MBA USP ESALQ

ANALYTICAL HIERARCHY PROCESS (AHP)

Prof. Dr. Marcos dos Santos

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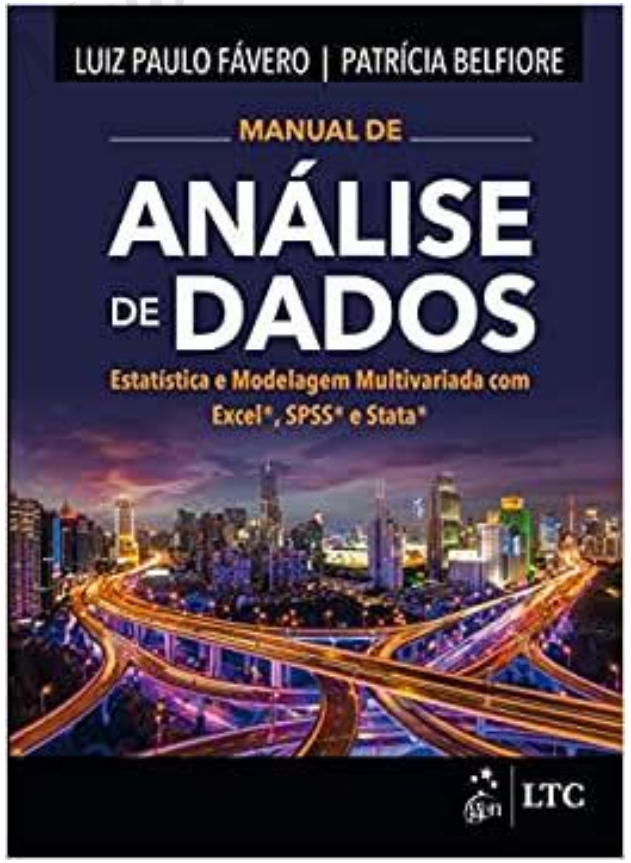
MATHEMATICAL STRUCTURING AND MODELING WITH THE AHP METHOD

Prof. Dr. Marcos dos Santos

*The responsibility for trustworthiness, originality, and legality of the didactic content presented is responsibility of the professor.

Acknowledgment



INTRODUCTION

(Decision making)

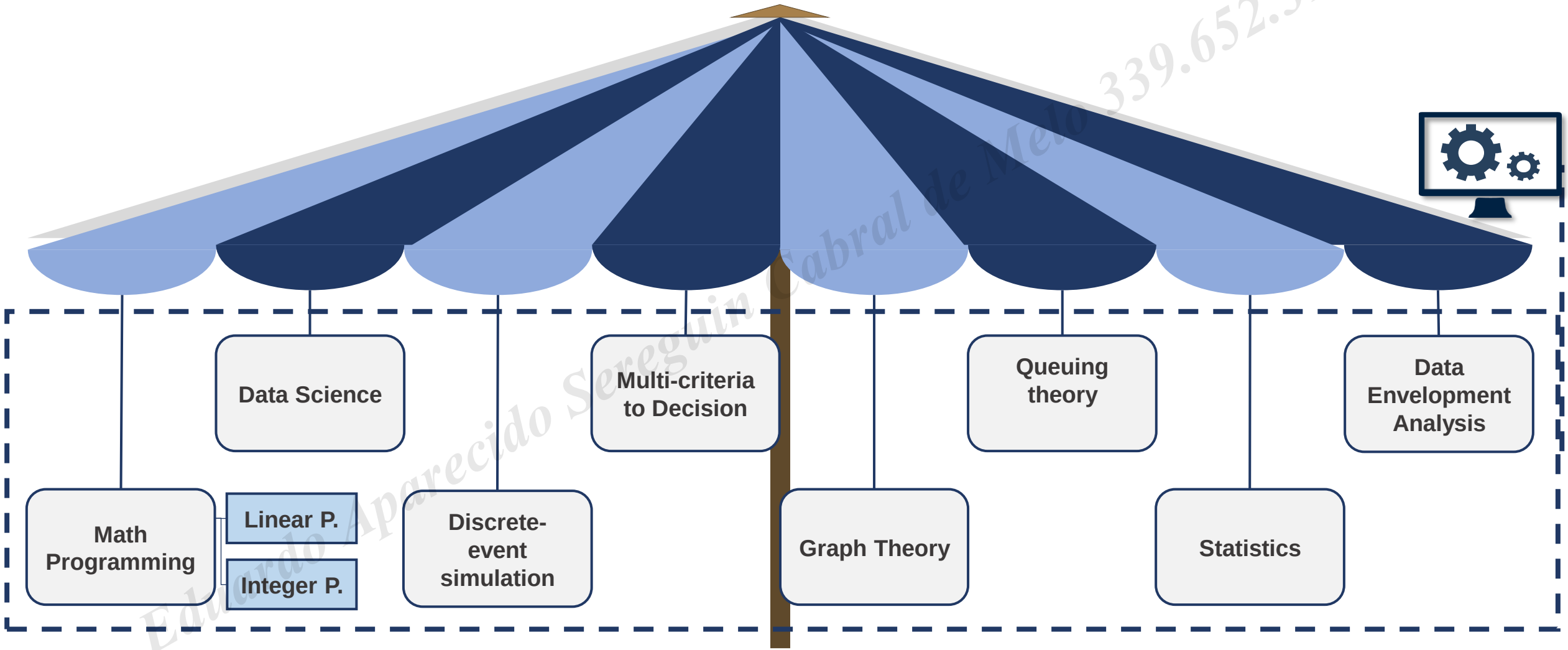
What is OPERATIONAL RESEARCH?

