Homework-seminar 08

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Homework 8

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2. For $x, y \in \mathbb{R}^m$ prove that the following identities hold: state-ments are $\equiv$:

a) $\langle x, y \rangle = 0$

b) $\|x+y\| = \|x-y\|$

c) $\|x+y\|^2 = \|x^2\| + \|y\|^2$.

I. assume a). is True $\Rightarrow$

$\Rightarrow \|x+y\|^2 = \|x\|^2 + 2\langle x, y \rangle + \|y\|^2 \Rightarrow \|x+y\|^2 = \|x\|^2 + \|y$

but $\langle x, y \rangle = 0$ (c) $\Rightarrow$

$\rightarrow$ a) $\Rightarrow$ c). (1).

assume b) is True :

$\|x+y\| = \|x-y\|$ ↑2 =)

=) $\|x+y\|^2 = \|x-y\|^2$ =)

=) $\|x\|^2 + 2\langle x,y\rangle + \|y\|^2 = \|x\|^2 - 2\langle x,y\rangle + \|y\|^2$ =)

=) $2\langle x,y\rangle + 2\langle x,y\rangle = 0$ =)

=) $\langle x,y\rangle = 0$ (a1) . =) b) => a) . ①

assume c) is True..

$\|x+y\|^2 = \|x\|^2 + \|y\|^2$.

$\|x+y\|^2 = \|x\|^2 + 2\langle x,y\rangle + \|y\|^2$ } =)

=) $2\langle x,y\rangle = 0$ =)

=) $\langle x,y\rangle = 0$. => c) -> a) ②.

assume a) is True :

$\langle x,y\rangle = 0$. =)

=) $\|x+y\|^2 = \|x-y\|^2$ =) $\|x+y\| = \|x-y\|$.

$\|x+y\|^2 = \|x\|^2 + \|y\|^2 + 2\underbrace{\langle x,y\rangle}_{0}$

$\|x-y\|^2 = \|x\|^2 + \|y\|^2 - 2\underbrace{\langle x,y\rangle}_{0}$

=) =)

=) a) => b) . ③ .

- from ① + ② + ③ + ④ => a), b), c) ≡ .

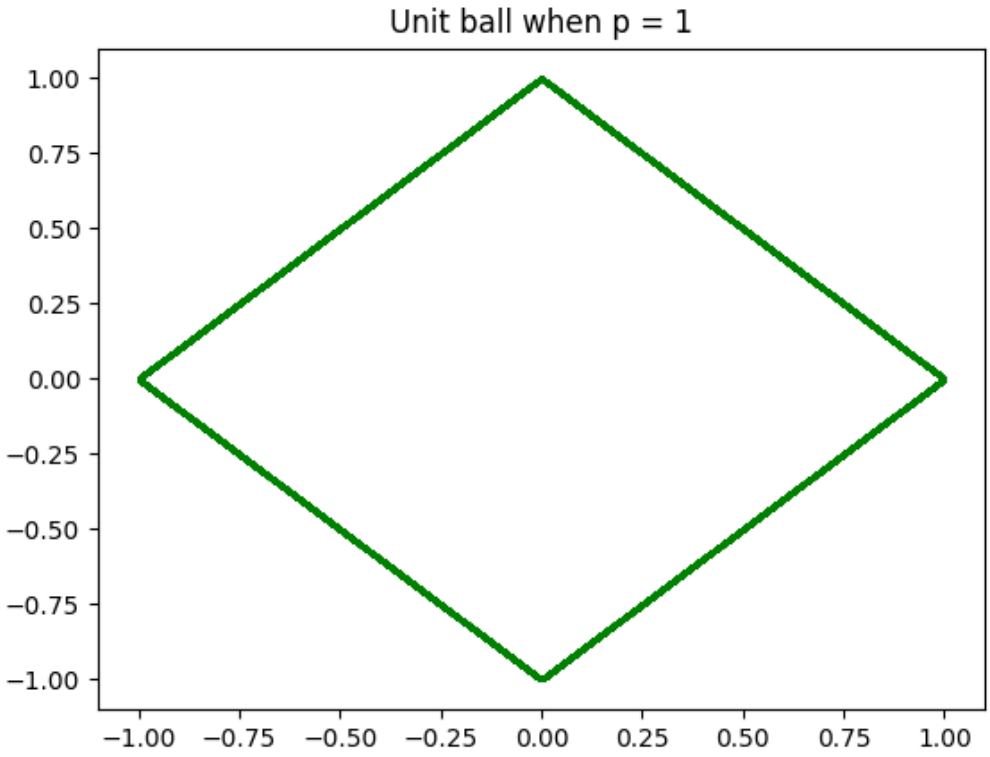
```python
import matplotlib.pyplot as plt
import numpy as np
# importing the necessary libraries:
# matplotlib.pyplot for plotting and
# numpy for numerical operations.

7 usages
def plot_value(p):

    # x is a NumPy array containing 10,000 random
    # samples from a uniform distribution between 0 and 1.
    # the samples are used as the x-coordinates
    # for points in the unit ball.
    x = np.random.random_sample(10000)

    # The loop structure repeats the elements of
    # x and -x twice
    # the result is a list 'final_x' that represents
    # the x-coordinates of points in the unit ball.
```

