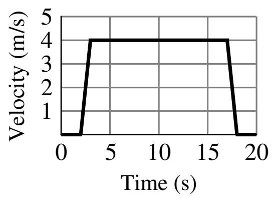
The Derivative

Exercise Set 2.1

1. (a) $m_{\tan} = (50 - 10)/(15 - 5) = 40/10 = 4$ m/s.

(b)



2. At $t = 4$ s, $m_{\tan} \approx (90 - 0)/(10 - 2) = 90/8 = 11.25$ m/s. At $t = 8$ s, $m_{\tan} \approx (140 - 0)/(10 - 4) = 140/6 \approx 23.33$ m/s.

3. (a) $(10 - 10)/(3 - 0) = 0$ cm/s.

(b) $t = 0$, $t = 2$, $t = 4.2$, and $t = 8$ (horizontal tangent line).

(c) maximum: $t = 1$ (slope > 0), minimum: $t = 3$ (slope < 0).

(d) $(3 - 18)/(4 - 2) = -7.5$ cm/s (slope of estimated tangent line to curve at $t = 3$).

4. (a) decreasing (slope of tangent line decreases with increasing time)

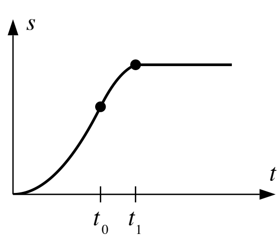
(b) increasing (slope of tangent line increases with increasing time)

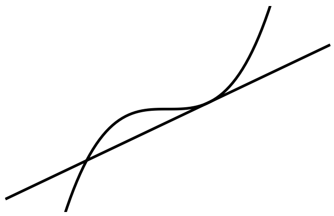
(c) increasing (slope of tangent line increases with increasing time)

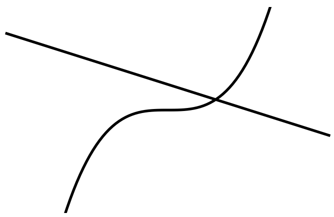
(d) decreasing (slope of tangent line decreases with increasing time)

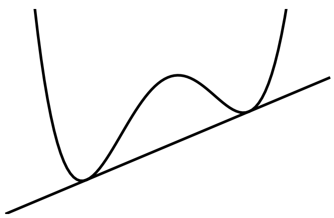
5. It is a straight line with slope equal to the velocity.

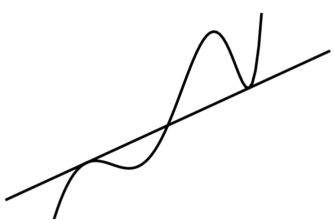
6. The velocity increases from time 0 to time t_0, so the slope of the curve increases during that time. From time t_0 to time t_1, the velocity, and the slope, decrease. At time t_1, the velocity, and hence the slope, instantaneously drop to zero, so there is a sharp bend in the curve at that point.



7. 

8. 

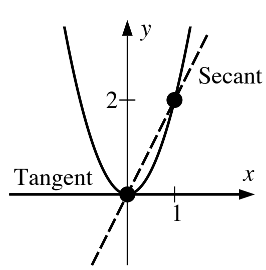
9. 

10. 

11. **(a)** $m_{\text{sec}} = \dfrac{f(1) - f(0)}{1 - 0} = \dfrac{2}{1} = 2$

(b) $m_{\tan} = \lim\limits_{x_1 \to 0} \dfrac{f(x_1) - f(0)}{x_1 - 0} = \lim\limits_{x_1 \to 0} \dfrac{2x_1^2 - 0}{x_1 - 0} = \lim\limits_{x_1 \to 0} 2x_1 = 0$

(c) $m_{\tan} = \lim\limits_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{2x_1^2 - 2x_0^2}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} (2x_1 + 2x_0) = 4x_0$



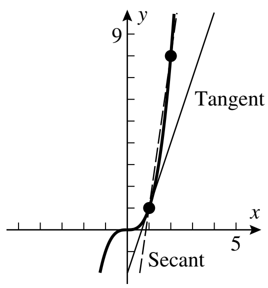
(d) The tangent line is the x-axis.

12. **(a)** $m_{\text{sec}} = \dfrac{f(2) - f(1)}{2 - 1} = \dfrac{2^3 - 1^3}{1} = 7$

(b) $m_{\tan} = \lim\limits_{x_1 \to 1} \dfrac{f(x_1) - f(1)}{x_1 - 1} = \lim\limits_{x_1 \to 1} \dfrac{x_1^3 - 1}{x_1 - 1} = \lim\limits_{x_1 \to 1} \dfrac{(x_1 - 1)(x_1^2 + x_1 + 1)}{x_1 - 1} = \lim\limits_{x_1 \to 1} (x_1^2 + x_1 + 1) = 3$

(c) $m_{\tan} = \lim_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{x_1^3 - x_0^3}{x_1 - x_0} = \lim_{x_1 \to x_0} (x_1^2 + x_1 x_0 + x_0^2) = 3x_0^2$

(d)

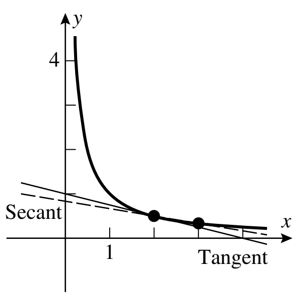


13. (a) $m_{\sec} = \dfrac{f(3) - f(2)}{3 - 2} = \dfrac{1/3 - 1/2}{1} = -\dfrac{1}{6}$

(b) $m_{\tan} = \lim_{x_1 \to 2} \dfrac{f(x_1) - f(2)}{x_1 - 2} = \lim_{x_1 \to 2} \dfrac{1/x_1 - 1/2}{x_1 - 2} = \lim_{x_1 \to 2} \dfrac{2 - x_1}{2x_1(x_1 - 2)} = \lim_{x_1 \to 2} \dfrac{-1}{2x_1} = -\dfrac{1}{4}$

(c) $m_{\tan} = \lim_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{1/x_1 - 1/x_0}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{x_0 - x_1}{x_0 x_1 (x_1 - x_0)} = \lim_{x_1 \to x_0} \dfrac{-1}{x_0 x_1} = -\dfrac{1}{x_0^2}$

(d)

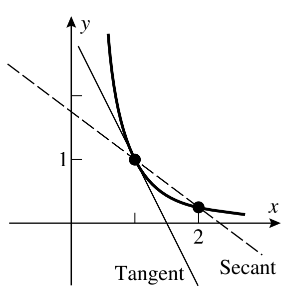


14. (a) $m_{\sec} = \dfrac{f(2) - f(1)}{2 - 1} = \dfrac{1/4 - 1}{1} = -\dfrac{3}{4}$

(b) $m_{\tan} = \lim_{x_1 \to 1} \dfrac{f(x_1) - f(1)}{x_1 - 1} = \lim_{x_1 \to 1} \dfrac{1/x_1^2 - 1}{x_1 - 1} = \lim_{x_1 \to 1} \dfrac{1 - x_1^2}{x_1^2(x_1 - 1)} = \lim_{x_1 \to 1} \dfrac{-(x_1 + 1)}{x_1^2} = -2$

(c) $m_{\tan} = \lim_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{1/x_1^2 - 1/x_0^2}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{x_0^2 - x_1^2}{x_0^2 x_1^2 (x_1 - x_0)} = \lim_{x_1 \to x_0} \dfrac{-(x_1 + x_0)}{x_0^2 x_1^2} = -\dfrac{2}{x_0^3}$

(d)



15. (a) $m_{\tan} = \lim_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{(x_1^2 - 1) - (x_0^2 - 1)}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{(x_1^2 - x_0^2)}{x_1 - x_0} = \lim_{x_1 \to x_0} (x_1 + x_0) = 2x_0$

(b) $m_{\tan} = 2(-1) = -2$

16. **(a)** $m_{\tan} = \lim_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{(x_1^2 + 3x_1 + 2) - (x_0^2 + 3x_0 + 2)}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{(x_1^2 - x_0^2) + 3(x_1 - x_0)}{x_1 - x_0} =$
$= \lim_{x_1 \to x_0} (x_1 + x_0 + 3) = 2x_0 + 3$

(b) $m_{\tan} = 2(2) + 3 = 7$

17. **(a)** $m_{\tan} = \lim_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{(x_1 + \sqrt{x_1}\,) - (x_0 + \sqrt{x_0}\,)}{x_1 - x_0} = \lim_{x_1 \to x_0} \left(1 + \dfrac{1}{\sqrt{x_1} + \sqrt{x_0}}\right) = 1 + \dfrac{1}{2\sqrt{x_0}}$

(b) $m_{\tan} = 1 + \dfrac{1}{2\sqrt{1}} = \dfrac{3}{2}$

18. **(a)** $m_{\tan} = \lim_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{1/\sqrt{x_1} - 1/\sqrt{x_0}}{x_1 - x_0} = \lim_{x_1 \to x_0} \dfrac{\sqrt{x_0} - \sqrt{x_1}}{\sqrt{x_0}\,\sqrt{x_1}\,(x_1 - x_0)} =$
$= \lim_{x_1 \to x_0} \dfrac{-1}{\sqrt{x_0}\,\sqrt{x_1}\,(\sqrt{x_1} + \sqrt{x_0}\,)} = -\dfrac{1}{2x_0^{3/2}}$

(b) $m_{\tan} = -\dfrac{1}{2(4)^{3/2}} = -\dfrac{1}{16}$

19. True. Let $x = 1 + h$.

20. False. A secant line meets the curve in at least two places, but a tangent line might meet it only once.

21. False. Velocity represents the rate at which position changes.

22. True. The units of the rate of change are obtained by dividing the units of $f(x)$ (inches) by the units of x (tons).

23. **(a)** 72°F at about 4:30 P.M. **(b)** About $(67 - 43)/6 = 4$°F/h.

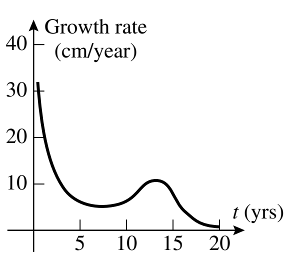
(c) Decreasing most rapidly at about 9 P.M.; rate of change of temperature is about -7°F/h (slope of estimated tangent line to curve at 9 P.M.).

24. For $V = 10$ the slope of the tangent line is about $(0 - 5)/(20 - 0) = -0.25$ atm/L, for $V = 25$ the slope is about $(1 - 2)/(25 - 0) = -0.04$ atm/L.

25. **(a)** During the first year after birth.

(b) About 6 cm/year (slope of estimated tangent line at age 5).

(c) The growth rate is greatest at about age 14; about 10 cm/year.

(d)


26. **(a)** The object falls until $s = 0$. This happens when $1250 - 16t^2 = 0$, so $t = \sqrt{1250/16} = \sqrt{78.125} > \sqrt{25} = 5$; hence the object is still falling at $t = 5$ sec.

(b) $\dfrac{f(6) - f(5)}{6 - 5} = \dfrac{674 - 850}{1} = -176$. The average velocity is -176 ft/s.

(c) $v_{\text{inst}} = \lim_{h\to 0} \dfrac{f(5+h)-f(5)}{h} = \lim_{h\to 0} \dfrac{[1250-16(5+h)^2]-850}{h} = \lim_{h\to 0} \dfrac{-160h-16h^2}{h} = \lim_{h\to 0}(-160-16h) = -160$ ft/s.

27. (a) $0.3 \cdot 40^3 = 19{,}200$ ft **(b)** $v_{\text{ave}} = 19{,}200/40 = 480$ ft/s

(c) Solve $s = 0.3t^3 = 1000; t \approx 14.938$ so $v_{\text{ave}} \approx 1000/14.938 \approx 66.943$ ft/s.

(d) $v_{\text{inst}} = \lim_{h\to 0} \dfrac{0.3(40+h)^3 - 0.3\cdot 40^3}{h} = \lim_{h\to 0} \dfrac{0.3(4800h+120h^2+h^3)}{h} = \lim_{h\to 0} 0.3(4800+120h+h^2) = 1440$ ft/s

28. (a) $v_{\text{ave}} = \dfrac{4.5(12)^2 - 4.5(0)^2}{12-0} = 54$ ft/s

(b) $v_{\text{inst}} = \lim_{t_1\to 6} \dfrac{4.5t_1^2 - 4.5(6)^2}{t_1-6} = \lim_{t_1\to 6} \dfrac{4.5(t_1^2-36)}{t_1-6} = \lim_{t_1\to 6} \dfrac{4.5(t_1+6)(t_1-6)}{t_1-6} = \lim_{t_1\to 6} 4.5(t_1+6) = 54$ ft/s

29. (a) $v_{\text{ave}} = \dfrac{6(4)^4 - 6(2)^4}{4-2} = 720$ ft/min

(b) $v_{\text{inst}} = \lim_{t_1\to 2} \dfrac{6t_1^4 - 6(2)^4}{t_1-2} = \lim_{t_1\to 2} \dfrac{6(t_1^4-16)}{t_1-2} = \lim_{t_1\to 2} \dfrac{6(t_1^2+4)(t_1^2-4)}{t_1-2} = \lim_{t_1\to 2} 6(t_1^2+4)(t_1+2) = 192$ ft/min

30. See the discussion before Definition 2.1.1.

31. The instantaneous velocity at $t=1$ equals the limit as $h\to 0$ of the average velocity during the interval between $t=1$ and $t=1+h$.

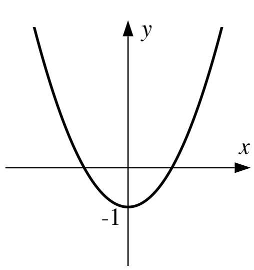
Exercise Set 2.2

1. $f'(1) = 2.5$, $f'(3) = 0$, $f'(5) = -2.5$, $f'(6) = -1$.

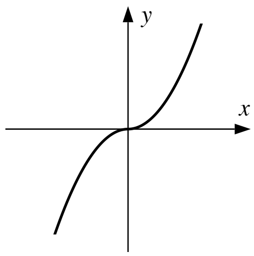
2. $f'(4) < f'(0) < f'(2) < 0 < f'(-3)$.

3. (a) $f'(a)$ is the slope of the tangent line. **(b)** $f'(2) = m = 3$ **(c)** The same, $f'(2) = 3$.

4. $f'(1) = \dfrac{2-(-1)}{1-(-1)} = \dfrac{3}{2}$



5.



6.

7. $y-(-1)=5(x-3),\ y=5x-16$

8. $y-3=-4(x+2),\ y=-4x-5$

9. $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\dfrac{2(x+h)^2-2x^2}{h}=\lim_{h\to 0}\dfrac{4xh+2h^2}{h}=4x$; $f'(1)=4$ so the tangent line is given by $y-2=4(x-1)$, $y=4x-2$.

10. $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\dfrac{1/(x+h)^2-1/x^2}{h}=\lim_{h\to 0}\dfrac{x^2-(x+h)^2}{hx^2(x+h)^2}=\lim_{h\to 0}\dfrac{-2xh-h^2}{hx^2(x+h)^2}=\lim_{h\to 0}\dfrac{-2x-h}{x^2(x+h)^2}=-\dfrac{2}{x^3}$; $f'(-1)=2$ so the tangent line is given by $y-1=2(x+1)$, $y=2x+3$.

11. $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\dfrac{(x+h)^3-x^3}{h}=\lim_{h\to 0}(3x^2+3xh+h^2)=3x^2$; $f'(0)=0$ so the tangent line is given by $y-0=0(x-0)$, $y=0$.

12. $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\dfrac{[2(x+h)^3+1]-[2x^3+1]}{h}=\lim_{h\to 0}(6x^2+6xh+2h^2)=6x^2$; $f(-1)=2(-1)^3+1=-1$ and $f'(-1)=6$ so the tangent line is given by $y+1=6(x+1)$, $y=6x+5$.

13. $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\dfrac{\sqrt{x+1+h}-\sqrt{x+1}}{h}=\lim_{h\to 0}\dfrac{\sqrt{x+1+h}-\sqrt{x+1}}{h}\dfrac{\sqrt{x+1+h}+\sqrt{x+1}}{\sqrt{x+1+h}+\sqrt{x+1}}=\lim_{h\to 0}\dfrac{h}{h(\sqrt{x+1+h}+\sqrt{x+1})}=\dfrac{1}{2\sqrt{x+1}}$; $f(8)=\sqrt{8+1}=3$ and $f'(8)=\dfrac{1}{6}$ so the tangent line is given by $y-3=\dfrac{1}{6}(x-8)$, $y=\dfrac{1}{6}x+\dfrac{5}{3}$.

14. $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\dfrac{\sqrt{2x+2h+1}-\sqrt{2x+1}}{h}\dfrac{\sqrt{2x+2h+1}+\sqrt{2x+1}}{\sqrt{2x+2h+1}+\sqrt{2x+1}}=$
$=\lim_{h\to 0}\dfrac{2h}{h(\sqrt{2x+2h+1}+\sqrt{2x+1})}=\lim_{h\to 0}\dfrac{2}{\sqrt{2x+2h+1}+\sqrt{2x+1}}=\dfrac{1}{\sqrt{2x+1}}$; $f(4)=\sqrt{2\cdot 4+1}=\sqrt{9}=3$ and $f'(4)=1/3$ so the tangent line is given by $y-3=\dfrac{1}{3}(x-4), y=\dfrac{1}{3}x+\dfrac{5}{3}$.

15. $f'(x)=\lim_{\Delta x\to 0}\dfrac{\dfrac{1}{x+\Delta x}-\dfrac{1}{x}}{\Delta x}=\lim_{\Delta x\to 0}\dfrac{\dfrac{x-(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}=\lim_{\Delta x\to 0}\dfrac{-\Delta x}{x\Delta x(x+\Delta x)}=\lim_{\Delta x\to 0}-\dfrac{1}{x(x+\Delta x)}=-\dfrac{1}{x^2}$.

16. $f'(x)=\lim_{\Delta x\to 0}\dfrac{\dfrac{1}{(x+\Delta x)+1}-\dfrac{1}{x+1}}{\Delta x}=\lim_{\Delta x\to 0}\dfrac{\dfrac{(x+1)-(x+\Delta x+1)}{(x+1)(x+\Delta x+1)}}{\Delta x}=\lim_{\Delta x\to 0}\dfrac{x+1-x-\Delta x-1}{\Delta x(x+1)(x+\Delta x+1)}=$
$=\lim_{\Delta x\to 0}\frac{-\Delta x}{\Delta x(x+1)(x+\Delta x+1)}=\lim_{\Delta x\to 0}\dfrac{-1}{(x+1)(x+\Delta x+1)}=-\dfrac{1}{(x+1)^2}$.

17. $f'(x)=\lim_{\Delta x\to 0}\dfrac{(x+\Delta x)^2-(x+\Delta x)-(x^2-x)}{\Delta x}=\lim_{\Delta x\to 0}\dfrac{2x\Delta x+(\Delta x)^2-\Delta x}{\Delta x}=\lim_{\Delta x\to 0}(2x-1+\Delta x)=2x-1$.

