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Operational Research and Optimization and Simulation Modeling II

Prof. Dr. Marcos dos Santos

Eduardo Aparecido Sereguin Cabral de Melo 339.652.318-04

BACKGROUND

- Senior Officer, 29 years of service in the Navy of Brazil;
- Navy Academy;
- Naval School;
- Viagem de instrução de Guardas-Marinha (VIGM) in 2001;
- 10 years on board of war vessels;
- 11 years in CASNAV: Researcher and Project Manager in the Corp of Operational Research;
- Professor of P.O. of CAAML, EsAO, CIASC and ECEME;
- Specialized in Mathematical Instrumentation (UFF)
- Specialization in Mathematics (IMPA);
- IT Governance (FGV-RJ);
- Graduate Course in Production Engineering - Operational Research (COPPE/UFRJ);
- PhD and post-doctoral in Systems, Support to Decision and Logistics (UFF);
- Post-doctoral in Sciences and Spatial Technologies (ITA);
- Board of the Brazilian Society of Operational Research (SOBRAPO);
- Professor of MBA in Data Science and Analytics (USP);
- Professor of the Graduate program in Production Engineering (UFF);
- Professor of the Graduate Program in Systems and Computing (IME).

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Exercise 2

In a grocer's, the price of a soda A is equal to R$ 1.20 while the other soda B is equal to R$ 1.40. What is the income obtained with the sale of certain amount x1 of a soda A and an amount x2 of soda B?

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Exercise 3

A craftsman gains R$16.00 for each ashtray that they sells and gains R$32.00 for each lampshade. What is the profit obtained with the sale of these objects?

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Exercise 4

An operator takes care of the maintenance of three machines : A, B and C. For each assistance provided to the machine A, it spends 10 min. For each assistance to the machine B, it spends 15 min and for each assistance to the machine C, it spends 18 min. What is the total time of assistances to machines, if each of them requires different numbers of repairs x1, x2 and x3?

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Exercise 5

In an bookstore, an average profit of R$ 30.00/book is obtained with the books of the legal area. An average profit of 48.00/book is obtained with the books of the medical area, and an average profit of R$ 25.00/book is obtained with the books of the management area. What is the profit function with the sale of books of these three areas?

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Exercise 6

The cost of a certain glazed tile per square meter for a building materials store is R$15.27. Demonstrate the cost C according to the requested amount in square meters (x).

Now, the accommodation for stock allows to accommodate up to $\mathbf{500}\boldsymbol{m^2}$ of such tiles. Express this condition in mathematical language.

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Exercise 7

In a grocer's, the price of a soda A is equal to R$ 1.20 while the one of soda B is equal to R$ 1.40. What is the income obtained with the sale of certain amount x1 of a soda A and an amount x2 of soda B?

Research showed that the demands for each soda are **30 units of A and 24 units of B**. In addition, the refrigerator of the grocery store admits at most **60 units** of soda. Express these conditions mathematically.

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Exercise 8

In an bookstore, a profit of R$ 30.00/book is obtained with the books of the legal area. An average profit of 48.00/book is obtained with the books of the medical area, and an average profit of R$ 25.00/book is obtained with the books of the management area. What is the profit function with the sale of books of

Considering a monthly demand of **400** legal books, **300** medical books and **250** management books, represent these restrictions mathematically.

Obs: the amount of books of each type to be acquired by the bookstore should be equal or lower than the monthly demand.

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Exercise 9

A soap industry makes 2 types of soaps A and B. The type A generates a profit of R$0.10 per unit and with the B type is R$ 0.15 per unit. The company spends 2 seconds to manufacture the type A and 3 seconds for the type B, which should be manufactured in a total of 2 hours. According to the previous demands, the maximum to manufacture is 2400 units of the soap A and 1800 units of the soap B. Model the problem mathematically with the goal of achieving the highest profit.

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FORMALIZATION OF A LPP

A mathematical programming problem is linear if the Objective Function and each of the functions that represent the restrictions are linear, that is, in the form below:

$$Max\ ,ou\ Min.$$

$$Z = c_1x_1 + c_2x_2 + \ \ldots + c_nx_n$$

$$a_{11}x_1 + a_{12}x_2 + \ \ldots + a_{1n}x_n \le b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ \ldots + a_{2n}x_n \le b_2$$
$$\ldots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ \ldots + a_{mn}x_n \le b_m$$

REDUCED MODEL

$$\underset{x_1,x_2,\ldots,x_n}{Max} \ L = \sum_{i=1}^{n} c_i x_i$$

$$s.a$$

$$\sum_{i=1}^{n} a_{ji}x_i \le b_j \quad j = 1, \ldots, p$$

$$x_i \ge 0 \qquad \forall i$$

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EXAMPLES OF LINEAR PROBLEMS

$$\max x_1 + x_2$$
s.r.
$$2x_1 + 4x_2 \leq 20$$
$$180x_1 + 20x_2 \leq 600$$
$$x_1, x_2 \geq 0$$

$$\min x_1 + 2x_2$$
s.r.
$$2x_1 + 3x_2 \geq 20$$
$$180x_1 + 20x_2 = 600$$
$$x_1, x_2 \geq 0$$

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STEP BY STEP TO MODEL A LPP

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1. Identifying the variables of the problem $x_{1,}\ x_{2,\ \ldots,}\ x_n$.
2. Identifying the constants of the problem c_1, $c_{2,}$ ..., c_n.
3. Identifying the values of the limiting factors of the problem b_i.
4. Determining the objective function (linear equation that relates x_n *to* c_n).
5. Establishing restrictions (linear inequalities and equation that relates x_n *to* b_i).
6. Do not forget that the variables must be *non-negatives:* $x_n \geq 0$.

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CLASSICAL PROBLEMS – LPP MIX OF PRODUCTION

Manufacturing of two toys models: B_1 and B_2.

Unit profit: R$8.00 for B_1 and R$5.00 for B_2

Available resources:

1000 kg of special plastic.

40 hours for weekly production.

Requisites of the Marketing Department:

Total production cannot exceed 700 units;

The amount produced of B_1 can not exceed 350 units of the produced amount of B_2.

Technical data:

B_1 requires 2 kg of plastic and 3 minutes per unit.

B_2 requires 1 kg of plastic and 4 minutes per unit.

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CLASSICAL PROBLEMS – LPP OF THE MIX OF PRODUCTION

The Administration is seeking a production program that **maximizes the Company's profit**.

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CLASSICAL PROBLEMS - LPP OF ALLOYS

- Foundries produce several types of steel from several inputs, such as: iron ingots, graffiti, industrial scrap of several types of steel, among others.

- These inputs are placed in a high temperature oven, where they, in liquid state, melt to form a alloy, that is, a mix.

- The composition of the alloy to be produced, in terms of carbon, silicon, manganese, among others, is determined by technical standards of metallurgy.

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CLASSICAL PROBLEMS - LPP OF ALLOYS

- The purchase prices of inputs and sale prices of the alloys can vary and they are known.

- The problem consists of determining the amounts of each input to be smelt, so that the composition of the obtained alloy satisfies the technical standards of metallurgy, to maximize the profit with the sales.

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CLASSICAL PROBLEMS - LPP OF ALLOYS

- A metallurgy produces alloys of two types, Alloy 1 and Alloy 2, from a mix of three raw materials: copper, zinc and lead.

- The following table clarifies the proportion of each raw material in the mix to obtain each type of alloy, as the availability in stock of each raw material (in ton) and the sales prices (by ton) of each type of alloy.

- What should be the amount to produce of each type of alloy, respecting the stock restrictions of each raw material, **<u>to maximize the profit of the metallurgy</u>**?

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CLASSICAL PROBLEMS - LPP OF ALLOYS

	Alloy 1	Alloy 2	Availability
Copper	0.5	0.2	16 ton
Zinc.	0.25	0,3	11 ton
Lead	0.25	0.5	15 ton
Profit	3,000	5,000	

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CLASSICAL PROBLEMS – LPP OF THE DIET

- The elaboration of a balanced diet, according to the palate of the patient and serving all daily needs of nutrients, it is a difficult manipulation problem when there aren't appropriate techniques of mathematical programming.