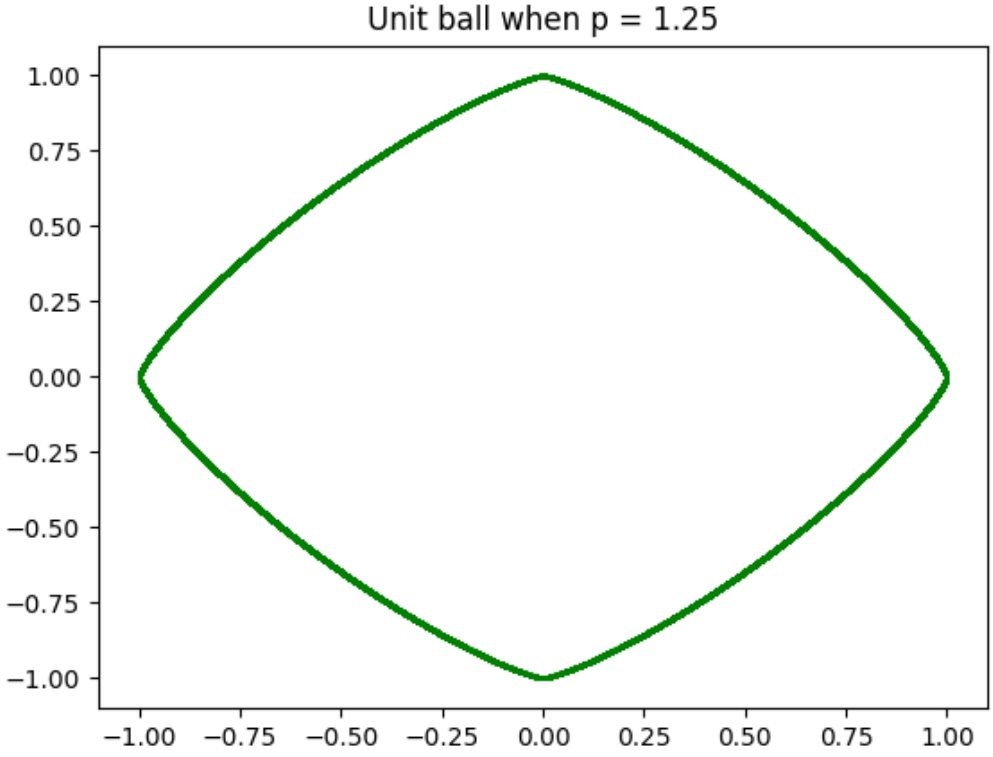
```python
  final_x = []

  for j in range(2):
      for i in x:
          final_x.append(i)

  for j in range(2):
      for i in x:
          final_x.append(-i)
```

