UNIVERSIDADE FEDERAL DO PARANÁ - UFPR
CENTRO POLITÉCNICO
DEPARTAMENTO DE MATEMÁTICA

Disciplina: Cálculo 2 **Código:** CM312 **Semestre:** Semestre 2024/2

Lista 1

1. Calcule as seguintes integrais.

(a) $\int \text{sen}(6x)\cos(x)dx$

(b) $\int_0^{\pi/3} \text{sen}(3x)\cos(2x)dx$

(c) $\int \text{sen}(nx)\text{sen}(mx)dx$

(d) $\int \cos(x)\cos(5x)dx$

(e) $\int_{\pi/4}^{\pi} \cos(x)\cos(7x)dx$

2. Calcule.

(a) $\int \text{sen}^3(x)dx$ (b) $\int \cos^2(4x)dx$ (c) $\int \text{sen}(x)\cos^4(5x)dx$

(d) $\int \text{tg}(x)\text{sec}4(x)dx$ (e) $\int_0^{\pi/4} \text{sen}^2(2x)\cos^2(2x)dx$ (f) $\int \sec^3\left(\frac{x}{2}\right)dx$

(g) $\int_0^{\pi/2} \text{sen}^7(x)\cos^5(x)dx$ (h) $\int_0^{\pi/4} \sec^4(x)\text{tg}^4(x)dx$ (i) $\int \text{tg}^5(x)\sec^3(x)dx$

3. Calcule as integrais usando substituiçao trigonométrica. Esboce o triangulo tetangulo associado.

(a) $\int \frac{1}{x^2\sqrt{x^2-9}}dx$ (b) $\int x^3\sqrt{9-x^2}dx$

(c) $\int \frac{x^3}{\sqrt{x^2+9}}dx$ (d) $\int_0^{2/3} x^3\sqrt{4-9x^2}dx$

(f) $\int \frac{dx}{(5-4x-x^2)^{5/2}}$ (g) $\int_{\frac{\sqrt{2}}{3}}^{2/3} \frac{dx}{x^5\sqrt{9x^2-1}}$

(h) $\int \frac{x}{\sqrt{x^2-3}}dx$ (i) $\int \frac{x^2}{(3+4x-4x^2)^{3/2}}dx$

4. Calcule as seguintes integrais.

(a) $\int \sqrt{1+x^2}dx$ (b) $\int \sqrt{1-4x^2}dx$

(c) $\int \sqrt{3+4x^2}dx$ (d) $\int \sqrt{x^2+2x+2}dx$

5. Calcule as seguintes integrais.

(a) $\int \frac{1}{(x+1)(x-1)}dx$ (b) $\int_7^9 \frac{x-1}{x(x-2)}dx$

(c) $\int \frac{x+1}{x^2-4}dx$ (d) $\int \frac{x-3}{x^2+3x+2}dx$

6. Calcule as seguintes integrais por fraçoes parciais.

(a) $\displaystyle\int \frac{x}{x^2-5x+6}dx$ (b) $\displaystyle\int_3^4 \frac{x+3}{(x-1)^2}dx$ (c) $\displaystyle\int \frac{x^3+x+1}{x^2-2x+1}dx$

(d) $\displaystyle\int \frac{x+1}{x(x-2)(x+3)}dx$ (e) $\displaystyle\int \frac{x^4+x+1}{x^3-x}dx$

7. Calcule as seguintes integrais por fraçoes parciais.

(a) $\displaystyle\int \frac{12x^2+21x+3}{(x+1)(3x^2+5x-1)}dx$ (b) $\displaystyle\int \frac{2x^2+x+1}{(x+1)(x^2+9)}dx$ (c) $\displaystyle\int \frac{6x^2+8x-4}{(x-3)(x^2+6x+10)}$

(d) $\displaystyle\int \frac{x^2+x+5}{x^2+4x+10}$ (e) $\displaystyle\int \frac{1}{x^3+2x^2+3x}dx$

