Competing with Equivocal Information: The Importance of Weak Candidates*

Eduardo Perez-Richet
perezedu@stanford.edu

Abstract

In the usual persuasion game framework, where an informed sender tries to persuade an uninformed receiver to take a certain action by selectively communicating verifiable information, all the relevant information is revealed in equilibrium because any action of the sender can be outguessed by the receiver. If the sender is unable to interpret her own information, however, this classical unraveling argument breaks down. When the receiver is sufficiently inclined to act as the sender wishes without any information, the sender has no incentive to inform her. This paper examines whether full disclosure can be restored with competition between multiple senders. In the model, the senders compete for a limited number of prizes allocated by the receiver. Full disclosure can be restored only in the presence of weak candidates, that is ex ante unpromising candidates. With sufficiently many weak candidates, it is always possible to ensure full disclosure.
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1 Introduction

Economic agents sometimes control the access of others to information but are not able to predict others' reactions to it. A climate expert may understand the environmental effects of a particular emission reduction policy, but lack the economic and political expertise to apprehend its electoral value to those in charge of approving it. A movie producer may find it impossible to predict how the information conveyed in a trailer will affect the willingness of any particular

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consumer to watch a movie, or an advertiser or a search engine to know how the information contained in a sponsored link will influence a particular consumer.

In spite of its opacity, the control of the information can grant its gatekeeper some power over the decisions of other parties. A job candidate with a good resume, for example, is unlikely to reveal additional information about herself in a statement of purpose. Since it is both difficult to appreciate how such information will be interpreted by the employer and easy to make the statement of purpose deliberately vague, a candidate who thinks that she will be hired on the basis of her resume alone will not communicate potentially detrimental information. Furthermore, common knowledge that information is equivocal to the candidate prevents the employer from drawing any unfavorable inferences from her behavior. Conversely, a candidate with a weaker resume has to provide as much additional information as possible in order to sway the employer's decision. Similarly, this analysis suggests that the best strategy to advertise a movie from a popular director is to keep the trailer elliptic and mysterious, while the trailer of a movie from an unknown director will feature all its best scenes in order to attract audiences.

Such reasoning on the part of a job candidate or a movie producer should, however, be altered by the presence of competitors. If an *ex ante* weaker job candidate has to provide more information about herself in order to stay in the race, this behavior could make her stronger than other candidates *ex post*, compelling *ex ante* stronger candidates to disclose more information as well, so as not to run the risk of losing their initial advantage. Thus, intuitively at least, the forces of competition may be expected to mitigate the *real authority*[1] of *ex ante* strong candidates over the employment decision and lead to more disclosure.

One of the main results of the literature on persuasion games, as most generally stated in Milgrom and Roberts (1986), is the identification of a set of assumptions that, by ensuring skepticism on the part of the receiver, leads to efficient provision of information by the sender. Such skepticism occurs when the receiver (buyer, decision maker) is capable of strate-

[1]As defined by Aghion and Tirole (1997), real authority is the effective control over decisions as determined by the information structure, rather than by the formal right to decide.

gic reasoning, informed about the interests of the sender (seller, candidate) and aware of the type of information that is available to the sender. Milgrom and Roberts (1986) and Milgrom (2008) argue that, even when these assumptions fail, competition among senders can sometimes provide an (imperfect) substitute and lead to more disclosure. My analysis of the single-sender/single-receiver case points at another important assumption in the usual persuasion games: that the sender is able to anticipate the impact of the information she holds on the receiver. Interestingly, the inability of the sender to interpret her information is an advantage as, by effectively eliminating the asymmetry of (interpretable) information between the sender and the receiver, it renders the observable actions of the sender completely uninformative for the receiver. This allows the sender to fully benefit from the control she exerts on information[2].

This paper investigates the effects of competition in a theoretical framework designed to fit several economic situations. Several candidates with heterogeneous prospects compete for a limited number of homogeneous prizes or slots (jobs, funding or political clearance to implement a project or a policy, purchase decisions of a buyer). The model incorporates the search strategy of the agent who decides how to allocate the slots (the *decision maker*). A candidate's *prospect* is her probability of being a good fit. I analyze the case in which the prospects of all candidates are common knowledge among them, as well as the case in which candidates know only their own prospect but not others' (asymmetric information among candidates).

The main finding is that in either case sufficient competition leads to full disclosure (*i.e.* disclosure by all candidates) only if some of them are *weak* (their prospects are sufficiently low that approving them without further investigation would be wasteful in expectation). This result emphasizes the importance of weak candidates in this type of contests. It may have policy implications for the preselection of pools of candidates in procurement contests or when hiring. The results can also be applied to the disclosure or information to buyers in a market. A market with strong competitors[3] only may harm the consumer by limiting disclosure. For instance, a

[2]Note that the single-sender/single-receiver case used as a benchmark in this paper is analyzed in Caillaud and Tirole (2007).

[3]The model will make the meaning of strong clearer. For this discussion, it means competitors whose products are sufficiently likely to be satisfactory to the consumer that she would make the purchase in the absence of

horizontal merger between two weak competitors is often considered to be pro-competitive if it creates a stronger player able to compete more aggressively with other strong players. The model suggests that such a merger may harm the consumer by reducing her information. The model is too limited to be used as a policy guide for mergers, but the consequences of horizontal mergers on information provision may be worth considering. Yet this aspect of mergers is not even mentioned in the 1992 Horizontal Merger Guidelines[4] jointly issued by the Federal Trade Commission and the Department of Justice.

The cost of processing or acquiring information can have important consequences in many environments. In the examples above, it is often costly to process additional information about candidates, even when they make this information available. If an engineer's project has sufficiently good prospects, the CEO of a company may be willing to fund it without further assessments because learning about these details would be too costly, or she may be constrained to *rubberstamp* the project in the absence of more detailed information that she would have preferred to consult. In the presence of many candidates with different prospects, about which additional information may or may not be available, and may be costly to process when available, the decision maker faces a complex search problem. For example, the presence of a second candidate with a sufficiently strong prospect, by changing the outside option of the decision maker, may make it valuable to conduct a further assessment that would otherwise have been wasteful. In the absence of processing costs, a decision maker would optimally start by processing all available information, before rubberstamping any project. A theoretical contribution of this paper is to provide conditions on the prospects under which she adopts a similar behavior in the presence of processing costs, and to characterize an optimal search algorithm for the decision maker in such cases, for any number of available slots. However, the presence of small processing costs also induces an optimal order in the treatment of available information: in order to save on processing costs, it is optimal to process projects with better prospects first. Therefore the set of optimal policies in the presence of these costs, even as they

additional information, even though she would prefer to consult this additional information.

[4] Available at http://www.ftc.gov/bc/docs/horizmer.htm.

go to zero, is a subset of the policies that are optimal when processing is costless.

In the model, a project is either good or bad for the decision maker. Each of the candidates has information that would allow the decision maker to perfectly figure out the value of her project, but is unable to process this information in the decision maker's place and therefore to anticipate its effect on her. A candidate decides whether to provide documentation about her project. The decision maker can process each piece of information at a cost. If the processing cost is sufficiently low, I show that it is an optimal policy for the decision maker to first learn sequentially about all the documented projects, and allocate a slot to each project that is found to be good. In this processing phase, she examines the projects in the order of decreasing prospects as long as there remains some slots to allocate. She starts allocating slots to undocumented projects only after having examined all the documented projects, even those less promising than the non-documented ones. Hence, by withholding information, a candidate loses her priority in the choice process of the decision maker. It might, however, still be beneficial to do so in equilibrium if the probability that the decision maker can find sufficiently many good projects in the set of documented projects is low . As for the decision maker, she is clearly best of when all information is made available to her since it expands her choice set.

When candidates are perfectly informed about their opponents, I show that there is no equilibrium with full disclosure in the absence of weak candidates. With one slot, increasing the number of weak candidates and improving their initial prospects both make full disclosure more likely. Furthermore, in the game with a single slot to fill, when a pure strategy equilibrium exists, it is unique. In such an equilibrium, at most one candidate withholds information. I show that increasing the number of weak candidates or improving their initial prospects always leads to a weaker candidate withholding information, to the benefit of the decision maker. I also analyze the bayesian game with imperfect information of the candidates about the prospects of their peers. Full disclosure never obtains in the absence of weak types, but is the unique equilibrium in their presence whenever there are sufficiently many candidates.

Related Literature. There is a large economic literature on the strategic communication of information that distinguishes between soft information (Crawford and Sobel (1982)), and hard information (Grossman (1981), Grossman and Hart (1980), Milgrom (1981)). The literature on *persuasion games*[5] studies the case of hard (certifiable) information in problems with a single sender trying to persuade a single receiver to take a certain action. For example, a seller tries to influence the decision of a buyer with verifiable information. I also focus on hard information[6].

Caillaud and Tirole (2007) analyze a single-sender/multiple-receivers model from a mechanism design perspective in order to understand optimal persuasion strategies when decisions affecting the sender are made by a committee under a qualified majority rule, with obvious political economy applications to lobbying situations. I analyze a multiple-senders/single-receiver version of the same benchmark model from a game-theoretic perspective to explore the effects of competition. Competition is an important feature of lobbying, and as such this paper is a contribution to a growing literature (e.g. Gul and Pesendorfer (2007)) that views lobbying as a competition in the provision of information.

The assumption that an economic agent can control access to information that she cannot process plays an important role in several other recent papers than Caillaud and Tirole (2007). Eso and Szentes (2003) propose an agency model where the principal can release, but not observe, information that would allow the agent to refine her knowledge of her own type. They show that when the full mechanism design problem is considered altogether, the optimal mechanism calls for full disclosure and allows the principal to appropriate the rents of the information she controls exactly as if it were observable to her. Eso and Szentes (2007) develop an auction model in which similar conclusions hold, reversing the results of earlier auction papers that considered the problem of disclosure and the design of the selling mechanism separately (Bergemann and Pesendorfer (2007)).

[5]For a review of this literature see Milgrom (2008). Sobel (2007) summarizes the literature on information transmission.

[6]The distinction between soft and hard information is less meaningful in the context of this paper, since it is not clear how and why the sender would falsify information that she cannot interpret. In this light, it seems natural to assume non-falsifiable information, as implicitly done in this paper.

This paper is also connected to the economic literature on advertising. It makes predictions about the relationship between product quality and the informativeness of advertising. This question is connected to the analysis of the relationship between product quality and levels of advertising in the literature. As summarized in Bagwell (2007), the empirical literature on the topic does not strongly support a systematic positive relationship. Bagwell and Overgaard (2006) and Bar-Isaac, Caruana and Cuñat (2008) offer possible theoretical explanations for a negative relationship. To the extent that the quantity of advertising is an acceptable measure of its informativeness, this paper offers an alternative and simple theoretical explanation, in the case of a monopolist. Furthermore, it makes it possible to analyze the case of competition which was not done with other models in the literature. This topic is discussed further in Section 6.

2 The Model

2.1 Setup

For clarity, the model is described in the language of project adoption, although it fits other situations as well. Finitely many candidates with a single project, indexed by the set $\mathcal{N} = \{1, \ldots, N\}$, seek to maximize the probability that their project adopted by a *decision maker* who can implement only $M \leq N$ of them. A project is either good or bad for the decision maker. A good project yields an expected gain $G > 0$ for the decision maker, whereas a bad project yields an expected loss $L > 0$[7].

All the players share the belief that the projects are of independent values from one another, and assign probability $\rho_n \in (0, 1)$ to the event that project n is good[8] . Without loss of generality[9], $\rho_1 > \cdots > \rho_N$. I refer to the order that underlies this ranking as the *strength*

[7]In the multi-seller/buyer interpretation of the model, projects are items for sale to a seller with demand for a fixed quantity M and these payoffs implicitly assume away any price heterogeneity across sellers.

[8]This assumption is relaxed in Section 5, where the prospect of a project is known by its sponsor and the decision maker, but not by other candidates.

[9]There is in fact a small loss of generality since ties are ruled out, but this is a measure 0 event as long as the probability profile is drawn from an atomless joint distribution on $[0, 1]^N$.

order on projects. Each candidate n controls information that would allow the decision maker to figure out the value of project n but are irrelevant to other projects. However, the candidate is unable to process this information[10]: she can only decide whether to communicate it to the decision maker, who can then process it at a cost $c > 0$. Investigation by the decision maker is not contractible. A project whose information is made available to the decision maker is said to be *documented.*

The timing is as follows. First, the candidates decide simultaneously whether to disclose their information. Then the decision maker decides which information to process and which projects to approve.

2.2 Assumptions and Notations

Assumptions. Approving a project with prospect ρ without learning about it provides the decision maker with an expected incremental payoff $\rho(G + L) - L$, whereas processing and conditionally approving the project gives the expected incremental payoff $\rho G - c$. Let $\underline{\rho} = c/G$, $\hat{\rho} = L/(L + G)$ and $\overline{\rho} = 1 - c/L$.

Assumption 1 (Affordable Learning **(AL)**). *The processing cost is sufficiently low to ensure that learning can take place*

$$c < LG/(L + G). \tag{1}$$

(AL) ensures that $\underline{\rho} < \hat{\rho} < \overline{\rho}$. It is easy to see (Figure 1) that the interval (0,1) can then be partitioned in four intervals such that:

(i) if $\rho \in (0, \underline{\rho})$, the project is not worth considering for either immediate (*i.e.* rubberstamping) or conditional approval (*i.e.* after investigation);

(ii) if $\rho \in (\underline{\rho}, \hat{\rho})$, the project only deserves conditional approval, but rubberstamping is wasteful;

[10] Alternatively, I could assume that the candidates must commit to a communication decision before observing the value of their project.

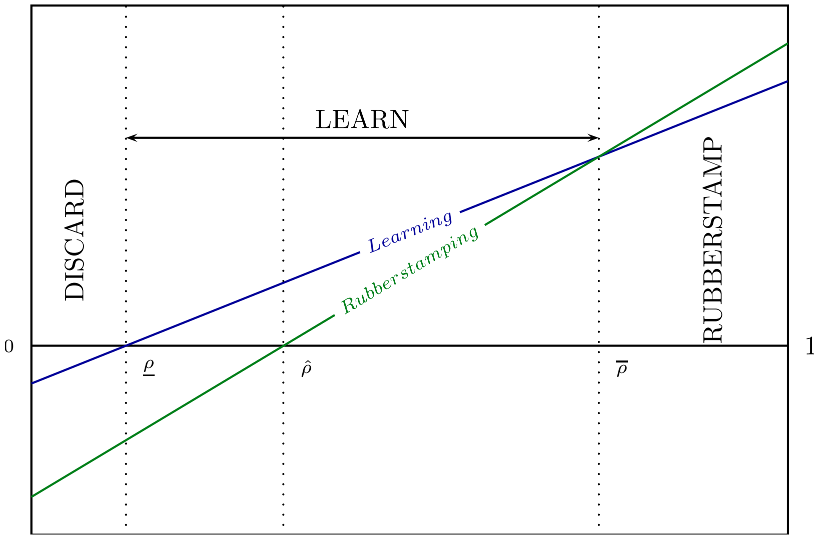

Figure 1: **D**iscarding, Learning and Approving with One Candidates: Payoffs.

(iii) if $\rho \in (\hat{\rho}, \overline{\rho})$, the first-best option is conditional approval, but rubberstamping beats mere rejection;

(iv) if $\rho > \overline{\rho}$, rubberstamping is the first-best option: learning about the project would be wasteful.

In this light, (AL) simply says that at least some projects are worth processing.