- Personal information such as sex, age, weight and height lead to restrictions of minimum and maximum daily requirements of nutrients necessary for each person.

- It is known that there are special cases that can modify such requirements, such as the recovery of some disease or in the case of athletes, who, in general, use more proteins and carbohydrates.

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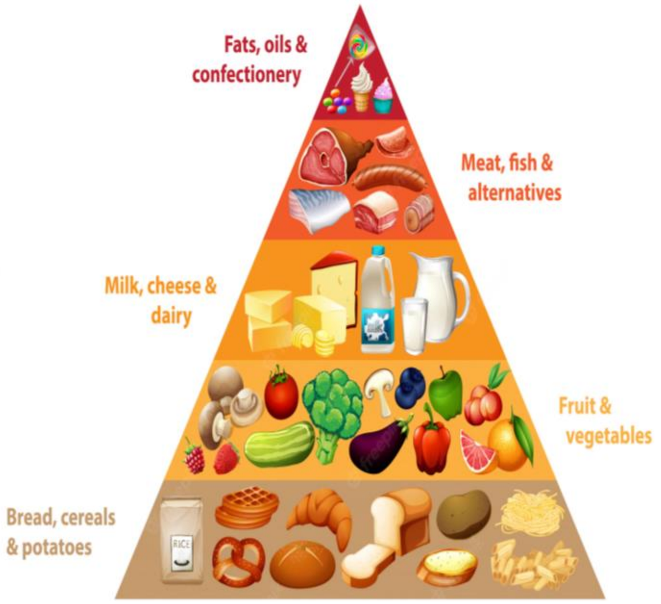
CLASSICAL PROBLEMS – LPP OF THE DIET

- The purchase prices of food can vary and are known. The average quantity of each type of nutrient in each food are also known.

- The problem consists of determining the amount of each food to ingest, to satisfy the minimum and maximum daily needs of consumption nutrients, **<u>to minimize the total cost of the diet</u>**.

- The daily nutritional requirements are expressed according to the minimum amount (in mg) of vitamins A, C and D that should be ingested.



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CLASSICAL PROBLEMS – LPP OF THE DIET

- The following table presents the available amount of each vitamin in each of the food allowed in the diet, the daily need of each vitamin and the cost (in real) of each food.

- What should be the **minimum cost** diet that satisfies the food needs?

Vitamin	Milk (liter)	Meat (kg)	Fish (kg)	Salad (100g)	Minimum Requirement
A	2 mg	2 mg	10 mg	20 mg	11 mg
C	50 mg	20 mg	10 mg	30 mg	70 mg
D	80 mg	70 mg	10 mg	80 mg	250 mg
Cost	6.70	46.00	28.00	4.10	

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CLASSICAL PROBLEMS – LPP OF THE TRANSPORT

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- The problem of transport consists of the transport of products from production centers to the consumer markets, so that **the total cost of transport is the lowest possible**.

- Usually, the amount produced or offered in each production center and the amount demanded in each consumer market are known.

- The transport **should be carried out respecting the supply limitations and meeting the demand**.

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CLASSICAL PROBLEMS – LPP OF THE TRANSPORT

- A soda factory has three producing centers in the baixada fluminense, and three main markets in the mountain region of the state.

- The following figure demonstrates the transport costs of each producer center to each consumer center, as well as the demands (in sodas' boxes) of each consumer market, in addition to the maximum amount of production (in sodas' box) in each production center.

- The transport problem associated to this example consists of products from production centers to the consumer markets, **so that the total cost of transport is the lowest possible**.

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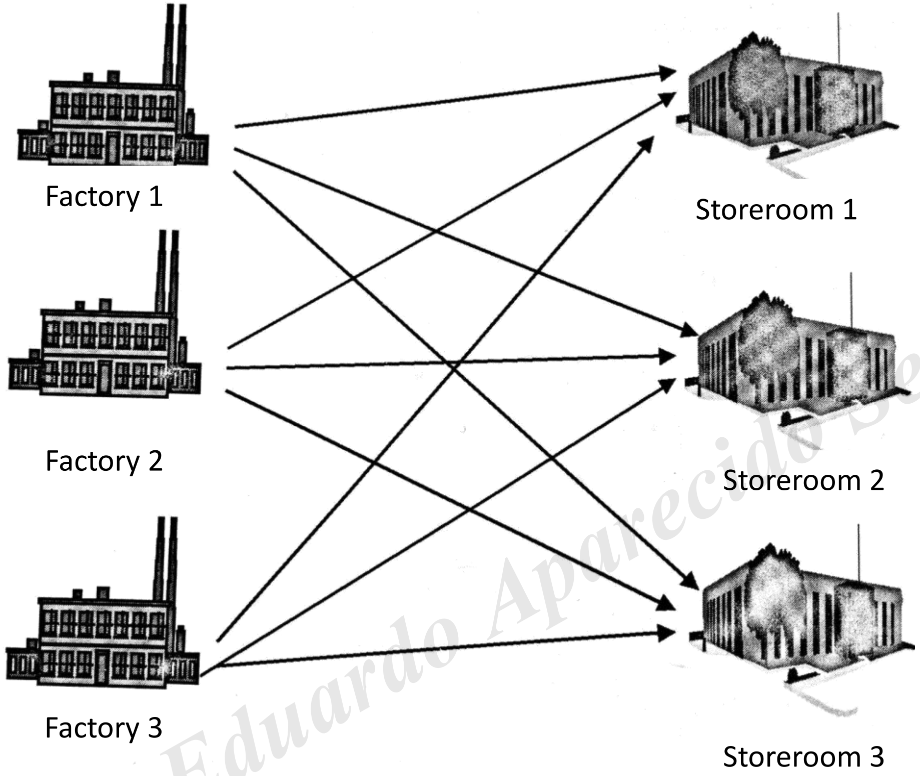
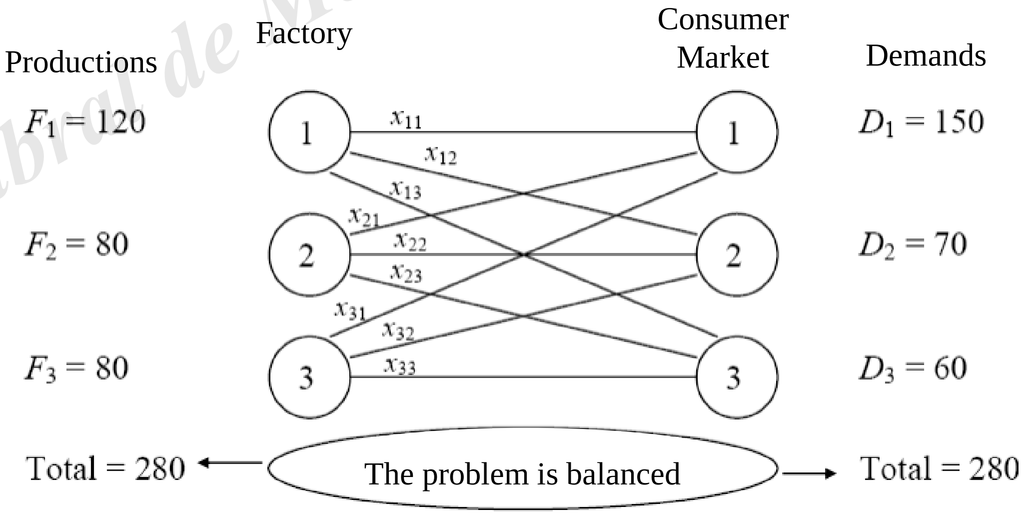
CLASSICAL PROBLEMS – LPP OF THE TRANSPORT

SCHEMATICALLY:





Costs $x_{(ij)}$	**Destination *(j)***		
Sources *(i)*	**1**	**2**	**3**
1	8	5	6
2	15	10	12
3	3	9	10

