MBA USP ESALQ

Operational Research and Optimization and Simulation Modeling III

Prof. Dr. Marcos dos Santos

BACKGROUND

- Senior Officer, 29 years of service in the Navy of Brazil;
- Navy Academy;
- Naval School;
- Viagem de instrução de Guardas-Marinha (VIGM) in 2001;
- 10 years on board of war vessels;
- 11 years in CASNAV: Researcher and Project Manager in the Corp of Operational Research;
- Professor of P.O. of CAAML, EsAO, CIASC and ECEME;
- Specialized in Mathematical Instrumentation (UFF)
- Specialization in Mathematics (IMPA);
- IT Governance (FGV-RJ);
- Graduate Course in Production Engineering - Operational Research (COPPE/UFRJ);
- PhD and post-doctoral in Systems, Support to Decision and Logistics (UFF);
- Post-doctoral in Sciences and Spatial Technologies (ITA);
- Board of the Brazilian Society of Operational Research (SOBRAPO);
- Professor of MBA in Data Science and Analytics (USP);
- Professor of the Graduate program in Production Engineering (UFF);
- Professor of the Graduate Program in Systems and Computing (IME).

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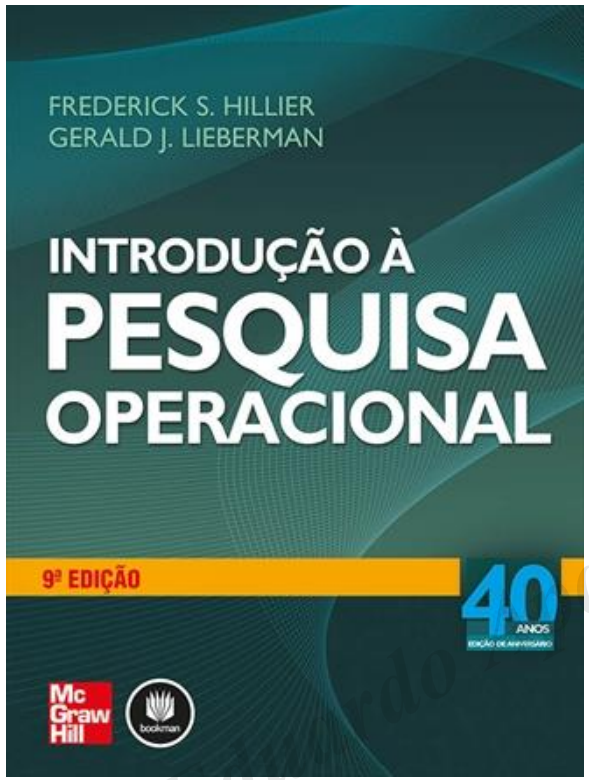
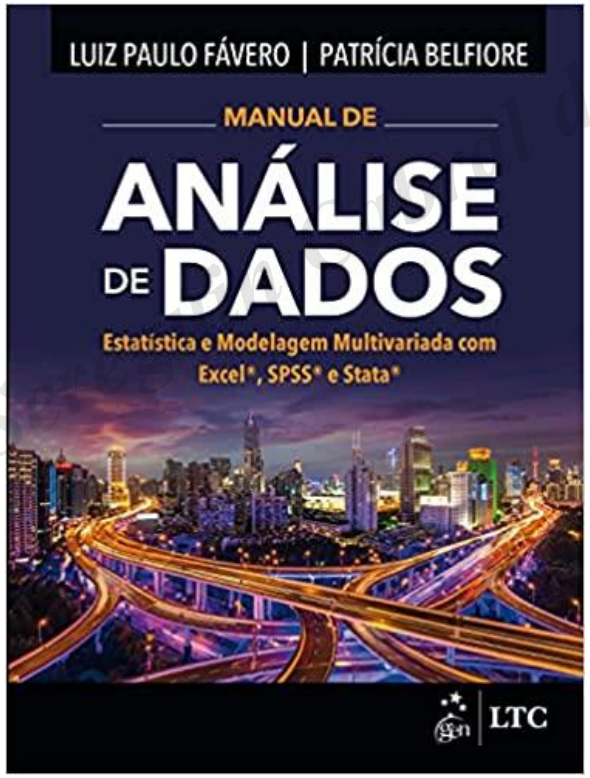
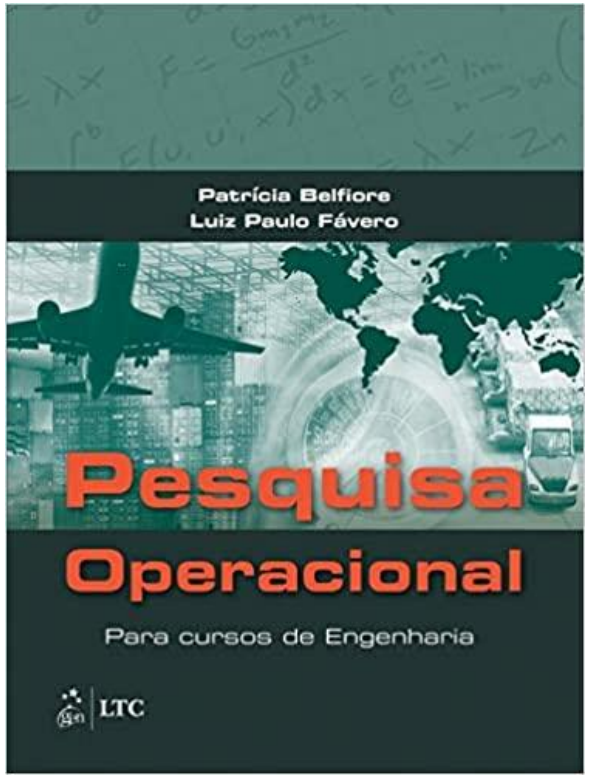
REFERENCES







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OBJECTIVE

To determine the "Optimal Solution" of a LPP through the Simplex Method, especially appropriate for problems with more than two D.V. (departing variable).

OVERVIEW

We can not forget that O.R. is a **multidisciplinary science** that gives several techniques of pr m ring and mathematical models tha to ment context that we

The O.R. is not even close of being a "remedy" of mathematical algorithms to be memorized!

