



Disciplina: Cálculo 2 **Código:** CM312 **Semestre:** Semestre 2024/2

Lista 2

1. Um tanque para armazenamento de gás propano é construído soldando dois hemisférios nas extremidades de um cilindro circular reto. Escreva o volume do tanque como uma função de r e l , onde r é o raio do cilindro e os hemisférios, e l é a largura do cilindro.
2. Seja $f(x, y) = x^2 - y^2 + 4xy - 7x + 10$, encontre:
(a) $f(2, 1)$ (b) $f(-3, 5)$ (c) $f(x + h, y)$
(d) $f(x, y + k)$ (e) $f(x, x)$
3. Seja $g(x, y) = \ln(xy + y - 1)$, encontre:
(a) $g(1, 1)$ (b) $g(e, 1)$ (c) $f(x, 1)$
(d) $g(x + h, y)$ (e) $g(x, y + k)$
4. Seja $F(x, y) = \frac{3xy}{x^2 + 2y^2}$, encontre:
(a) $F(1, 1)$ (b) $F(-1, 2)$ (c) $F(t, 1)$
(d) $F(-1, y)$ (e) $F(x, x^2)$
5. Seja $G(x, y) = x \sin(y) \cos(z)$, encontre:
(a) $G(2, \pi/6, \pi/3)$ (b) $G(4, \pi/4, 0)$ (c) $G(t, t, t)$
(d) $G(u, v, 0)$ (e) $G(x, x + y, x)$
6. Encontre o domínio e a imagem das funções
(a) $f(x, y) = x + 2y - 5$ (b) $f(x, y) = \sqrt{x - y}$ (c) $f(x, y) = \frac{2}{x + y}$
(d) $f(x, y) = \operatorname{tg}^{-1}\left(\frac{y}{x}\right)$ (e) $f(x, y, z) = \frac{x}{yz}$ (f) $f(x, y, y) = x \sin(y + z)$
7. seja $f(x, y) = e^{x^2 - y}$.
(a) Calcule $f(2, 4)$;
(b) Determine o domínio de f ;
(c) Determine a imagem de f .
8. seja $g(x, y) = \sqrt{36 - 9x^2 - 4y^2}$.
(a) Calcule $g(1, 2)$;
(b) Determine o domínio de g ;
(c) Determine a imagem de g .
9. seja $f(x, y, z) = x^2 \ln(x - y + z)$.

- (a) Calcule $f(3, 6, 4)$;
- (b) Determine o domínio de f ;
- (c) Determine a imagem de f .

10. seja $f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2 - 1}}$.

- (a) Calcule $f(1, 3, -4)$;
- (b) Determine o domínio de f ;
- (c) Determine a imagem de f .

11. Determine e esboce o domínio da função.

- (a) $f(x, y) = xy\sqrt{x^2 + y}$
- (b) $f(x, y) = \frac{\sqrt{9 - x^2 - y^2}}{x + 2y}$
- (c) $f(x, y) = \frac{x^2 + y^2}{x^2 - y^2}$
- (d) $f(x, y) = \operatorname{tg}(x - y)$
- (e) $f(x, y) = \ln(xy - 1)$
- (f) $f(x, y) = \ln(x^2 - y^2)$
- (g) $f(x, y) = x^2 \sec(y)$
- (h) $f(x, y) = \arcsen(x + y)$
- (i) $f(x, y) = \sqrt{4 - 2x^2 - y^2}$
- (j) $f(x, y) = \ln x + \ln \operatorname{sen}(y)$
- (k) $f(x, y) = \sqrt{y - x} \ln(y + x)$

12. Esboce o gráfico da função.

- (a) $f(x, y) = x$
- (b) $f(x, y) = \operatorname{sen}(y)$
- (c) $f(x, y) = x^2 + 9y^2$
- (d) $f(x, y) = y^2$
- (e) $f(x, y) = \sqrt{16 - x^2 - 16y^2}$
- (f) $f(x, y) = y^2 - x^2$
- (g) $f(x, y) = 1 - x^2$
- (h) $f(x, y) = x^2 + y^2 - 4x - 2y + 5$

13. Faça o mapa de contorno da função mostrando várias curvas de nível.

- (a) $f(x, y) = \frac{x}{y}$
- (b) $f(x, y) = \frac{x + y}{x - y}$
- (c) $f(x, y) = y - \cos(x)$
- (d) $f(x, y) = e^{1/(x^2 + y^2)}$
- (e) $f(x, y) = x^2 + 9y^2$
- (f) $f(x, y) = e^{xy}$

