# **M**Dorado

Documentation of Version 0.3.0

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# 1 Importing Testfiles

A number of files for unit testing and examples are shipped with the installation of MDorado. Their absolute paths can be obtained via a python import statement using the mdorado.data.datafilenames module.

# 1.1 File references

water_topology	Gromacs .tpr file (water.tpr) of a small SPC water simulation (125 molecules, $300\mathrm{K}$ , $10^5\mathrm{Pa}$ ) used as topology reference by MDAnalysis. Is used in a number of unit tests and examples throughout all functions.
water_trajectory	Gromacs .xtc file (water.xtc) of a small SPC water simulation (125 molecules, $300\mathrm{K},10^5\mathrm{Pa}$ ) used as trajectory reference by MDAnalysis. Is used in a number of unit tests and examples throughout all functions.
test_gofr_ss	Textfile (gofr_ss.dat) containing the reference output of the mdorado.gofr.Gofr calculation using the "site-site" mode. Is used in the unit test of the mdorado.gofr module.
test_gofr_cc	Textfile (gofr_cc.dat) containing the reference output of the mdorado.gofr.Gofr calculation using the "cms-cms" mode. Is used in the unit test of the mdorado.gofr module.
test_gofr_sc	Textfile (gofr_sc.dat) containing the reference output of the mdorado.gofr.Gofr calculation using the "site-cms" mode. Is used in the unit test of the mdorado.gofr module.
test_hbanalyze	Textfile (hb_analyze.dat) containing the reference output of the mdorado.hb_analyze.hb_analyze function. Is used in the unit test of the mdorado.hb_analyze module.

test\_lifetime Textfile (lifetime\_test.dat) containing the reference output of the mdorado.lifetime.calc\_lifetime function. Is

used in the unit test of the mdorado.lifetime module.

test\_unwrap File (msd\_molpos.npy) containing the reference output of the

mdorado.msd.unwrap function Is used in the unit test of the

mdorado.msd module.

test\_msd Textfile (msd\_h.dat) containing the reference output of the

mdorado.msd.msd function. Is used in the unit test of the

mdorado.msd module.

test\_getvecarray File (vecarray42.npy) containing the reference output of the

mdorado.vectors.get\_vecarray function. Is used in the

unit test of the mdorado.vectors module.

test\_getnormvecarray File (normal\_vecarray42.npy) containing the reference out-

put of the mdorado.vectors.get\_normal\_vecarray function. Is used in the unit test of the mdorado.vectors module.

test\_getvecmatrix File (vecmatrix4247.npy) containing the reference output of

the mdorado.vectors.get\_vectormatrix function. Is used

in the unit test of the mdorado vectors module.

test\_correlvec Textfile (anisolg2.dat) containing the reference output of

the mdorado.veccor.correlvec function. Is used in the unit

test of the mdorado.veccor module.

test\_vectors File (vectors\_isocorrel.npy) containing input for the

unittest of the mdorado.veccor.isocorrelvec function.

test\_isocorrelvec Textfile (slow\_isolg2.dat) containing the reference output

of the mdorado.veccor.isocorrelvec function. Is used in

the unit test of the mdorado.veccor module.

test\_isocorrelveclg1 Textfile (fast\_isolg1.dat) containing the reference output

of the mdorado.veccor.isocorrelveclg1 function. Is used

in the unit test of the mdorado.veccor module.

test\_isocorrelveclg2 Textfile (fast\_isolg2.dat) containing the reference output

of the mdorado.veccor.isocorrelveclg2 function. Is used

in the unit test of the mdorado.veccor module.

# 1.2 Example

In particular the water simuation can be used as an example input to test the functionality of MDorado. It is used in that way in many examples throughout this documentation, so the functions can be tested on an user-independent trajectory. This code, for example, is an example for the hb\_analyze module and imports the file paths of the water.tpr (water\_topology) and water.xtc (water\_trajectory):

```
import MDAnalysis
from mdorado.hb_analyze import hb_analyze
from mdorado.data.datafilenames import water_topology,
water_trajectory

u = MDAnalysis.Universe(water_topology, water_trajectory)
grp = u.select_atoms("name ow")
hgrp = u.select_atoms("name hw")[::2]

hb_analyze(universe=u, xgrp=xgrp, hgrp=hgrp, rmin=1.5, rmax=5,
cosalphamin=-1, cosalphamax=1, bins=50)
```

# 2 Correlate

This function uses scipy.signal.correlate to cross correlate two discrete functions a(t) and b(t). The function computes  $\langle a(0)b(t)\rangle$  directly via sums or using a Fast Fourier Transform algorithm, depending on which is faster (see scipy.signal.convolve). In addition to the original scipy functionality, the function tailors the correlation function so that only non-negative time values  $(\langle a(0)b(t)\rangle)$  for  $t\geq 0$  are returned. It may only be sensible to calculate such a function if a(t) and b(t) (and the corresponding array elements a[t] and b[t]) reference the same point in time and the arrays a and b are of equal length. The autocorrelation function  $\langle a(0)a(t)\rangle$  is computed if b=None (default).

### 2.1 Function

mdorado.correlations.correlate(a, b=None)

### Parameters:

a: one-dimensional ndarray or list

Discrete values of the function a(t).

b: one-dimensional ndarray or list, optional

Discrete values of the function b(t). If None the autocorrelation

function  $\langle a(0)a(t)\rangle$  is computed. Default is None.

Returns: ndarray

An indexervation and containing the correlation function  $\langle a(0)b(t)\rangle$  for  $t\geq 0$  is returned.

# 3 Lifetimes

To analyze the lifetime of a hydrogen bond or any other impermanent interaction we can define a bonding operator h(t) which is unity if the criteria for the interaction are fulfilled and zero otherwise:<sup>[1,2]</sup>

$$h(t) = \begin{cases} 1, & \text{if criteria are fulfilled} \\ 0, & \text{otherwise} \end{cases}$$
 (3.1)

The fluctuations of h(t) can be described by the autocorrelation function C(t)

$$C(t) = \frac{\langle h(0)h(t)\rangle - \langle h\rangle^2}{\langle h\rangle},\tag{3.2}$$

which describes the probability of the bond being intact at the time t if the bond was intact at t = 0. The so-called intermittent lifetime of the interaction can be estimated from C(t).<sup>[3–6]</sup>

The reactive flux approach<sup>[4,6–9]</sup> is another approach to estimate the lifetime of such interactions. To follow that approach we require the function  $k_{\rm in}(t)$ 

$$k_{\rm in}(t) = -\frac{\left\langle \dot{h}(0)[1 - h(t)]H(t)\right\rangle}{\langle h\rangle},\tag{3.3}$$

where  $\dot{h}$  denotes the time-derivative of h(t). H(t) is a vicinity operator closely related to h(t). If the donor and acceptor of the interaction are "near" each other H(t) equals unity otherwise it equals zero.

