed, suppose the decision maker has received documentation about N only, and is therefore contemplating two choices: (i) rubberstamping 1 directly obtaining the payoff $\rho_1(G+L)-L$, or (ii) first investigating N and then rubberstamping 1 if she finds that N is a bad project, yielding $\rho_N G + (1-\rho_N)(\rho_1(G+L)-L) - c$. The latter dominates the former if and only if (LP) is satisfied. Proposition 1 shows that this assumption ensures that the decision maker prioritizes learning in any situation. It also ensures that any optimal policy of the decision maker gives the same incentives to candidates in the disclosure game which is important for the analysis.

3.2 Discarding, Learning, Approving

The relevant variable to a decision maker with M slots to fill and a pool of candidates $\mathcal{N}$ is the subset of projects that she can investigate. Let $\mathcal{D} \subset \mathcal{N}$ denote this subset, and $\mathcal{H} = \mathcal{N} \setminus \mathcal{D}$. $\mathcal{D}$ is the *documented set* of the decision maker, while $\mathcal{H}$ is her *hidden set*. The partition $[\mathcal{D}, \mathcal{H}]$ of the set of projects $\mathcal{N}$ is her *learning partition*. The set of expected payoffs that she can reach given any policy is then completely characterized by the triple $(\mathcal{D}, \mathcal{H}, M)$. Let $\mathcal{H}_W \equiv \mathcal{H} \cap \mathcal{N}_W$, $\mathcal{H}_S \equiv \mathcal{H} \cap \mathcal{N}_S$, $\mathcal{D}_W \equiv \mathcal{D} \cap \mathcal{N}_W$ and $\mathcal{D}_S \equiv \mathcal{D} \cap \mathcal{N}_S$ denote the weak and strong subsets of these sets.

The policies available to the decision maker can be described as finite sequences of investigation and approval decisions with bounded lengths. Hence the decision maker's problem is to maximize a function over a finite set which implies the existence of an optimal policy. Because (NE) excludes projects for which it is not optimal to investigate on an individual basis, any policy is equivalent to a sequential policy where, at each stage, the decision maker can either rubberstamp a project from $\mathcal{H}$ or investigate and conditionally approve a project from $\mathcal{D}$[14]. A policy in state $(\mathcal{D}, \mathcal{H}, M)$ is therefore well described by a vector $\pi = \left(\pi_\ell\right)_{1 \leq \ell \leq \bar{\ell}}$ of dimension $\bar{\ell} \leq N$, listing elements of $\mathcal{N}$ in the order of their examination, with the understanding that if $\pi_\ell \in \mathcal{D}$, the policy means investigating π_ℓ at cost c, conditionally approving it, and then moving on to $\pi_{\ell+1}$ if some slots remain available, while if $\pi_\ell \in \mathcal{H}$, the policy means rubberstamping π_ℓ and then moving on to step $\pi_{\ell+1}$ if some slots remain available.

Let $\Pi(\mathcal{D}, \mathcal{H}, M)$ denote the set of optimal policies at $(\mathcal{D}, \mathcal{H}, M)$ and let $V(\mathcal{D}, \mathcal{H}, M)$ be the maximum achievable payoff. It is intuitive that, at each stage, the decision maker is best off by

[14] In principle, the decision maker could delay approval after observing that a project is good, but there is no advantage in doing so since all good projects yield the same expected payoffs.

choosing between the strongest project in $\mathcal{H}$ and the strongest project in $\mathcal{D}$. Hence in each state $(\mathcal{D}, \mathcal{H}, M)$, the choice of the decision maker can be described as a choice between (i) approving $h(1)$ and moving on to the state $(\mathcal{D}, \mathcal{H}^+(1), M-1)$; or (ii) investigating $d(1)$, approving it if it is good and moving on to the state $(\mathcal{D}^+(1), \mathcal{H}, M-1)$, or simply moving on to the state $(\mathcal{D}^+(1), \mathcal{H}, M)$ if $d(1)$ is bad. Of course there is also the option to rubberstamp a project in $\mathcal{D}$, but investigating it is always better because of (NE). The following lemma identifies some useful properties that optimal policies must satisfy.

Lemma 1.

(i) Projects in $\mathcal{H}_W$ are optimally discarded and only the first M projects in $\mathcal{H}_S$ are ever considered: $\Pi(\mathcal{D}, \mathcal{H}, M) = \Pi(\mathcal{D}, \mathcal{H}_S^-(M), M)$.

(ii) If $M > D$ it is optimal to rubberstamp the first $K = \min(M-D, H_S)$ projects in $\mathcal{H}_S$. More precisely, any policy in $\Pi(\mathcal{D}, \mathcal{H}, M)$ is a policy resulting from the combination of a policy $\pi \in \Pi(\mathcal{D}, \mathcal{H}_S^+(K), \max(D, M-H_S))$ with the rubberstamping of every project in $\mathcal{H}_S^-(K)$ in any order and at any point in the sequence.

(iii) If $M > H_S$, there is an optimal policy that consists in filling as many of the first $M - H_S$ slots as possible with projects in $\mathcal{D}$ that are found to be good after processing, and then solving for the continuation problem.

The next result shows that an optimal policy for the decision maker is to start by investigating all the documented projects in the order of decreasing strength, and then to fill the remaining slots with the most promising strong undocumented projects while discarding weak undocumented ones. Hence, withholding information implies losing one's *ex ante* priority in the order of attribution, generating a cost to non-disclosure.

Proposition 1. *Given any triple $(\mathcal{D}, \mathcal{H}, M)$, the following two-step procedure is an optimal policy for the decision maker. Furthermore, the probability that a certain project is approved is invariant across all optimal policies of the decision maker.*

Step 1 *Investigate and conditionally approve all projects in $\mathcal{D}$ sequentially in the order $d(1) \to d(2) \to \dots$ as long as there are some empty slots.*

Step 2 *Fill the $m \geq 0$ remaining slots after with the* $\min\{m, H_S\}$ *strongest projects in $\mathcal{H}_S$.*

The proof consists in a double induction on M and N that extends (LP) to more than two candidates and one slot. Induction works because of the recursive nature of the decision maker's problem, as is usual in search models.

As c approaches 0, any pool of projects satisfies the assumptions of the model ensuring that the policy is an optimal one. When it is costless to process information, it is natural for the decision maker to prioritize learning. The proposition says that this priority is maintained for sufficiently small processing costs where small is defined by (LP) and (AL). When $c = 0$, however, the order in which documented projects are investigated is irrelevant to the decision maker. Therefore, while the policy of the proposition is still an optimal one, it is not true anymore that the probability that a given project is implemented is invariant across all the optimal policies. For instance, another optimal policy would be to process all documented projects and then to implement as many of the good ones as possible, using a randomization device if their number is greater than the number of slots. This implies that different optimal policies give different incentives to the candidates for the disclosure game when $c = 0$. Taking the limit as c goes to 0 of the equilibria analyzed below provides a method for selecting equilibria in the game with $c = 0$.

As a consequence of the proposition, the probability that a project is approved depends on the probabilities of finding good projects in subsets of $\mathcal{D}$. Hence it is useful to introduce the following notations. For a subset $\mathcal{K}$ of $\mathcal{D}$, let $f(p, \mathcal{K})$ denote the probability of finding exactly p good projects in $\mathcal{K}$, and let

$$F(p, \mathcal{K}) \equiv \sum_{q=0}^{p} f(q, \mathcal{K})$$

be the probability that there are fewer than p good projects in $\mathcal{K}$.

$$f(p, \mathcal{K}) = \sum_{\substack{\mathcal{J} \subset \mathcal{K} \\ J = p}} \prod_{j \in \mathcal{J}} \prod_{l \in \mathcal{K} \setminus \mathcal{J}} \rho_j (1 - \rho_l), \tag{1}$$

and

$$F(p, \mathcal{K}) = \sum_{\substack{\mathcal{J} \subset \mathcal{K} \\ J \leq p}} \prod_{j \in \mathcal{J}} \prod_{l \in \mathcal{K} \setminus \mathcal{J}} \rho_j (1 - \rho_l). \tag{2}$$

$F(p, \mathcal{K})$ is clearly increasing in p. It is also decreasing in $\mathcal{K}$ for the set containment order and decreasing in ρ_k for any $k \in \mathcal{K}$, as (4) shows. It is therefore decreasing in $\mathcal{K}$ for the strength order. Intuitively, adding new candidates to a pool or increasing the probability that any project already in the pool is good increases the probability that at least p projects in the pool are good. These results are direct consequences of the following straightforward lemma.

Lemma 2. *For any $\mathcal{K} \subseteq \mathcal{N}$, $k \in \mathcal{K}$, $p \in \{1, \cdots, K\}$ and $q \leq K$,*

$$f(p, \mathcal{K}) = \rho_k f(p - 1, \mathcal{K} \setminus \{k\}) + (1 - \rho_k) f(p, \mathcal{K} \setminus \{k\}), \tag{3}$$

and

$$F(q, \mathcal{K}) = F(q, \mathcal{K} \setminus \{k\}) - \rho_k f(q, \mathcal{K} \setminus \{k\}). \tag{4}$$

Another useful property is that the probability that there are at least p good projects in a given set becomes higher after having made k good picks and no bad picks.

Lemma 3. *For fixed $p > 0$ and $\mathcal{J} \subseteq \mathcal{N}$, and any subset of projects $\mathcal{K} \subseteq \mathcal{N}$ such that $\mathcal{J} \cap \mathcal{K} = \emptyset$ and $0 < K < p$,*

$$F(p, \mathcal{J} \cup \mathcal{K}) > F(p - K, \mathcal{J}) \tag{5}$$

With these notations, and as a corollary of Proposition 1, the following expressions can be given to the probability that a project is implemented by the decision maker

Corollary 1. *The probability that $d(k)$, the k-th best project in $\mathcal{D}$, is implemented by the* *decision maker is equal to*

$$F(M-1,\mathcal{D}^{-}(k-1))\rho_{i(k)},$$

and the probability that $h(k)$, the k-th project in $\mathcal{H}$, is implemented is equal to

$$F(M-k,\mathcal{D})\cdot \mathbb{1}_{h(k)\in\mathcal{H}_S}.$$

3.3 Implied Preferences of the Decision Maker

Proposition 1 implies the following expression for the expected payoff of the decision maker $V(\mathcal{D},\mathcal{H},M)$, where $M'=\min(M,H_S)$

$$\begin{aligned} V(\mathcal{D},\mathcal{H},M) &= (1-F(M-1,\mathcal{D}))MG + G\sum_{p=1}^{M-1} pf(p,\mathcal{D}) \\ &\quad -c\sum_{q=0}^{D-1} F(M-1,\mathcal{D}^{-}(q)) + \sum_{r=1}^{M'} F(M-r,\mathcal{D})\left(\rho_{h(r)}(G+L)-L\right). \end{aligned} \tag{6}$$

The first term measures the payoff from implementing M good projects, weighted by the probability $1-F(M-1,\mathcal{D})$ of finding them. The second term measures the expected payoff obtained when less than $M-1$ good projects are found in $\mathcal{D}$. The third term measures the expected cost of the search in $\mathcal{D}$. If less than $M-1$ projects are found among the first $q<D$ projects in $\mathcal{D}$, at least one more project has to be investigated at the cost of c. Finally, the last term measures the payoff from filling the remaining slots with projects in $\mathcal{H}_S$.

Proposition 2. *Consider two documented sets $\mathcal{D}_0 \subseteq \mathcal{D}_1 \subseteq \mathcal{N}$. Then $V(\mathcal{D}_1,\mathcal{H}_1,M) \geq V(\mathcal{D}_0,\mathcal{H}_0,M)$.*

Clearly, a larger documented set gives more options to the decision maker who can investigate, rubberstamp or discard any project in her documented set while she can only rubberstamp or discard other projects.