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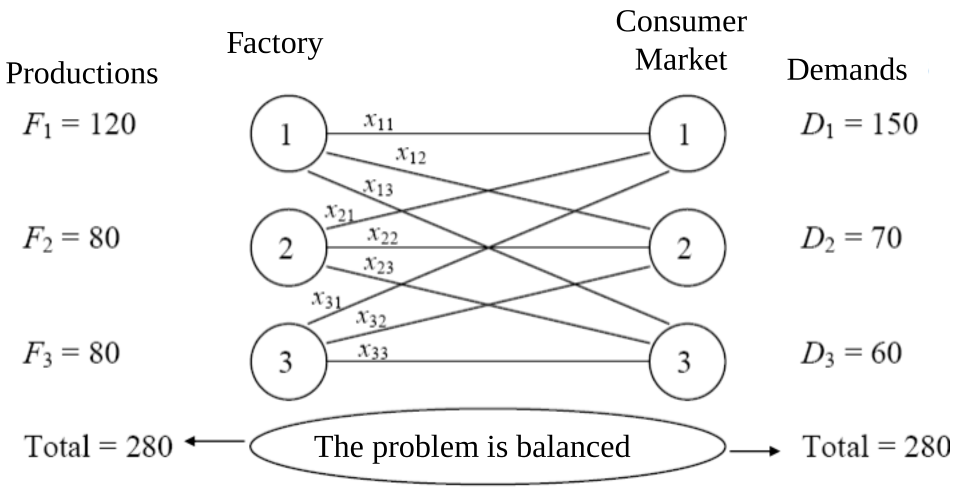
CLASSICAL PROBLEMS – LPP OF THE TRANSPORT



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TO FINISH:

1) **<u>A model cannot be better than the information contained in it</u>**. To demonstrate this principle, it is necessary to mention a maxim used by computer programmers: "Garbage in, ***Garbage out***" (**GIGO**) If the information contained in the model are uncertain and/or imprecise, there is no way to expect exact and trustworthy results that are able to support the decision.

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TO FINISH:

1) **<u>A model cannot be better than the information contained in it</u>**. To demonstrate this principle, it is necessary to mention a maxim used by computer programmers: "Garbage in, **Garbage out**" (***GIGO***) If the information contained in the model are uncertain and/or imprecise, there is no way to expect exact and trustworthy results that are able to support the decision.

2) **<u>Models cannot replace decision-making</u>**. A model, much as it is elaborated, represents only the logical equation of a problematic situation. It will never superimpose the human capacity for perception and analysis of things and facts. Therefore, we should maintain the models in the condition of good tools that provide help in decision-making. Do not forget that, after all, who will decide is a person or a group of people.

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Understanding how mathematical structuring of a Linear Programming Problem (LPP) happens; and

Performing the modeling of some classical LPP.

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GRAPHICAL SOLUTION OF A LPP

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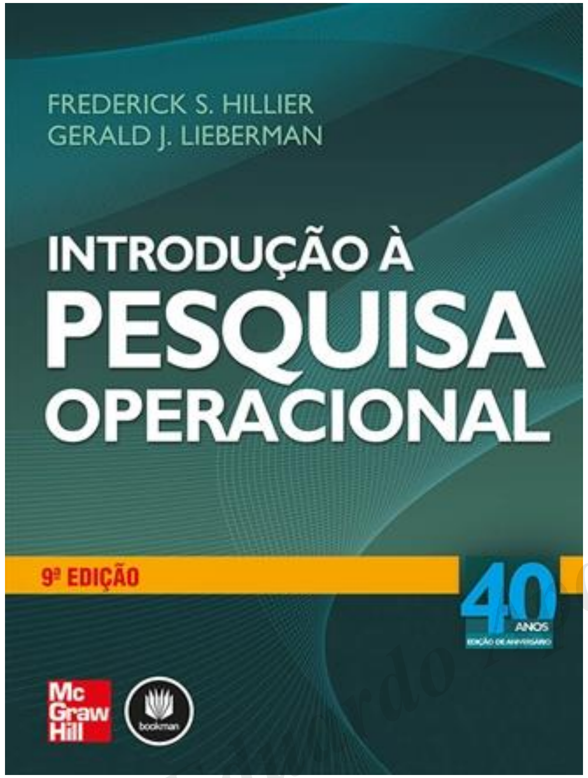

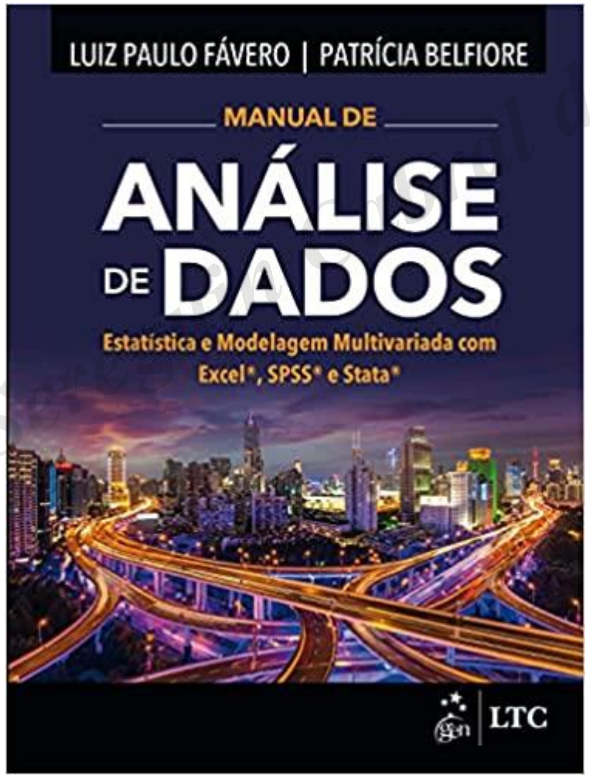

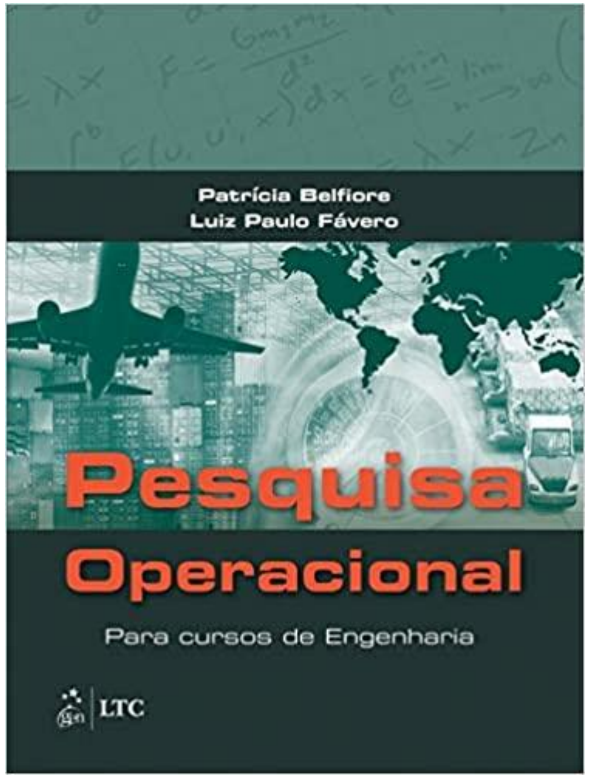

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OBJECTIVE

Determining graphically the "Optimal Solution" of a LPP with two Decision Variables.