The purpose of calc\_lifetime is to calculate the correlation functions  $\langle h(0)h(t)\rangle$  from equation 3.2 and  $-\langle \dot{h}(0)[1-h(t)]H(t)\rangle$  from equation 3.3. Normalizing the correlation functions will be up to the user, since several approaches are viable. [10] To obtain both correlations we first need to determine h(t) and H(t) for every donor-acceptor pair. After that, we are able to compute both correlation functions and average them over all donor-acceptor pairs.

### 3.1 Function

```
mdorado.lifetime.calc_lifetime(universe, timestep, xgrp, hgrp,
    cutoff_hy, cutoff_xy, angle_cutoff, ygrp=None, nproc=1,
    check_memory=True)
```

### Parameters:

universe: MD.Analysis.Universe

Universe containing the trajectory.

timestep: int or float

Timestep between configurations in the universe. The unit is

freely selectable and will influence the units of the output.

xgrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms X involved in the interaction

 $X-H\cdots Y$ . Has to be the same size as hgrp.

hgrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms H involved in the interaction

 $X-H\cdots Y$ . Has to be the same size as xgrp.

ygrp: AtomGroup from MDAnalysis or None, optional

MDAnalysis AtomGroup containing all atoms Y involved in the interaction  $X-H\cdots Y$ . If None is given, it is assumed that Y=X (interaction  $X-H\cdots X$ ) and xgrp is taken as acceptor group. The

default is None.

cutoff\_hy int or float

Criterion for the  $H \cdots Y$  distance in Å to define h(t). The criterion is fulfilled if the distance between a HY-pair is smaller than the

value specified.

cutoff\_xy int or float

Criterion for the  $X \cdots Y$  distance in Å to define H(t). The criterion is fulfilled if the distance between a XY-pair is smaller than the

value specified.

angle\_cutoff int or float

Criterion for the angle  $\alpha \angle XHY$  in in radian to define h(t). The cutoff is set so that if  $\alpha >$  angle cutoff the criterion is fulfilled.

nproc int, optional

Number of processors available to parallelize the execution of the

script. The default is 1.

check\_memory bool, optional

Perform an approximate check if the amout of memory is sufficient.

The default is True.

### **Output:**

For every donor i in  $\mathsf{xgrp}$  a file  $\mathsf{ct\_i.dat}$  will be created. The file contains the results in three columns. The first column contains the timestep t in the same unit given in the option  $\mathsf{timestep}$ . The second column contains  $\langle h(0)h(t)\rangle$  for that donor. The third column contains  $-\langle \dot{h}(0)[1-h(t)]H(t)\rangle$  for that donor in inverse units of  $\mathsf{timestep}$ . As long as the amount of acceptors (ygrp) is constant the data of multiple donors i can be averaged by computing the arithmetic mean of the desired quantity.

## 3.2 Example

Using a water simulation from the files of the module, we take the first 20 water molecules and calculate correlation functions using one of the hydrogen atoms as donor but all water oxygen atoms as donor. The timestep of this example trajectory is 0.2 ps. The hydrogen bond was here defined by a distance cutoff  $H \cdots O$  of 2.5 Å and and angle cutoff of  $\alpha > 2.27$  rad. For H(t) the distance criterion  $X \cdots Y$  was set to 3.5 Å.

```
import MDAnalysis
from mdorado.lifetime import calc_lifetime
from mdorado.data.datafilenames import water_topology,
water_trajectory

universe = MDAnalysis.Universe(water_topology, water_trajectory)

xgrp = universe.select_atoms("name ow")[:20]
hgrp = universe.select_atoms("name hw")[:40:2]
ygrp = universe.select_atoms("name ow")
```

- 11 calc\_lifetime(universe=universe, timestep=0.2, xgrp=xgrp, hgrp=hgrp,
  - ygrp=ygrp, cutoff\_hy=2.5, angle\_cutoff=2.27, cutoff\_xy=3.5)

# 4 gofr

The radial distribution function  $g_{AB}(r)$  describes the density of particle B in a spherical shell of width dr at distance r around particle A in relation to the average number density of B  $\langle \rho_B \rangle$  in the system

$$g_{\rm AB}(r) = \frac{\langle \rho_{\rm B}(r) \rangle}{\langle \rho_{\rm B} \rangle} = \frac{1}{\langle \rho_{\rm B} \rangle \cdot N_{\rm A}} \left\langle \sum_{i \in \mathcal{A}}^{N_{\rm A}} \sum_{j \in \mathcal{B}}^{N_{\rm B}} \frac{\delta(r_{ij} - r)}{4\pi r^2} \right\rangle. \tag{4.1}$$

Here,  $N_{\rm A}$  and  $N_{\rm B}$  references the number of particles A and B in the system, respectively. It should be noted that  $g_{\rm AB}(r) = g_{\rm BA}(r)$ .

From  $g_{AB}(r)$  and  $\langle \rho_B \rangle$  the average cumulative number of neighbors  $N_B(R)$  of paricles B in a sphere of radius R around a particle A is obtainable via

$$N_{\rm B}(R) = \rho_{\rm B} \cdot 4\pi \int_{0}^{R} g_{\rm AB}(r) r^2 dr$$
 (4.2)

Similarly,  $N_{\rm A}(R)$  can be computed using  $\langle \rho_{\rm A} \rangle$ .

Three modes (mode) are implemented at the moment: "site-site", "cms-cms", and "site-cms". The mode "site-site" computes the average  $g_{AB}(r)$  between all atoms A in agrp and all atoms B in bgrp. For example, if agrp contains atoms A0 and A1 while bgrp contains atoms B0 and B1, the pairs A0B0, A0B1, A1B0, and A1B1 will contribute to  $g_{AB}(r)$ .

