a) GRR = 20241770

B) $A = 1$ $\quad B = 7$ $\quad C = 7$ $\quad D = 0$

$\alpha = 2$ $\quad \beta = 8$ $\quad \gamma = 8$ $\quad \omega = 1$

$$f(x) = \frac{8}{x+8} + 1$$

c) Vamos provar por ε e δ que $\lim\limits_{x \to 2} \frac{8}{x+8} + 1 = \frac{8}{10} + 1$

Provando que $\forall\, \varepsilon > 0\ \exists\, \delta > 0$ TQ se

$0 < |x-2| < \delta$ então $\left|\frac{8}{x+8} + 1 - \left(\frac{8}{10} + 1\right)\right| < \varepsilon \to \left|\frac{16-8x}{10(x+8)}\right| < \varepsilon$

$\to \left|\frac{8(2-x)}{10(8+x)}\right| < \varepsilon \to \left|\frac{8}{10(8+x)}\right| \cdot |2-x| < \varepsilon \to \left|\frac{8}{10(8+x)}\right| \cdot |x-2| < \varepsilon$

Tomando $\delta \leq 1$:

$-1 < x-2 < 1 \to 9 < x+8 < 11 \to \frac{10 \cdot 9}{8} < \frac{10(8+x)}{8} < \frac{10 \cdot 11}{8} \to$

$\to \frac{8}{10 \cdot 11} < \frac{8}{10(8+x)} < \frac{8}{10 \cdot 9}$. Assim tome δ como MIN $\left\{1 ; \frac{9 \cdot 10\, \varepsilon}{8}\right\}$

Concluindo:

$$\left|\frac{8}{10(8+x)}\right| \cdot |x-2| < \frac{8}{10 \cdot 9} \cdot \delta = \frac{\cancel{8}}{\cancel{10 \cdot 9}} \cdot \frac{\cancel{9 \cdot 10}\, \varepsilon}{\cancel{8}} = \varepsilon.$$

$\therefore$ Provamos que $\lim\limits_{x \to 2} \frac{8}{x+8} + 1 = \frac{8}{10} + 1$

d) Usando a definição formal mostre que

$$\lim_{x \to -8^+} \frac{8}{x+8} + 1 = +\infty$$

$\forall M > 0 \ \exists \ \delta > 0$ TQ $0 < |x+8| < \delta$ e $x > -8$, então

$\frac{8}{x+8} + 1 > M \rightarrow \frac{8 + x + 8}{x+8} > M \rightarrow \frac{x+16}{x+8} > M$. Provando:

Tomando $\delta \leq 1$ temos que:

$-1 < x+8 < 1 \rightarrow 7 < x+16 \rightarrow \frac{7}{x+8} < \frac{x+16}{x+8}$

Sendo assim, como:

$\frac{x+16}{x+8} > \frac{7}{x+8}$ vamos provar que: $\frac{7}{x+8} > M$, para que: $\frac{x+16}{x+8} > M$

Descobrindo o δ:

$x+8 < \delta \rightarrow 1 < \frac{\delta}{x+8} \rightarrow \frac{1}{\delta} < \frac{1}{x+8} \rightarrow \frac{7}{\delta} < \frac{7}{x+8}$

Desse modo, $\delta = \min\left\{1, \frac{7}{M}\right\}$

Concluindo:

$x+8 < \delta \rightarrow x+8 < \frac{7}{M} \rightarrow 1 < \frac{7}{M(x+8)} \rightarrow \frac{7}{x+8} > M$. O que prova que

$$\lim_{x \to -8^+} \frac{8}{x+8} + 1 = +\infty$$