18. $f'(x) = \lim_{\Delta x \to 0} \dfrac{(x+\Delta x)^4 - x^4}{\Delta x} = \lim_{\Delta x \to 0} \dfrac{4x^3\Delta x + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4}{\Delta x} =$
$= \lim_{\Delta x \to 0} (4x^3 + 6x^2\Delta x + 4x(\Delta x)^2 + (\Delta x)^3) = 4x^3.$

19. $f'(x) = \lim_{\Delta x \to 0} \dfrac{\dfrac{1}{\sqrt{x+\Delta x}} - \dfrac{1}{\sqrt{x}}}{\Delta x} = \lim_{\Delta x \to 0} \dfrac{\sqrt{x} - \sqrt{x+\Delta x}}{\Delta x\sqrt{x}\sqrt{x+\Delta x}} = \lim_{\Delta x \to 0} \dfrac{x - (x+\Delta x)}{\Delta x\sqrt{x}\sqrt{x+\Delta x}(\sqrt{x} + \sqrt{x+\Delta x})} =$
$= \lim_{\Delta x \to 0} \dfrac{-1}{\sqrt{x}\sqrt{x+\Delta x}(\sqrt{x} + \sqrt{x+\Delta x})} = -\dfrac{1}{2x^{3/2}}.$

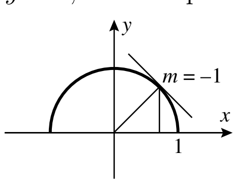
20. $f'(x) = \lim_{\Delta x \to 0} \dfrac{\dfrac{1}{\sqrt{x+\Delta x-1}} - \dfrac{1}{\sqrt{x-1}}}{\Delta x} = \lim_{\Delta x \to 0} \dfrac{\sqrt{x-1} - \sqrt{x+\Delta x-1}}{\Delta x\sqrt{x-1}\sqrt{x+\Delta x-1}} \dfrac{\sqrt{x-1} + \sqrt{x+\Delta x-1}}{\sqrt{x-1} + \sqrt{x+\Delta x-1}} =$
$= \lim_{\Delta x \to 0} \dfrac{-\Delta x}{\Delta x\sqrt{x-1}\sqrt{x+\Delta x-1}(\sqrt{x-1} + \sqrt{x+\Delta x-1})} = \lim_{\Delta x \to 0} \dfrac{-1}{\sqrt{x-1}\sqrt{x+\Delta x-1}(\sqrt{x-1} + \sqrt{x+\Delta x-1})} =$
$-\dfrac{1}{2(x-1)^{3/2}}.$

21. $f'(t) = \lim_{h \to 0} \dfrac{f(t+h) - f(t)}{h} = \lim_{h \to 0} \dfrac{[4(t+h)^2 + (t+h)] - [4t^2 + t]}{h} = \lim_{h \to 0} \dfrac{4t^2 + 8th + 4h^2 + t + h - 4t^2 - t}{h} =$
$\lim_{h \to 0} \dfrac{8th + 4h^2 + h}{h} = \lim_{h \to 0} (8t + 4h + 1) = 8t + 1.$

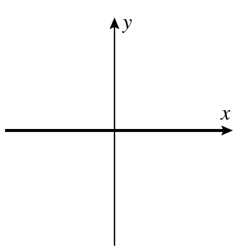
22. $\dfrac{dV}{dr} = \lim_{h \to 0} \dfrac{\frac{4}{3}\pi(r+h)^3 - \frac{4}{3}\pi r^3}{h} = \lim_{h \to 0} \dfrac{\frac{4}{3}\pi(r^3 + 3r^2h + 3rh^2 + h^3 - r^3)}{h} = \lim_{h \to 0} \dfrac{4}{3}\pi(3r^2 + 3rh + h^2) = 4\pi r^2.$

23. **(a)** D **(b)** F **(c)** B **(d)** C **(e)** A **(f)** E

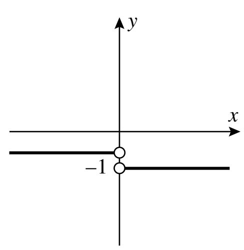
24. $f'(\sqrt{2}/2)$ is the slope of the tangent line to the unit circle at $(\sqrt{2}/2, \sqrt{2}/2)$. This line is perpendicular to the line $y = x$, so its slope is -1.



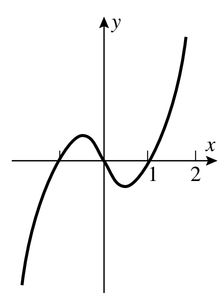
25. **(a)**



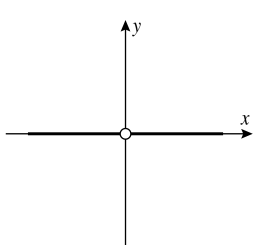
(b)



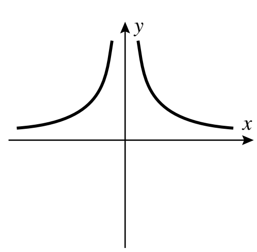
(c)



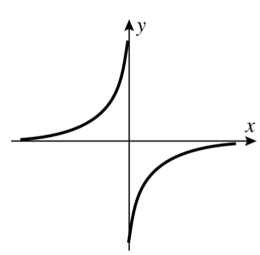
26. **(a)**



(b)



(c)



27. False. If the tangent line is horizontal then $f'(a) = 0$.

28. True. $f'(-2)$ equals the slope of the tangent line.

29. False. E.g. $|x|$ is continuous but not differentiable at $x = 0$.

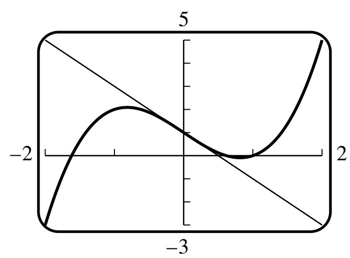
30. True. See Theorem 2.2.3.

31. **(a)** $f(x) = \sqrt{x}$ and $a = 1$ **(b)** $f(x) = x^2$ and $a = 3$

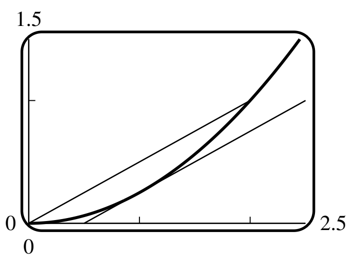
32. **(a)** $f(x) = \cos x$ and $a = \pi$ **(b)** $f(x) = x^7$ and $a = 1$

33. $\dfrac{dy}{dx} = \lim\limits_{h\to 0} \dfrac{(1-(x+h)^2)-(1-x^2)}{h} = \lim\limits_{h\to 0} \dfrac{-2xh-h^2}{h} = \lim\limits_{h\to 0}(-2x-h) = -2x$, and $\left.\dfrac{dy}{dx}\right|_{x=1} = -2$.

34. $\dfrac{dy}{dx} = \lim\limits_{h\to 0} \dfrac{\dfrac{x+2+h}{x+h} - \dfrac{x+2}{x}}{h} = \lim\limits_{h\to 0} \dfrac{x(x+2+h)-(x+2)(x+h)}{hx(x+h)} = \lim\limits_{h\to 0} \dfrac{-2}{x(x+h)} = \dfrac{-2}{x^2}$, and $\left.\dfrac{dy}{dx}\right|_{x=-2} = -\dfrac{1}{2}$.



35. $y = -2x + 1$



36.

37. **(b)**

w	1.5	1.1	1.01	1.001	1.0001	1.00001
$\dfrac{f(w)-f(1)}{w-1}$	1.6569	1.4355	1.3911	1.3868	1.3863	1.3863

w	0.5	0.9	0.99	0.999	0.9999	0.99999
$\dfrac{f(w)-f(1)}{w-1}$	1.1716	1.3393	1.3815	1.3858	1.3863	1.3863

38. **(b)**

w	$\frac{\pi}{4}+0.5$	$\frac{\pi}{4}+0.1$	$\frac{\pi}{4}+0.01$	$\frac{\pi}{4}+0.001$	$\frac{\pi}{4}+0.0001$	$\frac{\pi}{4}+0.00001$
$\dfrac{f(w)-f(\pi/4)}{w-\pi/4}$	0.50489	0.67060	0.70356	0.70675	0.70707	0.70710

w	$\frac{\pi}{4}-0.5$	$\frac{\pi}{4}-0.1$	$\frac{\pi}{4}-0.01$	$\frac{\pi}{4}-0.001$	$\frac{\pi}{4}-0.0001$	$\frac{\pi}{4}-0.00001$
$\dfrac{f(w)-f(\pi/4)}{w-\pi/4}$	0.85114	0.74126	0.71063	0.70746	0.70714	0.70711

39. **(a)** $\dfrac{f(3)-f(1)}{3-1}=\dfrac{2.2-2.12}{2}=0.04$; $\dfrac{f(2)-f(1)}{2-1}=\dfrac{2.34-2.12}{1}=0.22$; $\dfrac{f(2)-f(0)}{2-0}=\dfrac{2.34-0.58}{2}=0.88$.

(b) The tangent line at $x=1$ appears to have slope about 0.8, so $\dfrac{f(2)-f(0)}{2-0}$ gives the best approximation and $\dfrac{f(3)-f(1)}{3-1}$ gives the worst.

40. **(a)** $f'(0.5)\approx\dfrac{f(1)-f(0)}{1-0}=\dfrac{2.12-0.58}{1}=1.54$.

(b) $f'(2.5)\approx\dfrac{f(3)-f(2)}{3-2}=\dfrac{2.2-2.34}{1}=-0.14$.

41. **(a)** dollars/ft

(b) $f'(x)$ is roughly the price per additional foot.

(c) If each additional foot costs extra money (this is to be expected) then $f'(x)$ remains positive.

(d) From the approximation $1000=f'(300)\approx\dfrac{f(301)-f(300)}{301-300}$ we see that $f(301)\approx f(300)+1000$, so the extra foot will cost around \$1000.

42. **(a)** $\dfrac{\text{gallons}}{\text{dollars/gallon}}=\text{gallons}^2/\text{dollar}$

(b) The increase in the amount of paint that would be sold for one extra dollar per gallon.

(c) It should be negative since an increase in the price of paint would decrease the amount of paint sold.

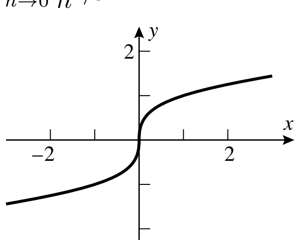
(d) From $-100=f'(10)\approx\dfrac{f(11)-f(10)}{11-10}$ we see that $f(11)\approx f(10)-100$, so an increase of one dollar per gallon would decrease the amount of paint sold by around 100 gallons.

43. **(a)** $F\approx 200$ lb, $dF/d\theta\approx 50$ **(b)** $\mu=(dF/d\theta)/F\approx 50/200=0.25$

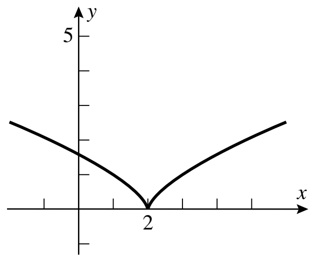
44. The derivative at time $t=100$ of the velocity with respect to time is equal to the slope of the tangent line, which is approximately $m\approx\dfrac{12500-0}{140-40}=125\text{ ft/s}^2$. Thus the mass is approximately $M(100)\approx\dfrac{T}{dv/dt}=\dfrac{7680982\text{ lb}}{125\text{ ft/s}^2}\approx$ 61000 slugs.

45. **(a)** $T\approx 115°\text{F}$, $dT/dt\approx -3.35°\text{F/min}$ **(b)** $k=(dT/dt)/(T-T_0)\approx(-3.35)/(115-75)=-0.084$

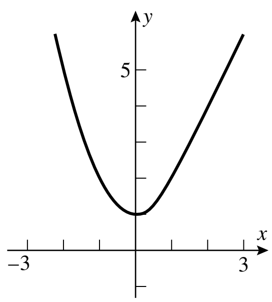
46. **(a)** $\lim_{x\to 0}f(x)=\lim_{x\to 0}\sqrt[3]{x}=0=f(0)$, so f is continuous at $x=0$. $\lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\dfrac{\sqrt[3]{h}-0}{h}=\lim_{h\to 0}\dfrac{1}{h^{2/3}}=+\infty$, so $f'(0)$ does not exist.



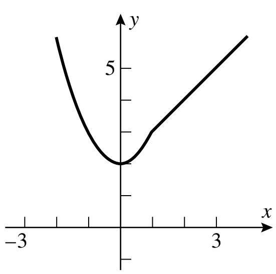
(b) $\lim_{x\to 2} f(x) = \lim_{x\to 2}(x-2)^{2/3} = 0 = f(2)$ so f is continuous at $x = 2$. $\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\dfrac{h^{2/3}-0}{h} = \lim_{h\to 0}\dfrac{1}{h^{1/3}}$ which does not exist so $f'(2)$ does not exist.



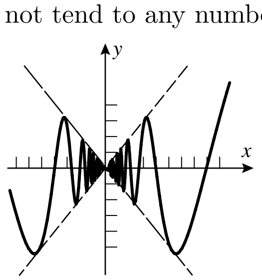
47. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$, so f is continuous at $x = 1$. $\lim_{h\to 0^-}\dfrac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-}\dfrac{[(1+h)^2+1]-2}{h} = \lim_{h\to 0^-}(2+h) = 2$; $\lim_{h\to 0^+}\dfrac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+}\dfrac{2(1+h)-2}{h} = \lim_{h\to 0^+} 2 = 2$, so $f'(1) = 2$.



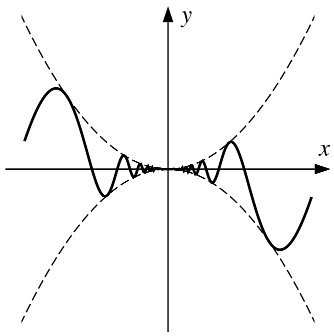
48. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$ so f is continuous at $x = 1$. $\lim_{h\to 0^-}\dfrac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-}\dfrac{[(1+h)^2+2]-3}{h} = \lim_{h\to 0^-}(2+h) = 2$; $\lim_{h\to 0^+}\dfrac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+}\dfrac{[(1+h)+2]-3}{h} = \lim_{h\to 0^+} 1 = 1$, so $f'(1)$ does not exist.



49. Since $-|x| \le x\sin(1/x) \le |x|$ it follows by the Squeezing Theorem (Theorem 1.6.4) that $\lim_{x\to 0} x\sin(1/x) = 0$. The derivative cannot exist: consider $\dfrac{f(x)-f(0)}{x} = \sin(1/x)$. This function oscillates between -1 and $+1$ and does not tend to any number as x tends to zero.



50. For continuity, compare with $\pm x^2$ to establish that the limit is zero. The difference quotient is $x\sin(1/x)$ and (see Exercise 49) this has a limit of zero at the origin.



51. Let $\epsilon = |f'(x_0)/2|$. Then there exists $\delta > 0$ such that if $0 < |x - x_0| < \delta$, then $\left|\dfrac{f(x) - f(x_0)}{x - x_0} - f'(x_0)\right| < \epsilon$. Since $f'(x_0) > 0$ and $\epsilon = f'(x_0)/2$ it follows that $\dfrac{f(x) - f(x_0)}{x - x_0} > \epsilon > 0$. If $x = x_1 < x_0$ then $f(x_1) < f(x_0)$ and if $x = x_2 > x_0$ then $f(x_2) > f(x_0)$.

52. $g'(x_1) = \lim\limits_{h \to 0} \dfrac{g(x_1 + h) - g(x_1)}{h} = \lim\limits_{h \to 0} \dfrac{f(m(x_1 + h) + b) - f(mx_1 + b)}{h} = m \lim\limits_{h \to 0} \dfrac{f(x_0 + mh) - f(x_0)}{mh} = mf'(x_0)$.

53. **(a)** Let $\epsilon = |m|/2$. Since $m \neq 0$, $\epsilon > 0$. Since $f(0) = f'(0) = 0$ we know there exists $\delta > 0$ such that $\left|\dfrac{f(0 + h) - f(0)}{h}\right| < \epsilon$ whenever $0 < |h| < \delta$. It follows that $|f(h)| < \frac{1}{2}|hm|$ for $0 < |h| < \delta$. Replace h with x to get the result.

(b) For $0 < |x| < \delta, |f(x)| < \frac{1}{2}|mx|$. Moreover $|mx| = |mx - f(x) + f(x)| \le |f(x) - mx| + |f(x)|$, which yields $|f(x) - mx| \ge |mx| - |f(x)| > \frac{1}{2}|mx| > |f(x)|$, i.e. $|f(x) - mx| > |f(x)|$.

(c) If any straight line $y = mx + b$ is to approximate the curve $y = f(x)$ for small values of x, then $b = 0$ since $f(0) = 0$. The inequality $|f(x) - mx| > |f(x)|$ can also be interpreted as $|f(x) - mx| > |f(x) - 0|$, i.e. the line $y = 0$ is a better approximation than is $y = mx$.

54. Let $g(x) = f(x) - [f(x_0) + f'(x_0)(x - x_0)]$ and $h(x) = f(x) - [f(x_0) + m(x - x_0)]$; note that $h(x) - g(x) = (f'(x_0) - m)(x - x_0)$. If $m \neq f'(x_0)$ then there exists $\delta > 0$ such that if $0 < |x - x_0| < \delta$ then $\left|\dfrac{f(x) - f(x_0)}{x - x_0} - f'(x_0)\right| < \dfrac{1}{2}|f'(x_0) - m|$. Multiplying by $|x - x_0|$ gives $|g(x)| < \dfrac{1}{2}|h(x) - g(x)|$. Hence $2|g(x)| < |h(x) + (-g(x))| \le |h(x)| + |g(x)|$, so $|g(x)| < |h(x)|$. In words, $f(x)$ is closer to $f(x_0) + f'(x_0)(x - x_0)$ than it is to $f(x_0) + m(x - x_0)$. So the tangent line gives a better approximation to $f(x)$ than any other line through $(x_0, f(x_0))$. Clearly any line not passing through that point gives an even worse approximation for x near x_0, so the tangent line gives the best linear approximation.

55. See discussion around Definition 2.2.2.

56. See Theorem 2.2.3.

Exercise Set 2.3

1. $28x^6$, by Theorems 2.3.2 and 2.3.4.

2. $-36x^{11}$, by Theorems 2.3.2 and 2.3.4.

3. $24x^7 + 2$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

4. $2x^3$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

5. 0, by Theorem 2.3.1.

6. $\sqrt{2}$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

7. $-\frac{1}{3}(7x^6+2)$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

8. $\frac{2}{5}x$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

9. $-3x^{-4}-7x^{-8}$, by Theorems 2.3.3 and 2.3.5.

10. $\dfrac{1}{2\sqrt{x}}-\dfrac{1}{x^2}$, by Theorems 2.3.3 and 2.3.5.

11. $24x^{-9}+1/\sqrt{x}$, by Theorems 2.3.3, 2.3.4, and 2.3.5.

12. $-42x^{-7}-\dfrac{5}{2\sqrt{x}}$, by Theorems 2.3.3, 2.3.4, and 2.3.5.

13. $f'(x)=ex^{e-1}-\sqrt{10}\,x^{-1-\sqrt{10}}$, by Theorems 2.3.3 and 2.3.5.

14. $f'(x)=-\frac{2}{3}x^{-4/3}$, by Theorems 2.3.3 and 2.3.4.

15. $(3x^2+1)^2=9x^4+6x^2+1$, so $f'(x)=36x^3+12x$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

16. $3ax^2+2bx+c$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

17. $y'=10x-3$, $y'(1)=7$.

18. $y'=\dfrac{1}{2\sqrt{x}}-\dfrac{2}{x^2}$, $y'(1)=-3/2$.

19. $2t-1$, by Theorems 2.3.2 and 2.3.5.

20. $\dfrac{1}{3}-\dfrac{1}{3t^2}$, by Theorems 2.3.3, 2.3.4, and 2.3.5.

21. $dy/dx=1+2x+3x^2+4x^3+5x^4$, $dy/dx|_{x=1}=15$.

22. $\dfrac{dy}{dx}=\dfrac{-3}{x^4}-\dfrac{2}{x^3}-\dfrac{1}{x^2}+1+2x+3x^2$, $\left.\dfrac{dy}{dx}\right|_{x=1}=0$.