The mode "cms-cms" can be used to compute center-of-mass radial distribution functions. It will calculate the center-of-mass of atoms belonging to the same molecule in agrp and bgrp and procede to calculate the radial distribution function of these centers-of-mass. For example, given an agrp containing four atoms belonging to two different molecules (A0 and A1 belonging to molecule M0, A2 and A3 belonging to M1) and the same for bgrp (B0 and B1 belonging to M2, B2 and B3 belonging to M3) it will first calculate the centers-of-mass  $cms_{M0}(A0,A1)$ ,  $cms_{M1}(A2,A3)$ ,  $cms_{M2}(B0,B1)$ , and  $cms_{M3}(B2,B3)$ . The radial distribution function will then contain contributions from the pairs  $cms_{M0}cms_{M2}$ ,  $cms_{M0}cms_{M3}$ ,  $cms_{M1}cms_{M2}$ , and  $cms_{M1}cms_{M3}$ .

The mode "site-cms" is a mix of both of the functions described above, where agrp is taken atom-wise as in gofr and for bgrp the center-of-mass of atoms belonging to the same molecule is calculated as in gofr\_cms.

### 4.1 Function

```
mdorado.gofr.Gofr(universe, agrp, bgrp, rmax, rmin=0, bins=100,
    mode="site-site", outfilename="gofr.dat")
```

#### **Parameters:**

universe: MDAnalysis.Universe

Universe containing the trajectory.

agrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms A.

bgrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms B.

rmax: int or float

The upper boundary of the  $A \cdots B$  distance used for the g(r) in

units of Å.

rmin: int or float, optional

The lower boundary of the  $A \cdots B$  distance used for the g(r) in

units of Å. The default is 0.

bins: int or sequence of scalars or str, optional

Specifies the number of points between rmin (inclueded) and rmax (excluded). Will be used directly by numpy.histogram. From the numpy documentation: "If bins is an int, it defines the number of equal-width bins in the given range. If bins is a sequence, it defines a monotonically increasing array of bin edges, including the rightmost edge, allowing for non-uniform bin widths. If bins is a string, it defines the method used to calculate the optimal bin width, as defined by histogram bin edges." The default is 100.

mode: str, optional

Sets the mode for calculating different radial distribution functions: "site-site", "cms-cms", "site-cms". If mode is set to "site-site", the average radial distribution function of all sites in agrp to all sites in bgrp will be computed. The mode "cms-cms" will first compute the center-of-mass of sites belonging to the same molecule in agrp and bgrp, respectively, and then determin the radial distribution function between those centers of mass. The mode "site-cms" is a mix between the two, where every site in agrp is taken individually but for bgrp the center-of-mass of sites belonging to the same molecule is computed firs. The default is "site-site".

outfilename:

str, optional

The name of the output file. The default is "gofr.dat".

### **Output:**

The program creates a file named outfilename with the distance r in Å (first column), the radial distribution function  $g_{AB}(r)$  (second column), the cumulative number of neighbors A in a sphere of radius r around particle B  $N_A(r)$  (third column), and the cumulative number of neighbors B in a sphere of radius r around particle A  $N_B(r)$  (fourth column).

### **Class Methods:**

rdat: Distance r (center of bins).

edges: Edges of the bins.

hist: Radial distribution function  $g_{AB}(r)$ .

annn: Average number of neighbors A in a sphere of radius r around particle B

 $N_{\rm A}(r)$ .

bnnn: Average number of neighbors B in a sphere of radius r around particle A

 $N_{\rm B}(r)$ .

avvol: Average volume of the universe.

na: Number of particles A in agrp. If mode is "site-site" or "site-cms", na is the number of sites in agrp. If mode is "cms-cms", na is the number of

molecules (centers-of-mass) in agrp.

nb: Number of particles B in bgrp. If mode is "site-site", na is the number of sites in agrp. If mode is "site-cms" or "cms-cms", nb is the number of molecules (centers-of-mass) in bgrp.

## 4.2 Example

We start with a simulation of water from the data files shipped with the module, where all oxygen atoms are named "ow" and all hydrogen atoms "hw". The resname of the water molecules is "sol" for solvent. We will compute three different radial distribution functions to show reveal the differences in "site-site", "cms-cms", and "site-cms": Firstly, we will use "site-site" to calculate the radial distribution function between all hydrogen and oxygen  $(H \cdots O)$  atoms, which could for example be used to define the hydrogen bond  $O-H \cdots O$ . Secondly, we will calculate the center-of-mass radial distribution function of all water molecules  $(cms \cdots cms)$  using "cms-cms". Thirdly, we use "site-cms" to compute the radial distribution function of all hydrogen atoms to the centers-of-mass of all water molecules  $(H \cdots cms)$ .

We first have to create a universe and select different AtomGroups to achieve the goals described above.

```
import MDAnalysis
from mdorado.gofr import Gofr
from mdorado.data.datafilenames import water_topology,
    water_trajectory

u = MDAnalysis.Universe(water_topology, water_trajectory)
hgrp = u.select_atoms("name hw")
ogrp = u.select_atoms("name ow")
watergrp = u.select_atoms("resname sol")

sitesite = Gofr(universe=u, agrp=hgrp, bgrp=ogrp, rmin=1.1, rmax=6,
    bins=200, mode="site-site", outfilename="h_o.dat")
cmscms = Gofr(universe=u, agrp=watergrp, bgrp=watergrp, rmin=1.1,
    rmax=6, bins=200, mode="cms-cms", outfilename="cms_cms.dat")
sitecms = Gofr(universe=u, agrp=hgrp, bgrp=watergrp, rmin=1.1,
    rmax=6, bins=200, mode="site-cms", outfilename="h_cms.dat")
rmax=6, bins=200, mode="site-cms", outfilename="h_cms.dat")
```

In Fig. 4.1 the three different g(r) are plotted. Additionally, we obtain the neighbour numbers  $N_{\rm A}(r)$   $N_{\rm B}(r)$  for each pair. In case of the H···O distribution  $N_{\rm A}(r)$  would be the average number of hydrogen atoms in a sphere of radius r around an oxygen atom. In case of the cms···cms distribution  $N_{\rm A}(r) = N_{\rm B}(r)$  denotes the number of water molecules in a sphere of radius r around a water molecule. In case of the H···cms distribution  $N_{\rm A}(r)$  is the number of water molecules around in a sphere of radius r around a hydrogen atom.

