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MIR program

AUV model equations

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Abstract

This document justifies the equations and the model that ECA Robotics provides as part of the MIR program. ECARobotics created underwater drones called AUV. The purpose of these drones is to independently scan areas. The AUV, equipped with a sonar, has the function of detecting underwater mines. Indeed, mines are low-cost and effective solutions to destroy a military ship or a submarine. The problem is that an AUV is equipped with a sonar and that this sensor requires high criteria of stability on all levels (vertical plan, horizontal plan, roll,...).A guide and a simplified simulator of an AUV will be provided. The control command used in this simulator is a PID and this control lacks robustness and stability. The aim of this practical work will be to write an optimal control law on one of the planes. Since the writing of the model is not the part sought in these works, this document provides and justifies the equations of the model which will be to linearize.

Bibliography

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Voluntarily applicable documents not provided in this document

Acronymns

AUV Autonomous Underwater Vehicle

COG Course Over Ground

DOF Degree Of Freedom

NED North East Down

STW Speed Throw Water

SOG Speed Over Ground

Nomenclature

ν Sea water viscosity, at 1,2.10^{-6} $m^2.s^{-1}$

ϕ, θ and ψ roll, pitch and heading of the vehicle

ρ Density of water, at 1026 kg.m^3

A Virtual horizontal helms angle

B Buoyancy force

B_1 Rudder angle for the upper right rudder (vehicle seen from behind)

B_2 Rudder angle for the lower right rudder (vehicle seen from behind)

B_3 Rudder angle for the lower left rudder (vehicle seen from behind)

B_4 Rudder angle for the upper left rudder (vehicle seen from behind)

BAR Virtual vertical helms angle

C_G Hydrodynamic moment coefficient of helms along the axis $\vec{x}$

C_M Hydrodynamic moment coefficient of helms along the axis $\vec{y}$

C_N Hydrodynamic moment coefficient of helms along the axis $\vec{z}$

C_X Drag coefficient of the helms along the axis $\vec{x}$

C_Y Drag coefficient of the helms along the axis $\vec{y}$

C_Z Drag coefficient of the helms along the axis $\vec{z}$

C_{Lup} Damping coefficient of the moment M_X by the product of u.p

C_{Muq} Damping coefficient of the moment M_Y by the product of u.q

C_{Muw} Damping coefficient of the moment M_Y by the product of u.w

C_{Nur} Damping coefficient of the moment M_Z by the product of u.r

C_{Nuv} Damping coefficient of the moment M_Z by the product of u.v

$C_{Xuu} = C_{X0}$ Lateral lift coefficient at zero incidence, proportional to u^2

C_{Yur} Force damping coefficient F_Y by the product of u.r

C_{Yuv} Force damping coefficient F_Y by the product of u.v

C_{Zuq} Force damping coefficient F_Z by the product of u.q

C_{Zuw} Force damping coefficient F_Z by the product of u.w

G Virtual roll helms angle

L_{ref} Vehicle reference length

p, q and r Vehicle angular speeds relative to water along its x, y and z axi

Re The number of reynolds

S_{ref} Vehicle reference surface

u,v and w Vehicle speeds relative to water along its x, y and z axis

W Gravitational force

X_{arv} Position according to X in Rv of the point of expression of the rudder speeds, for the lateral speed

X_{arw} Position according to X in Rv of the point of expression of the rudder speeds, for the vertical speed

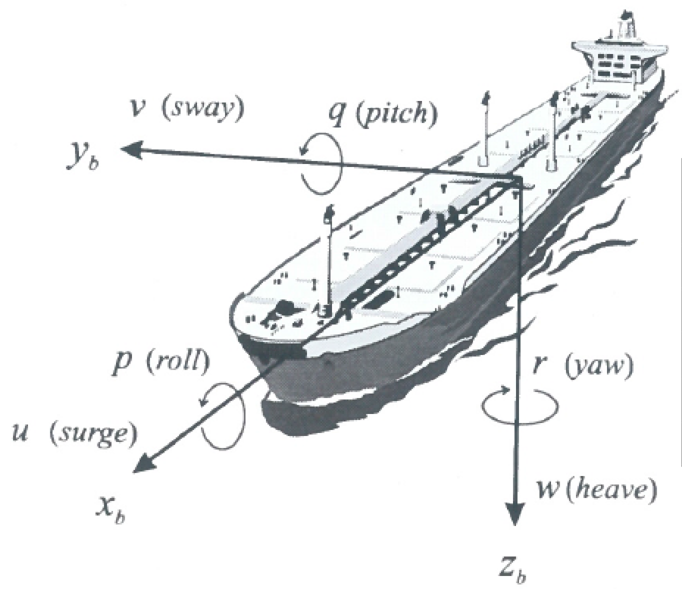
X_{ov} Position along X of the center of the vehicle marker

1 Introduction to non-linear modeling of an AUV

1.1 Introduction

As part of the control of marine systems, a control must be implemented. A modeling of the system must be done and to carry out this modeling, we use the documents cited in the bibliography(chapter). This document begins by taking up all the bases of calculations in a general context before emitting the simplifying hypotheses and writing the model in the context of the subject. This modeling takes up the basics described in the books of Thor I. Fossen [3], [2], [1] and [5]. The complete system can be modeled according to 6 states such as you can see on the figure 1, let us name Degrees of freedom: Degree Of Freedom (DOF). We also define 3 landmarks:

- An absolute landmark (O_T,X_T,Y_T,Z_T), North East Down (NED) landmark.
- A vehicle gravity landmark (O_G,X_G,Y_G,Z_G).
- A vehicle landmark (O_V,X_V,Y_V,Z_V).



DOF	States
u	Speed $\vec{x}$
v	Speed $\vec{y}$
w	Speed $\vec{z}$
p	Angular speed $\vec{x}$ (roll angle)
q	Angular spee $\vec{y}$ (pitch angle)
r	Angular speed $\vec{z}$ (yaw angle)

Figure 1: System variables, source [1]

The complete system results from physical relations with 6 degrees of freedom and 12 states, nonlinear and with coupled states. The complete system is valid at all speeds and contains the coupling between the states. It allows to have all the states of the system from kinematic and dynamic equations. This induces a state vector as defined by SNAME (World Organization: Society of Naval Architects and Marine Engineers).

$$\underline{x} = (x\ y\ z\ \phi\ \theta\ \psi\ \ u\ v\ w\ p\ q\ r)^T = (\eta\ \upsilon)^T \tag{1}$$

DOF	Force or Torque	Speed	Position or Angle
Movement along x	X	u	x
Movement along y	Y	v	y
Movement along z	Z	w	z
Rotation along x (roll angle)	K	p	ϕ
Rotation along y (pitch angle)	M	q	θ
Rotation along z (yaw angle)	N	r	ψ

Of course, a reduction of the complete model into several sub-models is valid but under certain assumptions and conditions (small angles, low speeds, ...). In this chapter, we are interested not in the reduced model but in the complete nonlinear model.