Proposition 3. *When $M = 1$, a decision maker who can choose from which candidate to get information between a stronger and a weaker one always opts for the stronger one.*

Consider a swap of the following type. For a fixed set of projects $\mathcal{N}$ take two projects n and m, with $n < m$ ($\rho_n > \rho_m$). Let $[\hat{\mathcal{D}}, \hat{\mathcal{H}}]$ be a partition of $\mathcal{N} \setminus \{n, m\}$. Let $\mathcal{D}_0 = \hat{\mathcal{D}} \cup \{m\}$, $\mathcal{H}_0 = \hat{\mathcal{H}} \cup \{n\}$, $\mathcal{D}_1 = \hat{\mathcal{D}} \cup \{n\}$ and $\mathcal{H}_1 = \hat{\mathcal{H}} \cup \{m\}$. Then $[\mathcal{D}_0, \mathcal{H}_0]$ and $[\mathcal{D}_1, \mathcal{H}_1]$ are both partitions of $\mathcal{N}$ that are obtained from one another by swapping the roles of n and m, so that $\mathcal{D}_1 > \mathcal{D}_0$ and $\mathcal{H}_1 < \mathcal{H}_0$. The decision maker of the proposition is asked to choose between $(\mathcal{D}_1, \mathcal{H}_1)$ and $(\mathcal{D}_0, \mathcal{H}_0)$, and the proposition says that it is optimal to choose $(\mathcal{D}_1, \mathcal{H}_1)$, that is $V(\mathcal{D}_1, \mathcal{H}_1, 1) \geq V(\mathcal{D}_0, \mathcal{H}_0, 1)$.

Surprisingly, the result does not hold in general for $M > 1$, as the following example shows.

Example 1. *Consider the case of three strong candidates for two slots. If the two most promising candidates disclose their information while the third one does not, the payoff of the decision maker is $V_1 = 2\rho_1\rho_2 G + G(\rho_1(1-\rho_2) + \rho_2(1-\rho_1)) - 2c + (1-\rho_1\rho_2)(\rho_3(G+L) - L)$. The first term gives her payoff if the search in her documented set is fully successful weighted by the probability of such a success; the second term is the weighted payoff of the search when it is only partially successful, the third term is the cost of the search, it is $2c$ for sure since there are two slots to fill and only two candidates in the documented set; the last term is the weighted payoff from rubberstamping the third project in case the search is not fully successful. The decision maker's payoff in the case that the first and the third candidate disclose their information while the second does not is obtained by symmetry $V_2 = 2\rho_1\rho_3 G + G(\rho_1(1-\rho_3) + \rho_3(1-\rho_1)) - 2c + (1-\rho_1\rho_3)(\rho_2(G+L) - L)$. Then $V_2 - V_1 = (1-\rho_1)(\rho_2 - \rho_3)L > 0$. Hence the decision maker prefers to get information from candidate 3 than from candidate 2 when candidate 1 is disclosing. In fact, it is easy to show that with 3 strong candidates and two slots, the decision maker always prefers to obtain her information from weaker candidates.*

4 The Disclosure game[15]

4.1 Benchmark: One Candidate

This case is also the benchmark case of Caillaud and Tirole (2007) who first analyzed it. The only problem for the decision maker is to know whether the single project is worth implementing. Let ρ be its prospect. If $\rho > \hat{\rho}$ the decision maker accepts the project based on her prior. She would, however, be willing to investigate the project whenever $\rho G - c > \rho G - (1-\rho)L$, or equivalently $\rho < \overline{\rho}$. If on the other hand $\rho < \hat{\rho}$, the decision maker is willing to investigate if $\rho G - c > 0$ that is $\rho > \underline{\rho}$.

Proposition 4 (Caillaud and Tirole 2007)**.** *If $\rho < \underline{\rho}$, the project is discarded without examination, if it is in the weak set, it is investigated and conditionally approved, while if $\rho > \hat{\rho}$ it is rubberstamped by the decision maker. If the project is in the strong set, $\rho \in (\hat{\rho}, \overline{\rho})$, the decision maker prefers to investigate, but the candidate is not willing to disclose her information. The expected payoff of the decision maker in equilibrim is given by*

$$\Big(\rho G - c\Big)\mathbb{1}_{\rho \in \mathcal{N}_W} + \Big(\rho(L+G) - L\Big)\mathbb{1}_{\rho \in \mathcal{N}_S}.$$

The result of Proposition 4 is illustrated in Figure 2. The candidate has real authority over the final decision when $\rho \in (\hat{\rho}, \overline{\rho})$. This generates a non-monotonicity in the expected payoff of the decision maker as a function of ρ.

4.2 Multiple Candidates

An action profile is equivalent to a partition $[\mathcal{D}, \mathcal{H}]$ of $\mathcal{N}$. Given a project $n \in \mathcal{N}$, denote by $[\mathcal{D}, \mathcal{H}]_{-n}$ an action profile of all the candidates except n. It is a partition of $\mathcal{N} \setminus \{n\}$. Since, by Lemma 1, any project in $\mathcal{H}_W$ is discarded by the decision maker, a candidate $n \in \mathcal{N}_W$ is certain that her project stands no chance if she refuses to disclose her information. If, on the

[15]Proofs that are not in the text can be found in Appendix B

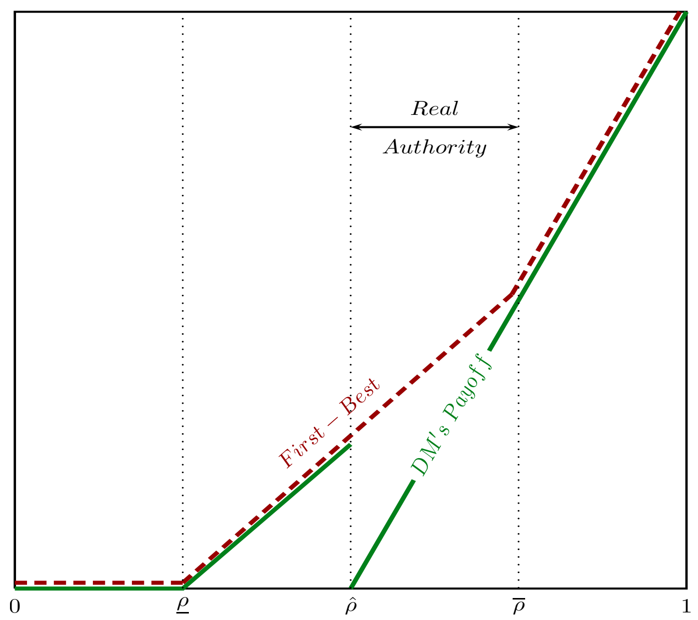

Figure 2: **Payoff of the Decision Maker with One Candidate.**

other hand, she disclosed this information, and given any action profile $[\mathcal{D}, \mathcal{H}]_{-n}$ of the other candidates, she would face a probability of adoption given by

$$F\Big(M-1, \mathcal{D}^{-}\big(r_{\mathcal{D}}(n)-1\big)\Big)\rho_n > 0.$$

Hence

Remark 1. *It is a dominant strategy for candidates in $\mathcal{N}_W$ to disclose their information.*

Therefore, in any equilibrium $\mathcal{H} = \mathcal{H}_S \subseteq \mathcal{N}_S$, and $\mathcal{N}_W \subseteq \mathcal{D}$. Note that disclosing always yields a positive probability of approval. Withholding, on the other hand, yields a null probability of approval for all but the first M projects in $\mathcal{H}_S$. Consequently, in any equilibrium, $H \leq M$, for otherwise project $h(M+1)$ would gain by disclosing.

Remark 2. *Any equilibrium action profile $[\mathcal{D}, \mathcal{H}]$ satisfies $H \leq M$.*

For the same reason, when $H = M$, no candidate weaker than $h(M)$ has any incentive to withhold.

Remark 3. *Given any action profile $[\mathcal{D}, \mathcal{H}]$ such that $H = M$, a candidate $n \in \mathcal{D}$ such that $r_{\mathcal{N}}(n) > r_{\mathcal{N}}(h(M))$ has no incentive to deviate.*

By definition, an equilibrium must satisfy as many incentive constraints as there are candidates. Fortunately, many of these constraints do not bind, as I show next after introducing some new definitions. A subset of projects $\mathcal{M} \subseteq \mathcal{N}$ is a *chain* if it consists of consecutive elements of $\mathcal{N}$ *i.e.* $\mathcal{M} = \{n, n+1, \ldots, n+k\} \subseteq \mathcal{N}$. A chain $\mathcal{M} \subseteq \mathcal{L} \subseteq \mathcal{N}$ is said to be *maximal* in $\mathcal{L}$ if any other chain $\mathcal{M}' \subseteq \mathcal{L}$ satisfies $\mathcal{M}' \subseteq \mathcal{M}$ or $\mathcal{M} \cap \mathcal{M}' = \emptyset$.

The next result states that along any chain of strong disclosing candidates, the incentive to deviate is higher for weaker projects.

Lemma 4. *Suppose $M = 1$ or $\rho_1 \leq 1/2$. Pick an action profile $[\mathcal{D}, \mathcal{H}]$, and a chain $\mathcal{J} \subseteq \mathcal{D}_S$. Then for any $p < J$, candidate $j(p+1)$ has a higher incentive to deviate from disclosure than $j(p)$.*

These results entail a practically useful characterization of the pure strategy equilibria of the disclosure game exposed in the following proposition. It implies that, in order to check whether a certain profile is an equilibrium, the number of incentive constraints to satisfy is less than $\min(2M, N_S) \leq N$ since only the incentives of the weakest candidates of the maximal chains of $\mathcal{D}_S$ need to be checked. The set of possible equilibria among all action profiles is also reduced by Remark 1 and Remark 2.

Proposition 5. *If $M = 1$ or $\rho_1 \leq 1/2$, $[\mathcal{D}, \mathcal{H}]$ is a pure strategy equilibrium of the disclosure game if and only if it satisfies*

(i) $\mathcal{H} \subseteq \mathcal{N}_S$.

(ii) $H \leq M$.

(iii) *For any maximal chain $\mathcal{J} \subseteq \mathcal{D}_S$,*

$$F\Big(M-1, \mathcal{D}^-\big(r_{\mathcal{D}}(j(J))-1\big)\Big)\rho_{j(J)} \geq F\Big(M - r_{\mathcal{H}}(j(J)), \mathcal{D} \smallsetminus \{j(J)\}\Big).$$

(iv) For any project $h \in \mathcal{H}$,

$$F\Big(M-1, \mathcal{D}^{-}\big(r_{\mathcal{D}}(h)-1\big)\Big)\rho_h \leq F\Big(M - r_{\mathcal{H}}(h), \mathcal{D}\Big).$$

Proof. The necessity is clear since (iii) and (iv) constitute a subset of the incentive conditions required by equilibrium. Sufficiency is a direct consequence of Lemma 4. □

In practice, situations where the number of slots available is small compared to the number of projects are more likely to be of interest. The reduction of the number of incentive constraints to check is particularly effective in these situations.

4.3 Full Disclosure

Since full disclosure is the optimal outcome of the disclosure game for the decision maker, it is important to characterize the conditions under which it obtains. The first result is that there must be at least one weak candidate in the pool for full disclosure to be possible.

Proposition 6. *Full disclosure is impossible in the absence of weak candidates.*

Proof. If all projects are strong and all candidates disclose, the weakest candidate has the same probability of being reached by the search whether she discloses or not. But, conditionally on being reached, her project is certain to be accepted if she withholds and not otherwise. □

When Proposition 5 applies, it is easy to provide a necessary and sufficient conditions for the existence of a full disclosure equilibrium. Indeed, the only condition to check is that the weakest candidate in $\mathcal{N}_S$ has no incentive to deviate.

