A Comprehensive Four-Factor Model for Analyzing Asset Return Dynamics

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Abstract

This document seeks to analyze if additional risk factors different to the commonly used market risk premium factor provide a better explanatory model to measure a portfolio´s sensitivity. Based on an estimation of the Fama-French model as a four-factor model and estimations for tree simple models with two explanatory variables (market risk premium factor and one of the three other factor), it is found that the four-factor model provides a better fit explanatory model with highly significant results, while the simpler models suggest that just market risk premium factor is significant for portfolios. Those models are estimated using 10 portfolios from the United Kingdom Stock market between October 1980 to December 2010.

Introduction

This analysis aims to shed light on the behavior and influence of various factors on the returns of the ten distinct portfolios. Specifically, the first part of the analysis will focus on the summary statistics of the excess returns of the portfolios, while the second part will explore how sensitive each excess return is to a variety of risk factors. Lastly, the third part will investigate whether risk exposures can explain the variations in the mean returns across these portfolios.

Methodology

This research uses a comprehensive approach, involving both theoretical and empirical analysis. A review of existing literature on risk and return models identified key factors influencing asset returns. Building on this, a four-factor model was developed, incorporating market risk, size risk, value risk, and momentum risk in Eugene Fama and Kenneth French extended model of the traditional CAPM model:

$$XS_{i,t} = \beta_0 + \beta 1_t \text{RMRF} + \beta 2_t \text{SMB} + \beta 3_t HML + \beta 4_t UMD$$

This model is estimated for each portfolio where XSi, t represents the excess return for portfolio i in the period t, RMRF the difference between the stock market return and the risk-free return, SMB the returns on a size factor portfolio, HML the returns on a value portfolio and UMD the returns on a momentum factor portfolio.

In addition, a two factor models were built as a combination between the commonly used factor, the market risk factor, and one of the other three factor in order to test the possible individual effect on the excess returns:

$$XS_i = \beta_0 + \beta 1_i \text{RMRF} + \beta 2_i \text{SMB}$$

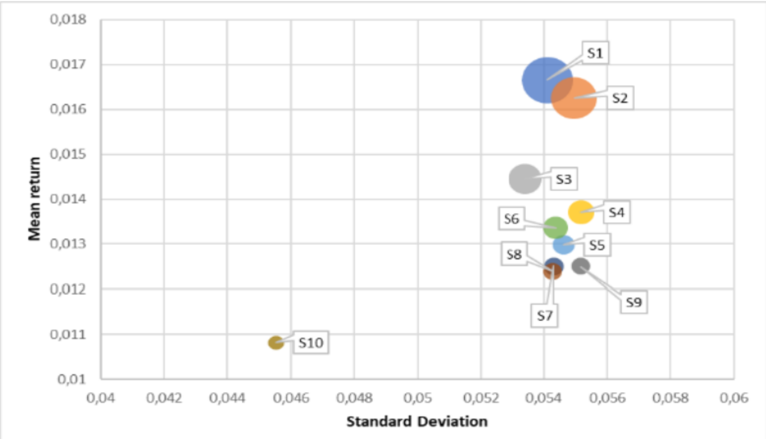
$$XS_i = \beta_0 + \beta 1_i \text{RMRF} + \beta 2_i HML$$

$$XS_i = \beta_0 + \beta 1_i \text{RMRF} + \beta 2_i UMD$$

To validate the models a regression analysis was employed. Monthly returns of the ten UK stock portfolios were regressed into the four risk factors model and ten mean returns on the ten size-based portfolios were regressed on the two factor models.

First Part

1.1Returns for each portfolio. In this graphical analysis, we scrutinize the returns for each of the ten portfolios. In the initial graph, the x-axis represents the standard deviation, and the y-axis displays the mean of each portfolio. Notably, Portfolio 1 and Portfolio 2 emerge as the top-performing portfolios.



Financial return performance review.