23. $y=(1-x^2)(1+x^2)(1+x^4)=(1-x^4)(1+x^4)=1-x^8$, $\dfrac{dy}{dx}=-8x^7$, $dy/dx|_{x=1}=-8$.

24. $dy/dx=24x^{23}+24x^{11}+24x^7+24x^5$, $dy/dx|_{x=1}=96$.

25. $f'(1)\approx\dfrac{f(1.01)-f(1)}{0.01}=\dfrac{-0.999699-(-1)}{0.01}=0.0301$, and by differentiation, $f'(1)=3(1)^2-3=0$.

26. $f'(1)\approx\dfrac{f(1.01)-f(1)}{0.01}\approx\dfrac{0.980296-1}{0.01}\approx-1.9704$, and by differentiation, $f'(1)=-2/1^3=-2$.

27. The estimate will depend on your graphing utility and on how far you zoom in. Since $f'(x)=1-\dfrac{1}{x^2}$, the exact value is $f'(1)=0$.

28. The estimate will depend on your graphing utility and on how far you zoom in. Since $f'(x) = \dfrac{1}{2\sqrt{x}} + 2$, the exact value is $f'(1) = 5/2$.

29. $32t$, by Theorems 2.3.2 and 2.3.4.

30. 2π, by Theorems 2.3.2 and 2.3.4.

31. $3\pi r^2$, by Theorems 2.3.2 and 2.3.4.

32. $-2\alpha^{-2} + 1$, by Theorems 2.3.2, 2.3.4, and 2.3.5.

33. True. By Theorems 2.3.4 and 2.3.5, $\dfrac{d}{dx}[f(x) - 8g(x)] = f'(x) - 8g'(x)$; substitute $x = 2$ to get the result.

34. True. $\dfrac{d}{dx}[ax^3 + bx^2 + cx + d] = 3ax^2 + 2bx + c$.

35. False. $\dfrac{d}{dx}[4f(x) + x^3]\Big|_{x=2} = (4f'(x) + 3x^2)\big|_{x=2} = 4f'(2) + 3 \cdot 2^2 = 32$

36. False. $f(x) = x^6 - x^3$ so $f'(x) = 6x^5 - 3x^2$ and $f''(x) = 30x^4 - 6x$, which is not equal to $2x(4x^3 - 1) = 8x^4 - 2x$.

37. **(a)** $\dfrac{dV}{dr} = 4\pi r^2$ **(b)** $\dfrac{dV}{dr}\Big|_{r=5} = 4\pi(5)^2 = 100\pi$

38. $\dfrac{d}{d\lambda}\left[\dfrac{\lambda\lambda_0 + \lambda^6}{2 - \lambda_0}\right] = \dfrac{1}{2 - \lambda_0}\dfrac{d}{d\lambda}(\lambda\lambda_0 + \lambda^6) = \dfrac{1}{2 - \lambda_0}(\lambda_0 + 6\lambda^5) = \dfrac{\lambda_0 + 6\lambda^5}{2 - \lambda_0}$.

39. $y - 2 = 5(x + 3)$, $y = 5x + 17$.

40. $y + 2 = -(x - 2)$, $y = -x$.

41. **(a)** $dy/dx = 21x^2 - 10x + 1$, $d^2y/dx^2 = 42x - 10$ **(b)** $dy/dx = 24x - 2$, $d^2y/dx^2 = 24$

(c) $dy/dx = -1/x^2$, $d^2y/dx^2 = 2/x^3$ **(d)** $dy/dx = 175x^4 - 48x^2 - 3$, $d^2y/dx^2 = 700x^3 - 96x$

42. **(a)** $y' = 28x^6 - 15x^2 + 2$, $y'' = 168x^5 - 30x$ **(b)** $y' = 3$, $y'' = 0$

(c) $y' = \dfrac{2}{5x^2}$, $y'' = -\dfrac{4}{5x^3}$ **(d)** $y' = 8x^3 + 9x^2 - 10$, $y'' = 24x^2 + 18x$

43. **(a)** $y' = -5x^{-6} + 5x^4$, $y'' = 30x^{-7} + 20x^3$, $y''' = -210x^{-8} + 60x^2$

(b) $y = x^{-1}$, $y' = -x^{-2}$, $y'' = 2x^{-3}$, $y''' = -6x^{-4}$

(c) $y' = 3ax^2 + b$, $y'' = 6ax$, $y''' = 6a$

44. **(a)** $dy/dx = 10x - 4$, $d^2y/dx^2 = 10$, $d^3y/dx^3 = 0$

(b) $dy/dx = -6x^{-3} - 4x^{-2} + 1$, $d^2y/dx^2 = 18x^{-4} + 8x^{-3}$, $d^3y/dx^3 = -72x^{-5} - 24x^{-4}$

(c) $dy/dx = 4ax^3 + 2bx$, $d^2y/dx^2 = 12ax^2 + 2b$, $d^3y/dx^3 = 24ax$

45. **(a)** $f'(x) = 6x$, $f''(x) = 6$, $f'''(x) = 0$, $f'''(2) = 0$

(b) $\dfrac{dy}{dx} = 30x^4 - 8x,\ \dfrac{d^2y}{dx^2} = 120x^3 - 8,\ \left.\dfrac{d^2y}{dx^2}\right|_{x=1} = 112$

(c) $\dfrac{d}{dx}\left[x^{-3}\right] = -3x^{-4},\ \dfrac{d^2}{dx^2}\left[x^{-3}\right] = 12x^{-5},\ \dfrac{d^3}{dx^3}\left[x^{-3}\right] = -60x^{-6},\ \dfrac{d^4}{dx^4}\left[x^{-3}\right] = 360x^{-7},\ \left.\dfrac{d^4}{dx^4}\left[x^{-3}\right]\right|_{x=1} = 360$

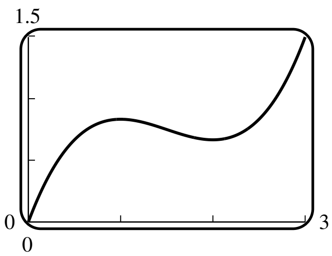
46. **(a)** $y' = 16x^3 + 6x^2$, $y'' = 48x^2 + 12x$, $y''' = 96x + 12$, $y'''(0) = 12$

(b) $y = 6x^{-4},\ \dfrac{dy}{dx} = -24x^{-5},\ \dfrac{d^2y}{dx^2} = 120x^{-6},\ \dfrac{d^3y}{dx^3} = -720x^{-7},\ \dfrac{d^4y}{dx^4} = 5040x^{-8},\ \left.\dfrac{d^4y}{dx^4}\right|_{x=1} = 5040$

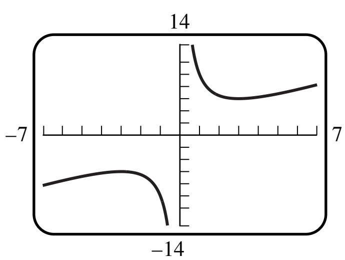
47. $y' = 3x^2 + 3$, $y'' = 6x$, and $y''' = 6$ so $y''' + xy'' - 2y' = 6 + x(6x) - 2(3x^2 + 3) = 6 + 6x^2 - 6x^2 - 6 = 0$.

48. $y = x^{-1}$, $y' = -x^{-2}$, $y'' = 2x^{-3}$ so $x^3y'' + x^2y' - xy = x^3(2x^{-3}) + x^2(-x^{-2}) - x(x^{-1}) = 2 - 1 - 1 = 0$.

49. The graph has a horizontal tangent at points where $\dfrac{dy}{dx} = 0$, but $\dfrac{dy}{dx} = x^2 - 3x + 2 = (x-1)(x-2) = 0$ if $x = 1, 2$. The corresponding values of y are $5/6$ and $2/3$ so the tangent line is horizontal at $(1, 5/6)$ and $(2, 2/3)$.



50. Find where $f'(x) = 0:\ f'(x) = 1 - 9/x^2 = 0$, $x^2 = 9$, $x = \pm 3$. The tangent line is horizontal at $(3, 6)$ and $(-3, -6)$.



51. The y-intercept is -2 so the point $(0, -2)$ is on the graph; $-2 = a(0)^2 + b(0) + c$, $c = -2$. The x-intercept is 1 so the point (1,0) is on the graph; $0 = a + b - 2$. The slope is $dy/dx = 2ax + b$; at $x = 0$ the slope is b so $b = -1$, thus $a = 3$. The function is $y = 3x^2 - x - 2$.

52. Let $P(x_0, y_0)$ be the point where $y = x^2 + k$ is tangent to $y = 2x$. The slope of the curve is $\dfrac{dy}{dx} = 2x$ and the slope of the line is 2 thus at P, $2x_0 = 2$ so $x_0 = 1$. But P is on the line, so $y_0 = 2x_0 = 2$. Because P is also on the curve we get $y_0 = x_0^2 + k$ so $k = y_0 - x_0^2 = 2 - (1)^2 = 1$.

53. The points $(-1, 1)$ and $(2, 4)$ are on the secant line so its slope is $(4-1)/(2+1) = 1$. The slope of the tangent line to $y = x^2$ is $y' = 2x$ so $2x = 1$, $x = 1/2$.

54. The points $(1, 1)$ and $(4, 2)$ are on the secant line so its slope is $1/3$. The slope of the tangent line to $y = \sqrt{x}$ is $y' = 1/(2\sqrt{x})$ so $1/(2\sqrt{x}) = 1/3$, $2\sqrt{x} = 3$, $x = 9/4$.

55. $y' = -2x$, so at any point (x_0, y_0) on $y = 1 - x^2$ the tangent line is $y - y_0 = -2x_0(x - x_0)$, or $y = -2x_0x + x_0^2 + 1$. The point $(2, 0)$ is to be on the line, so $0 = -4x_0 + x_0^2 + 1$, $x_0^2 - 4x_0 + 1 = 0$. Use the quadratic formula to get $x_0 = \dfrac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$. The points are $(2 + \sqrt{3}, -6 - 4\sqrt{3})$ and $(2 - \sqrt{3}, -6 + 4\sqrt{3})$.

56. Let $P_1(x_1, ax_1^2)$ and $P_2(x_2, ax_2^2)$ be the points of tangency. $y' = 2ax$ so the tangent lines at P_1 and P_2 are $y - ax_1^2 = 2ax_1(x - x_1)$ and $y - ax_2^2 = 2ax_2(x - x_2)$. Solve for x to get $x = \frac{1}{2}(x_1 + x_2)$ which is the x-coordinate of a point on the vertical line halfway between P_1 and P_2.

57. $y' = 3ax^2 + b$; the tangent line at $x = x_0$ is $y - y_0 = (3ax_0^2 + b)(x - x_0)$ where $y_0 = ax_0^3 + bx_0$. Solve with $y = ax^3 + bx$ to get

$$\begin{aligned}
(ax^3 + bx) - (ax_0^3 + bx_0) &= (3ax_0^2 + b)(x - x_0)\\
ax^3 + bx - ax_0^3 - bx_0 &= 3ax_0^2x - 3ax_0^3 + bx - bx_0\\
x^3 - 3x_0^2x + 2x_0^3 &= 0\\
(x - x_0)(x^2 + xx_0 - 2x_0^2) &= 0\\
(x - x_0)^2(x + 2x_0) &= 0, \text{ so } x = -2x_0.
\end{aligned}$$

58. Let (x_0, y_0) be the point of tangency. Note that $y_0 = 1/x_0$. Since $y' = -1/x^2$, the tangent line has the equation $y - y_0 = (-1/x_0^2)(x - x_0)$, or $y - \dfrac{1}{x_0} = -\dfrac{1}{x_0^2}x + \dfrac{1}{x_0}$ or $y = -\dfrac{1}{x_0^2}x + \dfrac{2}{x_0}$, with intercepts at $\left(0, \dfrac{2}{x_0}\right) = (0, 2y_0)$ and $(2x_0, 0)$. The distance from the y-intercept to the point of tangency is $\sqrt{(x_0 - 0)^2 + (y_0 - 2y_0)^2}$, and the distance from the x-intercept to the point of tangency is $\sqrt{(x_0 - 2x_0)^2 + (y_0 - 0)^2}$ so that they are equal (and equal the distance $\sqrt{x_0^2 + y_0^2}$ from the point of tangency to the origin).

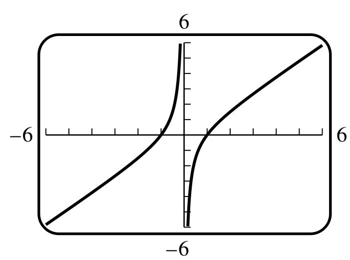
59. $y' = -\dfrac{1}{x^2}$; the tangent line at $x = x_0$ is $y - y_0 = -\dfrac{1}{x_0^2}(x - x_0)$, or $y = -\dfrac{x}{x_0^2} + \dfrac{2}{x_0}$. The tangent line crosses the x-axis at $2x_0$, the y-axis at $2/x_0$, so that the area of the triangle is $\dfrac{1}{2}(2/x_0)(2x_0) = 2$.

60. $f'(x) = 3ax^2 + 2bx + c$; there is a horizontal tangent where $f'(x) = 0$. Use the quadratic formula on $3ax^2 + 2bx + c = 0$ to get $x = (-b \pm \sqrt{b^2 - 3ac})/(3a)$ which gives two real solutions, one real solution, or none if

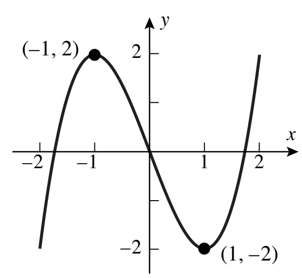
(a) $b^2 - 3ac > 0$ **(b)** $b^2 - 3ac = 0$ **(c)** $b^2 - 3ac < 0$

61. $F = GmMr^{-2}$, $\dfrac{dF}{dr} = -2GmMr^{-3} = -\dfrac{2GmM}{r^3}$

62. $dR/dT = 0.04124 - 3.558 \times 10^{-5}T$ which decreases as T increases from 0 to 700. When $T = 0$, $dR/dT = 0.04124\,\Omega/°\text{C}$; when $T = 700$, $dR/dT = 0.01633\,\Omega/°\text{C}$. The resistance is most sensitive to temperature changes at $T = 0°\text{C}$, least sensitive at $T = 700°\text{C}$.

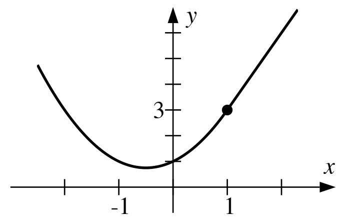


63. $f'(x) = 1 + 1/x^2 > 0$ for all $x \neq 0$



64. $f'(x) = 3x^2 - 3 = 0$ when $x = \pm 1$; $f'(x) > 0$ for $-\infty < x < -1$ and $1 < x < +\infty$

65. f is continuous at 1 because $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$; also $\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^-} (2x+1) = 3$ and $\lim_{x\to 1^+} f'(x) = \lim_{x\to 1^+} 3 = 3$ so f is differentiable at 1, and the derivative equals 3.



66. f is not continuous at $x = 9$ because $\lim_{x\to 9^-} f(x) = -63$ and $\lim_{x\to 9^+} f(x) = 3$. f cannot be differentiable at $x = 9$, for if it were, then f would also be continuous, which it is not.

67. f is continuous at 1 because $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$. Also, $\lim_{x->1^-} \dfrac{f(x)-f(1)}{x-1}$ equals the derivative of x^2 at $x = 1$, namely $2x\big|_{x=1} = 2$, while $\lim_{x->1^+} \dfrac{f(x)-f(1)}{x-1}$ equals the derivative of $\sqrt{x}$ at $x = 1$, namely $\left.\dfrac{1}{2\sqrt{x}}\right|_{x=1} = \dfrac{1}{2}$. Since these are not equal, f is not differentiable at $x = 1$.

68. f is continuous at 1/2 because $\lim_{x\to 1/2^-} f(x) = \lim_{x\to 1/2^+} f(x) = f(1/2)$; also $\lim_{x\to 1/2^-} f'(x) = \lim_{x\to 1/2^-} 3x^2 = 3/4$ and $\lim_{x\to 1/2^+} f'(x) = \lim_{x\to 1/2^+} 3x/2 = 3/4$ so $f'(1/2) = 3/4$, and f is differentiable at $x = 1/2$.

69. **(a)** $f(x) = 3x - 2$ if $x \geq 2/3$, $f(x) = -3x + 2$ if $x < 2/3$ so f is differentiable everywhere except perhaps at 2/3. f is continuous at 2/3, also $\lim_{x\to 2/3^-} f'(x) = \lim_{x\to 2/3^-} (-3) = -3$ and $\lim_{x\to 2/3^+} f'(x) = \lim_{x\to 2/3^+} (3) = 3$ so f is not differentiable at $x = 2/3$.

(b) $f(x) = x^2 - 4$ if $|x| \geq 2$, $f(x) = -x^2 + 4$ if $|x| < 2$ so f is differentiable everywhere except perhaps at ± 2. f is continuous at -2 and 2, also $\lim_{x\to 2^-} f'(x) = \lim_{x\to 2^-} (-2x) = -4$ and $\lim_{x\to 2^+} f'(x) = \lim_{x\to 2^+} (2x) = 4$ so f is not differentiable at $x = 2$. Similarly, f is not differentiable at $x = -2$.

70. **(a)** $f'(x) = -(1)x^{-2}$, $f''(x) = (2\cdot 1)x^{-3}$, $f'''(x) = -(3\cdot 2\cdot 1)x^{-4}$; $f^{(n)}(x) = (-1)^n \dfrac{n(n-1)(n-2)\cdots 1}{x^{n+1}}$

(b) $f'(x) = -2x^{-3}$, $f''(x) = (3\cdot 2)x^{-4}$, $f'''(x) = -(4\cdot 3\cdot 2)x^{-5}$; $f^{(n)}(x) = (-1)^n \dfrac{(n+1)(n)(n-1)\cdots 2}{x^{n+2}}$

71. **(a)**

$$\frac{d^2}{dx^2}[cf(x)] = \frac{d}{dx}\left[\frac{d}{dx}[cf(x)]\right] = \frac{d}{dx}\left[c\frac{d}{dx}[f(x)]\right] = c\frac{d}{dx}\left[\frac{d}{dx}[f(x)]\right] = c\frac{d^2}{dx^2}[f(x)]$$

$$\frac{d^2}{dx^2}[f(x)+g(x)] = \frac{d}{dx}\left[\frac{d}{dx}[f(x)+g(x)]\right] = \frac{d}{dx}\left[\frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]\right] = \frac{d^2}{dx^2}[f(x)] + \frac{d^2}{dx^2}[g(x)]$$

(b) Yes, by repeated application of the procedure illustrated in part **(a)**.

72. $\lim_{w\to 2} \dfrac{f'(w) - f'(2)}{w-2} = f''(2)$; $f'(x) = 8x^7 - 2$, $f''(x) = 56x^6$, so $f''(2) = 56(2^6) = 3584$.

73. **(a)** $f'(x) = nx^{n-1}$, $f''(x) = n(n-1)x^{n-2}$, $f'''(x) = n(n-1)(n-2)x^{n-3}, \ldots$, $f^{(n)}(x) = n(n-1)(n-2)\cdots 1$

(b) From part **(a)**, $f^{(k)}(x) = k(k-1)(k-2)\cdots 1$ so $f^{(k+1)}(x) = 0$ thus $f^{(n)}(x) = 0$ if $n > k$.

(c) From parts **(a)** and **(b)**, $f^{(n)}(x) = a_n n(n-1)(n-2)\cdots 1$.

74. **(a)** If a function is differentiable at a point then it is continuous at that point, thus f' is continuous on (a, b) and consequently so is f.