1.2 Kinematic and dynamic equations

The complete system and the reduced models are governed by the same physical equations however it is the simplifying hypotheses and the conditions of use which differ. To be able to reduce the complete system it is necessary first to study the physics which governs the complete system before making the simplifying hypotheses to have the reduced system. The kinematics of the system are governed by Vessel's equations:

System kinematics : $\dot{\eta} = J(\eta)\upsilon$ (2)

$J(\eta)$ is the matrix for passing from the system reference to the terrestrial reference, developed below.

Dynamic equation (2nd Newton law): $M\dot{\upsilon}_r = C(\upsilon_r)\upsilon_r + D_0 + D(\upsilon_r).(\upsilon_r) + g(\eta) + \tau + w$ (3)

With :

$$(x\ y\ z\ \phi\ \theta\ \psi\ )^T = (\eta) \qquad et \qquad (u\ \upsilon\ w\ p\ q\ r)^T = (\upsilon) \tag{4}$$

- M an inertia matrix (including added mass)
- $\upsilon_r = \upsilon - \upsilon_c$. We consider that there is no current : $\upsilon_r = \upsilon$
- C(υ) the Coriolis / Centripetal matrix
- D_0 et D(υ) the damping matrix of the dynamic system
- g(η) the vector of gravitation, forces and moments of floating
- τ the vector of actuators
- w the vector of non-measurable environmental disturbances (waves, wind, current, ...)
- J(η) the landmark change matrix

1.2.1 M(υ): Mass

When an Autonomous Underwater Vehicle (AUV) advances on water, a layer of water moves with it and creates an additional, virtual mass, to be added to the system called M_A.

$$M = M_{RB} + M_A \tag{5}$$

Where M_{RB} is the Rigid Body inertia matrix and M_A is the Added mass inertia matrix. To estimate M we therefore have to calculate M_A and M_{RB}.

1.2.1.1 Representation of the vehicle mass

Fossen book [3], Chapter 2.3 : Rigid-Body Dynamics, page 26, equation (2.91)
The matrix M_{RB} is unique and satisfied: $M_{RB} = M^T_{RB} > 0$ and $\dot{M}_{RB} = O_{6x6}$. Since the calculations are developed in the book (above in italics), we admit that:

$$M_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & mz_g & -my_g \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & my_g & -mx_g & 0 \\ 0 & -mz_g & my_g & I_x & I_{xy} & I_{xz} \\ mz_g & 0 & -mx_g & I_{yx} & I_y & I_{yz} \\ my_g & mx_g & 0 & I_{zx} & I_{zy} & I_z \end{bmatrix} \tag{6}$$

We take, as a convention for writing the inertia matrix, not that of Landau and Lifchitz in [4] but this one:

$$I_k = \begin{bmatrix} \sum m_k(y_k^2+z_k^2) & -\sum m_k.y_k.x_k & -\sum m_k.z_k.x_k \\ -\sum m_k.x_k.y_k & \sum m_k(x_k^2+z_k^2) & -\sum m_k.z_k.y_k \\ -\sum m_k.x_k.y_k & -\sum m_k.y_k.z_k & \sum m_k(x_k^2+y_k^2) \end{bmatrix} = \begin{bmatrix} I_x & I_{xy} & I_{xz} \\ I_{yx} & I_y & I_{yz} \\ I_{zx} & I_{zy} & I_z \end{bmatrix} \tag{7}$$

1.2.1.2 Representation ofthe added mass

Fossen book [3], Chapter 2.4 : Hydrodynamics Forces and Moments, page 33, equation (2.120).
As for M_{RB} the inertia matrix of the added mass is accepted as being the result of the equation (above in italics) provided that $M_A = M_A^T > 0$.

$$M_A = -\begin{bmatrix} X_{\dot u} & X_{\dot v} & X_{\dot w} & X_{\dot p} & X_{\dot q} & X_{\dot r} \\ Y_{\dot u} & Y_{\dot v} & Y_{\dot w} & Y_{\dot p} & Y_{\dot q} & Y_{\dot r} \\ Z_{\dot u} & Z_{\dot v} & Z_{\dot w} & Z_{\dot p} & Z_{\dot q} & Z_{\dot r} \\ K_{\dot u} & K_{\dot v} & K_{\dot w} & K_{\dot p} & K_{\dot q} & K_{\dot r} \\ M_{\dot u} & M_{\dot v} & M_{\dot w} & M_{\dot p} & M_{\dot q} & M_{\dot r} \\ N_{\dot u} & N_{\dot v} & N_{\dot w} & N_{\dot p} & N_{\dot q} & N_{\dot r} \end{bmatrix} \tag{8}$$

With, for example, $Y_{\dot u} := -\frac{\partial Y}{\partial \dot u}$. Y is seen as the hydrodynamic force of the mass added along the y axis due to the acceleration $\dot u$ in the direction x.

1.2.1.3 Conclusion

The added mass can also be understood as induced pressure forces and the moments created by the movement of the vessel body forces are themselves proportional to the acceleration. The acceleration and the added mass are phase shifted by 180°. This allows us to write that:

$$M = \begin{bmatrix} m - X_{\dot u} & -X_{\dot v} & -X_{\dot w} & -X_{\dot p} & mz_g - X_{\dot q} & -my_g - X_{\dot r} \\ -Y_{\dot u} & m - Y_{\dot v} & -Y_{\dot w} & -mz_g - Y_{\dot p} & -Y_{\dot q} & mx_g - Y_{\dot r} \\ -Z_{\dot u} & -Z_{\dot v} & m - Z_{\dot w} & my_g - Z_{\dot p} & -mx_g - Z_{\dot q} & -Z_{\dot r} \\ -K_{\dot u} & -mz_g - K_{\dot v} & my_g - K_{\dot w} & I_x - K_{\dot p} & I_{xy} - K_{\dot q} & I_{xz} - K_{\dot r} \\ mz_g - M_{\dot u} & -M_{\dot v} & -mx_g - M_{\dot w} & I_{yx} - M_{\dot p} & I_y - M_{\dot q} & I_{yz} - M_{\dot r} \\ -my_g - N_{\dot u} & mx_g - N_{\dot v} & -N_{\dot w} & I_{zx} - N_{\dot p} & I_{zy} - N_{\dot q} & I_z - N_{\dot r} \end{bmatrix} \tag{9}$$

The Coriolis/Centripetal matrix C is the Coriolis and Centripetal matrix of the Rigid Body.

