# Macroeconomía Dinámica
# Modelos DGSE

Educate
Edinson Tolentino
email: edinson.tolentino@gmail.com

# Contenido

# Modelo RBC

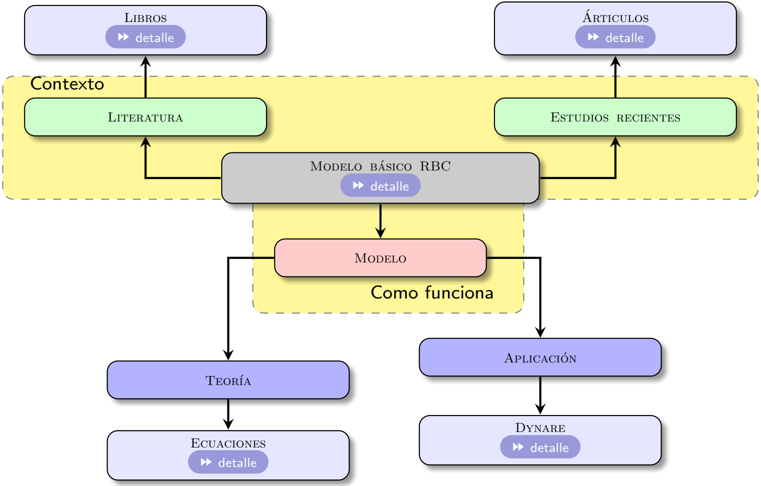


Figura: Clase Modelo

# Modelos RBC

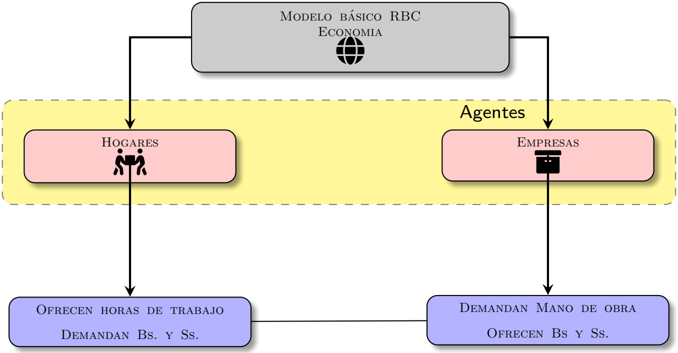


Figura: Modelo RBC básico

esquema

# Literatura

EDÚCATE PERÚ CONSULTORES

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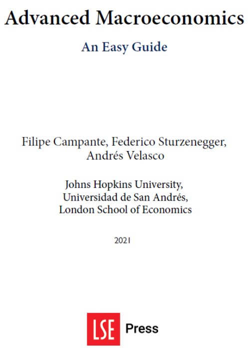
Figura 1:


Advanced Macroeconomics

An Easy Guide

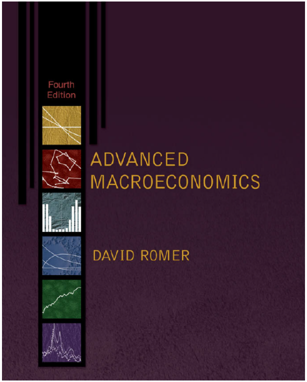
Filipe Campante, Federico Sturzenegger,
Andrés Velasco

Johns Hopkins University,
Universidad de San Andrés,
London School of Economics

2021

LSE Press

Figura 2:



# Dynare model file

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- The structure of a Dynare model file (`.mod`)
  - Declare endogenous variables
  - Declare exogenous variables
  - Declare parameters
  - Declare the model equations
  - Ask Dynare to solve for the steady state

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# Declare endogenous variables

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- Note productivity $z_t$ is treated as endogenous

Listing: Script

```

// (1) declare endogenous variables

var         c, k, l, z;
```

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# Declare exogenous variables

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- It is the innovations $\varepsilon_t$ that are fundamentally exogenous, given technology shock

Listing: Script

```

// (2) declare exogenous variables (shocks)

varexo     e;
```

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# Declare parameters

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- List of parameter names

Listing: Script

```

// (3) declare parameters

parameters alpha, beta, delta, sigma, phi, sigmaeps, varphi;
```

- Set parameter values

Listing: Script

```

alpha    = 0.485;
beta     = 0.925;
delta    = 0.078;
phi      = 0.95;
sigma    = 1;
sigmaeps = 0.01;
varphi   = 0.397;
```