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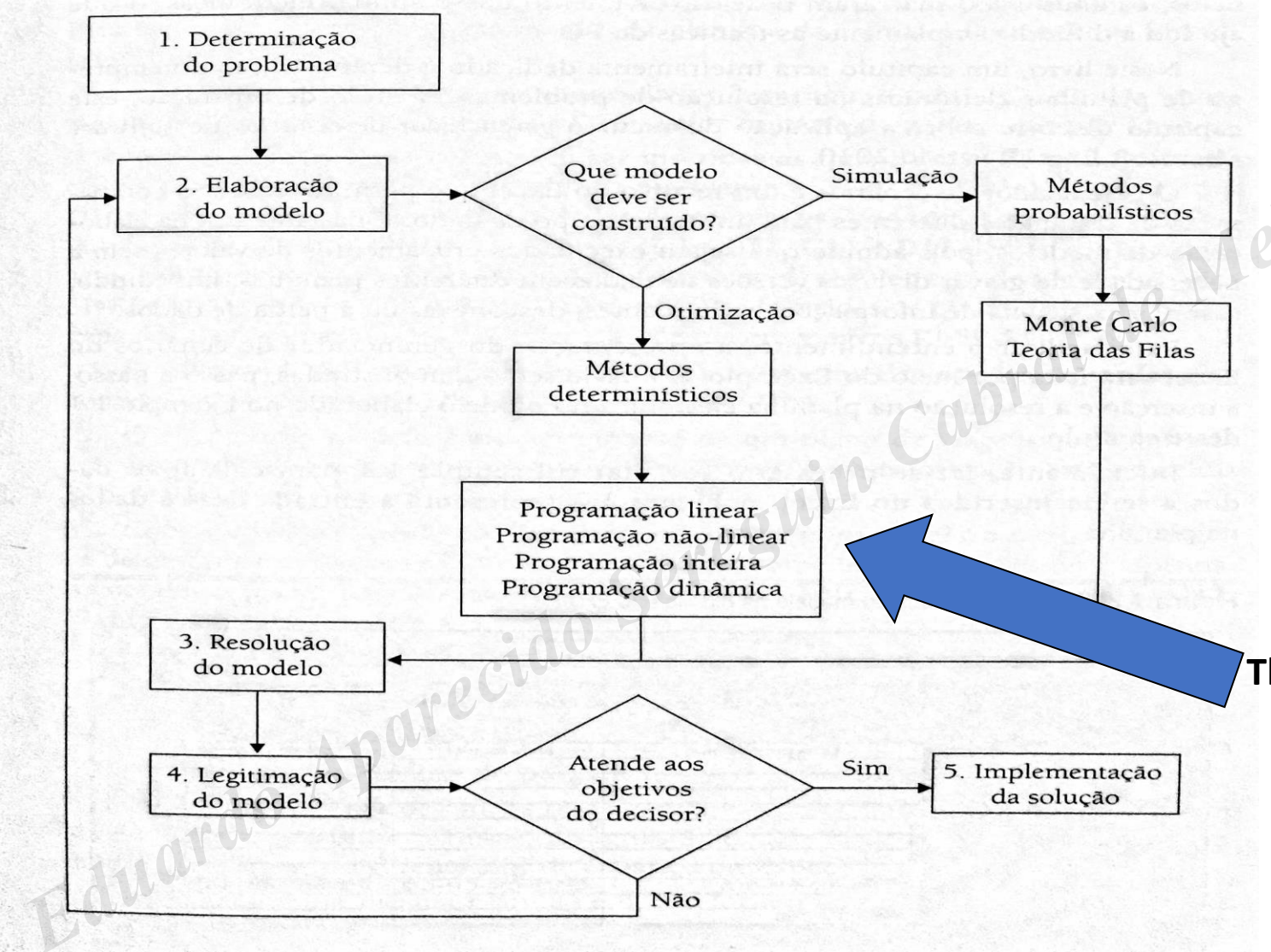
TYPES OF METHODS



The LP uses deterministic methods.

ALGEBRAIC METHODS

In the previous class we could solve a LP model using the **graphical method**. Now, it is possible to learn the procedures for determining the solution using **algebraic methods**.

The algebraic methods for solving Linear Programming are more robust than the graphical method. That's because its use does not implicate the limitation in the number of variables.

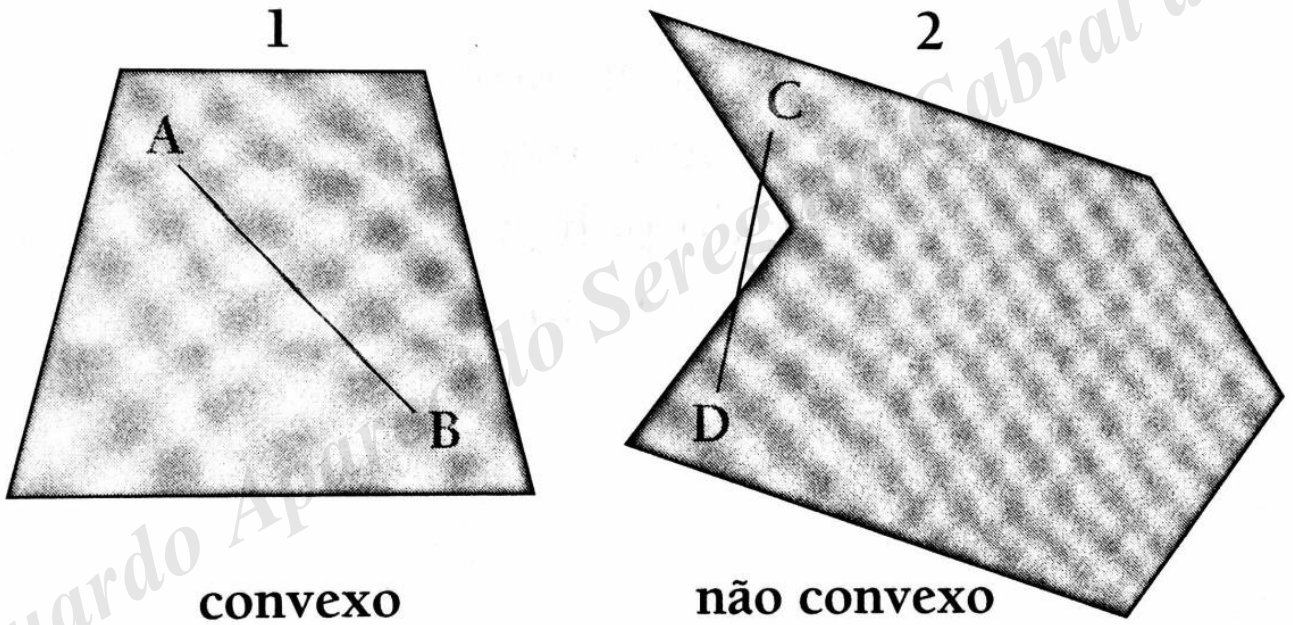
ALGORITHM

The development of the SIMPLEX Method occurs **through a standardized set of routines or instructions** that perform the mathematical calculation (algorithm).

Therefore, even the reader that is not used to the mathematics of vectors and the resolution of linear equation and inequation systems will not find greater difficulties in the use of SIMPLEX.