1.2.1.4 Representation of the vehicle coriolis

Fossen book [3], 2.3 : Rigid-Body Dynamics, page 27-28, equation (2.93) à (2.102).
The matrix C is unique and satisfied: $C(\upsilon) = -C^T(\upsilon), \forall \upsilon \in \mathbb{R}^6$. Since the calculations are developed in the book (above in italics), we admit that:

$$C(\upsilon) = \begin{bmatrix} 0 & 0 & 0 & -m(y_g q + z_g r) & m(x_g q - w) & m(x_g r + v) \\ 0 & 0 & 0 & m(y_g p + w) & -m(z_g r + x_g p) & m(y_g r - u) \\ 0 & 0 & 0 & m(z_g p - v) & m(z_g q + u) & -m(x_g p + y_g q) \\ m(y_g q + z_g r) & -m(y_g p + w) & -m(z_g p - v) & 0 & I_{yz}q + I_{xz}p - I_z r & -I_{yz}r - I_{xy}p + I_y q \\ -m(x_g q - w) & m(z_g r + x_g p) & -m(z_g q + u) & -I_{yz}q - I_{xz}p + I_{zr} & 0 & I_{xz}r + I_{xy}q - I_x p \\ -m(x_g r + v) & -m(y_g r - u) & m(x_g p + y_g q) & I_{yz}r + I_{xy}p - I_y q & -I_{xz}r - I_{xy}q + I_x p & 0 \end{bmatrix} \tag{10}$$

Consider inertia matrix as diagonal, so all terms are null except I_x, I_y, I_z. Moreover, consider that the gravity and buoyancy centers have only a term in z axis (no x or y axis terms).

$$C(\upsilon) = \begin{bmatrix} 0 & 0 & 0 & -mz_g r & -mw & mv \\ 0 & 0 & 0 & mw & -mz_g r & -mu \\ 0 & 0 & 0 & m(z_g p - v) & m(z_g q + u) & 0 \\ mz_g r & -mw & -m(z_g p - v) & 0 & -I_z r & I_y q \\ mw & mz_g r+ & -m(z_g q + u) & 0 & 0 & -I_x p \\ -mv & m & 0 & -I_y q & I_x p & 0 \end{bmatrix} \tag{11}$$

1.2.2 D(υ): Damping of the dynamic system

The damping matrix of dynamic forces can be expressed in a matrix of resistive forces only multiplied by speed. With reference to the book, the express the amortization matrix, we have to start from Morison's equation:

$$f(U) = -\frac{1}{2}\rho C_D(R_n) S |U| U \tag{12}$$

- f(U), the drag force
- ρ the density of seawater
- U the normalized speed
- R_n the Reynolds number
- $C_D(R_n)$ the coefficient of drag
- S the projected characteristic surface

1.2.2.1 Representation of the damping

Fossen book [3], 2.4 : Hydrodynamic Forces and Moments, page 42-46, equation (2.157) à (2.165).

$$D(\upsilon) = D_{lin}(\upsilon).\upsilon \tag{13}$$

With :

$$D_{lin}(\upsilon) = \frac{1}{2}.\rho.S_{ref}.U.\begin{bmatrix} \frac{C_{X_0}|U|}{U} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{Y_{uv}} & 0 & 0 & 0 & L_{ref}.C_{Y_{ur}} \\ 0 & 0 & C_{Z_{uw}} & 0 & L_{ref}.C_{Z_{uq}} & 0 \\ 0 & 0 & 0 & L_{ref}^2.C_{L_{up}} & 0 & 0 \\ 0 & 0 & L_{ref}.C_{M_{uw}} & 0 & L_{ref}^2.C_{M_{uq}} & 0 \\ 0 & L_{ref}.C_{N_{uv}} & 0 & 0 & 0 & L_{ref}^2.C_{N_{ur}} \end{bmatrix} \tag{14}$$

$$C_{X_0} = K_{sh}.C_{XF} \qquad C_{XF} = \frac{0.075}{(log_{10}(Re) - 2))^2} \; and \; Re = \frac{|U|.Lref}{\upsilon} \tag{15}$$

This hypothesis is available if $U > 0.5 m.s^{-1}$.

1.2.3 g(η): Hydrostatic (restoring) forces and torques

Fossen book [3], Chapter 2.34 : Hydrodynamic Forces and Moments, page 47, equation (2.168).
Consider the weight (W) applied to its center of gravity and the Archimedes thrust (B) applied to the volume center as egal (no buyoancy). Moreover, consider that the gravity and buoyancy centers have only a term in z axis (no x or y axis terms). We take again the equations in the book, quoted above but by changing the norm, that is to say:

$$g(\eta) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -(z_g - z_b).W.cos(\theta).sin(\phi) \\ -(z_g - z_b).W.sin(\theta) \\ 0 \end{bmatrix} \tag{16}$$

1.2.4 J(η): the landmark change matrix

$$J_1(\eta) = \begin{bmatrix} cos(\theta)cos(\psi) & sin(\theta)sin(\phi)cos(\psi) - sin(\psi)cos(\phi) & sin(\theta)cos(\phi)cos(\psi) + sin(\psi)sin(\phi) \\ cos(\phi)sin(\psi) & sin(\psi)sin(\theta)sin(\phi) + cos(\psi)cos(\phi) & sin(\theta)cos(\phi)sin(\psi) - cos(\psi)sin(\phi) \\ -sin(\theta) & cos(\theta)sin(\phi) & cos(\theta)cos(\phi) \end{bmatrix} \tag{17}$$

$$J_2(\eta) = \begin{bmatrix} 1 & sin(\phi)tan(\theta) & cos(\phi)tan(\theta) \\ 0 & cos(\phi) & -sin(\phi) \\ 0 & \frac{sin(\phi)}{cos(\theta)} & \frac{cos(\phi)}{cos(\theta)} \end{bmatrix} \Rightarrow J(\eta) = \begin{bmatrix} J_1(\eta) & 0 \\ 0 & J_2(\eta) \end{bmatrix} \tag{18}$$

1.3 Global nonlinear equation of an AUV

Nonlinear and complete state representation which models the dynamic behavior of a UUV.