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# Declare parameters

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- Set parameter values

Listing: `Script`

```

alpha    = 0.485;
beta     = 0.925;
delta    = 0.078;
phi      = 0.95;
sigma    = 1;
sigmaeps = 0.01;
varphi   = 0.397;
```

- Calibration table 3

| Parameter | Meaning of parameter | Calibrate value |
|---|---|---|
| $\alpha$ | Output elasticity of capital | 0.485 |
| $\beta$ | Discount factor | 0.925 |
| $\delta$ | Depreciation rate | 0.078 |
| $\sigma$ | Consumption elasticity | 1 |
| $z_{ss}$ | Technology shock | 1 |
| $\rho$ | Autoregressive coefficient for productivity | 0.95 |
| $\sigma_{\varepsilon}$ | Shock error | 0.01 |
| $\varphi$ | Labor supply elasticity | 0.397 |

Cuadro: Parameter values of the structural model.

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# Declare the model equations

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## Labor elasticity

- In usual notation

$$l_t^{\varphi} c_t^{\sigma} = (1-\alpha) z_t \left( \frac{k_t}{l_t} \right)^{\alpha}$$

Listing: `Script`

```
// labor supply
exp(l)^(varphi)*exp(c)^(sigma) =
(1-alpha)*exp(z)*(exp(k)^(alpha))*(exp(l)^(-alpha));
```

- Variables chosen at $t$ have no time argument
- Variables chosen at $t-1$ have $-1$ argument
- Variables chosen at $t+1$ have $+1$ argument

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# Declare the model equations



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**Consumption Euler equation**

- In usual notation

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \left[ \alpha z_{t+1} \left( \frac{k_{t+1}}{l_{t+1}} \right)^{\alpha - 1} + (1 - \delta) \right]$$

- In Dynare notation, supposing we want an approximation in logs

Listing: Script

```
// consumption Euler equation
exp(c)^(-sigma) = beta*(exp(c(+1))^(-sigma))*
 (alpha*exp(z(+1))*(exp(k(+1))^(alpha-1))*
 (exp(l(+1))^(1-alpha))+1-delta);
```

- Variables chosen at $t$ have no time argument
- Variables chosen at $t-1$ have $-1$ argument
- Variables chosen at $t+1$ have $+1$ argument

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# Declare the model equations

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## Resource constraint

- In usual notation

$$c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta)\, k_t$$

- In Dynare notation, supposing we want an approximation in logs

Listing: Script

```
// resource constraint
exp(c) + exp(k) = exp(z)*(exp(k(-1))^alpha)*
(exp(l)^(1-alpha))+(1-delta)*exp(k(-1));
```

- Variables chosen at $t$ have no time argument
- Variables chosen at $t-1$ have $-1$ argument
- Variables chosen at $t+1$ have $+1$ argument

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# Declare the model equations

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**Law of motion for productivity**

- In usual notation

$$\ln z_{t+1} = \rho \ln z_t + \varepsilon_{t+1}$$

- In Dynare notation, supposing we want an approximation in logs

Listing: Script

```
// law of motion productivity
z = phi*z(-1) + e;
```

- Variables chosen at $t$ have no time argument
- Variables chosen at $t-1$ have $-1$ argument
- Variables chosen at $t+1$ have $+1$ argument

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# Declare the model equations

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- Start block with `model` that list equations, then `end`

Listing: Script

```

// (4) declare the model equations

model;
// labor supply
exp(l)^(varphi)*exp(c)^(sigma) =
(1-alpha)*exp(z)*(exp(k)^(alpha))*(exp(l)^(-alpha));

// consumption Euler equation
exp(c)^(-sigma) = beta*(exp(c(+1))^(-sigma))*
(alpha*exp(z(+1))*(exp(k(+1))^(alpha-1))*
(exp(l(+1))^(1-alpha))+1-delta);

// resource constraint
exp(c) + exp(k) = exp(z)*(exp(k(-1))^alpha)*
(exp(l)^(1-alpha))+(1-delta)*exp(k(-1));


// law of motion productivity
z = phi*z(-1) + e;

end;
```

⏩ siguiente ⏩ esquema

# Solve for the steady state

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- Solve for steady state numerically (system of nonlinear equations)