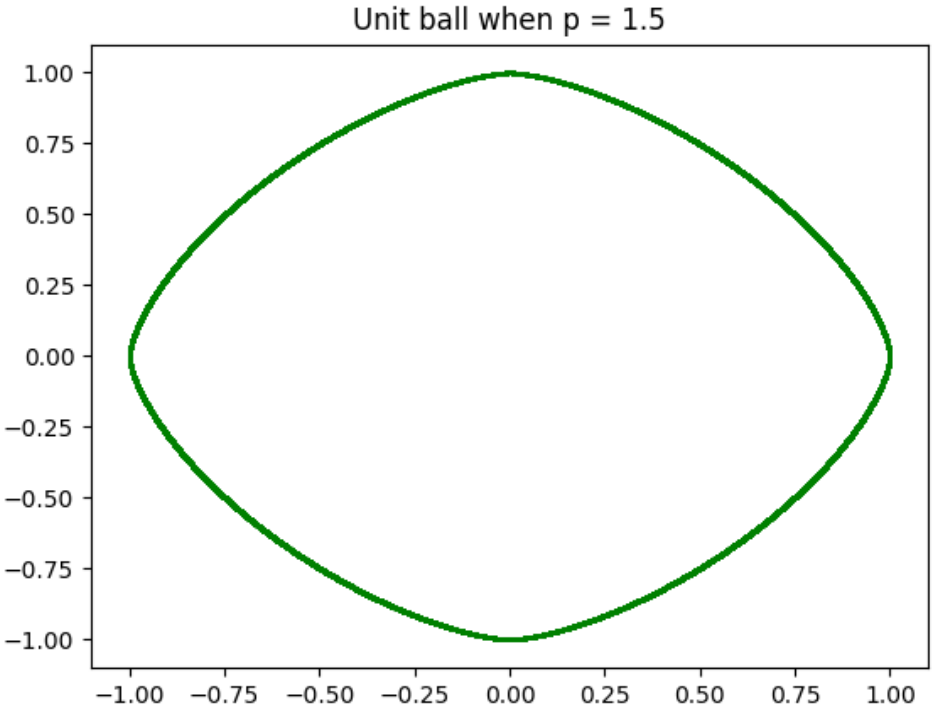
```
# the next part calculates the corresponding y-values
# based on the formula for the lp norm.
# it repeats the elements of y and -y twice to create
# a list 'final_y' representing the y-coordinates of
# points in the unit ball
y = []
final_y = []

for i in x:
    y.append(np.power(1 - np.power(i, p), 1.0000 / p))

for j in range(2):
    for i in y:
        final_y.append(i)

    for i in y:
        final_y.append(-i)

# there are generated 100,000 random samples from a
# uniform distribution between 0 and 1 and stored in
# the array y
y = np.random.random_sample(100000)
x = []

# the corresponding x-coordinates based on the
# formula for the Lp norm are computed and appended
# to the list final_x, repeating them twice (p & n)
```

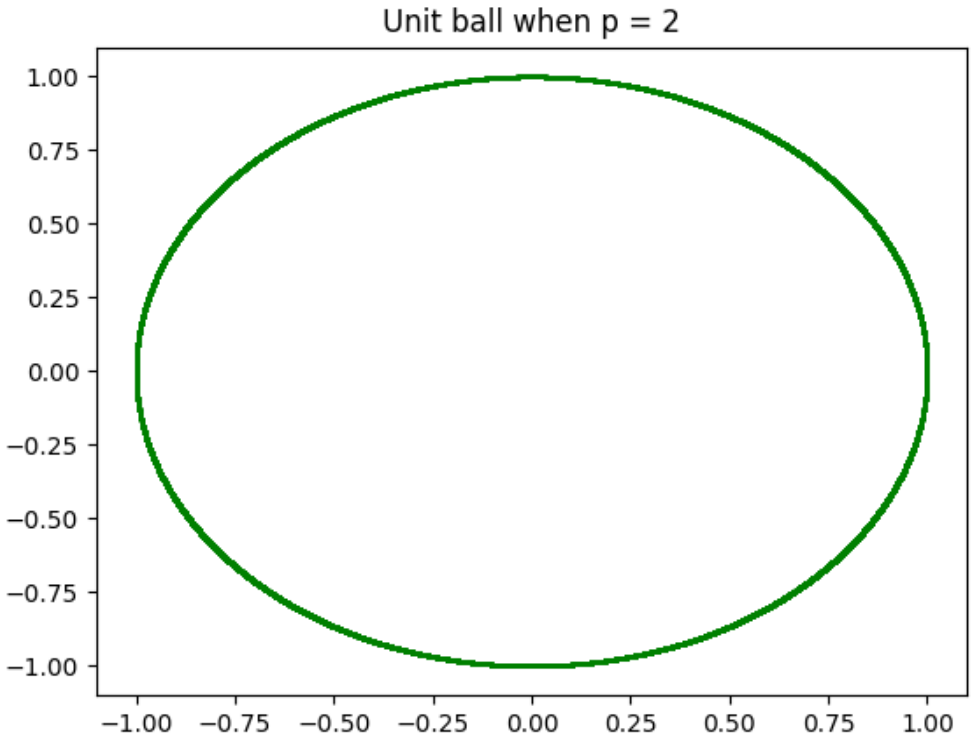
```
for i in y:
    x.append(np.power(1 - np.power(i, p), 1.0000 / p))

for j in range(2):
    for i in x:
        final_x.append(i)

for j in range(2):
    for i in x:
        final_x.append(-i)

# the elements of y (positive and negative) are
# appended to the list final_y

for j in range(2):
    for i in y:
        final_y.append(i)

    for i in y:
        final_y.append(-i)

plt.title("Unit ball when p = " + str(p))
plt.plot( *args: final_x, final_y, "go", markersize=0.5)
plt.show()
```