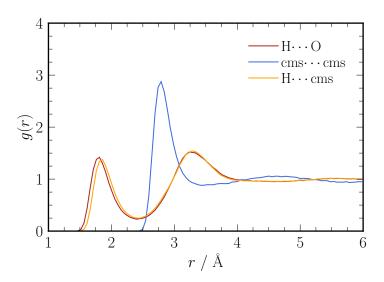


Fig. 4.1: Radial distribution functions obtained from the example above.

# 5 hb analyze

Geometric criteria can be used to define a hydrogen bond. Distance criteria can often be derived from pair-correlation functions but it may be required to include angular restrictions on the interaction. Two dimensional potentials of mean force (PMFs) can be used to obtain such criteria. The PMF is calculated using the probability density of finding a donor-acceptor pair with the respective donor-acceptor ( $\mathbf{H} \cdots \mathbf{Y}$ ) distance r and angle  $\alpha$  ( $\alpha \angle \mathbf{XHY}$ ).

This density can be derived from populations from equilibrium molecular dynamics trajectories. Therefore, donor-acceptor pairs with a distance r between rmin and rmax and an angle  $\cos(\alpha)$  between cosalphamin and cosalphamax will be counted in a twodimensional histogram according to the number of bins specified with the option bins. Each count is weighted with the respective  $r^{-2}$  to account for the growth of the spherical volume element with increasing r. The histogram is then normalized to the respective probabilty density function  $P(r, \cos(\alpha))$  using the area of each bin  $dr \cdot d\cos(\alpha)$  and the sum of all counts so that the integral over P is unity. At the end, the natural logarithm of P in each bin is calculated, due to the connection between P and the PMF via

$$F = -k_{\rm B}T\ln(P) + c,\tag{5.1}$$

with the Boltzmann constant  $k_{\rm B}$ , the temperature T, and an unknown constant c. The output is a two-dimensional grid where each bin contains  $\ln(P)$  of the respective bin.

### 5.1 Function

```
mdorado.hb_analyze.hb_analyze(universe, xgrp, hgrp, rmax, ygrp=None,
    rmin=0, cosalphamin=-1, cosalphamax=1, bins=50,
    outfilename="hb_analyze.dat", ralphalist=False)
```

### **Parameters:**

universe: MDAnalysis.Universe

Universe containing the trajectory.

xgrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms X involved in the interaction

 $X-H\cdots Y$ . Has to be the same size as hgrp.

hgrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms H involved in the interaction

 $X-H\cdots Y$ . Has to be the same size as xgrp.

ygrp: AtomGroup from MDAnalysis or None, optional

MDAnalysis AtomGroup containing all atoms Y involved in the interaction X–H $\cdots$ Y. If None is given, it is assumed that Y=X (interaction X–H $\cdots$ X) and xgrp is taken as acceptor group. The

default is None.

rmax: int or float

The upper boundary of the  $H \cdots Y$  distance in units of Å.

rmin: int or float, optional

The lower boundary of the  $H \cdots Y$  distance in units of Å. The de-

fault is 0.

cosalphamin: int or float, optional

The lower boundary of  $\cos(\alpha)$  ( $\alpha \angle XHY$ ). The default is -1.

cosalphamax: int or float, optional

The upper boundary of  $\cos(\alpha)$  ( $\alpha \angle XHY$ ). The default is 1.

bins: int or array\_like or [int, int] or [array, array], optional

Bins used for the 2D-histogram. Will be used directly by numpy.histogram2d. For two numbers the first will specify the bins of the  $H \cdots Y$  distance (x\_edges) and the second will specify the bins of  $\cos(\alpha)$  (y\_edges) The default is 50. Specifications:

- If int, the number of bins for the two dimensions (nx=ny=bins).
- If array\_like, the bin edges for the two dimensions (x edges=y edges=bins).
- If [int, int], the number of bins in each dimension (nx, ny = bins).
- If [array, array], the bin edges in each dimension (x\_edges, y edges = bins).
- A combination [int, array] or [array, int], where int is the number of bins and array is the bin edges.

outfilename: str, optional

The name of the outputfile. The default is hb\_analyze.dat.

ralphalist: bool, optional

Changes the output from the weighted probability density matrix to the list containing all the  $H \cdots Y$  distances and corresponding  $\cos(\alpha)$  from which the probability density is calculated. The default is False.

### **Output:**

The program creates a file named outfilename with the weighted two dimensional histogram. The first axis represents the  $H \cdots Y$  distance and the second axis represents  $\cos(\alpha)$  ( $\alpha \angle XHY$ ).

If ralphalist=True the file contains the distances and corresponding angles of HY-pairs as a list: in the first column the distances are written in units of Å and the second column indicates the cosine of the corresponding angle  $\cos(\alpha)$ , both in the respective range rmin to rmax and cosalphamin to cosalphamax.

## 5.2 Example and Visualization

To use hb\_analyze we first have to create a universe, define xgrp and hgrp, the range of the histogram, and the amount of bins in each dimension. If no ygrp is given the

program will use xgrp as acceptor group and analyze the interaction  $X-H\cdots X$  instead. Here an example for a water simulation from the datafiles of mdorado where the oxygen atoms are named "ow" and the hydrogen atoms "hw":

```
import MDAnalysis
from mdorado.hb_analyze import hb_analyze
from mdorado.data.datafilenames import water_topology,
    water_trajectory

u = MDAnalysis.Universe(water_topology, water_trajectory)
grp = u.select_atoms("name ow")
hgrp = u.select_atoms("name hw")[::2]

hb_analyze(universe=u, xgrp=xgrp, hgrp=hgrp, rmin=1.5, rmax=5,
    cosalphamin=-1, cosalphamax=1, bins=50)
```