Notations. For any subset $\mathcal{J} \subseteq \mathcal{N}$, denote its cardinality by J, and let $j(1) < \cdots < j(J)$ be the ordered elements of this subset, so that $\rho_{j(1)} > \cdots > \rho_{j(J)}$ gives the decision maker's *ex ante* preference order over this subset of projects.

Definition 1 (Truncated Subsets)**.** *For any subset $\mathcal{J} = \{j(1), \ldots, j(J)\} \subseteq \mathcal{N}$ and any $k < J$, let $\mathcal{J}^-(k) \equiv \{j(1), \ldots, j(k)\}$ and $\mathcal{J}^+(k) \equiv \{j(k+1), \ldots, j(J)\}$ be the left and right truncations of $\mathcal{J}$ at k. Also for $1 \leq k < k + r \leq J$, write $\mathcal{J}(k, k + r) = \{j(k + 1), \ldots, j(k + r)\}$. By convention, $\mathcal{J}^-(0) = \mathcal{J}^+(J) = \emptyset$.*

For a project $n \in \mathcal{N}$, and a subset of projects $\mathcal{J} \subseteq \mathcal{N}$, let $r_{\mathcal{J}}(n)$ be the *rank* of n in $\mathcal{J}$. This does not require n to be an element of $\mathcal{J}$: if n is not in $\mathcal{J}$ then $r_{\mathcal{J}}(n)$ is the rank that n would have in $\mathcal{J} \cup \{n\}$. For example, if $\mathcal{N}$ consists of three projects 1, 2 and 3 such that $\rho_1 > \rho_2 > \rho_3$

and $\mathcal{J} = \{1, 3\}$, then $r_{\mathcal{J}}(3) = r_{\mathcal{J}}(2) = 2$ as project 3 is the second strongest project in $\mathcal{J}$ and project 2 would be the second strongest project in $\mathcal{J} \cup \{2\}$.

For comparisons between sets of projects, I will use the usual *set containment order* $\subset$, as well as the *strength order* for sets of the same cardinality, as defined below.

Definition 2 (Strength Order on Sets)**.** *For two sets of projects $\mathcal{K}, \mathcal{K}' \subseteq \mathcal{N}$ with the same cardinality $K = K'$, $\mathcal{K}$ is stronger than $\mathcal{K}'$, denoted $\mathcal{K} > \mathcal{K}'$ if for every $\kappa = 1, \ldots, K$, $\rho_{k(\kappa)} \geq \rho_{k'(\kappa)}$, with at least one of these inequalities holding strictly.*

3 The Decision Maker's Choice[11]

3.1 Game Reduction and an Additional Assumption

Game Reduction. As a preliminary, notice that projects with $\rho < \underline{\rho}$ will never be considered for either processing or approval. Indeed, since such a project n's prospect satisfies $\rho_n < c/G < L/(L+G)$, the expected incremental payoff from examination, $\rho_n G - c$, is negative, as is the expected incremental payoff from rubberstamping $\rho_n(G+L) - L$. The presence of these projects is also irrelevant to the communication game, since they have no effects on the payoffs of other candidates.

In the remainder of the paper, I therefore assume without loss of generality that there are no such projects: $\rho_n > \underline{\rho}$ $(\forall n \in \mathcal{N})$. I also assume, though not without loss of generality, that no candidate is sufficiently strong to make the incremental payoff from learning less than the incremental payoff from rubberstamping[12]: $\rho_n < \overline{\rho}$, $(\forall n \in \mathcal{N})$. Let $\mathcal{N}_W \equiv \left\{n \in \mathcal{N}; \underline{\rho} < \rho_n < \hat{\rho}\right\}$ be the set of *weak candidates* or *weak set*, and define $\mathcal{N}_S \equiv \left\{n \in \mathcal{N}; \hat{\rho} < \rho_n < \overline{\rho}\right\}$ as the set of *strong candidates* or *strong set*.

[11]See Appendix A for proofs that are not in the text.

[12]It is intuitive that projects with very high prospects can be eliminated without loss of generality because they would be rubberstamped without affecting the rest of the game. This intuition is addressed in Proposition 11 of Section 4.4 for the single slot case, showing that when the prospect of the best project *ex ante* exceeds a certain threshold $\rho^+ > \overline{\rho}$, project 1 is rubberstamped in equilibrium.

Additional Assumption. In the remainder of the paper, I also make the following assumption to simplify the analysis.

Assumption 2 (Learning Priority **(LP)**).
Whenever $N \geq 2$,

$$\rho_N(1-\rho_1) > c/(L+G) \tag{2}$$

This assumption is satisfied if $\mathcal{N}_S = \emptyset$ or if $\mathcal{N}_W = \emptyset$, but it is not satisfied in general. If, for example, $\rho_1 \simeq \overline{\rho} = 1 - c/L$ and $\rho_N \simeq \underline{\rho} = c/G$, then $\rho_N(1-\rho_1) \simeq c^2/(LG) < c/(L+G)$, where the last inequality is a consequence of (AL). Note that the assumption is satisfied when learning is costless for the decision maker ($c = 0$). (LP) can be interpreted as bounding processing costs above, or as ruling out extreme prospects.

It is called a learning priority assumption because it implies, a decision maker with one slot to fill and a pool of projects reduced to the best and the worst projects from the original pool would always choose to process all available information before rubberstamping a project. Indeed, suppose the decision maker has received documentation about N only, and is therefore contemplating two choices: either rubberstamping 1 directly obtaining the payoff $\rho_1(G+L)-L$, or processing the information of N first and rubberstamping 1 if she finds that N is a bad project, yielding $\rho_N G + (1-\rho_N)$. The latter dominates the former if and only if (LP) is satisfied. Proposition 1 in the next section shows that this assumption ensures that the decision maker prioritizes learning in any situation.

3.2 Discarding, Learning, Approving

The only variable relevant to a decision maker with M slots to fill and a pool of candidates $\mathcal{N}$ is her learnable set: the subset of projects that are documented. Let $\mathcal{I} \subset \mathcal{N}$ denote this subset, and $\mathcal{H} = \mathcal{N} \setminus \mathcal{I}$. $\mathcal{I}$ is called the *information set* of the decision maker, while $\mathcal{H}$ is her *hidden set*. The partition $[\mathcal{I}, \mathcal{H}]$ of the set of projects $\mathcal{N}$ is her *learning partition*. The set of expected payoffs that she can reach given any policy is completely characterized by the triple $(\mathcal{I}, \mathcal{H}, M)$, which consists of the two sets of her learning partition and the number of available slots. Let $\mathcal{H}_W \equiv \mathcal{H} \cap \mathcal{N}_W$, $\mathcal{H}_S \equiv \mathcal{H} \cap \mathcal{N}_S$, $\mathcal{I}_W \equiv \mathcal{I} \cap \mathcal{N}_W$ and $\mathcal{I}_S \equiv \mathcal{I} \cap \mathcal{N}_S$ denote the weak and strong subsets of the two elements of the learning partition.

The policies available to the decision maker can be described as finite sequences of processing and approval decisions with bounded lengths. Hence the decision maker's problem is to maximize a function over a finite set: this implies the existence of an optimal policy. Because I excluded projects for which it is not optimal to learn on an individual basis ($\rho < \underline{\rho}$ or $\rho > \overline{\rho}$), any policy is equivalent to a sequential policy where, at each stage, the decision maker can either approve a project from $\mathcal{H}$ or process and conditionally approve a project from $\mathcal{I}$[13]. A policy in state $(\mathcal{I}, \mathcal{H}, M)$ is therefore well-described by a vector $\pi = \left(\pi_d\right)_{1 \leq d \leq D}$ of dimension $D \leq N$, listing elements of $\mathcal{N}$ in the order of their examination, where it is understood that if $\pi_d \in \mathcal{I}$, the policy involves processing π_d at cost c, conditionally approving it, and then moving on to π_{d+1}, while if $\pi_d \in \mathcal{H}$, the policy consists in rubberstamping π_d at step d and then moving on to step $d + 1$.

Let $\Pi(\mathcal{I}, \mathcal{H}, M)$ denote the set of optimal policies at $(\mathcal{I}, \mathcal{H}, M)$ and let $V(\mathcal{I}, \mathcal{H}, M)$ be the maximum achievable payoff. It is intuitive that, at each stage, the decision maker is best off by choosing between the strongest project in $\mathcal{H}$ and the strongest project in $\mathcal{I}$. Hence in each state $(\mathcal{I}, \mathcal{H}, M)$, the choice of the decision maker can be described as a choice between (i) approving $h(1)$ and moving on to the state $(\mathcal{I}, \mathcal{H}^+(1), M - 1)$; or (ii) processing $i(1)$, then approving it if it is good and moving on to the state $(\mathcal{I}^+(1), \mathcal{H}, M - 1)$, or simply moving on to the state $(\mathcal{I}^+(1), \mathcal{H}, M)$ if $i(1)$ is bad. Of course there is also the option to rubberstamp a project in $\mathcal{I}$, but processing it is always better because of the assumption that all projects have prospects below $\overline{\rho}$. The following lemma presents some useful remarks about optimal policies.

Lemma 1.

(i) Projects in $\mathcal{H}_W$ are optimally discarded and only the first M projects in $\mathcal{H}_S$ are ever

[13]In principle, the decision maker could delay approval after observing that a project is of good value, but there is no advantage in doing so since all projects of the same value yield equal expected payoffs.

considered: $\Pi(\mathcal{I},\mathcal{H},M)=\Pi(\mathcal{I},\mathcal{H}_S^-(M),M)$.

(ii) If $M>I$ *it is optimal to rubberstamp the first* $K=\min(M-I,H_S)$ *projects in* $\mathcal{H}_S$*: any policy in* $\Pi(\mathcal{I},\mathcal{H},M)$ *is a policy resulting from the combination of a policy* $\pi\in\Pi(\mathcal{I},\mathcal{H}_S^+(K),\max(I,M-H_S))$ *with the rubberstamping of every project in* $\mathcal{H}_S^-(K)$ *in any order and at any point in the sequence.*

(iii) If $M>H_S$*, there is an optimal policy that consists in filling as many of the first* $M-H_S$ *slots as possible with projects in* $\mathcal{I}$ *that are found to be good after processing, and then solving for the continuation problem.*

The next result shows that an optimal policy for the decision maker is to start by learning about all the documented projects, and then to fill the remaining slots by rubberstamping strong undocumented projects with positive expected incremental payoff in the order of their *ex ante* ranking. Hence, withholding information implies losing one's *ex ante* priority in the order of attribution, generating a cost to non-disclosure.

Proposition 1. *Given any triple* $(\mathcal{I},\mathcal{H},M)$*, the procedure described below is an optimal policy for the decision maker.*

An Optimal Policy	
Step 1	*Process and conditionally approve all projects in* $\mathcal{I}$ *sequentially in the increasing order* ($i(1)\to i(2)\to\dots$)*, as long as there are some empty slots.*
Step 2	*Fill the* $m\geq 0$ *remaining slots after step 1 with the* $\min\{m,H_S\}$ *strongest projects in* $\mathcal{H}_S$*.*

Furthermore, given $\mathcal{I}$,$\mathcal{H}$ *and* M*, the probability that a given project is implemented is invariant across all optimal policies of the decision maker.*

The proof of the proposition consists in showing by a double induction on M and N that the result of LP extends to more candidates and more slots. Induction works because of the recursive nature of the decision maker's problem, as is usual in search models.

As c approaches 0, any pool of projects satisfies the assumptions of the model ensuring that the policy is an optimal one. When it is costless to process information, it is natural for the decision maker to prioritize learning. The proposition says that this priority is maintained for sufficiently small processing costs where small is defined by (LP). Note, however, that when $c = 0$, the order in which the documented projects are processed is irrelevant to the decision maker. Therefore, while the policy of the proposition is still an optimal one, it is not true anymore that the probability that a given project is implemented is invariant across all the optimal policies. For instance, another optimal policy would be to process all the documented and then implement as many of the good ones as possible, using a randomization device if their number is greater than the number of slots. This implies that different optimal policies give different incentives to the candidates for the communication game when $c = 0$. Taking the limit as c goes to 0 of the equilibria analyzed below provides a method for selecting equilibria in the game with $c = 0$.

As a consequence of the proposition, the probability that a project is implemented therefore depends on the probabilities of finding good projects in subsets of $\mathcal{I}$. Hence it is useful to introduce the following notations. For a subset $\mathcal{K}$ of $\mathcal{I}$, where $f(p, \mathcal{K})$ denotes the probability of finding exactly p good projects in $\mathcal{K}$, let

$$F(p, \mathcal{K}) \equiv \sum_{q=0}^{p} f(q, \mathcal{K})$$

be the probability that there are fewer than p good projects in $\mathcal{K}$. These probabilities can be expressed as follows:

$$f(p, \mathcal{K}) = \sum_{\substack{\mathcal{J} \subset \mathcal{K} \\ J = p}} \prod_{j \in \mathcal{J}} \prod_{l \in \mathcal{K} \setminus \mathcal{J}} \rho_j (1 - \rho_l), \tag{3}$$

and

$$F(p, \mathcal{K}) = \sum_{\substack{\mathcal{J} \subset \mathcal{K} \\ J \leq p}} \prod_{j \in \mathcal{J}} \prod_{l \in \mathcal{K} \setminus \mathcal{J}} \rho_j (1 - \rho_l). \tag{4}$$

$F(p,\mathcal{K})$ is obviously increasing in p. It is also decreasing in $\mathcal{K}$ for the set containment order and decreasing in ρ_k for any $k \in \mathcal{K}$, as (6) shows. It is therefore decreasing in $\mathcal{K}$ for the strength order. Intuitively, adding new candidates to a pool or increasing the probability that any project already in the pool is of good value reduces the probability that there are fewer than p good projects in the pool, that is, it increases the probability that at least p projects in the pool are good. These results are direct consequences of the following lemma.

Lemma 2. *For any $\mathcal{K} \subseteq \mathcal{N}$, $k \in \mathcal{K}$, $p \in \{1, \cdots, K\}$ and $q \leq K$,*

$$f(p,\mathcal{K}) = \rho_k f(p-1, \mathcal{K} \smallsetminus \{k\}) + (1-\rho_k) f(p, \mathcal{K} \smallsetminus \{k\}), \tag{5}$$

and

$$F(q,\mathcal{K}) = F(q, \mathcal{K} \smallsetminus \{k\}) - \rho_k f(q, \mathcal{K} \smallsetminus \{k\}). \tag{6}$$

Another useful property of these probabilities is given in the following lemma. It says that the probability of finding at least p good projects in a given initial set becomes higher after having made k good picks and no bad picks.

Lemma 3. *For fixed $p > 0$ and $\mathcal{J} \subseteq \mathcal{N}$, and any subset of projects $\mathcal{K} \subseteq \mathcal{N}$ such that $\mathcal{J} \cap \mathcal{K} = \emptyset$ and $0 < K < p$,*

$$F(p, \mathcal{J} \cup \mathcal{K}) > F(p-K, \mathcal{J}) \tag{7}$$

With these notations, and as a corollary of Proposition 1, the following expressions can be given to the probability that a project is implemented by the decision maker

Corollary 1. *The probability that $i(k)$, the k-th project in $\mathcal{I}$, is implemented by an optimizing decision maker is equal to*

$$F(M-1, \mathcal{I}^{-}(k-1))\rho_{i(k)},$$

and the probability that $h(k)$, the k-th project in $\mathcal{H}$, is implemented is equal to

$$F(M-k, \mathcal{I}) \cdot \mathbb{1}_{h(k) \in \mathcal{H}_S}.$$

These formulas give the payoffs of the candidates when the decision maker acts optimally.