Proposition 7. *If $M = 1$ of $\rho_1 \leq 1/2$, full disclosure is an equilibrium of the disclosure game if and only if the weakest of the strong candidates has no incentive to deviate, that is*

$$\rho_{N_S} \geq \frac{F(M-1, \mathcal{N} \setminus \{N_S\})}{F(M-1, \mathcal{N}^{-}(N_S-1))}. \tag{7}$$

Furthermore, the disclosure game is dominance solvable whenever the inequality in (7) holds strictly. In particular, full disclosure is then the unique equilibrium.

Proposition 6 and 7 show the importance of weak candidates. It is competition from weak candidates, who cannot afford secrecy, that puts pressure on stronger candidates to reveal their information. More generally, the right-hand side of (7) is decreasing in $\mathcal{N}_W$ both for the set order and for the strength order, implying that a better pool of weak candidates makes condition (7) easier to satisfy.

For the single-slot case, a sufficient condition for full disclosure can be provided in the form of a lower bound on the number N_W of weak candidates.

Proposition 8. *If $M = 1$, full disclosure is an equilibrium if and only if*

$$\rho_{N_S} \geq \prod_{n \in \mathcal{N}_W} (1 - \rho_n). \tag{8}$$

In particular $N_W \geq B(\rho_{N_S})$ is a sufficient condition for the existence of a full disclosure equilibrium, where

$$B(\rho) \equiv \min \left\{ k \in \mathbb{N} : (1 - c/G)^k < \rho \right\} = \left\lceil \frac{\log \rho}{\log(1 - c/G)} \right\rceil .$$

An alternative sufficient condition that does not depend on the prospect of any particular project is $N_W > B(\hat{\rho})$.

4.4 The Single-Slot Case

Let $n^* = \min\{n \in \mathcal{N}_S : \rho_n \leq (1 - \rho_{n+1}) \dots (1 - \rho_N)\}$, and $n^* = \emptyset$ when the set is empty. n^* is the strongest candidate of the strong set whose prospect is less than the probability that none of the projects with lower prospects is good. It is also the strongest candidate of the strong set who prefers to withhold when everyone else discloses. If there is no such candidate, full disclosure is the equilibrium outcome. If $n^* = 1$, then candidate 1 is the only one withholding information in equilibrium. Otherwise, either $n^* - 1$ has no incentive to withhold given that n^* does and other candidates disclose, and then this is an equilibrium, or there is no pure strategy equilibrium.

Proposition 9. *In the case $M = 1$ there exists an equilibrium in pure strategies if $n^* \in \{1, \infty\}$ or if n^* satisfies $\rho_{n^*-1} \geq (1 - \rho_{n^*+1}) \ldots (1 - \rho_N)$. When it exists, this equilibrium is unique. It is full disclosure if $n^* = \emptyset$, and otherwise the only withholding candidate in equilibrium is n^*.*

The next proposition shows that improving the set of weak candidates $\mathcal{N}_W$ can only lead to a better pure strategy equilibrium of the disclosure game from the point of view of the decision maker.

Proposition 10. *Let $\mathcal{N}_0 = \mathcal{N}_S \cup \mathcal{N}_{W0}$ and $\mathcal{N}_1 = \mathcal{N}_S \cup \mathcal{N}_{W1}$ be two sets of projects such that each of them leads to a pure strategy equilibrium $[\mathcal{D}, \mathcal{H}]_\gamma$ of the corresponding disclosure games Γ_γ. Then, if either $\mathcal{N}_{W0} < \mathcal{N}_{W1}$ or $\mathcal{N}_{W0} \subset \mathcal{N}_{W1}$, the decision maker prefers $\mathcal{N}_1$ to $\mathcal{N}_0$, that is*

$$V([\mathcal{D}, \mathcal{H}]_1, 1) > V([\mathcal{D}, \mathcal{H}]_0, 1).$$

With a single slot, it is also possible to know in which case the strongest project would be optimally rubberstamped by the decision maker as the outcome of the game. In this proposition only, (NE) and (LP) are no longer assumed to hold.

Proposition 11 (Outstanding Candidates)**.** *If $M = 1$, there exists $\rho^+ > \overline{\rho}$ such that for every profile of prospects, the decision maker optimally rubberstamps project 1 given any documented set that excludes project 1 whenever $\rho_1 > \rho^+$, where*

$$\rho^+ = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4c}{L+G}}\right).$$

Hence in any equilibrium project 1 is rubberstamped, and candidate 1 either withholds or is indifferent between withholding and disclosing.

Interestingly, ρ^+ does not depend on the number of candidates, or the particular profile of prospects. Naturally, it is decreasing in the processing cost c.

To conclude this section, I consider the example of two strong candidates, and possibly many weak candidates. In the general case, mixed strategy equilibria are difficult to characterize and may involve mixing by more than two candidates, but they can be analyzed in a simple way in this example.

Example 2 (Two Strong Candidates (and Many Weak Ones) for One Slot)**.** *When $N_S = 2$, a mixed strategy equilibrium obtains whenever $(1-\rho_2)f(0,\mathcal{N}_W) < \rho_1 < f(0,\mathcal{N}_W)$ (from Proposition 9). In this case, the mixed strategy equilibrium is unique and the two strong candidates play as follows. The stronger one discloses her information with probability $\lambda_1 = \frac{\rho_2}{\rho_1\rho_2+(1-\rho_1)f(0,\mathcal{N}_W)}$, while the weaker one discloses her information with probability $\lambda_2 = \frac{f(0,\mathcal{N}_W)-\rho_1}{\rho_2 f(0,\mathcal{N}_W)}$ (see the argument in Appendix B).*

5 Incomplete Information[16]

In some applications, it may be unreasonable to assume that candidates know each other's initial prospects, especially when their number is large. I assume that the prospects of the projects are drawn independently from an atomless distribution with cumulative density function H and full support $\mathcal{S} = [\underline{x}, \overline{x}] \subseteq [\underline{\rho}, \overline{\rho}]$ such that $\underline{x}(1-\overline{x}) > c/(L+G)$[17]. The corresponding probability density function h is assumed to be bounded away from 0 by some $m > 0$[18]. All prospects are observed by the decision maker so that the policy of Proposition 1 remains optimal. N is common knowledge. The *type* of candidate n is her realized prospect $\rho_n \in \mathcal{S}$. Types lying in $\mathcal{S} \cap (0,\hat{\rho})$ are weak, and types in $\mathcal{S} \cap (\hat{\rho}, \overline{x})$ are strong. If $\hat{\rho} \leq \underline{x}$, weak types are *absent*, and otherwise they are *present*. A distributional strategy of candidate n is a probability measure λ_n on the Borelians of $\mathcal{S} \times \{0,1\}$ for which the marginal distribution of $\mathcal{S}$ is

[16] All proofs are in Appendix C.
[17] Hence any particular realization of the vector of prospects satisfies (LP).
[18] The only result that relies on this assumption is Proposition 16

h, where $\{0,1\}$ is a description of the action set and 1 corresponds to disclosing. This formalism introduced by Milgrom and Weber (1985) allows one to describe mixing behaviors by the players while avoiding the measurability issue noted in Aumann (1964). The probability that player n discloses information given that her type is ρ is then $\lambda_n(1|\rho)$. To simplify the notations, I denote this probability by $\lambda_n(\rho)$. The equilibrium notion for the disclosure game is Bayesian Nash equilibrium in distributional strategies. I generally consider symmetric equilibria. In this section, full disclosure denotes the strategy profile such that all the candidates disclose with probability 1 regardless of their type. Finally, as is well known, if $(\lambda_1, \ldots, \lambda_N)$ is an equilibrium, then so is any strategy profile $(\lambda'_1, \ldots, \lambda'_N)$ that such that λ'_n and λ_n differ on a subset of measure 0 of the set on which n is indifferent between disclosing or not. The characterization results in what follows are up to this known issue. Finally $Int(X)$ and $Cl(X)$ denote the interior and the closure of a set $X \subseteq \mathbb{R}$.

5.1 The Single-Slot Case

Let $\boldsymbol{\lambda} = (\lambda_1, \cdots, \lambda_N)$ be a strategy profile. Supposing that all other candidates are playing according to $\boldsymbol{\lambda}$, the payoffs of candidate n are given by

$$\begin{aligned} V^{\boldsymbol{\lambda}}_{D,n}(\rho) &\equiv \rho E\left[\prod_{m\neq n}\left(1-\lambda_m(\rho_m)\rho_m \mathbb{1}_{\rho_m>\rho}\right)\right] \\ &= \rho \prod_{m\neq n}\left(1-\int_{\rho}^{\overline{x}} x\lambda_m(x)dH(x)\right), \end{aligned} \tag{9}$$

if she discloses, and

$$\begin{aligned} V^{\boldsymbol{\lambda}}_{H,n}(\rho) &\equiv E\left[\prod_{m\neq n}\lambda_m(\rho_m)\left(1-\rho_m\right)+\left(1-\lambda_m(\rho_m)\right)\mathbb{1}_{\rho_m<\rho}\right]\mathbb{1}_{\rho\geq\hat{\rho}} \\ &= \mathbb{1}_{\rho\geq\hat{\rho}}\prod_{m\neq n}\left(\int_{\underline{x}}^{\overline{x}}\lambda_m(x)(1-x)dH(x)+\int_{\underline{x}}^{\rho}\left(1-\lambda_m(x)\right)dH(x)\right), \end{aligned} \tag{10}$$

if she withholds[19]. $V^{\boldsymbol{\lambda}}_{D,n}$ is continuous in type[20] while $V^{\boldsymbol{\lambda}}_{H,n}$ has a single discontinuity at $\hat{\rho}$ if weak types are present. $V^{\boldsymbol{\lambda}}_{D,n}$ is also strictly increasing in ρ, while $V^{\boldsymbol{\lambda}}_{H,n}$ is only weakly increasing in ρ. In particular, it is constant on any interval of types on which all other players disclose with probability 1. The reason is that the probability of being considered by the decision maker when withholding depends on a candidate's own type ρ only through the implied probability that a candidate with type higher than ρ also withholds, which is invariant while ρ stays within an interval on which other candidates disclose.

If $\boldsymbol{\lambda} = (\lambda, \ldots, \lambda)$ is a symmetric strategy profile, dropping the n index for the payoff functions,

$$V^{\lambda}_D(\rho) = \rho \left(1 - \int_{\rho}^{\overline{x}} x\lambda(x)dH(x)\right)^{N-1}, \tag{11}$$

and,

$$V^{\lambda}_H(\rho) = \left(\int_{\underline{x}}^{\overline{x}} \lambda(x)(1-x)dH(x) + \int_{\underline{x}}^{\rho} (1-\lambda(x))\, dH(x)\right)^{N-1} \mathbb{1}_{\rho \geq \hat{\rho}}. \tag{12}$$

A profile $\boldsymbol{\lambda}$ is an equilibrium if n is willing to play according to λ_n when other candidates follow $\boldsymbol{\lambda}$. Let $\Lambda_n \equiv \{\rho \in \mathcal{S} : \lambda_n(\rho) \in (0,1)\}$, $\Lambda^0_n \equiv \{\rho \in \mathcal{S} : \lambda_n(\rho) = 0\}$ and $\Lambda^1_n \equiv \{\rho \in \mathcal{S} : \lambda_n(\rho) = 1\}$. Then λ is an equilibrium strategy if and only if

(i) $\forall \rho \in \Lambda^0_n, \quad V^{\boldsymbol{\lambda}}_{D,n}(\rho) \leq V^{\lambda}_{H,n}(\rho)$,

(ii) $\forall \rho \in \Lambda_n, \quad V^{\boldsymbol{\lambda}}_{D,n}(\rho) = V^{\lambda}_{H,n}(\rho)$,

(iii) $\forall \rho \in \Lambda^1_n, \quad V^{\boldsymbol{\lambda}}_{D,n}(\rho) \geq V^{\lambda}_{H,n}(\rho)$.