(b) f and all its derivatives up to $f^{(n-1)}(x)$ are continuous on (a, b).

75. Let $g(x) = x^n, f(x) = (mx + b)^n$. Use Exercise 52 in Section 2.2, but with f and g permuted. If $x_0 = mx_1 + b$ then Exercise 52 says that f is differentiable at x_1 and $f'(x_1) = mg'(x_0)$. Since $g'(x_0) = nx_0^{n-1}$, the result follows.

76. $f(x) = 4x^2 + 12x + 9$ so $f'(x) = 8x + 12 = 2 \cdot 2(2x + 3)$, as predicted by Exercise 75.

77. $f(x) = 27x^3 - 27x^2 + 9x - 1$ so $f'(x) = 81x^2 - 54x + 9 = 3 \cdot 3(3x - 1)^2$, as predicted by Exercise 75.

78. $f(x) = (x - 1)^{-1}$ so $f'(x) = (-1) \cdot 1(x - 1)^{-2} = -1/(x - 1)^2$.

79. $f(x) = 3(2x + 1)^{-2}$ so $f'(x) = 3(-2)2(2x + 1)^{-3} = -12/(2x + 1)^3$.

80. $f(x) = \dfrac{x + 1 - 1}{x + 1} = 1 - (x + 1)^{-1}$, and $f'(x) = -(-1)(x + 1)^{-2} = 1/(x + 1)^2$.

81. $f(x) = \dfrac{2x^2 + 4x + 2 + 1}{(x + 1)^2} = 2 + (x + 1)^{-2}$, so $f'(x) = -2(x + 1)^{-3} = -2/(x + 1)^3$.

82. **(a)** If $n = 0$ then $f(x) = x^0 = 1$ so $f'(x) = 0$ by Theorem 2.3.1. This equals $0x^{0-1}$, so the Extended Power Rule holds in this case.

(b) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x + h) - f(x)}{h} = \lim\limits_{h\to 0} \dfrac{1/(x + h)^m - 1/x^m}{h} = \lim\limits_{h\to 0} \dfrac{x^m - (x + h)^m}{hx^m(x + h)^m} =$

$$= \lim_{h\to 0} \frac{(x + h)^m - x^m}{h} \cdot \lim_{h\to 0} \left(-\frac{1}{x^m(x + h)^m}\right) = \frac{d}{dx}(x^m) \cdot \left(-\frac{1}{x^{2m}}\right) = mx^{m-1} \cdot \left(-\frac{1}{x^{2m}}\right) = -mx^{-m-1} = nx^{n-1}.$$

Exercise Set 2.4

1. **(a)** $f(x) = 2x^2 + x - 1$, $f'(x) = 4x + 1$ **(b)** $f'(x) = (x + 1) \cdot (2) + (2x - 1) \cdot (1) = 4x + 1$

2. **(a)** $f(x) = 3x^4 + 5x^2 - 2$, $f'(x) = 12x^3 + 10x$ **(b)** $f'(x) = (3x^2 - 1) \cdot (2x) + (x^2 + 2) \cdot (6x) = 12x^3 + 10x$

3. **(a)** $f(x) = x^4 - 1$, $f'(x) = 4x^3$ **(b)** $f'(x) = (x^2 + 1) \cdot (2x) + (x^2 - 1) \cdot (2x) = 4x^3$

4. **(a)** $f(x) = x^3 + 1$, $f'(x) = 3x^2$ **(b)** $f'(x) = (x + 1)(2x - 1) + (x^2 - x + 1) \cdot (1) = 3x^2$

5. $f'(x) = (3x^2 + 6)\dfrac{d}{dx}\left(2x - \dfrac{1}{4}\right) + \left(2x - \dfrac{1}{4}\right)\dfrac{d}{dx}(3x^2 + 6) = (3x^2 + 6)(2) + \left(2x - \dfrac{1}{4}\right)(6x) = 18x^2 - \dfrac{3}{2}x + 12$

6. $f'(x) = (2 - x - 3x^3)\dfrac{d}{dx}(7 + x^5) + (7 + x^5)\dfrac{d}{dx}(2 - x - 3x^3) = (2 - x - 3x^3)(5x^4) + (7 + x^5)(-1 - 9x^2) = -24x^7 - 6x^5 + 10x^4 - 63x^2 - 7$

7. $f'(x) = (x^3 + 7x^2 - 8)\dfrac{d}{dx}(2x^{-3} + x^{-4}) + (2x^{-3} + x^{-4})\dfrac{d}{dx}(x^3 + 7x^2 - 8) = (x^3 + 7x^2 - 8)(-6x^{-4} - 4x^{-5}) +$
$+(2x^{-3} + x^{-4})(3x^2 + 14x) = -15x^{-2} - 14x^{-3} + 48x^{-4} + 32x^{-5}$

8. $f'(x) = (x^{-1} + x^{-2})\dfrac{d}{dx}(3x^3 + 27) + (3x^3 + 27)\dfrac{d}{dx}(x^{-1} + x^{-2}) = (x^{-1} + x^{-2})(9x^2) + (3x^3 + 27)(-x^{-2} - 2x^{-3}) =$
$3 + 6x - 27x^{-2} - 54x^{-3}$

9. $f'(x) = 1 \cdot (x^2 + 2x + 4) + (x - 2) \cdot (2x + 2) = 3x^2$

10. $f'(x) = (2x+1)(x^2-x) + (x^2+x)(2x-1) = 4x^3 - 2x$

11. $f'(x) = \dfrac{(x^2+1)\frac{d}{dx}(3x+4) - (3x+4)\frac{d}{dx}(x^2+1)}{(x^2+1)^2} = \dfrac{(x^2+1)\cdot 3 - (3x+4)\cdot 2x}{(x^2+1)^2} = \dfrac{-3x^2-8x+3}{(x^2+1)^2}$

12. $f'(x) = \dfrac{(x^4+x+1)\frac{d}{dx}(x-2) - (x-2)\frac{d}{dx}(x^4+x+1)}{(x^4+x+1)^2} = \dfrac{(x^4+x+1)\cdot 1 - (x-2)\cdot(4x^3+1)}{(x^4+x+1)^2} = \dfrac{-3x^4+8x^3+3}{(x^4+x+1)^2}$

13. $f'(x) = \dfrac{(3x-4)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(3x-4)}{(3x-4)^2} = \dfrac{(3x-4)\cdot 2x - x^2\cdot 3}{(3x-4)^2} = \dfrac{3x^2-8x}{(3x-4)^2}$

14. $f'(x) = \dfrac{(3x-4)\frac{d}{dx}(2x^2+5) - (2x^2+5)\frac{d}{dx}(3x-4)}{(3x-4)^2} = \dfrac{(3x-4)\cdot 4x - (2x^2+5)\cdot 3}{(3x-4)^2} = \dfrac{6x^2-16x-15}{(3x-4)^2}$

15. $f(x) = \dfrac{2x^{3/2}+x-2x^{1/2}-1}{x+3}$, so

$$f'(x) = \frac{(x+3)\frac{d}{dx}(2x^{3/2}+x-2x^{1/2}-1) - (2x^{3/2}+x-2x^{1/2}-1)\frac{d}{dx}(x+3)}{(x+3)^2} =$$
$$= \frac{(x+3)\cdot(3x^{1/2}+1-x^{-1/2}) - (2x^{3/2}+x-2x^{1/2}-1)\cdot 1}{(x+3)^2} = \frac{x^{3/2}+10x^{1/2}+4-3x^{-1/2}}{(x+3)^2}$$

16. $f(x) = \dfrac{-2x^{3/2}-x+4x^{1/2}+2}{x^2+3x}$, so

$$f'(x) = \frac{(x^2+3x)\frac{d}{dx}(-2x^{3/2}-x+4x^{1/2}+2) - (-2x^{3/2}-x+4x^{1/2}+2)\frac{d}{dx}(x^2+3x)}{(x^2+3x)^2} =$$
$$= \frac{(x^2+3x)\cdot(-3x^{1/2}-1+2x^{-1/2}) - (-2x^{3/2}-x+4x^{1/2}+2)\cdot(2x+3)}{(x^2+3x)^2} =$$
$$= \frac{x^{5/2}+x^2-9x^{3/2}-4x-6x^{1/2}-6}{(x^2+3x)^2}$$

17. This could be computed by two applications of the product rule, but it's simpler to expand $f(x)$: $f(x) = 14x + 21 + 7x^{-1} + 2x^{-2} + 3x^{-3} + x^{-4}$, so $f'(x) = 14 - 7x^{-2} - 4x^{-3} - 9x^{-4} - 4x^{-5}$.

18. This could be computed by two applications of the product rule, but it's simpler to expand $f(x)$: $f(x) = -6x^7 - 4x^6 + 16x^5 - 3x^{-2} - 2x^{-3} + 8x^{-4}$, so $f'(x) = -42x^6 - 24x^5 + 80x^4 + 6x^{-3} + 6x^{-4} - 32x^{-5}$.

19. In general, $\dfrac{d}{dx}\left[g(x)^2\right] = 2g(x)g'(x)$ and $\dfrac{d}{dx}\left[g(x)^3\right] = \dfrac{d}{dx}\left[g(x)^2 g(x)\right] = g(x)^2g'(x) + g(x)\dfrac{d}{dx}\left[g(x)^2\right] = g(x)^2g'(x) + g(x)\cdot 2g(x)g'(x) = 3g(x)^2g'(x)$.
Letting $g(x) = x^7+2x-3$, we have $f'(x) = 3(x^7+2x-3)^2(7x^6+2)$.

20. In general, $\dfrac{d}{dx}\left[g(x)^2\right] = 2g(x)g'(x)$, so $\dfrac{d}{dx}\left[g(x)^4\right] = \dfrac{d}{dx}\left[\left(g(x)^2\right)^2\right] = 2g(x)^2\cdot\dfrac{d}{dx}\left[g(x)^2\right] = 2g(x)^2\cdot 2g(x)g'(x) = 4g(x)^3g'(x)$
Letting $g(x) = x^2+1$, we have $f'(x) = 4(x^2+1)^3\cdot 2x = 8x(x^2+1)^3$.

21. $\dfrac{dy}{dx} = \dfrac{(x+3)\cdot 2 - (2x-1)\cdot 1}{(x+3)^2} = \dfrac{7}{(x+3)^2}$, so $\left.\dfrac{dy}{dx}\right|_{x=1} = \dfrac{7}{16}$.

22. $\dfrac{dy}{dx} = \dfrac{(x^2-5)\cdot 4 - (4x+1)\cdot(2x)}{(x^2-5)^2} = \dfrac{-4x^2-2x-20}{(x^2-5)^2}$, so $\left.\dfrac{dy}{dx}\right|_{x=1} = -\dfrac{26}{16} = -\dfrac{13}{8}$.

23. $\dfrac{dy}{dx} = \left(\dfrac{3x+2}{x}\right)\dfrac{d}{dx}\left(x^{-5}+1\right)+\left(x^{-5}+1\right)\dfrac{d}{dx}\left(\dfrac{3x+2}{x}\right) = \left(\dfrac{3x+2}{x}\right)\left(-5x^{-6}\right)+\left(x^{-5}+1\right)\left[\dfrac{x(3)-(3x+2)(1)}{x^2}\right] = \left(\dfrac{3x+2}{x}\right)\left(-5x^{-6}\right)+\left(x^{-5}+1\right)\left(-\dfrac{2}{x^2}\right)$; so $\left.\dfrac{dy}{dx}\right|_{x=1} = 5(-5)+2(-2) = -29$.

24. $\dfrac{dy}{dx} = (2x^7-x^2)\dfrac{d}{dx}\left(\dfrac{x-1}{x+1}\right)+\left(\dfrac{x-1}{x+1}\right)\dfrac{d}{dx}(2x^7-x^2) = (2x^7-x^2)\left[\dfrac{(x+1)(1)-(x-1)(1)}{(x+1)^2}\right]+$
$+\left(\dfrac{x-1}{x+1}\right)(14x^6-2x) = (2x^7-x^2)\cdot\dfrac{2}{(x+1)^2}+\left(\dfrac{x-1}{x+1}\right)(14x^6-2x)$; so $\left.\dfrac{dy}{dx}\right|_{x=1} = (2-1)\dfrac{2}{4}+0(14-2) = \dfrac{1}{2}$.

25. $f'(x) = \dfrac{(x^2+1)\cdot 1 - x\cdot 2x}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}$, so $f'(1)=0$.

26. $f'(x) = \dfrac{(x^2+1)\cdot 2x-(x^2-1)\cdot 2x}{(x^2+1)^2} = \dfrac{4x}{(x^2+1)^2}$, so $f'(1)=1$.

27. **(a)** $g'(x) = \sqrt{x}f'(x)+\dfrac{1}{2\sqrt{x}}f(x)$, $g'(4) = (2)(-5)+\dfrac{1}{4}(3) = -37/4$.

(b) $g'(x) = \dfrac{xf'(x)-f(x)}{x^2}$, $g'(4) = \dfrac{(4)(-5)-3}{16} = -23/16$.

28. **(a)** $g'(x) = 6x-5f'(x)$, $g'(3) = 6(3)-5(4) = -2$.

(b) $g'(x) = \dfrac{2f(x)-(2x+1)f'(x)}{f^2(x)}$, $g'(3) = \dfrac{2(-2)-7(4)}{(-2)^2} = -8$.

29. **(a)** $F'(x) = 5f'(x)+2g'(x), F'(2) = 5(4)+2(-5) = 10$.

(b) $F'(x) = f'(x)-3g'(x), F'(2) = 4-3(-5) = 19$.

(c) $F'(x) = f(x)g'(x)+g(x)f'(x), F'(2) = (-1)(-5)+(1)(4) = 9$.

(d) $F'(x) = [g(x)f'(x)-f(x)g'(x)]/g^2(x), F'(2) = [(1)(4)-(-1)(-5)]/(1)^2 = -1$.

30. **(a)** $F'(x) = 6f'(x)-5g'(x), F'(\pi) = 6(-1)-5(2) = -16$.

(b) $F'(x) = f(x)+g(x)+x(f'(x)+g'(x)),\ F'(\pi) = 10-3+\pi(-1+2) = 7+\pi$.

(c) $F'(x) = 2f(x)g'(x)+2f'(x)g(x) = 2(20)+2(3) = 46$.

(d) $F'(x) = \dfrac{(4+g(x))f'(x)-f(x)g'(x)}{(4+g(x))^2} = \dfrac{(4-3)(-1)-10(2)}{(4-3)^2} = -21$.

31. $\dfrac{dy}{dx} = \dfrac{2x(x+2)-(x^2-1)}{(x+2)^2}$, $\dfrac{dy}{dx}=0$ if $x^2+4x+1=0$. By the quadratic formula, $x = \dfrac{-4\pm\sqrt{16-4}}{2} = -2\pm\sqrt{3}$. The tangent line is horizontal at $x=-2\pm\sqrt{3}$.

32. $\dfrac{dy}{dx} = \dfrac{2x(x-1)-(x^2+1)}{(x-1)^2} = \dfrac{x^2-2x-1}{(x-1)^2}$. The tangent line is horizontal when it has slope 0, i.e. $x^2-2x-1=0$ which, by the quadratic formula, has solutions $x = \dfrac{2\pm\sqrt{4+4}}{2} = 1\pm\sqrt{2}$, the tangent line is horizontal when $x=1\pm\sqrt{2}$.

33. The tangent line is parallel to the line $y = x$ when it has slope 1. $\dfrac{dy}{dx} = \dfrac{2x(x+1) - (x^2+1)}{(x+1)^2} = \dfrac{x^2+2x-1}{(x+1)^2} = 1$ if $x^2 + 2x - 1 = (x+1)^2$, which reduces to $-1 = +1$, impossible. Thus the tangent line is never parallel to the line $y = x$.

34. The tangent line is perpendicular to the line $y = x$ when the tangent line has slope -1. $y = \dfrac{x+2+1}{x+2} = 1 + \dfrac{1}{x+2}$, hence $\dfrac{dy}{dx} = -\dfrac{1}{(x+2)^2} = -1$ when $(x+2)^2 = 1$, $x^2+4x+3 = 0$, $(x+1)(x+3) = 0$, $x = -1, -3$. Thus the tangent line is perpendicular to the line $y = x$ at the points $(-1, 2), (-3, 0)$.

35. Fix x_0. The slope of the tangent line to the curve $y = \dfrac{1}{x+4}$ at the point $(x_0, 1/(x_0+4))$ is given by $\dfrac{dy}{dx} = \left.\dfrac{-1}{(x+4)^2}\right|_{x=x_0} = \dfrac{-1}{(x_0+4)^2}$. The tangent line to the curve at (x_0, y_0) thus has the equation $y - y_0 = \dfrac{-(x-x_0)}{(x_0+4)^2}$, and this line passes through the origin if its constant term $y_0 - x_0\dfrac{-1}{(x_0+4)^2}$ is zero. Then $\dfrac{1}{x_0+4} = \dfrac{-x_0}{(x_0+4)^2}$, so $x_0 + 4 = -x_0$, $x_0 = -2$.

36. $y = \dfrac{2x+5}{x+2} = \dfrac{2x+4+1}{x+2} = 2 + \dfrac{1}{x+2}$, and hence $\dfrac{dy}{dx} = \dfrac{-1}{(x+2)^2}$, thus the tangent line at the point (x_0, y_0) is given by $y - y_0 = \dfrac{-1}{(x_0+2)^2}(x - x_0)$, where $y_0 = 2 + \dfrac{1}{x_0+2}$. If this line is to pass through $(0, 2)$, then $2 - y_0 = \dfrac{-1}{(x_0+2)^2}(-x_0)$, $\dfrac{-1}{x_0+2} = \dfrac{x_0}{(x_0+2)^2}$, $-x_0 - 2 = x_0$, so $x_0 = -1$.

37. **(a)** Their tangent lines at the intersection point must be perpendicular.

(b) They intersect when $\dfrac{1}{x} = \dfrac{1}{2-x}$, $x = 2 - x$, $x = 1$, $y = 1$. The first curve has derivative $y = -\dfrac{1}{x^2}$, so the slope when $x = 1$ is -1. Second curve has derivative $y = \dfrac{1}{(2-x)^2}$ so the slope when $x = 1$ is 1. Since the two slopes are negative reciprocals of each other, the tangent lines are perpendicular at the point $(1, 1)$.

38. The curves intersect when $a/(x-1) = x^2 - 2x + 1$, or $(x-1)^3 = a, x = 1 + a^{1/3}$. They are perpendicular when their slopes are negative reciprocals of each other, i.e. $\dfrac{-a}{(x-1)^2}(2x-2) = -1$, which has the solution $x = 2a + 1$. Solve $x = 1 + a^{1/3} = 2a + 1, 2a^{2/3} = 1, a = 2^{-3/2}$. Thus the curves intersect and are perpendicular at the point $(2a + 1, 1/2)$ provided $a = 2^{-3/2}$.

39. $F'(x) = xf'(x) + f(x)$, $F''(x) = xf''(x) + f'(x) + f'(x) = xf''(x) + 2f'(x)$.

40. **(a)** $F'''(x) = xf'''(x) + 3f''(x)$.

(b) Assume that $F^{(n)}(x) = xf^{(n)}(x) + nf^{(n-1)}(x)$ for some n (for instance $n = 3$, as in part **(a)**). Then $F^{(n+1)}(x) = xf^{(n+1)}(x) + (1+n)f^{(n)}(x) = xf^{(n+1)}(x) + (n+1)f^{(n)}(x)$, which is an inductive proof.

41. $R'(p) = p \cdot f'(p) + f(p) \cdot 1 = f(p) + pf'(p)$, so $R'(120) = 9000 + 120 \cdot (-60) = 1800$. Increasing the price by a small amount Δp dollars would increase the revenue by about $1800\Delta p$ dollars.

