Dynamic Macroeconomics using Matlab
Lecture 4

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Content

Real business cycles

- Endogenous labor supply:
- Proper RBC model with employment fluctuations
- Period utility over consumption c and labor supply l
- We have a series of optimal decisions that give rise to what we will call `General Competitive Equilibrium`.
- We have two agents: household and firms (no Government and International sectors)

Representative household

- Period utility over consumption c and labor supply l

$$u(c, l)$$

- Strictly increasing in c, strictly decreasing in l

Representative household

- Social planner maximizes expected intertemporal utility

$$U\left([c_t]_{t=0}^{\infty}\right) = E_0\left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)\right] \tag{1}$$

subject to sequence of budget constraints, for each date and state

$$c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta) k_t \tag{2}$$

- Initial $k_0 > 0$ and stochastic processes for wage w_t and rental rate r_t taken as given
- `Rational expectations` : household understands mapping from exogenous stochastic processes like productivity to endogenous stochastic processes like wage and rental rate

Representative household: planner's problem

- Using Eq (1) and Eq (2):

$$\max_{c_t, k_{t+1}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right]$$

$$\text{s.t: } c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta) k_t$$

- There are two ways to solve this problem: using dynamic solution and F.O.C

Representative household

- Lagrangian with multiplier $\lambda_t > 0$ for each resource constraint

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) + \sum_{t=0}^{\infty} \lambda_t \left[w_t l_t + (r_t + 1 - \delta) k_t - c_t - k_{t+1} \right] \right\}$$

- Some key first order conditions

$$\begin{aligned}
c_t : \quad & \beta^t u_c(c_t, l_t) - \lambda_t = 0 \\
l_t : \quad & \beta^t u_l(c_t, l_t) + \lambda_t w_t = 0 \\
k_{t+1} : \quad & -\lambda_t + E_t \left\{ \lambda_{t+1} \left[r_t + (1 - \delta) \right] \right\} = 0 \\
\lambda_t : \quad & w_t l_t + (r_t + 1 - \delta) k_t - c_t - k_{t+1} = 0
\end{aligned}$$

- Using first and second equation

$$\lambda_t = \beta^t u_c(c_t, l_t) \qquad \lambda_{t+1} = \beta^{t+1} u_c(c_{t+1}, l_{t+1})$$

$$-\lambda_t w_t = \beta^t u_l(c_t, l_t)$$

Representative household

- Using labor F.O.C

$$\lambda_t = \beta^t u_c(c_t, l_t) \qquad \lambda_{t+1} = \beta^{t+1} u_c(c_{t+1}, l_{t+1})$$
$$-\lambda_t w_t = \beta^t u_l(c_t, l_t)$$

- New intratemporal condition governing optimal labor supply

$$-\lambda_t w_t = \beta^t u_l(c_t, l_t)$$
$$-\beta^t u_c(c_t, l_t) w_t = \beta^t u_l(c_t, l_t)$$
$$w_t = -\frac{\beta^t u_l(c_t, l_t)}{\beta^t u_c(c_t, l_t)}$$

- Usual consumption (`Euler equation`)

$$u_c(c_t, l_t) = \beta u_c(c_{t+1}, l_{t+1}) \left[r_{t+1} + (1-\delta)\right]$$

- and budget constraint

$$c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta) k_t$$

Representative firm

- Production function

$$y_t = f(k_t, l_t)$$

where y_t will take as example $y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$

- Maximize profits

$$\pi = zf(k,l) - rk - wl$$

- At an optimum, marginal products equal factor prices

$$\begin{aligned} zf_k(k,l) &= r \\ zf_l(k,l) &= w \end{aligned}$$

Where:

  - partial derivate:

$$f_k = \frac{\partial f}{\partial k} \qquad f_l = \frac{\partial f}{\partial l}$$

- Note by constant returns of $f(k,l)$

$$rk + wl = zf(k,l)$$

Equilibrium

- The `equilibrium` condition:

$$y_t = c_t + i_t$$

- and

$$w_t = -\frac{\beta^t u_l(c_t, l_t)}{\beta^t u_c(c_t, l_t)}$$
$$zf_l = -\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)}$$

- and

$$u_c(c_t, l_t) = \beta u_c(c_{t+1}, l_{t+1}) \left[r_{t+1} + (1-\delta)\right]$$
$$u_c(c_t, l_t) = \beta u_c(c_{t+1}, l_{t+1}) \left[z_{t+1} f_k(k_{t+1}, l_{t+1}) + (1-\delta)\right]$$

- and

$$c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta) k_t$$
$$c_t + k_{t+1} = w_t l_t + r k_t + (1-\delta) k_t$$
$$c_t + k_{t+1} = z_t f(k_t, l_t) + (1-\delta) k_t$$