$$M\dot{\upsilon} = C(\upsilon)\upsilon + D(\upsilon)\upsilon + g(\eta) + \tau + w \qquad \dot{\eta} = J(\eta)\upsilon$$

$$
\underbrace{\begin{bmatrix}
m-X_{\dot u} & -X_{\dot v} & -X_{\dot w} & -X_{\dot p} & mz_g-X_{\dot q} & -my_g-X_{\dot r}\\
-Y_{\dot u} & m-Y_{\dot v} & -Y_{\dot w} & -mz_g-Y_{\dot p} & -Y_{\dot q} & mx_g-Y_{\dot r}\\
-Z_{\dot u} & -Z_{\dot v} & m-Z_{\dot w} & my_g-Z_{\dot p} & -mx_g-Z_{\dot q} & -Z_{\dot r}\\
-K_{\dot u} & -mz_g-K_{\dot v} & my_g-K_{\dot w} & I_x-K_{\dot p} & I_{xy}-K_{\dot q} & I_{xz}-K_{\dot r}\\
mz_g-M_{\dot u} & -M_{\dot v} & -mx_g-M_{\dot w} & I_{yx}-M_{\dot p} & I_y-M_{\dot q} & I_{yz}-M_{\dot r}\\
-my_g-N_{\dot u} & mx_g-N_{\dot v} & -N_{\dot w} & I_{zx}-N_{\dot p} & I_{zy}-N_{\dot q} & I_z-N_{\dot r}
\end{bmatrix}}_{Mass\ matrix}
\begin{pmatrix}\dot u\\ \dot v\\ \dot w\\ \dot p\\ \dot q\\ \dot r\end{pmatrix}
$$

$$
=\underbrace{\begin{bmatrix}
0 & 0 & 0 & -mz_gr & -mw & mv\\
0 & 0 & 0 & mw & -mz_gr & -mu\\
0 & 0 & 0 & m(z_gp-v) & m(z_gq+u) & 0\\
mz_gr & -mw & -m(z_gp-v) & 0 & -I_zr & I_yq\\
mw & mz_gr & -m(z_gq+u) & 0 & 0 & -I_xp\\
-mv & mu & 0 & -I_yq & I_xp & 0
\end{bmatrix}}_{Coriolis/Centripetal\ vehicle\ and\ added\ water\ mass}
\begin{pmatrix}u\\ v\\ w\\ p\\ q\\ r\end{pmatrix}
\tag{19}
$$

$$
+\underbrace{\frac{1}{2}\rho S_{ref}\begin{bmatrix}
C_{X_0}|u| & 0 & 0 & 0 & 0 & 0\\
0 & C_{Yuv}u & 0 & 0 & 0 & C_{Yur}L_{ref}u\\
0 & 0 & C_{Zuw}u & 0 & C_{Zuq}L_{ref}u & 0\\
0 & 0 & 0 & L^2_{ref}C_{Lup}u & 0 & 0\\
0 & 0 & C_{Muw}L_{ref}u & 0 & L^2_{ref}C_{Muq}u & 0\\
0 & C_{Nuv}L_{ref}u & 0 & 0 & 0 & L^2_{ref}C_{Nur}u
\end{bmatrix}
\begin{pmatrix}u\\ v\\ w\\ p\\ q\\ r\end{pmatrix}}_{Hydrodynamic\ damping}
+\underbrace{\begin{bmatrix}
0\\ 0\\ 0\\ -(z_g-z_b).W.cos(\theta).sin(\phi)\\ -(z_g-z_b).W.sin(\theta)\\ 0
\end{bmatrix}}_{Hydrostatic\ restoring\ forces}
+\tau
$$

2 AUV actuator model

2.1 Purpose

The actuators are the helms and the motor. The angular position (in degrees or radians) of the four fins is noted B1, B2, B3 and B4. This part is used to justify the writing of the vector of the actuators $\begin{pmatrix} X \\ Y \\ Z \\ L \\ M \\ N \end{pmatrix}$

2.2 Modelisation of the helms

2.2.1 Introduction

The vehicle is equipped with four rear fins, in the "Saint-André" configuration (X-shaped, see image 2).

- The position B1 is the helms on sur starboard, up (called TR = Top, Right).
- The position B2 is the helms on sur starboard, down (called BR = Bottom, Right).
- The position B3 is the helms on sur port side, down (called BR = Bottom, Right).
- The position B4 is the helms on sur port side, up (called TL = Top, Left).

Figure 2: Rear view of an AUV

The "physical" positions of the four helms are used to define three "virtual" positions, called BAR (vertical plane), A (horizontal plane) and G (roll plane), of three "virtual" ailerons.

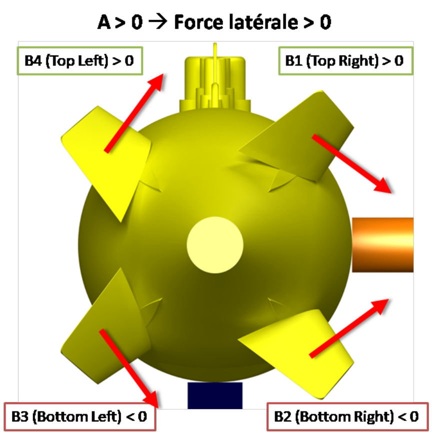


Figure 3: Modeling of the command in virtual bar (horizontal): A

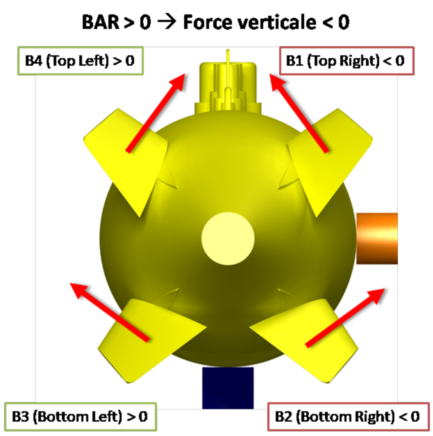


Figure 4: Modeling of the command in virtual bar (vertical): BAR

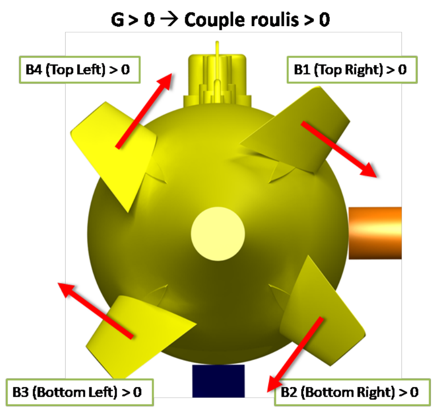


Figure 5: Modeling of the command in virtual bar (roulis): G

Which give :

