$\boxed{1}$

c) . $\displaystyle\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\sin(y)}{y}\, dy\, dx$. ?

$0 \le x \le \frac{\pi}{2}$; $x \le y \le \frac{\pi}{2}$ $\Rightarrow$

$\Rightarrow$ $0 \le y \le \frac{\pi}{2}$; $0 \le x \le y$.

$$= \int_0^{\frac{\pi}{2}} \int_0^{y} \frac{\sin(y)}{y}\, dx\, dy = \int_0^{\frac{\pi}{2}} \frac{\sin(y)}{\cancel{y}} \cdot \cancel{y}\, dy = -\cos y \Big|_0^{\frac{\pi}{2}} =$$

$$= -\cos\frac{\pi}{2} + \cos 0 = 0 + 1 = 1.$$

3 d). $\iint_D \ln(x^2+y^2)\,dx\,dy$, D- the region between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$, $0<a<b$, first quadrant.

$$\left.\begin{array}{l} x = r\cdot\cos\theta \\ y = r\cdot\sin\theta \end{array}\right\} \Rightarrow \left\{\begin{array}{l} x^2 = r^2\cdot\cos^2\theta \\ y^2 = r^2\cdot\sin^2\theta \end{array}\right| \Rightarrow x^2+y^2 = r^2(\cos^2\theta+\sin^2\theta)$$

$$\left.\begin{array}{l} = r^2. \\ x^2+y^2 = a^2 \\ x^2+y^2 = b^2 \\ 0<a<b \end{array}\right\} \Rightarrow r \in [a,b].$$

$$J = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix} \Rightarrow$$

$$\Rightarrow \det(J) = r$$

first quadrant $\Rightarrow \theta \in [0, \frac{\pi}{2}]$

$$\iint_D \ln(x^2+y^2)\,dx\,dy = \int_a^b \int_0^{\frac{\pi}{2}} \ln(r^2)\cdot r\,d\theta\,dr = \frac{\pi}{2}\cdot\int_a^b \ln(r^2)\cdot r\,dr =$$

$\boxed{2}$ d) $\iint_D xy\,dx\,dy$, D- parallelogram with vertices $(0,0)$, $(2,2)$ $(1,2)$, $(3,4)$ $\Rightarrow$

$\rightarrow$ equations: $\begin{cases} y-x=1 \\ y-x=0 \\ y-2x=0 \\ y-2x=-2 \end{cases} \Bigg| \Rightarrow \begin{array}{l} u=y-x \\ v=2x-y \end{array} \Bigg| \rightarrow \begin{array}{l} x=u+v \\ y=2u+v \end{array}$

$$J=\begin{bmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \Rightarrow \det(J)=-1$$

$$\iint_D xy\,dx\,dy = \int_0^2 \int_0^1 (u+v)(2u+v)\cdot(-1)\,du\,dv =$$

$$= \int_0^2 \int_0^1 (-1)\cdot(2u^2+3uv+v^2)\,du\,dv =$$

$$= \int_0^2 \left(-\frac{2}{3}-\frac{3}{2}v-v^2\right)dv = -\frac{2}{3}\cdot 2+\left(-\frac{3}{2}\right)\cdot\frac{2^2}{2}+\left(-\frac{2^3}{2}\right) =$$

$$= -\frac{4}{3}\overset{3}{/}-3\overset{3}{/}-4 = -\frac{25}{3}.$$