3.3 Implied Preferences for the Decision Maker

Proposition 1 implies the following expression for the expected payoff of the decision maker $V(\mathcal{I},\mathcal{H},M)$, where $M' = \min(M, H_S)$

$$V(\mathcal{I},\mathcal{H},M) = (1 - F(M-1,\mathcal{I}))MG + G\sum_{p=1}^{M-1} pf(p,\mathcal{I})$$
$$-c\sum_{q=0}^{I-1} F(M-1,\mathcal{I}^{-}(q)) + \sum_{r=1}^{M'} F(M-r,\mathcal{I})\left(\rho_{h(r)}(G+L) - L\right). \tag{8}$$

The first term measures the payoff from implementing M good projects, weighted by the probability $1 - F(M-1,\mathcal{I})$ of finding them. The second term measures the expected payoff obtained when fewer than $M-1$ good projects are found in $\mathcal{I}$. The third term measures the expected cost of the search in $\mathcal{I}$. If fewer than $M-1$ projects are found among the first $q < I$ projects in $\mathcal{I}$, at least one more project has to be investigated at the cost of c. Finally, the last term measures the payoff from filling with projects in $\mathcal{H}_S$ the slots that are still unallocated after the search in $\mathcal{I}$.

Proposition 2. *Consider two information sets $\mathcal{I}_0 \subseteq \mathcal{I}_1 \subseteq \mathcal{N}$. Then $V(\mathcal{I}_1,\mathcal{H}_1,M) \geq V(\mathcal{I}_0,\mathcal{H}_0,M)$.*

The intuition (and the proof) is that a larger information set gives more options to the decision maker who can process, rubberstamp or discard any project in her information set while she can only rubberstamp or discard other projects.

Proposition 3. *When $M = 1$, a decision maker who can choose from which candidate to get information between a stronger and a weaker one always opts for the stronger one.*

Consider an exchange of the following type: for a fixed set of projects $\mathcal{N}$ take two projects n and m, with $n < m$, so that $\rho_n > \rho_m$, and let $[\hat{\mathcal{I}},\hat{\mathcal{H}}]$ be a partition of $\mathcal{N} \setminus \{n,m\}$. Let

$\mathcal{I}_0 = \hat{\mathcal{I}} \cup \{m\}$, $\mathcal{H}_0 = \hat{\mathcal{H}} \cup \{n\}$, $\mathcal{I}_1 = \hat{\mathcal{I}} \cup \{n\}$ and $\mathcal{H}_1 = \hat{\mathcal{H}} \cup \{m\}$. Then $[\mathcal{I}_0, \mathcal{H}_0]$ and $[\mathcal{I}_1, \mathcal{H}_1]$ are both partitions of $\mathcal{N}$ that are obtained from one another by exchanging the roles of n and m, so that $\mathcal{I}_1 > \mathcal{I}_0$ and $\mathcal{H}_1 < \mathcal{H}_0$. The decision maker of the proposition is asked to choose between $(\mathcal{I}_1, \mathcal{H}_1)$ and $(\mathcal{I}_0, \mathcal{H}_0)$, and the proposition says that it is optimal to choose $(\mathcal{I}_1, \mathcal{H}_1)$, that is $V(\mathcal{I}_1, \mathcal{H}_1, 1) \geq V(\mathcal{I}_0, \mathcal{H}_0, 1)$.

Note that, maybe surprisingly, the result does not hold in general for $M > 1$, as the following example shows.

Example 1. Consider the case of three strong candidates for two slots. If the two most promising candidates disclose their information while the third one does not, the payoff of a decision maker is $V_1 = 2\rho_1\rho_2 G + G(\rho_1(1-\rho_2) + \rho_2(1-\rho_1)) - 2c + (1-\rho_1\rho_2)(\rho_3(G+L) - L)$. The first term gives the payoff of the decision maker if the search in her information set is fully successful weighted by the probability of such a success; the second term is the weighted payoff of the search when it is only partially successful, the third term is the cost of the search, it is $2c$ for sure since there are two slots to fill and only two candidates in the information set; the last term is the weighted payoff from rubberstamping the third project in case the search is not fully successful. The decision maker's payoff in the case that the first and the third candidate disclose their information while the second does not is obtained by symmetry $V_2 = 2\rho_1\rho_3 G + G(\rho_1(1-\rho_3) + \rho_3(1-\rho_1)) - 2c + (1-\rho_1\rho_3)(\rho_2(G+L) - L)$. Substracting the first expression from the second one yields $V_2 - V_1 = (1-\rho_1)(\rho_2 - \rho_3)L > 0$ and this proves that in this example the decision maker prefers to get information from candidate 3 than from candidate 2 when candidate 1 is disclosing her information anyway. In fact, it is easy to show that with 3 strong candidates and two slots, the decision maker always prefers to obtain her information from weaker candidates.

4 The Simultaneous Communication Game[14]

4.1 Benchmark: One Candidate

This benchmark case is also used as a benchmark in Caillaud and Tirole (2007), which studies the problem of decision making by a committee with a single candidate. The only problem for the decision maker is to know whether project 1 is worth implementing. Let ρ be the prospect of the single project. If $\rho > \hat{\rho}$ the decision maker accepts the project based on her prior. She would, however, be willing to get more information about the project when $\rho G - c > \rho G - (1-\rho)L$, or equivalently $\rho < \overline{\rho}$. If on the other hand $\rho < \hat{\rho}$, the decision maker is willing to acquire information about 1 if $\rho G - c > 0$, *i.e.* $\rho > \underline{\rho}$.

Proposition 4 (Caillaud and Tirole (2007))**.** *If $\rho < \underline{\rho}$, the project is refused without examination, if the project is in the weak set, it is examined and approved if it is good, while if $\rho > \hat{\rho}$ it is rubberstamped by the decision maker. If however the project is in the strong set ($\rho \in (\hat{\rho}, \overline{\rho})$), the decision maker would prefer to examine the project, but the candidate is not willing to disclose her information. The expected payoff of the decision maker is given by*

$$\Big(\rho G - c\Big)\mathbb{1}_{\rho\in\mathcal{N}_W} + \Big(\rho(L+G) - L\Big)\mathbb{1}_{\rho\in\mathcal{N}_S}.$$

The result of Proposition 4 is illustrated in Figure 2. The candidate has real authority over the final decision when $\rho \in (\hat{\rho}, \overline{\rho})$. This generates a non-monotonicity in the expected payoff of the decision maker as a function of ρ.

4.2 Multiple Candidates

An action profile is equivalent to a partition $[\mathcal{I}, \mathcal{H}]$ of $\mathcal{N}$. Given a project $n \in \mathcal{N}$, denote by $[\mathcal{I}, \mathcal{H}]_{-n}$ an action profile of all the candidates except n. It is a partition of $\mathcal{N} \setminus \{n\}$. Since, by Lemma 1, any project in $\mathcal{H}_W$ is discarded by the decision maker, a candidate $n \in \mathcal{N}_W$ is

[14] Proofs that are not in the text can be found in Appendix B

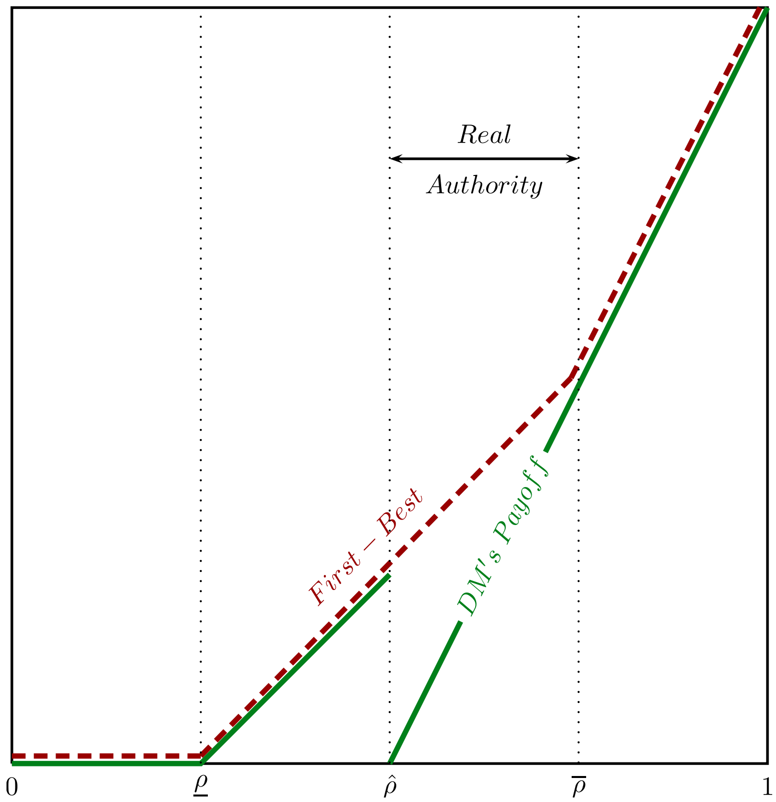

Figure 2: **Payoff of the Decision Maker with One Project.**

certain that her project stands no chance of being implemented if she refuses to communicate her information. Were she, on the other hand, to disclose this information, and given any action profile $[\mathcal{I},\mathcal{H}]_{-n}$ of the other candidates, that she would face a probability of adoption given by

$$F\Big(M-1,\mathcal{I}^{-}\big(r_{\mathcal{I}}(n)-1\big)\Big)\rho_n > 0.$$

Hence:

Remark 1. *It is a dominant strategy for candidates in $\mathcal{N}_W$ to disclose their information.*

Therefore, in any equilibrium $\mathcal{H} = \mathcal{H}_S \subseteq \mathcal{N}_S$, and $\mathcal{N}_W \subseteq \mathcal{I}$. Note that the argument of the proof relies on the fact that disclosing information always yields a positive probability of implementation. Withholding information, on the other hand, yields a null probability of implementation for all but the first M projects in $\mathcal{H}_S$. The consequence of this observation is that in any equilibrium $H \leq M$, for otherwise project $h(M+1)$ would stand no chance of being implemented and the candidate would benefit by disclosing her information.

Remark 2. *Any equilibrium action profile $[\mathcal{I},\mathcal{H}]$ satisfies $H \leq M$.*

A closely related result is that, when $H = M$, no candidate weaker than $h(M)$ has any incentive to hide information.

Remark 3. *Given any action profile $[\mathcal{I},\mathcal{H}]$ such that $H = M$, a candidate $n \in \mathcal{I}$ such that $r_{\mathcal{N}}(n) > r_{\mathcal{N}}(h(M))$ has no incentive to deviate.*

In principle there are as many incentive constraints to satisfy in any equilibrium as there are candidates. Fortunately, many of these constraints do not bind, as I show next after introducing some new definitions. A subset of projects $\mathcal{M} \subseteq \mathcal{N}$ is a *chain* if it consists of consecutive elements of $\mathcal{N}$ *i.e.* $\mathcal{M} = \{n, n+1, \ldots, n+k\} \subseteq \mathcal{N}$. A chain $\mathcal{M} \subseteq \mathcal{L} \subseteq \mathcal{N}$ is said to be *maximal* in $\mathcal{L}$ if any other chain $\mathcal{M}' \subseteq \mathcal{L}$ satisfies $\mathcal{M}' \subseteq \mathcal{M}$ or $\mathcal{M} \cap \mathcal{M}' = \emptyset$.

The next result states that, if $M = 1$ or $\rho_1 \leq 1/2$, for any action profile, and along any subset of $\mathcal{I}_S$ that is also a chain, the incentive to deviate from disclosure is higher for weaker projects.

Lemma 4. *Suppose $M = 1$ or $\rho_1 \leq 1/2$. Pick an action profile $[\mathcal{I}, \mathcal{H}]$, and a chain $\mathcal{J} \subseteq \mathcal{I}_S$. Then for any $p < J$, candidate $j(p+1)$ has a higher incentive to deviate from disclosure than $j(p)$.*

The preceding series of results entails a practically useful characterization of the pure strategy equilibria of the perfect information communication game. The following proposition provides this characterization. It implies in particular that in order to check whether a certain action profile is an equilibrium, the number of incentive constraints to satisfy is less than $\min(2M, N_S) \leq N$. Indeed among projects in $\mathcal{I}$, only the incentives of the weakest projects of the maximal chains of $\mathcal{I}_S$ need to be checked. The set of possible equilibria among all action profiles is also considerably reduced by Remark 1 and Remark 2.

Proposition 5. *If $M = 1$ or $\rho_1 \leq 1/2$, an action profile $[\mathcal{I}, \mathcal{H}]$ is a pure strategy equilibrium of the communication game if and only if it satisfies*

(i) $\mathcal{H} \subset \mathcal{N}_S$.

(ii) $H \leq M$.

(iii) For any maximal chain $\mathcal{J} \subseteq \mathcal{I}_S$,

$$F\Big(M-1, \mathcal{I}^-\big(r_{\mathcal{I}}(j(J))-1\big)\Big)\rho_{j(J)} \geq F\Big(M - r_{\mathcal{H}}(j(J)), \mathcal{I} \setminus \{j(J)\}\Big).$$

(iv) For any project $h \in \mathcal{H}$,

$$F\Big(M-1, \mathcal{I}^-\big(r_{\mathcal{I}}(h)-1\big)\Big)\rho_h \leq F\Big(M - r_{\mathcal{H}}(h), \mathcal{I}\Big).$$

Proof. The necessity is obvious since (iii) and (iv) constitute a subset of the incentive conditions required by the equilibrium. Sufficiency holds as a direct consequence of Lemma 4. □

Note that in practice, situations where the number of slots available is small compared to the number of projects are more likely to be of interest. The reduction on the number of incentives to check is particularly effective in these situations. For example, if $M = 2$, $N_S = 7$ and $N_W = 5$, supposing that $\rho_1 \leq 1/2$, at most $1 + N_S + (N_S - 1)N_S/2 = 29$ action profiles are candidate equilibria, and the total number of incentive conditions to check in order to find all the pure strategy equilibria is at most $1+2N_S+4N_S(N_S-1)/2 = 99$ instead of $N2^N = 12\times 2^{12}$. Note that a similar reduction of the set of equilibrium incentive constraints for candidates in $\mathcal{H}$ can be provided under different conditions. This result can be found in Appendix D.

4.3 Full Disclosure

The results of Section 3.3 show that full disclosure is the optimal outcome of the communication game from the point of view of the decision maker. It is therefore interesting to know under what conditions full disclosure obtains. The first result states that there must be at least one weak candidate in the pool for full disclosure to be possible. The intuition behind this result is simple: if all the projects are strong and every other candidate discloses, the candidate with the lowest prospect has the same probability of being reached by the search whether she discloses her information or not, but conditionally on being reached her project is certain to be accepted if she withholds her information and not otherwise. This proves the proposition.

Proposition 6. *Full disclosure is impossible in the absence of weak candidates.*

When Proposition 5 applies, it is easy to provide a necessary and sufficient conditions for the existence of a full disclosure equilibrium. Indeed, the only condition to check is that the weakest candidate in $\mathcal{N}_S$ has no incentive to deviate from the full disclosure profile.