42. $R'(p) = p \cdot f'(p) + f(p) \cdot 1 = f(p) + pf'(p)$, so $R'(120) = 9000 + 120 \cdot (-80) = -600$. Increasing the price by a small amount Δp dollars would decrease the revenue by about $600\Delta p$ dollars.

43. $f(x) = \dfrac{1}{x^n}$ so $f'(x) = \dfrac{x^n \cdot (0) - 1 \cdot (nx^{n-1})}{x^{2n}} = -\dfrac{n}{x^{n+1}} = -nx^{-n-1}$.

Exercise Set 2.5

1. $f'(x) = -4\sin x + 2\cos x$

2. $f'(x) = \dfrac{-10}{x^3} + \cos x$

3. $f'(x) = 4x^2 \sin x - 8x\cos x$

4. $f'(x) = 4\sin x \cos x$

5. $f'(x) = \dfrac{\sin x(5+\sin x) - \cos x(5-\cos x)}{(5+\sin x)^2} = \dfrac{1+5(\sin x - \cos x)}{(5+\sin x)^2}$

6. $f'(x) = \dfrac{(x^2+\sin x)\cos x - \sin x(2x+\cos x)}{(x^2+\sin x)^2} = \dfrac{x^2\cos x - 2x\sin x}{(x^2+\sin x)^2}$

7. $f'(x) = \sec x \tan x - \sqrt{2}\sec^2 x$

8. $f'(x) = (x^2+1)\sec x\tan x + (\sec x)(2x) = (x^2+1)\sec x \tan x + 2x\sec x$

9. $f'(x) = -4\csc x\cot x + \csc^2 x$

10. $f'(x) = -\sin x - \csc x + x\csc x\cot x$

11. $f'(x) = \sec x(\sec^2 x) + (\tan x)(\sec x\tan x) = \sec^3 x + \sec x\tan^2 x$

12. $f'(x) = (\csc x)(-\csc^2 x) + (\cot x)(-\csc x\cot x) = -\csc^3 x - \csc x\cot^2 x$

13. $f'(x) = \dfrac{(1+\csc x)(-\csc^2 x) - \cot x(0-\csc x\cot x)}{(1+\csc x)^2} = \dfrac{\csc x(-\csc x - \csc^2 x + \cot^2 x)}{(1+\csc x)^2}$, but $1+\cot^2 x = \csc^2 x$ (identity), thus $\cot^2 x - \csc^2 x = -1$, so $f'(x) = \dfrac{\csc x(-\csc x - 1)}{(1+\csc x)^2} = -\dfrac{\csc x}{1+\csc x}$.

14. $f'(x) = \dfrac{(1+\tan x)(\sec x\tan x) - (\sec x)(\sec^2 x)}{(1+\tan x)^2} = \dfrac{\sec x\tan x + \sec x\tan^2 x - \sec^3 x}{(1+\tan x)^2} =$
$= \dfrac{\sec x(\tan x + \tan^2 x - \sec^2 x)}{(1+\tan x)^2} = \dfrac{\sec x(\tan x - 1)}{(1+\tan x)^2}$

15. $f(x) = \sin^2 x + \cos^2 x = 1$ (identity), so $f'(x) = 0$.

16. $f'(x) = 2\sec x\tan x\sec x - 2\tan x\sec^2 x = \dfrac{2\sin x}{\cos^3 x} - 2\dfrac{\sin x}{\cos^3 x} = 0$; also, $f(x) = \sec^2 x - \tan^2 x = 1$ (identity), so $f'(x) = 0$.

17. $f(x) = \dfrac{\tan x}{1+x\tan x}$ (because $\sin x\sec x = (\sin x)(1/\cos x) = \tan x$), so
$f'(x) = \dfrac{(1+x\tan x)(\sec^2 x) - \tan x[x(\sec^2 x) + (\tan x)(1)]}{(1+x\tan x)^2} = \dfrac{\sec^2 x - \tan^2 x}{(1+x\tan x)^2} = \dfrac{1}{(1+x\tan x)^2}$ (because $\sec^2 x - \tan^2 x = 1$).

18. $f(x) = \dfrac{(x^2+1)\cot x}{3-\cot x}$ (because $\cos x\csc x = (\cos x)(1/\sin x) = \cot x$), so
$f'(x) = \dfrac{(3-\cot x)[2x\cot x - (x^2+1)\csc^2 x] - (x^2+1)\cot x\csc^2 x}{(3-\cot x)^2} = \dfrac{6x\cot x - 2x\cot^2 x - 3(x^2+1)\csc^2 x}{(3-\cot x)^2}$.

19. $dy/dx = -x\sin x + \cos x$, $d^2y/dx^2 = -x\cos x - \sin x - \sin x = -x\cos x - 2\sin x$

20. $dy/dx = -\csc x \cot x$, $d^2y/dx^2 = -[(\csc x)(-\csc^2 x) + (\cot x)(-\csc x \cot x)] = \csc^3 x + \csc x \cot^2 x$

21. $dy/dx = x(\cos x) + (\sin x)(1) - 3(-\sin x) = x\cos x + 4\sin x$,
$d^2y/dx^2 = x(-\sin x) + (\cos x)(1) + 4\cos x = -x\sin x + 5\cos x$

22. $dy/dx = x^2(-\sin x) + (\cos x)(2x) + 4\cos x = -x^2\sin x + 2x\cos x + 4\cos x$,
$d^2y/dx^2 = -[x^2(\cos x) + (\sin x)(2x)] + 2[x(-\sin x) + \cos x] - 4\sin x = (2 - x^2)\cos x - 4(x+1)\sin x$

23. $dy/dx = (\sin x)(-\sin x) + (\cos x)(\cos x) = \cos^2 x - \sin^2 x$,
$d^2y/dx^2 = (\cos x)(-\sin x) + (\cos x)(-\sin x) - [(\sin x)(\cos x) + (\sin x)(\cos x)] = -4\sin x \cos x$

24. $dy/dx = \sec^2 x, d^2y/dx^2 = 2\sec^2 x \tan x$

25. Let $f(x) = \tan x$, then $f'(x) = \sec^2 x$.

(a) $f(0) = 0$ and $f'(0) = 1$, so $y - 0 = (1)(x - 0)$, $y = x$.

(b) $f\left(\frac{\pi}{4}\right) = 1$ and $f'\left(\frac{\pi}{4}\right) = 2$, so $y - 1 = 2\left(x - \frac{\pi}{4}\right)$, $y = 2x - \frac{\pi}{2} + 1$.

(c) $f\left(-\frac{\pi}{4}\right) = -1$ and $f'\left(-\frac{\pi}{4}\right) = 2$, so $y + 1 = 2\left(x + \frac{\pi}{4}\right)$, $y = 2x + \frac{\pi}{2} - 1$.

26. Let $f(x) = \sin x$, then $f'(x) = \cos x$.

(a) $f(0) = 0$ and $f'(0) = 1$, so $y - 0 = (1)(x - 0)$, $y = x$.

(b) $f(\pi) = 0$ and $f'(\pi) = -1$, so $y - 0 = (-1)(x - \pi)$, $y = -x + \pi$.

(c) $f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ and $f'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, so $y - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)$, $y = \frac{1}{\sqrt{2}}x - \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}}$.

27. **(a)** If $y = x\sin x$ then $y' = \sin x + x\cos x$ and $y'' = 2\cos x - x\sin x$ so $y'' + y = 2\cos x$.

(b) Differentiate the result of part **(a)** twice more to get $y^{(4)} + y'' = -2\cos x$.

28. **(a)** If $y = \cos x$ then $y' = -\sin x$ and $y'' = -\cos x$, so $y'' + y = (-\cos x) + (\cos x) = 0$; if $y = \sin x$ then $y' = \cos x$ and $y'' = -\sin x$ so $y'' + y = (-\sin x) + (\sin x) = 0$.

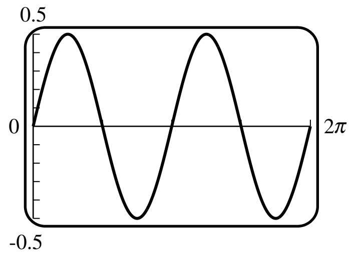
(b) $y' = A\cos x - B\sin x$, $y'' = -A\sin x - B\cos x$, so $y'' + y = (-A\sin x - B\cos x) + (A\sin x + B\cos x) = 0$.

29. **(a)** $f'(x) = \cos x = 0$ at $x = \pm\pi/2, \pm 3\pi/2$.

(b) $f'(x) = 1 - \sin x = 0$ at $x = -3\pi/2, \pi/2$.

(c) $f'(x) = \sec^2 x \geq 1$ always, so no horizontal tangent line.

(d) $f'(x) = \sec x \tan x = 0$ when $\sin x = 0$, $x = \pm 2\pi, \pm\pi, 0$.

30. **(a)** 

(b) $y = \sin x \cos x = (1/2)\sin 2x$ and $y' = \cos 2x$. So $y' = 0$ when $2x = (2n+1)\pi/2$ for $n = 0, 1, 2, 3$ or $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

31. $x = 10\sin\theta$, $dx/d\theta = 10\cos\theta$; if $\theta = 60°$, then $dx/d\theta = 10(1/2) = 5$ ft/rad $= \pi/36$ ft/deg ≈ 0.087 ft/deg.

32. $s = 3800\csc\theta, ds/d\theta = -3800\csc\theta\cot\theta$; if $\theta = 30°$, then $ds/d\theta = -3800(2)(\sqrt{3}) = -7600\sqrt{3}$ ft/rad $= -380\sqrt{3}\pi/9$ ft/deg ≈ -230 ft/deg.

33. $D = 50\tan\theta, dD/d\theta = 50\sec^2\theta$; if $\theta = 45°$, then $dD/d\theta = 50(\sqrt{2})^2 = 100$ m/rad $= 5\pi/9$ m/deg ≈ 1.75 m/deg.

34. **(a)** From the right triangle shown, $\sin\theta = r/(r+h)$ so $r + h = r\csc\theta$, $h = r(\csc\theta - 1)$.

(b) $dh/d\theta = -r\csc\theta\cot\theta$; if $\theta = 30°$, then $dh/d\theta = -6378(2)(\sqrt{3}) \approx -22,094$ km/rad ≈ -386 km/deg.

35. False. $g'(x) = f(x)\cos x + f'(x)\sin x$

36. True, if $f(x)$ is continuous at $x = 0$, then $g'(0) = \lim_{h\to 0}\dfrac{g(h) - g(0)}{h} = \lim_{h\to 0}\dfrac{f(h)\sin h}{h} = \lim_{h\to 0} f(h) \cdot \lim_{h\to 0}\dfrac{\sin h}{h} = f(0)\cdot 1 = f(0)$.

37. True. $f(x) = \dfrac{\sin x}{\cos x} = \tan x$, so $f'(x) = \sec^2 x$.

38. False. $g'(x) = f(x)\cdot\dfrac{d}{dx}(\sec x) + f'(x)\sec x = f(x)\sec x\tan x + f'(x)\sec x$, so $g'(0) = f(0)\sec 0\tan 0 + f'(0)\sec 0 = 8\cdot 1\cdot 0 + (-2)\cdot 1 = -2$. The second equality given in the problem is wrong: $\lim_{h\to 0}\dfrac{f(h)\sec h - f(0)}{h} = -2$ but $\lim_{h\to 0}\dfrac{8(\sec h - 1)}{h} = 0$.

39. $\dfrac{d^4}{dx^4}\sin x = \sin x$, so $\dfrac{d^{4k}}{dx^{4k}}\sin x = \sin x$; $\dfrac{d^{87}}{dx^{87}}\sin x = \dfrac{d^3}{dx^3}\dfrac{d^{4\cdot 21}}{dx^{4\cdot 21}}\sin x = \dfrac{d^3}{dx^3}\sin x = -\cos x$.

40. $\dfrac{d^{100}}{dx^{100}}\cos x = \dfrac{d^{4k}}{dx^{4k}}\cos x = \cos x$.

41. $f'(x) = -\sin x$, $f''(x) = -\cos x$, $f'''(x) = \sin x$, and $f^{(4)}(x) = \cos x$ with higher order derivatives repeating this pattern, so $f^{(n)}(x) = \sin x$ for $n = 3, 7, 11, \ldots$

42. $f(x) = \sin x, f'(x) = \cos x, f''(x) = -\sin x, f'''(x) = -\cos x, f^{(4)}(x) = \sin x$, and the right-hand sides continue with a period of 4, so that $f^{(n)}(x) = \sin x$ when $n = 4k$ for some k.

43. **(a)** all x **(b)** all x **(c)** $x \neq \pi/2 + n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

(d) $x \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ **(e)** $x \neq \pi/2 + n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ **(f)** $x \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

(g) $x \neq (2n+1)\pi$, $n = 0, \pm 1, \pm 2, \ldots$ **(h)** $x \neq n\pi/2$, $n = 0, \pm 1, \pm 2, \ldots$ **(i)** all x

44. **(a)** $\dfrac{d}{dx}[\cos x] = \lim\limits_{h\to 0} \dfrac{\cos(x+h)-\cos x}{h} = \lim\limits_{h\to 0} \dfrac{\cos x \cos h - \sin x \sin h - \cos x}{h} =$
$= \lim\limits_{h\to 0} \left[\cos x \left(\dfrac{\cos h - 1}{h}\right) - \sin x \left(\dfrac{\sin h}{h}\right)\right] = (\cos x)(0) - (\sin x)(1) = -\sin x.$

(b) $\dfrac{d}{dx}[\cot x] = \dfrac{d}{dx}\left[\dfrac{\cos x}{\sin x}\right] = \dfrac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x} = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \dfrac{-1}{\sin^2 x} = -\csc^2 x.$

(c) $\dfrac{d}{dx}[\sec x] = \dfrac{d}{dx}\left[\dfrac{1}{\cos x}\right] = \dfrac{0\cdot\cos x - (1)(-\sin x)}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x} = \sec x \tan x.$

(d) $\dfrac{d}{dx}[\csc x] = \dfrac{d}{dx}\left[\dfrac{1}{\sin x}\right] = \dfrac{(\sin x)(0) - (1)(\cos x)}{\sin^2 x} = -\dfrac{\cos x}{\sin^2 x} = -\csc x \cot x.$

45. $\dfrac{d}{dx}\sin x = \lim\limits_{w\to x} \dfrac{\sin w - \sin x}{w - x} = \lim\limits_{w\to x} \dfrac{2\sin\frac{w-x}{2}\cos\frac{w+x}{2}}{w-x} = \lim\limits_{w\to x} \dfrac{\sin\frac{w-x}{2}}{\frac{w-x}{2}} \cos\dfrac{w+x}{2} = 1\cdot\cos x = \cos x.$

46. $\dfrac{d}{dx}[\cos x] = \lim\limits_{w\to x} \dfrac{\cos w - \cos x}{w-x} = \lim\limits_{w\to x} \dfrac{-2\sin(\frac{w-x}{2})\sin(\frac{w+x}{2})}{w-x} = -\lim\limits_{w\to x} \sin\left(\dfrac{w+x}{2}\right) \lim\limits_{w\to x} \dfrac{\sin(\frac{w-x}{2})}{\frac{w-x}{2}} = -\sin x.$

47. **(a)** $\lim\limits_{h\to 0} \dfrac{\tan h}{h} = \lim\limits_{h\to 0} \dfrac{\left(\dfrac{\sin h}{\cos h}\right)}{h} = \lim\limits_{h\to 0} \dfrac{\left(\dfrac{\sin h}{h}\right)}{\cos h} = \dfrac{1}{1} = 1.$

(b) $\dfrac{d}{dx}[\tan x] = \lim\limits_{h\to 0} \dfrac{\tan(x+h) - \tan x}{h} = \lim\limits_{h\to 0} \dfrac{\dfrac{\tan x + \tan h}{1 - \tan x \tan h} - \tan x}{h} = \lim\limits_{h\to 0} \dfrac{\tan x + \tan h - \tan x + \tan^2 x \tan h}{h(1 - \tan x \tan h)} =$
$\lim\limits_{h\to 0} \dfrac{\tan h(1+\tan^2 x)}{h(1 - \tan x\tan h)} = \lim\limits_{h\to 0} \dfrac{\tan h \sec^2 x}{h(1-\tan x \tan h)} = \sec^2 x \lim\limits_{h\to 0} \dfrac{\dfrac{\tan h}{h}}{1 - \tan x \tan h} = \sec^2 x \dfrac{\lim\limits_{h\to 0} \dfrac{\tan h}{h}}{\lim\limits_{h\to 0}(1 - \tan x \tan h)} = \sec^2 x.$

48. $\lim\limits_{x\to 0} \dfrac{\tan(x+y) - \tan y}{x} = \lim\limits_{h\to 0} \dfrac{\tan(y+h) - \tan y}{h} = \dfrac{d}{dy}(\tan y) = \sec^2 y.$

49. By Exercises 49 and 50 of Section 1.6, we have $\lim\limits_{h\to 0} \dfrac{\sin h}{h} = \dfrac{\pi}{180}$ and $\lim\limits_{h\to 0} \dfrac{\cos h - 1}{h} = 0$. Therefore:

(a) $\dfrac{d}{dx}[\sin x] = \lim\limits_{h\to 0} \dfrac{\sin(x+h) - \sin x}{h} = \sin x \lim\limits_{h\to 0} \dfrac{\cos h - 1}{h} + \cos x \lim\limits_{h\to 0} \dfrac{\sin h}{h} = (\sin x)(0) + (\cos x)(\pi/180) = \dfrac{\pi}{180}\cos x.$

(b) $\dfrac{d}{dx}[\cos x] = \lim\limits_{h\to 0} \dfrac{\cos(x+h) - \cos x}{h} = \lim\limits_{h\to 0} \dfrac{\cos x\cos h - \sin x \sin h - \cos x}{h} = \cos x \lim\limits_{h\to 0} \dfrac{\cos h - 1}{h} - \sin x \lim\limits_{h\to 0} \dfrac{\sin h}{h} = 0\cdot\cos x - \dfrac{\pi}{180}\cdot\sin x = -\dfrac{\pi}{180}\sin x.$

50. If f is periodic, then so is f'. Proof: Suppose $f(x+p) = f(x)$ for all x. Then $f'(x+p) = \lim\limits_{h\to 0} \dfrac{f(x+p+h) - f(x+p)}{h} = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = f'(x)$. However, f' may be periodic even if f is not. For example, $f(x) = x + \sin x$ is not periodic, but $f'(x) = 1 + \cos x$ has period 2π.

Exercise Set 2.6

1. $(f\circ g)'(x) = f'(g(x))g'(x)$, so $(f\circ g)'(0) = f'(g(0))g'(0) = f'(0)(3) = (2)(3) = 6.$

2. $(f \circ g)'(2) = f'(g(2))g'(2) = 5(-3) = -15$.

3. (a) $(f \circ g)(x) = f(g(x)) = (2x-3)^5$ and $(f \circ g)'(x) = f'(g(x))g'(x) = 5(2x-3)^4(2) = 10(2x-3)^4$.

(b) $(g \circ f)(x) = g(f(x)) = 2x^5 - 3$ and $(g \circ f)'(x) = g'(f(x))f'(x) = 2(5x^4) = 10x^4$.

4. (a) $(f \circ g)(x) = 5\sqrt{4+\cos x}$ and $(f \circ g)'(x) = f'(g(x))g'(x) = \dfrac{5}{2\sqrt{4+\cos x}}(-\sin x)$.

(b) $(g \circ f)(x) = 4 + \cos(5\sqrt{x})$ and $(g \circ f)'(x) = g'(f(x))f'(x) = -\sin(5\sqrt{x})\dfrac{5}{2\sqrt{x}}$.