14. Determine o domínio da função vetorial

$$\mathbf{r}(t) = \left(\ln(t), \frac{t}{t-1}, e^{-t} \right)$$

15. Esboce o gráfico da curva cuja equação vetorial é dada. Indique com setas a direção na qual o parâmetro t cresce

- (a) $\mathbf{r}(t) = \langle t^2, t, 2 \rangle$
- (b) $\mathbf{r}(t) = \langle t, -t, 2t \rangle$
- (c) $\mathbf{r}(t) = \langle \operatorname{sen}(t), t, \cos(t) \rangle$

16. Utilize um computador para traçar a curva da equação vetorial dada. Escolha o domínio do parâmetro e ponto de vista de forma a revelar a verdadeira natureza da curva.

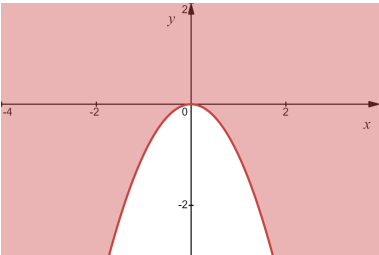
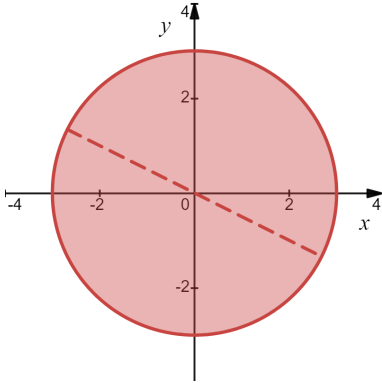
- (a) $\mathbf{r}(t) = \langle t^2, t^3 - t, t \rangle$
- (b) $\mathbf{r}(t) = \langle \sqrt{t}, t, t^2 - 2 \rangle$

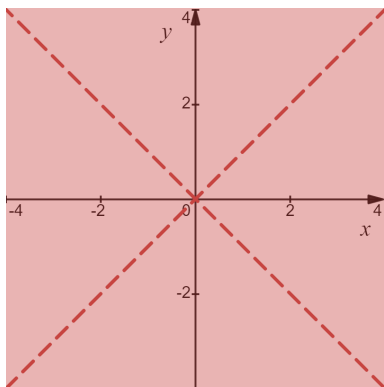
17. Se dois objetos viajam pelo espaço ao longo de duas curvas diferentes, é sempre importante saber se eles vão colidir. (Um míssil vai atingir seu alvo móvel? Duas aeronaves vão colidir?) As curvas podem se interceptar, mas precisamos saber se os objetos estarão na mesma posição no mesmo instante. Suponha que as trajetórias de duas partículas sejam dadas pelas seguintes funções vetoriais:

$$\mathbf{r}_1(t) = \langle t^2, 7t - 12, t^2 \rangle \quad \mathbf{r}_2(t) = \langle 4t - 3, t^2, 5t - 6 \rangle$$

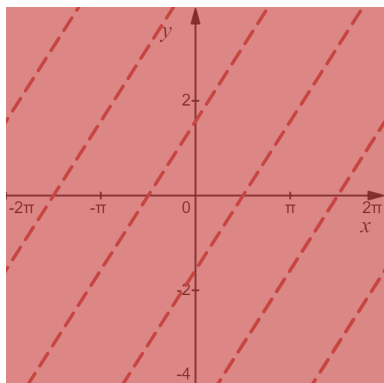
para $t \geq 0$. As partículas colidem?

Respostas:

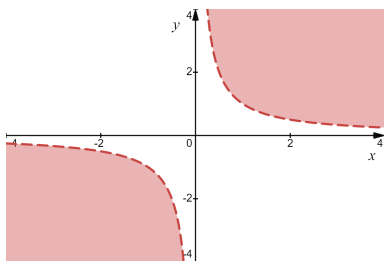
1. $V(r, l) = \pi r^2 \left(l + \frac{4}{3}r \right)$
2. (a) 7 (d) $x^2 - y^2 - 2ky - k^2 + 4xy + 4xk - 7x + 10$
 (b) -45
 (c) $x^2 + 2xh + h^2 - y^2 + 4xy + 4hy - 7x - 7h + 10$ (e) $4x^2 - 7x + 10$
3. (a) 0 (c) $\ln x$ (e) $\ln(xy + kx + y + k - 1)$
 (b) $\ln e - 1$ (d) $\ln(xy + hy + y - 1)$
4. (a) 1 (c) $\frac{3t}{t^2 + 2}$ (e) $\frac{3x}{1 + 2x^2}$
 (b) $-\frac{2}{3}$ (d) $-\frac{3y}{1 + 2y^2}$
5. (a) $\frac{1}{2}$ (d) $u \operatorname{sen}(v)$
 (b) $2\sqrt{2}$ (e) $x \cos(x) [\operatorname{sen}(x) \cos(y) + \operatorname{sen}(y) \cos(x)]$
 (c) $t \operatorname{sen}(t) \cos(t)$
6. (a) \mathbb{R}^2, \mathbb{R} (d) $\{(x, y) : x \neq 0\}, \{z : -\frac{\pi}{2} < z < \frac{\pi}{2}\}$
 (b) $\{(x, y) : x \geq y\}, \{z : z \geq 0\}$ (e) $\{(x, y, z) : yz \neq 0\}, \mathbb{R}$
 (c) $\{(x, y) : x + y \neq 0\}, \{z : z \neq 0\}$ (c) \mathbb{R}^3, \mathbb{R}
7. (a) 1 (b) \mathbb{R}^2 (c) $\{z : z > 0\}$
8. (a) $\sqrt{11}$ (b) $\{(x, y) : \frac{1}{4}x^2 + \frac{1}{9}y^2 \leq 1\}$ (c) $\{z : 0 \leq z \leq 6\}$
9. (a) 0 (b) $\{(x, y, z) : x + z > y\}$ (c) \mathbb{R}
10. (a) $\frac{1}{5}$ (b) $\{(x, y, z) : x^2 + y^2 + z^2 > 1\}$ (c) $(0, \infty)$
11. (a) $\{(x, y) : y \leq -x^2\}$

 (b) $\{(x, y) : y \neq -\frac{1}{2}x \text{ e } x^2 + y^2 \leq 9\}$

 (c) $\{(x, y) : y \neq x \text{ e } y \neq -x\}$



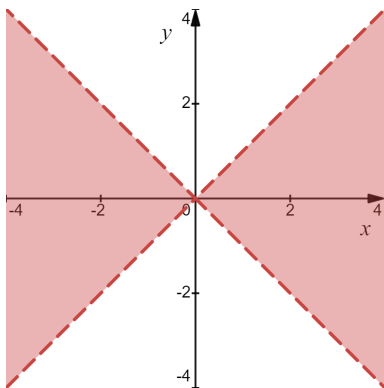
(d) $\{(x, y) : x - y \neq \frac{\pi}{2} + n\pi, n \text{ um inteiro} \}$



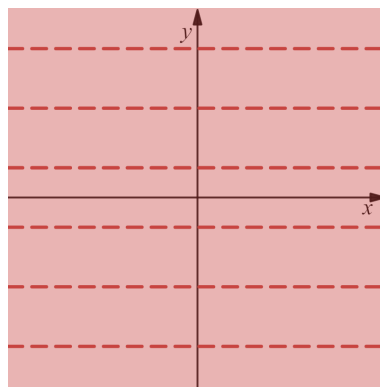
(e) $\{(x, y) : xy > 1\}$



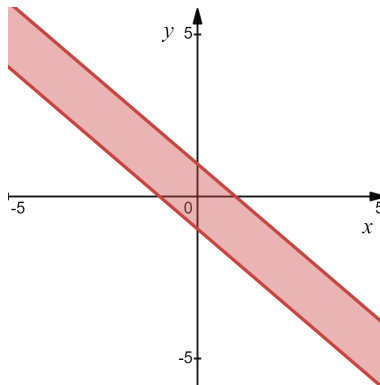
(f) $\{(x, y) : |y| < |x|\}$



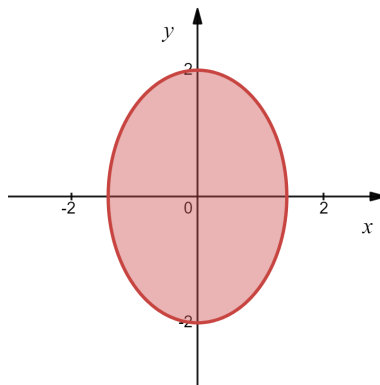
(g) $\{(x, y) : y \neq \frac{\pi}{2} + n\pi, n \text{ um inteiro} \}$