[19] In both equations, the equality is a consequence of the independence of the prospects.

[20] The only non-obvious part of the argument consists in showing that $\int_{\rho}^{\overline{x}} x\lambda_m(x)dH(x)$ is continuous in ρ. Because λ_m is bounded between 0 and 1, $\left|\int_{\rho}^{\overline{x}} x\lambda_m(x)dH(x) - \int_{\rho'}^{\overline{x}} x\lambda(x)dH(x)\right| \leq \left|\int_{\rho}^{\rho'} dH(x)\right| = |H(\rho) - H(\rho')|$ for any pair (ρ, ρ'). Hence, H being atomless, this difference goes to 0 when $\rho' \to \rho$, and that concludes the argument. A similar argument works for $V^{\boldsymbol{\lambda}}_H$.

Before going further, note that there exists a symmetric Bayesian Nash equilibrium in distributional strategies. As I show below, symmetric pure strategy equilibria, however, do not always exist[21].

Proposition 12. *There exists a symmetric Bayesian Nash Equilibrium in distributional strategies for the disclosure game.*

As in the case with perfect information, it is clear that any strategy that prescribes to disclose with probability less than 1 for weak types is strictly dominated.

Lemma 5. *Any strategy λ_n such that for some $\rho < \hat{\rho}$, $\lambda_n(\rho) < 1$ is strictly dominated.*

Therefore in equilibrium $[\underline{x}, \hat{\rho}) \subseteq \Lambda_n^1$. It is easy to prove that when weak types are present, sufficient competition yields full disclosure.

Proposition 13. *If weak types are present, full disclosure is an equilibrium if and only if*

$$N \geq \hat{N} \equiv 1 + \frac{\log(1/\hat{\rho})}{\log\left(1 - \int_{\hat{\rho}}^{\overline{x}} x dH(x)\right) - \log\left(1 - \int_{\underline{x}}^{\overline{x}} x dH(x)\right)}. \tag{13}$$

Furthermore the game is dominance solvable whenever this inequality holds strictly. In particular, full disclosure is then the unique equilibrium. In the absence of weak types, full disclosure is never an equilibrium.

In order to understand the role of weak and strong candidates, it is interesting to look at the effect of the distribution of types on $\hat{N}$. The next result shows that increasing the weight on stronger types in the weak set and decreasing the weight on stronger types in the strong set while keeping the relative weights of these sets constant, hence concentrating the distribution around $\hat{\rho}$, leads to a lower threshold $\hat{N}$. Obviously it is also better to have stronger candidates for the decision maker, therefore decreasing the weight on stronger types in the strong set has

[21] Note that since payoffs depend on both the types and actions of other candidates, the purification theorem of Milgrom and Weber (1985) does not apply.

an additional detrimental effect. Increasing the weight on stronger types in the weak set is unambiguously better for the decision maker.

Proposition 14. *Consider two distributions G and H with the same support $\mathcal{S}$ that includes weak types, $\hat{\rho} \in \mathcal{S}$, and such that for every $x \in \mathcal{S}$, $G(x) \leq H(x)$ if $x \leq \hat{\rho}$, and $G(x) \geq H(x)$ if $x \geq \hat{\rho}$. Then $\hat{N}_G \leq \hat{N}_H$.*

The next lemma shows that if a strong type ρ discloses information with probability 1 in a symmetric equilibrium, then all the types above ρ also disclose with probability 1. The intuition is the following. If there exists an interval Ω on which λ is equal to 1, then as already noted V_H^λ is constant on Ω while V_D^λ is strictly increasing. If λ is an equilibrium strategy, $V_D^\lambda > V_H^\lambda$ on Ω. But then, the continuity of the two payoff functions implies that V_H^λ can never catch up with V_D^λ as ρ increases, so that disclosing must be strictly better than withholding.

Lemma 6. *If λ is a symmetric equilibrium such that there exists a strong type $\rho > \hat{\rho}$ satisfying $\rho \in Int(\Lambda^1)$, then $[\rho, \overline{x}] \subseteq \Lambda^1$.*

Therefore, in any equilibrium $Int(\Lambda^1) = (\underline{x}, \hat{\rho}) \cup (\rho^*, \overline{x})$ for some $\rho^* \in [\hat{\rho}, \overline{x}]$, or $Int(\Lambda_1) = (\underline{x}, \overline{x})$. In the absence of weak types, $Int(\Lambda^1) = (\rho^*, \overline{x})$. The next proposition, which characterizes the symmetric equilibria in pure strategies, is an immediate corollary of this lemma.

Proposition 15. *If λ is a symmetric equilibrium in pure strategies, it must take the form*

$$\lambda(\rho) = \mathbb{1}_{\rho \in \Lambda^1},$$

where

$$\Lambda^1 = [\underline{x}, \hat{\rho}) \cup \langle \rho^*, \overline{x}]$$

for some $\rho^* \geq \hat{\rho}$, and where $\langle$ denotes either $($ or $[$.

When ρ^* is interior, $\rho^* \in (\hat{\rho}, \overline{x})$, it must be a solution to the following equation in ρ

$$\left(1 - \rho^{\frac{1}{N-1}}\right)\left(1 - \int_\rho^{\overline{x}} x dH(x)\right) = \int_{\underline{x}}^{\hat{\rho}} x dH(x). \tag{14}$$

Furthermore, in the absence of weak types, there is no symmetric equilibrium in pure strategies.

Hence, symmetric pure strategy equilibria other than full disclosure, when they exist, must be non-monotonic in the presence of weak types. The characterization of the threshold ρ^* in (14) derives from the fact that a player with type ρ^* must be indifferent between disclosing and withholding. It is not difficult to find examples of such equilibria. A simple one is with $c = 0$, $\hat{\rho} = 0.25$, $[\underline{x}, \overline{x}] = [0, 1]$, the uniform distribution and less than 3 candidates.

Knowing that sufficient competition leads to full disclosure as long as weak types are present, one may wonder about the effect of competition in the absence of weak types. The following proposition shows that, even though full disclosure is never an equilibrium, any sequence of symmetric equilibria approaches full disclosure as N goes to infinity.

Proposition 16. *In the absence of weak types, there exists some $\tilde{N}$ such that for all $N > \tilde{N}$ every symmetric equilibrium λ is of the form $Cl(\Lambda) = [\underline{x}, y]$ and $\Lambda^1 \supseteq (y, \overline{x}]$, with $y \in (\underline{x}, \overline{x})$. If $\{\lambda_N\}_{N=1}^{\infty}$ is a sequence of equilibria for N candidates then y_N is defined for $N > \tilde{N}$ and*

$$\lim_{N\to\infty} y_N = \underline{x}.$$

Furthermore for every $\varepsilon > 0$ there exists some $N' > \tilde{N}$ such that if $N > N'$, then for almost every $\rho \in \Lambda_N$,

$$\left| \lambda_N(\rho) - \frac{1}{1+\rho} \right| < \varepsilon.$$

These equilibria approach full disclosure in the following sense.

Corollary 2. *In the absence of weak types, if $\{\lambda_N\}_{N=1}^{\infty}$ is a sequence of symmetric equilibria, then for every $\rho \in (\underline{x}, \overline{x}]$*

$$\lim_{N\to\infty} \lambda_N(\rho) = 1.$$

5.2 Multiple Slots $M \geq 1$

The case with multiple slots is far less tractable. However some of the results extend to this case.

Proposition 17. *With multiple slots, any symmetric pure strategy equilibrium must take the form*

$$\lambda(\rho) = \mathbb{1}_{\rho \in \Lambda^1},$$

where

$$\Lambda^1 = [\underline{x}, \hat{\rho}) \cup \langle \rho^*, \overline{x}]$$

for some $\rho^* \geq \hat{\rho}$, and where $\langle$ denotes either (or [. In the absence of weak types, there is no pure strategy equilibrium. In particular, full disclosure is impossible in the absence of weak types.

The arguments used to prove this proposition extend those of the single slot case. In the remaining of the section, I describe how the essential intuitions translate in the multiple-slots case.

For any Borel set $\mathcal{K} \subseteq \mathcal{S}$, let $\eta\left(\mathcal{K}\right) = \int_{\mathcal{K}} dH(x)$ be the measure of this set according to the measure implied by the distribution H, and let $x_e(\mathcal{K}) = \frac{1}{\eta\left(\mathcal{K}\right)} \int_{\mathcal{K}} x dH(x)$ denote the expected type of a candidate knowing that her type lies in $\mathcal{K}$. When $M \geq 1$ and all the candidates except i play according to the pure strategy: disclose on Λ^1, withhold on $\Lambda^0 = \mathcal{S} \setminus \Lambda^1$, the payoff from disclosing for candidate n as a function of her type ρ is given by

$$\begin{aligned} V_D(\rho) &= \rho \cdot \Pr\Big(\text{there are less than } M-1 \text{ good projects in } \Lambda^1 \cap (\rho, \overline{x})\Big) \\ &= \rho \cdot \left\{ \sum_{m=0}^{N-1} \binom{m}{N-1} \eta\big(\Lambda^1 \cap (\rho, \overline{x})\big)^m \big(1 - \eta\big(\Lambda^1 \cap (\rho, \overline{x})\big)\big)^{N-1-m} \right. \\ &\quad \left. \times \sum_{k=0}^{\min(m,M-1)} \binom{k}{m} x_e\big(\Lambda^1 \cap (\rho, \overline{x})\big)^k \big(1 - x_e\big(\Lambda^1 \cap (\rho, \overline{x})\big)\big)^{m-k} \right\}, \end{aligned} \tag{15}$$

and the payoff from withholding

$$
\begin{aligned}
V_H(\rho) &= \mathbb{1}_{\rho>\hat\rho}\Pr\Big(\text{the number of good projects in } \Lambda^1 + \text{the number of projects in } \Lambda^0\cap(\rho,\overline{x}) \le M-1\Big)\\
&= \mathbb{1}_{\rho>\hat\rho}\sum_{m=0}^{N-1}\binom{m}{N-1}\eta\big(\Lambda^1\big)^m\big(1-\eta\big(\Lambda^1\big)\big)^{N-1-m}\\
&\quad\times\sum_{k=0}^{\min(m,M-1)}\binom{k}{m}x_e\big(\Lambda^1\big)^k\big(1-x_e\big(\Lambda^1\big)\big)^{m-k}\\
&\quad\times\sum_{l=0}^{M-1-k}\binom{l}{N-1-m}\left(\frac{\eta\big(\Lambda^0\cap(\rho,\overline{x})\big)}{\eta\big(\Lambda^0\big)}\right)^l\left(\frac{\eta\big(\Lambda^0\big)-\eta\big(\Lambda^0\cap(\rho,\overline{x})\big)}{\eta\big(\Lambda^0\big)}\right)^{N-1-m-l}.
\end{aligned}
\tag{16}
$$

The intuition works in the same way as in the case with $M=1$. V_D is strictly increasing in ρ everywhere (because the set $\Lambda^1\cap(\rho,\overline{x})$ is shrinking as ρ increases implying that if there are less than $M-1$ good projects in that set for a certain ρ then there are also less than $M-1$ good projects in that set for a higher ρ), whereas V_H is constant in ρ on Λ^1 and increasing elsewhere. Both functions are continuous on $(\hat\rho,\overline{x})$. Therefore $\Lambda^1=[\underline{x},\hat\rho)\cup\langle\rho^*,\overline{x}]$ for some $\rho^*\in[\hat\rho,\overline{x}]$. The threshold ρ^* is now characterized by the following equation which says that $V_D(\rho^*)=V_H(\rho^*)$ and makes use of the particular form of Λ^1.