Standard parameterization

- Using Eq (1) and Eq (2):

$$\max_{c_t, k_{t+1}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right]$$

$$\text{s.t: } c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta) k_t$$

- Where:

$$u(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{l^{1+\varphi}}{1+\varphi}$$

where as we will see, φ controls elasticity of labor supply

- Production function

$$y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$$

Standard parameterization

- With these functional forms, system becomes

$$zf_l = -\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)}$$

$$z_t(1-\alpha)k_t^{\alpha} l_t^{-\alpha} = -\left(\frac{-l^{\varphi}}{c^{-\sigma}}\right)$$

$$l_t^{\varphi} c_t^{\sigma} = (1-\alpha) z_t \left(\frac{k_t}{l_t}\right)^{\alpha}$$

- and

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma}\left[\alpha z_{t+1}\left(\frac{k_{t+1}}{l_{t+1}}\right)^{\alpha-1} + (1-\delta)\right]$$

- and

$$c_t + k_{t+1} = z_t k_t^{\alpha} l_t^{1-\alpha} + (1-\delta)\, k_t$$

- and

$$w_t = z_t(1-\alpha)k_t^{\alpha} l_t^{-\alpha} \qquad r_t = z_t(\alpha)k_t^{\alpha-1} l_t^{1-\alpha}$$

Standard parameterization

Equation definition
$l_t^{\varphi} c_t^{\sigma} = (1-\alpha) z_t \left(\frac{k_t}{l_t} \right)^{\alpha}$ (Labor supply)
$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \left[\alpha z_{t+1} \left(\frac{k_{t+1}}{l_{t+1}} \right)^{\alpha-1} + (1-\delta) \right]$ (Euler equation)
$w_t = z_t (1-\alpha) k_t^{\alpha} l_t^{-\alpha}$ $r_t = z_t (\alpha) k_t^{\alpha-1} l_t^{1-\alpha}$ (return for capital and labor)
$k_{t+1} = (1-\delta) k_t + i_t$ (Law of motion for capital)
$y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$ (Production function)
$y_t = c_t + i_t$ (Equilibrium condition)

Table: Model structure.

Steady state

- Steady state where $\triangle c_t = 0$ and $\triangle k_t = 0$, Let c_{ss} k_{ss} denote steady state values. These are determined by table 2

Equation definition
$l_{ss}^{\varphi} c_{ss}^{\sigma} = (1-\alpha) z_{ss} \left(\frac{k_{ss}}{l_{ss}}\right)^{\alpha}$ (Labor supply)
$1 = \beta \left[\alpha z_{ss} \left(\frac{k_{ss}}{l_{ss}}\right)^{\alpha-1} + (1-\delta)\right]$ (Euler equation)
$w_{ss} = z_{ss}(1-\alpha)\left(\frac{k_{ss}}{l_{ss}}\right)^{\alpha}$ $r_{ss} = z_{ss}(\alpha)\left(\frac{k_{ss}}{l_{ss}}\right)^{1-\alpha}$ (return for capital and labor)
$\delta = \frac{i_{ss}}{k_{ss}}$ (Law of motion for capital)
$y_{ss} = z_{ss} k_{ss}^{\alpha} l_{ss}^{1-\alpha}$ (Production function)
$y_{ss} = c_{ss} + i_{ss}$ (Equilibrium condition)

Table: Model steady state.

Steady state

- Euler equation and ratio capital-labor perworker

$$1 = \beta \left[\alpha z_{ss} \left(\frac{k_{ss}}{l_{ss}} \right)^{\alpha - 1} + (1 - \delta) \right]$$

$$\frac{1}{\beta} = \alpha z_{ss} \left(\frac{k_{ss}}{l_{ss}} \right)^{\alpha - 1} + (1 - \delta)$$

$$\frac{1}{\beta} - (1 - \delta) = \alpha z_{ss} \left(\frac{k_{ss}}{l_{ss}} \right)^{\alpha - 1}$$

$$\frac{\alpha z_{ss}}{\beta - (1 - \delta)} = \left(\frac{k_{ss}}{l_{ss}} \right)^{1 - \alpha}$$

$$\frac{k_{ss}}{l_{ss}} = \left(\frac{\alpha z_{ss}}{\beta - (1 - \delta)} \right)^{\frac{1}{(1 - \alpha)}}$$

Calibration

- Calibration table 3

Parameter	Meaning of parameter	Calibrate value
φ	Labor supply elasticity	1
α	Output elasticity of capital	0.33
β	Discount factor	0.99
δ	Depreciation rate	0.025
z_{ss}	Technology shock	1

Table: Parameter values of the structural model.