Proposition 7. *If $M = 1$ of $\rho_1 \leq 1/2$, full disclosure is an equilibrium of the communication game if and only if the weakest of the strong candidates has no incentive to deviate, that is if* *and only if*

$$\rho_{N_S} \geq \frac{F(M-1, \mathcal{N} \smallsetminus \{N_S\})}{F(M-1, \mathcal{N}^{-}(N_S-1))}. \tag{9}$$

Furthermore, the communication game is dominance solvable whenever the inequality in (9) holds strictly.

Proposition 6 and Proposition 7 are the first results to shed light on one of the main intuitions of the paper: the important role of weak candidates in pushing for disclosure. Indeed, it states that when all candidates are strong (*i.e.* $\mathcal{N}_W = \emptyset$) full disclosure can never be an equilibrium outcome, irrespective of the degree of competition as measured by N/M. It is competition from weak candidates who cannot afford secrecy that puts pressure on stronger candidates to reveal their information. More generally, the right-hand side of (9) is decreasing in $\mathcal{N}_W$ both for the set order and for the strength order, implying that a better pool of weak candidates makes condition (9) easier to satisfy.

For the single-slot case, a sufficient condition for full disclosure can be provided in the form of a lower bound on the number N_W of weak candidates.

Proposition 8. *If $M = 1$, full disclosure is an equilibrium if and only if*

$$\rho_{N_S} \geq \prod_{k \in \mathcal{N}_W} (1 - \rho_k). \tag{10}$$

In particular $N_W \geq B(\rho_{N_S})$ is a sufficient condition for the existence of a full disclosure equilibrium, where

$$B(\rho) \equiv \min \left\{ P \in \mathbb{N}; (1 - c/G)^P < \rho \right\} = \left\lceil \frac{\log \rho}{\log(1 - c/G)} \right\rceil.$$

An alternative sufficient condition that does not depend on the prospect of any particular project is

$$N_W > B(\hat{\rho}).$$

4.4 The Single-Slot Case

Let $n^* = \min\{n \in \mathcal{N}_S; \rho_n \leq (1-\rho_{n+1}) \ldots (1-\rho_N)\}$, and $n^* = \infty$ when the set on the right-hand side is empty. n^* is the strongest candidate of the strong set whose prospect is less than the probability that none of the projects with lower prospects is good. It is also the strongest candidate of the strong set who prefers to withhold when everyone else discloses. If there is no such candidate, full disclosure is the equilibrium outcome. If $n^* = 1$, then candidate 1 is the only one withholding information in equilibrium. Otherwise, either the candidate immediately above n^* has no incentive to withhold information given that n^* withholds and other candidates disclose, and then this is an equilibrium, or there is no pure strategy equilibrium.

Proposition 9. *In the case $M = 1$ there exists an equilibrium in pure strategies if $n^* \in \{1, \infty\}$ or if n^* satisfies $\rho_{n^*-1} \geq (1-\rho_{n^*+1}) \ldots (1-\rho_N)$. When it exists, this equilibrium is unique. It is full disclosure if $n^* = \infty$, and otherwise the only candidate withholding information in equilibrium is n^*.*

The next proposition shows that improving the set of weak candidates $\mathcal{N}_W$ can only lead to a better pure strategy equilibrium of the communication game from the point of view of the decision maker.

Proposition 10. *Let $\mathcal{N}_0 = \mathcal{N}_S \cup \mathcal{N}_{W0}$ and $\mathcal{N}_1 = \mathcal{N}_S \cup \mathcal{N}_{W1}$ be two sets of projects such that each of them leads to a pure strategy equilibrium $[\mathcal{I}, \mathcal{H}]_\gamma$ ($\gamma = 0, 1$), of the corresponding communication games Γ_γ, with $M = 1$. Then, if either $\mathcal{N}_{W0} < \mathcal{N}_{W1}$ or $\mathcal{N}_{W0} \subset \mathcal{N}_{W1}$, the decision maker prefers $\mathcal{N}_1$ to $\mathcal{N}_0$, that is*

$$V([\mathcal{I}, \mathcal{H}]_1, M = 1) > V([\mathcal{I}, \mathcal{H}]_0, M = 1).$$

With a single slot, it is also possible to know in which case the strongest project would be optimally rubberstamped by the decision maker as the outcome of the game. There exists a threshold $\rho^+ > \overline{\rho}$ such that project 1 is rubberstamped whenever $\rho_1 > \rho^+$.

Proposition 11 (Outstanding Candidates). *If $\rho_1 > \rho^+$, the decision maker optimally rubberstamps project 1 given any information set that excludes project 1 (that is for every action profile in which 1 withholds her information), where*

$$\rho^+ = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4c}{L+G}}\right).$$

As a consequence, in any equilibrium of the game, project 1 is rubberstamped, and candidate 1 either withholds her information or is indifferent between withholding and disclosing.

It is interesting to note that the threshold ρ^+ is does not depend on the number of candidates, or the particular profile of prospects. Naturally, ρ^+ is decreasing in the processing cost c.

To conclude this section, it is useful to consider the example of two strong candidates, and possibly many weak candidates. In the general case, mixed strategy equilibria are difficult to characterize and may involve mixing by more than two candidates, but they can be analyzed in a simple way in this example.

Example 2 (Two Strong Candidates (and Many Weak) for One Slot). When $N_S = 2$, a mixed strategy equilibrium obtains under (AL) and (LP) whenever $(1-\rho_2)f(0, \mathcal{N}_W) < \rho_1 < f(0, \mathcal{N}_W)$ (from Proposition 9). In this case, the mixed strategy equilibrium is unique and the two strong candidates play as follows. The leading candidate discloses her information with probability $\lambda = \frac{\rho_2}{\rho_1\rho_2 + (1-\rho_1)f(0, \mathcal{N}_W)}$, while the second candidate discloses her information with probability $\mu = \frac{f(0, \mathcal{N}_W) - \rho_1}{\rho_2 f(0, \mathcal{N}_W)}$ (see the argument in Appendix B).

5 Incomplete Information of the candidates[15]

In some applications, it may be unreasonable to assume that the candidates know the initial prospects of each other, especially when their number is large. The candidates share the common belief that the prospects of the projects are drawn independently from an atomless

[15] All proofs are in Appendix C.

distribution with cumulative density function H and full support[16] $\mathcal{S} = [\underline{x}, \overline{x}] \subseteq [\underline{\rho}, \overline{\rho}]$ such that $\underline{x}(1-\overline{x}) > c/(L+G)$[17]. The corresponding probability density function h is assumed to be bounded away from 0 by some $m > 0$, hence for every $\rho \in \mathcal{S}$, $h(\rho) \geq m$. The prospects of all the candidate projects are observed by the decision maker so that the optimal policy of the decision maker established in Section 3.2 remains optimal. The number N of candidates is common knowledge. The *type* of candidate n is her realized prospect $\rho_n \in \mathcal{S}$. Types lying in $\mathcal{S} \cap (0, \hat{\rho})$ are weak, and types in $\mathcal{S} \cap (\hat{\rho}, \overline{x})$ are strong. If $\hat{\rho} \leq \underline{x}$, weak types are *absent*, and otherwise they are *present*. A distributional strategy of candidate n is a probability measure λ_n on the Borelians of $\mathcal{S} \times \{0,1\}$ for which the marginal distribution of $\mathcal{S}$ is h, where $\{0,1\}$ is a description of the action set and 1 corresponds to disclosing information. This formalism introduced by Milgrom and Weber (1985) allows the modeler to describe mixing behaviors by the players while avoiding the measurability issue noted in Aumann (1964). The probability that player n discloses information given that her type is ρ is then $\lambda_n(1|\rho)$. To simplify the notations, I denote this probability by $\lambda_n(\rho)$. The equilibrium notion for the communication game is Bayesian Nash equilibrium in distributional strategies. I will generally consider symmetric equilibria in which $\lambda_1 = \lambda_2 = \cdots = \lambda_N$. In this section, I call full disclosure the strategy profile such that all the candidates disclose with probability 1. When full disclosure is not an equilibrium outcome, it is still possible that the realized types of the candidates results in each candidate disclosing information *ex post*, but the *ex ante* probability of this happening is less than 1.

5.1 The single slot case $M = 1$

Let $\lambda = (\lambda_1, \cdots, \lambda_N)$ be a strategy profile. Supposing that all other candidates are playing according to λ, the contingent payoffs of candidate n are given by[18]

[16] That is $h > 0$ everywhere on the support.
[17] Hence any particular realization of the vector of prospects satisfies (LP).
[18] In both equations, the equality is a consequence of the independence of the prospects.

$$
\begin{aligned}
V_{I,n}^{\lambda}(\rho) &\equiv \rho E\left[\prod_{m\neq n}(1-\lambda_m(\rho_m)\rho_m \mathbb{1}_{\rho_m>\rho})\right] \\
&= \rho\prod_{m\neq n}\left(1-\int_{\rho}^{\overline{x}} x\lambda_m(x)dH(x)\right),
\end{aligned} \tag{11}
$$

if she discloses, and

$$
\begin{aligned}
V_{H,n}^{\lambda}(\rho) &\equiv E\left[\prod_{m\neq n}\lambda_m(\rho_m)\left(1-\rho_m\right)+\left(1-\lambda_m(\rho_m)\right)\mathbb{1}_{\rho_m<\rho}\right]\mathbb{1}_{\rho\geq\hat{\rho}} \\
&= \mathbb{1}_{\rho\geq\hat{\rho}}\prod_{m\neq n}\left(\int_{\underline{x}}^{\overline{x}}\lambda_m(x)(1-x)dH(x)+\int_{\underline{x}}^{\rho}\left(1-\lambda_m(x)\right)dH(x)\right),
\end{aligned} \tag{12}
$$

if she withholds. Note that $V_{I,n}^{\lambda}$ is continuous in type[19] on $\mathcal{S}$ while $V_{H,n}^{\lambda}$ has a single discontinuity at $\hat{\rho}$ if weak types are present. $V_{I,n}^{\lambda}$ is also strictly increasing in ρ, while $V_{H,n}^{\lambda}$ is only weakly increasing in ρ. In particular, it is constant on any interval of the type space on which all other players disclose with probability 1. This is intuitive as the probability of being ever considered by the decision maker when withholding depends on a player's own type ρ only through the implied probability that a player with type higher than ρ also withholds which is invariant while ρ stays on an interval on which all the other players disclose.

If λ is a symmetric strategy profile, dropping the n index for the payoff functions,

$$
V_I^{\lambda}(\rho) = \rho\left(1-\int_{\rho}^{\overline{x}} x\lambda(x)dH(x)\right)^{N-1}, \tag{13}
$$

and,

[19]The only non-obvious part of the argument consists in showing that $\int_{\rho}^{\overline{x}} x\lambda_m(x)dH(x)$ is continuous in ρ. Because λ_m is bounded between 0 and 1, $\left|\int_{\rho}^{\overline{x}} x\lambda_m(x)dH(x)-\int_{\rho'}^{\overline{x}} x\lambda(x)dH(x)\right| \leq \left|\int_{\rho}^{\rho'} dH(x)\right| = |H(\rho)-H(\rho')|$ for any pair (ρ,ρ'). Hence, H being atomless, this difference goes to 0 when $\rho'\to\rho$, and that concludes the argument. A similar argument works for V_H^{λ}.

$$V_H^\lambda(\rho) = \left(\int_{\underline{x}}^{\overline{x}} \lambda(x)(1-x)dH(x) + \int_{\underline{x}}^{\rho} (1-\lambda(x))\, dH(x) \right)^{N-1} \mathbb{1}_{\rho \geq \hat{\rho}}. \tag{14}$$

A profile λ is an equilibrium if n is willing to play according to λ_n when other candidates follow λ. I define the sets $\Lambda_n = \{\rho \in \mathcal{S} | \lambda_n(\rho) \in (0,1)\}$, $\Lambda_n^0 = \{\rho \in \mathcal{S} | \lambda_n(\rho) = 0\}$ and $\Lambda_n^1 = \{\rho \in \mathcal{S} | \lambda_n(\rho) = 1\}$, and denote the interior of a set with the operator $Int(.)$. Then λ is an equilibrium strategy if and only if:

(i) $\forall \rho \in Int\left(\Lambda_n^0\right), \quad V_{I,n}^\lambda(\rho) < V_{H,n}^\lambda(\rho)$,

(ii) $\forall \rho \in \Lambda_n, \quad V_{I,n}^\lambda(\rho) = V_{H,n}^\lambda(\rho)$,

(iii) $\forall \rho \in Int\left(\Lambda_n^1\right), \quad V_{I,n}^\lambda(\rho) > V_{H,n}^\lambda(\rho)$.

Before going further, note that there exists a Bayesian Nash equilibrium in distributional strategies. This is a direct application of Milgrom and Weber (1985), Proposition 1, and Theorem 1. This equilibrium, however, is not necessarily symmetric. At this point, it may be worthwhile to notice that the payoff of a candidate depends on both the types and actions of other candidates, and as a consequence, the purification theorem of Milgrom and Weber (1985) does not apply.

Proposition 12. *There exists a Bayesian Nash Equilibrium in distributional strategies for the communication game.*

As in the case with perfect information, a weak candidate always discloses with probability 1 in equilibrium. More precisely:

Lemma 5. *Any strategy λ_n such that for some $\rho < \hat{\rho}$, $\lambda_n(\rho) < 1$ is strictly dominated.*

Therefore in equilibrium $\mathcal{S} \cap [\underline{x}, \hat{\rho}) \subseteq \Lambda_n^1$. Before going further, it is easy to prove that when weak types are present, sufficient competition yields full disclosure. The proposition shows that when full disclosure is an equilibrium it is generically unique, and the game is actually dominance solvable, meaning that full disclosure is the only strategy profile that survives the iterated deletion of strictly dominated strategies

Proposition 13. *If weak types are present, full disclosure is an equilibrium if and only if* $N \geq \hat{N}$*, where*

$$\hat{N} = 1 + \frac{\log(1/\hat{\rho})}{\log\left(1 - \int_{\hat{\rho}}^{\overline{x}} x dH(x)\right) - \log\left(1 - \int_{\underline{x}}^{\overline{x}} x dH(x)\right)}, \tag{15}$$

furthermore the game is dominance solvable whenever this inequality holds strictly (in particular, full disclosure is the unique equilibrium). In the absence of weak types, full disclosure is never an equilibrium.

In order to understand the role of weak and strong candidates, it is interesting to look at the effect of the distribution of types on the threshold $\hat{N}$. The next result shows that increasing the weight on stronger types in the weak set and decreasing the weight on stronger types in the strong set while keeping the relative weights of these two sets constant leads to a lower threshold $\hat{N}$. Hence it is easier to obtain full disclosure with a distribution concentrated around the frontier type $\hat{\rho}$. Obviously it is also better to have stronger candidates for the decision maker, therefore decreasing the weight on stronger types in the strong set has an additional detrimental effect. Increasing the weight on stronger types in the weak set is unambiguously better for the decision maker.