$$
\begin{aligned}
&\rho^*\Bigg\{\sum_{m=0}^{N-1}\binom{m}{N-1}\Big(1-H(\rho^*)\Big)^m H(\rho^*)^{N-1-m}\\
&\quad\times\sum_{k=0}^{\min(m,M-1)}\binom{k}{m}\left(\frac{1}{1-H(\rho^*)}\int_{\rho^*}^{\overline{x}}xdH(x)\right)^k\left(1-\frac{1}{1-H(\rho^*)}\int_{\rho^*}^{\overline{x}}xdH(x)\right)^{m-k}\Bigg\}\\
&\quad=\sum_{m=0}^{N-1}\binom{m}{N-1}\Big(1-H(\rho^*)+H(\hat\rho)\Big)^m\Big(H(\rho^*)-H(\hat\rho)\Big)^{N-1-m}\\
&\quad\quad\times\sum_{k=0}^{\min(m,M-1)}\binom{k}{m}\left(\frac{1}{1-H(\rho^*)+H(\hat\rho)}\int_{(\underline{x},\hat\rho)\cup(\rho^*,\overline{x})}xdH(x)\right)^k\\
&\quad\quad\times\left(1-\frac{1}{1-H(\rho^*)+H(\hat\rho)}\int_{(\underline{x},\hat\rho)\cup(\rho^*,\overline{x})}xdH(x)\right)^{m-k}.
\end{aligned}
\tag{17}
$$

(17) simply states that the frontier type ρ^* must be indifferent between disclosing (on the left-hand side) and withholding (on the right-hand side). If for $\rho^* = \hat{\rho}$, the left-hand side is greater than the right-hand side, then full disclosure is an equilibrium. If for $\rho^* = \overline{x}$, the left-hand side is smaller than the right-hand side, then no disclosure is an equilibrium. If the left-hand side is strictly greater than the right-side for every $\rho^* \in (\hat{\rho}, \overline{x})$, full disclosure is the unique symmetric equilibrium in pure strategies, and if the opposite inequality holds on $(\hat{\rho}, \overline{x})$, no disclosure is the unique equilibrium. In the absence of weak candidates $H(\hat{\rho}) = 0$ and the left-hand side of (17) is then equal to its right-hand side multiplied by $\rho^* < 1$. Therefore, in the absence of weak types, no disclosure is the only possible symmetric equilibrium in pure strategies. However it is clear that no disclosure cannot be an equilibrium as the lowest type would be better off by disclosing, and therefore there is no symmetric pure strategy equilibrium in the absence of weak types.

6 Conclusion

Common knowledge that information is unequivocal to its owners is a crucial assumption the unraveling argument to work in strategic disclosure games. When information is equivocal, the receiver cannot second-guess the actions of the sender, and skepticism does not ensure full revelation. Competition can mitigate this problem, but only under certain conditions. The results of this paper highlight the importance of *ex ante* weaker candidates to elicit information transmission in certain types of contests. They are less important, however, when candidates have imperfect information about their competitors since then competition works asymptotically.

The equivocal information assumption, that agents control information but cannot predict its effect on others, deserves further examination. In the context of this model, the inability of the sender to understand the consequences of her information gives her an advantage. This may give an incentive to a sender who can interpret information to pretend she cannot, a question that it may be interesting to explore in a reputation model, for example, where it could be valuable to establish a reputation of limited understanding. The model offers several insights into practical situations of advertising in particular. It would be interesting to analyze this link with advertising further, notably by considering heterogeneous consumers and including price competition. This is only one way of introducing heterogeneity on the receiving side.

Another interesting but difficult extension would be to blend the framework of this paper with that of Caillaud and Tirole (2007), with multiple senders and a committee of receivers with heterogeneous beliefs. Correlations in the preferences of the committee members give the opportunity for a sender to engineer cascades of information among members to push her case. These forces seriously complicate the analysis of the competition among senders.

Appendix A Optimal Policy of the Decision Maker

Proof of Lemma 1. The first part of (i) comes from the fact that projects in $\mathcal{H}_W$ cannot be investigated, and the incremental expected payoff of rubberstamping them is negative. The second part of the statement is obvious given the existence of the cap M on the number of projects that can be implemented. (ii) is true because any project in $\mathcal{H}_S$ has a positive expected incremental payoff, and therefore the first $\min(M-D, H_S)$ projects in $\mathcal{H}_S$ should be used to fill the slots that cannot be filled by projects in $\mathcal{D}$ since $D < M$. Finally (iii) holds because it never hurts to fill slots that cannot be filled by projects in $\mathcal{H}_S$ with projects in $\mathcal{D}$. □

Proof of Proposition 1. If $\mathcal{D} = \mathcal{N}$ it is clearly optimal to investigate projects starting from the strongest one and then moving down in the strength order, approving a project each time it is found to be good, until all slots are filled. The policy of the proposition clearly does this. If $\mathcal{H} = \mathcal{N}$ it is also clear that the policy of the proposition is optimal: since projects in $\mathcal{N}_S$ have positive expected incremental payoffs the strongest ones should be approved according to the availability of slots.

Consider states that satisfy $D \geq M = H$ and $\mathcal{H} \subseteq \mathcal{N}_S$. Below, I show, by a double induction on D and $M = H$, that the policy described in the proposition is optimal for all such states, and that it is the unique optimal policy up to some details in the order of learning explained below. I will take this result as given for now and argue that it implies that the policy of the proposition is optimal, although not uniquely, in any other state. This is a consequence of Lemma 1. Indeed, by point (i) of the lemma, projects in $\mathcal{H}_W$ are irrelevant. Hence I can assume that $M \leq N - H_W = D + H_S$ for additional slots would never be filled. By point (ii), I can assume $D \geq M$ for otherwise the optimal policy consists in rubberstamping projects in $\mathcal{H}_S$ until a state where $D = M$ is reached and then continuing with the optimal policy. Furthermore, the lemma says that this rubberstamping can occur at any place in the sequence describing the optimal policy. Hence it can be done at the end of the sequence so that the policy of the proposition is indeed optimal. This is the first source of non uniqueness of the optimal policy in general. Since the projects rubberstamped in this operation are the strongest in $\mathcal{H}$, and are certain to get approved, however, they are also irrelevant to the probability that any given project is approved across different optimal policies. Point (i) and (iii) of the lemma allow me to consider only the cases where $H = M$. As a consequence, an optimal policy can always be described as: always start by learning as much as possible, and then rubberstamp. The order of investigation never affects the probability of having to rubberstamp some projects in the end. As a consequence, it is always optimal to investigate stronger projects first as it minimizes the cost of the search. This is not uniquely optimal, however, for the following reason. If there are M slots available then the order in which the first M projects in $\mathcal{D}$ are investigated is irrelevant since at least M projects will be investigated regardless of what is found. Hence the argument below shows optimality, and uniqueness up to this subtlety. But the argument just made implies that the probability that a given project is approved is unaltered by which particular optimal policy is used.

Initiation $\mathcal{D} = \{d\}$, $\mathcal{H} = \{h\}$, $M = 1$. In this case the choice is between (i) rubberstamping h, and (ii) investigating d, approving d if it is of the good type, rejecting d and rubberstamping h if it is of the bad type. The first choice pays $\rho_h(G + L) - L$ while the second one pays

$\rho_d G - c + (1-\rho_d)(\rho_h(G+L) - L)$. Letting Δ be the gain from learning,

$$\Delta = \rho_d(1-\rho_h)(G+L) - c > 0.$$

where the inequality holds because of (LP). Hence the unique optimal policy is to learn first.

Induction Step, $D > M = H = 1$. Suppose the result holds for any triple $(\mathcal{D}, \mathcal{H}, M)$ such that $D = K > H = M = 1$ and $\mathcal{H} \subseteq \mathcal{N}_S$, and consider a state $(\mathcal{D}, \mathcal{H}, M)$ with $D = K + 1$. The decision maker can either (i) rubberstamp h and end, or (ii) investigatea project d in $\mathcal{D}$, approve d and end if it is of the good type, and move on to the state $(\mathcal{D} \setminus \{d\}, \{h\}, 1)$ otherwise. Hence we only need to compare the payoff of (ii)

$$\rho_d G - c + (1-\rho_d)V(\mathcal{D} \setminus \{d\}, \{h\}, 1),$$

to the payoff $\rho_h(G+L) - L$ of (i). Letting Δ denote the gain from learning

$$\Delta = \rho_d(1-\rho_h)(G+L) - c + (1-\rho_d)V(\mathcal{D} \setminus \{d\}, \{h\}, 1) > 0,$$

where the first two terms add up to something positive by (LP), and the last term is non-negative because the decision maker always has the option to discard all remaining projects and get 0. Hence, investigating first is optimal, and by the induction hypothesis it is also the best continuation policy. Because investigating in the order of decreasing strength minimizes the cost of search, it is optimal to do so. This proves the claim. It is unique up to the subtlety about the order of learning explained above.

Induction Step, $D > M = H > 1$. Suppose the result holds for all $(\mathcal{D}, \mathcal{H}, M)$ with $\mathcal{H} \subseteq \mathcal{N}_S$ such that $H = M \leq K$, or $H = M = K + 1$ and $D \leq J$. Consider a triple $(\mathcal{D}, \mathcal{H}, M)$ where $H = M = K + 1$ and $D = J + 1$. Consider the choice between (i) investigating (and conditionally approving) project $d(1)$ in $\mathcal{D}$, and (ii) rubberstamping a project $h \in \mathcal{H}$. Then the decision can move on with an optimal continuation policy. I only consider the investigation of

$d(1)$ since it is clearly the best option among policies that start with investigation. The first option yields

$$\rho_{d(1)}\Big(G + V(\mathcal{D} \setminus \{d(1)\}, \mathcal{H} \setminus \{h(H)\}, M-1)\Big) + (1-\rho_{d(1)})V(\mathcal{D} \setminus \{d(1)\}, \mathcal{H}, M) - c,$$

while the second option yields

$$\rho_h(G+L) - L + V(\mathcal{D}, \mathcal{H} \setminus \{h\}, M-1),$$

which by induction can be rewritten as

$$\rho_h(G+L)-L+\rho_{d(1)}\Big(G+V(\mathcal{D}\setminus\{d(1), \mathcal{H}\setminus\{h, h(H)\}, M-2)\Big)+(1-\rho_{d(1)})V(\mathcal{D}\setminus\{d(1)\}, \mathcal{H}\setminus\{h\}, M-1)-c$$

The gain from learning is then

$$\Delta = \rho_{d(1)} \underbrace{\Big(V(\mathcal{D} \setminus \{d(1)\}, \mathcal{H}, M-1) - \big(V(\mathcal{D} \setminus \{d(1)\}, \mathcal{H} \setminus \{h, h(H)\}, M-2) + \rho_h(G+L) - L\big)\Big)}_{A}$$
$$+(1-\rho_{d(1)}) \underbrace{\Big(V(\mathcal{D} \setminus \{d(1)\}, \mathcal{H}, M) - \big(V(\mathcal{D} \setminus \{d(1)\}, \mathcal{H} \setminus \{h\}, M-1) + \rho_h(G+L) - L\big)\Big)}_{B}.$$

$A > 0$. Indeed in state $(\mathcal{D} \setminus \{d(1)\}, \mathcal{H} \setminus \{h(H)\}, M-1)$ an available policy is to rubberstamp h and then move on to state $(\mathcal{D} \setminus \{d(1)\}, \mathcal{H} \setminus \{h, h(H)\}, M-2)$ and continue with the optimal policy. Because of the induction hypothesis, this is not optimal at $(\mathcal{D}\setminus\{d(1)\}, \mathcal{H}\setminus\{h(H)\}, M-1)$, and A is exactly the difference of payoffs between the former policy and the optimal one. A similar argument shows that $B > 0$. Therefore $\Delta > 0$ implying that learning is optimal at $(\mathcal{D}, \mathcal{H}, M)$. Once again this implies that the policy of the proposition is uniquely optimal up to the order of learning.