- $B_1 = \frac{G-BAR+A-D}{4}$
- $B_2 = \frac{G-BAR-A+D}{4}$
- $B_3 = \frac{G+BAR-A-D}{4}$
- $B_4 = \frac{G+BAR+A+D}{4}$
- $G = B_1 + B_2 + B_3 + B_4$
- $BAR = -B_1 - B_2 + B_3 + B_4$
- $A = B_1 - B_2 - B_3 + B_4$
- $D = -B_1 + B_2 - B_3 + B_4$

2.2.2 Equations of forces and moments of the helms

We modelised the forces created by the actuators in 2 possible forms, we will only retain the form using the angles of virtual bars.

$$\begin{cases} F_{xGouv} = \frac{1}{2}\rho S_{ref}\left(C_{X1}B_1^2 + C_{X2}B_2^2 + C_{X3}B_3^2 + C_{X4}B_4^2\right) u\,|u| \\ F_{yGouv} = \frac{1}{2}\rho S_{ref}\left(C_{Y1}B_1 + C_{Y2}B_2 + C_{Y3}B_3 + C_{Y4}B_4\right) u\,|u| \\ F_{zGouv} = \frac{1}{2}\rho S_{ref}\left(C_{Z1}B_1 + C_{Z2}B_2 + C_{Z3}B_3 + C_{Z4}B_4\right) u\,|u| \\ M_{xGouv} = \frac{1}{2}\rho S_{ref}L_{ref}\left(C_{M1}B_1 + C_{M2}B_2 + C_{M3}B_3 + C_{M4}B_4\right) u\,|u| \\ M_{yGouv} = \frac{1}{2}\rho S_{ref}L_{ref}\left(C_{M1}B_1 + C_{M2}B_2 + C_{M3}B_3 + C_{M4}B_4\right) u\,|u| \\ M_{zGouv} = \frac{1}{2}\rho S_{ref}L_{ref}\left(C_{N1}B_1 + C_{N2}B_2 + C_{N3}B_3 + C_{N4}B_4\right) u\,|u| \end{cases} \tag{20}$$

$$\begin{cases} F_{xGouv} = \frac{1}{8}\rho S_{ref}C_X\left(B_{ar}^2 + A^2 + G^2 + D^2\right) u\,|u| \\ F_{yGouv} = \frac{1}{2}\rho S_{ref}C_Y A\,u\,|u| \\ F_{zGouv} = \frac{1}{2}\rho S_{ref}C_Z BAR\,u\,|u| \\ M_{xGouv} = \frac{1}{2}\rho S_{ref}L_{ref}C_L G\,u\,|u| \\ M_{yGouv} = \frac{1}{2}\rho S_{ref}L_{ref}C_M BAR\,u\,|u| \\ M_{zGouv} = \frac{1}{2}\rho S_{ref}L_{ref}C_N A\,u\,|u| \end{cases} \tag{21}$$

2.3 Modelisation of the thruster

Fossen book [3], Chapter 4.1.1 : Thruster Model, page 94, equation (4.4 and 4.5).
The thruster create a force T and a Torque Q, which are respectively F_x and M_x.

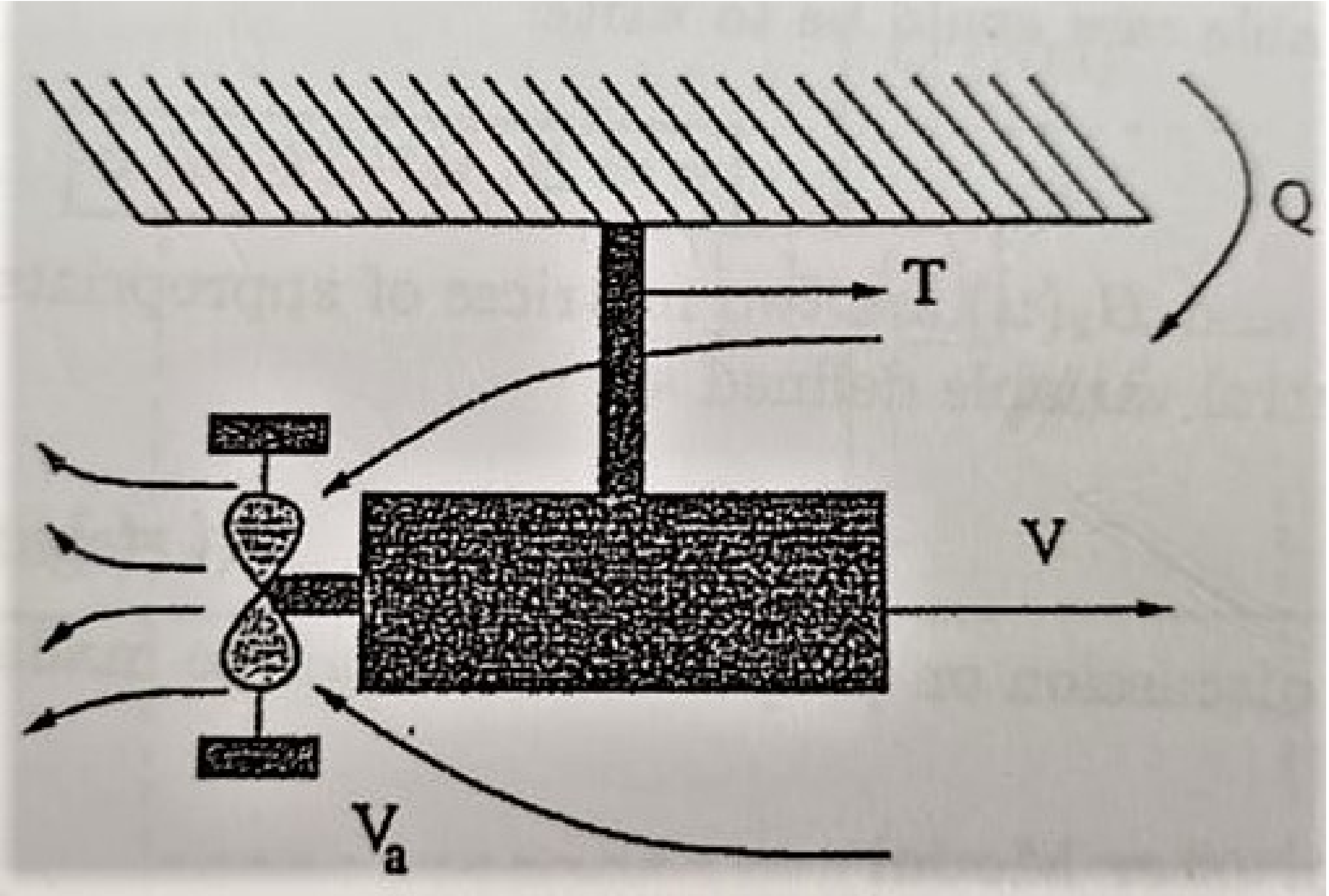


Figure 6: Schematic drawing of propeller, source [3]