After excecution a file hb\_analyze.dat (changable by the option outfilename) can be found in the current folder. It contains the  $50 \times 50$  (bins) matrix of the weighted probability function. This matrix can be plotted by matplotlibs contour and similar programs. Here an example using matplotlibs contourf:

```
15 rmin=1.5
_{16} rmax=5
17 cosalphamin=-1
18 cosalphamax=1
20 fig, ax = plt.subplots()
21 histo_matrix = np.loadtxt("hb_analyze.dat")
22 levels = ticker.MaxNLocator(nbins=60).tick_values(-3, 2)
23 cax = ax.contourf(histo_matrix, extent=(cosalphamin, cosalphamax,
    rmin, rmax), levels=levels, extend='both', cmap=cmapown)
24 plt.xlabel('$\\cos(\\alpha)$')
25 plt.ylabel('$r$ / \\AA')
26 plt.axis([cosalphamin, cosalphamax, rmin, rmax])
27 cbar = fig.colorbar(cax, ticks=[-3, -2, -1, 0, 1, 2])
28 cbar.ax.set_ylabel('$\\log[W(\\cos(\\alpha), r)]$')
29 plt.tight_layout()
30 plt.savefig("histo.pdf")
31 plt.clf()
```

After importing the necessary modules, we first define or own colormap cmapown (line 6 to 13). Standard colormaps can be found here. The output of hb\_analyze only contains the weighted probabilty densities for each bin and not their position, so we have to tell the program in line 15–18 in which range the histogram is plotted (option extent of contourf line 23 and x- and y-axis limits line 26).

The actual plotting happens onwards from line 20. Using numpys loadtxt we load the histogram matrix into the array histo\_matrix (line 21). In line 22 we define the amount of bins (nbis=60) and the range (-3, 2) of the coloraxis. The array can directly be processed by contourf where we also input the range, levels, and colormap. The option extend='both' enables the colors beyond the levels definded before (arrows above and below the coloraxis). The lines 24 to 28 are defining the axis-ticks and -labels. After that the plot is already finished and can be saved or shown directly. An example plot for a small watersimulation is shown in Fig. 5.1.

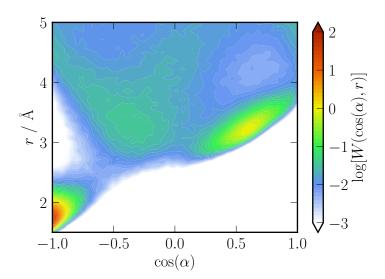


Fig. 5.1: Example plot of a twodimensional histogram computed with hb\_analyze.

# 6 Mean Square Displacement

First, to compute the mean square displacement (MSD) of a particle the trajectory has to be "unwrapped", so that the effects of the periodic boudary conditions (PBCs) are reversed. The function mdorado.msd.unwrap is able to accomplish this task for cuboid boxes (all box angles are 90°).

Given a trajectory (universe) AtomGroup (agrp) the positions of the atoms of interest are recomputed so that each atom starts in the origin of the coordinate system at t=0. The displacement in each time step is then added incrementally to obtain a trajectory relative to this starting position. If the magnitude of the displacement in one dimension (x, y or z) is larger than half a box length in that dimension, indicating a jump of the atom due to PBCs, the displacement is adjusted by adding or subtracting the box length to ensure "jump-free" movement.

From these coordinates mdorado.msd.msd is able to compute the MSD using an efficient algorithm<sup>[13]</sup>

$$MSD(t) = \langle |\boldsymbol{r}(t) - \boldsymbol{r}(0)|^2 \rangle, \qquad (6.1)$$

where r(t) denotes a positional vector of the unwrapped trajectory.

## 6.1 Functions

mdorado.msd.unwrap(universe, agrp, dimensionskey="xyz", cms=False)

### **Parameters:**

universe: MDAnalysis.Universe

Universe containing the trajectory.

agrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms for which the trajectory should

be unwrapped.

dimensionskey: str, optional

Dimensions in which the trajectory is unwrapped. The keywords are:

- "xyz" for all dimensions
- "x", "y", and "z" for one of the three principal box axes
- "xy", "xz", and "yz" for a combination of two of the three box axes

Changes the shape of the output array. The default value is "xyz".

cms:

bool, optional

If cms=True the program computes the movement of the center-of-mass of atoms belonging to the same residue in agrp. If, for example, agrp would contain all atoms of water molecules in the simulation this option allows for the calculation of the average center-of-mass MSD of these water molecules. If for the same case cms=False is chosen, the program calculates the MSD of all the various atoms individually, averaging over hydrogen as well as oxygen atoms. The default value is False.

### Returns: ndarray

An index containing the unwrapped positions. The shape of the array is  $(N_A, N_{\text{dim}}, N_{\text{steps}})$ , where  $N_A$  denotes the number of atoms in agrp (or the number of residues if cms=True),  $N_{\text{dim}}$  denotes the number of dimensions according to the option dimensionskey, and  $N_{\text{steps}}$  is the number of timesteps in the universe.

```
mdorado.msd.msd(positions, dt, outfilename="msd.dat")
```

#### **Parameters:**

positions: ndarray

ndarray of d

dt: int or float

Difference in time between two configurations in positions.

outfilename: str, optional

The name of the outputfile. The default is "msd.dat".

### **Output:**

The function writes a file with two columns, where the first column is the time delay t and the second column is the average MSD for that time delay  $MSD(t) = \langle |(\mathbf{r}(t) - \mathbf{r}(0))|^2 \rangle$ 

## 6.2 Example

We are going to compute two slightly different MSDs from the example water trajectory. First, we are interested in the movement of the center of mass of the water molecules. Therefore, we create an atom group with all oxygen and hydrogen atoms (solgrp). The msd.unwrap function repairs the jumps due to the periodic boundary conditions ("unwrapping"). Because we are interested in the 3D movement of the center-of-mass, we set the option dimensionskey to "xyz" and the option cms to "True" (line 10). The function atomatically computes the center-of-mass of atoms belonging to the same molecule and returns the trajectory of these centers in the ndarray cmspos of shape (128, 3, 2501)  $[(N_A, N_{\text{dim}}, N_{\text{steps}})]$ . This array can be used as input for the msd.msd function along with the timestep (dt) to calculate the average MSD of these centers-of-mass.