Listing: Script

```


// (5) solve the steady state
initval;
c = 0.75;
k = 3.5;
l = 0.3;
z = 1;
e = 0;
end;
```

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# Set the shocks

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- Set the variance/covariance structure of shocks

Listing: Script

```
// specify variance of shocks

shocks;
var e = 100*sigmaeps^2;
end;
```

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# Solve for the dynamics

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- Solve for coefficients, obtain moments, plot impulse responses etc

Listing: Script

```
// (6) solve the dynamics
stoch_simul(order=2,irf=60);
```

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# Representative household

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- Social planner maximizes expected intertemporal utility

$$U\left([c_t]_{t=0}^{\infty}\right) = E_0\left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)\right] \tag{1}$$

subject to sequence of budget constraints, for each date and state

$$c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta)k_t \tag{2}$$

- Initial $k_0 > 0$ and stochastic processes for wage $w_t$ and rental rate $r_t$ taken as given
- `Rational expectations` : household understands mapping from exogenous stochastic processes like productivity to endogenous stochastic processes like wage and rental rate

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# Representative household: planner's problem

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- Using Eq (1) and Eq (2):

$$\max_{c_t, k_{t+1}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right]$$

$$\text{s.t: } c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta) k_t$$

- There are two ways to solve this problem: using dynamic solution and F.O.C

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# Representative household

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- Lagrangian with multiplier $\lambda_t > 0$ for each resource constraint

$$\mathcal{L} = E_0\left\{\sum_{t=0}^{\infty}\beta^t u(c_t, l_t) + \sum_{t=0}^{\infty}\lambda_t\left[w_t l_t + (r_t + 1 - \delta)k_t - c_t - k_{t+1}\right]\right\}$$

- Some key first order conditions

$$\begin{aligned}
c_t : &\quad \beta^t u_c(c_t, l_t) - \lambda_t = 0 \\
l_t : &\quad \beta^t u_l(c_t, l_t) + \lambda_t w_t = 0 \\
k_{t+1} : &\quad -\lambda_t + E_t\left\{\lambda_{t+1}\left[r_t + (1-\delta)\right]\right\} = 0 \\
\lambda_t : &\quad w_t l_t + (r_t + 1 - \delta)k_t - c_t - k_{t+1} = 0
\end{aligned}$$

- Using first and second equation

$$\lambda_t = \beta^t u_c(c_t, l_t) \qquad \lambda_{t+1} = \beta^{t+1} u_c(c_{t+1}, l_{t+1})$$

$$-\lambda_t w_t = \beta^t u_l(c_t, l_t)$$

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# Representative household

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- Using labor F.O.C

$$\lambda_t = \beta^t u_c(c_t, l_t) \qquad \lambda_{t+1} = \beta^{t+1} u_c(c_{t+1}, l_{t+1})$$
$$-\lambda_t w_t = \beta^t u_l(c_t, l_t)$$

- New intratemporal condition governing optimal labor supply

$$\begin{aligned} -\lambda_t w_t &= \beta^t u_l(c_t, l_t) \\ -\beta^t u_c(c_t, l_t) w_t &= \beta^t u_l(c_t, l_t) \\ w_t &= -\frac{\beta^t u_l(c_t, l_t)}{\beta^t u_c(c_t, l_t)} \end{aligned}$$

- Usual consumption (`Euler equation`)

$$u_c(c_t, l_t) = \beta u_c(c_{t+1}, l_{t+1}) \left[ r_{t+1} + (1-\delta) \right]$$

- and budget constraint

$$c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta) k_t$$

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# Representative firm

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- Production function

$$y_t = f(k_t, l_t)$$

where $y_t$ will take as example $y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$

- Maximize profits

$$\pi = zf(k,l) - rk - wl$$

- At an optimum, marginal products equal factor prices

$$\begin{aligned} zf_k(k,l) &= r \\ zf_l(k,l) &= w \end{aligned}$$

Where:

- partial derivate:

$$f_k = \frac{\partial f}{\partial k} \qquad f_l = \frac{\partial f}{\partial l}$$

- Note by constant returns of $f(k,l)$

$$rk + wl = zf(k,l)$$

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# Equilibrium

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- The equilibrium condition:

$$y_t = c_t + i_t$$

- and

$$w_t = -\frac{\beta^t u_l(c_t, l_t)}{\beta^t u_c(c_t, l_t)}$$
$$zf_l = -\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)}$$