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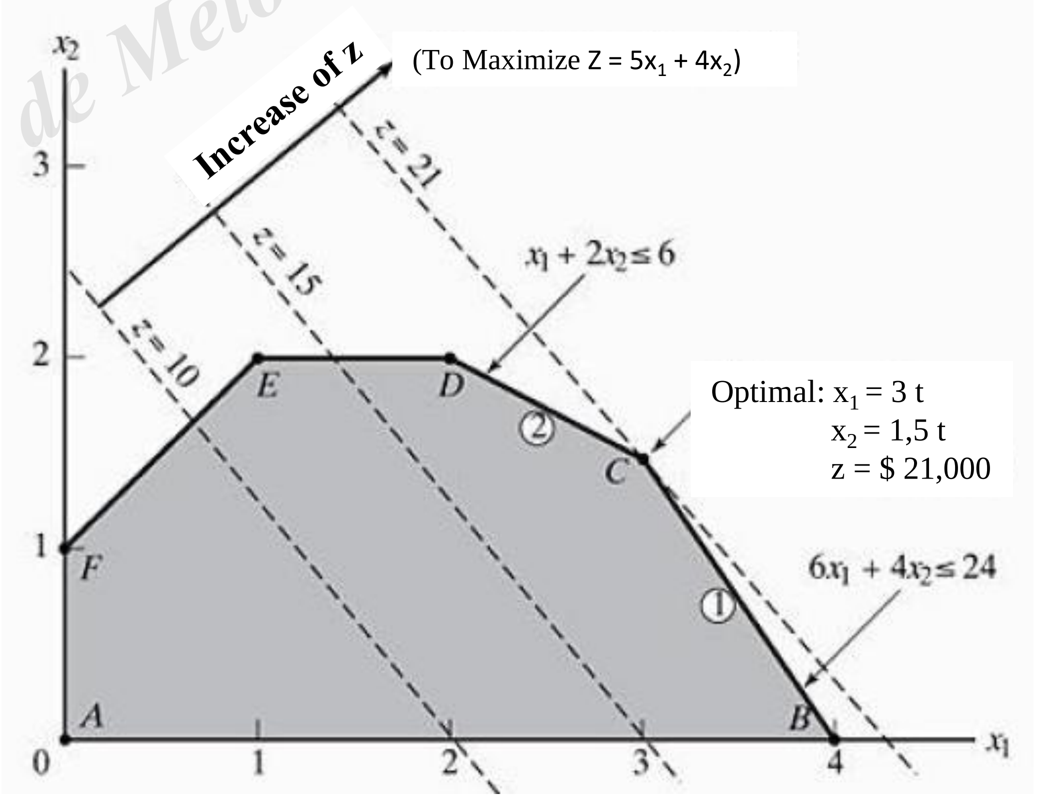
SOLUTION OF A LPP

Any specification of values for decision variables (x_1, $_{xi}$..., x_n) is called solution, independently if it is desirable or not.

A solution can be:



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FEASIBLE SOLUTION OF A LPP

A **feasible** solution is the one which **<u>all restrictions</u>**

<u>are satisfied</u>.

To Maximize $Z = 3x_1 + 5x_2$

Subject to restrictions $x_1 \leq 4$

2x2 $\leq$ 12

3x1 + 2x2 ≤ $_1$8

$x_1 \geq 0, x_2 \geq 0$

$S = (x_1, x_2)$

$S_1 = (4, 1)$

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UNVIABLE SOLUÇÃO OF A LPP

An **infeasible** solution is the one which **at least one of the restrictions is not satisfied**.

To Maximize $Z = 3x_1 + 5x_2$

Subject to restrictions $x_1 \leq 4$

$2x2 \leq 12$

$3x1 + 2x2 \leq {}_1 8$

$x_1 \geq 0, x_2 \geq 0$

$S = (x_1, x_2)$

$S_1 = (4, 4)$

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OPTIMAL SOLUTION OF A LPP

An **optimal solution is the one that all restrictions are met, optimizing the value of the objective function.**

To Maximize $Z = 3x_1 + 5x_2$

S_1 =(2, 6)

Subject to restrictions $x_1 \leq 4$

$2x2 \leq 12$

$3x1 + 2x2 \leq {}_18$

$x_1 \geq 0, x_2 \geq 0$

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REVIEW OF ANALYTIC GEOMETRY
(Equation of the Line)

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REVIEW OF ANALYTIC GEOMETRY (Half planes)

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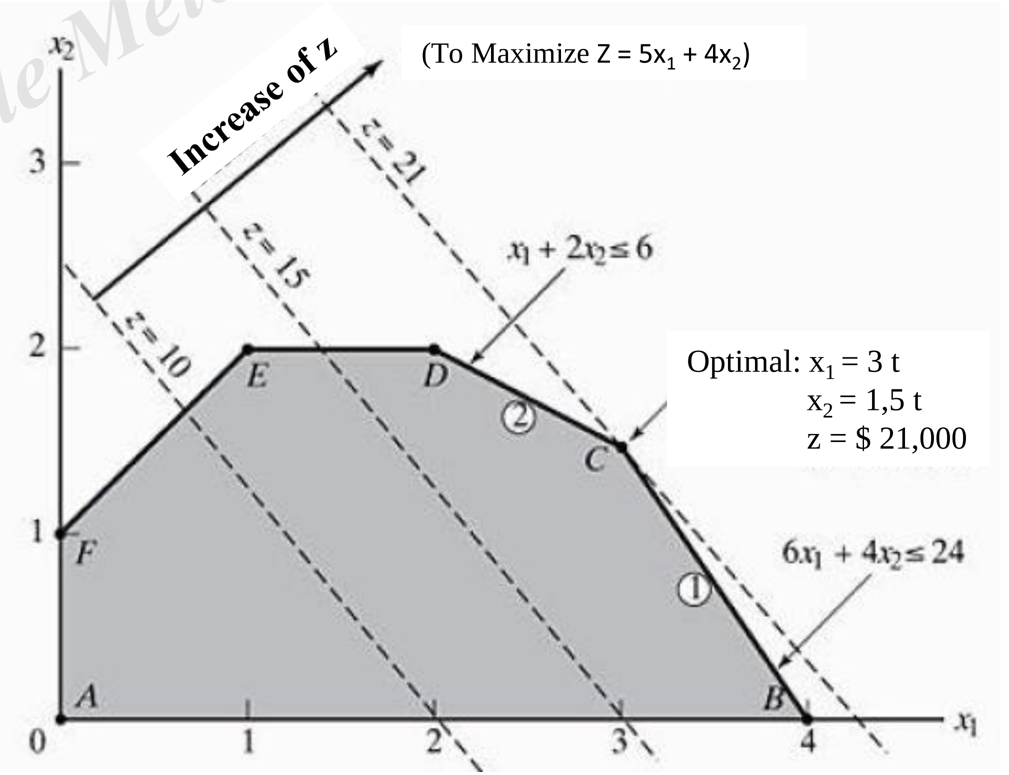
GRAPHIC SOLUTION OF A LPP

In the graphic resolution of a Linear Programming model, first, the **space of feasible** solutions or **feasible region** is determined.

A feasible solution is the one that satisfies all the the model restrictions, including those of non-negativity.

If a given solution violates at least one of the restrictions of the model, it is called **infeasible solution.**