5. (a) $F'(x) = f'(g(x))g'(x)$, $F'(3) = f'(g(3))g'(3) = -1(7) = -7$.

(b) $G'(x) = g'(f(x))f'(x)$, $G'(3) = g'(f(3))f'(3) = 4(-2) = -8$.

6. (a) $F'(x) = f'(g(x))g'(x)$, $F'(-1) = f'(g(-1))g'(-1) = f'(2)(-3) = (4)(-3) = -12$.

(b) $G'(x) = g'(f(x))f'(x), G'(-1) = g'(f(-1))f'(-1) = -5(3) = -15$.

7. $f'(x) = 37(x^3+2x)^{36}\dfrac{d}{dx}(x^3+2x) = 37(x^3+2x)^{36}(3x^2+2)$.

8. $f'(x) = 6(3x^2+2x-1)^5\dfrac{d}{dx}(3x^2+2x-1) = 6(3x^2+2x-1)^5(6x+2) = 12(3x^2+2x-1)^5(3x+1)$.

9. $f'(x) = -2\left(x^3 - \dfrac{7}{x}\right)^{-3}\dfrac{d}{dx}\left(x^3 - \dfrac{7}{x}\right) = -2\left(x^3 - \dfrac{7}{x}\right)^{-3}\left(3x^2 + \dfrac{7}{x^2}\right)$.

10. $f(x) = (x^5 - x + 1)^{-9}$, $f'(x) = -9(x^5 - x + 1)^{-10}\dfrac{d}{dx}(x^5 - x + 1) = -9(x^5 - x + 1)^{-10}(5x^4 - 1) = \dfrac{-9(5x^4-1)}{(x^5-x+1)^{10}}$.

11. $f(x) = 4(3x^2 - 2x + 1)^{-3}$, $f'(x) = -12(3x^2 - 2x + 1)^{-4}\dfrac{d}{dx}(3x^2 - 2x + 1) = -12(3x^2 - 2x + 1)^{-4}(6x - 2) = \dfrac{24(1-3x)}{(3x^2-2x+1)^4}$.

12. $f'(x) = \dfrac{1}{2\sqrt{x^3-2x+5}}\dfrac{d}{dx}(x^3 - 2x + 5) = \dfrac{3x^2-2}{2\sqrt{x^3-2x+5}}$.

13. $f'(x) = \dfrac{1}{2\sqrt{4+\sqrt{3x}}}\dfrac{d}{dx}(4 + \sqrt{3x}) = \dfrac{\sqrt{3}}{4\sqrt{x}\sqrt{4+\sqrt{3x}}}$.

14. $f'(x) = \dfrac{1}{3}\left(12+\sqrt{x}\right)^{-2/3} \cdot \dfrac{1}{2}\dfrac{1}{\sqrt{x}} = \dfrac{1}{6(12+\sqrt{x})^{2/3}\sqrt{x}}$.

15. $f'(x) = \cos(1/x^2)\dfrac{d}{dx}(1/x^2) = -\dfrac{2}{x^3}\cos(1/x^2)$.

16. $f'(x) = \left(\sec^2\sqrt{x}\right)\dfrac{d}{dx}\sqrt{x} = \left(\sec^2\sqrt{x}\right)\dfrac{1}{2\sqrt{x}}$.

17. $f'(x) = 20\cos^4 x\dfrac{d}{dx}(\cos x) = 20\cos^4 x(-\sin x) = -20\cos^4 x \sin x$.

18. $f'(x) = 4 + 20(\sin^3 x)\dfrac{d}{dx}(\sin x) = 4 + 20\sin^3 x\cos x$.

19. $f'(x) = 2\cos(3\sqrt{x})\dfrac{d}{dx}[\cos(3\sqrt{x})] = -2\cos(3\sqrt{x})\sin(3\sqrt{x})\dfrac{d}{dx}(3\sqrt{x}) = -\dfrac{3\cos(3\sqrt{x})\sin(3\sqrt{x})}{\sqrt{x}}.$

20. $f'(x) = 4\tan^3(x^3)\dfrac{d}{dx}[\tan(x^3)] = 4\tan^3(x^3)\sec^2(x^3)\dfrac{d}{dx}(x^3) = 12x^2\tan^3(x^3)\sec^2(x^3).$

21. $f'(x) = 4\sec(x^7)\dfrac{d}{dx}[\sec(x^7)] = 4\sec(x^7)\sec(x^7)\tan(x^7)\dfrac{d}{dx}(x^7) = 28x^6\sec^2(x^7)\tan(x^7).$

22. $f'(x) = 3\cos^2\left(\dfrac{x}{x+1}\right)\dfrac{d}{dx}\cos\left(\dfrac{x}{x+1}\right) = 3\cos^2\left(\dfrac{x}{x+1}\right)\left[-\sin\left(\dfrac{x}{x+1}\right)\right]\dfrac{(x+1)(1)-x(1)}{(x+1)^2} =$
$= -\dfrac{3}{(x+1)^2}\cos^2\left(\dfrac{x}{x+1}\right)\sin\left(\dfrac{x}{x+1}\right).$

23. $f'(x) = \dfrac{1}{2\sqrt{\cos(5x)}}\dfrac{d}{dx}[\cos(5x)] = -\dfrac{5\sin(5x)}{2\sqrt{\cos(5x)}}.$

24. $f'(x) = \dfrac{1}{2\sqrt{3x-\sin^2(4x)}}\dfrac{d}{dx}[3x-\sin^2(4x)] = \dfrac{3-8\sin(4x)\cos(4x)}{2\sqrt{3x-\sin^2(4x)}}.$

25. $f'(x) = -3\left[x+\csc(x^3+3)\right]^{-4}\dfrac{d}{dx}\left[x+\csc(x^3+3)\right] =$
$= -3\left[x+\csc(x^3+3)\right]^{-4}\left[1-\csc(x^3+3)\cot(x^3+3)\dfrac{d}{dx}(x^3+3)\right] =$
$= -3\left[x+\csc(x^3+3)\right]^{-4}\left[1-3x^2\csc(x^3+3)\cot(x^3+3)\right].$

26. $f'(x) = -4\left[x^4-\sec(4x^2-2)\right]^{-5}\dfrac{d}{dx}\left[x^4-\sec(4x^2-2)\right] =$
$= -4\left[x^4-\sec(4x^2-2)\right]^{-5}\left[4x^3-\sec(4x^2-2)\tan(4x^2-2)\dfrac{d}{dx}(4x^2-2)\right] =$
$= -16x\left[x^4-\sec(4x^2-2)\right]^{-5}\left[x^2-2\ \sec(4x^2-2)\tan(4x^2-2)\right].$

27. $\dfrac{dy}{dx} = x^3(2\sin 5x)\dfrac{d}{dx}(\sin 5x) + 3x^2\sin^2 5x = 10x^3\sin 5x\cos 5x + 3x^2\sin^2 5x.$

28. $\dfrac{dy}{dx} = \sqrt{x}\left[3\tan^2(\sqrt{x})\sec^2(\sqrt{x})\dfrac{1}{2\sqrt{x}}\right] + \dfrac{1}{2\sqrt{x}}\tan^3(\sqrt{x}) = \dfrac{3}{2}\tan^2(\sqrt{x})\sec^2(\sqrt{x}) + \dfrac{1}{2\sqrt{x}}\tan^3(\sqrt{x}).$

29. $\dfrac{dy}{dx} = x^5\sec\left(\dfrac{1}{x}\right)\tan\left(\dfrac{1}{x}\right)\dfrac{d}{dx}\left(\dfrac{1}{x}\right) + \sec\left(\dfrac{1}{x}\right)(5x^4) = x^5\sec\left(\dfrac{1}{x}\right)\tan\left(\dfrac{1}{x}\right)\left(-\dfrac{1}{x^2}\right) + 5x^4\sec\left(\dfrac{1}{x}\right) =$
$= -x^3\sec\left(\dfrac{1}{x}\right)\tan\left(\dfrac{1}{x}\right) + 5x^4\sec\left(\dfrac{1}{x}\right).$

30. $\dfrac{dy}{dx} = \dfrac{\sec(3x+1)\cos x - 3\sin x\sec(3x+1)\tan(3x+1)}{\sec^2(3x+1)} = \cos x\cos(3x+1) - 3\sin x\sin(3x+1).$

31. $\dfrac{dy}{dx} = -\sin(\cos x)\dfrac{d}{dx}(\cos x) = -\sin(\cos x)(-\sin x) = \sin(\cos x)\sin x.$

32. $\dfrac{dy}{dx} = \cos(\tan 3x)\dfrac{d}{dx}(\tan 3x) = 3\sec^2 3x\cos(\tan 3x).$

33. $\dfrac{dy}{dx} = 3\cos^2(\sin 2x)\dfrac{d}{dx}[\cos(\sin 2x)] = 3\cos^2(\sin 2x)[-\sin(\sin 2x)]\dfrac{d}{dx}(\sin 2x) = -6\cos^2(\sin 2x)\sin(\sin 2x)\cos 2x.$

34. $\dfrac{dy}{dx} = \dfrac{(1-\cot x^2)(-2x\csc x^2\cot x^2)-(1+\csc x^2)(2x\csc^2 x^2)}{(1-\cot x^2)^2} = -2x\csc x^2\ \dfrac{1+\cot x^2+\csc x^2}{(1-\cot x^2)^2}$, since $\csc^2 x^2 = 1+\cot^2 x^2$.

35. $\dfrac{dy}{dx} = (5x+8)^7\dfrac{d}{dx}(1-\sqrt{x})^6 + (1-\sqrt{x})^6\dfrac{d}{dx}(5x+8)^7 = 6(5x+8)^7(1-\sqrt{x})^5\dfrac{-1}{2\sqrt{x}} + 7\cdot 5(1-\sqrt{x})^6(5x+8)^6 = \dfrac{-3}{\sqrt{x}}(5x+8)^7(1-\sqrt{x})^5 + 35(1-\sqrt{x})^6(5x+8)^6$.

36. $\dfrac{dy}{dx} = (x^2+x)^5\dfrac{d}{dx}\sin^8 x + (\sin^8 x)\dfrac{d}{dx}(x^2+x)^5 = 8(x^2+x)^5\sin^7 x\cos x + 5(\sin^8 x)(x^2+x)^4(2x+1)$.

37. $\dfrac{dy}{dx} = 3\left[\dfrac{x-5}{2x+1}\right]^2\dfrac{d}{dx}\left[\dfrac{x-5}{2x+1}\right] = 3\left[\dfrac{x-5}{2x+1}\right]^2\cdot\dfrac{11}{(2x+1)^2} = \dfrac{33(x-5)^2}{(2x+1)^4}$.

38. $\dfrac{dy}{dx} = 17\left(\dfrac{1+x^2}{1-x^2}\right)^{16}\dfrac{d}{dx}\left(\dfrac{1+x^2}{1-x^2}\right) = 17\left(\dfrac{1+x^2}{1-x^2}\right)^{16}\dfrac{(1-x^2)(2x)-(1+x^2)(-2x)}{(1-x^2)^2} = 17\left(\dfrac{1+x^2}{1-x^2}\right)^{16}\dfrac{4x}{(1-x^2)^2} = \dfrac{68x(1+x^2)^{16}}{(1-x^2)^{18}}$.

39. $\dfrac{dy}{dx} = \dfrac{(4x^2-1)^8(3)(2x+3)^2(2)-(2x+3)^3(8)(4x^2-1)^7(8x)}{(4x^2-1)^{16}} = \dfrac{2(2x+3)^2(4x^2-1)^7[3(4x^2-1)-32x(2x+3)]}{(4x^2-1)^{16}} = -\dfrac{2(2x+3)^2(52x^2+96x+3)}{(4x^2-1)^9}$.

40. $\dfrac{dy}{dx} = 12[1+\sin^3(x^5)]^{11}\dfrac{d}{dx}[1+\sin^3(x^5)] = 12[1+\sin^3(x^5)]^{11}3\sin^2(x^5)\dfrac{d}{dx}\sin(x^5) =$
$= 180x^4[1+\sin^3(x^5)]^{11}\sin^2(x^5)\cos(x^5)$.

41. $\dfrac{dy}{dx} = 5\left[x\sin 2x+\tan^4(x^7)\right]^4\dfrac{d}{dx}\left[x\sin 2x\tan^4(x^7)\right] =$
$= 5\left[x\sin 2x+\tan^4(x^7)\right]^4\left[x\cos 2x\dfrac{d}{dx}(2x)+\sin 2x+4\tan^3(x^7)\dfrac{d}{dx}\tan(x^7)\right] =$
$= 5\left[x\sin 2x+\tan^4(x^7)\right]^4\left[2x\cos 2x+\sin 2x+28x^6\tan^3(x^7)\sec^2(x^7)\right]$.

42. $$\dfrac{dy}{dx} = 4\tan^3\left(2+\dfrac{(7-x)\sqrt{3x^2+5}}{x^3+\sin x}\right)\sec^2\left(2+\dfrac{(7-x)\sqrt{3x^2+5}}{x^3+\sin x}\right)$$
$$\times\left(-\dfrac{\sqrt{3x^2+5}}{x^3+\sin x}+3\dfrac{(7-x)x}{\sqrt{3x^2+5}\,(x^3+\sin x)}-\dfrac{(7-x)\sqrt{3x^2+5}\,(3x^2+\cos x)}{(x^3+\sin x)^2}\right)$$

43. $\dfrac{dy}{dx} = \cos 3x - 3x\sin 3x$; if $x=\pi$ then $\dfrac{dy}{dx} = -1$ and $y=-\pi$, so the equation of the tangent line is $y+\pi = -(x-\pi)$, or $y=-x$.

44. $\dfrac{dy}{dx} = 3x^2\cos(1+x^3)$; if $x=-3$ then $y=-\sin 26$, $\dfrac{dy}{dx} = 27\cos 26$, so the equation of the tangent line is $y+\sin 26 = 27(\cos 26)(x+3)$, or $y = 27(\cos 26)x + 81\cos 26 - \sin 26$.

45. $\dfrac{dy}{dx} = -3\sec^3(\pi/2-x)\tan(\pi/2-x)$; if $x=-\pi/2$ then $\dfrac{dy}{dx} = 0, y=-1$, so the equation of the tangent line is $y+1=0$, or $y=-1$

46. $\dfrac{dy}{dx} = 3\left(x - \dfrac{1}{x}\right)^2\left(1 + \dfrac{1}{x^2}\right)$; if $x = 2$ then $y = \dfrac{27}{8}$, $\dfrac{dy}{dx} = 3\dfrac{9}{4}\dfrac{5}{4} = \dfrac{135}{16}$, so the equation of the tangent line is $y - 27/8 = (135/16)(x - 2)$, or $y = \dfrac{135}{16}x - \dfrac{27}{2}$.

47. $\dfrac{dy}{dx} = \sec^2(4x^2)\dfrac{d}{dx}(4x^2) = 8x\sec^2(4x^2)$, $\left.\dfrac{dy}{dx}\right|_{x=\sqrt{\pi}} = 8\sqrt{\pi}\sec^2(4\pi) = 8\sqrt{\pi}$. When $x = \sqrt{\pi}$, $y = \tan(4\pi) = 0$, so the equation of the tangent line is $y = 8\sqrt{\pi}(x - \sqrt{\pi}) = 8\sqrt{\pi}x - 8\pi$.

48. $\dfrac{dy}{dx} = 12\cot^3 x\dfrac{d}{dx}\cot x = -12\cot^3 x\csc^2 x$, $\left.\dfrac{dy}{dx}\right|_{x=\pi/4} = -24$. When $x = \pi/4, y = 3$, so the equation of the tangent line is $y - 3 = -24(x - \pi/4)$, or $y = -24x + 3 + 6\pi$.

49. $\dfrac{dy}{dx} = 2x\sqrt{5 - x^2} + \dfrac{x^2}{2\sqrt{5 - x^2}}(-2x)$, $\left.\dfrac{dy}{dx}\right|_{x=1} = 4 - 1/2 = 7/2$. When $x = 1, y = 2$, so the equation of the tangent line is $y - 2 = (7/2)(x - 1)$, or $y = \dfrac{7}{2}x - \dfrac{3}{2}$.

50. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1 - x^2}} - \dfrac{x}{2}(1 - x^2)^{3/2}(-2x)$, $\left.\dfrac{dy}{dx}\right|_{x=0} = 1$. When $x = 0, y = 0$, so the equation of the tangent line is $y = x$.

51. $\dfrac{dy}{dx} = x(-\sin(5x))\dfrac{d}{dx}(5x) + \cos(5x) - 2\sin x\dfrac{d}{dx}(\sin x) = -5x\sin(5x) + \cos(5x) - 2\sin x\cos x =$
$= -5x\sin(5x) + \cos(5x) - \sin(2x)$,
$\dfrac{d^2y}{dx^2} = -5x\cos(5x)\dfrac{d}{dx}(5x) - 5\sin(5x) - \sin(5x)\dfrac{d}{dx}(5x) - \cos(2x)\dfrac{d}{dx}(2x) = -25x\cos(5x) - 10\sin(5x) - 2\cos(2x)$.

52. $\dfrac{dy}{dx} = \cos(3x^2)\dfrac{d}{dx}(3x^2) = 6x\cos(3x^2)$, $\dfrac{d^2y}{dx^2} = 6x(-\sin(3x^2))\dfrac{d}{dx}(3x^2) + 6\cos(3x^2) = -36x^2\sin(3x^2) + 6\cos(3x^2)$.

53. $\dfrac{dy}{dx} = \dfrac{(1 - x) + (1 + x)}{(1 - x)^2} = \dfrac{2}{(1 - x)^2} = 2(1 - x)^{-2}$ and $\dfrac{d^2y}{dx^2} = -2(2)(-1)(1 - x)^{-3} = 4(1 - x)^{-3}$.

54. $\dfrac{dy}{dx} = x\sec^2\left(\dfrac{1}{x}\right)\dfrac{d}{dx}\left(\dfrac{1}{x}\right) + \tan\left(\dfrac{1}{x}\right) = -\dfrac{1}{x}\sec^2\left(\dfrac{1}{x}\right) + \tan\left(\dfrac{1}{x}\right)$,
$\dfrac{d^2y}{dx^2} = -\dfrac{2}{x}\sec\left(\dfrac{1}{x}\right)\dfrac{d}{dx}\sec\left(\dfrac{1}{x}\right) + \dfrac{1}{x^2}\sec^2\left(\dfrac{1}{x}\right) + \sec^2\left(\dfrac{1}{x}\right)\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = \dfrac{2}{x^3}\sec^2\left(\dfrac{1}{x}\right)\tan\left(\dfrac{1}{x}\right)$.

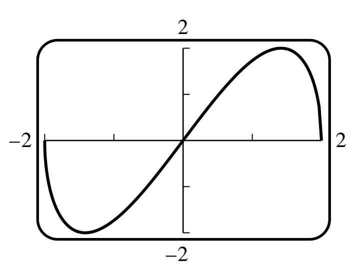
55. $y = \cot^3(\pi - \theta) = -\cot^3\theta$ so $dy/dx = 3\cot^2\theta\csc^2\theta$.

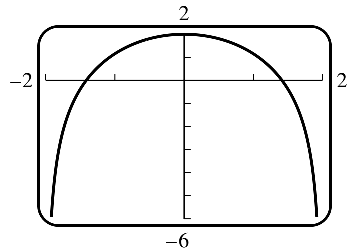
56. $6\left(\dfrac{au + b}{cu + d}\right)^5\dfrac{ad - bc}{(cu + d)^2}$.