- and

$$u_c(c_t, l_t) = \beta u_c(c_{t+1}, l_{t+1})\left[r_{t+1} + (1-\delta)\right]$$
$$u_c(c_t, l_t) = \beta u_c(c_{t+1}, l_{t+1})\left[z_{t+1} f_k(k_{t+1}, l_{t+1}) + (1-\delta)\right]$$

- and

$$c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta)\, k_t$$
$$c_t + k_{t+1} = w_t l_t + r k_t + (1-\delta)\, k_t$$
$$c_t + k_{t+1} = z_t f(k_t, l_t) + (1-\delta)\, k_t$$

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# Standard parameterization

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- Using Eq (1) and Eq (2):

$$\max_{c_t, k_{t+1}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right]$$

$$\text{s.t: } c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta) k_t$$

- Where:

$$u(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{l^{1+\varphi}}{1+\varphi}$$

where as we will see, $\varphi$ controls elasticity of labor supply

- Production function

$$y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$$

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# Standard parameterization

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- With these functional forms, system becomes

$$
\begin{aligned}
zf_l &= -\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} \\
z_t(1-\alpha)k_t^{\alpha} l_t^{-\alpha} &= -\left(\frac{-l^{\varphi}}{c^{-\sigma}}\right) \\
l_t^{\varphi} c_t^{\sigma} &= (1-\alpha) z_t \left(\frac{k_t}{l_t}\right)^{\alpha}
\end{aligned}
$$

- and

$$
c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \left[\alpha z_{t+1} \left(\frac{k_{t+1}}{l_{t+1}}\right)^{\alpha-1} + (1-\delta)\right]
$$

- and

$$
c_t + k_{t+1} = z_t k_t^{\alpha} l_t^{1-\alpha} + (1-\delta)\, k_t
$$

- and

$$
w_t = z_t(1-\alpha)k_t^{\alpha} l_t^{-\alpha} \qquad r_t = z_t(\alpha) k_t^{\alpha-1} l_t^{1-\alpha}
$$

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# Standard parameterization

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| Equation definition |
|---|
| $l_t^{\varphi} c_t^{\sigma} = (1-\alpha) z_t \left(\frac{k_t}{l_t}\right)^{\alpha}$<br>(Labor supply) |
| $c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \left[\alpha z_{t+1} \left(\frac{k_{t+1}}{l_{t+1}}\right)^{\alpha-1} + (1-\delta)\right]$<br>(Euler equation) |
| $w_t = z_t (1-\alpha) k_t^{\alpha} l_t^{-\alpha}$<br>$r_t = z_t (\alpha) k_t^{\alpha-1} l_t^{1-\alpha}$<br>(return for capital and labor) |
| $k_{t+1} = (1-\delta) k_t + i_t$<br>(Law of motion for capital) |
| $y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$<br>(Production function) |
| $y_t = c_t + i_t$<br>(Equilibrium condition) |
| $log(A_t) = \rho log(A_{t-1}) + \varepsilon$<br>(Productivity shock) |

Cuadro: Model structure.

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# Calibration



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- Calibration table 3

| Parameter | Meaning of parameter | Calibrate value |
|---|---|---|
| $\varphi$ | Labor supply elasticity | 1 |
| $\alpha$ | Output elasticity of capital | 0.33 |
| $\beta$ | Discount factor | 0.99 |
| $\delta$ | Depreciation rate | 0.025 |
| $z_{ss}$ | Technology shock | 1 |

Cuadro: Parameter values of the structural model.

esquema

# Evidencia Empirica

- **Contexto**: Modelos macroeconomia dinámicos
- **Literatura**: revisión de documentos

| Autor | Pais | Descripción |
|---|---|---|
| Teaching | | |
| Costa et al. (2018) | - | Teaching DSGE models to undergraduates |
| Bongers et al. (2020) | - | Teaching dynamic General equilibrium macroeconomics to undergraduates using a spreadsheet |
| Jenkins Brian. (2022) | - | A Python-based undergraduate course in computational macroeconomics |
| Aplicación | | |
| Gauthier Vermandel. (2020) | - | Real Business Cycle Models (RBC) using Matlab |
| GDSGE toolbox | - | Getting Started - A Simple RBC Model |

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