Induction Step, $D = M = H$. Suppose now that the result holds for all $(\mathcal{D}, \mathcal{H}, M)$ with $\mathcal{H} \subseteq \mathcal{N}_S$ and $H = M \leq K$, and consider a triple $(\mathcal{D}, \mathcal{H}, M)$ such that $D = H = M = K+1$.

Then the payoff of learning about $d(1)$ is

$$\rho_{d(1)}\Big(G+V(\mathcal{D}\smallsetminus\{d(1)\},\mathcal{H}\smallsetminus\{h(H)\},M-1)\Big)+\big(1-\rho_{d(1)}\big)\ \Big(\rho_{h(1)}(G+L)-L+V(\mathcal{D}^+(1),\mathcal{H}^+(1),M-1)\Big)-c,$$

and the payoff of rubberstamping $h(1)$ ($h(1)$ is clearly better than any other h here) is

$$\rho_{h(1)}(G+L)-L+V(\mathcal{D},\mathcal{H}\smallsetminus\{h(1)\},M-1),$$

or, because of the induction hypothesis,

$$\begin{aligned}\rho_{h(1)}(G+L)-L+\rho_{d(1)}&\Big(G+V(\mathcal{D}\smallsetminus\{d(1)\},\mathcal{H}\smallsetminus\{h(1),h(H)\},M-2)\Big)\\&+\big(1-\rho_{d(1)}\big)\,V(\mathcal{D}\smallsetminus\{d(1)\},\mathcal{H}\smallsetminus\{h(1)\},M-1)-c.\end{aligned}$$

Hence the gain from learning is

$$\begin{aligned}\Delta=\rho_{d(1)}\Big(&V(\mathcal{D}\smallsetminus\{d(1)\},\mathcal{H}\smallsetminus\{h(H)\},M-1)\\&-\big(V(\mathcal{D}\smallsetminus\{d(1)\},\mathcal{H}\smallsetminus\{h(1),h(H)\},M-2)+\rho_{h(1)}(G+L)-L\big)\Big).\end{aligned}$$

By the induction hypothesis, rubberstamping $h(1)$ is an available but non optimal policy at $(\mathcal{D}\smallsetminus\{d(1)\},\mathcal{H}\smallsetminus\{h(H)\},M-1)$, hence $\Delta>0$. This concludes the proof. □

Proof of Lemma 3. I show the result for $K=1$. The general result follows by iteration. Let k be the unique project in $\mathcal{K}$. Then, by Lemma 2

$$F(p,\mathcal{J}\cup\{k\})=F(p,\mathcal{J})-\rho_k f(p,\mathcal{J}).$$

Therefore

$$
\begin{aligned}
F(p, \mathcal{J} \cup \{k\}) - F(p-1, \mathcal{J}) &= F(p, \mathcal{J}) - F(p-1, \mathcal{J}) - \rho_j f(p, \mathcal{J}) \\
&= (1-\rho_j) f(p, \mathcal{J}) > 0.
\end{aligned}
$$

□

Proof of Proposition 3. I prove the proposition for the case where the two swapped projects are adjacent in $\mathcal{N}$, that is $m = n+1$. Evidently this proves the general result, as any other swap can be decomposed in a series of swaps between adjacent projects. When $M = 1$ and the learning partition of the decision maker is given by $[\mathcal{D}, \mathcal{H}]$, the expected payoff of the decision maker is

$$
V(\mathcal{D}, \mathcal{H}, 1) = (1 - f(0, \mathcal{D}))G + f(0, \mathcal{D}) \Big(\rho_{h(1)}(G+L) - L \Big) - c \sum_{q=0}^{D-1} F(0, \mathcal{D}^-(q)).
$$

Let Δ be the change in payoff due to the swap $\Delta = V(\mathcal{D}_1, \mathcal{H}_1, 1) - V(\mathcal{D}_0, \mathcal{H}_0, 1)$. Using Lemma 2, and supposing $r_{\mathcal{H}}(n) > 1$ or $n \notin \mathcal{N}_S$, it is equal to

$$
\Delta = (\rho_n - \rho_{n+1}) f(0, \hat{\mathcal{D}})(1 - \rho_{h(1)})(G+L) + c \sum_{q=Q}^{D} \Big(F(0, \mathcal{D}_0^-(q)) - F(0, \mathcal{D}_1^-(q)) \Big),
$$

where $Q = r_{\mathcal{D}}(n)$. The first term is clearly positive, and the second term, that corresponds to the decrease of the search cost when the set of searchable projects is improved, is positive because for any $q = Q, \ldots, D$, it is true that $\mathcal{D}_0^-(q) < \mathcal{D}_1^-(q)$ and $F(.,.)$ is decreasing in its second argument for the strength order.

If $\{n, n+1\} \subseteq \mathcal{N}_S$ and $r_{\mathcal{H}}(n) = r_{\mathcal{H}}(n+1) = 1$, then

$$\Delta = (\rho_n - \rho_{n+1}) f(0, \hat{\mathcal{D}}) G - (\rho_n - \rho_{n+1}) f(0, \hat{\mathcal{D}})(G+L) + L f(0, \hat{\mathcal{D}})(\rho_n - \rho_{n+1}) \\ + c \sum_{q=Q}^{D} \Big(F(0, \mathcal{D}_0^-(q)) - F(0, \mathcal{D}_1^-(q)) \Big),$$

and the first three terms sum up to 0 so that $\Delta > 0$.

Finally, if $n \in \mathcal{N}_S$, $n+1 \notin \mathcal{N}_S$ and $r_{\mathcal{H}}(n) = 1$, then

$$\Delta = (\rho_n - \rho_{n+1}) f(0, \hat{\mathcal{D}}) G - f(0, \hat{\mathcal{D}})(1-\rho_{n+1}) \Big(\rho_n (G+L) - L \Big) + c \sum_{q=Q}^{D} \Big(F(0, \mathcal{D}_0^-(q)) - F(0, \mathcal{D}_1^-(q)) \Big),$$

so that

$$\Delta = f(0, \hat{\mathcal{D}}) \rho_{n+1} (1-\rho_n)(G+L) + c \sum_{q=Q}^{D} \Big(F(0, \mathcal{D}_0^-(q)) - F(0, \mathcal{D}_1^-(q)) \Big) > 0.$$

□

Appendix B The disclosure game

The incentive of a candidate n to deviate from an action profile $[\mathcal{D}, \mathcal{H}]$ is defined as the ratio of her deviation payoff over her current payoff, and it is denoted by $\delta(n, [\mathcal{D}, \mathcal{H}])$ or simply $\delta(n)$ when the context is clear.

Proof of Lemma 4. Let n and $n+1$ be two adjacent projects in $\mathcal{N}$ in $\mathcal{J}$, let $r = r_{\mathcal{H}}(n) = r_{\mathcal{H}}(n+1)$ be the rank that any of these projects would occupy in $\mathcal{H}$ and $d = r_{\mathcal{D}}(n)$ be the rank of n in $\mathcal{D}$, so that $r_{\mathcal{D}}(n+1) = d+1$. Then the incentives to deviate of the two candidates are given by

$$\delta(n) = \frac{F(M-r, \mathcal{D} \smallsetminus \{n\})}{F(M-1, \mathcal{D}^-(d-1)) \rho_n},$$

and

$$\delta(n+1) = \frac{F(M-r, \mathcal{D} \setminus \{n+1\})}{F(M-1, \mathcal{D}^{-}(d))\rho_{n+1}}.$$

Therefore, with the help of Lemma 2,

$$\frac{\delta(n+1)}{\delta(n)} = \frac{\rho_n\big(X - Y\rho_n\big)}{\rho_{n+1}\big(X - Y\rho_{n+1}\big)} \frac{F(M-1, \mathcal{D}^{-}(d-1))}{F(M-1, \mathcal{D}^{-}(d))},$$

where

$$X = F(M-r, \mathcal{D} \setminus \{n, n+1\}) > 0,$$

and

$$Y = f(M-r, \mathcal{D} \setminus \{n, n+1\}) > 0.$$

The second fraction is clearly greater than 1 because $F(P, .)$ is decreasing in its second argument for the set order. As for the first fraction, notice that the function $\rho(X - \rho Y)$ is increasing in ρ on $(0, 1/2)$ whenever $X/Y \geq 1$, and the latter is obviously satisfied. Since $\rho_n > \rho_{n+1}$, this fraction is also greater than 1 when $\rho_1 \leq 1/2$. Therefore $\delta(n+1) > \delta(n)$, which concludes the proof for this case.

When $M = 1$, $\delta(n+1)/\delta(n)$ is equal to $\rho_n/(\rho_{n+1}(1-\rho_n)) > 1$. $\square$

Proof of Proposition 7. By Proposition 5, the only incentive condition that needs to be checked is that of N_S, the weakest candidate in $\mathcal{N}_S$, which is done in (7). When $\mathcal{N}_W = \emptyset$, the right-hand side of (7) becomes equal to 1, proving the second statement. For the last point, remember that it is a dominant strategy for all the candidates in $\mathcal{N}_W$ to disclose their information. When (7) holds strictly, the proof is immediate if $N_S = 1$, while if $N_S > 1$ it is strictly optimal for N_S to disclose when all the candidates in $\mathcal{N}^{-}(N_S - 1)$ disclose as well. If on the other hand M or more candidates in $\mathcal{N}^{-}(N_S - 1)$ were to withhold, it would clearly be strictly optimal for N_S to disclose her information as she would stand no chance of being rubberstamped otherwise. Finally, suppose that $K < M$ candidates in $\mathcal{N}^{-}(N_S - 1)$ withhold and denote by $\mathcal{K} \subseteq \mathcal{N}^{-}(N_S - 1)$ this set of candidates, and $\mathcal{J} = \mathcal{N}^{-}(N_S - 1) \setminus \mathcal{K}$. In this case, N_S strictly prefers to disclose if and only if

$$\rho_{N_S} > \frac{F(M-K-1, \mathcal{N}^+(N_S) \cup \mathcal{J})}{F(M-1, \mathcal{J})}. \tag{18}$$

Because F is decreasing in its second argument for the set order, $F(M-1,\mathcal{J}) > F(M-1,\mathcal{N}^-(N_S-1))$. And by Lemma 3, $F(M-K-1,\mathcal{N}^+(N_S)\cup\mathcal{J}) < F(M-1,\mathcal{N}^+(N_S)\cup\mathcal{J}\cup\mathcal{K}) = F(M-1,\mathcal{N}\setminus\{N_S\})$. Therefore, (7) implies (18), showing that it is a dominant strategy for N_S to disclose. Now consider candidate $N_S - 1$. By Lemma 4, the equation obtained by replacing N_S by $N_S - 1$ in (7) is satisfied. Hence repeating the argument implies that it is also a dominant strategy for $N_S - 1$ to disclose. By induction, this shows that the game is dominance solvable. □

Proof of Proposition 9. The fact that the equilibrium described in the proposition exists under the condition given is a direct consequence of Proposition 5. In fact there is an equilibrium such that n is the only candidate withholding information if and only if $\rho_n \leq (1-\rho_{n+1})\dots(1-\rho_N)$ and $\rho_{n-1} \geq (1-\rho_{n+1})\dots(1-\rho_N)$. The only point to prove is therefore uniqueness. Let $F_n = (1-\rho_{n+1})\dots(1-\rho_N)$. $\{F_n\}$ is an increasing sequence whereas $\{\rho_n\}$ is a decreasing sequence. Then, by definition of n^*, $\rho_n \leq F_n$ if and only if $n \geq n^*$. But since for there to be an equilibrium in which n is the only withholding candidate $\rho_{n-1} \geq F_n$ must also hold, n^* is the only possibility. Indeed if $n \geq n^*$, $\rho_{n-1} \leq F_{n-1} < F_n$ so that the second condition for an equilibrium cannot hold. □