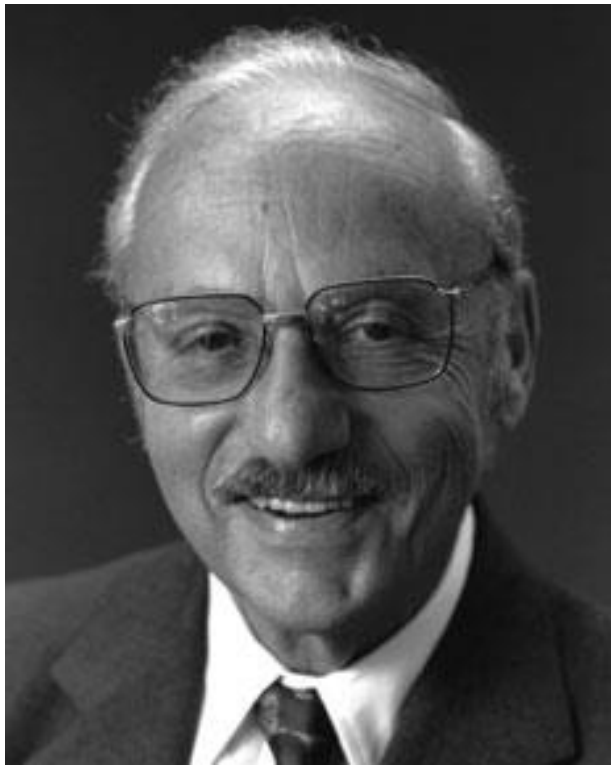
CONVEXITY

A set is called convex when it contains all segments that join any two points of this set.



CREATION OF SIMPLEX METHOD

The SIMPLEX Method for LPP's resolution was developed and improved by the American mathematical George Dantzig (1914– 2005). Because of his work, that was published in the year of 1947 and presented to the world mathematical community of SIMPLEX, Dantzig became known as the "Dad of Linear Programming".



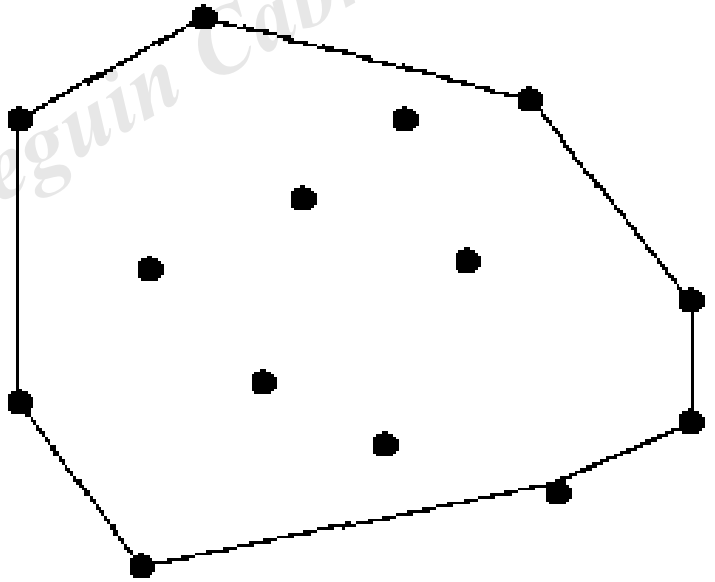
ITERATION

1) Iteration = repetition.

2) Resolution process of an equation through operations in which successively the object of each one is the result of the previous one.

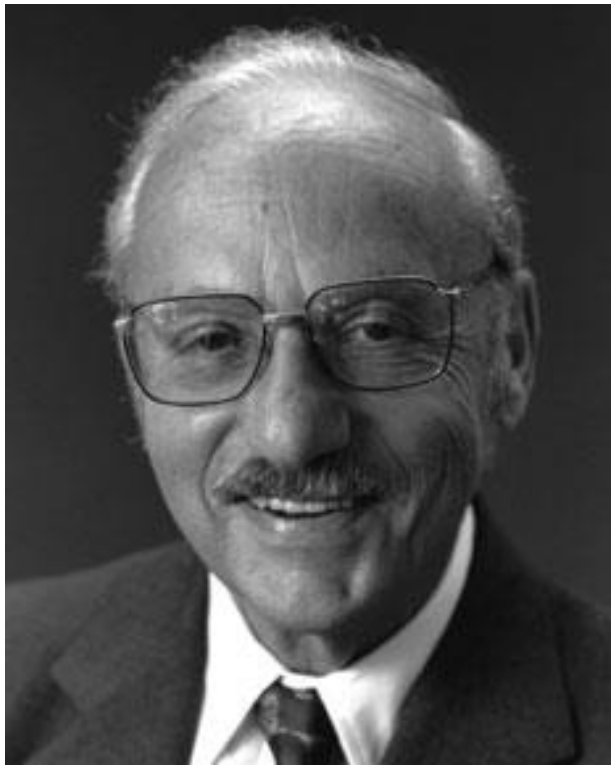
STAGES OF THE SIMPLEX METHOD

In general, the SIMPLEX seeks, from a first feasible solution, **to examine in an iterative way the vertices of a polygon** to reach a solution considered optimal for the problem.



STAGES OF THE SIMPLEX METHOD

In practice, the use of SIMPLEX is performed through a sequence of steps. They are: (01) reduction of the linear system to **the canonical form**; (02) construction of **the SIMPLEX Tableau**; (03) determination of **the Initial Basic Solution**; (04) choice of the variables that enter into the base and the variables that don't; (5) calculation of the basic solution and (06) test of this new solution.



LPP IN THE STANDARD FORM

1. The Objective Function is of Maximization;

2. The restrictions have the sign of inequality (less than or equal to);

3. The constants of all restrictions are non-negative;

4. The decision variables can only assume non-negative values.

LPP IN THE STANDARD FORM

To maximize $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

Subject to :

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2$$

→ Non-negative

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m$$

$$x_1, x_2, x_3, \ldots x_n \geq 0$$

LPP IN THE STANDARD FORM

Standard Form

$$\text{Max } X_1 + X_2$$

s.r.

$$2X_1 + 4X_2 \leq 20$$

$$180X_1 + 20X_2 \leq 600$$

$$X_1, X_2 \geq 0$$

Non-Standard Form

$$\min x_1 + 2x_2$$

s.r.

$$2x_1 + 3x_2 \geq 20$$

$$180x_1 + 20x_2 = 600$$

$$x_1, x_2 \geq 0$$

TRANSFORMING AN INEQUATIONS SYSTEM INTO A SYSTEM OF EQUATIONS

SLACK VARIABLE:

$$\max z = 300x_1 + 200x_2$$

$$\begin{aligned} \text{s.a.} \quad & 3x_1 + x_2 \leq 15 \\ & 3x_1 + x_2 \leq 10 \\ & 50x_1 + 10x_2 \leq 70 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Attention: Variables of Slack or Surplus

Given a Linear Programming Problem, we need to insert the slack or surplus variables so that the inequalities become equities.

TRANSFORMING AN INEQUATIONS SYSTEM INTO A SYSTEM OF EQUATIONS

SURPLUS VARIABLE:

$$\min x_1 + 2x_2$$

s.r.

$$2x_1 + 3x_2 \geq 20$$

$$180x_1 + 20x_2 = 600$$

$$x_1, x_2 \geq 0$$

Let's practice!