Proposition 14. *Consider two distributions* G *and* H *with the same support* $\mathcal{S}$ *and such that for every* $x \in \mathcal{S}$*,* $G(x) \leq H(x)$ *for every* $x \leq \hat{\rho}$ *and* $G(x) \geq H(x)$ *for every* $x \geq \hat{\rho}$*. Suppose also that the support includes weak types,* $\hat{\rho} \in \mathcal{S}$*, and that the two distributions put the same weights on the weak and the strong sets* $H(\hat{\rho}) = G(\hat{\rho})$*. Then* $\hat{N}_G \leq \hat{N}_H$*.*

The next lemma shows that if a strong type ρ discloses information with probability 1 in a symmetric equilibrium, then all the types above ρ also disclose with probability 1. The intuition for this is as follows. If there exists some interval Ω on which λ is equal to 1, then V_H^λ

is independent of ρ as can be seen on the expression of V_H^λ. Intuitively this is because when ρ is moving in a neighborhood on which every type discloses, the probability that there is a withholding candidate with a type higher than ρ does not decrease with ρ. Therefore V_H^λ is constant on Ω while V_I^λ is strictly increasing. If λ is an equilibrium strategy, $V_I^\lambda > V_H^\lambda$ on Ω. But then, the continuity of the two payoff functions implies that V_H^λ can never catch up with V_I^λ as ρ increases, so that disclosing must be strictly better than withholding.

Lemma 6. *If λ is a strategy that defines a symmetric equilibrium strategy profile such that there exists a strong type $\rho \in \mathcal{S} \cap (\hat{\rho}, \overline{x})$ satisfying $\rho \in Int(\Lambda^1)$, then $\mathcal{S} \cap (\rho, \overline{x}) \subseteq \Lambda^1$.*

Therefore, in any equilibrium $Int(\Lambda^1) = (\underline{x}, \hat{\rho}) \cup (\rho^*, \overline{x})$ for some $\rho^* \in [\hat{\rho}, \overline{x}]$. In the absence of weak types, $Int(\Lambda^1) = (\rho^*, \overline{x})$. With the help of the lemma, I show that with sufficient competition almost no type discloses with probability 1. The next proposition, which characterizes the symmetric equilibria in pure strategies, is an immediate corollary of this lemma.

Proposition 15. *If λ is a strategy that defines a symmetric equilibrium in pure strategies, it must take the form*

$$\lambda(\rho) = \mathbb{1}_{\rho \in \Lambda^1},$$

where

$$\Lambda^1 = [\underline{x}, \hat{\rho}) \cup \langle \rho^*, \overline{x}]$$

for some $\rho^* \geq \hat{\rho}$, and where $\langle$ denotes either $($ or $[$. When the threshold ρ^* is interior, $\rho^* \in (\hat{\rho}, \overline{x})$, it is a solution to the following equation in ρ

$$\left(1 - \rho^{\frac{1}{N-1}}\right)\left(1 - \int_\rho^{\overline{x}} x dH(x)\right) = \int_{\underline{x}}^{\hat{\rho}} x dH(x). \tag{16}$$

Furthermore, in the absence of weak types, no such equilibrium exists.

Hence, symmetric pure strategy equilibria other than full disclosure, when they exist, must be non-monotonic in the presence of weak types. The characterization of the threshold ρ^* in

(16) derives from the fact that a player with type ρ^* must be indifferent between disclosing and withholding. The next proposition shows that in fact, with sufficient competition and in the absence of weak types, in any symmetric equilibrium, all the types are strictly mixing.

Proposition 16. *In the absence of weak types, if $\{\lambda_N\}_{N\geq 1}$ is a sequence of symmetric equilibrium strategies for the N-candidates communication game, there exists some $\tilde{N}$ such that $Int(\Lambda_N^1) = Int(\Lambda_N^0) = \emptyset$ for every $N > \tilde{N}$. That is in equilibrium, almost every type is strictly mixing. For $N > \tilde{N}$, λ_N is almost everywhere equal to a continuous function. Furthermore, for almost every $\rho \in \mathcal{S}$,*

$$\lim_{N\to\infty} \lambda_N(\rho) = \frac{1}{1+\rho}.$$

In particular, stronger candidates are less likely to disclose information than weaker ones in the limit. Interestingly, the limit equilibrium strategy is independent of the particular distribution of types. Note however, that the proposition does not establish the existence of a symmetric equilibrium (equilibria exist from Proposition 12, but they are not necessarily symmetric) for each N, but merely describes the asymptotic behavior of a sequence of equilibria if they were to exist.

5.2 Multiple Slots $M \geq 1$

The case with multiple slots is far less tractable. However some of the results extend to this case.

Proposition 17. *With multiple slots, any symmetric pure strategy equilibrium must take the form*

$$\lambda(\rho) = \mathbb{1}_{\rho\in\Lambda^1},$$

where

$$\Lambda^1 = [\underline{x}, \hat{\rho}) \cup \langle \rho^*, \overline{x}]$$

for some $\rho^* \geq \hat{\rho}$, and where $\langle$ denotes either $($ or $[$. In the absence of weak types, there is no pure *strategy equilibrium. In particular, full disclosure is impossible in the absence of weak types.*

The arguments used to prove this proposition extend those of the single slot case. In the remaining of the section, I describe how the essential intuitions translate in the multiple-slots case.

For any Borel set $\mathcal{K} \subseteq \mathcal{S}$, let $\eta(\mathcal{K}) = \int_{\mathcal{K}} dH(x)$ be the measure of this set according to the measure implied by the distribution H, and let $x_e(\mathcal{K}) = \frac{1}{\eta(\mathcal{K})} \int_{\mathcal{K}} x dH(x)$ denote the expected type of a candidate knowing that her type lies in $\mathcal{K}$. When $M \geq 1$ and all the candidates except i play according to the pure strategy: disclose on Λ^1, withhold on $\Lambda^0 = \mathcal{S} \setminus \Lambda^1$, the payoff from disclosing for candidate n as a function of her type ρ is given by

$$\begin{aligned}
V_I(\rho) &= \rho \cdot \Pr\Big(\text{there are less than } M-1 \text{ good projects in } \Lambda^1 \cap (\rho, \overline{x})\Big) \\
&= \rho \cdot \Bigg\{ \sum_{m=0}^{N-1} \binom{m}{N-1} \eta\big(\Lambda^1 \cap (\rho, \overline{x})\big)^m \big(1 - \eta\big(\Lambda^1 \cap (\rho, \overline{x})\big)\big)^{N-1-m} \\
&\quad \times \sum_{k=0}^{\min(m, M-1)} \binom{k}{m} x_e\big(\Lambda^1 \cap (\rho, \overline{x})\big)^k \big(1 - x_e\big(\Lambda^1 \cap (\rho, \overline{x})\big)\big)^{m-k} \Bigg\},
\end{aligned} \tag{17}$$

and the payoff from withholding

$$\begin{aligned}
V_H(\rho) &= \mathbb{1}_{\rho > \hat{\rho}} \Pr\Big(\text{the number of good projects in } \Lambda^1 + \text{ the number of projects in } \Lambda^0 \cap (\rho, \overline{x}) \leq M-1\Big) \\
&= \mathbb{1}_{\rho > \hat{\rho}} \sum_{m=0}^{N-1} \binom{m}{N-1} \eta\big(\Lambda^1\big)^m \big(1 - \eta\big(\Lambda^1\big)\big)^{N-1-m} \\
&\quad \times \sum_{k=0}^{\min(m, M-1)} \binom{k}{m} x_e\big(\Lambda^1\big)^k \big(1 - x_e\big(\Lambda^1\big)\big)^{m-k} \\
&\quad \times \sum_{l=0}^{M-1-k} \binom{l}{N-1-m} \left(\frac{\eta\big(\Lambda^0 \cap (\rho, \overline{x})\big)}{\eta\big(\Lambda^0\big)}\right)^l \left(\frac{\eta\big(\Lambda^0\big) - \eta\big(\Lambda^0 \cap (\rho, \overline{x})\big)}{\eta\big(\Lambda^0\big)}\right)^{N-1-m-l}.
\end{aligned} \tag{18}$$

The intuition works in the same way as in the case with $M = 1$: V_I is increasing in ρ

everywhere (because the set $\Lambda^1 \cap (\rho, \overline{x})$ is shrinking as ρ increases implying that if there are less than $M-1$ good projects in that set for a certain ρ then there are also less than $M-1$ good projects in that set for a higher ρ), whereas V_H is constant in ρ on Λ^1 and increasing elsewhere. Both functions are continuous on $(\hat{\rho}, \overline{x})$. Therefore $\Lambda^1 = [\underline{x}, \hat{\rho}) \cup \langle \rho^*, \overline{x}]$ for some $\rho^* \in [\hat{\rho}, \overline{x}]$. The threshold ρ^* is now characterized by the following equation which says that $V_I(\rho^*) = V_H(\rho^*)$ and makes use of the particular form of Λ^1.

$$
\begin{aligned}
\rho^* \Bigg\{ & \sum_{m=0}^{N-1} \binom{m}{N-1} \Big(1 - H(\rho^*)\Big)^m H(\rho^*)^{N-1-m} \\
& \times \sum_{k=0}^{\min(m,M-1)} \binom{k}{m} \left(\frac{1}{1-H(\rho^*)} \int_{\rho^*}^{\overline{x}} x dH(x) \right)^k \left(1 - \frac{1}{1-H(\rho^*)} \int_{\rho^*}^{\overline{x}} x dH(x) \right)^{m-k} \Bigg\} \\
& = \sum_{m=0}^{N-1} \binom{m}{N-1} \Big(1 - H(\rho^*) + H(\hat{\rho})\Big)^m \Big(H(\rho^*) - H(\hat{\rho})\Big)^{N-1-m} \\
& \quad \times \sum_{k=0}^{\min(m,M-1)} \binom{k}{m} \left(\frac{1}{1-H(\rho^*)+H(\hat{\rho})} \int_{(\underline{x},\hat{\rho}) \cup (\rho^*, \overline{x})} x dH(x) \right)^k \\
& \quad \times \left(1 - \frac{1}{1-H(\rho^*)+H(\hat{\rho})} \int_{(\underline{x},\hat{\rho}) \cup (\rho^*, \overline{x})} x dH(x) \right)^{m-k} .
\end{aligned}
\tag{19}
$$

(19) simply states that the frontier type ρ^* must be indifferent between disclosing (on the left-hand side) and withholding (on the right-hand side). If for $\rho^* = \hat{\rho}$, the left-hand side is greater than the right-hand side, then full disclosure is an equilibrium. If for $\rho^* = \overline{x}$, the left-hand side is less than the right-hand side, then no disclosure is an equilibrium. If the left-hand side is strictly greater than the right-side for every $\rho^* \in (\hat{\rho}, \overline{x})$, full disclosure is the unique symmetric equilibrium in pure strategies, and if the opposite inequality holds on $(\hat{\rho}, \overline{x})$, no disclosure is the unique equilibrium. Note that in the absence of weak candidates $H(\hat{\rho}) = 0$ and the left-hand side of (19) is then equal to its right-hand side multiplied by $\rho^* < 1$. Therefore, in the absence of weak types, no disclosure is the only possible symmetric equilibrium in pure strategies. However it is clear that no disclosure cannot be an equilibrium as the lowest type would clearly be better off by disclosing, and therefore there is no symmetric pure strategy

equilibrium in the absence of weak types.

6 Advertising and Product Information

The model developed in this paper provides an original framework to study advertising. It gives a new theoretical explanation of a negative relationship between product quality and the informational content of advertising. Consider the case of a monopolist, for example. The model can be interpreted in two ways in an advertising context. The first way consists in considering that ρ is a proxy for quality itself and that it is public information. The experience of the consumer with the good is more likely to be pleasing when the quality is higher, but it may depend on other factors as well. By advertising informatively, the producer of the good can inform the consumer about these other factors but she cannot perfectly infer how well the information will please the taste of the consumer. After all, marketing and advertising would not be so important if consumers' tastes were not so elusive. Then the model clearly predicts that higher quality producers (higher ρ) advertise less (or at least less informatively). A second way to interpret the model that leads to the same conclusion is to interpret the value of the good for the consumer ("good" or "bad") as its quality, and ρ as the empirically-rooted *ex ante* probability that the product is of good quality. ρ is then a proxy for the reputation of the producer and is inferred from her product history. Then assuming that the distribution of ρ across time and industries is given by a density function g over $(0, 1)$, the cross-probability that a product that is advertised informatively is of good quality is $q_a = \Pr[G|\text{advertising}] = \int_0^{\hat{\rho}} \rho g(\rho) d\rho$ and it is less than the cross-probability that a product that is not advertised informatively is of good quality $q_{na} = \Pr[G|\text{no advertising}] = \int_{\hat{\rho}}^1 \rho g(\rho) d\rho > q_a$.

Apart from the different nature of the explanation, this model has the advantage over other models in the literature (Bagwell and Overgaard (2006), Bar-Isaac, Caruana and Cuñat (2008)) that it accounts for the effects of competition on advertising content. The imperfect information framework of Section 5 in particular, shows that, in the absence of weak sellers, increasing competition asymptotically leads to a negative relationship between the strength of candidates and their probability to disclose. The inability of competition to generate full disclosure in the absence of weak producers, in particular, may be a source of concern for consumers. A limitation of the model, however, is that it does not take pricing into account.

7 Conclusion

Common knowledge that information is unequivocal to its owners is a crucial assumption in problems of strategic transmission of hard information. If it does not hold, the receiver cannot second-guess the actions of the sender, and skepticism does not ensure full revelation. Competition can mitigate this problem, but only under certain conditions. The results of this paper highlight the importance of *ex ante* weaker candidates to elicit information transmission in certain types of contests.

The equivocal information assumption, that agents control information but cannot predict its effect on others, deserves further examination. In the context of this model, the inability of the sender to understand the consequences of her information gives her an advantage. This may give an incentive to a sender who can interpret information to pretend she cannot, a question that it may be interesting to explore in a reputation model, for example, where it could be valuable to establish a reputation of limited understanding. The model offers several insights into practical situations of advertising in particular. It would be interesting to analyze this link with advertising further, notably by considering heterogeneous consumers and including price competition. This is only one way of introducing heterogeneity on the receiving side.

Another obvious, but nonetheless interesting, and difficult extension, for example, would be to blend the framework of this paper with that of Caillaud and Tirole (2007), with multiple senders and a committee of receivers with heterogeneous beliefs. As analyzed in Caillaud and Tirole (2007), this correlations in the preferences of the committee members give the opportunity for a sender to engineer cascades of information among members to push her case.

These forces seriously complicate the analysis of the competition among senders.

In addition to the connections to the literature drawn in the introduction, this paper is also related, although more tenuously, to the search literature, and to multi-armed bandit problems which have been used to model the tradeoff between exploration and exploitation. It is clear that the problem of the decision maker in this paper is a search problem, and the dynamic programming techniques that I use to solve it are typical of this literature. But it can also be understood as a multi-armed-bandit problem in which rubberstamping a project that cannot be learned about is akin to exploitation, while learning more about one of the other projects is akin to exploration. Under this interpretation, this paper is related to recent work that makes the value of the tradeoff on each arm endogenous through pricing by the owner of the arm (Bergemann and Välimäki (1996), Felli and Harris (1996), Bergemann and Välimäki (2006)). Looking at the model of this paper from the point of view of this literature raises the following question: what happens to the exploration/exploitation tradeoff when in a bandit problem the information available on each arm is the fruit of an endogenous decision by an interested gatekeeper with limited processing abilities?