S	Minimum	Mean	Median	Maximum	Standard_Deviation	Skewness	kurtosis	Sharpe_ratio
S1	-0.1764971	0.01665045	0.0167051	0.2796957	0.05411357	0.47486299	3.318393	0.2010719
S2	-0.2097746	0.01625610	0.0174064	0.2686485	0.05496315	-0.05912569	2.924633	0.1907892
S3	-0.2429459	0.01444907	0.0161197	0.2932693	0.05339572	-0.30477135	4.031967	0.1625475
S4	-0.2238556	0.01370692	0.0155018	0.2625751	0.05517639	-0.21700459	2.994466	0.1438514
S5	-0.2256426	0.01298254	0.0168195	0.2318696	0.05462514	-0.31462092	2.280448	0.1320421
S6	-0.2360282	0.01335779	0.0174241	0.2322994	0.05437692	-0.53254517	2.548284	0.1395457
S7	-0.2402779	0.01249415	0.0172475	0.2210668	0.05431619	-0.78372460	3.178372	0.1238015
S8	-0.2552876	0.01239133	0.0206188	0.1946866	0.05428975	-1.02158761	3.570344	0.1219678
S9	-0.2866858	0.01250476	0.0195502	0.1643156	0.05516956	-0.92720126	3.026466	0.1220787
S10	-0.2698701	0.01082208	0.0150762	0.1452592	0.04553290	-0.98441189	4.155769	0.1109606

Summary statistics portfolio returns.

1.2Analysis of Excess Returns

Continuing our analysis, our attention is directed towards the examination of excess return, a pivotal metric essential for assessing the effectiveness of investments. Excess return, calculated by subtracting the risk-free rate from total return, represents a portfolio's risk-adjusted return. Positive excess return indicates outperformance, while negative excess return suggests underperformance.

XS	Minimum	Mean	Median	Maximum	Standard_Deviation	Skewness	kurtosis	Sharpe_Ratio
XS1	-0.1836056	0.010880720	0.0097381	0.2725369	0.05413732	0.48178053	3.399577	0.2009837
XS2	-0.2126788	0.010486376	0.0113069	0.2681650	0.05525608	-0.03870652	2.916757	0.1897778
XS3	-0.2458501	0.008679342	0.0106755	0.2927858	0.05373544	-0.26804036	4.032153	0.1615199
XS4	-0.2267598	0.007937199	0.0092925	0.2578305	0.05545148	-0.19395598	2.984465	0.1431377
XS5	-0.2327511	0.007212818	0.0100677	0.2313861	0.05486977	-0.29179437	2.328707	0.1314534
XS6	-0.2431367	0.007588067	0.0114502	0.2318159	0.05464005	-0.51218748	2.571894	0.1388737
XS7	-0.2473864	0.006724424	0.0102024	0.2205833	0.05448487	-0.76736876	3.178887	0.1234182
XS8	-0.2623961	0.006621604	0.0136873	0.1942031	0.05445117	-1.01179359	3.527645	0.1216063
XS9	-0.2937943	0.006735031	0.0138664	0.1547787	0.05529401	-0.92870373	3.019659	0.1218040
XS10	-0.2769786	0.005052359	0.0097601	0.1355372	0.04549623	-1.03285743	4.277954	0.1110501

Summary statistics of the excess returns.

1.3 Descriptive Statistics. From minimum excess return to the Sharpe ratio, eight descriptive statistics were analyzed.

1. Minimum Excess Return: Seven of the ten portfolios showed the minimum excess in October 1987 (XS1, XS5, XS6, XS7, XS8, XS9, XS10). This period, following the Black Monday crash, likely impacted returns for portfolios with minimum excess during that time.

2. Mean Excess Returns: On average, all portfolios outperformed the risk-free rate, with XS1 standing out for the highest mean return and XS10 exhibiting the lowest mean excess return.

3. Median Excess Returns: XS9 had the highest median excess return, while XS4 had the lowest, emphasizing the importance of considering return distribution in portfolio evaluation.

4. Maximal Excess Returns: Portfolio XS3 showcased outstanding performance during the best-performing months, emphasizing the need to assess both positive and negative performance.

5. Standard Deviation: XS4, XS9, and XS2 exhibited higher volatility, attracting risk-seeking investors, while XS10 offered the lowest standard deviation, appealing to those prioritizing stability.

