Homework-seminar 11

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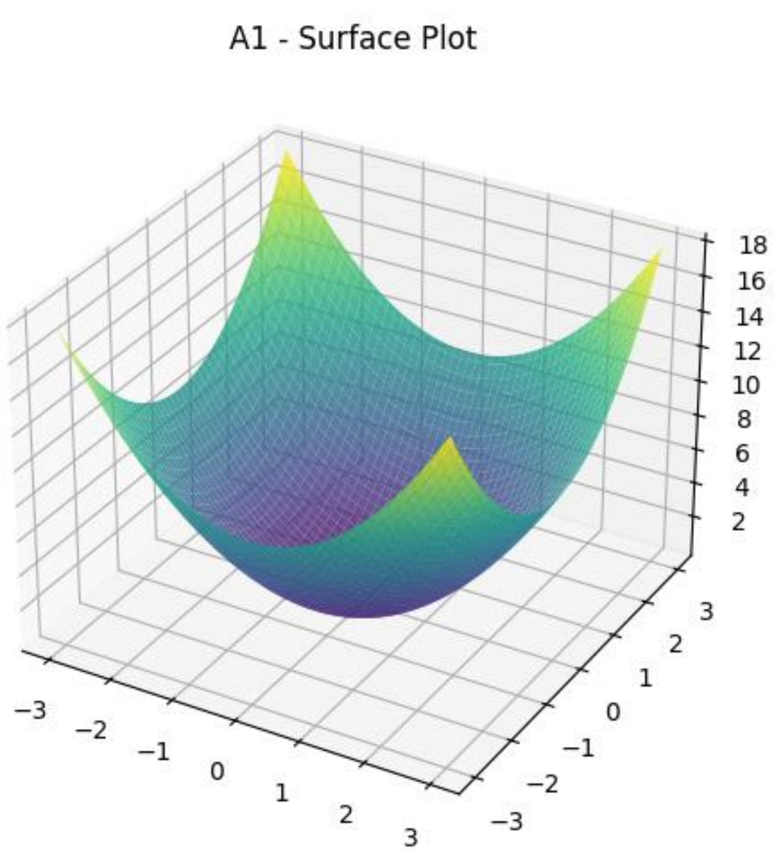
```
import numpy as np
import matplotlib.pyplot as plt


# we have the given quadratic function f(x) = 1/2 * x^T * A * x
1 usage
def quadratic_function(x, A):
    return 1 / 2 * np.dot(x.T, np.dot(A, x))


# we define a function to compute the gradient of f(x)
1 usage
def gradient(x, A):
    return np.dot(A, x)


# we create a 2x2 matrix A for each task: a), b) and c)
A_min = np.array([[2, 0], [0, 2]])  # ->unique minimum
A_max = np.array([[2, 0], [0, -2]])  # ->unique maximum
A_saddle = np.array([[2, 1], [1, -2]])  # ->saddle point
```

```
# we plot 3D surface, contour lines, and gradients for each case
fig = plt.figure(figsize=(18, 15))

for i, (A, color) in enumerate(zip([A_min, A_max, A_saddle], ['blue', 'green', 'orange']), start=1):
    ax = fig.add_subplot(3, 3, i * 3 - 2, projection='3d')
```