Proof of Proposition 10. Because $F(0,.)$ is decreasing in its second argument for the set order as well as for the strength order, and for any $\mathcal{N}$ and any $n \in \mathcal{N}_S$ it is true that $\mathcal{N}_W \subseteq \mathcal{N}^+(n)$, and therefore for every $n \in \mathcal{N}_S$, $F(0,\mathcal{N}_1^+(n)) < F(0,\mathcal{N}_0^+(n))$. Hence if n_γ is the unique candidate who withholds information in the equilibrium of the game $\gamma \in \{0,1\}$, it must be true that $n_1 \geq n_0$ that is the withholding candidate is a weaker candidate in game 1 than in game 0. Since all the candidates in $\mathcal{N}_{W\gamma}$ disclose their information in equilibrium, this implies that the decision maker prefers Γ_1 to Γ_0. □

Proof of Proposition 11. First note that $\rho^+ > \overline{\rho}$ so that rubberstamping project 1 beats learning about it. Since 1 is the best project, any alternative policy of the decision maker that stands a chance of being optimal given that 1 is not providing information consists in learning about k projects and rubberstamping 1 only if this search proves unfruitful. The payoff of such a policy is of the form $V = P_1 + P_2$ where $P_1 = \rho^1 G - c + (1-\rho^1)(\rho^2 G - c + (1-\rho^2)(\rho^3 G - c + \dots))$ where the sum stops at $\rho^k G - c$, and $P_2 = (1-\rho^1)\dots(1-\rho^k)(\rho_1(G+L) - L)$, and where $\rho^1, \dots, \rho^k$ denote the ordered prospects of the k projects investigated by the decision maker. A little bit of algebra shows that

$$\frac{\partial V}{\partial \rho^i} = (1-\rho^1)\dots(1-\rho^{i-1})\left(c(1+(1-\rho^{i+1})+\dots(1-\rho^{k-1})) + (1-\rho^{i+1})\dots(1-\rho^k)(1-\rho_1)(G+L)\right) > 0.$$

Hence V is strictly increasing in each ρ^i, and $V < (\rho_1 G - c)(1 + (1-\rho_1) + \dots + (1-\rho_1)^{k-1}) + (1-\rho_1)^k(\rho_1(G+L) - L) = (\rho_1 G - c)(1-(1-\rho_1)^k)/\rho_1 + (1-\rho_1)^k(\rho_1(G+L) - L)$. The payoff of rubberstamping 1 without going through the preliminary search is $\rho_1(G+L) - L$, and it is greater than the former expression if and only if (with some algebra)

$$\rho_1^2 - \rho_1 + \frac{c}{G+L} > 0.$$

The greatest root of the second degree equation associated with the former is

$$\rho^+ = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4c}{L+G}}\right),$$

where $4c/(L+G) < 1$ is implied by (AL) . Therefore $\rho_1 > \rho^+$ implies that rubberstamping 1 beats the alternative strategy. Because, by withholding, candidate 1 can force the decision maker to rubberstamp her project irrespective of the behavior of other candidates, rubberstamping 1 has to be the outcome of the game. $\square$

Proof of the Characterization of the Mixed Strategy Equilibrium in Example 2. In order to make 1 indifferent between disclosing and withholding, λ_2 must satisfy $(\lambda_2(1-\rho_2)+(1-\lambda_2))f(0,\mathcal{N}_W) =$

ρ_1 where the left-hand side is 1's payoff when withholding and the right-hand side is her payoff when she discloses. The same indifference condition for candidate 2 gives $\lambda_1(1-\rho_1)\rho_2 + (1-\lambda_1)\rho_2 = \lambda_1(1-\rho_1)f(0, \mathcal{N}_W)$. □

Appendix C Incomplete Information

Proof of Proposition 12. Propositions 1 and 3 and Theorem 1 of Milgrom and Weber (1985) imply that when the set of distributional strategies is topologized by weak convergence, the players' strategy sets are compact, convex metric spaces and the payoff functions are continuous and linear in each single player's strategy. Then the best response function $\phi : \Sigma \to 2^{\Sigma}$ that maps a strategy σ into the set of best responses of any player (the game is symmetric) to σ is a Kakutani map (that is upper-semicontinuous, non-empty valued and convex valued), where Σ is the set of distributional strategies of each player. Then the Kakutani-Glicksberg-Fan fixed point theorem applies implying that there exists a symmetric equilibrium of the disclosure game in distributional strategies . For references, see Glicksberg (1952); Fan (1952); Milgrom and Weber (1985) and Dugundji and Granas (2003, Chapter II, Section 7.8). □

Proof of Proposition 13. First note that (13) is equivalent to

$$\hat{\rho}\left(\frac{1-\int_{\hat{\rho}}^{\overline{x}} x dH(x)}{1-\int_{\underline{x}}^{\overline{x}} x dH(x)}\right)^{N-1} \geq 1,$$

and the left-hand side is equal to the ratio $V_D^1(\hat{\rho})/V_H^1(\hat{\rho})$ of the payoffs of a player with type $\hat{\rho}$ when all other players disclose with probability 1. Therefore, (13) implies that there is no incentive of a player with type $\hat{\rho}$ to deviate from full disclosure. It is therefore clearly a necessary condition for equilibrium.

To show that it is also sufficient, note that for a candidate with type $\rho > \hat{\rho}$, when all the other candidates disclose with probability 1,

$$\frac{V_D^1(\rho)}{V_H^1(\rho)} = \rho \left(\frac{1 - \int_\rho^{\overline{x}} x dH(x)}{1 - \int_{\underline{x}}^{\overline{x}} x dH(x)} \right)^{N-1} > \hat{\rho} \left(\frac{1 - \int_{\hat{\rho}}^{\overline{x}} x dH(x)}{1 - \int_{\underline{x}}^{\overline{x}} x dH(x)} \right)^{N-1} \geq 1,$$

implying that there is no incentive to deviate from the full disclosure profile for such a candidate. Since it is a dominant strategy for the types below $\hat{\rho}$ to disclose, this proves that (13) is also a sufficient condition. In the absence of weak types, $\hat{N}$ is infinite and full disclosure cannot be an equilibrium.

Now suppose that (13) holds with a strict inequality, and that there exists $\rho_k \geq \hat{\rho}$ such that all strategies prescribing to disclose with probability less than 1 for some $\rho < \rho_k$ have been eliminated and call $\mathcal{L}_k$ the set of remaining strategies. Because the function

$$\rho \left(\frac{1 - \int_\rho^{\overline{x}} x dH(x)}{1 - \int_{\underline{x}}^{\overline{x}} x dH(x)} \right)^{N-1}$$

is increasing in ρ, it must be strictly greater than 1 when evaluated at ρ_k. Now given a strategy profile $\lambda \in \mathcal{L}_k$, the incentive to disclose for a type $\rho > \rho_k$ is given by

$$\begin{aligned}
\frac{V_{D,n}^\lambda(\rho)}{V_{H,n}^\lambda(\rho)} &= \rho \prod_{m \neq n} \frac{1 - \int_\rho^{\overline{x}} x \lambda_m(x) dH(x)}{\int_{\underline{x}}^{\overline{x}} \lambda_m(x)(1-x) dH(x) + \int_{\rho_k}^{\rho} (1 - \lambda_m(x))\, dH(x)} \\
&= \rho \prod_{m \neq n} \frac{1 - \int_\rho^{\overline{x}} x \lambda_m(x) dH(x)}{\int_{\underline{x}}^{\rho_k} \lambda_m(x)(1-x) dH(x) + \int_{\rho_k}^{\rho} (1 - \lambda_m(x) x)\, dH(x) + \int_\rho^{\overline{x}} (1-x)\lambda_m(x) dH(x)} \\
&\geq \rho \left(\frac{1 - \int_\rho^{\overline{x}} x dH(x)}{1 - \int_{\underline{x}}^{\overline{x}} dH(x) + \int_{\rho_k}^{\rho} x dH(x)} \right)^{N-1} \equiv L(\rho_k, \rho),
\end{aligned}$$

where the lower bound is attained on $\mathcal{L}_k$ by the strategy $\lambda(x) = 1 - \mathbb{1}_{x \in (\rho_k, \rho)}$. $L(.,.)$ is clearly continuous and the limit of $L(\rho_k, \rho)$ as $\rho \to \rho_k$ is

$$\rho_k \left(\frac{1 - \int_{\rho_k}^{\overline{x}} x dH(x)}{1 - \int_{\underline{x}}^{\overline{x}} x dH(x)} \right)^{N-1} > 1.$$

By continuity, it must also be strictly greater than 1 on a neighborhood to the right of ρ_k. Thus we can define

$$\rho_{k+1} = \sup \left\{ \rho \in (\rho_k, \overline{x}) : L(\rho_k, \rho) > 1 \right\}.$$

If $\rho_{k+1} < \overline{x}$, it must be true that $L(\rho_k, \rho_{k+1}) = 1$. We can define $\mathcal{L}_{k+1}$ to be the set of strategies that prescribe to disclose with probability 1 whenever $\rho < \rho_{k+1}$. The construction of ρ_{k+1} implies that, provided that players are restricted to use strategies in $\mathcal{L}_k$, strategies in $\mathcal{L}_k \setminus \mathcal{L}_{k+1}$ are strictly dominated.

Let $\rho_0 = \hat{\rho}$ and define $\mathcal{L}_0$ accordingly. I have already proved in Lemma 5 that strategies that are not in $\mathcal{L}_0$ are strictly dominated. Therefore I can apply the construction above to find an increasing sequence $\{\rho_0, \rho_1, \dots\}$ and the corresponding shrinking sequence $\{\mathcal{L}_0, \mathcal{L}_1, \dots\}$ stopping whenever $\rho_k = \overline{x}$. If this happens, the construction implies that the only strategy that survives the iterated elimination of strictly dominated strategies is to disclose with probability 1 everywhere except perhaps at $\rho = \overline{x}$.

Suppose that it is not the case so that for every k, $\rho_k < \overline{x}$. Because the sequence $(\rho_k)_{k \geq 0}$ is increasing and bounded, it admits a limit $\rho_\infty \leq \overline{x}$. By construction, the relationship $L(\rho_k, \rho_{k+1}) = 1$ must hold for every k. Then, taking k to infinity and using the continuity of $L(.,.)$

$$L(\rho_\infty, \rho_\infty) = \rho_\infty \left(\frac{1 - \int_{\rho_\infty}^{\overline{x}} x dH(x)}{1 - \int_{\underline{x}}^{\overline{x}} dH(x)} \right)^{N-1} = 1.$$

But the function

$$\rho \left(\frac{1 - \int_{\rho}^{\overline{x}} x dH(x)}{1 - \int_{\underline{x}}^{\overline{x}} x dH(x)} \right)^{N-1}$$

is increasing in ρ and strictly greater than 1 at $\hat{\rho}$ and therefore also at $\rho_\infty > \hat{\rho}$. A contradiction.