$$\begin{cases} F_x = T_{nn}\,|n|\,n + T_{nu}\,|n|\,u \\ F_y = 0 \\ F_z = 0 \\ M_x = Q_{nn}\,|n|\,n + Q_{nu}\,|n|\,u \\ M_y = 0 \\ M_z = 0 \end{cases} \tag{22}$$

with u the Speed Throw Water (STW) longitudinal speed in $m.s^{-1}$ and n the number of revolutions per minute (rpm). STW and Speed Over Ground (SOG) should not be confused but in the context of this simplified practical work there is no current so STW = SOG.

3 Non-linear model in the horizontal plane of an AUV

3.1 Purpose

This chapter is used to explain the development of the horizontal model of an AUV. To start the full model will be repeated, then the simplifying hypothesis of the horizontal plane will be listed to present the nonlinear model from the horizontal plane.

3.2 Simplification of the dynamic equation

Here we model the dynamics in the horizontal plane. Let us therefore detail the simplifying hypotheses, those specific to the lateral plane, as well as that which make it possible to reduce the state vector:

- Center of gravity and center of volume are aligned: $x_g = x_b$ et $y_g = y_b$

- The system is considered to be balanced : $W = B$

- The distribution is considered to be homogeneous and (xz) is a plane of symmetry $\rightarrow I_{xy} = I_{yz} = 0, I_{xz} \neq 0,$ *and* $Y_{\dot{u}} = X_{\dot{v}} \approx 0, X_{\dot{r}} = N_{\dot{u}} \approx 0, Y_{\dot{r}} = N_{\dot{v}}$

- There are 2 types of masses: the mass of the vehicle weighed in the air and the mass calculated using ρV here we take $m = \rho V$ therefore the mass of the vehicle + the mass of water on board of the vehicle. The matrix C_A is therefore already included in C_{RB} and is therefore not to be taken into account.

- The model is restricted to the horizontal plane, the roll and pitch angles are considered small: θ and ϕ small but not ψ.

- Since the movements along the roll axis are considered to be negligible in the horizontal plane: : p and $\dot{p}$ are insignifiant.

- Since the movements along the pitch axis are considered to be negligible in the horizontal plane : q and $\dot{q}$ are insignifiant.

- We study the system in the horizontal plane in order to linearize it, so we consider that there is no disturbance : $w = 0$.

With those hypothesis, the système $\underline{x} = (x\ y\ z\ \phi\ \theta\ \psi\ u\ v\ p\ q\ q\ r)^T$ becomes $\underline{x_{horizontal}} = (x\ y\ \psi\ v\ r)^T$.

Now let's go back to the equation 1.3 and apply the assumptions in this section, the equation becomes:
(Remember: $\tau_1 = X, \tau_2 = Y$ et $\tau_6 = N$))

$$\begin{bmatrix} m-X_{\dot u} & -X_{\dot v} & -my_g - X_{\dot r}\\ -Y_{\dot u} & m-Y_{\dot v} & mx_g - Y_{\dot r}\\ -my_g - N_{\dot u} & mx_g - N_{\dot v} & I_z - N_{\dot r}\end{bmatrix}\begin{pmatrix}\dot u\\ \dot v\\ \dot r\end{pmatrix} = \begin{bmatrix}0 & 0 & m\left(x_g r+v\right)\\ 0&0&m\left(y_g r - u\right)\\ -m\left(x_g r+v\right) & -m\left(y_g r-u\right)u & 0\end{bmatrix}\begin{bmatrix}u\\v\\r\end{bmatrix}$$
$$+\frac{1}{2}\rho S_{ref}\begin{bmatrix} C_{X_{uu}}.|u| & 0 & 0\\ 0 & C_{Y_{uv}}.u & C_{Y_{ur}}.L_{ref}.u \\ 0 & C_{N_{uv}}.L_{ref}.u & L_{ref}^2 C_{N_{ur}}.u\end{bmatrix}\begin{pmatrix}u\\v\\r\end{pmatrix} + \begin{bmatrix}0\\0\\0\end{bmatrix}\begin{pmatrix}u\\v\\r\end{pmatrix} + \begin{pmatrix}\tau_1\\ \tau_2\\ \tau_6\end{pmatrix} \tag{23}$$

In order to obtain the reduced dynamic equation, let us take the simplified model written in 23 and apply the following hypotheses to it :

- The model is restricted to the horizontal plane, the roll and pitch angles are considered small: θ and ϕ small but not ψ .
- Since the movements along the roll axis are considered to be negligible in the horizontal plane: : p and $\dot p$ are insignifiant.
- Since the movements along the pitch axis are considered to be negligible in the horizontal plane : q and $\dot q$ are insignifiant.
- In addition we consider to be at constant speed : $u = u_0$.
- These assumptions also imply that : $Y_{\dot u} = X_{\dot v} \approx 0, X_{\dot r} = N_{\dot u} \approx 0, Y_{\dot r} = N_{\dot v}$
- the term on x axis and y axis are still null for the gravity and buyoancy centers. ($x_g = 0, y_g = 0, x_b = 0, y_b = 0$).
- τ_1, τ_2, τ_6 are the actuators forces F_x, F_y and M_z so forces and torques of helms and the thruster.