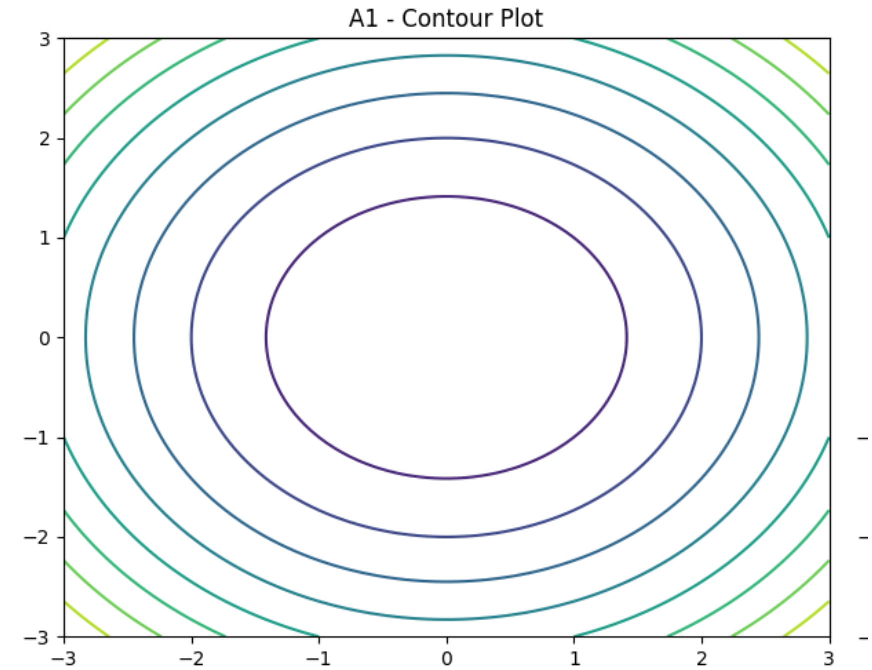
```
# the 3D Plot surface:
x_range = np.linspace(-3,  stop: 3,  num: 100)
y_range = np.linspace(-3,  stop: 3,  num: 100)
X, Y = np.meshgrid( *xi: x_range, y_range)
Z = np.array([quadratic_function(np.array([x, y]), A) for x, y in zip(X.flatten(), Y.flatten())])
Z = Z.reshape(X.shape)
ax.plot_surface(X, Y, Z, cmap='viridis', alpha=0.8)
ax.set_title(f'A{i} - Surface Plot')
```

```python
    # the plot contour linesD
    ax = fig.add_subplot(3, 3, i * 3 - 1)
    ax.contour( *args: X, Y, Z, levels=10, cmap='viridis')
    ax.set_title(f'A{i} - Contour Plot')

    # we choose three different points for gradients
    points = np.array([[-2, -2], [0, 0], [2, 2]])

    # the gradients are plotted in the folowing way:
    ax = fig.add_subplot(3, 3, i * 3)
    ax.contour( *args: X, Y, Z, levels=10, cmap='viridis')
    gradients = np.array([gradient(point, A) for point in points])

    # the quiver line needs to be corrected in order to use gradient vectors directly
    ax.quiver( *args: points[:, 0], points[:, 1], gradients[:, 0], gradients[:, 1],
              angles='xy', scale_units='xy', scale=1, color=color)
    ax.set_title(f'A{i} - Gradients')

plt.tight_layout()
plt.show()
```

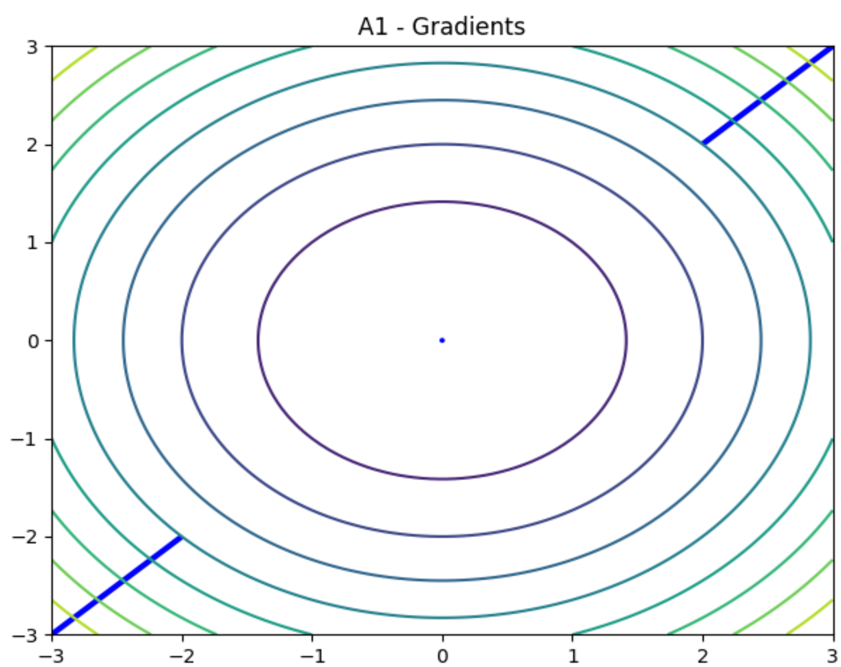
Results:



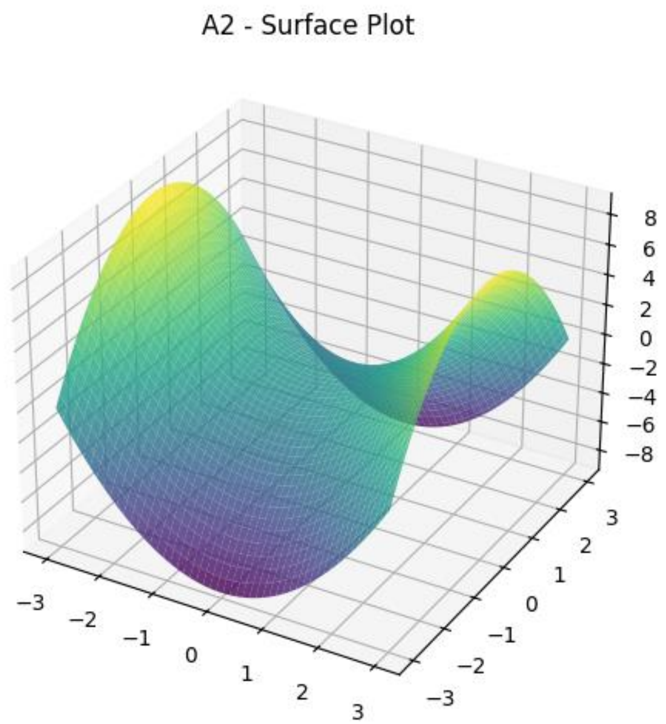
Unique Minimum. The 3D surface plot illustrates a quadratic function, where *A1* is a 2x2 matrix resulting in a distinct minimum. This plot displays the function values in 3D space, showcasing a descending surface towards a singular minimum point.



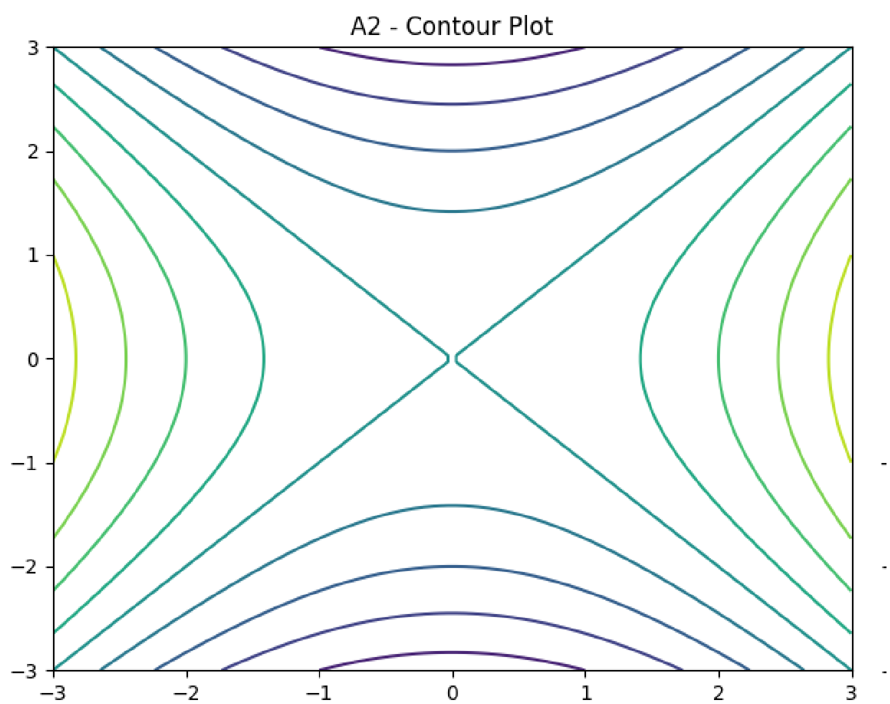
A 2D representation of the surface plot, the contour plot exhibits curves of constant function values. For a unique minimum, concentric circles emerge, delineating level sets of the quadratic function.