The second example computes the average MSD of all hydrogen atoms in the trajectory. Therefore, we create an atom group containing the hydrogen atoms (hgrp) and unwrap their trajectory with cms set to False (default) (line 13). The array hpos is of shape (256, 3, 2501) (two hydrogen atoms per water molecule) and can again be used as an input for the msd.msd function to compute the MSD.

```
import MDAnalysis
from mdorado import msd
from mdorado.data.datafilenames import water_topology,
water_trajectory

u = MDAnalysis.Universe(water_topology, water_trajectory)
solgrp = u.select_atoms("resname sol")
hgrp = u.select_atoms("name hw")
dt = 0.2

cmspos = msd.unwrap(universe=u, agrp=solgrp, dimensionskey="xyz",
cms=True)
msd.msd(positions=cmspos, dt=dt, outfilename="msd_cms.dat")

import MDAnalysis
import msd
start mader msd
import msd
start mader msd
import msd
start mader msd
start mader msd
start mader msd
start ms
```

# 7 vectors

# 7.1 pbc\_vecarray

Applies periodic boundary condition to an array of vectors so that the resulting array satisfies the minimum image convention for cuboid simulation boxes (all angles are 90°).

```
mdorado.vectors.pbc_vecarray(vecarray, box)
```

### Parameters:

vecarray: ndarray

Array of shape  $N_{\text{vec}}$  (number of vectors),  $N_{\text{dim}}$  (dimensionality of the vectors) on which the periodic boundary condition will be ap-

plied.

box: array-like

One-dimensional array-like where the elements contain the length

of the simulation box in the particular dimension.

### Returns: ndarray

Array of the same shape as **vecarray** containing the vectors corrected for the minimum image convention.

### Example:

```
import numpy as np
from mdorado.vectors import pbc_vecarray

vectorarray = np.array([[11,4,-4], [4,1,8], [-6,-7, 2], [3,0, -1]])
box = [10,5,7]
pbc_vectorarray = pbc_vecarray(vecarray=vectorarray, box=box)
```

The resulting array (pbc\_vectorarray) contains the corrected vectors originally listed in vectorarray: [[1, -1, 3], [4, 1, 1], [4, -2, 2], [3, 0, -1]].

## 7.2 norm\_vecarray

Given an array of vectors, normalizes each vector and returns the array of normalized vectors as well as the length of each vector.

```
mdorado.vectors.norm_vecarray(vecarray)
```

#### Parameters:

vecarray: ndarray

Array of shape  $N_{\text{vec}}$  (number of vectors),  $N_{\text{dim}}$  (dimensionality of

the vectors) containing the vectors that will be normalized.

### Returns: unitvecarray, norm: ndarray, ndarray

Returns two arrays, the first (unitvecarray) is of the same shape as the input array and contains the normalized vectors and the second (norm) is of the shape  $(N_{\text{vec}},)$  and contains the length of the corresponding original vector.

### Example:

```
import numpy as np
from mdorado.vectors import pbc_vecarray

vectorarray = np.array([[11,4,-4], [4,1,8], [-6,-7, 2], [3,0, -1]])
unitvectors, length = norm_vecarray(vecarray=vectorarray)
```

The array unitvectors contains the unit vectors of the vectors originally listed in vectorarray and length the corresponding lengths of the original vectors.

### 7.3 vectormatrix

Given two sets of particle positions  $\mathbf{A} = [A_1, A_2, \dots, A_n]$  and  $\mathbf{B} = [B_1, B_2, \dots, B_m]$  computes a matrix  $\mathbf{C}$  containing vectors for all combinations  $C_{i,j} = B_j - A_i$ .

```
mdorado.vectors.vectormatrix(apos, bpos)
```

### **Parameters:**

apos: ndarray

Array of shape (n, 3) containing the position vectors of all particles

A.

bpos: ndarray

Array of shape (m, 3) containing the position vectors of all particles

В.

### Returns: ndarray

Returns an array (abmat) of shape (n, m, 3) containing all vectors  $\overrightarrow{A_iB_j}$ :

abmat[i, j] = bpos[j] - apos[i].

# 7.4 get\_vectormatrix

Uses mdorado.vectors.vectormatrix to compute a vector matrix containing all vectors between all combinations of particles A and B for every timestep in a universe.

mdorado.vectors.get\_vectormatrix(universe, agrp, bgrp, pbc=True)

### **Parameters:**

universe: MDAnalysis.Universe

Universe containing the trajectory.

agrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms of A.

bgrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms of B.

pbc: bool, optional

Specifies whether periodic boundary conditions should be applied to find the shortest vector from A to an image of B. Calls the mdorado.vectors.pbc\_vecarray function. Only works for cuboid

boxes (all angles are 90°). Default is True.

### Returns: ndarray

Array of the shape  $(N_{\rm steps}, N_{\rm A}, N_{\rm B}, 3)$ , where  $N_{\rm steps}$  is the number of timesteps in the

universe,  $N_A$  the number of particles A,  $N_B$  the number of particles B, and the last axis refers to the three directions in space x, y and z. For example, the array element AB[i, j, k, 2] refers to the z-component of the vector pointing from  $A_j$  to  $B_k$  at the *i*-th timestep of the simulation.

## **7.5** get\_vecarray

Given to AtomGroups agrp and bgrp, it computes the time evolution of every vector bgrp[i]-agrp[i] for the whole trajectory. Can account for periodic boundary conditions for cuboid boxes.

mdorado.vectors.get\_vecarray(universe, agrp, bgrp, pbc=True)

### Parameters:

universe: MDAnalysis.Universe

Universe containing the trajectory.

agrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms of type A for which a vector  $\overrightarrow{AB}$ 

should be computed.

bgrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms of type B for which a vector  $\overrightarrow{AB}$ 

should be computed.

pbc: bool, optional

Specifies whether periodic boundary conditions should be applied to find the shortest vector from A to an image of B. Calls the mdorado.vectors.pbc\_vecarray function. Only works for cuboid

boxes (all angles are 90°). Default is True.

### Returns: ndarray

Array containing the trajectory of every vector bgrp[i]-agrp[i] of the shape  $N_{\text{vec}}$  (number of vectors),  $N_{\text{ts}}$  (number of timesteps in the universe),  $N_{\text{dim}}$  (number of dimensions).