The equation 23 becomes :

$$\underbrace{\begin{bmatrix} m-X_{\dot u_0} & -X_{\dot v} & -X_{\dot r}\\ -Y_{\dot u} & m-Y_{\dot v} & -Y_{\dot r}\\ -N_{\dot u} & -N_{\dot v} & I_z - N_{\dot r}\end{bmatrix}}_{\text{Mass Matrice}}\begin{pmatrix}\dot u_0\\ \dot v\\ \dot r\end{pmatrix} = \underbrace{\begin{bmatrix}0 & 0 & m.v\\ 0 & 0 & -m.u\\ -m.v & m.u & 0\end{bmatrix}}_{\text{Coriolis/Centripete vehicle and added water mass}}\begin{bmatrix}u_0\\ v\\ r\end{bmatrix}$$
$$\underbrace{+\frac{1}{2}\rho S_{ref}\begin{bmatrix} C_{X_0}.|u_0| & 0 & 0\\ 0| & C_{Y_{uv}}.u_0 & C_{Y_{ur}}.L_{ref}.u_0 \\ 0 & C_{N_{uv}}.L_{ref}.u_0 & L_{ref}^2 C_{N_{ur}}.u_0\end{bmatrix}\begin{pmatrix}u\\v\\r\end{pmatrix}}_{\text{Hydrodynamic damping}} + \underbrace{\begin{pmatrix} T_{nn}\,|n|\,n + T_{nu}\,|n|\,u_0 + \frac{1}{8}\rho S_{ref} C_X (A^2 + BAR^2 + G^2 + D^2) u_0\,|u_0| \\ \frac{1}{2}\rho S_{ref} C_Y A\, u_0\,|u_0| \\ \frac{1}{2}\rho S_{ref} L_{ref} C_N A\, u_0\,|u_0| \end{pmatrix}}_{\text{Actuators}} \tag{24}$$

3.3 Kinematic equation

The kinematic equation is :

$$\begin{pmatrix}\dot x\\ \dot y\\ \dot\psi\end{pmatrix} = \begin{bmatrix}\cos(\theta)\cos(\psi) & \sin(\theta)\sin(\phi)\cos(\psi) - \sin(\psi)\cos(\phi) & 0\\ \cos(\phi)\sin(\psi) & \sin(\psi)\sin(\theta)\sin(\phi) + \cos(\psi)\cos(\phi) & 0\\ 0 & 0 & \frac{\cos(\phi)}{\cos(\theta)}\end{bmatrix}\begin{pmatrix}u\\v\\r\end{pmatrix} \tag{25}$$

By taking up the preceding simplifying hypotheses in "Final equation", the equation below becomes:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{pmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} u \\ v \\ r \end{pmatrix} \tag{26}$$

3.4 Final Equations

Using the equation 24 we can write the equations below and we recall that the hypothesis $u = u_0$ therefore we can extend the system $\begin{pmatrix} u \\ v \end{pmatrix} \rightarrow \begin{pmatrix} u_0 \\ v \\ r \end{pmatrix}$. It will therefore be necessary to model the system for the speed of advance u. We consider that we are always at $u > 0.5ms^{-1}$, $u = u_0$. One models an AUV compared to water therefore the terms of currents are not to be taken into account.
Moreover, we want to create control only in the horizontal plane, so we do not consider the force F_x of the thruster in this plan.

$$\begin{pmatrix} \dot{u_0} \\ \dot{v} \\ \dot{r} \end{pmatrix} = \underbrace{M^{-1}}_{\text{inverse of the mass matrix}} \Bigg[\underbrace{\frac{1}{2}\rho S_{ref} \begin{pmatrix} \frac{1}{4}C_X(A^2 + BAR^2 + G^2 + D^2)u_0 \left|u_0\right| \\ C_Y A u_0 \left|u_0\right| \\ L_{ref} C_N A u_0 \left|u_0\right| \end{pmatrix}}_{\text{Actuators}} + \underbrace{\begin{bmatrix} 0 & 0 & m.v \\ 0 & 0 & -m.u_0 \\ -m.v & m.u_0 & 0 \end{bmatrix}}_{\text{Coriolis/Centripete vehicle and added water mass}} \begin{pmatrix} u_0 \\ v \\ r \end{pmatrix}$$
$$+ \underbrace{\frac{1}{2}\rho S_{ref} \begin{bmatrix} C_{X0} & 0 & 0 \\ 0 & C_{Y_{uv}}.u_0 & C_{Y_{ur}}.L_{ref}.u_0 \\ 0 & C_{Nuv}.L_{ref}.u_0 & L_{ref}^2 C_{N_{ur}}.u_0 \end{bmatrix}}_{\text{Hydrodynamic damping}} \begin{pmatrix} u_0 \\ v \\ r \end{pmatrix} \Bigg] \tag{27}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{bmatrix} cos(\psi) & -sin(\psi) & 0 \\ sin(\psi) & cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} u_0 \\ v \\ r \end{pmatrix}$$

4 Non-linear model in the vertical plane of an AUV

4.1 Purpose

This chapter is used to explain the development of the vertical model of an AUV. To start the full model will be repeated, then the simplifying hypothesis of the vertical plane will be listed to present the nonlinear model from the vertical plane.

4.2 Simplification of the dynamic equation

Here we model the dynamics in the vertical plane. Let us therefore detail the simplifying hypotheses, those specific to the vertical plane, as well as that which make it possible to reduce the state vector:

- Center of gravity and center of volume are aligned: $x_g = x_b$ et $y_g = y_b$

- The system is considered to be balanced : $W = B$

- The distribution is considered to be homogeneous and (xz) is a plane of symmetry $\rightarrow I_{xy} = I_{yz} = 0, I_{xz} \neq 0,$ *and* $Z_{\dot{u}} = X_{\dot{w}} \approx 0, X_{\dot{q}} = M_{\dot{u}} \approx 0, Z_{\dot{q}} = M_{\dot{w}}$

- There are 2 types of masses: the mass of the vehicle weighed in the air and the mass calculated using ρV here we take $m = \rho V$ therefore the mass of the vehicle + the mass of water on board of the vehicle. The matrix C_A is therefore already included in C_{RB} and is therefore not to be taken into account.

- The model is restricted to the vertical plane, the roll and yaw angles are considered small: ψ and ϕ small but not θ.

- Since the movements along the roll axis are considered to be negligible in the vertical plane: : p and $\dot{p}$ are insignifiant.

- Since the movements along the yaw axis are considered to be negligible in the vertical plane : r and $\dot{r}$ are insignifiant.

- We study the system in the vertical plane in order to linearize it, so we consider that there is no disturbance : $w = 0$.

- In addition we consider to be at constant speed : $u = u_0$.

- These assumptions also imply that : $Z_{\dot{u}} = X_{\dot{w}} \approx 0, X_{\dot{q}} = M_{\dot{u}} \approx 0, Z_{\dot{q}} = M_{\dot{q}}$

- the term on x axis and y axis are still null for the gravity and buyoancy centers. ($x_g = 0, y_g = 0, x_b = 0, y_b = 0$).

- τ_1, τ_2, τ_6 are the actuators forces F_x, F_z and M_y so forces and torques of helms and the thruster.

With those hypothesis, the système $\underline{x} = (x\ y\ z\ \phi\ \theta\ \psi\ u\ v\ p\ q\ q\ r)^T$ becomes $\underline{x_{vertical}} = (x\ y\ \theta\ w\ q)^T$.