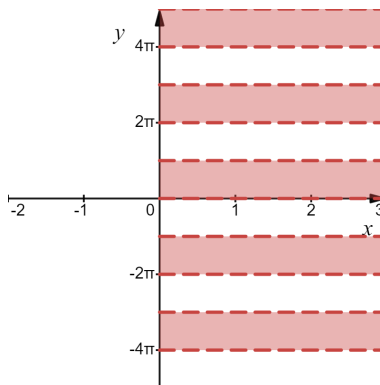
(h) $\{(x, y) : -1 - x \leq y \leq 1 - x\}$



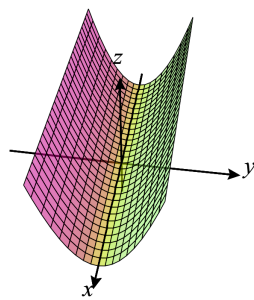
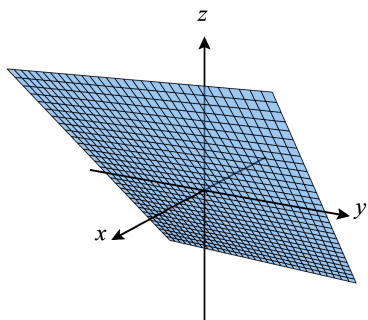
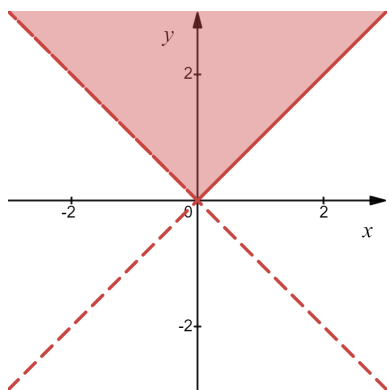
(i) $\{(x, y) : 2x^2 + y^2 \leq 4\}$



(j) $\{(x, y) : x > 0 \text{ e } 2\pi n < y < (2n + 1)\pi, n \text{ um inteiro} \}$

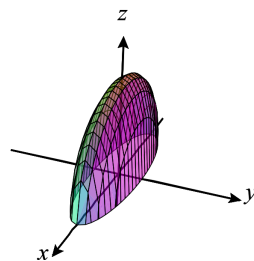
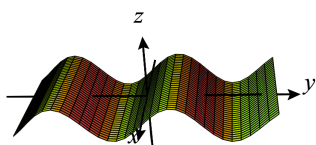


(k) $\{(x, y) : -y < x \leq y, y > 0\}$



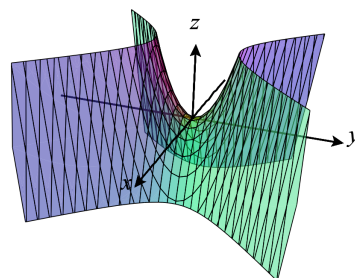
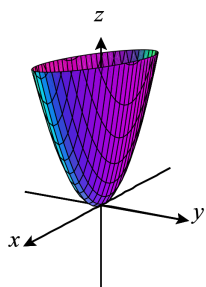
12. (a)

(d)



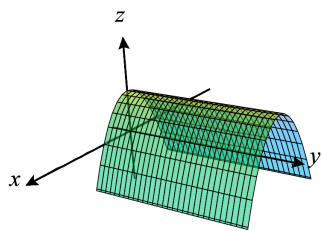
(b)

(e)

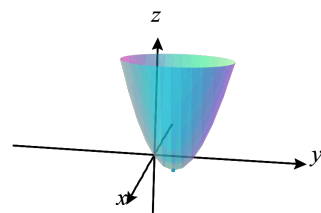


(c)

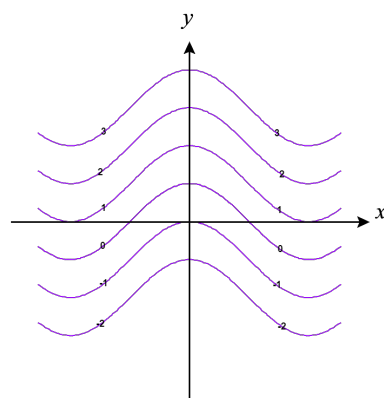
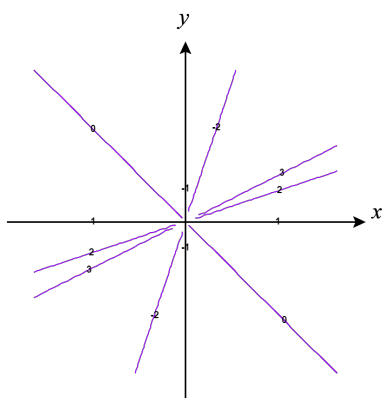
(f)