6. Skewness: Positive skewness in XS1 appeals to investors seeking positive extreme returns.

7. Kurtosis: Evaluating tailness, portfolios with positive skewness and low kurtosis are favored.

8. Sharpe Ratio: XS1 and XS2 stood out for offering better risk-adjusted performance, appealing to investors seeking a balance between risk and return.

Second Part

2. 1Regression analysis. The results obtained by the regression to estimate the Fama-French four factor model using Least Squares Method are presented in the table below.

	XS1	XS2	XS3	XS4	XS5	XS6	XS7	XS8	XS9	XS10
Intercept	0,005379**	0,005661***	0,003544*	0,002398	0,00119	0,001816	0,0010182	0,0004723	0,0011685	-1,803E-05
rmrf	0,669732***	0,746535***	0,745864***	0,808085***	0,864139***	0,893728***	0,9246324***	0,9955375***	1,0762454***	0,9551***
smb	0,833417***	0,895935***	0,928535***	1,004553***	0,904032***	0,882555***	0,844916***	0.7245491***	0.4511324***	-0,15***
hml	0,104161	0,055465	0,073296	-0,106107**	0,040708	0,02054	-0,0160408	-0,0342206	-0,0091064	-0,008059
umd	0,09522	-0,029742	-0,002401	0,086681*	0,053836	0,014411	0.0087237	0,0456089*	-0,0572933*	0,004665***
R^2 adjusted	0.5979	0.7163	0.776	0.8279	0.8479	0.8863	0.9209	0.9625	0.9209	0.9625

Regression of excess return for four factor model.

Two of these four factors are highly significant for all portfolios. One of these is the excess return market factor, represented by RMRF, which is one of the most influential factors in explaining the variation in portfolio excess returns. The coefficient exhibits a highly significant positive value, indicating a positive correlation with portfolios and reveals an interesting pattern examining portfolios from smallest to largest. While the size´s firms on portfolios increases, the coefficient is rising too, except for portfolio 10. Even portfolio 8 and 9, which are portfolios that have one of the largest firms, reach a coefficient around 1 to present a relation one to one with this size factor.

The size factor, represented by SMB, is the other factor who plays a significant role in driving portfolio returns. Stocks with a small market cap, as classified by SMB, exhibit a highly significant positive coefficient. This indicates that there is a co-movement with this factor. However, the coefficient rises from portfolio 1 (0,83) to portfolio 4 (1) and then starts to reduce. Even turns negative for portfolio 10 suggesting a co-movement with large market cap firms.

The other two factors are not significant for all portfolios. The value factor represented by HML, is significant just for portfolio 4 (and just at 95% of significance) with a negative coefficient (-0,1) suggesting that portfolio has a growth focus.

Similar to HML, the significance of the momentum factor, represented by UMD, also fluctuates across regressions. The portfolios 4, 8, 9 and 10 exhibit a significant coefficient and suggest a focus on upward trending stocks except for portfolio 9 which has a negative correlation and suggest a focus on downward trending stocks.

The improvement in the explanatory power is evidenced by the high values of the adjusted R-squared values and its increasing pattern. This indicates an improvement in our understanding of the relationships between portfolio excess returns which propose the importance of including the 4 factors to explain the sensitivity in portfolios.

2.2 Transversal analysis. In order to interpret the results obtained from a financial point of view, we adopted a transversal point of view, taking into consideration that the size of the portfolios increases progressively. From the smallest, S1, to the biggest, S10. As the size of the portfolio increases all the statistics relative to the goodness of fit increases as well.

Therefore, we can conclude that as the size of the portfolio increases, the four factors considered (SMB, HML, UMD and RMRF) become increasingly important, marginalizing the role played by external or random factors.