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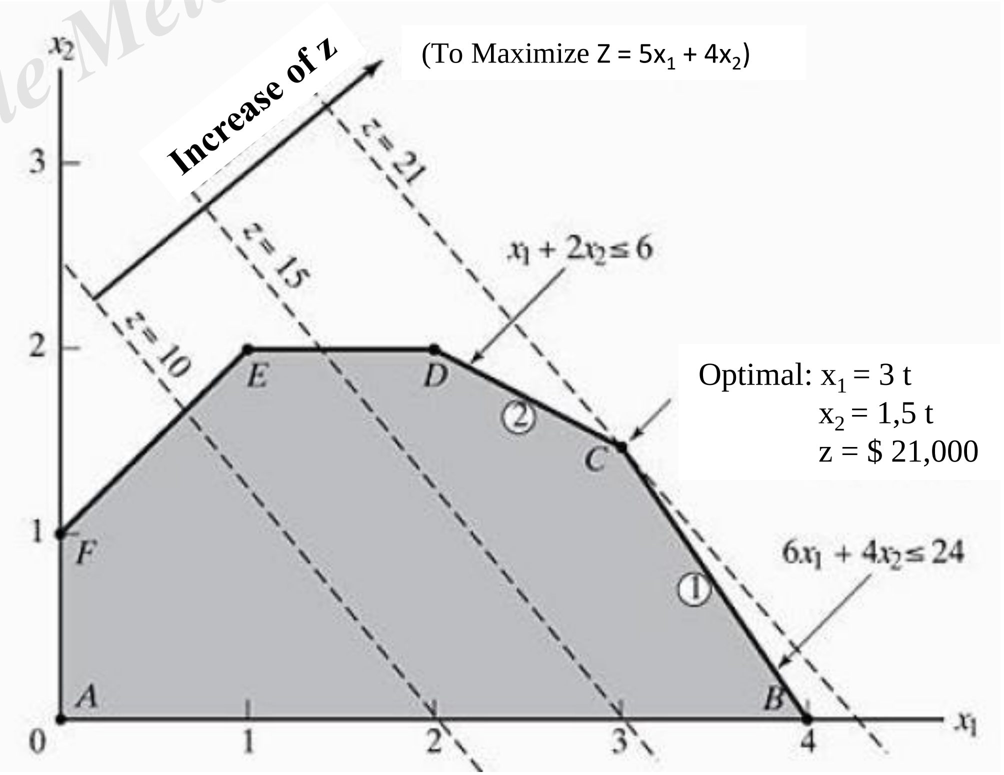
GRAPHIC SOLUTION OF A LPP

The next step consists of determining the **optimal solution** of the model, that is, the feasible solution that presents the best value of the objective function.

For a **maximization** problem, after determining the set of feasible solutions, the optimal solution is the one that provides **the highest value to the objective function** within this set.

For a **minimization** problem, the optimal solution is the one that **minimizes the objective function**.



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GRAPHIC SOLUTION OF A LPP

Types of Restrictions

To Maximize $Z = 3x_1 + 5x_2$

Subject to restrictions $x_1 \leq 4$

$2x2 _{\leq} 12$

$3x1 + 2x2 \leq {}_{1}8$

Functional Restrictions

$x_1 \geq 0, x_2 \geq 0$

Non-negative restrictions

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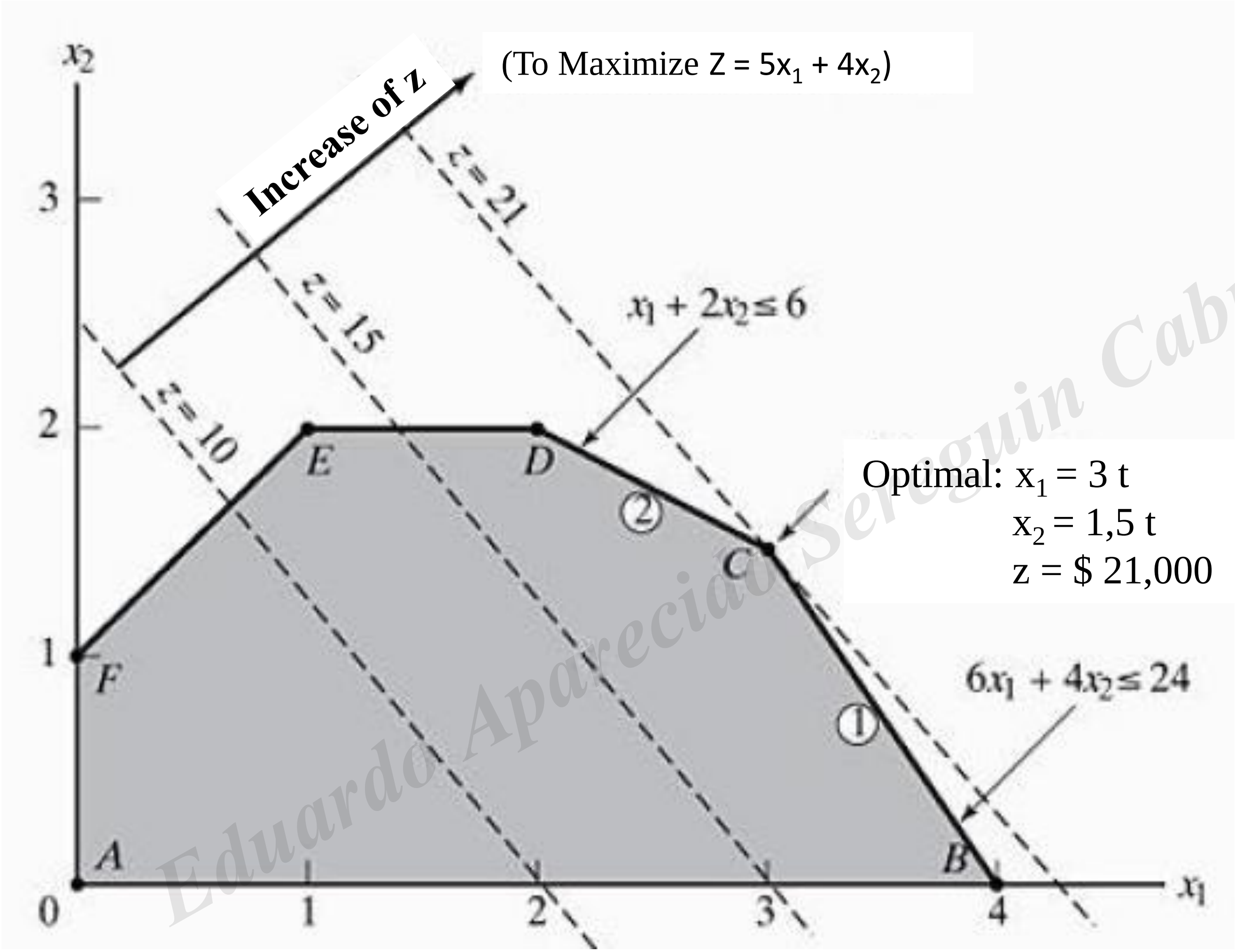
GRAPHIC SOLUTION OF A LPP
Types of Restrictions



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GRAPHIC SOLUTION OF A LPP

When the problem involves only two decision variables, the optimal solution of a linear programming problem can be graphically found.

$$
\begin{aligned}
&Max \quad Z = 5X_1 + 2X_2 \\
&s.r. \quad x_1 \leq 3 (a) \\
&\qquad X_2 \leq 4 (b) \\
&\qquad X_1 + 2X_2 \leq 9 (c) \\
&\qquad X_1 \geq 0, X_2 \geq 0 (d)
\end{aligned}
$$

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GRAPHIC SOLUTION OF A LPP

Step by step...

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$$
\begin{aligned}
&Max \quad Z = 5x_1 + 2x_2 \\
&s.r. \qquad x_1 \le 3 \quad (a) \\
&\qquad\qquad x_2 \le 4 \quad (b) \\
&\qquad x_1 + 2x_2 \le 9 \quad (c) \\
&\qquad x_1 \ge 0, x_2 \ge 0 \quad (d)
\end{aligned}
$$