(g)

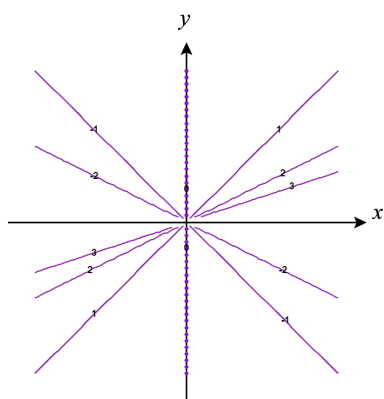


(h)

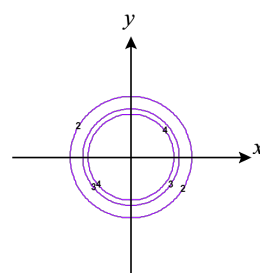


13. (a)

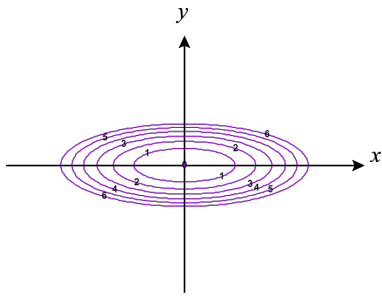
(c)



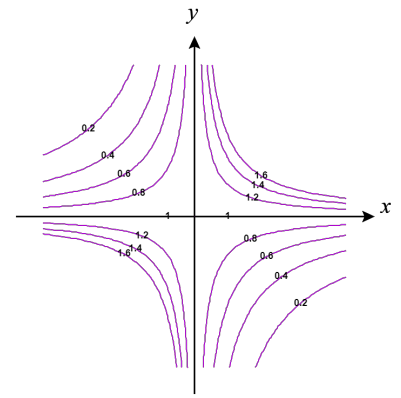
(b)



(d)

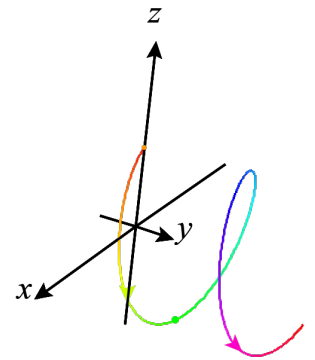
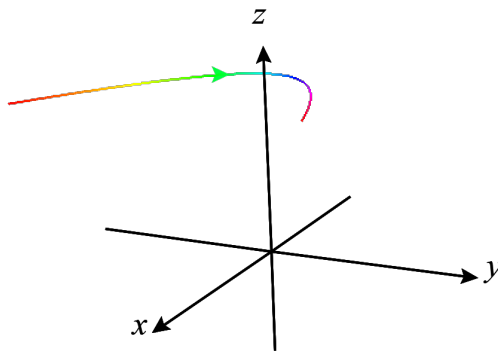


(e)



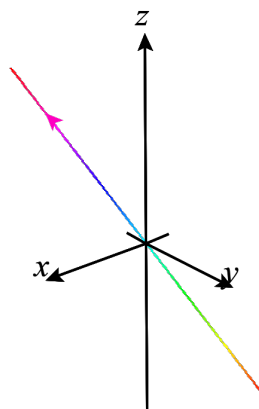
(f)

14. $(0, 1) \cup (1, \infty)$



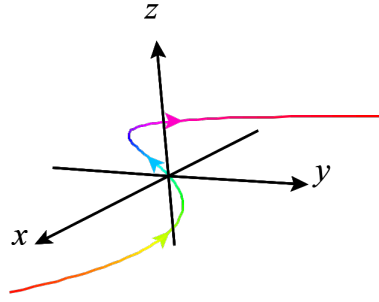
15. (a)

(c)

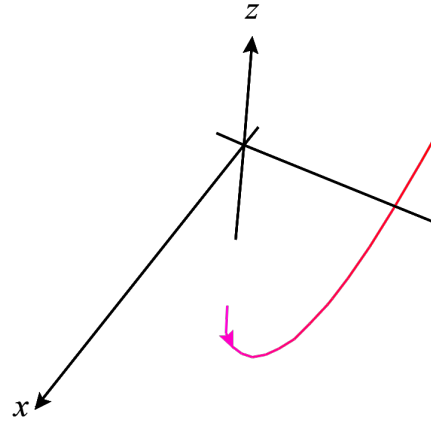


(b)

16.



(a)



(b)

17. Sim