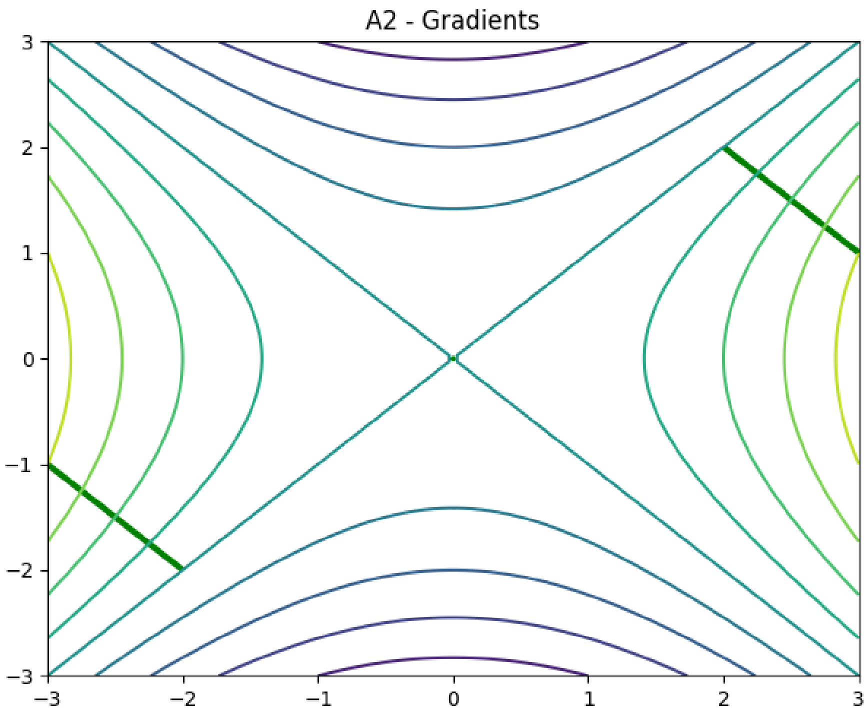
The gradient plot depicts vectors at three distinct points (-2, -2), (0, 0), and (2, 2). Each vector represents the function's gradient at that point. In the case of a unique minimum, these gradients extend outward, indicating the direction of the steepest ascent.



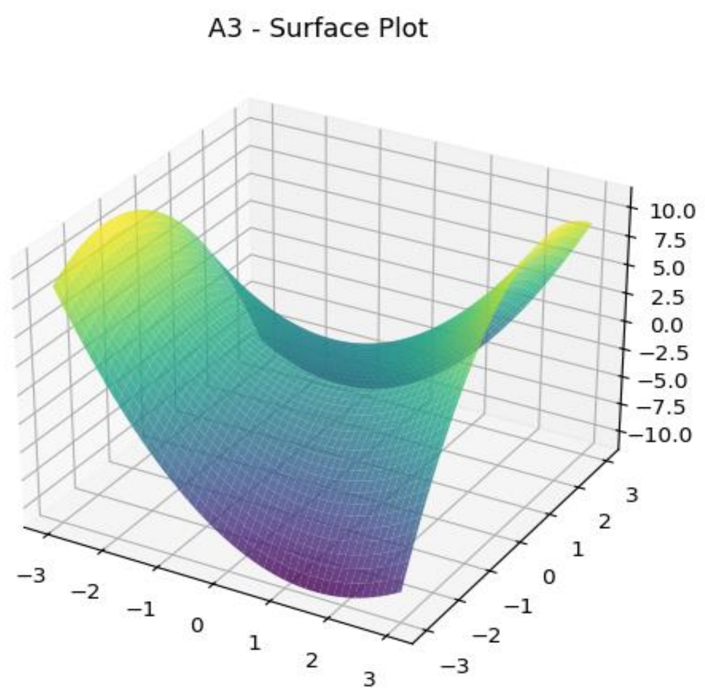
Unique Maximum. The 3D surface plot illustrates the quadratic function, where A2 is a 2x2 matrix resulting in a unique maximum. The surface in this case ascends, depicting a trajectory toward a maximum point.