TRANSFORMING AN INEQUATIONS SYSTEM INTO A SYSTEM OF EQUATIONS

Exercise
Introduce the variables of slack and/or surplus in the the following LPP:

$$\begin{aligned} \text{Maximize} \quad & Z = 5x_1 + 6x_2 \\ \text{Subject to} \quad & x_1 \leq 6 \\ & 2x_2 \leq 12 \\ & 3x_1 + 2x_2 \leq 18 \\ \text{To} \quad & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

SIMPLEX TABLEAU

PRODUTO	LUCRO UNITÁRIO	HOMENS HORA POR UNIDADE PRODUZIDA	
		DEPARTAMENTO DE MONTAGEM	DEPARTAMENTO DE ACABAMENTO
Mesas	R$ 20	3	4
Bancos	R$ 24	6	2
Homens hora disponíveis		60	32

SIMPLEX TABLEAU

PRODUCT	UNITARY PROFIT	Men hours/produced unit	
		MOUNTING DEPARTMENT	FINISHING DEPARTMENT
Tables	R$ 20	3	4
Stools	R$ 24	6	2
Available men per hour		60	32

Let's practice!

SIMPLEX TABLEAU

Exercise

Build Simplex Tableau of the following LPP:

$$\max z = 300x_1 + 200x_2$$

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SIMPLEX METHOD

Step by step...

Empregando o Método do Simplexo, resolva:

$$\text{Max } Z = 7x_1 + 3x_2 + 2x_3$$

sujeito a

$$5x_1 + 2x_2 + 2x_3 \leqslant 19$$

$$2x_1 + x_2 + 2x_3 \leqslant 8$$

$$x_1 \geqslant 0,\ x_2 \geqslant 0,\ x_3 \geqslant 0$$

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REAL PROBLEMS USING THE SIMPLEX METHOD

ESTRUTURAÇÃO E MODELAGEM MATEMÁTICA NO APOIO À TOMADA DE DECISÃO: ESTUDO DE CASO DE UMA FÁBRICA DE SACOS DE LIXO

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MARCOS DOS SANTOS - marcosdossantos_doutorado_uff@yahoo.com.br
INSTITUTO MILITAR DE ENGENHARIA - IME

Com a finalidade de proporcionar o lucro mediante a maximização da receitas da empresa, por meio do mix produtivo, a função objetivo fica definida pelo somatório das receitas líquidas geradas pela venda de cada um dos produtos. Assim, tem-se:

$$F.O. = Max \{14,79X_1 + 14,06X_2 + 23,06X_3 + 20,84X_4 + 12,51X_5 + 12,07X_6 + 19,37X_7 + 17,47X_8 + 15,61X_9 + 13,69X_{10} + 22,07X_{11} + 21,05X_{12} + 13,4X_{13} + 11,95X_{14} + 18,83X_{15} + 17,65X_{16} + 18,43X_{17} + 17,3X_{18} + 18,11X_{19} + 18,15X_{20} + 7,15X_{21} + 8,71X_{22} + 11,45X_{23} + 7,42X_{24} + 9,08X_{25} + 11,88X_{26}\}$$

*The responsibility for

REAL PROBLEMS USING THE SIMPLEX METHOD

Aplicação da Programação Linear na formulação de uma dieta de custo mínimo: estudo de caso de uma empresa de refeições coletivas no Estado do Rio de Janeiro

5.1.1. Função Objetivo

Compondo o custo de cada cota e a respectiva variável de decisão, pode-se definir a FO, apresentada na expressão 1 a seguir:

$$
\begin{aligned}
F.O. = \min \{ & 0{,}197x_1 + 0{,}195x_2 + 0{,}3x_3 + 0{,}189x_4 + 0{,}242x_5 + 0{,}199x_6 + 0{,}469x_7 \\
& + 0{,}340x_8 + 0{,}139x_9 + 0{,}532x_{10} + 0{,}109x_{11} + 1{,}190x_{12} + 0{,}329x_{13} \\
& + 0{,}00138x_{14} + 1{,}490x_{15} + 0{,}0525x_{16} + 0{,}200x_{17} + 0{,}200x_{18} + 0{,}150x_{19} \\
& + 0{,}220x_{20} + 0{,}650x_{21} + 0{,}28x0_{22} + 0{,}250x_{23} + 0{,}250x_{24} + 0{,}222x_{25} \\
& + 0{,}300x_{26} + 0{,}275x_{27} + 0{,}360x_{28} + 0{,}150x_{29} + 0{,}140x_{30} + 0{,}292x_{31} \\
& + 0{,}150x_{32} + 0{,}300x_{33} + 0{,}350x_{34} + 0{,}200x_{35} + 0{,}375x_{36} + 1{,}590x_{37} \\
& + 0{,}870x_{38} + 1{,}100x_{39} + 0{,}857x_{40} + 0{,}898x_{41} + 2{,}630x_{42} + 0{,}930x_{43} \\
& + 0{,}880x_{44} \}
\end{aligned}
\qquad (1)
$$

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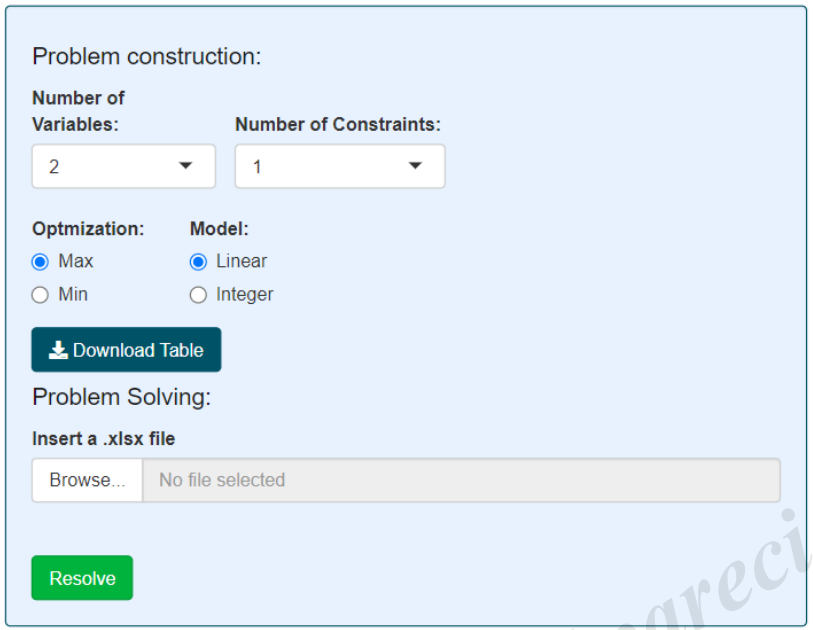
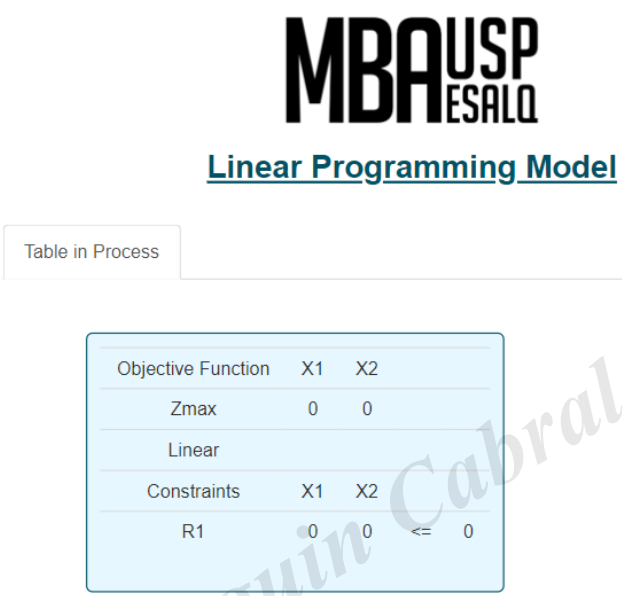
SIMPLEX METHOD USING R





https://marcosdossantos.shinyapps.io/pl_shiny/

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SIMPLEX METHOD USING R

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Homens hora disponíveis		60	32

