Ramesy-Cass-Koopmans growth model
Setup
Intensive form
Intertemporal utility
Social planner's problem
F.O.C
Dynamical system
Steady state
Phase diagram

Dynamic Macroeconomics using Matlab Lecture 2

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Ramesy-Cass-Koopmans growth model

- Optimal savings, not an exogenous constant
- Solve the problem of a `benevolent social planner`
  - how should society save?
  - In the absence of frictions, the outcome chosen by the social planner can generally be implemented using market arrangements (a version of the second welfare theorem)
  - Will see how to do this decentralization later

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Setup

- Discrete time $t = 0, 1, 2, \cdots$
- Aggregate production function:

$$Y_t = F(K_t, L)$$

  (for now, keep things simply by setting $A_t = 1$ and $L_t = L$)
- Physical capital depreciates at rate δ

$$K_{t+1} = (1-\delta)K_t + I_t$$

  Where $0 < \delta < 1,\ K_0 > 0$
- Goods may be either consumed or invested

$$C_t + I_t = Y_t$$

- Gives the sequence of `resource constraints`, one for each date

$$C_t + K_{t+1} = F(K_t, L) + (1-\delta)K_t$$

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Intensive form

- In per worker units

$$y \equiv \frac{Y}{L} \; k \equiv \frac{K}{L}$$

- Aggregate production function

$$y = f(k)$$

- Resource constraints

$$c_t + k_{t+1} = f(k_t) + (1-\delta)k_t \tag{1}$$

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Intertemporal utility

- Social planner seeks to maximize intertemporal utility

$$U\left([c_t]_{t=0}^{\infty}\right) = \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2}$$

  with strictly concave period utility $u'(c_t) > 0$ and $u''(c_t) < 0$
- Future is discounted by constant factor β

$$1, \beta, \beta^2, \cdots$$

- $U(\cdot)$ is `time-separable` marginal utility of date-t consumption

$$\frac{\partial U(\cdot)}{\partial c_t} = \beta^t u'(c_t)$$

  depends only on c_t , not consumption on other dates
- Infinite horizon keeps model `stationary`, no life-cycle effects

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Social planner's problem

- Using Eq (2) and Eq (1):

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t: } c_t + k_{t+1} = f(k_t) + (1-\delta)k_t$$

- There are two ways to solve this problem
  - Find to F.O.C
  - Using dynamic solution (programacion dinamica)

Social planner's problem: F.O.C

- Lagrangian with multiplier $\lambda_t > 0$ for each resource constraint

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \lambda_t \left[f(k_t) - c_t + (1-\delta)k_t - k_{t+1} \right]$$

- Some key first order conditions

$$\begin{aligned} c_t : \quad & \beta^t u'(c_t) - \lambda_t = 0 \\ k_{t+1} : \quad & -\lambda_t + \lambda_{t+1} \left[f'(k_{t+1}) + (1-\delta) \right] = 0 \\ \lambda_t : \quad & f(k_t) - c_t + (1-\delta)k_t - k_{t+1} = 0 \end{aligned}$$

- Using first and second equation

$$\lambda_t = \beta^t u'(c_t) \qquad \lambda_{t+1} = \beta^{t+1} u'(c_{t+1})$$

- Eliminating the Lagrange multipliers (`Euler equation`)

$$u'(c_t) = \beta u'(c_{t+1}) \left[f'(k_{t+1}) + (1-\delta) \right]$$

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Social planner's problem: F.O.C

- The `Euler equation`:

$$u'(c_t) = \beta u'(c_{t+1}) \left[f'(k_{t+1}) + (1 - \delta) \right]$$

MRS between t and t + 1

$$\frac{u'(c_t)}{\beta u'(c_{t+1})}$$

MRT between t and t + 1

$$f'(k_{t+1}) + (1 - \delta)$$

Planner equates MRS and MRT

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Dynamical system

- Gives a system of two nolinear difference equations in c_t, k_t

$$u'(c_t) = \beta u'(c_{t+1}) \left[f'(k_{t+1}) + (1-\delta) \right]$$
$$c_t + k_{t+1} = f(k_t) + (1-\delta) k_t$$

- Two boundary conditions: (i) initial $k_0 > 0$ given, and (ii) `transversality condition`

$$\lim_{T \to \infty} \beta^T \lambda_T k_{T+1} = 0$$

Steady state

- Steady state where $\triangle c_t = 0$ and $\triangle k_t = 0$, Let c_{ss} k_{ss} denote steady state values. These are determined by

$$u'(c_t) = \beta u'(c_{t+1})\left[f'(k_{t+1}) + (1-\delta)\right]$$
$$c_t + k_{t+1} = f(k_t) + (1-\delta)k_t$$

$$u'(c_{ss}) = \beta u'(c_{ss})\left[f'(k_{ss}) + (1-\delta)\right]$$
$$1 = \beta\left[f'(k_{ss}) + (1-\delta)\right]$$

$$c_{ss} + k_{ss} = f(k_{ss}) + (1-\delta)k_{ss}$$
$$c_{ss} = f(k_{ss}) - \delta k_{ss}$$

Phase diagram in k_t, c_t space

- Consumption dynamics

$$c_{t+1} > c_t \qquad \Leftrightarrow k_{t+1} < k_{ss}$$

- Capital dynamics

$$k_{t+1} > k_t \qquad \Leftrightarrow c_t < C(k_t)$$

- Divides k_t, c_t space into four regions. Flows can be analyzed with a two-dimensional phase diagram

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Phase diagram in k_t, c_t space

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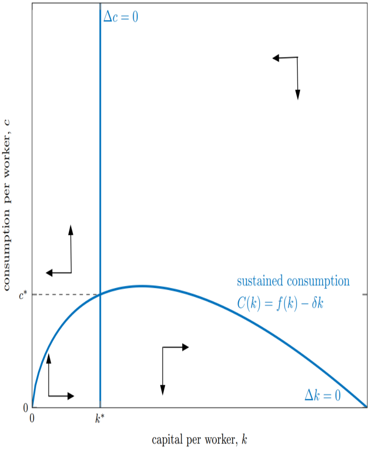
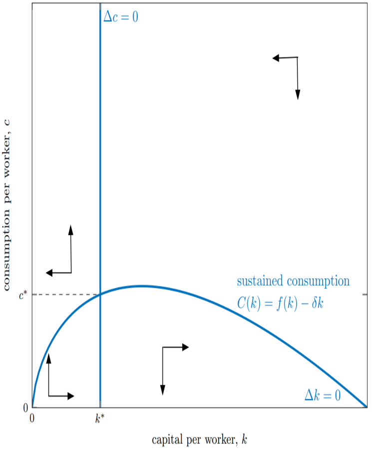


Phase diagram in k_t, c_t space

- Capital dynamics

$$k_{t+1} > k_t \quad \Leftrightarrow c_t < C(k_t)$$



Phase diagram in k_t, c_t space