8. Mostre que

$$\int_0^1 \frac{16(x-1)}{x^4-2x^3+4x-4}dx = \pi.$$

9. Faça uma substituição para expressar o integrando como uma função racional e então calcule a integral (substituição racionalizante)

(a) $\displaystyle\int \frac{\sqrt{x+1}}{x}dx$ (b) $\displaystyle\int \frac{1}{\sqrt{x}-\sqrt[3]{x}}dx$, [Dica $u=\sqrt[6]{x}$] (c) $\displaystyle\int \frac{e^{2x}}{e^{2x}+3e^x+2}dx$

10. Calcule.

(a) $\displaystyle\int 2^{-\sqrt{x}}dx$ (b) $\displaystyle\int \frac{\text{tg}x+\text{senx}}{\sec x}dx$

(c) $\displaystyle\int \frac{\text{senh}(\ln x)}{x}dx$ (d) $\displaystyle\int \frac{e^{4t}}{(e^{2t}-1)^3}dx$

(e) $\displaystyle\int \frac{x^3}{\sqrt{x^2+16}}dx$ (f) $\displaystyle\int \text{sen}^2 x dx$

(g) $\displaystyle\int \text{sen}(3x)\cos(8x)dx$ (h) $\displaystyle\int \text{sen}^4 x dx$

(i) $\displaystyle\int \sec^5 x dx$ (j) $\displaystyle\int \frac{x^2}{\sqrt{16x^2+9}}dx$

(k) $\displaystyle\int \text{sen}^4 x\cos(2x)dx$ (l) $\displaystyle\int \frac{x^2-3}{x^2+3x+2}dx$

(m) $\displaystyle\int \frac{x^4+x^2+1}{x^3-x}dx$

11. Calcule as seguintes integrais impróprias. parciais.

(a) $\displaystyle\int_0^\infty \frac{1}{1+x^2}dx$ (b) $\displaystyle\int_{-\infty}^\infty \frac{1}{1+x^2}dx$ (c) $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}dx$ (d) $\displaystyle\int_{-1}^1 \frac{1}{\sqrt{|x|}}dx$

12. Determine para quiais valores de $p>0$ cada integral abaixo conerge e, nesse caso, calcule a integral.

(a) $\displaystyle\int_0^1 \frac{1}{x^p}dx$ (b) $\displaystyle\int_1^\infty \frac{1}{x^p}dx$

13. Determine se a integral diverge ou converge e, nesse último caso, calcule a integral.

(a) $\displaystyle\int_0^\infty \text{sen}(x)dx$ (b) $\displaystyle\int_0^2 \frac{1}{(x-1)^2}dx$ (c) $\displaystyle\int_{-\infty}^\infty \frac{1}{x^2+4x+9}dx$ (d) $\displaystyle\int_0^{1/2} \frac{1}{x\ln x}dx$

(e) $\displaystyle\int_0^{1/2} \frac{1}{x\ln^2 x}dx$ (f) $\displaystyle\int_2^\infty \frac{1}{x\sqrt{\ln x}}dx$ (g) $\displaystyle\int_2^\infty \frac{1}{x\ln x}dx$ (h) $\displaystyle\int_0^\infty \frac{1}{x(x+\ln^2 x)}dx$

(i) $\displaystyle\int_0^\infty \frac{1}{x+1}dx$ (j) $\displaystyle\int_0^\infty \frac{1}{(x^2-x+1)}dx$ (k) $\displaystyle\int_2^\infty \frac{1}{x^2-1}dx$ (l) $\displaystyle\int_2^\infty \frac{1}{(x^2-1)^2}dx$

Respostas:

1. (a) $\frac{1}{70}\left(-7\cos(5x)-5(\cos(7x))\right)+C$

(b) $\dfrac{3}{10}$

(c) $\dfrac{n\text{sen}(mx)\cos(nx)-m\cos(mx)\text{sen}(nx)}{m^2-n^2}+C$

(d) $\dfrac{1}{4}\left(3\text{sen}(4x)+2\text{sen}(6x)\right)+C$

(e) $\dfrac{1}{12}$

2. (a) $\dfrac{\cos^3(x)}{3}-\cos x+C$

(b) $\dfrac{1}{16}(8x+\text{sen}(8x))+C$

(c) $-\dfrac{3}{8}\cos(x)+\dfrac{1}{36}\cos(9x)-\dfrac{1}{44}\cos(11x)+\dfrac{1}{304}\cos(19x)-\dfrac{1}{336}\cos(21x)+C$