THE OPERATIONAL RESEARCH SOCIETY

Operational research (OR) **is a scientific approach to the solution of problems** in the management of complex systems **that enables decision makers to make better decisions.**

Most of the problems OR tackles are messy and complex, often involving considerable uncertainty. OR uses advanced analytics, modelling, problem structuring, simulation, optimization and data science to determine the best solution to the problem and the best practical course of action.

THE AREAS OF THE OPERATIONAL RESEARCH



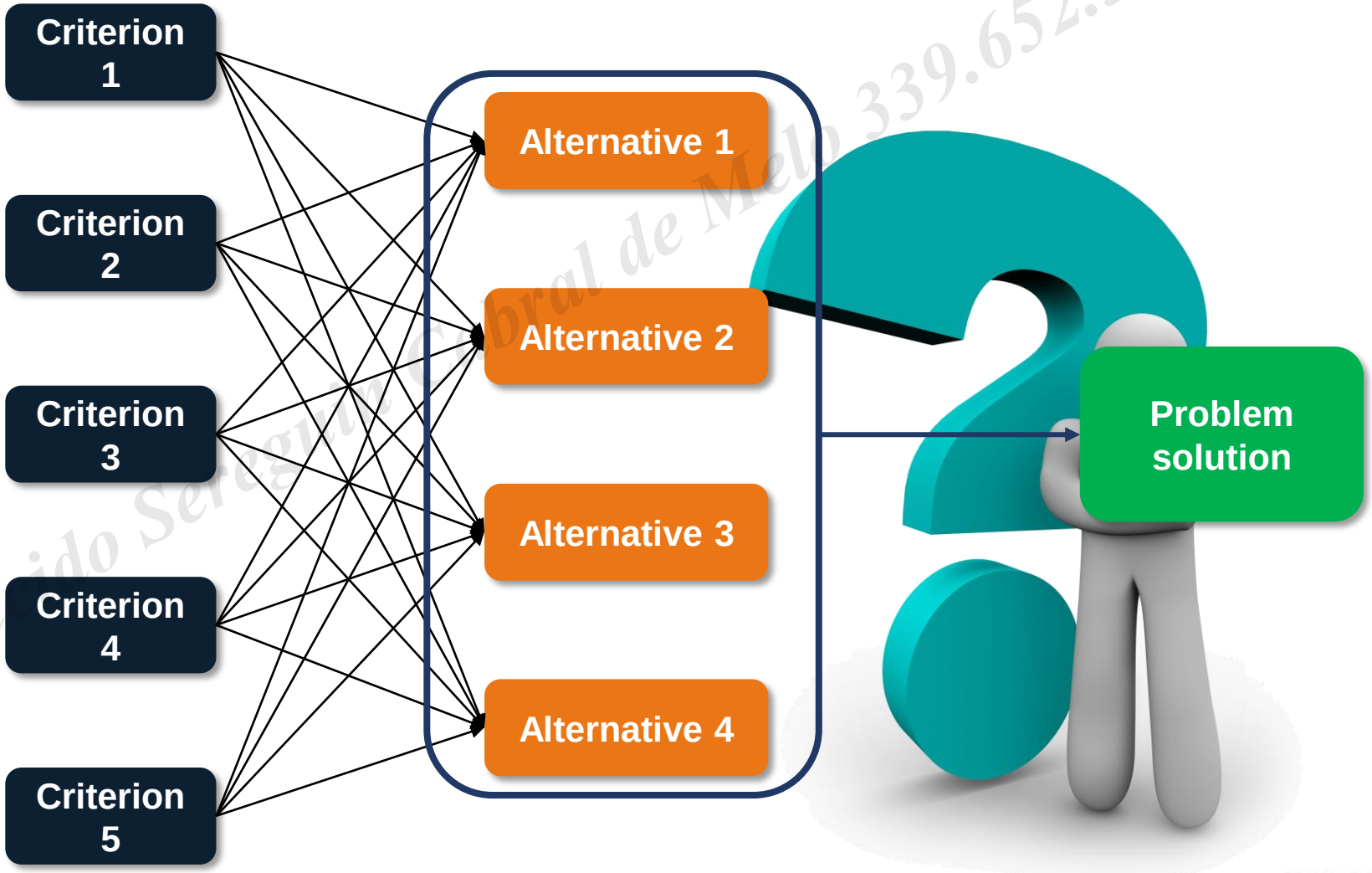
DECISION MAKING

A decision needs to be made **whenever we face a problem that has more than one alternative** for its solution. Even in order to solve a problem, we have a single action to make, we have the alternatives to make or not make this action.

DECISION MAKING

The decision process requires a set of feasible alternatives for its composition, in which **each decision has a gain and a loss associated with** it.

MULTI-CRITERIA METHODS