SIMPLEX METHOD USING R

Empregando o Método do Simplexo, resolva:

$$\text{Max } Z = 7x_1 + 3x_2 + 2x_3$$

sujeito a

$$5x_1 + 2x_2 + 2x_3 \leqslant 19$$

$$2x_1 + x_2 + 2x_3 \leqslant 8$$

$$x_1 \geqslant 0,\ x_2 \geqslant 0,\ x_3 \geqslant 0$$

SIMPLEX METHOD USING R

Resolva o problema de Programação Linear

$$\text{Min } Z = x_1 + 2x_2 - x_3$$

sujeito a

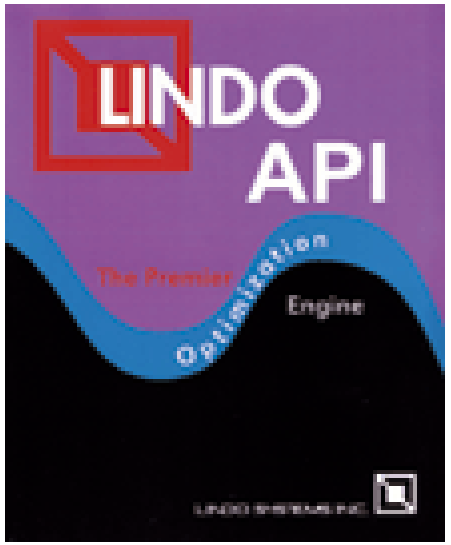
$$x_1 - x_2 + 4x_3 \geqslant -30$$

$$2x_1 - x_2 + 2x_3 \leqslant 10$$

e

$$x_1, x_2, x_3 \geqslant 0$$

OTHER SOFTWARES



python™

PHPSimplex

GUROBI
OPTIMIZATION

And the question remains:

which one is the best?

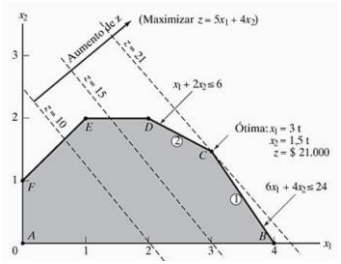
CONCLUDING

It does not matter the software that you will use. This is a question of personal preference. **The focus must be the solution of the problem of your organization**.

And for this, **you must know the methods**.

The software are meres facilitators!

I learned to solve a LPP using the Graphical Solution.



I learned to solve a LPP, using the Simplex Method.

	z	x_1	x_2	s_1	s_2	s_3	b
z	1	0	0	1.6	2.2	0	544
s_1	0	1	1	0.4	−0.2	0	16
s_2	0	0	1	−0.2	0.6	0	72
s_3	0	0	0	−0.4	0.2	1	19

I learned to solve a LPP using R.

OBJECTIVE

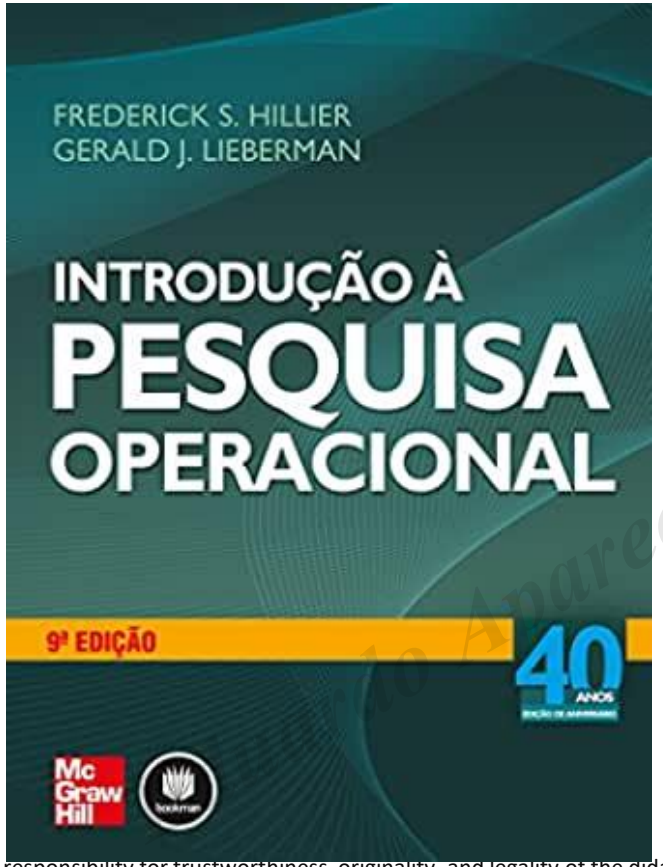
To determine the "Optimal Solution" of a LPP through the Simplex Method, especially appropriate for lems with more than two D.V. (departing variable).