Therefore all strategies that disclose with probability less than 1 anywhere except at $\rho = \overline{x}$ are eliminated in a finite number of steps. But then the incentive to disclose for type $\overline{x}$ is given by

$$\overline{x}\left(\frac{1}{1-\int_{\underline{x}}^{\overline{x}} x dH(x)}\right)^{N-1} \geq \hat{\rho}\left(\frac{1-\int_{\hat{\rho}}^{\overline{x}} x dH(x)}{1-\int_{\underline{x}}^{\overline{x}} x dH(x)}\right)^{N-1} > 1.$$

Hence strategies that prescribe to disclose with probability less than 1 for $\overline{x}$ can be eliminated as well. □

Proof of Proposition 14. To show that $\hat{N}_G \leq \hat{N}_H$, note that I can rewrite

$$\hat{N}_H = 1 + \frac{\log(1/\hat{\rho})}{-\log R_H},$$

where

$$R_H = \frac{1-\int_{\underline{x}}^{\overline{x}} x dH(x)}{1-\int_{\hat{\rho}}^{\overline{x}} x dH(x)} = 1 - \frac{\int_{\underline{x}}^{\hat{\rho}} x dH(x)}{1-\int_{\hat{\rho}}^{\overline{x}} x dH(x)},$$

and $\hat{N}_H$ is increasing in R_H. Integrating by parts, I obtain

$$R_H = 1 + \frac{\int_{\underline{x}}^{\hat{\rho}} H(x)dx - \hat{\rho}H(\hat{\rho})}{1-\overline{x}+\hat{\rho}H(\hat{\rho})+\int_{\hat{\rho}}^{\overline{x}} H(x)dx}.$$

And clearly, $R_G \leq R_H$ since $G(x) \leq H(x)$ on $[\underline{x}, \hat{\rho}]$ and $G(x) \geq H(x)$ on $[\hat{\rho}, \overline{x}]$. Hence $\hat{N}_G \leq \hat{N}_H$. □

Proof of Lemma 6. Let $\Omega \subseteq (\hat{\rho}, \overline{x})$ be an open interval such that $\lambda = 1$ on Ω and $x \in \Omega$. Let $y = \sup\{\rho : \forall \rho' \in [x, \rho],\ \lambda(\rho') = 1\}$. Suppose $y < \overline{x}$. By continuity of the payoff functions, it must be true that $V_D^\lambda(y) = V_H^\lambda(y)$. However, $V_D^\lambda(.)$ is strictly increasing on (x, y) while $V_H^\lambda(.)$ is constant on the same interval. Furthermore, since λ is an equilibrium strategy, $V_D^\lambda(x) > V_H^\lambda(x)$, but then by continuity $V_D^\lambda(y) > V_D^\lambda(x) > V_H^\lambda(x) = V_H^\lambda(y)$, a contradiction. □

Proof of Proposition 15. The only claim that needs to be proved is the last point of the proposition. By Proposition 13, full disclosure is not an equilibrium. No disclosure cannot be an equilibrium either as then $V_D^0(\underline{x}) = \underline{x} > 0 = V_H^0(\underline{x})$. By Lemma 6, a pure strategy equilibrium must be of the type $\lambda(\rho) = \mathbb{1}_{\rho > \rho^*}$ for some $\rho^* \in (\underline{x}, \overline{x})$. But then, $V_D^\lambda(\rho^*) = \rho^* \left(1 - \int_{\rho^*}^{\overline{x}} x dH(x)\right)^{N-1} < \left(1 - \int_{\rho^*}^{\overline{x}} x dH(x)\right)^{N-1} = V_H^\lambda(\rho^*)$ which is a contradiction since by continuity the type ρ^* should be indifferent between the two actions. $\square$

Before proving Proposition 16 I prove a few useful lemmas.

Lemma 7. *Suppose weak types are absent. There exists some N_0 such that for $N > N_0$, if the strategy λ defines a symmetric Bayesian Nash equilibrium of the disclosure game such that there exists $\rho \in Int(\Lambda^0)$, then $[\rho, \overline{x}] \subseteq \Lambda^0$.*

Proof. There must be some $\eta > 0$ such that $\Omega = [\rho, \rho + \eta) \subseteq \Lambda^0$. I can differentiate $V_D^\lambda(.)$ and $V_H^\lambda(.)$ on Ω to obtain

$$\begin{aligned}\left(V_D^\lambda\right)'(\rho) &= \left(1 - \int_\rho^{\overline{x}} x\lambda(x) dH(x)\right)^{N-1} = \frac{V_D^\lambda(\rho)}{\rho} \\ &< \frac{V_H^\lambda(\rho)}{\underline{x}},\end{aligned}$$

where the inequality is due to the fact that $\rho \in \Lambda^0$ and that λ is an equilibrium. Also,

$$\begin{aligned}\left(V_H^\lambda\right)'(\rho) &= (N-1)h(\rho)\left(\int_{\underline{x}}^\rho (1 - x\lambda(x)) dH(x) + \int_\rho^{\overline{x}} \lambda(x)(1-x) dH(x)\right)^{N-2} \\ &= (N-1)h(\rho) V_H^\lambda(\rho)^{1-\frac{1}{N-1}} \\ &\geq (N-1) m V_H^\lambda(\rho),\end{aligned}$$

where m is the lower bound of h and the last inequality obtains because V_H is always bounded above by 1.

It is clear then that for $N > 1 + \frac{1}{\underline{x}m}$, $V_H^\lambda(.)$ grows faster than $V_D^\lambda(.)$ on Ω. But since $V_H^\lambda(\rho) > V_D^\lambda(\rho)$, it means that V_H^λ must remain above V_D^λ on Ω so that the payoff from disclosing can never catch up with the payoff from withholding. This implies that no type above ρ would want to disclose with positive probability, hence $[\rho, \overline{x}] \subseteq \Lambda^0$. $\square$

Lemma 8. *For every $\varepsilon > 0$, there exists some N_1 such that if λ defines a symmetric Bayesian Nash equilibrium of the disclosure game for some $N > N_1$ such that $Int(\Lambda) \neq \emptyset$, then for every $\rho \in Int(\Lambda)$,*

$$\left|\lambda(\rho) - \frac{1}{1+\rho}\right| < \varepsilon.$$

Proof. For every $\rho \in Int(\Lambda)$, $V_H^\lambda(\rho) = V_D^\lambda(\rho)$. Since the two functions are differentiable on $Int(\Lambda)$, their derivatives must be equal as well, implying

$$(N-1)(1-\lambda(\rho))h(\rho)V_H^\lambda(\rho)^{1-\frac{1}{N-1}} = \frac{V_D^\lambda(\rho)}{\rho} + \rho^2(N-1)\lambda(\rho)h(\rho)\left(\frac{V_D^\lambda(\rho)}{\rho}\right)^{1-\frac{1}{N-1}}.$$

Noting that $V_H^\lambda(\rho) = V_D^\lambda(\rho)$, and after some algebra, I obtain

$$\frac{1}{1+\rho} - \lambda(\rho) = \frac{V_D^\lambda(\rho)^{\frac{1}{N-1}}}{(N-1)\rho(1+\rho)h(\rho)} + \frac{\rho\lambda(\rho)}{1+\rho}\left(\rho^{\frac{1}{N-1}} - 1\right).$$

Hence

$$\left|\frac{1}{1+\rho} - \lambda(\rho)\right| \leq \frac{1}{(N-1)\underline{x}(1+\underline{x})m} + \frac{\overline{x}}{1+\overline{x}}\left(1 - \underline{x}^{\frac{1}{N-1}}\right).$$

Since both terms on the right-hand side go to 0 and are independent of ρ, this proves the lemma. □

Lemma 9. *There exists some $\ell > 0$ and some N_2 such that if λ defines a symmetric Bayesian Nash equilibrium of the disclosure game for some $N > N_2$ such that $Int(\Lambda) \neq \emptyset$, then for every $\rho \in Int(\Lambda)$,*

$$1 - \ell > \lambda(\rho) > \ell.$$

Proof. This is a corollary of Lemma 8 obtained by choosing $\ell = \varepsilon = \min\left(\frac{1}{2(1+\overline{x})}, \frac{1}{2}\left(1 - \frac{1}{1+\underline{x}}\right)\right)$. □

Proof of Proposition 16. By Lemma 6 and Lemma 7, there are five possible types of symmetric Bayesian Nash equilibria when N is sufficiently high: (i) $\lambda = 1$; (ii) $\lambda = 0$; (iii) $\Lambda = [\underline{x}, y)$ and $\Lambda^1 = (y, \overline{x}]$ for some $y \in (\underline{x}, \overline{x})$; (iv) $\Lambda = [\underline{x}, y)$ and $\Lambda^0 = (y, \overline{x}]$ for some $y \in (\underline{x}, \overline{x})$; (v)

$\Lambda = [\underline{x}, \overline{x}]$. By Proposition 15, (i) and (ii) are impossible.

Suppose that there exists a sequence $(\lambda_k)_{k=1}^{\infty}$ of equilibria of type (iv) for the disclosure game with $N(k)$ candidates, where $N(k)$ increases strictly with k. To each λ_k is associated a $y_k \in (\underline{x}, \overline{x})$ and since $[\underline{x}, \overline{x}]$ is compact, I can assume, up to an extraction, that the sequence $y_{N(k)}$ converges to some $y_\infty \in [\underline{x}, \overline{x}]$. Because it is an equilibrium to withhold with the type $\overline{x}$, I can write

$$V_D^k(\overline{x}) = \overline{x} < \left(1 - \int_{\underline{x}}^{\overline{x}} x\lambda_k(x)dH(x)\right)^{N(k)-1} = V_H^k(\overline{x}).$$

Using Lemma 9, for a sufficiently high k

$$\left(1 - \int_{\underline{x}}^{\overline{x}} x\lambda_k(x)dH(x)\right)^{N(k)-1} \leq \left(1 - \ell\underline{x}H(y_k)\right)^{N(k)-1},$$

and for the right-hand side to be greater than $\overline{x}$ for every k it must be true that $y_\infty = \underline{x}$. But then, looking at the incentives of type y_k, we have

$$V_D^k(y_k) = y_k \xrightarrow[k\to\infty]{} \underline{x},$$

and

$$V_H^k(y_k) = \left(\int_{\underline{x}}^{y_k} (1 - x\lambda(x))dH(x)\right)^{N(k)-1} \leq ((1 - \ell\underline{x})H(y_k))^{N(k)-1} \xrightarrow[k\to\infty]{} 0,$$

implying that for k sufficiently high $V_D^k(y_k) > V_H^k(y_k)$ which is a contradiction since these payoffs should be equal for λ_k to be an equilibrium.

Suppose now that there exists a similar sequence $(\lambda_k)_{k=1}^{\infty}$ of equilibria of type (v). Then for k sufficiently high,

$$V_H^k(\overline{x}) = \left(1 - \int_{\underline{x}}^{\overline{x}} x\lambda_k(x)dH(x)\right)^{N(k)-1} < (1 - \ell\underline{x})^{N(k)-1} \xrightarrow[k\to\infty]{} 0.$$

Hence for k sufficiently high $V_H^k(\overline{x}) < \overline{x} = V_D^k(\overline{x})$, which contradicts the fact that λ_k is an equilibrium.

Hence, for N sufficiently large, the only possible equilibria are of type (iii). They exist by Proposition 12. Let $(\lambda_k)_{k=1}^{\infty}$ be a sequence of equilibria of this type. The payoffs of type y_k are given by

$$V_D^k(y_k) = y_k \left(1 - \int_{y_k}^{\overline{x}} x dH(x)\right)^{N(k)-1},$$

and

$$V_H^k(y_k) = \left(1 - \int_{y_k}^{\overline{x}} x dH(x) - \int_{\underline{x}}^{y_k} x\lambda_k(x) dH(x)\right)^{N(k)-1},$$

and are equal since λ_k is an equilibrium. Hence, for every $k > 0$,

$$\left(1 - y_k^{\frac{1}{N(k)-1}}\right)\left(1 - \int_{y_k}^{\overline{x}} x dH(x)\right) = \int_{\underline{x}}^{y_k} x\lambda_k(x) dH(x).$$

Since $\left(1 - \int_{y_k}^{\overline{x}} x dH(x)\right)$ is bounded and $0 < \underline{x} < y_k < \overline{x} < 1$, the left-hand side goes to 0 as $k \to \infty$. Because the right-hand side is bounded below by $\ell \underline{x} H(y_k)$, it must be true that $y_\infty = \underline{x}$. The remaining of the proposition is a consequence of Lemma 8. □

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