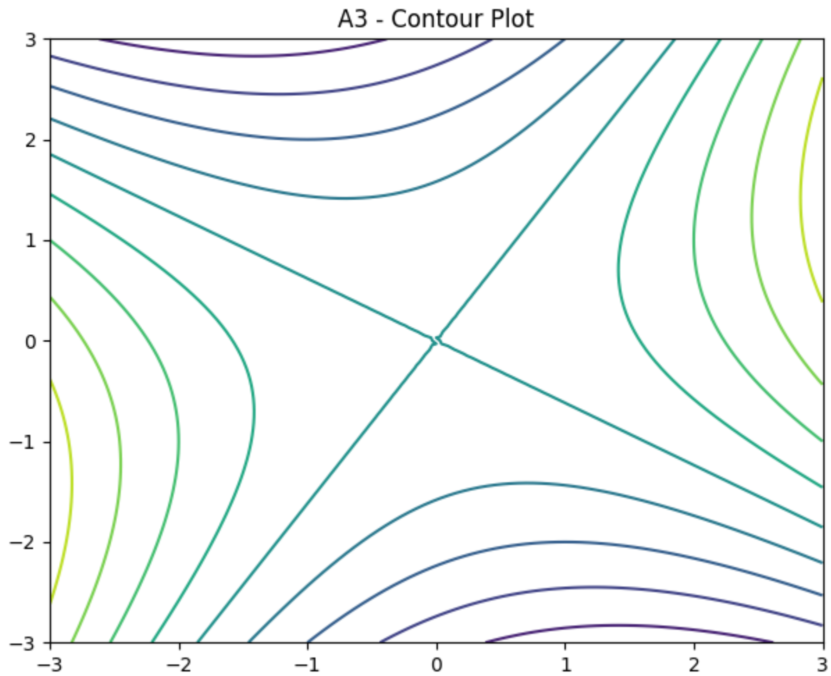
The contour plot for A2 displays upward-sloping contour lines, outlining the level sets of the quadratic function. As opposed to the unique minimum case, the lines are now more widely spaced as you move away from the maximum point.



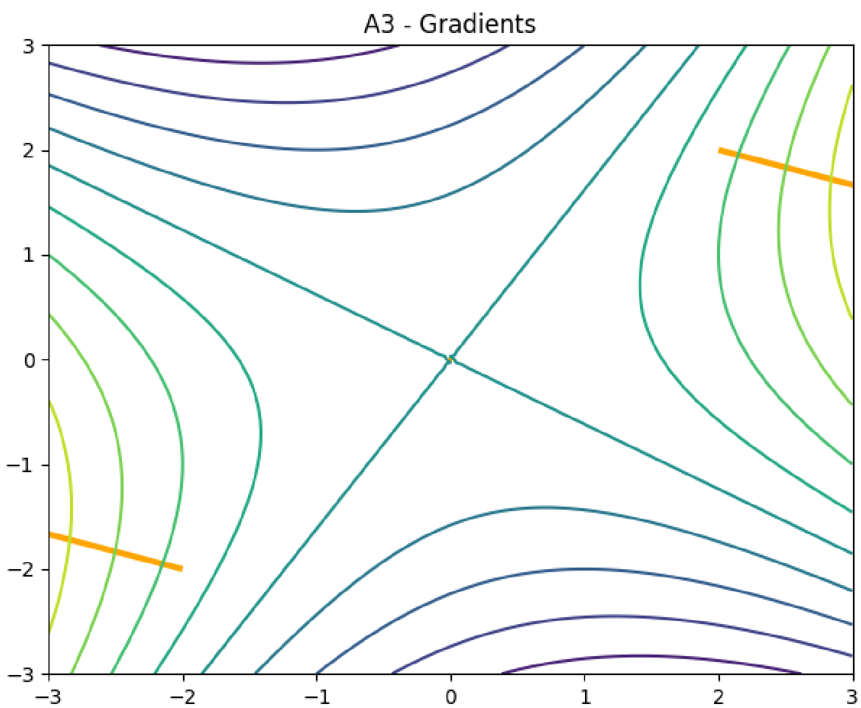
The gradient plot exhibits vectors at three distinct points (-2, -2), (0, 0), and (2, 2). In this instance, the gradients point inward towards the maximum, signaling the direction of the steepest ascent.



Unique Saddle Point. The 3D surface plot illustrates the quadratic function where A3 is a 2x2 matrix leading to a unique saddle point. The surface exhibits a combination of both upward and downward slopes, indicating the presence of a saddle point.



The contour plot for A3 showcases a mixture of upward and downward-sloping contour lines. These lines depict the level sets of the quadratic function in the vicinity of the saddle point.



In the gradient plot, vectors are depicted at three specific points (-2, -2), (0, 0), and (2, 2). At the saddle point, the gradients point in diverse directions, signifying the absence of a strict ascent or descent at that particular location.