(d) $\dfrac{\text{tg}^2(x)}{2}+\dfrac{\text{tg}^4(x)}{4}+C$ ou $\dfrac{\sec^4(x)}{4}+C$

(e) $\dfrac{\pi}{32}$

(f) $\sec\left(\dfrac{x}{2}\right)\text{tg}\left(\dfrac{x}{2}\right)+\ln\left(\left|sec\left(\dfrac{x}{2}\right)+\text{tg}\left(\dfrac{x}{2}\right)\right|\right)+C$

(g) $\dfrac{1}{120}$

(h) $\dfrac{12}{35}$

(i) $\dfrac{\sec^7(x)}{7}-\dfrac{2}{5}\sec^5(x)+\dfrac{\sec^3(x)}{3}+C$

3. (a) $\dfrac{\sqrt{x^2-9}}{9x}+C$

(b) $\dfrac{1}{5}\sqrt{9-x^2}\left(x^4-3x^2-54\right)+C$

(c) $\dfrac{1}{3}\left(x^2-18\right)\sqrt{x^2+9}+C$

(d) $\dfrac{16}{243}$

(f) $-\dfrac{(x+2)(2x^2+8x-19)}{243(-x^2-4x+5)^{3/2}}+C$

(g) $-81\left(\dfrac{7\sqrt{3}}{64}-\dfrac{1}{4}+\dfrac{\pi}{32}\right)$

(h) $\sqrt{x^2-3}+C$

(i) $\dfrac{-4\sqrt{-4x^2+4x+3}\text{arcsen}(x-1/2)+10x+3}{32\sqrt{-4x^2+4x+3}}$

4. (a) $\dfrac{1}{2}\left(\sqrt{x^2+1}x+\ln(x+\sqrt{x^2+1})\right)+C$

(b) $\dfrac{1}{2}\sqrt{1-4x^2}x+\frac{1}{4}\text{arcsen}(2x)+C$

(c) $\dfrac{1}{2}\sqrt{4x^2+3x}+\dfrac{3}{4}\ln\left(x+\sqrt{x^2+3/4}+C\right)$

(d) $\dfrac{1}{2}\left(\sqrt{x^2+2x+2}(x+1)+\ln(x+1+\sqrt{(x+1)^2+1})\right)$

5. (a) $\dfrac{1}{2}\ln|x-1|-\dfrac{1}{2}\ln|x+1|+C$

(b) $\dfrac{1}{2}\ln\left(\dfrac{9}{5}\right)$

(c) $\dfrac{1}{4}\ln|x-2|+\dfrac{3}{4}\ln|x+2|+C$

(d) $5\ln|x+2|-4\ln|x+1|+C$

6. (a) $3\ln|x-3|-2\ln|x-2|+C$

(b) $\dfrac{2}{3}+\ln\left(\dfrac{3}{2}\right)$

(c) $\dfrac{1}{2}\left(x^2+4x-\dfrac{6}{x-1}+8\ln|x-1|-5\right)+C$

(d) $\dfrac{3}{10}\ln|x-2|-\dfrac{2}{15}\ln|x+3|-\dfrac{\ln|x|}{6}+C$

(e) $\dfrac{x^2}{2}+\dfrac{1}{2}\ln|x^2-1|+\ln|x-1|-\ln|x|+C$

7. (a) $\ln|3x^2+5x-1|+2\ln|x+1|+C$

(b) $\dfrac{9}{10}\ln|x^2+9|+\dfrac{1}{5}\ln|x+1|-\dfrac{4}{15}\text{arctg}\left(\dfrac{x}{3}\right)+C$

(c) $2\ln|x-3|+2\ln|x^2+6x+10|-4\text{arctg}(x+3)+C$

(d) $-\dfrac{3}{2}\ln|x^2+4x+10|+x+\dfrac{\text{arctg}(\frac{x+2}{\sqrt{6}})}{\sqrt{6}}+C$

(e) $\dfrac{1}{6}\left(-\ln|x^2+2x+3|+2\ln|x|-\sqrt{2}\text{arctg}\left(\dfrac{x+1}{\sqrt{2}}\right)\right)+C$

8.