## 7.6 get\_normal\_vecarray

Computes given to AtomGroups agrp, bgrp and cgrp, it computes the normal vectors

$$\boldsymbol{n}_i(t) = [\boldsymbol{b}_i(t) - \boldsymbol{a}_i(t)] \times [\boldsymbol{c}_i(t) - \boldsymbol{a}_i(t)]$$
(7.1)

for every triple of atoms positions  $(a_i, b_i, c_i)$  and for every timestep t of the trajectory.

mdorado.vectors.get\_normal\_vecarray(universe, agrp, bgrp, cgrp,
 pbc=True)

#### **Parameters:**

universe: MDAnalysis.Universe

Universe containing the trajectory.

agrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms of type A used to define the plane

containing the atoms  $A_i$ ,  $B_i$  and  $C_i$ .

bgrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms of type B used to define the plane

containing the atoms  $A_i$ ,  $B_i$  and  $C_i$ .

cgrp: AtomGroup from MDAnalysis

AtomGroup containing all atoms of type C used to define the plane

containing the atoms  $A_i$ ,  $B_i$  and  $C_i$ .

pbc: bool, optional

Specifies whether periodic boundary conditions should be applied to find the shortest vector from A to an image of B and C. Calls the mdorado.vectors.pbc\_vecarray function. Only works for cuboid

boxes (all angles are 90°). Default is True.

### Returns: ndarray

An ndarray of shape  $N_{\text{vec}}$  (number of normal vectors  $\overrightarrow{n}$ ),  $N_{\text{steps}}$  (number of timesteps in universe),  $N_{\text{dim}}$  (number of dimensions) containing the time evolution of all normal vectors  $\overrightarrow{n} = \overrightarrow{AB} \times \overrightarrow{AC}$  in the trajectory.

# 8 Reorientational Correlation

The dynamic of the loss of reorientational correlation can be determined via reorientational correlation functions  $R_i(t)$ . In general, it is defined via

$$R_i(t) = \langle P_i \{ \cos[\theta(0)] \} P_i \{ \cos[\theta(t)] \} \rangle, \tag{8.1}$$

where  $P_i\{\cos[\theta(t)]\}$  denotes the *i*-th Legendre polynominal of the cosine of the angle  $\theta$  between a vector and a fixed external reference vector at time t. The brackets  $\langle \ldots \rangle$  indicate averaging vectors of the same kind as well as all times "0".

In an isotropic medium like unordered liquids, the choice of reference vector does not influence  $R_i(t)$ . By choosing the orientation of the vector at time "0" as the reference vector equation 8.1 simplifies to

$$R_i(t) = \langle P_i \{ \cos[\theta(t)] \} \rangle,$$
 (8.2)

as  $\cos[\theta(0)] = 1$  and therefore  $P_i\{\cos[\theta(0)]\} = 1$  in that case. Compared to using a fixed reference vector this simplification also improves the statistics of  $R_i(t)$  immensly. Both, the anisotropic as well as the isotropic case, can be investigated with MDORADO.

To obtain  $P_i\{\cos[\theta(t)]\}$ , we first need to compute  $\cos[\theta(t)]$  via

$$\cos[\theta(t)] = \boldsymbol{u}(t) \cdot \boldsymbol{u}_{\text{ref}},\tag{8.3}$$

where u(t) is the unit vector of interest and  $u_{ref}$  is a fixed external reference unit vector. Thus,  $u_{ref}$  for all timesteps is needed. The get\_vecarray and norm\_vecarray functions of the mdorado.vectors module can be used to obtain a trajectory of unit vectors. For the computation of  $R_i(t)$ , one should also make sure that the vector of interest does not suddenly "flip" due to atoms crossing the boundary of the simulation box, as this reorientation would be an artifact of the periodic boundary condition.

Given a trajectory of vectors, the functions correlvec and isocorrelvec can be used to obtain  $R_i(t)$ . The correlvec function follows equation 8.1 and needs a fixed external vector to obtain  $R_i(t)$ . The computation makes use of correlations via a Fast-Fourier-transform algorithm and is therefore fast even for long trajectories. But compared to the isotropic simplification via isocorrelvec, the statistics of  $R_i(t)$  is worse for correlvec.

Vice versa, the isocorrelvec function can be very slow for long trajectories, as we are not able to use a Fast-Fourier-transform algorithm for an arbitrary Legendre polyomial. For this reason, fast algorithms for the correlation functions of the first and second Legendre polynomials using the isotropic simplification are included in MDORADO via the functions isocorrelveclg1  $(R_1(t))$  and isocorrelveclg2  $(R_2(t))$ . These functions combine the superior statistics of the isotropic simplification and the computational speed of a Fast-Fourier-transform algorithm.

### 8.1 Functions

#### **Parameters:**

vecarray: ndarray

Array of shape  $N_{\rm vec}$  (number of vectors),  $N_{\rm steps}$  (number of timesteps),  $N_{\rm dim}$  (number of dimensions) containing the time evolution of all unit vectors of interest. See functions get\_vecarray, get\_normal\_vecarray and norm\_vecarray of the

mdorado.vectors module for ways to obtain such an array.

refvec: ndarray or array-like

Fixed external reference vector. Will be normalized internally.

dt: int or float

Timestep used in vecarray.

nlegendre: int

Specifies which Legendre polynomial should be computed. For example nlegendre = 2 references the second legendre polynomial  $P_2\{\cos[\theta(t)]\} = \frac{3}{2}\cos^2[\theta(t)] - \frac{1}{2}$  which will be used to computed

 $R_2(t)$  according to equation 8.1.

outfilename: str

If specified an xy-file with the name str(outfilename) containing t and  $R_i(t)$  will be written. If False no file will be written. Default

is False

normed: boolean

Specifies whether the function  $R_i(t)$  should be normalized or not. Default is True

Returns: timesteps, allcorrel: ndarray, ndarray

Returns two arrays, the first containing information about the timestep t and the second containing the averaged function  $R_i(t)$ .

mdorado.veccor.isocorrelvec(vecarray, dt, nlegendre, outfilename=False)

#### **Parameters:**

vecarray: ndarray

Array of shape  $N_{\rm vec}$  (number of vectors),  $N_{\rm steps}$  (number of timesteps),  $N_{\rm dim}$  (number of dimensions) containing the time evolution of all unit vectors of interest. See functions get\_vecarray, get\_normal\_vecarray and norm\_vecarray of the mdorado.vectors module for ways to obtain such an array.

