Universidade Federal do Paraná - UFPR
Centro Politécnico
Departamento de Matemática

Disciplina: Cálculo 2 **Código:** CM312 **Semestre:** Semestre 2024/2

Lista 4

1. Determine as derivadas parciais indicadas.

(a) $f(x,y) = x^3y^5$; $f_x(3,-1)$

(b) $f(x,y) = \sqrt{2x+3y}$; $f_y(2,4)$

(c) $f(x,y) = xe^{-y} + 3y$; $\dfrac{\partial f}{\partial y}(1,0)$

(d) $f(x,y) = \operatorname{sen}(y-x)$; $\dfrac{\partial f}{\partial y}(3,3)$

(e) $z = \dfrac{x^3+y^3}{x^2+y^2}$, $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

(f) $z = x\sqrt{y} - \dfrac{y}{\sqrt{x}}$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

(g) $z = \dfrac{x}{y} + \dfrac{y}{x}$; $\dfrac{\partial z}{\partial x}$

(h) $z = (3xy^2 - x^4 + 1)^4$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

(i) $u = xy\sec(xy)$; $\dfrac{\partial u}{\partial x}$

(j) $u = \dfrac{x}{x+t}$; $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial t}$

(k) $f(x,y,z) = xyz$; $f_y(0,1,2)$

(l) $f(x,y,z) = \sqrt{x^2+y^2+z^2}$; $f_z(0,3,4)$

(m) $u = xy + yz + zx$; u_x, u_y, u_z

(n) $u = x^2y^3t^4$; u_x, u_y, u_t

2. Determine as derivadas parciais de primeira ordem da função.

(a) $f(x,y) = x^3y^5 - 2x^2y + x$

(b) $f(x,y) = x^2y^2(x^4+y^4)$

(c) $f(x,y) = e^x \operatorname{tg}(x-y)$

(d) $f(s,t) = s/\sqrt{s^2+t^2}$

(e) $g(x,y) = y\operatorname{tg}(x^2y^3)$

(f) $g(x,y) = \ln(x + \ln y)$

(g) $f(x,y) = e^{xy}\cos x \operatorname{sen} y$

(h) $f(s,t) = \sqrt{2 - 3s^2 - 5t^2}$

(i) $f(u,v) = \operatorname{arctg}(u/v)$

(j) $f(x,t) = e^{\operatorname{sen}(t/x)}$

(k) $z = \ln\left(x + \sqrt{x^2+y^2}\right)$

(l) $z = x^{x^y}$

(m) $f(x,y) = \int_x^y e^{t^2}\,dt$

(n) $f(x,y) = \int_y^x e^t/t dt$

(o) $f(x,y,z) = x^{yz}$

(p) $f(x,y,z) = xe^y + ye^z + ze^x$

(q) $u = z\operatorname{sen}\frac{y}{x+z}$

(r) $u = xy^2z^3\ln(x+2y+3z)$

(s) $u = x^{y^z}$

(t) $f(x,y,z,t) = \frac{x-y}{z-t}$

(u) $f(x,y,z,t) = xy^2z^3t^4$

3. Use a derivação implícita para encontrar $\partial z/\partial z$ e $\partial z/\partial y$.

(a) $xy + yz = xz$

(b) $xyz = \cos(x+y+z)$

(c) $x^2 + y^2 - z^2 = 2x(y+z)$

(d) $xy^2z^3 + x^3y^2z = x + y + z$

4. Determine $\partial z/\partial x$ e $\partial z/\partial y$ se $z = f(ax+by)$.

5. Determine todas as derivadas parciais de segunda ordem .

(a) $f(x,y) = x^2y + x\sqrt{y}$ (b) $f(x,y) = \operatorname{sen}(x+y) + \cos(x-y)$