APPENDIX

Appendix A Optimal Policy of the Decision Maker

Proof of Lemma 1. The first part of (i) comes from the fact that projects in $\mathcal{H}_W$ cannot be learnt about, and the incremental expected payoff of rubberstamping them is negative. The second part of the statement is obvious given the existence of the cap M on the number of projects that can be implemented. (ii) is true because any project in $\mathcal{H}_S$ has a positive expected incremental payoff, and therefore the first $\min(M - I, H_S)$ projects in $\mathcal{H}_S$ should be used to fill the slots that cannot be filled by projects in $\mathcal{I}$ since $I < M$. Finally (iii) holds because it cannot hurt to fill slots that cannot be filled by projects in $\mathcal{H}_S$ with projects in $\mathcal{I}$. □

Proof of Proposition 1. If $\mathcal{I} = \mathcal{N}$ it is clearly optimal to learn about each of the projects starting from the strongest one and then moving down in the strength order, approving a project each time it is found to be good, and continuing until all slots are filled. The policy of the proposition clearly fulfills these criteria. If $\mathcal{H} = \mathcal{N}$ it is also clear that the policy of the proposition is optimal: since projects in $\mathcal{N}_S$ have positive expected incremental payoffs they should be approved orderly and according to the availability of slots.

Consider states that satisfy $I \geq M = H$ and $\mathcal{H} \subseteq \mathcal{N}_S$. Below, I show, by a double induction on I and $M = H$, that the policy described in the proposition is optimal for all such states, and that it is the unique optimal policy up to some details in the order of learning explained below. I will take this result as given for now and argue that it implies that the policy of the proposition is optimal, although not uniquely, in any other state. This is a consequence of Lemma 1. Indeed, by point (i) of the lemma, projects in $\mathcal{H}_W$ are irrelevant. By point (ii), I can assume $I \geq M$ for otherwise the optimal policy consists in rubberstamping projects in $\mathcal{H}$ until a state where $I = M$ is reached and then continuing with the optimal policy. Furthermore, the lemma says that this rubberstamping can occur at any place in the sequence describing the optimal policy. Hence they can be placed in the sequence so that the optimal policy is as described in the proposition. This is the first source of non uniqueness of the optimal policy in general. Since the projects rubberstamped in this operation are the strongest in $\mathcal{H}$, and are certain to get approved, they are also irrelevant to the probability that any given project is approved in any of the optimal policies. Point (i) and (iii) of the lemma allow me to consider only the cases where $H = M$. As a consequence, an optimal policy can always be described as: always start by learning as much as possible, and then rubberstamp. Therefore, the order of learning never affects the probability of having to rubberstamp some projects in the end. As a consequence, it is always optimal to learn about stronger projects first as it minimizes the cost of the search. This is not uniquely optimal, however, for the following reason: if there are M slots available then the order in which the first M projects in $\mathcal{I}$ are processed is irrelevant. Hence the argument below shows optimality, and uniqueness up to this subtlety. Note that the argument above also proves that the probability that a given project is approved is unaltered by which particular optimal policy is used.

Initiation $\mathcal{I} = \{i\}$, $\mathcal{H} = \{h\}$, $M = 1$. In this case the choice is between rubberstamping h, or examining i, approving i if it is of the good type, rejecting i and rubberstamping h if it is of the bad type. The first choice pays $\rho_h(G + L) - L$ while the second one pays $\rho_i G - c + (1 - \rho_i)(\rho_h(G + L) - L)$. Letting Δ be the gain from learning,

$$\Delta = \rho_i(1 - \rho_h)(G + L) - c > 0.$$

where the inequality holds because of the learning priority assumption (LP). Hence the unique optimal policy is to learn first.

Induction Step, $I > M = H = 1$. Suppose the result holds for any triple $(\mathcal{I}, \mathcal{H}, M)$ such that $I = K > H = M = 1$ and $\mathcal{H} \subseteq \mathcal{N}_S$, and consider a state $(\mathcal{I}, \mathcal{H}, M)$ with $I = K + 1$. The decision maker can either rubberstamp h and end, or choose to learn about a project i in $\mathcal{I}$, approve i and end if it is of the good type, and move on to the state $(\mathcal{I} \setminus \{i\}, \{h\}, 1)$ otherwise. Hence we only need to compare the payoffs of these choices

$$\rho_i G - c + (1 - \rho_i)V(\mathcal{I} \setminus \{i\}, \{h\}, 1),$$

to the payoff $\rho_h(G + L) - L$ of rubberstamping h. Letting Δ denote the gain from learning

$$\Delta = \rho_i(1 - \rho_h)(G + L) - c + (1 - \rho_i)V(\mathcal{I} \setminus \{i\}, \{h\}, 1) > 0,$$

where the first term is positive by the learning priority assumption (LP), and the second term is non-negative because the decision maker always has the option to discard all remaining projects and get 0. Hence, learning before moving on is once again optimal policy, and by the induction hypothesis it is also the best continuation policy. Because learning in the strength order minimizes the cost of search, it is optimal to do so. This proves the claim. It is unique up to the subtlety about the order of learning explained above.

Induction Step, $I > M = H > 1$. Suppose the result holds for all $(\mathcal{I}, \mathcal{H}, M)$ with $\mathcal{H} \subseteq \mathcal{N}_S$ and $H = M \leq K$ or $H = M = K+1$ but $I \leq J$, and consider a triple $(\mathcal{I}, \mathcal{H}, M)$ where $H = M = K+1$ and $I = J+1$. Consider the choice between examining (and conditionally approving) project $i(1)$ in $\mathcal{I}$, and rubberstamping a project $h \in \mathcal{H}$, and then move on with the optimal policy in the new state. The first option leads to the continuation value

$$\rho_{i(1)}\Big(G + V(\mathcal{I} \setminus \{i(1)\}, \mathcal{H} \setminus \{H\}, M-1)\Big) + (1-\rho_{i(1)})V(\mathcal{I} \setminus \{i(1)\}, \mathcal{H}, M) - c,$$

while the second option yields

$$\rho_h(G+L) - L + V(\mathcal{I}, \mathcal{H} \setminus \{h\}, M-1),$$

which by induction can be rewritten as

$$\rho_h(G+L) - L + \rho_{i(1)}\Big(G + V(\mathcal{I} \setminus \{i(1), \mathcal{H} \setminus \{h, H\}, M-2)\Big) + (1-\rho_{i(1)})V(\mathcal{I} \setminus \{i(1)\}, \mathcal{H} \setminus \{h\}, M-1) - c$$

The gain from learning is then

$$\begin{aligned}\Delta = &\rho_{i(1)} \underbrace{\Big(V(\mathcal{I} \setminus \{i(1)\}, \mathcal{H}, M-1) - \big(V(\mathcal{I} \setminus \{i(1)\}, \mathcal{H} \setminus \{h, H\}, M-2) + \rho_h(G+L) - L\big)\Big)}_{A} \\ &+ (1-\rho_{i(1)}) \underbrace{\Big(V(\mathcal{I} \setminus \{i(1)\}, \mathcal{H}, M) - \big(V(\mathcal{I} \setminus \{i(1)\}, \mathcal{H} \setminus \{h\}, M-1) + \rho_h(G+L) - L\big)\Big)}_{B}.\end{aligned}$$

$A > 0$. Indeed in state $(\mathcal{I} \setminus \{i(1)\}, \mathcal{H} \setminus \{H\}, M-1)$ an available policy is to rubberstamp h and then move on to state $(\mathcal{I} \setminus \{i(1)\}, \mathcal{H} \setminus \{h, H\}, M-2)$ and continue with the optimal policy. Because of the induction hypothesis, this is not optimal at $(\mathcal{I} \setminus \{i(1)\}, \mathcal{H} \setminus \{H\}, M-1)$, and A is exactly the difference of payoffs between the former policy and the optimal one. A similar argument shows that $B > 0$. Therefore $\Delta > 0$ implying that learning is optimal at $(\mathcal{I}, \mathcal{H}, M)$.

Once again this implies that the policy of the proposition is uniquely optimal up to the order of learning.

Induction Step, $I = M = H$. Suppose now that the result holds for all $(\mathcal{I}, \mathcal{H}, M)$ with $\mathcal{H} \subseteq \mathcal{N}_S$ and $H = M \leq K$, and consider a triple $(\mathcal{I}, \mathcal{H}, M)$ such that $I = H = M = K + 1$. Then the payoff of learning about $i(1)$ is

$$\rho_{i(1)}\Big(G+V(\mathcal{I}\smallsetminus\{i(1)\}, \mathcal{H}\smallsetminus\{(H\}, M-1)\Big)+\big(1-\rho_{i(1)}\big)\,\Big(\rho_{h(1)}(G+L)-L+V(\mathcal{I}^{+}(1), \mathcal{H}^{+}(1), M-1)\Big)-c,$$

and the payoff of rubberstamping $h(1)$ ($h(1)$ is clearly better than any other h here) is

$$\rho_{h(1)}(G+L) - L + V(\mathcal{I}, \mathcal{H} \smallsetminus \{h(1)\}, M-1),$$

or, because of the induction hypothesis,

$$\rho_{h(1)}(G+L)-L+\rho_{i(1)}\Big(G+V(\mathcal{I}\smallsetminus\{i(1)\}, \mathcal{H}\smallsetminus\{h(1), H\}, M-2)\Big)+\big(1-\rho_{i(1)}\big)\,V(\mathcal{I}\smallsetminus\{i(1)\}, \mathcal{H}\smallsetminus\{h(1)\}, M-1)-$$

Hence the gain from learning is

$$\Delta = \rho_{i(1)}\Big(V(\mathcal{I}\smallsetminus\{i(1)\}, \mathcal{H}\smallsetminus\{H\}, M-1) - \big(V(\mathcal{I}\smallsetminus\{i(1)\}, \mathcal{H}\smallsetminus\{h(1), H\}, M-2) + \rho_{h(1)}(G+L) - L\big)\Big).$$

By the induction hypothesis, rubberstamping $h(1)$ is an available but non optimal policy at $(\mathcal{I} \smallsetminus \{i(1)\}, \mathcal{H} \smallsetminus \{H\}, M-1)$, hence $\Delta > 0$. This concludes the proof. $\square$

Proof of Lemma 2. For $p \geq 1$ the probability of finding p good projects in $\mathcal{K}$ is equal to the probability of finding $p-1$ good projects in $\mathcal{K} \smallsetminus \{k\}$ times the probability that k is a good project plus the probability of finding p good projects in $\mathcal{K} \smallsetminus \{k\}$ times the probability that k is not a good project. This is exactly (5). (6) is obtained by summation of (5) for $p \leq q$.

$\square$

Proof of Lemma 3. I show the result for $K = 1$. The general result follows by iteration. Let k

be the unique project in $\mathcal{K}$. Then, by Lemma 2

$$F(p, \mathcal{J} \cup \{k\}) = F(p, \mathcal{J}) - \rho_k f(p, \mathcal{J}).$$

Therefore

$$\begin{aligned} F(p, \mathcal{J} \cup \{k\}) - F(p-1, \mathcal{J}) &= F(p, \mathcal{J}) - F(p-1, \mathcal{J}) - \rho_j f(p, \mathcal{J}) \\ &= (1-\rho_j) f(p, \mathcal{J}) > 0. \end{aligned}$$

□

Proof of Proposition 3. I prove the proposition for the case where the two projects exchanged are consecutive projects in $\mathcal{N}$, that is $m = n+1$. Evidently this proves the general result, as any other exchange can be decomposed in a series of exchanges between consecutive projects. When $M = 1$ and the learning partition of the decision maker is given by $[\mathcal{I}, \mathcal{H}]$, the expected payoff of the decision maker is

$$V(\mathcal{I}, \mathcal{H}, 1) = (1 - f(0, \mathcal{I}))G + f(0, \mathcal{I})\Big(\rho_{h(1)}(G+L) - L\Big) - c\sum_{q=0}^{I-1} F(0, \mathcal{I}^{-}(q)).$$

Let Δ be the change in payoffs due to the exchange of projects $\Delta = V(\mathcal{I}_1, \mathcal{H}_1, 1) - V(\mathcal{I}_0, \mathcal{H}_0, 1)$. Using Lemma 2, and supposing $r_{\mathcal{H}}(n) > 1$ or $n \notin \mathcal{N}_S$, it is equal to

$$\Delta = (\rho_n - \rho_{n+1}) f(0, \hat{\mathcal{I}})(1 - \rho_{h(1)})(G+L) + c\sum_{q=Q}^{I}\Big(F(0, \mathcal{I}_0^{-}(q)) - F(0, \mathcal{I}_1^{-}(q))\Big),$$

where $Q = r_{\mathcal{I}}(n)$. The first term is clearly positive, and the second term, that corresponds to the decrease of the search cost when the set of searchable projects is improved, is positive because for any $q = Q, \ldots, I$, it is true that $\mathcal{I}_0^{-}(q) < \mathcal{I}_1^{-}(q)$ and $F(.,.)$ is decreasing in its second argument for the strength order.

If $\{n, n+1\} \subseteq \mathcal{N}_S$ and $r_{\mathcal{H}}(n) = r_{\mathcal{H}}(n+1) = 1$, then

$$\Delta = (\rho_n - \rho_{n+1}) f(0, \hat{\mathcal{I}}) G - (\rho_n - \rho_{n+1}) f(0, \hat{\mathcal{I}})(G+L) + L f(0, \hat{\mathcal{I}})(\rho_n - \rho_{n+1}) + c \sum_{q=Q}^{I} \Big(F(0, \mathcal{I}_0^-(q)) - F(0, \mathcal{I}_1^-(q)) \Big),$$

and the first three terms sum up to 0 so that $\Delta > 0$.

Finally, if $n \in \mathcal{N}_S$, $n+1 \notin \mathcal{N}_S$ and $r_{\mathcal{H}}(n) = 1$, then

$$\Delta = (\rho_n - \rho_{n+1}) f(0, \hat{\mathcal{I}}) G - f(0, \hat{\mathcal{I}})(1 - \rho_{n+1}) \Big(\rho_n (G+L) - L \Big) + c \sum_{q=Q}^{I} \Big(F(0, \mathcal{I}_0^-(q)) - F(0, \mathcal{I}_1^-(q)) \Big),$$

so that

$$\Delta = f(0, \hat{\mathcal{I}}) \rho_{n+1} (1 - \rho_n)(G + L) + c \sum_{q=Q}^{I} \Big(F(0, \mathcal{I}_0^-(q)) - F(0, \mathcal{I}_1^-(q)) \Big) > 0.$$

□

Appendix B The Communication Game

The incentive of a candidate n to deviate from an action profile $[\mathcal{I}, \mathcal{H}]$ is defined as the ratio of her deviation payoff over her current payoff, and it is denoted by $\delta(n, [\mathcal{I}, \mathcal{H}])$ or simply $\delta(n)$ when the context is clear.

Proof of Lemma 4. Let n and $n+1$ be two consecutive elements of $\mathcal{N}$ in $\mathcal{J}$, let $r = r_{\mathcal{H}}(n) = r_{\mathcal{H}}(n+1)$ be the rank that any of these projects would occupy in $\mathcal{H}$ and $i = r_{\mathcal{I}}(n)$ be the rank of n in $\mathcal{I}$, so that $r_{\mathcal{I}}(n+1) = i+1$. Then the incentives to deviate of the two candidates are given by

$$\delta(n) = \frac{F(M - r, \mathcal{I} \setminus \{n\})}{F(M-1, \mathcal{I}^-(i-1)) \rho_n},$$

and

$$\delta(n+1) = \frac{F(M - r, \mathcal{I} \setminus \{n+1\})}{F(M-1, \mathcal{I}^-(i)) \rho_{n+1}}.$$

Therefore, with the help of Lemma 2

$$\frac{\delta(n+1)}{\delta(n)} = \frac{\rho_n\big(X - Y\rho_n\big)}{\rho_{n+1}\big(X - Y\rho_{n+1}\big)} \frac{F(M-1, \mathcal{I}^-(i-1))}{F(M-1, \mathcal{I}^-(i))},$$

where

$$X = F(M - r, \mathcal{I} \setminus \{n, n+1\}) > 0,$$

and

$$Y = f(M - r, \mathcal{I} \setminus \{n, n+1\}) > 0.$$

The second fraction is clearly greater than 1 because $F(P, .)$ is decreasing in its second argument for the set order. As for the first fraction, notice that the function $\rho(X - \rho Y)$ is increasing in ρ on $(0, 1/2)$ whenever $X/Y \geq 1$, and the latter is obviously satisfied. Since $\rho_n > \rho_{n+1}$, this fraction is also greater than 1 when $\rho_1 \leq 1/2$. Therefore $\delta(n+1) > \delta(n)$, which concludes the proof for this case.