dt: int or float

Timestep used in vecarray.

nlegendre: int

Specifies which Legendre polynomial should be computed. For example nlegendre = 2 references the second legendre polynomial  $P_2\{\cos[\theta(t)]\} = \frac{3}{2}\cos^2[\theta(t)] - \frac{1}{2}$  which will be used to computed

 $R_2(t)$  according to equation 8.2.

outfilename: str

If specified an xy-file with the name str(outfilename) containing t and  $R_1(t)$  will be written. If False no file will be written. Default

is False

Returns: timesteps, allcorrel: ndarray, ndarray

Returns two arrays, the first containing information about the timestep t and the second containing the averaged function  $R_i(t)$  using the isotropic simplification in equation 8.2.

mdorado.veccor.isocorrelveclg1(vecarray, dt, outfilename=False)

#### **Parameters:**

vecarray: ndarray

Array of shape  $N_{\rm vec}$  (number of vectors),  $N_{\rm steps}$  (number of timesteps),  $N_{\rm dim}$  (number of dimensions) containing the time evolution of all unit vectors of interest. See functions get\_vecarray, get\_normal\_vecarray and norm\_vecarray of the mdorado.vectors module for ways to obtain such an array.

int or float

Timestep used in vecarray.

outfilename: str

dt:

If specified an xy-file with the name str(outfilename) containing t and  $R_i(t)$  will be written. If False no file will be written. Default is False

Returns: timesteps, allcorrel: ndarray, ndarray

Returns two arrays, the first containing information about the timestep t and the second containing the averaged function  $R_1(t)$  using the isotropic simplification in equation 8.2.

mdorado.veccor.isocorrelveclg2(vecarray, dt, outfilename=False)

#### **Parameters:**

vecarray: ndarray

Array of shape  $N_{\rm vec}$  (number of vectors),  $N_{\rm steps}$  (number of timesteps),  $N_{\rm dim}$  (number of dimensions) containing the time evolution of all unit vectors of interest. See functions get\_vecarray, get\_normal\_vecarray and norm\_vecarray of the mdorado.vectors module for ways to obtain such an array.

dt: int or float

Timestep used in vecarray.

outfilename: str

If specified an xy-file with the name str(outfilename) containing t and  $R_2(t)$  will be written. If False no file will be written. Default

is False

Returns: timesteps, allcorrel: ndarray, ndarray

Returns two arrays, the first containing information about the timestep t and the second

## 8.2 Example

First, we need the time evolution of the vector of interest in all cases. The function  $\mathtt{get\_vecarray}$  of the  $\mathtt{mdorado.vectors}$  module can be used to compute a time-series of molecular vectors  $\overrightarrow{AB}$  given the  $\mathtt{AtomGroups}$  agrp and  $\mathtt{bgrp}$ . The function  $\mathtt{mdorado.vectors.get\_normal\_vecarray}$  can be used to compute normal vectors of the plane given by the particles in the  $\mathtt{AtomGroups}$  agrp,  $\mathtt{bgrp}$  and  $\mathtt{cgrp}$ . For both functions the  $\mathtt{AtomGroups}$  have to be of equal length, so that for every element i a vector  $\overrightarrow{A_iB_i} = \mathtt{bgrp}[i] - \mathtt{agrp}[i]$  (or normal vector  $\overrightarrow{n} = \overrightarrow{A_iB_i} \times \overrightarrow{A_iC_i}$ ) can be computed.

```
import MDAnalysis
2 from mdorado import vectors as mvec
3 from mdorado import veccor
4 from mdorado.data.datafilenames import water_topology,
     water_trajectory
6 u = MDAnalysis.Universe(water_topology, water_trajectory)
7 ogrp = u.select_atoms("name ow")
8 hgrp = u.select_atoms("name hw")[::2]
9 dt = 0.2
vectors = mvec.get_vecarray(universe=u, agrp=ogrp, bgrp=hgrp)
vectors = mvec.norm_vecarray(vectors)[0]
14 ts_aniso, correl_anisolg2 = veccor.correlvec(vectors, refvec=[1,1,1],

    dt=dt, nlegendre=2, outfilename="111_vec.dat", normed=True)

15 ts_iso, correl_iso = veccor.isocorrelvec(vectors, dt=dt, nlegendre=2,
  → outfilename="slow_isolg2.dat")
16 ts_isolg2, correl_isolg2 = veccor.isocorrelveclg2(vectors, dt=dt,
    outfilename="fast_isolg2.dat")
```

In our example, we want to investigate the reorientational behavior of the O-H bond vector in water. After initializing the universe (line 5), we create one AtomGroup with all oxygen atoms (line 6, ogrp) and another AtomGroup where we choose every second

hydrogen atom in our system, meaning one hydrogen atom per water molecule (line 7, hgrp). Using the get\_vecarray function, we are able to compute an array containing one O–H vector per water molecule for every timestep (line 11). The normalized (line 12) vector array can then be used to compute the correlation function according to equations 8.1 or 8.2.

Without assuming an isotropic phase or when it is desirable to investigate the reorientation in comparison to a fixed external vector, we can use equation 8.1 and employ the correlvec function to compute the autocorrelation function of the second Legendre polynomial (nlegendre=2, line 14). The reference vector, here [1,1,1], can be of arbitrary length, as it will be normalized internally. The normed keyword ensures, that the resulting correlation function starts at 1 for t=0.

In line 15, we compute the reorientation correlation function using the isotropic simplification (equation 8.2) employing the **isocorrelvec**. The **refvec** keyword is omitted here, as the orientation of the vectors at every time "0" will be used as reference at time t. This also makes normalization unnecessary, as  $\cos[\theta(t=0)] = 1$ . As mentioned, although **isocorrelvec** yields a better statistic for  $R_i(t)$ , it is significantly slower than the **correlvec** function as we can not easily use FFT correlation for every arbitrary Legendre polynomial.

To circumvent this problem, faster algorithms to compute the first and second Legendre polynomials using the isotropic simplification in equation 8.2 are available in MDo-RADO via the functions isocorrelveclg1 and isocorrelveclg2, respectively. Line 16 exemplifies the usage of these functions. The nlegendre keyword is omitted in both functions, as the type of Legendre polynomial is hard-coded in the respective function.

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