(c) $z = (x^2+y^2)^{3/2}$ (d) $z = \cos^2(5x+2y)$

(e) $z = t \operatorname{arcsen} \sqrt{x}$ (f) $z = x^{\ln t}$

6. Verifique que a conclusão do Teorema de Clairaut é válida, isto é, $u_{xy} = u_{yx}$.

(a) $u = x^5y^4 - 3x^2y^3 + 2x^2$ (b) $u = \operatorname{sen}^2 x \cos y$

(c) $u = \operatorname{arcsen}(xy^2)$ (d) $u = x^2y^3z^4$

7. Determine as derivadas parciais indicadas.

(a) $f(x,y) = x^2y^3 - 2x^4y$; f_{xxx} (b) $f(x,y) = e^{xy^2}$; f_{xxy}

(c) $f(x,y,z) = x^5 + x^4y^4z^3 + yz^2$; f_{xyz} (d) $f(x,y,z) = e^{xyz}$; f_{yzy}

(e) $z = x \operatorname{sen} y$; $\frac{\partial z}{\partial y^2 \partial x}$ (f) $z = \ln \operatorname{sen}(x-y)$; $\frac{\partial z}{\partial y \partial x^2}$

(g) $u = \ln\left(x + 2y^2 + 3z^3\right)$; $\frac{\partial^3 u}{\partial x \partial y \partial z}$

8. Se f e g são funções duas vezes deriváveis de uma única variável, mostre que a função

$$u(x,y) = xf(x+y) + yg(x+y)$$

satisfaz a equação $u_{xx} - 2u_{xy} + u_{yy} = 0$.

9. Mostre que a função

$$f(x,\cdots,x_n) = (x_1^2 + \cdots x_n)^{(2-n)/2}$$

satisfaz a equação

$$\frac{\partial^2 f}{\partial x_1^2} + \cdots + \frac{\partial^2 f}{\partial x_n^2} = 0$$

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10. Determine uma equação do plano tangente à superfície no ponto especificado.

(a) $z = x^2 + 4y^2$, $(2,1,8)$ (b) $z = x^2 - y^2$, $(3,-2,5)$

(c) $z = 5 + (x-1)^2 + (y+2)^2$, $(2,0,10)$ (d) $z = xy$, $(-1,2,-2)$

(e) $z = \sqrt{x-y}$, $(5,1,2)$ (f) $z = y^2 - x^2$, $(-4,5,9)$

(g) $z = \operatorname{sen}(x+y)$, $(1,-1,0)$ (h) $z = \ln(2x+y)$, $(-1,3,0)$

(i) $z = e^x \ln y$, $(3,1,0)$

11. Desenhe a superfície e o plano tangente no ponto dado. (Escolha o domínio e o ponto de vista de modo a ver tanto a superfície quanto o plano tangente.) Em seguida, dê zoom até que a superfície e o plano tangente se tornem indistinguíveis.

(a) $z = xy$, $(-1,2,-2)$ (b) $z = \sqrt{x-y}$, $(5,1,2)$

12. Explique por que a função é diferenciável no ponto dado. Em seguida, encontre a linearização $L(x,y)$ da função naquele ponto.