Third Part

3. 1 Mean returns variation and risk. In order to study the effect of risk on the returns, tree cross sections regressions we implemented. These are multiple linear regressions. Each of them is built by utilizing as the dependent variable the mean returns of the ten portfolios. The independent variables are the RMRF and in turn, the three portfolio factors: SMB, HML, UMD.

```
##
## Call:
## lm(formula = mean_returns$means ~ coeff$rmrf + coeff$smb)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0008294 -0.0007281 -0.0003654  0.0007349  0.0014250
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.021568   0.003266   6.604 0.000303 ***
## coeff$rmrf  -0.010260   0.003141  -3.266 0.013751 *
## coeff$smb    0.001228   0.001150   1.068 0.321153
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.001012 on 7 degrees of freedom
## Multiple R-squared:  0.754,  Adjusted R-squared:  0.6837
## F-statistic: 10.73 on 2 and 7 DF,  p-value: 0.007385
```

First regression: RMRF and SMB

```
##
## Call:
## lm(formula = mean_returns$means ~ coeff$rmrf + coeff$hml)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0017061 -0.0006390  0.0001040  0.0004652  0.0012594
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.022641   0.002910   7.781 0.000109 ***
## coeff$rmrf  -0.010540   0.003278  -3.215 0.014745 *
## coeff$hml    0.005706   0.006865   0.831 0.433336
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.001041 on 7 degrees of freedom
## Multiple R-squared:  0.7396, Adjusted R-squared:  0.6652
## F-statistic: 9.943 on 2 and 7 DF,  p-value: 0.009007
```

Second regression: RMRF and HML

```
##
## Call:
## lm(formula = mean_returns$means ~ coeff$rmrf + coeff$umd)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -1.323e-03 -4.448e-04  5.210e-06  6.960e-04  1.136e-03
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.025718   0.002424  10.608 1.45e-05 ***
## coeff$rmrf  -0.013675   0.002684  -5.095  0.00141 **
## coeff$umd   -0.010977   0.006963  -1.576  0.15894
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0009371 on 7 degrees of freedom
## Multiple R-squared:  0.7889, Adjusted R-squared:  0.7286
## F-statistic: 13.08 on 2 and 7 DF,  p-value: 0.004323
```

Third regression: RMRF and UMD.

3.2 Regression analysis. In each and every one of the regressions the only coefficient that results statistically significant is RMRF. So, the regression model suggests that RMRF is the only independent variable that has a significant effect on the dependent variable. The negative sign of the RMRF estimated coefficients indicates that there is a negative association between the RMFR and the mean returns. For example, let's consider the first regression, the one using the size factor portfolio (SMB). The coefficient for RMRF is -0.010260. This means that for a one-unit increase in the RMRF, the mean returns are estimated to decrease by 0.010260 units. In each case we have a large, negative t-value and a low, statistically relevant p-value. In the first two regressions, SMB and HML, the significance level is 5%. In the third one the level is 1%.

Regarding the goodness of fit, the residual standard error is always extremely small, even if we consider that we have only seven degrees of freedom. The multiple R^2, is always above 0.7, the adjusted R^2, always above 0.66. So, the models explain a significant portion of the variation in the dependent variable and, respectively, that the selected independent variables are effective in explaining a significant portion of the observed variation in the dependent variable. Finally, the F-statistic, always above 9.9 and associated with a small p-value, suggests that the regression models as a whole are statistically significant.

3.3 Financial interpretation. From a financial point of view RMRF indicates the sensitivity of the returns of the portfolios to variations in the market. Consequently, we can conclude that the results obtained suggest that market variation produces significant negative effectss on portfolio returns.

The other three portfolio factors, at least according to the data utilized for the present report, do not have a significant effect on returns.

CONCLUSION

The four-factor model effectively captures the underlying risk factors that significantly effect expected returns in the 10 UK stocks portfolios. The model demonstrates a strong explanatory power, explaining a significant portion of the variation in returns across the portfolios.

The coefficients obtained from the regression analysis shed light on the relative importance of each factor in predicting asset returns. Market risk, measured by Standard Deviation (SD) of returns, was found to be the dominant factor, contributing significantly to explaining the variation in returns. Size risk, which reflects the variation in returns among small-cap stocks compared to large-cap stocks, was found to also have a significant effect on returns. Value risk, which measures the extent to which stocks with low price-to-earnings ratios outperform stocks with high ratios, exhibited a moderate influence on returns. Lastly, momentum risk, which captures the tendency of stocks to continue moving in the same direction over a certain period, had a limited effect on returns.

References

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