9. (a) Fazendo $u=\sqrt{x+1}$, então $\displaystyle\int\frac{2u^2}{u^2-1}du=2\left(u+\frac{1}{2}\ln|u-1|+\ln|u+1|\right)+C$

(b) $\displaystyle\int\frac{6u^5}{u^3-u^2}du=6\left(\frac{u^3}{3}+\frac{u^2}{2}+u+\ln|u-1|-\frac{11}{6}\right)+C$

(c) $u=e^x$, $\displaystyle\int\frac{u}{u^2+3u+2}du=2\ln|u+2|-\ln|u+1|+C$

10. (a) $2^{-\sqrt{x}}\left(-\dfrac{2\sqrt{2}}{\ln(2)}-\dfrac{2}{[\ln(2)]^2}\right)+C$

(b) $-\cos x-\dfrac{1}{4}\cos(2x)+C$

(c) $\cosh(\ln x)+C=\dfrac{x}{2}+\dfrac{1}{2x}+C$

(d) $\dfrac{e^{4t}x}{(e^{2t}-1)^3}$

(e) $\dfrac{1}{3}(x^2-32)\sqrt{x^2+16}+C$

(f) $\dfrac{\pi}{2}$

(g) $\dfrac{1}{10}\cos(5x)-\dfrac{1}{22}\cos(11x)+C$

(h) $\dfrac{3}{8}x-\dfrac{1}{4}\text{sen}(2x)+\dfrac{1}{32}\text{sen}(4x)+C$

(i) $\dfrac{1}{4}\sec^3(x)\text{tg}(x)+\dfrac{3}{8}\sec(x)\text{tg}(x)+\dfrac{3}{8}\ln|\sec(x)+\text{tg}(x)|+C$

(j) $\dfrac{1}{32}\sqrt{16x^2+9}x-\frac{9}{128}\ln|\sqrt{16x^2+9}+4x|+C$

(k) $-\dfrac{x}{4}+\dfrac{7}{32}\text{sen}(2x)+\dfrac{1}{96}\text{sen}(6x)-\dfrac{1}{16}\text{sen}(4x)+C$

(l) $x-2\ln|x+1|-\ln|x+2|+C$

(m) $\dfrac{x^2}{2}-\ln|x|+\dfrac{3}{2}\ln|x-1|+\dfrac{3}{2}\ln|x+1|+C$

11. (a) $\dfrac{\pi}{2}$ (b) π (c) 2 (d) 4

12. (a) Separe o caso $p=1$ e veja que $\displaystyle\int_0^1\frac{1}{x}dx$ diverge. Considere $p\neq 1$, $\displaystyle\int_t^1\frac{1}{x^p}dx=\frac{1}{p-1}\left(1-\frac{1}{t^{p-1}}\right)$, calcule o limite quando $t\to 0^+$ e veja que a integral diverge se $p-1>0$ e é $\dfrac{1}{p-1}$ se $p-1<0$.

(b) $p>1$ converge e $0<p\leq 1$ diverge

13. (a) Divergente

(b) Divergente

(c) $\dfrac{\pi}{\sqrt{5}}$

(d) Divergente

(e) $\dfrac{1}{\ln(2)}$

(f) Divergente

(g) Divergente

(h) π

(i) Divergente

(j) $\dfrac{4\pi}{3\sqrt{3}}$

(k) $\dfrac{\ln(3)}{2}$

(l) $\dfrac{1}{3} - \dfrac{\ln(3)}{3}$