(a) $f(x,y) = y \ln x$, $(2,1)$ (b) $f(x,y) = \sqrt{1 + x^2y^2}$, $(0,2)$

13. Determine a diferencial da função.

(a) $z = x^2y^3$

(b) $v = \ln(2x - 3y)$

(c) $w = x\,\mathrm{sen}(yz)$

(d) $z = x^4 - 5x^2y + 6xy^3 + 10$

(e) $z = \dfrac{1}{x^2 + y^2}$

(f) $z = ye^{xy}$

(g) $z = e^x \cos(xy)$

(h) $w = x^2y + y^2z$

(i) $w = \dfrac{x + y}{y + z}$

14. Use diferenciais para aproximar o valor de f em um dado ponto.

(a) $f(x, y) = \sqrt{20 - x^2 - 7y^2}$, $(1,95; 1,08)$

(b) $f(x, y) = \ln(x - 3y)$, $(6,9; 2,06)$

(c) $f(x, y, z) = x^2y^3z^4$, $(1,05; 0,9; 3,01)$

(d) $f(x, y, z) = xy^2\,\mathrm{sen}(\pi z)$, $(3,99; 4,98; 4,03)$

15. Use diferenciais para aproximar o número.

(a) $8,94\sqrt{9,99 - (1,01)^3}$

(b) $(\sqrt{99} + \sqrt[3]{124})^4$

(c) $\sqrt{0,99}e^{0,02}$

(d) $\sqrt{(3,02)^2 + (1,97)^2 + (5,99)^2}$

Respostas:

1. (a) -27

(b) $\frac{3}{8}$

(c) 2

(d) -1

(e) $\frac{x^4+3x^2y^2-2xy^3}{(x^2+y^2)^2}$, $\frac{3x^2y^2+y^4-2yx^3}{(x^2+y^2)^2}$

(f) $\sqrt{y} + \frac{y}{2x^{3/2}}$, $\frac{x}{2\sqrt{y}} - \frac{1}{\sqrt{x}}$

(g) $\frac{1}{y} - \frac{y}{x^2}$

(h) $4(3xy^2 - x^4 + 1)^3(3y^2 - 4x^3)$, $24xy(3xy^2 - x^4 + 1)^3$

(i) $y\sec(xy)[1 + xy\,\mathrm{tg}(xy)]$

(j) $\frac{t}{(x+t)^2}$, $-\frac{x}{(x+t)^2}$

(k) 0

(l) $\frac{4}{5}$

(m) $y + z$, $x + z$, $y + x$

(n) $2xy^3t^4$, $3x^2y^2t^4$,$4x^2y^3t^3$

2. (a) $f_x(x, y) = 3x^2y^5 - 4xy + 1$, $f_y(x, y) = 5x^3y^4 - 2x^2$

(b) $f_x(x, y) = 6x^5y^2 + 2xy^6$, $f_y(x, y) = 6y^5x^2 + 2yx^6$

(c) $e^x[\mathrm{tg}(x - y) + \sec^2(x - y)]$, $f_y(x, y) = -e^x\sec^2(x - y)$

(d) $f_s(s, t) = \frac{t^2}{(t^2+s^2)^{3/2}}$, $f_t(s, t) = -\frac{st}{(t^2+s^2)^{3/2}}$

(e) $g_x(x, y) = 2xy^4\sec^2(x^2y^3)$, $g_y(x, y) = \mathrm{tg}(x^2y^3) + 3x^2y^3\sec^2(x^2y^3)$

(f) $g_x(x, y) = \frac{1}{x+\ln y}$, $g_y(x, y) = \frac{1}{y(x+\ln y)}$

(g) $f_x(x, y) = e^{xy}\,\mathrm{sen}\,y(y\cos x - \mathrm{sen}\,x)$, $f_y(x, y) = e^{xy}\cos x(x\,\mathrm{sen}\,y - \cos y)$

(h) $f_s(s, t) = -\frac{3s}{\sqrt{2-3s^2-5t^2}}$, $f_t(s, t) = -\frac{5t}{\sqrt{2-3s^2-5t^2}}$

(i) $f_u(u, v) = \frac{v}{u^2+v^2}$, $f_v(u, v) = -\frac{u}{u^2+v^2}$

(j) $f_x(x, t) = -t\cos\left(\frac{t}{x}\right)\frac{e^{\mathrm{sen}\left(\frac{t}{x}\right)}}{x^2}$, $f_t(x, t) = \frac{e^{\mathrm{sen}\left(\frac{t}{x}\right)}}{x}\cos\left(\frac{t}{x}\right)$

(k) $\frac{\partial z}{\partial x} = \frac{1}{\sqrt{x^2+y^2}}$, $\frac{\partial z}{\partial y} = \frac{y}{x\sqrt{x^2+y^2}+x^2+y^2}$