When $M = 1$, $\delta(n+1)/\delta(n)$ is equal to $\rho_n/(\rho_{n+1}(1-\rho_n)) > 1$. □

Proof of Proposition 7. By Proposition 5, the only incentive condition that needs to be checked is that of N_S, the weakest candidate in $\mathcal{N}_S$, which is done in (9). When $\mathcal{N}_W = \emptyset$, the right-hand side of (9) becomes equal to 1, proving the second statement. For the last point, remember that it is a dominant strategy forall the candidates in $\mathcal{N}_W$ to disclose their information. Consider, N_S, the weakest candidate in $\mathcal{N}_S$. When (9) holds strictly, the proof is immediate if $N_S = 1$, while if $N_S > 1$ it is strictly optimal for N_S to disclose when all the candidates in $\mathcal{N}^-(N_S - 1)$ disclose as well. If on the other hand M or more candidates in $\mathcal{N}^-(N_S - 1)$ were to withhold their information, it would clearly be strictly optimal for N_S to disclose her information as she would stand no chance of being rubberstamped otherwise. Finally, suppose that $K < M$ candidates in $\mathcal{N}^-(N_S - 1)$ withhold their information and denote by $\mathcal{K} \subseteq \mathcal{N}^-(N_S - 1)$ this set of candidates, and $\mathcal{J} = \mathcal{N}^-(N_S - 1) \setminus \mathcal{K}$. In this case, N_S strictly prefers to disclose her information if and only if

$$\rho_{N_S} > \frac{F(M-K-1, \mathcal{N}^+(N_S) \cup \mathcal{J})}{F(M-1, \mathcal{J})}. \tag{20}$$

Because F is decreasing in its second argument for the set order, $F(M-1, \mathcal{J}) > F(M-1, \mathcal{N}^-(N_S-1))$. And by Lemma 3, $F(M-K-1, \mathcal{N}^+(N_S) \cup \mathcal{J}) < F(M-1, \mathcal{N}^+(N_S) \cup \mathcal{J} \cup \mathcal{K}) = F(M-1, \mathcal{N} \setminus \{N_S\})$. Therefore, (9) implies (20), showing that it is a dominant strategy for N_S to disclose. Now consider candidate $N_S - 1$. By Lemma 4, the equation obtained by replacing N_S by $N_S - 1$ in (9) is satisfied. Hence, repeating the above argument implies that it is also a dominant strategy for $N_S - 1$ to disclose. By induction, this shows that the game is dominance solvable. $\square$

Proof of Proposition 9. The fact that the equilibrium described in the proposition exists under the condition given is a direct consequence of Proposition 5. In fact there is an equilibrium such that n is the only candidate withholding information if and only if $\rho_n \leq (1-\rho_{n+1}) \ldots (1-\rho_N)$ and $\rho_{n-1} \geq (1-\rho_{n+1}) \ldots (1-\rho_N)$. The only point to prove is therefore uniqueness. Let $F_n = (1-\rho_{n+1}) \ldots (1-\rho_N)$, and note that F_n is an increasing sequence while ρ_n is a decreasing sequence. Then, by definition of n^*, $\rho_n \leq F_n$ if and only if $n \geq n^*$. But since for an equilibrium at n $\rho_{n-1} \geq F_n$ must also hold, n^* is the only possible n. Indeed if $n \geq n^*$, $\rho_{n-1} \leq F_{n-1} < F_n$ so that the second condition for an equilibrium cannot hold. $\square$

Proof of Proposition 10. Because $F(0, .)$ is decreasing in its second argument for the set order as well as for the strength order, and for any $\mathcal{N}$ and any $n \in \mathcal{N}_S$ it is true that $\mathcal{N}_W \subseteq \mathcal{N}^+(n)$, and therefore for every $n \in \mathcal{N}_S$, $F(0, \mathcal{N}_1^+(n)) < F(0, \mathcal{N}_0^+(n))$. Hence if n_γ is the unique candidate who withholds information in the equilibrium of the game $\gamma \in \{0, 1\}$, it must be true that $n_1 \geq n_0$ that is the withholding candidate is a weaker candidate in game 1 than in game 0. Since all the candidates in $\mathcal{N}_{W_\gamma}$ disclose their information in equilibrium, this implies that the decision maker prefers Γ_1 to Γ_0. $\square$

Proof of Proposition 11. First note that $\rho^+ > \overline{\rho}$ so that rubberstamping project 1 beats learning about it. Since 1 is the best project, any alternative policy of the decision maker that stands a chance of being optimal given that 1 is not providing information consists in learning about k projects and rubberstamping 1 only if this search proves unfruitful. The payoff of such a policy is of the form $P_1 + P_2$ where $P_1 = \rho^1 G - c + (1-\rho^1)(\rho^2 G - c + (1-\rho^2)(\rho^3 G - c + \dots))$ where the sum stops at $\rho^k G - c$, and $P_2 = (1-\rho^1)\dots(1-\rho^k)(\rho_1(G+L)-L)$, and where $\rho^1, \dots, \rho^k$ denote the unordered prospects of the k projects in the seacrh of the decision maker. Because $\rho^1, \dots, \rho^k < \rho_1$ and because the payoff $P_1 + P_2$ is increasing in each ρ^i, it is true that $P_1 + P_2 < (\rho_1 G - c)(1 + (1-\rho^1) + \dots + (1-\rho_1)^{k-1}) + (1-\rho_1)^k(\rho_1(G+L)-L) = (\rho_1 G - c)(1-(1-\rho_1)^k)/\rho_1 + (1-\rho_1)^k(\rho_1(G+L)-L)$. The payoff of rubberstamping 1 without going through the preliminary search is $\rho_1(G+L)-L$, and it is greater than the former expression if and only if (with some algebra)

$$\rho_1^2 - \rho_1 + \frac{c}{G+L} > 0.$$

The greatest root of the second degree equation associated with the former is

$$\rho^+ = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4c}{L+G}}\right),$$

where $1 > 4c/(L+G)$ is implied by the assumption that $c < LG/(L+G)$ as it is easy to see that $LG/(L+G) < (L+G)/4$. Therefore $\rho_1 > \rho^+$ implies that rubberstamping 1 beats the alternative strategy. Because, by withholding her information candidate 1 can force the decision maker to rubberstamp her project irrespective of the behavior of other candidates and therefore have her project implemented with probability 1, rubberstamping 1 has to be the outcome of the game. □

Proof of the Characterization of the Mixed Strategy Equilibrium in Example 2. Let λ be the probability of disclosure for candidate 1 and μ the same probability for candidate 2. In order to make 1 indifferent between disclosing and withholding her information, μ must satisfy $(\mu(1-\rho_2)+(1-\mu))f(0,\mathcal{N}_W)=\rho_1$ where the left-hand side is 1's payoff when withholding her information and the right-hand side is her payoff when she plays transparently. The same indifference condition for candidate 2 gives $\lambda(1-\rho_1)\rho_2+(1-\lambda)\rho_2=\lambda(1-\rho_1)f(0,\mathcal{N}_W)$. □

Appendix C Proofs for the Incomplete Information Case

Proof of Lemma 5. A weak type is never rubberstamped by the decision maker. □

Proof of Proposition 13. First note that (15) is equivalent to

$$\hat{\rho}\left(\frac{1-\int_{\hat{\rho}}^{\overline{x}} x dH(x)}{1-\int_{\underline{x}}^{\overline{x}} x dH(x)}\right)^{N-1} \geq 1,$$

and the left-hand side is equal to the ratio $V_I^1(\hat{\rho})/V_H^1(\hat{\rho})$ of the payoffs of a player with type $\hat{\rho}$ when all other players disclose with probability 1. Therefore, (15) means that there is no incentive of a player with type $\hat{\rho}$ to deviate from the full disclosure profile. It is therefore clearly a necessary condition for equilibrium.

To show that it is also sufficient, note that for a candidate with type $\rho>\hat{\rho}$, when all the other candidates disclose with probability 1,

$$\frac{V_I^1(\rho)}{V_H^1(\rho)}=\rho\left(\frac{1-\int_{\rho}^{\overline{x}} x dH(x)}{1-\int_{\underline{x}}^{\overline{x}} x dH(x)}\right)^{N-1} > \hat{\rho}\left(\frac{1-\int_{\hat{\rho}}^{\overline{x}} x dH(x)}{1-\int_{\underline{x}}^{\overline{x}} x dH(x)}\right)^{N-1} \geq 1,$$

implying that there is no incentive to deviate from the full disclosure profile for such a candidate. Since it is a dominant strategy for the types below $\hat{\rho}$ to disclose, this proves that (15) is also a sufficient condition.

In the absence of weak types, $\hat{N}$ is infinite and full disclosure cannot be an equilibrium.

Now suppose that (15) holds with a strict inequality. Consider a profile λ such that for every candidate n, $\rho<\tilde{\rho}$ implies $\lambda_n(\rho)=1$, that is all players adopt a strategy that prescribes to disclose with probability 1 when $\rho<\tilde{\rho}$. Suppose also that $\tilde{\rho}\geq\hat{\rho}$. Then consider the same ratio of payoffs as above for a player n with type $\rho \geq \tilde{\rho}$ when other players are playing according to λ

$$
\begin{aligned}
\frac{V_{I,n}^{\lambda}(\rho)}{V_{H,n}^{\lambda}(\rho)} &= \rho \prod_{m \neq n} \frac{1 - \int_{\rho}^{\overline{x}} x \lambda_m(x)}{\int_{\underline{x}}^{\overline{x}} \lambda_m(x)(1-x) dH(x) + \int_{\rho^0}^{\rho} (1 - \lambda_m(x)) \, dH(x)} \\
&\geq \hat{\rho} \left(\frac{1 - \int_{\hat{\rho}}^{\overline{x}} x dH(x)}{1 - \int_{\underline{x}}^{\overline{x}} x dH(x)} + H(\rho) - H(\tilde{\rho}) \right)^{N-1}. \qquad (21)
\end{aligned}
$$

where the lower bound is obtained from (11) and (14) by choosing λ_m adequately in each of the integrals.

The strict inequality in (15) implies by continuity that for some $\eta > 0$,

$$
\hat{\rho} \left(\frac{1 - \int_{\hat{\rho}}^{\overline{x}} x dH(x)}{1 - \int_{\underline{x}}^{\overline{x}} x dH(x)} + \eta) \right)^{N-1} > 1.
$$

I have already proved that it is a dominant strategy for weak types to disclose with probability 1. Therefore I can eliminate all strategies that do not satisfy this. Now, taking $\tilde{\rho} = \hat{\rho}$ and applying (21) to the types $\rho \geq \hat{\rho}$ such that $H(\rho) \leq H(\hat{\rho}) + \eta$ shows that it is also a dominant strategies to disclose with probability one at these types. But then, taking $\tilde{\rho}$ to be the type such that $H(\tilde{\rho}) = H(\hat{\rho}) + \eta$, and reapplying the same idea to the types $\rho > \tilde{\rho}$ such that $H(\rho) \leq H(\tilde{\rho}) + \eta = H(\hat{\rho}) + 2\eta$ shows that it is a dominant strategy to disclose with probability one for these types as well. Obviously, reiterating this procedure a finite number of times is sufficient to cover all the types in the type space $\mathcal{S}$. This proves that full disclosure is the unique strategy profile that survives the iterated elimination of strictly dominated strategies. □

Proof of Proposition 14. To show that $\hat{N}_G \leq \hat{N}_H$, note that I can rewrite

$$
\hat{N}_H = 1 + \frac{\log(1/\hat{\rho})}{-\log R_H},
$$

where

$$R_H = \frac{1 - \int_{\underline{x}}^{\overline{x}} x dH(x)}{1 - \int_{\hat{\rho}}^{\overline{x}} x dH(x)} = 1 - \frac{\int_{\underline{x}}^{\hat{\rho}} x dH(x)}{1 - \int_{\hat{\rho}}^{\overline{x}} x dH(x)},$$

and $\hat{N}$ is increasing in R_H.

Integrating by parts, I obtain

$$R_H = 1 + \frac{\int_{\underline{x}}^{\hat{\rho}} H(x)dx - \hat{\rho}H(\hat{\rho})}{1 - \overline{x} + \hat{\rho}H(\hat{\rho}) + \int_{\hat{\rho}}^{\overline{x}} H(x)dx}.$$

And clearly, $R_G \leq R_H$ since $G(x) \leq H(x)$ on $[\underline{x}, \hat{\rho}]$ and $G(x) \geq H(x)$ on $[\hat{\rho}, \overline{x}]$. Hence $\hat{N}_G \leq \hat{N}_H$.

$\square$

Proof of Lemma 6. Let $\Omega \subseteq \mathcal{S} \cap (\hat{\rho}, \overline{x})$ be an open interval such that $\lambda = 1$ on Ω and $x \in \Omega$. Let $y = \sup\{\rho | \forall \rho' \in [x, \rho],\ \lambda(\rho') = 1\}$. Suppose $y < \overline{x}$. By continuity of the payoff functions, it must be true that $V_I^\lambda(y) = V_H^\lambda(y)$. However, $V_I^\lambda(.)$ is strictly increasing on (x, y) while V_H^λ is constant on the same interval. Furthermore, since λ is an equilibrium strategy, $V_I^\lambda(x) > V_H^\lambda(x)$, but then by continuity $V_I^\lambda(y) > V_I^\lambda(x) > V_H^\lambda(x) = V_H^\lambda(y)$, a contradiction. $\square$

Proof of Proposition 15. The only claim that needs to be proved is the last point of the proposition. By Proposition 13, full disclosure is not an equilibrium. No disclosure at all cannot be an equilibrium either as then $V_I^0(\underline{x}) = \underline{x} > 0 = V_H^0(\underline{x})$. By Lemma 6, a pure strategy equilibrium must be of the type $\lambda(\rho) = \mathbb{1}_{\rho > \rho^*}$ for some $\rho^* \in (\underline{x}, \overline{x})$. But then, $V_I^\lambda(\rho^*) = \rho^* \left(1 - \int_{\rho^*}^{\overline{x}} x dH(x)\right)^{N-1} < \left(1 - \int_{\rho^*}^{\overline{x}} x dH(x)\right)^{N-1} = V_H^\lambda(\rho^*)$ which is a contradiction since the type ρ^* should be indifferent between the two actions. $\square$

Before proving Proposition 16 I establish the following lemma.