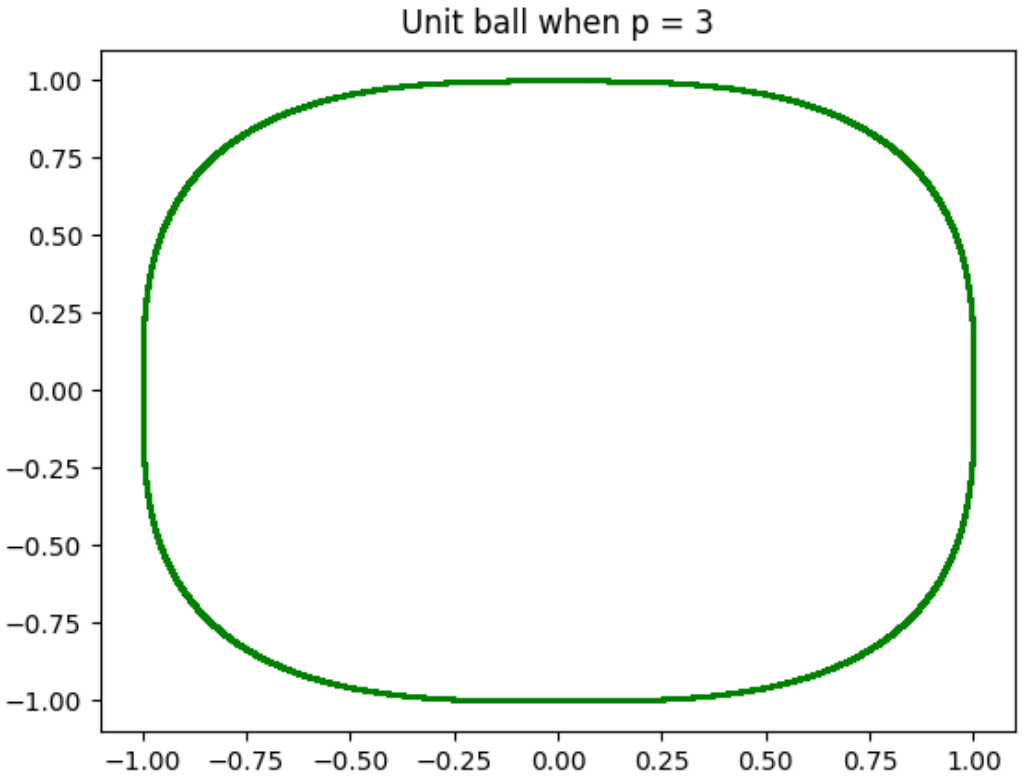
Results:

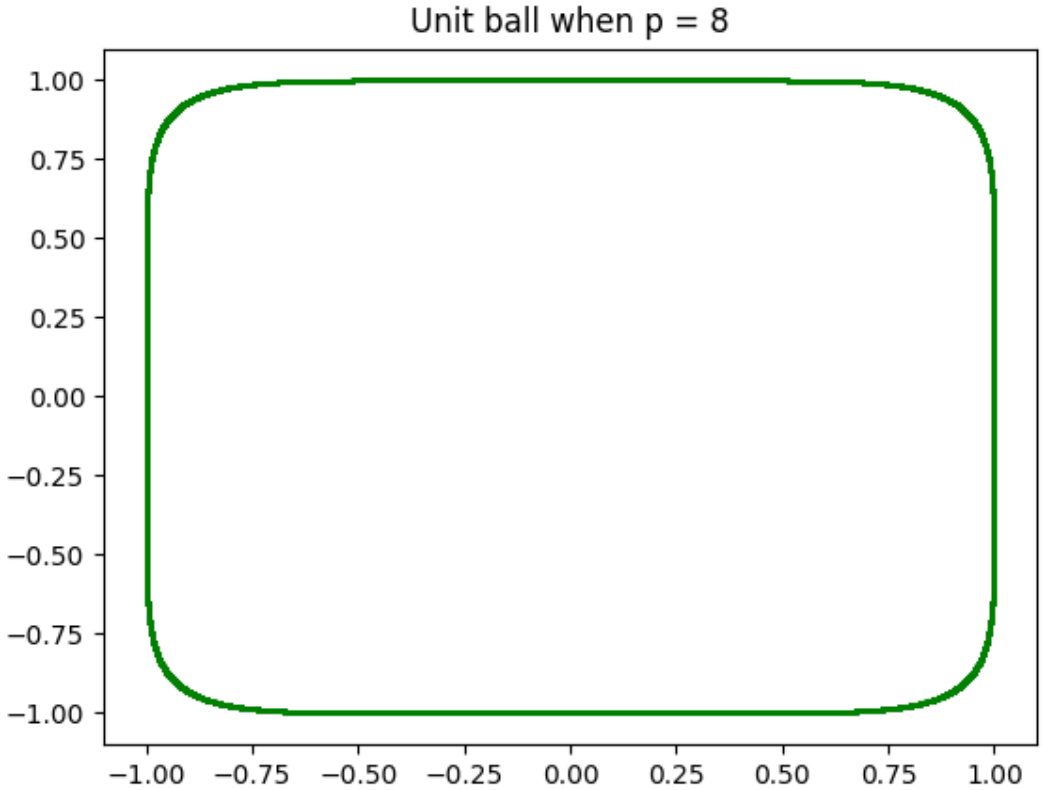

Unit ball when p = 1
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

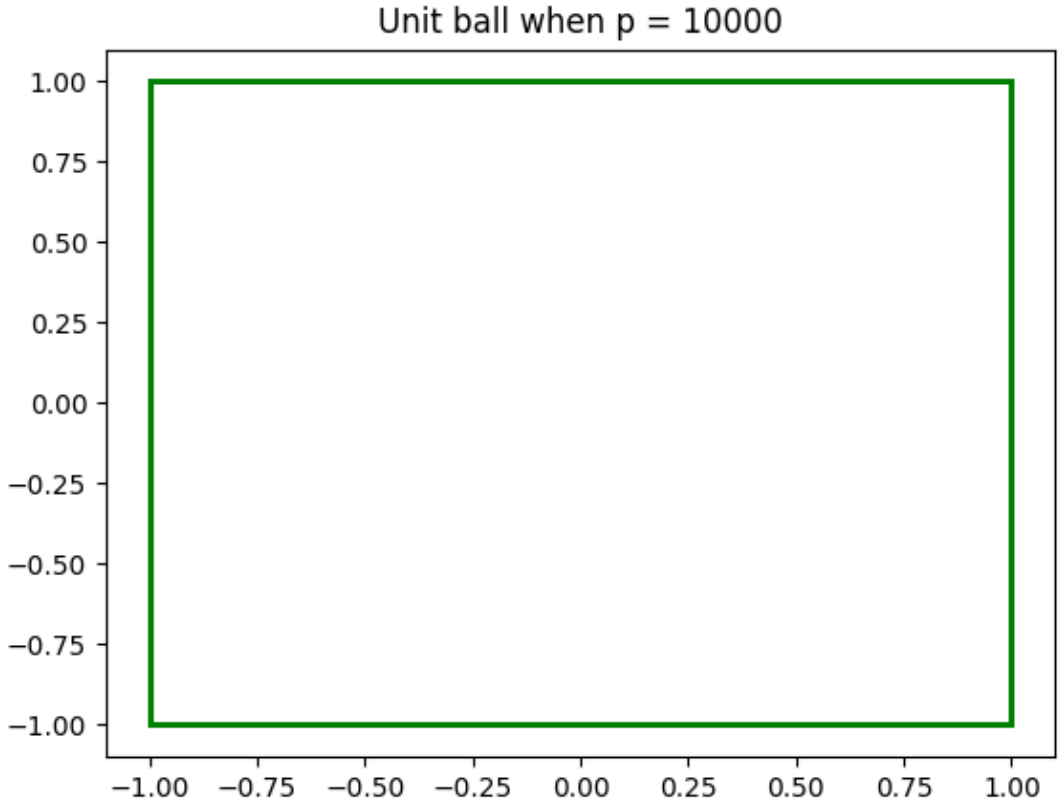

Unit ball when p = 1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


Unit ball when p = 1.5
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


Unit ball when p = 2
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


Unit ball when p = 3
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00


Unit ball when p = 8
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00


Unit ball when p = 10000
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00