MBA USP ESALQ

INTRODUCTION TO THE SIMULATION

Prof. Dr. Marcos dos Santos

REFERENCES



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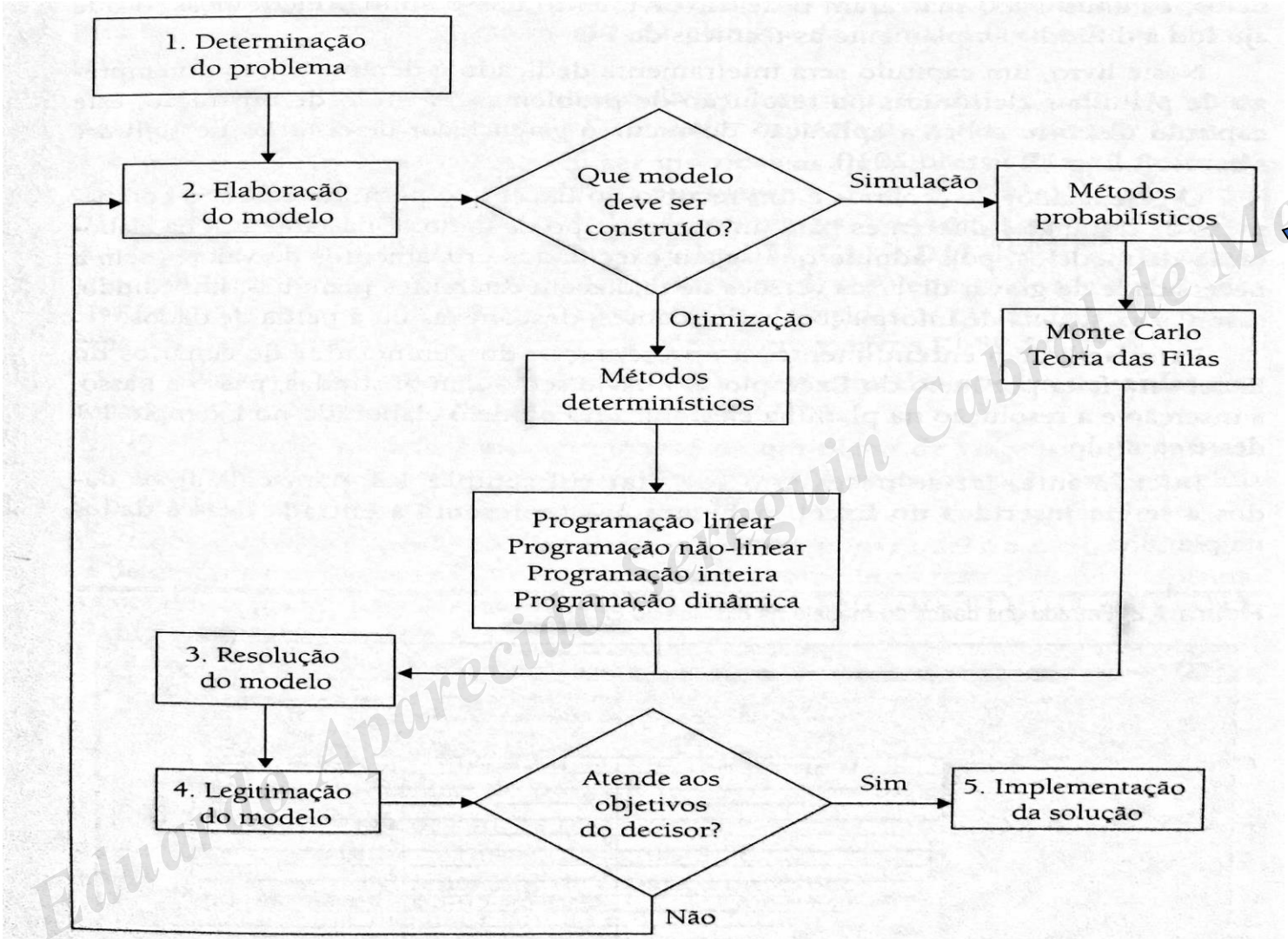
OBJECTIVE

To present the concept of Simulation, in what types of problem it must be used and what a Generator of Random Numbers is.

TYPES OF METHODS



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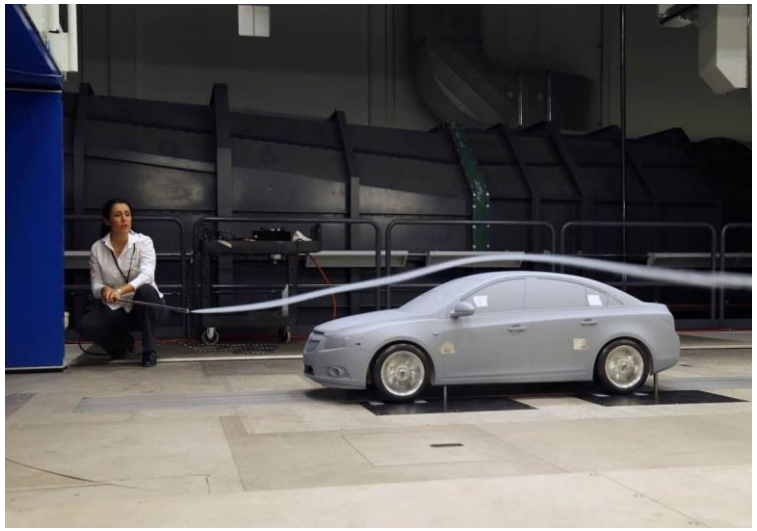
SIMULATION

The term "Simulation" can be classified into two categories: **the computational simulation** and **the non-computational simulation**.



The computational simulation is the one that needs a computer to be performed. The non-computational simulation does not need a computer to be performed; for example, a designer using a prototype in a reduced scale of an aircraft in a wind tunnel.

COMPUTER SIMULATION

Simulation is a stochastic process that uses **models that imitate or simulate the reality** through probabilistic or random values that feed these models.

The models used by Simulation are algorithms that generate, from random values or drawn values, **a large number of problem results**. The set of obtained results allows to calculate the average values, the deviations, the maximum or minimum values of the variables of the simulated problem.

COMPUTER SIMULATION

Law of Large Numbers.

The **law of large numbers** (LLN) is a fundamental theorem of probability theory, which describes the result of the same experience repeated many times. According to the LLN, the arithmetic mean of the results of the realization of the same experiences repeatedly tends to get closer to the expected value as the attempts succeed.

Ex: Flipping a coin N times:

N = 10 throws

N = 100 throws

N = 1000 throws

Expected Value:

P (heads) =

P(tails) =

TYPES OF COMPUTER SIMULATION

1. Discrete Event Simulation

It is the one in which the changes of state of the system occurs instantaneously in random points of the time as a result of the discrete events occurrence.

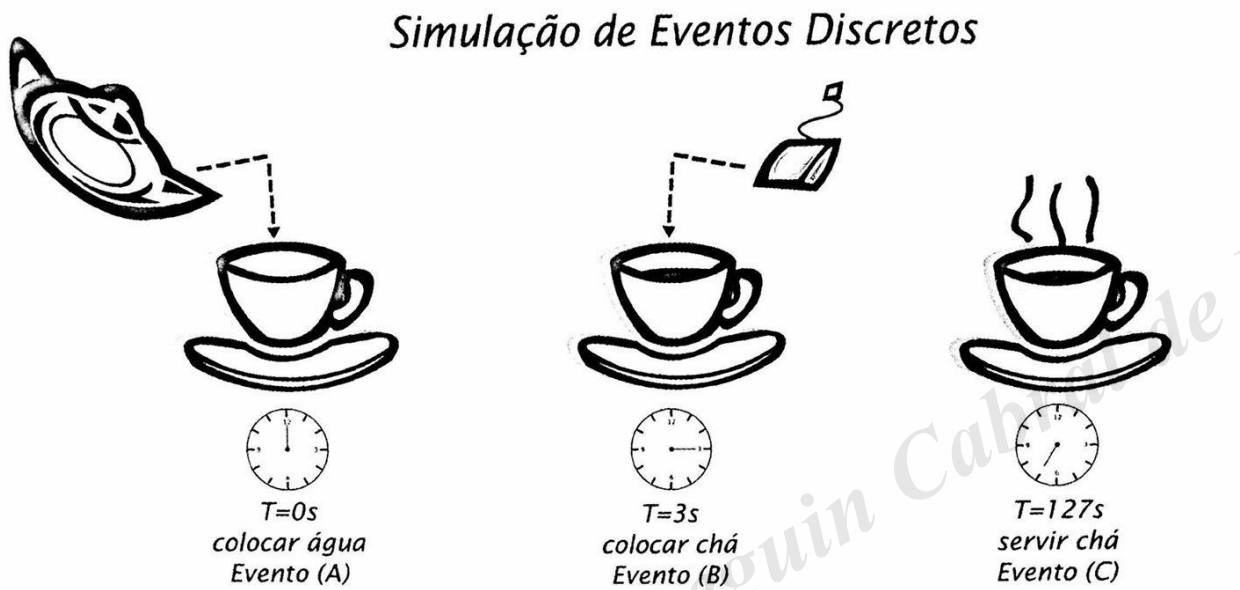
2. Continuous Simulation:

It is the one in which the changes in the system's state occurs continuously over time.

3. Monte Carlo Simulation:

It creates a model of possible results, using a probability distribution, such as an uniform or normal distribution, for any variable that has a degree of uncertainty. Then, it recalculates the results successively, each time using a different set of random numbers between the minimum and maximum values.

TYPES OF COMPUTER SIMULATION



Chwif and Medina (2007)

TYPES OF COMPUTER SIMULATION

Discrete-event Simulation

Available online at www.sciencedirect.com

ScienceDirect

Procedia Computer Science 55 (2015) 931 – 938

Procedia
Computer Science

Information Technology and Quantitative Management (ITQM 2015)

Simulation of Operation of An Integrated Information for Emergency Pre-Hospital Care In Rio de Janeiro Municipality

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[a] Master of Science in Production Engineering (UFRJ) [b] Master of Science in Accounting (UERJ) [c] Master in Civil Engineering (UFF) [d] PhD in Production Engineering (UFRJ)

Abstract

The health department of Rio de Janeiro has required a project to assess the service capacity of the emergency service, in view of the urban and natural disasters that are happening in many places, and large events that are about to happen in the city of Rio de Janeiro, as the Olympics in 2016. The objective of this work is to propose a system of information and coordination involving three agencies: the Operations Center of the GSE / SAMU, the Operations Center of the Municipality of Rio de Janeiro and the Internal regulation nucleus (NIR) of municipal hospitals. The model is based on the information flow, focusing on highly complex patients. The discrete event simulation is used to identify how the system responds to the increased demand. It is stated that the "use of information" is the key to improving overall system performance.

Monte Carlo Simulation

During the World War II, the Hungarian mathematical John Von Neumann, in his work on the Manhattan Project, created the concept called "Monte Carlo Simulation".

Each atom is a random variable. The effect generated by a set of atoms is the composition of several random variables.

The name was originated from randomness and the repetitive nature of the activities performed in Monte Carlo casinos. The roulette of a casino plays the role of a **Random Number Generator** (RNG).

Monte Carlo Simulation

GERENCIAMENTO DE RISCOS
NO PLANEJAMENTO DE EMPREENDIMENTOS:
UMA ABORDAGEM A PARTIR
DA SIMULAÇÃO DE MONTE CARLO

Marcos dos Santos
Capitão de Corveta. Pesquisador e Gerente de Projetos no Centro de Análises de Sistemas Navais (CASNAV).
Doutorando em Engenharia de Produção (UFF).

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RANDOM NUMERS

A random number can represent arbitrary decisions or serve as an input for times generation according to several distributions.

To produce random numbers:

- Physical devices (Ex: data, roulettes, coins etc.) ;
- Table of random numbers;
- Mathematical processes.

RANDOM NUMERS

Physical Devices

RANDOM NUMERS

Table of Random Numbers

Tabela de Números Aleatórios

57	72	00	39	84	84	41	79	67	71	40	21	13	97	56	49	86	54	08	93	29	68	74	54	83
28	80	53	51	59	09	93	98	87	58	70	27	71	77	17	06	32	02	78	62	16	74	69	65	17
92	59	18	52	87	30	48	86	97	48	35	25	18	88	74	03	62	98	38	58	65	86	42	41	03
90	38	12	91	74	30	19	75	89	07	50	64	15	59	71	88	13	74	95	30	52	78	30	11	75
80	91	16	94	67	58	60	82	06	66	90	47	56	18	46	45	11	12	35	32	45	50	41	13	43
22	01	70	31	32	96	91	92	75	40	16	54	29	72	74	99	00	95	97	61	00	98	24	30	07
56	24	10	04	30	20	46	29	90	53	53	11	05	84	41	21	64	79	19	76	29	51	62	60	66
79	44	92	62	02	96	86	64	30	00	94	56	69	30	20	59	87	87	35	44	22	50	97	78	19
53	99	66	45	08	89	78	50	77	53	37	25	77	41	27	62	38	02	23	57	62	01	41	60	35
18	92	87	35	88	56	05	21	36	51	39	28	50	14	66	85	79	30	19	79	72	66	64	31	45
53	08	58	96	63	05	61	25	70	22	50	41	28	96	62	66	43	63	06	63	01	32	79	85	22
03	58	80	29	28	76	89	51	18	24	88	89	46	47	48	59	19	29	87	03	10	33	99	67	12
27	07	81	88	65	69	49	98	00	28	04	70	51	30	01	47	18	97	33	21	85	82	45	43	24
05	21	08	59	01	06	22	24	98	91	81	17	55	44	66	16	07	73	07	66	10	12	31	78	58
40	36	13	27	84	30	82	33	36	39	69	42	05	58	64	61	12	33	89	27	89	52	66	71	93
54	60	25	28	85	88	20	00	10	59	61	05	36	61	33	72	01	01	19	01	61	10	51	20	91
71	51	63	40	76	71	11	73	73	52	37	31	60	45	88	92	73	43	71	28	04	98	09	02	48
61	02	01	81	73	92	60	66	73	58	53	34	42	68	26	38	34	03	27	44	96	04	46	65	93
82	55	93	13	46	30	95	26	55	06	96	17	65	91	72	39	79	96	12	49	52	80	63	26	99
89	98	54	14	21	74	13	57	68	19	86	28	60	89	47	33	15	26	28	77	45	38	48	08	08
00	99	84	84	14	67	95	13	77	58	90	14	50	79	42	73	63	31	06	60	43	40	12	55	04
62	41	50	78	20	48	05	88	43	52	98	03	19	93	92	03	04	97	25	84	95	95	03	63	31
94	27	90	69	24	68	09	92	11	86	07	63	83	19	32	99	51	15	55	71	09	27	02	67	00
44	89	29	28	84	36	28	25	15	82	87	74	18	97	25	76	10	63	26	76	02	26	74	53	28
97	30	76	95	33	21	10	54	26	95	66	65	52	04	99	36	58	48	03	08	93	63	58	17	96
39	16	58	04	44	80	15	59	59	83	90	95	54	66	81	84	39	60	85	38	88	66	33	35	69
60	78	11	03	26	67	50	34	09	61	31	30	20	76	93	66	30	83	51	09	33	83	64	76	05
03	19	23	47	62	89	57	77	91	33	88	47	60	59	37	54	39	48	77	67	49	85	38	43	91
41	28	52	67	56	25	39	59	96	65	51	36	90	32	22	39	33	05	22	99	03	39	97	96	99
77	54	98	50	39	25	37	42	52	97	10	03	56	04	92	81	66	86	70	01	48	89	55	82	10
28	63	41	61	91	64	24	83	81	37	34	48	83	27	96	38	71	69	73	06	77	50	25	64	60
74	24	48	85	40	12	33	59	67	50	14	98	14	26	42	79	79	13	52	89	69	78	80	44	71
00	24	03	37	96	46	68	75	05	32	42	16	63	33	28	97	26	36	47	27	73	65	38	34	46
05	41	47	69	69	45	36	16	71	18	95	51	97	22	04	13	23	96	58	60	03	69	48	79	83
62	69	84	97	97	47	23	66	51	56	13	08	69	11	52	75	59	26	86	81	80	43	00	98	92