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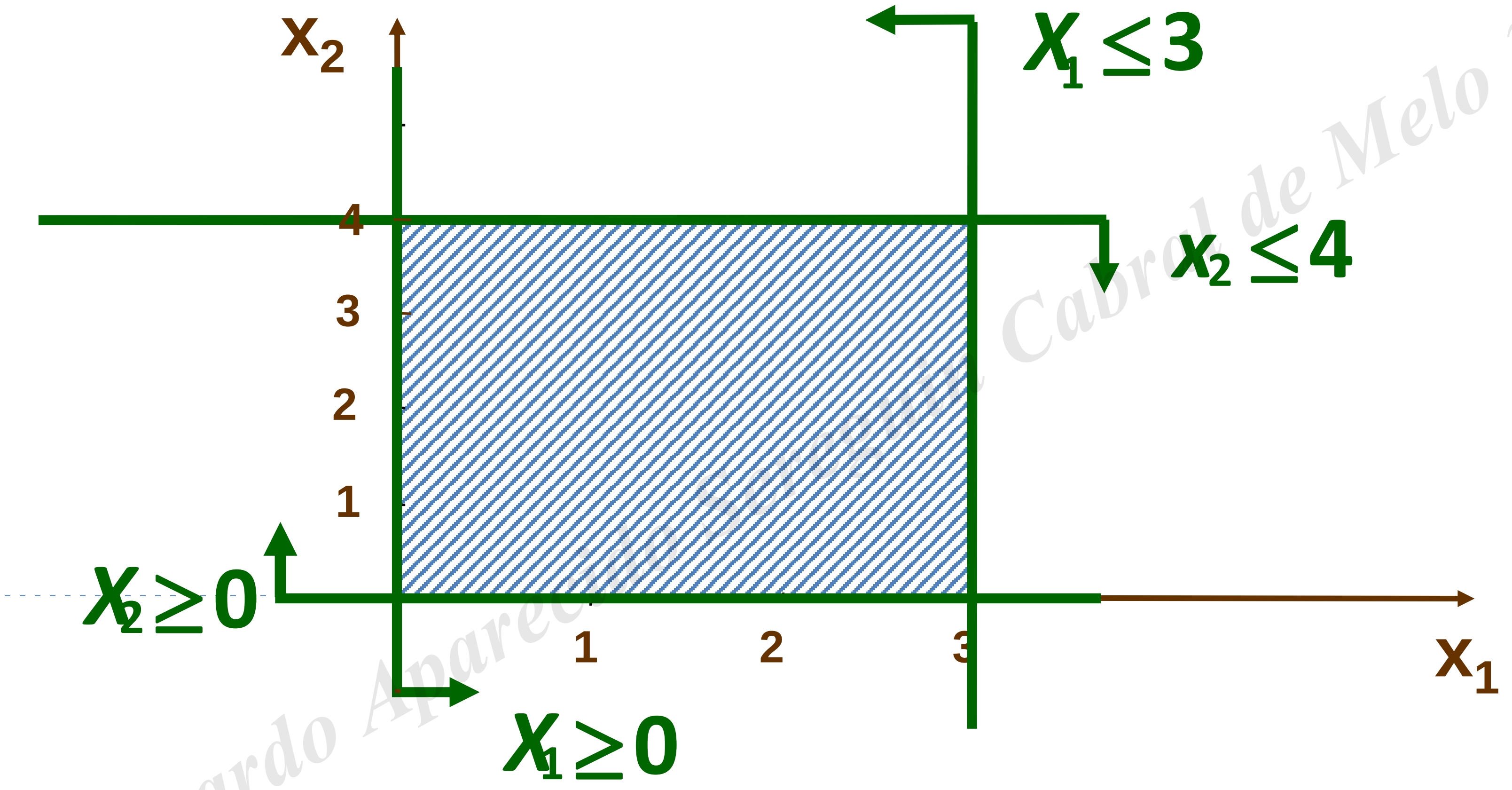
GRAPHIC SOLUTION OF A LPP



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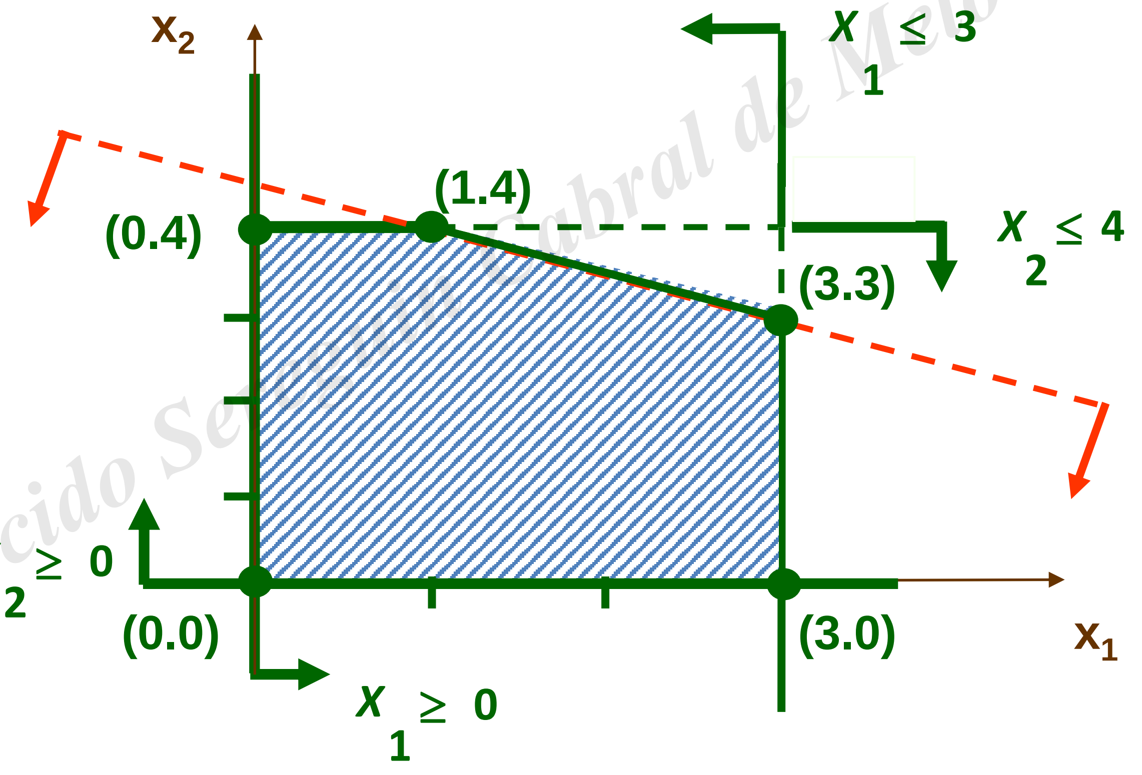
GRAPHIC SOLUTION OF A LPP

$$X_1 + 2X_2 \leq 9$$
$$X_1 + 2X_2 = 9$$



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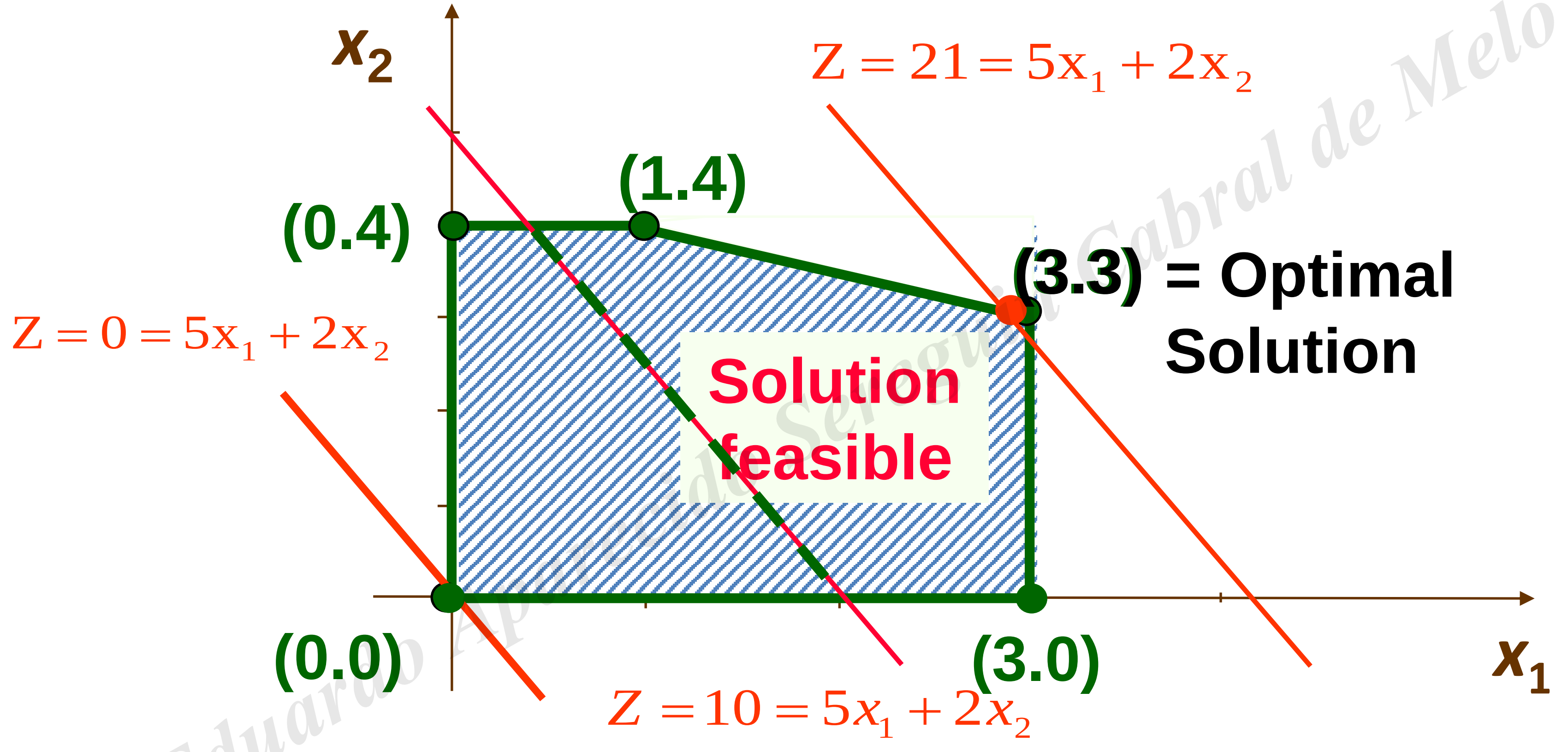
GRAPHIC SOLUTION OF A LPP



