Universidade Federal do Paraná - UFPR
Centro Politécnico
Departamento de Matemática

Disciplina: Cálculo 2 **Código:** CM312 **Semestre:** Semestre 2024/2

Lista 6

1. Use a Regra da Cadeia para determinar dz/dt ou dw/dt.

(a) $z = x^2 + y^2$, $x = t^3$, $y = 1 + t^2$

(b) $z = x^2y^3$, $x = 1 + \sqrt{t}$, $y = 1 - \sqrt{t}$

(c) $z = \ln(x + y^2)$,$x = \sqrt{1 + t}$, $y = 1 + \sqrt{t}$

(d) $z = xe^{x/y}$, $x = \cos t$, $y = e^{2t}$

(e) $z = 6x^3 - 3xy + 2y^2$,$x = e^t$, $y = \cos t$

(f) $z = x\sqrt{1 + y^2}$, $x = te^{2t}$, $y = e^{-t}$

(g) $w = xy^2z^3$,$x = \operatorname{sen} t$, $y = \cos t$, $z = 1 + e^{2t}$

(h) $w = \frac{x}{y} + \frac{y}{z}$, $x = \sqrt{t}$, $y = \cos(2t)$, $z = e^{-3t}$

2. Use a Regra da Cadeia para determinar $\partial z/\partial s$ e $\partial z/\partial t$.

(a) $z = x^2 \operatorname{sen} y$, $x = s^2 + t^2$, $y = 2st$

(b) $z = \operatorname{sen} x \cos y$, $x = (s - t)^2$, $y = s^2 - t^2$

(c) $z = x^2 - 3x^2y^3$, $x = se^t$, $y = se^{-t}$

(d) $z = x \operatorname{arctg}(xy)$, $x = t^2$, $y = se^t$

(e) $z = 2^{x-3y}$, $x = s^2t$, $y = st^2$

(f) $z = xe^y + ye^{-x}$, $x = e^t$, $y = st^2$

3. Use a Regra da Cadeia para determinar as derivadas parciais indicadas.

(a) $w = x^2 + y^2 + z^2$, $x = st$, $y = s\cos t$, $z = s\operatorname{sen} t$; $\frac{\partial w}{\partial s}$, $\frac{\partial w}{\partial t}$ quando $s = 1$, $t = 0$

(b) $u = xy + yz + zx$, $x = st$, $y = e^{st}$, $z = t^2$; $\frac{\partial u}{\partial s}$, $\frac{\partial u}{\partial t}$ quando $s = 0$, $t = 1$

(c) $z = y^2 \operatorname{tg} x$, $x = t^2uv$, $y = u + tv^2$; $\frac{\partial z}{\partial t}$, $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$ quando $t = 2$, $u = 1$, $v = 0$

(d) $z = \frac{x}{y}$, $x = re^{st}$, $y = rse^t$; $\frac{\partial z}{\partial r}$, $\frac{\partial z}{\partial s}$, $\frac{\partial z}{\partial t}$ quando $r = 1$, $s = 2$, $t = 0$

(e) $u = \frac{x+y}{y+z}$, $x = p + r + t$, $y = p - r + t$, $z = p + r - t$, $\frac{\partial u}{\partial p}$, $\frac{\partial u}{\partial r}$, $\frac{\partial u}{\partial t}$

(f) $t = z\sec(xy)$, $x = uv$, $y = vw$, $z = wu$, $\frac{\partial t}{\partial u}$, $\frac{\partial t}{\partial v}$, $\frac{\partial t}{\partial w}$

(g) $w = \cos(x - y)$, $x = rs^2t^3 \operatorname{sen}\theta$, $y = r^2st\cos\theta$, $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial s}$, $\frac{\partial w}{\partial t}$, $\frac{\partial w}{\partial \theta}$

(h) $u = pq - p^2r^2s$, $p = x + 2y$, $q = x - 2y$, $r = \frac{x}{y^4}$,, $s = 2xy^{3/2}$; $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$

4. Determine dy/dx.

(a) $x^2 - xy + y^3 = 8$

(b) $y^5 + 3x^2y^2 + 5x^4 = 12$

(c) $x\cos y + y\cos x = 1$

(d) $2y^2 + \sqrt[3]{xy} = 3x^2 + 17$

5. Determine $\partial z/\partial x$ e $\partial z/\partial y$.

(a) $xy + yz - xz = 0$

(b) $x^2 + y^2 - z^2 = 2x(y + z)$

(c) $xy^2z^3 + x^3y^2z = x + y + z$

(d) $y^2ze^{x+y} - \operatorname{sen}(xyz) = 0$

(e) $xy^2 + yz^2 + zx^2 = 3$

(f) $xe^y + yz + ze^x = 0$

(g) $\ln(x + yz) = 1 + xy^2z^3$

6. O raio de um cilindro reto está aumentando em uma taxa de $1,2$ cm/s enquanto a sua altura está decrescendo em uma taxa de 3 cm/s. Em qual taxa o volume do cilindro está variando quando o raio é 80 cm e a altura é 150 cm?