57. $\dfrac{d}{d\omega}[a\cos^2\pi\omega + b\sin^2\pi\omega] = -2\pi a\cos\pi\omega\sin\pi\omega + 2\pi b\sin\pi\omega\cos\pi\omega = \pi(b - a)(2\sin\pi\omega\cos\pi\omega) = \pi(b - a)\sin 2\pi\omega$.

58. $2\csc^2(\pi/3 - y)\cot(\pi/3 - y)$.

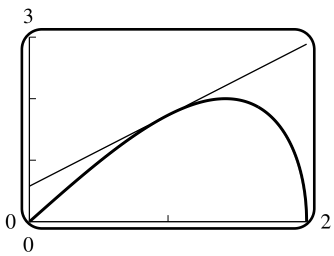
59. **(a)**



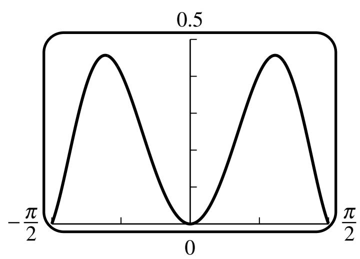


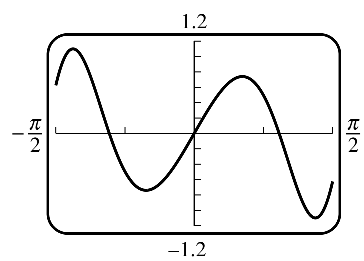
(c) $f'(x) = x\dfrac{-x}{\sqrt{4-x^2}} + \sqrt{4-x^2} = \dfrac{4-2x^2}{\sqrt{4-x^2}}$.

(d) $f(1) = \sqrt{3}$ and $f'(1) = \dfrac{2}{\sqrt{3}}$ so the tangent line has the equation $y - \sqrt{3} = \dfrac{2}{\sqrt{3}}(x-1)$.



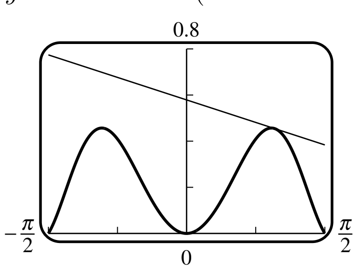
60. **(a)**





(c) $f'(x) = 2x\cos(x^2)\cos x - \sin x \sin(x^2)$.

(d) $f(1) = \sin 1 \cos 1$ and $f'(1) = 2\cos^2 1 - \sin^2 1$, so the tangent line has the equation $y - \sin 1 \cos 1 = (2\cos^2 1 - \sin^2 1)(x-1)$.



61. False. $\dfrac{d}{dx}[\sqrt{y}\,] = \dfrac{1}{2\sqrt{y}}\dfrac{dy}{dx} = \dfrac{f'(x)}{2\sqrt{f(x)}}$.

62. False. $dy/dx = f'(u)g'(x) = f'(g(x))\,g'(x)$.

63. False. $dy/dx = -\sin[g(x)]\,g'(x)$.

64. True. Let $u = 3x^3$ and $v = \sin u$, so $y = v^3$. Then $\dfrac{dy}{dx} = \dfrac{dy}{dv}\dfrac{dv}{du}\dfrac{du}{dx} = 3v^2 \cdot (\cos u) \cdot 9x^2 = 3\sin^2(3x^3) \cdot \cos(3x^3) \cdot 9x^2 = 27x^2 \sin^2(3x^3)\cos(3x^3)$.

65. **(a)** $dy/dt = -A\omega \sin \omega t, d^2y/dt^2 = -A\omega^2 \cos \omega t = -\omega^2 y$

(b) One complete oscillation occurs when ωt increases over an interval of length 2π, or if t increases over an interval of length $2\pi/\omega$.

(c) $f = 1/T$

(d) Amplitude $= 0.6$ cm, $T = 2\pi/15$ s/oscillation, $f = 15/(2\pi)$ oscillations/s.

66. $dy/dt = 3A\cos 3t, d^2y/dt^2 = -9A\sin 3t$, so $-9A\sin 3t + 2A\sin 3t = 4\sin 3t$, $-7A\sin 3t = 4\sin 3t, -7A = 4$, and $A = -4/7$

67. By the chain rule, $\dfrac{d}{dx}\left[\sqrt{x + f(x)}\right] = \dfrac{1 + f'(x)}{2\sqrt{x + f(x)}}$. From the graph, $f(x) = \dfrac{4}{3}x + 5$ for $x < 0$, so $f(-1) = \dfrac{11}{3}$, $f'(-1) = \dfrac{4}{3}$, and $\dfrac{d}{dx}\left[\sqrt{x + f(x)}\right]\Big|_{x=-1} = \dfrac{7/3}{2\sqrt{8/3}} = \dfrac{7\sqrt{6}}{24}$.

68. $2\sin(\pi/6) = 1$, so we can assume $f(x) = -\frac{5}{2}x + 5$. Thus for sufficiently small values of $|x - \pi/6|$ we have $\dfrac{d}{dx}[f(2\sin x)]\Big|_{x=\pi/6} = f'(2\sin x)\dfrac{d}{dx}2\sin x\Big|_{x=\pi/6} = -\dfrac{5}{2}2\cos x\Big|_{x=\pi/6} = -\dfrac{5}{2}2\dfrac{\sqrt{3}}{2} = -\dfrac{5}{2}\sqrt{3}$.

69. **(a)** $p \approx 10$ lb/in^2, $dp/dh \approx -2$ lb/in^2/mi. **(b)** $\dfrac{dp}{dt} = \dfrac{dp}{dh}\dfrac{dh}{dt} \approx (-2)(0.3) = -0.6$ lb/in^2/s.

70. **(a)** $F = \dfrac{45}{\cos\theta + 0.3\sin\theta}, \dfrac{dF}{d\theta} = -\dfrac{45(-\sin\theta + 0.3\cos\theta)}{(\cos\theta + 0.3\sin\theta)^2}$; if $\theta = 30^\circ$, then $dF/d\theta \approx 10.5$ lb/rad ≈ 0.18 lb/deg.

(b) $\dfrac{dF}{dt} = \dfrac{dF}{d\theta}\dfrac{d\theta}{dt} \approx (0.18)(-0.5) = -0.09$ lb/s.

71. With $u = \sin x$, $\dfrac{d}{dx}(|\sin x|) = \dfrac{d}{dx}(|u|) = \dfrac{d}{du}(|u|)\dfrac{du}{dx} = \dfrac{d}{du}(|u|)\cos x = \begin{cases} \cos x, & u > 0 \\ -\cos x, & u < 0 \end{cases} = \begin{cases} \cos x, & \sin x > 0 \\ -\cos x, & \sin x < 0 \end{cases}$ $= \begin{cases} \cos x, & 0 < x < \pi \\ -\cos x, & -\pi < x < 0 \end{cases}$

72. $\dfrac{d}{dx}(\cos x) = \dfrac{d}{dx}[\sin(\pi/2 - x)] = -\cos(\pi/2 - x) = -\sin x$.

73. **(a)** For $x \neq 0, |f(x)| \leq |x|$, and $\lim\limits_{x\to 0}|x| = 0$, so by the Squeezing Theorem, $\lim\limits_{x\to 0} f(x) = 0$.

(b) If $f'(0)$ were to exist, then the limit (as x approaches 0) $\dfrac{f(x) - f(0)}{x - 0} = \sin(1/x)$ would have to exist, but it doesn't.

(c) For $x \neq 0$, $f'(x) = x\left(\cos\dfrac{1}{x}\right)\left(-\dfrac{1}{x^2}\right) + \sin\dfrac{1}{x} = -\dfrac{1}{x}\cos\dfrac{1}{x} + \sin\dfrac{1}{x}$.

(d) If $x = \dfrac{1}{2\pi n}$ for an integer $n \neq 0$, then $f'(x) = -2\pi n\cos(2\pi n) + \sin(2\pi n) = -2\pi n$. This approaches $+\infty$ as $n \to -\infty$, so there are points x arbitrarily close to 0 where $f'(x)$ becomes arbitrarily large. Hence $\lim\limits_{x\to 0} f'(x)$ does not exist.

74. **(a)** $-x^2 \le x^2 \sin(1/x) \le x^2$, so by the Squeezing Theorem $\lim_{x\to 0} f(x) = 0$.

(b) $f'(0) = \lim_{x\to 0} \dfrac{f(x) - f(0)}{x - 0} = \lim_{x\to 0} x \sin(1/x) = 0$ by Exercise 73, part **(a)**.

(c) For $x \neq 0$, $f'(x) = 2x\sin(1/x) + x^2 \cos(1/x)(-1/x^2) = 2x\sin(1/x) - \cos(1/x)$.

(d) If $f'(x)$ were continuous at $x = 0$ then so would $\cos(1/x) = 2x\sin(1/x) - f'(x)$ be, since $2x\sin(1/x)$ is continuous there. But $\cos(1/x)$ oscillates at $x = 0$.

75. **(a)** $g'(x) = 3[f(x)]^2 f'(x)$, $g'(2) = 3[f(2)]^2 f'(2) = 3(1)^2(7) = 21$.

(b) $h'(x) = f'(x^3)(3x^2)$, $h'(2) = f'(8)(12) = (-3)(12) = -36$.

76. $F'(x) = f'(g(x))g'(x) = \sqrt{3(x^2-1)+4} \cdot 2x = 2x\sqrt{3x^2+1}$.

77. $F'(x) = f'(g(x))g'(x) = f'(\sqrt{3x-1})\dfrac{3}{2\sqrt{3x-1}} = \dfrac{\sqrt{3x-1}}{(3x-1)+1}\,\dfrac{3}{2\sqrt{3x-1}} = \dfrac{1}{2x}$.

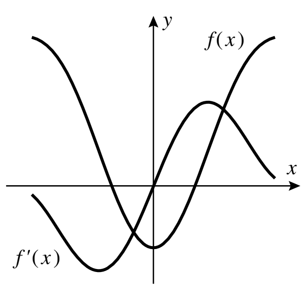
78. $\dfrac{d}{dx}[f(x^2)] = f'(x^2)(2x)$, thus $f'(x^2)(2x) = x^2$ so $f'(x^2) = x/2$ if $x \neq 0$.

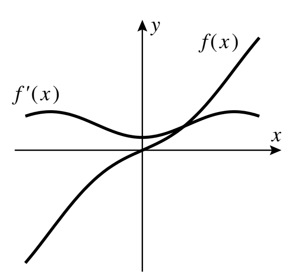
79. $\dfrac{d}{dx}[f(3x)] = f'(3x)\dfrac{d}{dx}(3x) = 3f'(3x) = 6x$, so $f'(3x) = 2x$. Let $u = 3x$ to get $f'(u) = \dfrac{2}{3}u$; $\dfrac{d}{dx}[f(x)] = f'(x) = \dfrac{2}{3}x$.

80. **(a)** If $f(-x) = f(x)$, then $\dfrac{d}{dx}[f(-x)] = \dfrac{d}{dx}[f(x)]$, $f'(-x)(-1) = f'(x)$, $f'(-x) = -f'(x)$ so f' is odd.

(b) If $f(-x) = -f(x)$, then $\dfrac{d}{dx}[f(-x)] = -\dfrac{d}{dx}[f(x)]$, $f'(-x)(-1) = -f'(x)$, $f'(-x) = f'(x)$ so f' is even.

81. For an even function, the graph is symmetric about the y-axis; the slope of the tangent line at $(a, f(a))$ is the negative of the slope of the tangent line at $(-a, f(-a))$. For an odd function, the graph is symmetric about the origin; the slope of the tangent line at $(a, f(a))$ is the same as the slope of the tangent line at $(-a, f(-a))$.





82. $\dfrac{dy}{dx} = \dfrac{dy}{du}\,\dfrac{du}{dv}\,\dfrac{dv}{dw}\,\dfrac{dw}{dx}$.

83. $\dfrac{d}{dx}[f(g(h(x)))] = \dfrac{d}{dx}[f(g(u))]$, $u = h(x)$, $\dfrac{d}{du}[f(g(u))]\dfrac{du}{dx} = f'(g(u))g'(u)\dfrac{du}{dx} = f'(g(h(x)))g'(h(x))h'(x)$.

84. $g'(x) = f'\left(\dfrac{\pi}{2} - x\right) \cdot \dfrac{d}{dx}\left(\dfrac{\pi}{2} - x\right) = -f'\left(\dfrac{\pi}{2} - x\right)$, so g' is the negative of the co-function of f'.

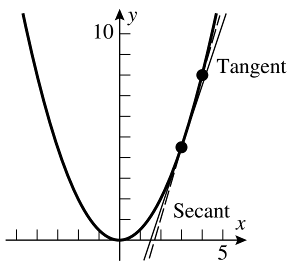
The derivatives of $\sin x$, $\tan x$, and $\sec x$ are $\cos x$, $\sec^2 x$, and $\sec x \tan x$, respectively. The negatives of the co-functions of these are $-\sin x$, $-\csc^2 x$, and $-\csc x \cot x$, which are the derivatives of $\cos x$, $\cot x$, and $\csc x$, respectively.

Chapter 2 Review Exercises

2. **(a)** $m_{\text{sec}} = \dfrac{f(4) - f(3)}{4 - 3} = \dfrac{(4)^2/2 - (3)^2/2}{1} = \dfrac{7}{2}$

(b) $m_{\tan} = \lim\limits_{w \to 3} \dfrac{f(w) - f(3)}{w - 3} = \lim\limits_{w \to 3} \dfrac{w^2/2 - 9/2}{w - 3} = \lim\limits_{w \to 3} \dfrac{w^2 - 9}{2(w - 3)} = \lim\limits_{w \to 3} \dfrac{(w + 3)(w - 3)}{2(w - 3)} = \lim\limits_{w \to 3} \dfrac{w + 3}{2} = 3.$

(c) $m_{\tan} = \lim\limits_{w \to x} \dfrac{f(w) - f(x)}{w - x} = \lim\limits_{w \to x} \dfrac{w^2/2 - x^2/2}{w - x} = \lim\limits_{w \to x} \dfrac{w^2 - x^2}{2(w - x)} = \lim\limits_{w \to x} \dfrac{w + x}{2} = x.$

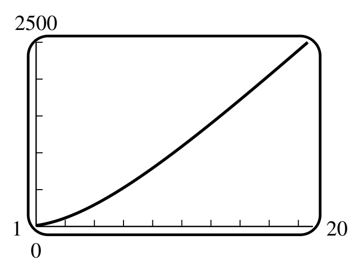
(d) 

3. **(a)** $m_{\tan} = \lim\limits_{w \to x} \dfrac{f(w) - f(x)}{w - x} = \lim\limits_{w \to x} \dfrac{(w^2 + 1) - (x^2 + 1)}{w - x} = \lim\limits_{w \to x} \dfrac{w^2 - x^2}{w - x} = \lim\limits_{w \to x} (w + x) = 2x.$

(b) $m_{\tan} = 2(2) = 4.$

4. To average 60 mi/h one would have to complete the trip in two hours. At 50 mi/h, 100 miles are completed after two hours. Thus time is up, and the speed for the remaining 20 miles would have to be infinite.

5. $v_{\text{inst}} = \lim\limits_{h \to 0} \dfrac{3(h + 1)^{2.5} + 580h - 3}{10h} = 58 + \dfrac{1}{10} \dfrac{d}{dx} 3x^{2.5} \Big|_{x=1} = 58 + \dfrac{1}{10}(2.5)(3)(1)^{1.5} = 58.75$ ft/s.

6. 164 ft/s 

7. **(a)** $v_{\text{ave}} = \dfrac{[3(3)^2 + 3] - [3(1)^2 + 1]}{3 - 1} = 13$ mi/h.

(b) $v_{\text{inst}} = \lim\limits_{t_1 \to 1} \dfrac{(3t_1^2 + t_1) - 4}{t_1 - 1} = \lim\limits_{t_1 \to 1} \dfrac{(3t_1 + 4)(t_1 - 1)}{t_1 - 1} = \lim\limits_{t_1 \to 1} (3t_1 + 4) = 7$ mi/h.

9. **(a)** $\dfrac{dy}{dx} = \lim\limits_{h \to 0} \dfrac{\sqrt{9 - 4(x + h)} - \sqrt{9 - 4x}}{h} = \lim\limits_{h \to 0} \dfrac{9 - 4(x + h) - (9 - 4x)}{h(\sqrt{9 - 4(x + h)} + \sqrt{9 - 4x})} =$

$= \lim\limits_{h \to 0} \dfrac{-4h}{h(\sqrt{9 - 4(x + h)} + \sqrt{9 - 4x})} = \dfrac{-4}{2\sqrt{9 - 4x}} = \dfrac{-2}{\sqrt{9 - 4x}}.$

(b) $\dfrac{dy}{dx} = \lim\limits_{h \to 0} \dfrac{\dfrac{x + h}{x + h + 1} - \dfrac{x}{x + 1}}{h} = \lim\limits_{h \to 0} \dfrac{(x + h)(x + 1) - x(x + h + 1)}{h(x + h + 1)(x + 1)} = \lim\limits_{h \to 0} \dfrac{h}{h(x + h + 1)(x + 1)} = \dfrac{1}{(x + 1)^2}.$

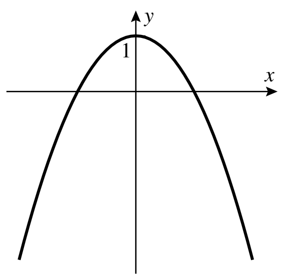
10. $f(x)$ is continuous and differentiable at any $x \neq 1$, so we consider $x = 1$.

(a) $\lim_{x\to 1^-}(x^2-1) = \lim_{x\to 1^+} k(x-1) = 0 = f(1)$, so any value of k gives continuity at $x = 1$.

(b) $\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^-} 2x = 2$, and $\lim_{x\to 1^+} f'(x) = \lim_{x\to 1^+} k = k$, so only if $k = 2$ is $f(x)$ differentiable at $x = 1$.

11. **(a)** $x = -2, -1, 1, 3$ **(b)** $(-\infty, -2)$, $(-1, 1)$, $(3, +\infty)$ **(c)** $(-2, -1)$, $(1, 3)$

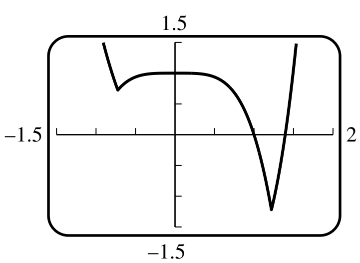
(d) $g''(x) = f''(x)\sin x + 2f'(x)\cos x - f(x)\sin x$; $g''(0) = 2f'(0)\cos 0 = 2(2)(1) = 4$

12. 

13. **(a)** The slope of the tangent line $\approx \dfrac{10-2.2}{2050-1950} = 0.078$ billion, so in 2000 the world population was increasing at the rate of about 78 million per year.

(b) $\dfrac{dN/dt}{N} \approx \dfrac{0.078}{6} = 0.013 = 1.3$ %/year

14. When $x^4 - x - 1 > 0$, $f(x) = x^4 - 2x - 1$; when $x^4 - x - 1 < 0$, $f(x) = -x^4 + 1$, and f is differentiable in both cases. The roots of $x^4 - x - 1 = 0$ are $x_1 \approx -0.724492$, $x_2 \approx 1.220744$. So $x^4 - x - 1 > 0$ on $(-\infty, x_1)$ and $(x_2, +\infty)$, and $x^4 - x - 1 < 0$ on (x_1, x_2). Then $\lim_{x\to x_1^-} f'(x) = \lim_{x\to x_1^-}(4x^3 - 2) = 4x_1^3 - 2$ and $\lim_{x\to x_1^+} f'(x) = \lim_{x\to x_1^+} -4x^3 = -4x_1^3$ which is not equal to $4x_1^3 - 2$, so f is not differentiable at $x = x_1$; similarly f is not differentiable at $x = x_2$.