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GRAPHIC SOLUTION OF A LPP

Exercise:
Consider the following problem of LP

$$
\begin{aligned}
Max\ & 3x_1 + 3x_2 \\
s.r.\ & 2x_1 + 4x_2 \leq 12 \\
& 6x_1 + 4x_2 \leq 24 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Find the optimal solution using the graphic method.

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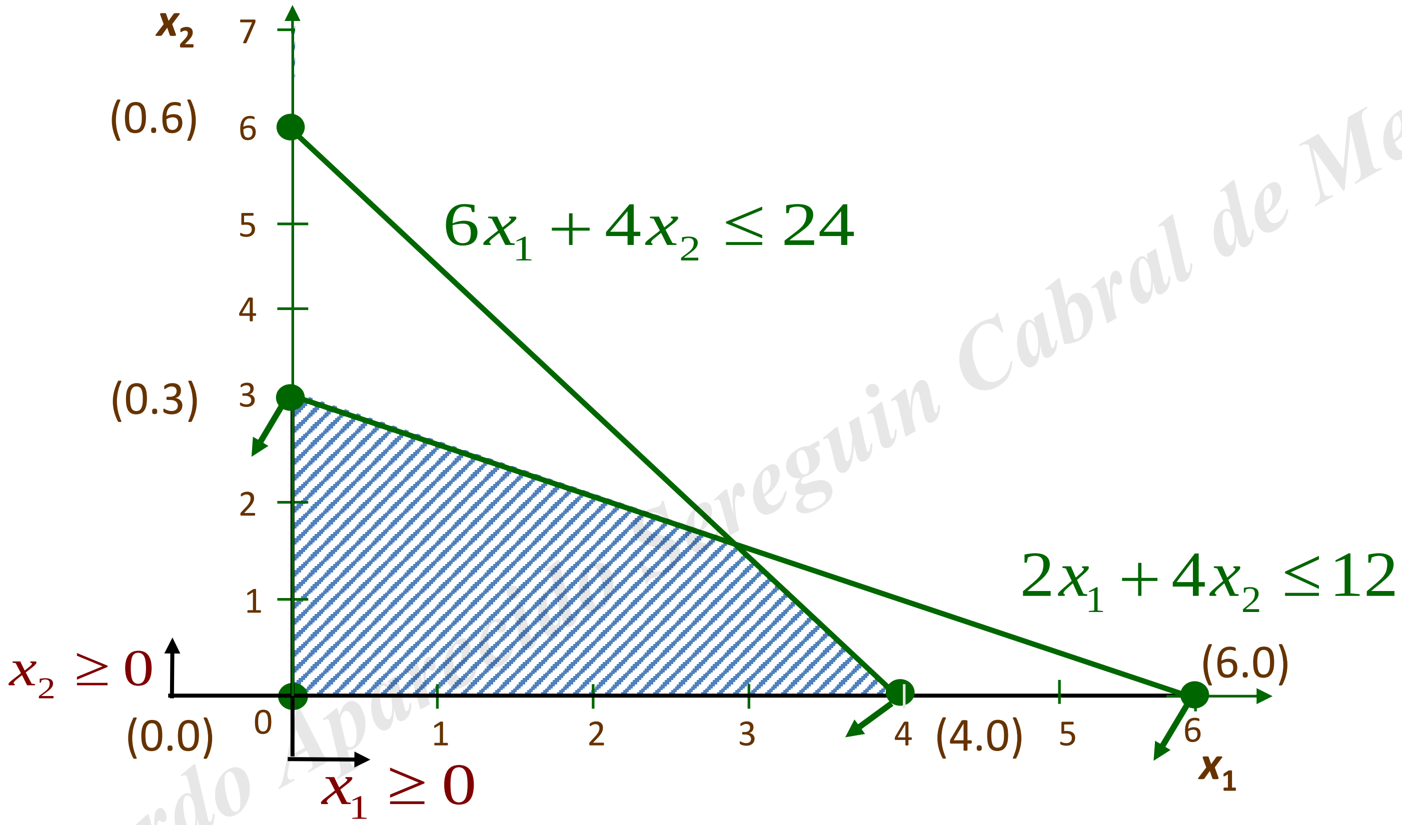
GRAPHIC SOLUTION OF A LPP



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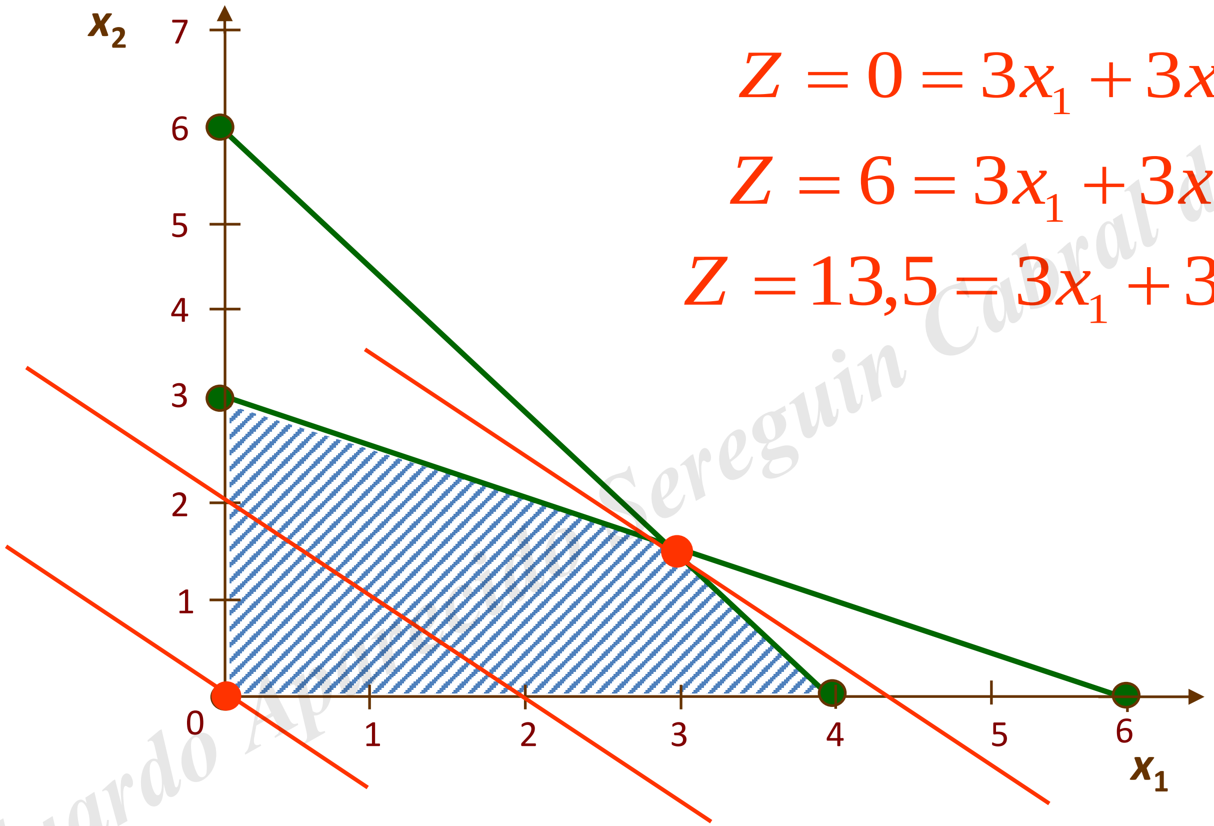
GRAPHIC SOLUTION OF A LPP