RANDOM NUMERS

Mathematical Methods

Middle-Square Method (Von Neumann – 1946)

From a "seed" of 2 digits, there is:

$$r_0 = 76 \Rightarrow 76^2 = 5776$$

$$r_1 = 77 \Rightarrow 77^2 = 5929$$

$$r_2 = 92 \Rightarrow \ldots\ldots\ldots\ldots\ldots.$$

Generated sequence: 76, 77, 92...

Observations:

1. When it results in 0, we must assign another seed.
2. These methods are called "pseudorandom".

Let's practice!

Exercise:

By the Middle-Square Method, generate the first numbers from a seed of 2 digits.

RANDOM NUMERS

Congruence Method

The most popular method for the generation of random numbers is the Congruence Method. This method works from four parameters which are:

1. The multiplier, a;
2. The increment, c;
3. The modulus, M;
4. r_0 The initial seed,

First we will understand what is the congruence operation:

x mod y = r

RANDOM NUMERS

Congruence Method

Step 1: Choose the values a, c and M.

Step 2: Choose the $r_0$seed, such that 1 r_0≤ M.

Step 3: Calculate the next random number by the expression:$r_1 = (a.r_0 + c)\ mod\ M$.

Step 4: Replace $r_0$with $r_1$and go back to the previous step to build the sequence of desired random numbers.

Let's practice!

Exercise:

By the Congruence Method, generate the first random numbers

using the values: a = 9; M = 17 and r_0= 8.

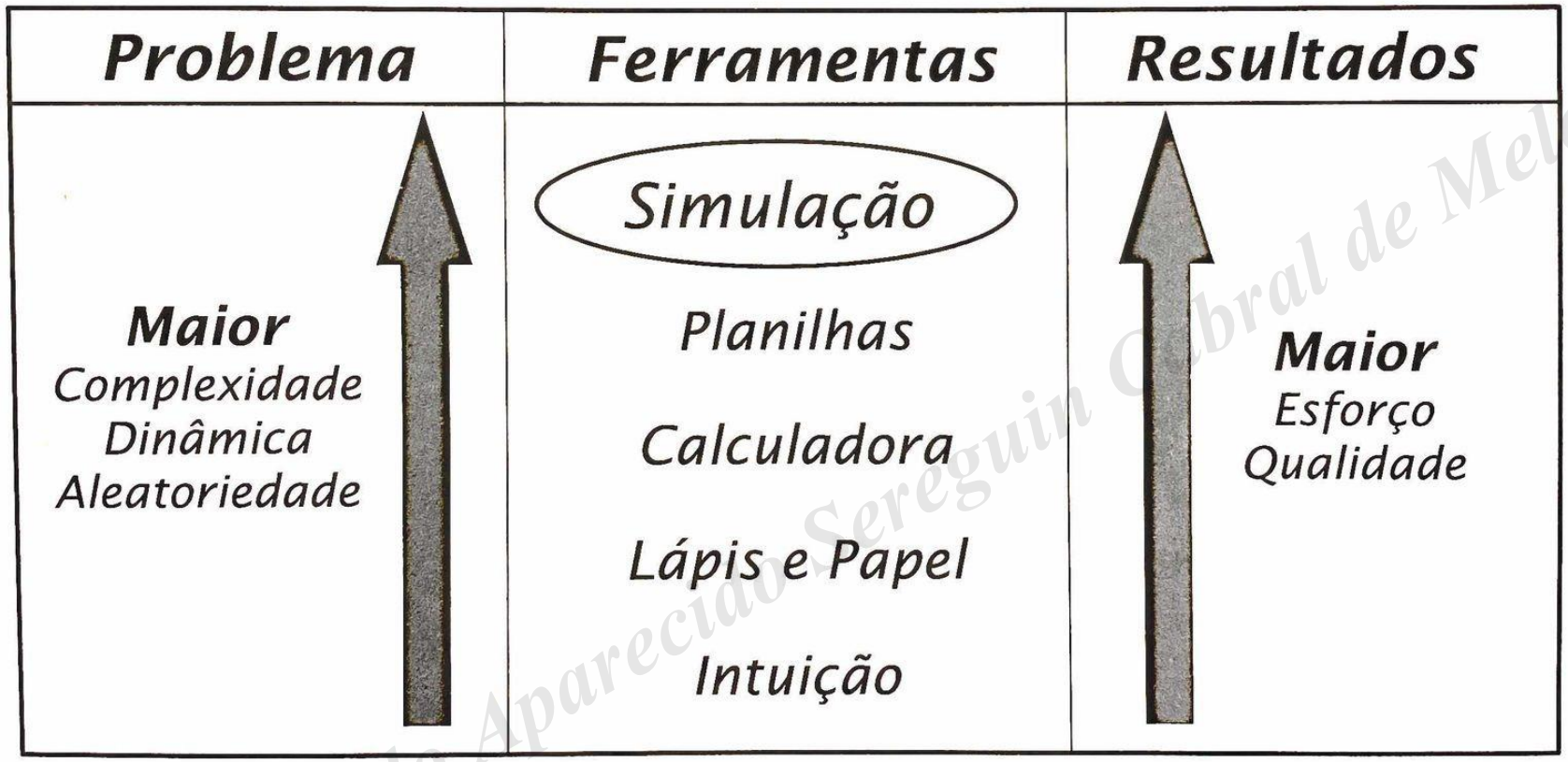
RANDOM NUMERS

In Excel, there are the functions:

1. = RANDOM(): generates the random numbers between 0 and 1.

2. = RANDBETWEEN(): generates the random numbers in a given interval.

WHEN TO SIMULATE?



Chwif and Medina (2007)

OBJECTIVE

To present the concept of Simulation, in what types of problem it must be used and what a Generator of Random Numbers is.

THANK YOU

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responsibility of the professor.