Now let's go back to the equation 1.3 and apply the assumptions in this section, the equation becomes:
(Remember: $\tau_1 = X, \tau_3 = Z$ et $\tau_5 = M$))

$$\underbrace{\begin{bmatrix} m - X_{\dot{u}} & -X_{\dot{w}} & mz_g - X_{\dot{q}} \\ -Z_{\dot{u}} & m - Z_{\dot{w}} & -mx_g - Z_{\dot{q}} \\ mz_g - M_{\dot{u}} & -M_{\dot{w}} & I_{zy} - N_{\dot{q}} \end{bmatrix}}_{\text{Mass Matrice}} \begin{pmatrix} \dot{u}_0 \\ \dot{w} \\ \dot{q} \end{pmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & -m.w \\ 0 & 0 & m.\left(z_g q + u\right) \\ m.w & -m.\left(z_g q + u\right) & 0 \end{bmatrix}}_{\text{Coriolis/Centripete vehicle and added water mass}} \begin{bmatrix} u_0 \\ w \\ q \end{bmatrix}$$

$$+ \underbrace{\frac{1}{2}\rho S_{ref} \begin{bmatrix} C_{X_{uu}}.|u| & 0 & 0 \\ 0 & C_{Zuw}.u & C_{Z_{uq}}.L_{ref}.u \\ 0 & C_{Muw}.L_{ref}.u & L_{ref}^2 C_{M_{uq}}.u \end{bmatrix}}_{\text{Hydrodynamic damping}} \begin{pmatrix} u \\ w \\ q \end{pmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ -(z_g - z_b).w.sin(\theta) \end{bmatrix}}_{\text{Hydrostatic}} \begin{pmatrix} u \\ w \\ q \end{pmatrix}$$

$$+ \underbrace{\begin{pmatrix} T_{nn}\,|n|\,n + T_{nu}\,|n|\,u_0 + \frac{1}{8}\rho S_{ref} C_X (A^2 + BAR^2 + G^2 + D^2) u_0\,|u_0| \\ \frac{1}{2}\rho S_{ref} C_Z BAR\,u_0\,|u_0| \\ \frac{1}{2}\rho S_{ref} L_{ref} C_M BAR\,u_0\,|u_0| \end{pmatrix}}_{\text{Actuators}} \tag{28}$$

4.3 Kinematic equation

The kinematic equation is :

$$\begin{pmatrix} \dot{x} \\ \dot{w} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} \cos(\theta)\cos(\psi) & \sin(\theta)\sin(\phi)\cos(\psi) - \sin(\psi)\cos(\phi) & 0 \\ -sin(\theta) & cos(\theta)cos(\phi) & 0 \\ 0 & 0 & cos(\phi) \end{bmatrix} \begin{pmatrix} u \\ w \\ q \end{pmatrix} \tag{29}$$

By taking up the preceding simplifying hypotheses in "Final equation", the equation below becomes:

$$\begin{pmatrix} \dot{x} \\ \dot{w} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} \cos(\theta) & 0 & 0 \\ -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} u \\ w \\ q \end{pmatrix} \tag{30}$$

4.4 Final Equations

Using the equation 28 we can write the equations below and we recall that the hypothesis $u = u_0$ therefore we can extend the system $\begin{pmatrix} u \\ w \end{pmatrix} \rightarrow \begin{pmatrix} u_0 \\ w \\ q \end{pmatrix}$. It will therefore be necessary to model the system for the speed of advance u. We consider that we are always at $u > 0.5ms^{-1}$, $u = u_0$. One models an AUV compared to water therefore the terms of currents are not to be taken into account. Moreover, we want to create control only in the vertical plane, so we do not consider the force F_x of the thruster in this plan

$$\begin{pmatrix} \dot{u_0} \\ \dot{w} \\ \dot{q} \end{pmatrix} =$$

$$\underbrace{M^{-1}}_{\text{inverse of the mass matrix}} \Bigg[\underbrace{\frac{1}{2}\rho S_{ref} \begin{pmatrix} \frac{1}{4} C_X (A^2 + BAR^2 + G^2 + D^2) u_0 \left| u_0 \right| \\ C_Z BAR u_0 \left| u_0 \right| \\ L_{ref} C_M BAR u_0 \left| u_0 \right| \end{pmatrix}}_{\text{Forces/Moments}} + \underbrace{\begin{bmatrix} 0 & 0 & -m.w \\ 0 & 0 & m.\left(z_g q + u_0\right) \\ m.w & -m.\left(z_g q + u_0\right) & 0 \end{bmatrix}}_{\text{Coriolis/Centripete vehicle and added water mass}} \begin{pmatrix} u_0 \\ w \\ q \end{pmatrix}$$

$$+ \underbrace{\frac{1}{2}\rho S_{ref} \begin{bmatrix} C_{X0} & 0 & 0 \\ 0 & C_{Z_{uw}}.u_0 & C_{Z_{uq}}.L_{ref}.u_0 \\ 0 & C_{Muw}.L_{ref}.u_0 & L_{ref}^2 C_{M_{uq}}.u_0 \end{bmatrix}}_{\text{Hydrodynamic damping}} \begin{pmatrix} u_0 \\ w \\ q \end{pmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ -(z_g - z_b).W.sin(\theta) \end{bmatrix}}_{\text{Hydrostatic}} \begin{pmatrix} u_0 \\ w \\ q \end{pmatrix} \Bigg]$$

$$\begin{pmatrix} \dot{x} \\ \dot{w} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} \cos(\theta) & 0 & 0 \\ -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} u_0 \\ w \\ q \end{pmatrix} \tag{31}$$

5 Model in the forward plane of an AUV

5.1 Purpose

This chapter is used to write the formula of an AUV in the plan in advance. This is a simple formula that will help you identify the parameter K_{sh}.

5.2 Dynamic equation in forward plan

From the equation 1.3, we can easily find that :

$$F_X = \frac{1}{2}.\rho.S_{ref}.C_{X_0}.u.\left|u\right| \tag{32}$$

However, 3 others equations will help you identify K_{sh}, with R_e the Reynolds number and ν the cinematic viscosity of sea water at 15°(1.04e-6 Pa.s)

$$\begin{aligned} C_{X_0} &= K_{sh}.C_{XF} \\ C_{XF} &= \frac{0.075}{(log_{10}(R_e) - 2)^2} \\ R_e &= \frac{u.L_{ref}}{\nu} \end{aligned} \tag{33}$$