$$Z = 0 = 3x_1 + 3x_2$$

$$Z = 6 = 3x_1 + 3x_2$$

$$Z = 13{,}5 = 3x_1 + 3x_2$$

Observe that an optimal solution of a linear programming problem is always associated with a vertex or extreme point of the solution space.

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Let's practice!

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EXERCISE

Obtain the graphic solution for the following LP program:

To Maximize Z = $3x_1 + 5x_2$

Subject to restrictions $x_1 \leq 4$

$2x2 \leq 12$

$3x1 + 2x2 \leq 18$

$x_1 \geq 0$, $x2 \geq 0$

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EXERCISE

Max $Z = 3x_1 + 5x_2$

s.a.

$x_1 \leq 4$

$2x_2 \leq 12$

$3x_1 + 2x_2 \leq 18$

$x_1 \geq 0,\ x_2 \geq 0$

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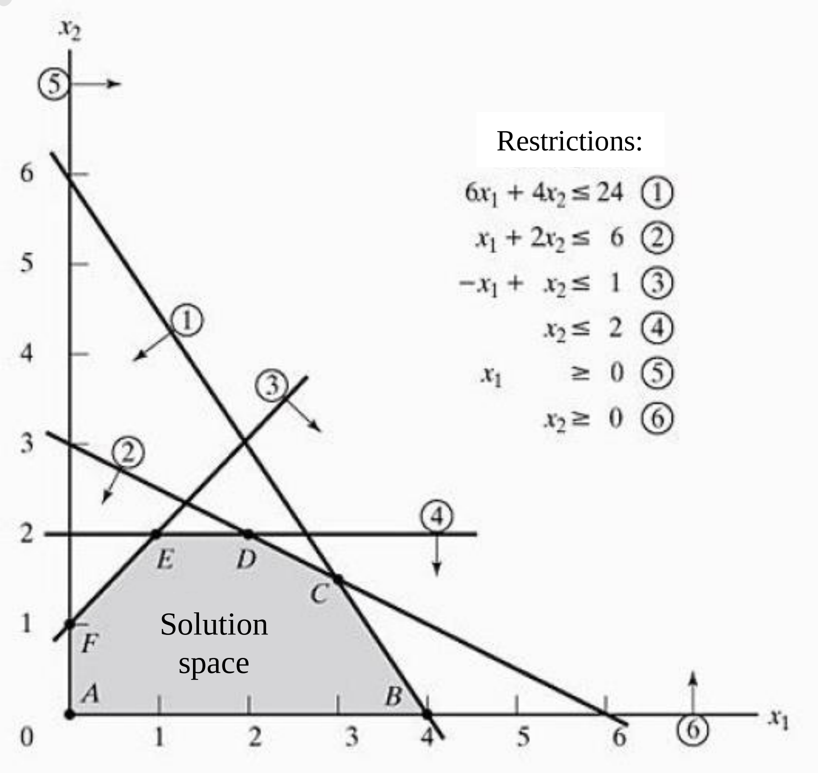
LIMITATION OF THE GRAPHICAL SOLUTION

According to the problems presented, the graphical solution of a LPP has great limitation of **solving problems with only two decision variables**.

For problems with three or more decision variables, we will use the Simplex Method, which will be presented in next classes.



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LIMITATION OF THE GRAPHICAL SOLUTION

MATHEMATICS STRUCTURING AND MODELLING TO SUPPORT THE DECISION-MAKING: CASE STUDY OF A FACTORY OF GARBAGE BAG

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INSTITUTO MILITAR DE ENGENHARIA - IME

Com a finalidade de proporcionar o lucro mediante a maximização da receitas da empresa, por meio do mix produtivo, a função objetivo fica definida pelo somatório das receitas líquidas geradas pela venda de cada um dos produtos. Assim, tem-se:

$$\text{F.O.} = \text{Max}\ \{14{,}79X_1 + 14{,}06X_2 + 23{,}06X_3 + 20{,}84X_4 + 12{,}51X_5 + 12{,}07X_6 + 19{,}37X_7 + 17{,}47X_8 + 15{,}61X_9 + 13{,}69X_{10} + 22{,}07X_{11} + 21{,}05X_{12} + 13{,}4X_{13} + 11{,}95X_{14} + 18{,}83X_{15} + 17{,}65X_{16} + 18{,}43X_{17} + 17{,}3X_{18} + 18{,}11X_{19} + 18{,}15X_{20} + 7{,}15X_{21} + 8{,}71X_{22} + 11{,}45X_{23} + 7{,}42X_{24} + 9{,}08X_{25} + 11{,}88X_{26}\}$$

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LIMITATION OF THE GRAPHICAL

Application of the LP in the formulation of a diet with low cost: case study of a company of collective meal in Rio de Janeiro

5.1.1. Função Objetivo

Compondo o custo de cada cota e a respectiva variável de decisão, pode-se definir a FO, apresentada na expressão 1 a seguir:

$$
\begin{aligned}
F.O. = \min \{ & 0{,}197x_1 + 0{,}195x_2 + 0{,}3x_3 + 0{,}189x_4 + 0{,}242x_5 + 0{,}199x_6 + 0{,}469x_7 \\
& + 0{,}340x_8 + 0{,}139x_9 + 0{,}532x_{10} + 0{,}109x_{11} + 1{,}190x_{12} + 0{,}329x_{13} \\
& + 0{,}00138x_{14} + 1{,}490x_{15} + 0{,}0525x_{16} + 0{,}200x_{17} + 0{,}200x_{18} + 0{,}150x_{19} \\
& + 0{,}220x_{20} + 0{,}650x_{21} + 0{,}28x0_{22} + 0{,}250x_{23} + 0{,}250x_{24} + 0{,}222x_{25} \\
& + 0{,}300x_{26} + 0{,}275x_{27} + 0{,}360x_{28} + 0{,}150x_{29} + 0{,}140x_{30} + 0{,}292x_{31} \\
& + 0{,}150x_{32} + 0{,}300x_{33} + 0{,}350x_{34} + 0{,}200x_{35} + 0{,}375x_{36} + 1{,}590x_{37} \\
& + 0{,}870x_{38} + 1{,}100x_{39} + 0{,}857x_{40} + 0{,}898x_{41} + 2{,}630x_{42} + 0{,}930x_{43} \\
& + 0{,}880x_{44} \}
\end{aligned} \quad (1)
$$

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OBJECTIVE

Determining graphically the "Optimal Solution" of a LPP with two Decision Variables.

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THANK YOU

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researchgate.net/profile/Marcos_Dos_Santos6

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