(l) $\frac{\partial z}{\partial x} = x^{y-1}x^{x^y}(1 + y\ln x)$, $\frac{\partial z}{\partial y} = x^{x^y+y}(\ lnx)^2$

(m) $f_x(x, y) = -e^{x^2}$, $f_y(x, y) = e^{y^2}$

(n) $f_x(x, y) = \frac{e^x}{x}$, $f_y(x, y) = -\frac{e^y}{y}$

(o) $f_x(x, y, z) = yzx^{yz-1}$, $f_y(x, y, z) = zx^{yz}\ln x$, $f_z(x, y, z) = yx^{yz}\ln x$

(p) $f_x(x,y,z) = e^y + ze^x$, $f_y(x,y,z) = xe^y + e^z$, $f_z(x,y,z) = ye^z + e^x$

(q) $\frac{\partial u}{\partial x} = \frac{-yz}{(x+z)^2}\cos\left(\frac{y}{x+z}\right)$, $\frac{\partial u}{\partial y} = \frac{z}{x+z}\cos\left(\frac{y}{x+z}\right)$, $\frac{\partial u}{\partial z} = \operatorname{sen}(\frac{y}{x+z}) - \frac{yz}{(x+z)^2}\cos\left(\frac{y}{x+z}\right)$

(r) $u_x = y^2z^3[\ln(x+2y+3z)+\frac{x}{x+2y+3z}]$, $u_y = 2xyz^3[\ln(x+2y+3z)+\frac{y}{x+2y+3z}]$, $u_z = 3xy^2z^2[\ln(x+2y+3z)+\frac{z}{x+2y+3z}]$

(s) $u_x = y^z x^{y^z-1}$, $u_y = x^{y^z}y^{z-1}z\ln x$, $u_z = x^{y^z}y^z\ln x\ln y$

(t) $f_x(x,y,z,t) = \frac{1}{z-t}$, $f_y(x,y,z,t) = -\frac{1}{z-t}$, $f_z(x,y,z,t) = \frac{y-x}{(z-t)^2}$, $f_t(x,y,z,t) = \frac{x-y}{(z-t)^2}$

(u) $f_x(x,y,z,t) = y^2z^3t^4$, $f_y(x,y,z,t) = 2xy^2z^3t^4$, $f_z(x,y,z,t) = 3xy^2z^2t^4$, $f_t(x,y,z,t) = 4xy^2z^3t^3$

3. (a) $\frac{z-y}{y-x}$, $\frac{x+z}{y-x}$

(b) $-\frac{yz+\operatorname{sen}(x+y+z)}{xy+\operatorname{sen}(x+y+z)}$, $-\frac{xz+\operatorname{sen}(x+y+z)}{xy+\operatorname{sen}(x+y+z)}$

(c) $\frac{x-y-z}{x+z}$,$\frac{y-x}{x+z}$

(d) $\frac{1-y^2z^3-3x^2y^2z}{3xy^2z^2+x^3y^2-1}$, $\frac{1-2xyz^3-2x^3yz}{3xy^2z^2+x^3y^2-1}$

4. $af'(ax+by)$, $bf'(ax+by)$

5. (a) $f_{xx} = 2y$, $f_{xy} = 2x + \frac{1}{2\sqrt{y}}$, $f_{yx} = 2x + \frac{1}{2\sqrt{y}}$, $f_{yy} = -\frac{x}{4y^{3/2}}$

(b) $f_{xx} = -\operatorname{sen}(x+y) - \cos(x-y)$, $f_{xy} = -\operatorname{sen}(x+y) + \cos(x-y)$, $f_{yx} = -\operatorname{sen}(x+y) + \cos(x-y)$, $f_{yy} = -\operatorname{sen}(x+y) - \cos(x-y)$