Lemma 7. *Let λ be an equilibrium strategy. V_I^λ and V_H^λ are continuously differentiable, and λ is continuous almost everywhere on $\mathcal{S}$ Furthermore, for almost every $\rho \in Int(\Lambda)$,*

$$\lambda(\rho)(1 + \rho^{1+\frac{1}{N-1}}) = 1 - \frac{1}{(N-1)h(\rho)} \left(1 - \int_\rho^{\overline{x}} x\lambda(x)dH(x)\right) \rho^{\frac{1}{N-1}-1}. \tag{22}$$

Proof. For any strategy λ, V_I^λ and V_H^λ are clearly differentiable, and their derivative are continuous on $\Lambda^0 \cup \Lambda^1$. On $Int(\Lambda)$, because V_I^λ is equal to V_H^λ if λ is an equilibrium, their derivatives are also equal. This equality yields (22), which shows that λ is continuous on $Int(\Lambda)$. Since λ is clearly continuous on Λ^1 and Λ^0, it is continuous on $\mathcal{S}$. □

Proof of Proposition 16. I proved in Proposition 15 that there is no equilibrium in pure strategies in the absence of weak types. Therefore $Int(\Lambda) \neq \emptyset$. Let (y, z) be a maximal interval of $Int(\Lambda)$. If $y > \underline{x}$ then there exists some $\eta > 0$ such that $(y - \eta, y) \subseteq \Lambda^0$ (λ could not be equal to 1 on $(y - \eta, y)$ because of Lemma 6). By continuity of the payoff functions $V_I^\lambda(y) = V_H^\lambda(y)$, and by Lemma 7 and the equilibrium condition (i), the left derivatives of the payoff functions at y must be ordered as follows

$$\left(V_H^\lambda\right)'_L(y) = (N-1)h(y)\left(V_H^\lambda(y)\right)^{\frac{N-2}{N-1}} < \left(V_I^\lambda\right)'_L(y) = V_I^\lambda(y)/y,$$

because $V_I^\lambda(y) = V_H^\lambda(y)$ I can rewrite

$$(N-1)h(y) < \left(V_H^\lambda(y)\right)^{\frac{1}{N-1}} = y^{\frac{1}{N-1}}\left(1 - \int_y^{\overline{x}} x\lambda(x)dH(x)\right) < y^{\frac{1}{N-1}} < 1,$$

because $h(y)$ and y are bounded away from 0 and from 1, the last equation clearly does not hold for N greater than some $\tilde{N}_1$. Therefore, for $N > \tilde{N}_1$, $y = \underline{x}$, that is the lowest types are strictly mixing.

Similarly, if $z < \overline{x}$, there exists $\eta > 0$ such that $(z, z + \eta) \subseteq \Lambda^1$ or $(z, z + \eta) \subseteq \Lambda^0$.

Start with the first case. Then, by Lemma 6, $(z, \overline{x}) \subseteq \Lambda^1$, and

$$V_I^\lambda(z) = z\left(1 - \int_z^{\overline{x}} xdH(x)\right)^{N-1},$$

and

$$V_H^\lambda(z) = \left(1 - \int_z^{\overline{x}} xdH(x) - \int_{\underline{x}}^z \lambda(x)xdH(x)\right)^{N-1}$$

Hence

$$\frac{V_H^\lambda(z)}{V_I^\lambda(z)} = \frac{1}{z}\left(1 - \frac{\int_{\underline{x}}^{z}\lambda(x)xdH(x)}{1-\int_{z}^{\overline{x}}xdH(x)}\right)^{N-1}.$$

And this ratio is equal to 1 if and only if $z^{\frac{1}{N-1}} = 1 - \frac{\int_{\underline{x}}^{z}\lambda(x)xdH(x)}{1-\int_{z}^{\overline{x}}xdH(x)}$. Since the left hand-side goes to 1 as $N \to \infty$ because $0 < \underline{x} < z < \overline{x} < 1$, the right hand-side must go to 1 as well. This occurs if and only if $z \to \underline{x}$ as $N \to \infty$. But in this case

$$\lim_{N\to\infty}\frac{V_H^\lambda(\underline{x})}{V_I^\lambda(\underline{x})} = 1/\underline{x} > 1,$$

which is a contradiction because it implies that for N sufficiently high, there is a neighborhood of $\underline{x}$ on which all types are withholding with probability 1.

Now suppose $(z, z+\eta) \subseteq \Lambda^0$. Then the derivatives of the payoff functions on $(z, z+\eta)$ are given by

$$\left(V_I^\lambda\right)'(\rho) = \left(1 - \int_{\rho}^{\overline{x}}\lambda(x)xdH(x)\right)^{N-1} = V_I^\lambda(z)/z,$$

and

$$\begin{aligned}\left(V_H^\lambda\right)'(\rho) &= (N-1)h(\rho)\left(\int_{\underline{x}}^{\overline{x}}\lambda(x)(1-x)dH(x) + \int_{\underline{x}}^{\rho}(1-\lambda(x))dH(x)\right)^{N-2}\\ &\geq (N-1)m\left(V_H^\lambda(z)\right)^{\frac{N-2}{N-1}},\end{aligned}$$

where $m > 0$ is the lower bound of h (see the setup of the incomplete information model in Section 5).

Hence, using $V_I^\lambda(z) = V_H^\lambda(z)$:

$$\frac{\left(V_H^\lambda\right)'(\rho)}{\left(V_I^\lambda\right)'(\rho)} \geq (N-1)\underline{x}^{1-\frac{1}{N-1}}m.$$

This implies that for N sufficiently high, V_H^λ grows faster than V_I^λ on $(z, z+\eta)$. Since $V_H^\lambda(z) = V_I^\lambda(z)$, V_I^λ and V_H^λ cannot cross on $(z, \overline{x})$, implying that $Int(\Lambda^0) = (z, \overline{x})$.

Then I can write

$$\frac{V_H^\lambda(z)}{V_I^\lambda(z)} = \frac{1}{z}\left(1 - \int_{\underline{x}}^{z} x\lambda(x)dH(x)\right)^{N-1}$$

This ratio must be equal to 1. This cannot happen as $N \to \infty$ unless $z \to \underline{x}$. But if that is the case, picking some $\rho \in (\underline{x}, \overline{x})$, I have

$$\lim_{N\to\infty} \frac{V_H^\lambda(\rho)}{V_I^\lambda(\rho)} = \lim_{N\to\infty} \frac{1}{\rho}\left(1 - \int_{\underline{x}}^{z} \lambda(x)xdH(x)\right)^{N-1} = 0,$$

implying that for N sufficiently high ρ is not disclosing (because for N sufficiently high $z < \rho$), but would prefer to disclose, which contradicts the fact that λ is an equilibrium. Note that the intuition comes from the fact that no disclosure at all cannot be an equilibrium for any candidate would prefer to disclose if nobody else does.

This concludes the first part of the argument, showing that for N sufficiently high, an equilibrium strategy λ must be strictly mixing almost everywhere.

Then, taking the limit when N goes to ∞ in (22), and because $\frac{1}{h(\rho)}\left(1 - \int_\rho^{\overline{x}} x\lambda(x)dH(x)\right)\rho^{\frac{1}{N-1}-1}$ is bounded,

$$\lim_{N\to\infty} \lambda(\rho) = \frac{1}{1+\rho} \quad a.e.$$

□

Appendix D Further Reduction of Equilibrium Incentive Constraints

In this appendix, I prove a result similar to Lemma 4 in that it allows me to reduce the incentive constraints that need to be checked for chains of projects in $\mathcal{H}$. This result does not hold in general however (counterexamples can indeed be exhibited), and some restricting assumptions are needed. The next lemma is conducive to these assumptions, and the lemma and its corollary present some intrinsic interest as they provide a sufficient condition on the primitives of the framework under which the probability of finding exactly p good projects in a given set of projects is increasing in p.

Lemma 8. *Let $\mathcal{K} \subseteq \mathcal{N}$ such that there is a lower bound b satisfying $\rho_k/(1-\rho_k) > b$, for every $k \in \mathcal{K}$. Then, $f(p, \mathcal{K})$ is increasing in p for $p < K + 1 - 1/b$. That is the probability of finding exactly p good projects in $\mathcal{K}$ is increasing in p for low values of p.*

Proof.

$$f(0, \mathcal{K}) = \prod_{k \in \mathcal{K}} (1 - \rho_k),$$

$$\frac{f(1, \mathcal{K})}{f(0, \mathcal{K})} = \sum_{k \in \mathcal{K}} \frac{\rho_k}{1 - \rho_k},$$

more generally

$$\frac{f(p, \mathcal{K})}{f(0, \mathcal{K})} = \sum_{k_1 \in \mathcal{K}} \frac{\rho_{k_1}}{1 - \rho_{k_1}} \sum_{k_2 \in \mathcal{K} \setminus \{k_1\}} \frac{\rho_{k_2}}{1 - \rho_{k_2}} \cdots \sum_{k_p \in \mathcal{K} \setminus \{k_1, \ldots, k_{p-1}\}} \frac{\rho_{k_p}}{1 - \rho_{k_p}}.$$

Therefore, for $p > 1$

$$\frac{f(p, \mathcal{K})}{f(p-1, \mathcal{K})} = \frac{\sum_{k_1 \in \mathcal{K}} \frac{\rho_{k_1}}{1-\rho_{k_1}} \sum_{k_2 \in \mathcal{K} \setminus \{k_1\}} \frac{\rho_{k_2}}{1-\rho_{k_2}} \cdots \sum_{k_{p-1} \in \mathcal{K} \setminus \{k_1, \ldots, k_{p-2}\}} \frac{\rho_{k_{p-1}}}{1-\rho_{k_{p-1}}} \sum_{k_p \in \mathcal{K} \setminus \{k_1, \ldots, k_{p-1}\}} \frac{\rho_{k_p}}{1-\rho_{k_p}}}{\sum_{k_1 \in \mathcal{K}} \frac{\rho_{k_1}}{1-\rho_{k_1}} \sum_{k_2 \in \mathcal{K} \setminus \{k_1\}} \frac{\rho_{k_2}}{1-\rho_{k_2}} \cdots \sum_{k_{p-1} \in \mathcal{K} \setminus \{k_1, \ldots, k_{p-2}\}} \frac{\rho_{k_{p-1}}}{1-\rho_{k_{p-1}}}}.$$

The sum at the numerator is a weighted sum of the same terms as the sum in the denominator with weights of the form $\sum_{k_p \in \mathcal{K} \setminus \{k_1, \ldots, k_{p-1}\}} \frac{\rho_{k_p}}{1-\rho_{k_p}}$ on each term. With $\rho/(1-\rho) > b$ for each term of the sum in the latter, and $K - p + 1$ terms, each of these weights is greater than $(K - p + 1)b > 1$ whenever $p < K + 1 - 1/b$. Therefore $f(p, \mathcal{K}) > f(p-1, \mathcal{K})$. □

This leads to.

Corollary 2. *If $[\mathcal{I}, \mathcal{H}]$ is an action profile such that $H \leq M$ and $\mathcal{H} \subseteq \mathcal{N}_S$ (hence in particular if it is an equilibrium action profile) and $2M < N + 1 - 1/\hat{\rho}$ then the probability of finding*

$p \leq M$ good projects in $\mathcal{I}$, $f(p, \mathcal{I})$, is increasing in p for $p \leq M$.

Proof. $\mathcal{H} \subseteq \mathcal{N}'$ implies that for every project $h \in \mathcal{H}$, $\rho_h > \hat{\rho}$. By Lemma 8, $f(p, \mathcal{I})$ is increasing in p whenever $p < I + 1 - 1/\hat{\rho} = N - H + 1 - 1/\hat{\rho}$. Since $p \leq M$ and $H \leq M$, it must be true that $p + H \leq 2M < N + 1 - 1/\hat{\rho}$ which concludes the proof. □

Therefore, using the terminology of Appendix B, the main result of this appendix can be stated as

Lemma 9. *Take an action profile $[\mathcal{I}, \mathcal{H}]$ satisfying $H \leq M$ and $\mathcal{H}_S \subseteq \mathcal{N}_S$ and a chain $\mathcal{K} \subseteq \mathcal{H}$. Then, assuming*

$$M < \min\left(\frac{N + 1 - 1/\hat{\rho}}{2}, \min_{(n,n+1) \in \mathcal{N}_S^2}\left(\frac{\rho_n}{\rho_n - \rho_{n+1}}\right)\right), \tag{23}$$

for any $p < H$, $k(p+1)$ has a higher incentive to deviate to disclosure than $k(p)$.

Proof. Let n and $n+1$ be two consecutive projects of $\mathcal{N}$ in the chain $\mathcal{K}$, let $h = r_{\mathcal{H}}(n)$ so that $r_{\mathcal{H}}(n+1) = h+1$ be the ranks of these projects in $\mathcal{H}$ and $i = r_{\mathcal{I}}(n) = r_{\mathcal{I}}(n+1)$ be the rank either project would occupy in $\mathcal{I}$ if the candidate were to switch to disclosure. Then the incentives to deviate of the two candidates are given by

$$\delta(n) = \frac{\rho_n F\big(M - 1, \mathcal{I}^-(i-1)\big)}{F\big(M - h, \mathcal{I}\big)},$$

and

$$\delta(n+1) = \frac{\rho_{n+1} F\big(M - 1, \mathcal{I}^-(i-1)\big)}{F\big(M - h - 1, \mathcal{I}\big)}.$$

Hence

$$\frac{\delta(n+1)}{\delta(n)} = \frac{\rho_{n+1}}{\rho_n} \frac{F\big(M - h, \mathcal{I}\big)}{F\big(M - h - 1, \mathcal{I}\big)}.$$

Notice that the second fraction satisfies

$$\frac{F\big(M - h, \mathcal{I}\big)}{F\big(M - h - 1, \mathcal{I}\big)} = 1 + \frac{f(M - h, \mathcal{I})}{F(M - h - 1, \mathcal{I})} = 1 + \frac{1}{\sum_{p=0}^{M-h-1} f(p, \mathcal{I})/(f(M - h, \mathcal{I}))}.$$

By Corollary 2, under the assumption that $M < (N+1-1/\hat{\rho})/2$, each term in the sum is less than 1, and therefore the sum is less than $M-h \leq M-1$. Therefore the second fraction is greater than $1+1/(M-1)$ and $M < \min_{(n,n+1)\in\mathcal{N}_S^2}(\rho_n/(\rho_n-\rho_{n+1}))$ implies that $\delta(n+1)/\delta(n) > 1$. This proves the claim. $\square$

And this leads to a new characterization of equilibrium action profiles.

Proposition 18. *If (23) is satisfied and $M=1$ or $\rho_1 < 1/2$, an action profile $[\mathcal{I},\mathcal{H}]$ is a pure strategy equilibrium of the communication game if and only if it satisfies*

(i) $\mathcal{H} \subset \mathcal{N}_S$.

(ii) $H \leq M$.

(iii) For any maximal chain $\mathcal{J} \subseteq \mathcal{I}_S$,

$$F\Big(M-1, \mathcal{I}^-\big(r_{\mathcal{I}}(j(J))-1\big)\Big)\rho_{j(J)} \geq F\Big(M - r_{\mathcal{H}}(j(J)), \mathcal{I} \setminus \{j(J)\}\Big).$$

(iv) For any maximal chain $\mathcal{K} \subseteq \mathcal{H}$,

$$F\Big(M-1, \mathcal{I}^-\big(r_{\mathcal{I}}(k(K))-1\big)\Big)\rho_{k(K)} \leq F\Big(M - r_{\mathcal{H}}(k(K)), \mathcal{I}\Big).$$

Note that condition (23) can be interpreted in two ways: the upper bound on M says that the environment must be sufficiently competitive for the result to hold; it is also more easily satisfied when strong projects are close to one another, that is when the *ex ante* prospects of the projects in $\mathcal{N}_S$ are homogeneous.

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