15. **(a)** $f'(x) = 2x\sin x + x^2\cos x$ **(c)** $f''(x) = 4x\cos x + (2 - x^2)\sin x$

16. **(a)** $f'(x) = \dfrac{1 - 2\sqrt{x}\sin 2x}{2\sqrt{x}}$ **(c)** $f''(x) = \dfrac{-1 - 8x^{3/2}\cos 2x}{4x^{3/2}}$

17. **(a)** $f'(x) = \dfrac{6x^2 + 8x - 17}{(3x+2)^2}$ **(c)** $f''(x) = \dfrac{118}{(3x+2)^3}$

18. **(a)** $f'(x) = \dfrac{(1+x^2)\sec^2 x - 2x\tan x}{(1+x^2)^2}$

(c) $f''(x) = \dfrac{(2 + 4x^2 + 2x^4)\sec^2 x\tan x - (4x + 4x^3)\sec^2 x + (-2 + 6x^2)\tan x}{(1+x^2)^3}$

19. **(a)** $\dfrac{dW}{dt} = 200(t-15)$; at $t=5$, $\dfrac{dW}{dt} = -2000$; the water is running out at the rate of 2000 gal/min.

(b) $\dfrac{W(5)-W(0)}{5-0} = \dfrac{10000-22500}{5} = -2500$; the average rate of flow out is 2500 gal/min.

20. **(a)** $\dfrac{4^3-2^3}{4-2} = \dfrac{56}{2} = 28$ **(b)** $(dV/d\ell)|_{\ell=5} = 3\ell^2\big|_{\ell=5} = 3(5)^2 = 75$

21. **(a)** $f'(x) = 2x, f'(1.8) = 3.6$ **(b)** $f'(x) = (x^2-4x)/(x-2)^2, f'(3.5) = -7/9 \approx -0.777778$

22. **(a)** $f'(x) = 3x^2-2x, f'(2.3) = 11.27$ **(b)** $f'(x) = (1-x^2)/(x^2+1)^2, f'(-0.5) = 0.48$

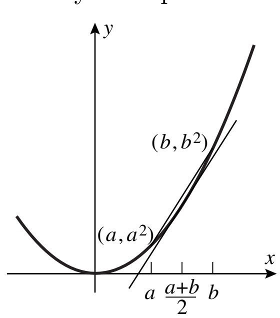
23. f is continuous at $x=1$ because it is differentiable there, thus $\lim_{h\to 0} f(1+h) = f(1)$ and so $f(1) = 0$ because $\lim_{h\to 0}\dfrac{f(1+h)}{h}$ exists; $f'(1) = \lim_{h\to 0}\dfrac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\dfrac{f(1+h)}{h} = 5$.

24. Multiply the given equation by $\lim_{x\to 2}(x-2) = 0$ to get $0 = \lim_{x\to 2}(x^3 f(x) - 24)$. Since f is continuous at $x=2$, this equals $2^3 f(2) - 24$, so $f(2) = 3$. Now let $g(x) = x^3 f(x)$. Then $g'(2) = \lim_{x\to 2}\dfrac{g(x)-g(2)}{x-2} = \lim_{x\to 2}\dfrac{x^3 f(x) - 2^3 f(2)}{x-2} = \lim_{x\to 2}\dfrac{x^3 f(x) - 24}{x-2} = 28$. But $g'(x) = x^3 f'(x) + 3x^2 f(x)$, so $28 = g'(2) = 2^3 f'(2) + 3\cdot 2^2 f(2) = 8f'(2) + 36$, and $f'(2) = -1$.

25. The equation of such a line has the form $y = mx$. The points (x_0, y_0) which lie on both the line and the parabola and for which the slopes of both curves are equal satisfy $y_0 = mx_0 = x_0^3 - 9x_0^2 - 16x_0$, so that $m = x_0^2 - 9x_0 - 16$. By differentiating, the slope is also given by $m = 3x_0^2 - 18x_0 - 16$. Equating, we have $x_0^2 - 9x_0 - 16 = 3x_0^2 - 18x_0 - 16$, or $2x_0^2 - 9x_0 = 0$. The root $x_0 = 0$ corresponds to $m = -16, y_0 = 0$ and the root $x_0 = 9/2$ corresponds to $m = -145/4, y_0 = -1305/8$. So the line $y = -16x$ is tangent to the curve at the point $(0,0)$, and the line $y = -145x/4$ is tangent to the curve at the point $(9/2, -1305/8)$.

26. The slope of the line $x + 4y = 10$ is $m_1 = -1/4$, so we set the negative reciprocal $4 = m_2 = \dfrac{d}{dx}(2x^3 - x^2) = 6x^2 - 2x$ and obtain $6x^2 - 2x - 4 = 0$ with roots $x = \dfrac{1 \pm \sqrt{1+24}}{6} = 1, -2/3$.

27. The slope of the tangent line is the derivative $y' = 2x\Big|_{x=\frac{1}{2}(a+b)} = a+b$. The slope of the secant is $\dfrac{a^2-b^2}{a-b} = a+b$, so they are equal.



28. **(a)** $f'(1)g(1) + f(1)g'(1) = 3(-2) + 1(-1) = -7$ **(b)** $\dfrac{g(1)f'(1) - f(1)g'(1)}{g(1)^2} = \dfrac{-2(3) - 1(-1)}{(-2)^2} = -\dfrac{5}{4}$

(c) $\dfrac{1}{2\sqrt{f(1)}}f'(1) = \dfrac{1}{2\sqrt{1}}(3) = \dfrac{3}{2}$ **(d)** 0 (because $f(1)g'(1)$ is constant)

29. **(a)** $8x^7 - \dfrac{3}{2\sqrt{x}} - 15x^{-4}$ **(b)** $2 \cdot 101(2x+1)^{100}(5x^2-7) + 10x(2x+1)^{101} = (2x+1)^{100}(1030x^2+10x-1414)$

30. **(a)** $\cos x - 6\cos^2 x \sin x$ **(b)** $(1+\sec x)(2x - \sec^2 x) + (x^2 - \tan x)\sec x \tan x$

31. **(a)** $2(x-1)\sqrt{3x+1} + \dfrac{3}{2\sqrt{3x+1}}(x-1)^2 = \dfrac{(x-1)(15x+1)}{2\sqrt{3x+1}}$

(b) $3\left(\dfrac{3x+1}{x^2}\right)^2 \dfrac{x^2(3)-(3x+1)(2x)}{x^4} = -\dfrac{3(3x+1)^2(3x+2)}{x^7}$

32. **(a)** $-\csc^2\left(\dfrac{\csc 2x}{x^3+5}\right)\dfrac{-2(x^3+5)\csc 2x \cot 2x - 3x^2 \csc 2x}{(x^3+5)^2}$ **(b)** $-\dfrac{2+3\sin^2 x\cos x}{(2x+\sin^3 x)^2}$

33. Set $f'(x) = 0$: $f'(x) = 6(2)(2x+7)^5(x-2)^5 + 5(2x+7)^6(x-2)^4 = 0$, so $2x+7=0$ or $x-2=0$ or, factoring out $(2x+7)^5(x-2)^4$, $12(x-2)+5(2x+7)=0$. This reduces to $x=-7/2$, $x=2$, or $22x+11=0$, so the tangent line is horizontal at $x = -7/2, 2, -1/2$.

34. Set $f'(x) = 0$: $f'(x) = \dfrac{4(x^2+2x)(x-3)^3 - (2x+2)(x-3)^4}{(x^2+2x)^2}$, and a fraction can equal zero only if its numerator equals zero. So either $x-3=0$ or, after factoring out $(x-3)^3$, $4(x^2+2x)-(2x+2)(x-3)=0$, $2x^2+12x+6=0$, whose roots are (by the quadratic formula) $x = \dfrac{-6\pm\sqrt{36-4\cdot 3}}{2} = -3\pm\sqrt{6}$. So the tangent line is horizontal at $x = 3, -3\pm\sqrt{6}$.

35. Suppose the line is tangent to $y = x^2+1$ at (x_0, y_0) and tangent to $y = -x^2-1$ at (x_1, y_1). Since it's tangent to $y = x^2+1$, its slope is $2x_0$; since it's tangent to $y=-x^2-1$, its slope is $-2x_1$. Hence $x_1 = -x_0$ and $y_1 = -y_0$. Since the line passes through both points, its slope is $\dfrac{y_1-y_0}{x_1-x_0} = \dfrac{-2y_0}{-2x_0} = \dfrac{y_0}{x_0} = \dfrac{x_0^2+1}{x_0}$. Thus $2x_0 = \dfrac{x_0^2+1}{x_0}$, so $2x_0^2 = x_0^2+1$, $x_0^2=1$, and $x_0 = \pm 1$. So there are two lines which are tangent to both graphs, namely $y=2x$ and $y=-2x$.

36. **(a)** Suppose $y = mx+b$ is tangent to $y = x^n+n-1$ at (x_0,y_0) and to $y = -x^n-n+1$ at (x_1,y_1). Then $m = nx_0^{n-1} = -nx_1^{n-1}$; since n is even this implies that $x_1 = -x_0$. Again since n is even, $y_1 = -x_1^n - n + 1 = -x_0^n - n + 1 = -(x_0^n + n - 1) = -y_0$. Thus the points (x_0,y_0) and (x_1,y_1) are symmetric with respect to the origin and both lie on the tangent line and thus $b=0$. The slope m is given by $m = nx_0^{n-1}$ and by $m = y_0/x_0 = (x_0^n+n-1)/x_0$, hence $nx_0^n = x_0^n+n-1$, $(n-1)x_0^n = n-1$, $x_0^n = 1$. Since n is even, $x_0 = \pm 1$. One easily checks that $y = nx$ is tangent to $y = x^n+n-1$ at $(1,n)$ and to $y=-x^n-n+1$ at $(-1,-n)$, while $y=-nx$ is tangent to $y = x^n+n-1$ at $(-1,n)$ and to $y = -x^n-n+1$ at $(1,-n)$.

(b) Suppose there is such a common tangent line with slope m. The function $y = x^n+n-1$ is always increasing, so $m \ge 0$. Moreover the function $y = -x^n-n+1$ is always decreasing, so $m \le 0$. Thus the tangent line has slope 0, which only occurs on the curves for $x=0$. This would require the common tangent line to pass through $(0, n-1)$ and $(0, -n+1)$ and do so with slope $m=0$, which is impossible.

37. The line $y - x = 2$ has slope $m_1 = 1$ so we set $m_2 = \dfrac{d}{dx}(3x - \tan x) = 3 - \sec^2 x = 1$, or $\sec^2 x = 2$, $\sec x = \pm\sqrt{2}$ so $x = n\pi \pm \pi/4$ where $n = 0, \pm 1, \pm 2, \ldots$.

38. Solve $3x^2 - \cos x = 0$ to get $x = \pm 0.535428$.

39. $3 = f(\pi/4) = (M+N)\sqrt{2}/2$ and $1 = f'(\pi/4) = (M-N)\sqrt{2}/2$. Add these two equations to get $4 = \sqrt{2}M$, $M = 2^{3/2}$. Subtract to obtain $2 = \sqrt{2}N$, $N = \sqrt{2}$. Thus $f(x) = 2\sqrt{2}\sin x + \sqrt{2}\cos x$. $f'\left(\dfrac{3\pi}{4}\right) = -3$, so the tangent line is $y - 1 = -3\left(x - \dfrac{3\pi}{4}\right)$.

40. $f(x) = M\tan x + N\sec x, f'(x) = M\sec^2 x + N\sec x\tan x$. At $x = \pi/4$, $2M + \sqrt{2}N, 0 = 2M + \sqrt{2}N$. Add to get $M = -2$, subtract to get $N = \sqrt{2} + M/\sqrt{2} = 2\sqrt{2}$, $f(x) = -2\tan x + 2\sqrt{2}\sec x$. $f'(0) = -2$, so the tangent line is $y - 2\sqrt{2} = -2x$.

41. $f'(x) = 2xf(x), f(2) = 5$

(a) $g(x) = f(\sec x), g'(x) = f'(\sec x)\sec x\tan x = 2\cdot 2f(2)\cdot 2\cdot\sqrt{3} = 40\sqrt{3}$.

(b) $h'(x) = 4\left[\dfrac{f(x)}{x-1}\right]^3 \dfrac{(x-1)f'(x) - f(x)}{(x-1)^2}, h'(2) = 4\dfrac{5^3}{1}\dfrac{f'(2) - f(2)}{1} = 4\cdot 5^3\dfrac{2\cdot 2f(2) - f(2)}{1} = 4\cdot 5^3\cdot 3\cdot 5 = 7500$

Chapter 2 Making Connections

1. (a) By property (ii), $f(0) = f(0+0) = f(0)f(0)$, so $f(0) = 0$ or 1. By property (iii), $f(0) \neq 0$, so $f(0) = 1$.

(b) By property (ii), $f(x) = f\left(\dfrac{x}{2} + \dfrac{x}{2}\right) = f\left(\dfrac{x}{2}\right)^2 \geq 0$. If $f(x) = 0$, then $1 = f(0) = f(x + (-x)) = f(x)f(-x) = 0\cdot f(-x) = 0$, a contradiction. Hence $f(x) > 0$.

(c) $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0}\dfrac{f(x)f(h) - f(x)}{h} = \lim\limits_{h\to 0} f(x)\dfrac{f(h) - 1}{h} = f(x)\lim\limits_{h\to 0}\dfrac{f(h) - f(0)}{h} = f(x)f'(0) = f(x)$

2. (a) By the chain rule and Exercise 1(c), $y' = f'(2x)\cdot\dfrac{d}{dx}(2x) = f(2x)\cdot 2 = 2y$.

(b) By the chain rule and Exercise 1(c), $y' = f'(kx)\cdot\dfrac{d}{dx}(kx) = kf'(kx) = kf(kx)$.

(c) By the product rule and Exercise 1(c), $y' = f(x)g'(x) + g(x)f'(x) = f(x)g(x) + g(x)f(x) = 2f(x)g(x) = 2y$, so $k = 2$.

(d) By the quotient rule and Exercise 1(c), $h'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2} = \dfrac{g(x)f(x) - f(x)g(x)}{g(x)^2} = 0$. As we will see in Theorem 4.1.2(c), this implies that $h(x)$ is a constant. Since $h(0) = f(0)/g(0) = 1/1 = 1$ by Exercise 1(a), $h(x) = 1$ for all x, so $f(x) = g(x)$.

3. (a) For brevity, we omit the "(x)" throughout.

$(f\cdot g\cdot h)' = \dfrac{d}{dx}[(f\cdot g)\cdot h] = (f\cdot g)\cdot\dfrac{dh}{dx} + h\cdot\dfrac{d}{dx}(f\cdot g) = f\cdot g\cdot h' + h\cdot\left(f\cdot\dfrac{dg}{dx} + g\cdot\dfrac{df}{dx}\right)$
$= f'\cdot g\cdot h + f\cdot g'\cdot h + f\cdot g\cdot h'$

(b) $(f\cdot g\cdot h\cdot k)' = \dfrac{d}{dx}[(f\cdot g\cdot h)\cdot k] = (f\cdot g\cdot h)\cdot\dfrac{dk}{dx} + k\cdot\dfrac{d}{dx}(f\cdot g\cdot h)$
$= f\cdot g\cdot h\cdot k' + k\cdot(f'\cdot g\cdot h + f\cdot g'\cdot h + f\cdot g\cdot h') = f'\cdot g\cdot h\cdot k + f\cdot g'\cdot h\cdot k + f\cdot g\cdot h'\cdot k + f\cdot g\cdot h\cdot k'$

(c) Theorem: If $n \geq 1$ and $f_1, \cdots, f_n$ are differentiable functions of x, then

$(f_1\cdot f_2\cdot\cdots\cdot f_n)' = \sum\limits_{i=1}^{n} f_1\cdot\cdots\cdot f_{i-1}\cdot f_i'\cdot f_{i+1}\cdot\cdots\cdot f_n$.

Proof: For $n = 1$ the statement is obviously true: $f_1' = f_1'$. If the statement is true for $n-1$, then

$(f_1\cdot f_2\cdot\cdots\cdot f_n)' = \dfrac{d}{dx}[(f_1\cdot f_2\cdot\cdots\cdot f_{n-1})\cdot f_n] = (f_1\cdot f_2\cdot\cdots\cdot f_{n-1})\cdot f_n' + f_n\cdot(f_1\cdot f_2\cdot\cdots\cdot f_{n-1})'$

$= f_1\cdot f_2\cdot\cdots\cdot f_{n-1}\cdot f_n' + f_n\cdot\sum\limits_{i=1}^{n-1} f_1\cdot\cdots\cdot f_{i-1}\cdot f_i'\cdot f_{i+1}\cdot\cdots\cdot f_{n-1} = \sum\limits_{i=1}^{n} f_1\cdot\cdots\cdot f_{i-1}\cdot f_i'\cdot f_{i+1}\cdot\cdots\cdot f_n$

so the statement is true for n. By induction, it's true for all n.

4. (a) $[(f/g)/h]' = \dfrac{h\cdot(f/g)' - (f/g)\cdot h'}{h^2} = \dfrac{h\cdot\frac{g\cdot f' - f\cdot g'}{g^2} - \frac{f\cdot h'}{g}}{h^2} = \dfrac{f'\cdot g\cdot h - f\cdot g'\cdot h - f\cdot g\cdot h'}{g^2h^2}$

(b) $[(f/g)/h]' = [f/(g\cdot h)]' = \dfrac{(g\cdot h)\cdot f' - f\cdot(g\cdot h)'}{(g\cdot h)^2} = \dfrac{f'\cdot g\cdot h - f\cdot(g\cdot h' + h\cdot g')}{g^2h^2} =$

$= \dfrac{f'\cdot g\cdot h - f\cdot g'\cdot h - f\cdot g\cdot h'}{g^2h^2}$

(c) $[f/(g/h)]' = \dfrac{(g/h)\cdot f' - f\cdot(g/h)'}{(g/h)^2} = \dfrac{\frac{f'\cdot g}{h} - f\cdot\frac{h\cdot g' - g\cdot h'}{h^2}}{(g/h)^2} = \dfrac{f'\cdot g\cdot h - f\cdot g'\cdot h + f\cdot g\cdot h'}{g^2}$

(d) $[f/(g/h)]' = [(f\cdot h)/g]' = \dfrac{g\cdot(f\cdot h)' - (f\cdot h)\cdot g'}{g^2} = \dfrac{g\cdot(f\cdot h' + h\cdot f') - f\cdot g'\cdot h}{g^2} =$

$= \dfrac{f'\cdot g\cdot h - f\cdot g'\cdot h + f\cdot g\cdot h'}{g^2}$

5. (a) By the chain rule, $\dfrac{d}{dx}\left([g(x)]^{-1}\right) = -[g(x)]^{-2}g'(x) = -\dfrac{g'(x)}{[g(x)]^2}$. By the product rule,

$h'(x) = f(x).\dfrac{d}{dx}\left([g(x)]^{-1}\right) + [g(x)]^{-1}.\dfrac{d}{dx}[f(x)] = -\dfrac{f(x)g'(x)}{[g(x)]^2} + \dfrac{f'(x)}{g(x)} = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$.

(b) By the product rule, $f'(x) = \dfrac{d}{dx}[h(x)g(x)] = h(x)g'(x) + g(x)h'(x)$. So

$h'(x) = \dfrac{1}{g(x)}[f'(x) - h(x)g'(x)] = \dfrac{1}{g(x)}\left[f'(x) - \dfrac{f(x)}{g(x)}g'(x)\right] = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$.