(c) $z_{xx} = \frac{3(2x^2+y^2)}{\sqrt{x^2+y^2}}$, $z_{xy} = \frac{3xy}{\sqrt{x^2+y^2}}$, $z_{yx} = \frac{3xy}{\sqrt{x^2+y^2}}$, $z_{yy} = \frac{3(x^2+2y^2)}{\sqrt{x^2+y^2}}$

(d) $z_{xx} = 50[\operatorname{sen}^2(5x+2y) - \cos^2(5x+2y)]$, $z_{xy} = 20[\operatorname{sen}^2(5x+2y) - \cos^2(5x+2y)]$, $z_{yx} = 20[\operatorname{sen}^2(5x+2y) - \cos^2(5x+2y)]$, $z_{yy} = 8[\operatorname{sen}^2(5x+2y) - \cos^2(5x+2y)]$

(e) $z_{xx} = \frac{t(2x-1)}{4(x-x^2)^{3/2}}$, $z_{xt} = \frac{1}{2\sqrt{x-x^2}}$, $z_{tx} = \frac{1}{2\sqrt{x-x^2}}$, $z_{tt} = 0$

(f) $z_{xx} = (\ln t)[(\ln t)-1]x^{(\ln t)-2}$, $z_{xt} = x^{(\ln t)-1}\frac{1+\ln t\ln x}{t}$, $z_{tx} = x^{(\ln t)-1}\frac{1+\ln t\ln x}{t}$, $z_{tt} = x^{\ln t}\ln x\frac{(\ln x)-1}{t^2}$

6. (a)

(b)

(c)

(d)

7. (a) $-48xy$

(b) $2y^3e^{xy^2}(2+xy^2)$

(c) $f_{xyz} = 48x^3y^3z^2$

(d) $x^2z(2+xyz)e^{xyz}$

(e) $-\operatorname{sen} y$

(d[f]) $-2\operatorname{cossec}^2(x-y)\operatorname{cotg}(x-y)$

(g) $\frac{72yz^2}{(x+2y^2+3z^3)^3}$

8.

9.

10. (a) $4x+8y-z=8$

(b) $6x+4y-z=5$

(c) $2x+4y-z=-6$

(d) $2x-y-z=-2$

(e) $x-y-4z=-4$

(f) $z = 8x+10y-9$

(g) $z = x+y$

(h) $z = 2x+y-1$

(i) $z = e^3y - e^3$

11. (a)

(b)

12. (a) $\frac{1}{2}x + (\ln 2)y - 1$

(b) 1

13.

(a) $2xy^3\mathrm{dx} + 3x^2y^2\mathrm{dy}$

(b) $\frac{1}{2x-3y}(2\mathrm{dx} - 3\mathrm{dy})$

(c) $(\operatorname{sen} yz)\mathrm{dx} + (xz\cos yz)\mathrm{dy} + (xy\cos yz)\mathrm{dz}$

(d) $(4x^3 - 10xy + 6y^3)\mathrm{dx} + (-5x^2 + 18xy^2)\mathrm{dy}$

(e) $-\frac{2}{(x^2+y^2)^2}(x\mathrm{dx} + y\mathrm{dy})$

(f) $y^2e^{xy}\mathrm{dx} + e^{xy}(1 + xy)\mathrm{dy}$

(g) $e^x(\cos xy - y\operatorname{sen} xy)\mathrm{dx} - (xe^x \operatorname{sen} xy)\mathrm{dy}$

(h) $2xy\mathrm{dx} + (x^2 + 2yz)\mathrm{dy} + y^2\mathrm{dz}$

(i) $\frac{(y+z)\mathrm{dx}+(z-x)\mathrm{dy}-(x+y)\mathrm{dz}}{(y+z)^2}$

14. (a) $2,84\overline{6}$

(b) $-0,28$

(c) $65,88$

(d) 3π

15. (a) $26,76$

(b) $49,770$

(c) $1,015$

(d) $6,9914$