Respostas:

1. (a) $6t^5 + 4t^3 + 4t$

(b) $\frac{(1-t)(1-\sqrt{t})^2}{\sqrt{t}} - \frac{3(1-t)^2}{2\sqrt{t}}$

(c) $\frac{1}{\sqrt{1+t}+1+\sqrt{t}}\left(\frac{1}{2\sqrt{1+t}} + \frac{1+\sqrt{t}}{\sqrt{t}}\right)$

(d) $-e^{\cos t/e^{2t}}\left[\left(+\frac{\cos t}{e^{2t}}\right)\operatorname{sen} t - \frac{2e^{2t}\cos^2 t}{e^{4t}}\right]$

(e) $(18e^{2t} - 3\cos t)e^t + (3e^t - 4\cos t)\operatorname{sen} t$

(f) $e^{2t}\sqrt{1+e^{-2t}}(1+2t) - t\sqrt{1+e^{-2t}}$

(g) $y^2z^3(\cos t) + 2xyz^3(-\operatorname{sen} t) + 3xy^2z^2(2e^{2t})$

(h) $\frac{1}{2y\sqrt{t}} + 2(\operatorname{sen}(2t))\left(\frac{x}{y^2} - \frac{1}{z}\right) + \frac{3y}{z^2e^{3t}}$

2. (a) $4sx\operatorname{sen} y + 2tx^2\cos y,\ 4xt\operatorname{sen} y + 3sx^2\cos y$

(b) $2(s-t)\cos x\operatorname{sen} y - 2s\operatorname{sen} x\operatorname{sen} y,\ 2(t-s)\cos x\cos y + 2t\operatorname{sen} x\operatorname{sen} y$

(c) $(2x - 6xy^3)e^t - 9x^2y^2e^{-t},\ (2x - 6xy^3)se^t + 9x^2y^2se^{-t}$

(d) $\frac{x^2e^t}{1+x^2y^2},\ \left[\operatorname{arctg}(xy) + \frac{xy}{1+x^2y^2}\right](2t) + \frac{x^2}{1+x^2y^2}se^t$

(e) $(2^{x-3y}\ln 2)(2st - 3t^2),\ (2^{x-3y}\ln 2)(s^2 - 6st)$

(f) $(xe^y + e^{-x})t^2,\ (e^y - ye^{-x})e^t + 2(xe^y + e^{-x})st$

3. (a) $2, 0$

(b) $3, 2$

(c) $0, 0, 4$

(d) $0, -\frac{1}{4}, \frac{1}{2}$

(e) $-t/(p^2), 0, 1/p$

(f) $\sec(xy)[w + vzy\operatorname{tg}(xy)],\ z\sec(xy)\operatorname{tg}(xy)[yu + xw],\ \sec(xy)[u + vzx\operatorname{tg}(xy)]$

(e) $st\operatorname{sen}(x-y)[2r\cos\theta - st^2\operatorname{sen}\theta],\ [rt\operatorname{sen}(x-y)](r\cos\theta - 2st^2\operatorname{sen}\theta),\ [sr\operatorname{sen}(x-y)](r\cos\theta - 3st^2\operatorname{sen}\theta),\ [-rst\operatorname{sen}(x-y)](st^2\cos\theta + r\operatorname{sen}\theta)$

(f) $2x - \frac{8x^2(x+2y)(x+y)}{y^{13/2}},\ -8y + \frac{(x+2y)x^3y^{1/2}(5x+2y)}{y^8}$

4. (a) $\frac{y-2x}{3y^2-x}$

(b) $-\frac{6xy^2+20x^3}{5y^4+6x^2y}$

(c) $\frac{y\operatorname{sen} x - \cos y}{\cos x - x\operatorname{sen} y}$

(d) $\frac{18x - x^{-2/3}y^{1/3}}{12y + x^{1/3}y^{-2/3}}$

5. (a) $\frac{z-y}{y-x},\ \frac{x+z}{x-y}$

(b) $\frac{x-y-z}{z+x},\ \frac{y-x}{z+x}$

(c) $-\frac{y^2z^3+3x^2y^2z-1}{3xy^2z^2+x^3y^2-1},\ -\frac{2xyz^3+2x^3yz-1}{3xy^2z^2+x^3y^2-1}$

(d) $\frac{z\cos(xyz) - yze^{x+y}}{ye^{x+y} - x\cos(xyz)},\ \frac{xz\cos(xyz) - e^{x+y}(2yz+y^2z)}{y^2e^{x+y} - xy\cos(xyz)}$

(e) $-\frac{y^2+2zx}{2yz+x^2},\ -\frac{2xy+z^2}{2yz+x^2}$

(f) $-\frac{e^y + ze^x}{y+e^x},\ -\frac{xe^y + z}{y+e^x}$

(g) $\frac{y^2z^3(x+yz)-1}{y-3xy^2z^2(x+yz)},\ \frac{2xyz^3(x+yz)-z}{y-3xy^2z^2(x+yz)}$

6. $-9600\pi\ cm^3/s$