MULTI-CRITERIA METHODS

Problemas of *multi-criteria* decision making (MCDM) are characterized by a Decision Matrix, which is composed by weighted alternatives and criteria in accord with the decision maker. **Methods to solve the MCDM problems have been widely used to select the best alternative between a finite number of alternatives**. AHP, ANP, PROMETHEE, THOR, SAPEVO and TOPSIS may be mentioned as examples.

TOPSIS

THOR

SAPEVO-M

AHP

PROMETHEE

ELECTRE

DECISION MATRIX

AHP METHOD

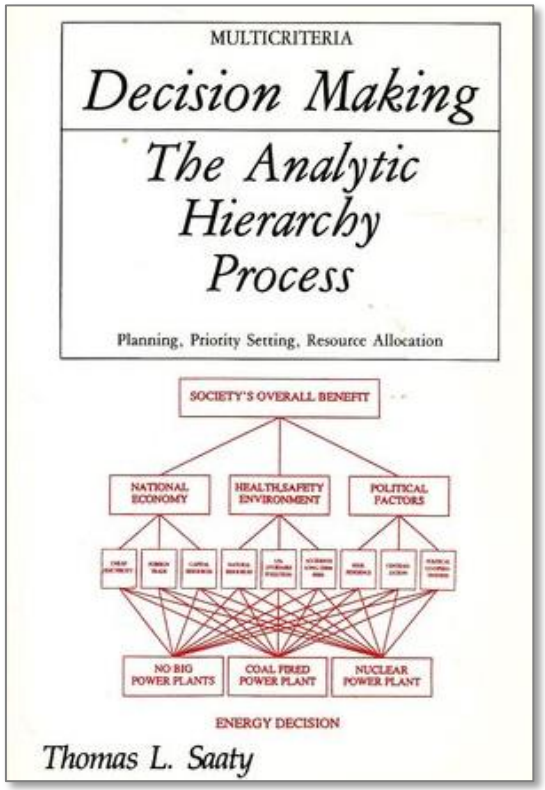
(Analytic Hierarchy Process)

AHP METHOD

The **Analytic Analytic Hierarchy Process (AHP)** is a method to help people on complex decision-making. More than just determining the correct decision, **AHP helps people choose and justify their choice**. Based on mathematics and psychology, it was developed in the 1970s by **Prof. Thomas Saaty**, at the Wharton School of the University of Pennsylvania.

AHP METHOD

AHP is a method for multi-criteria decision making and it's simple and flexible to use. It allows the evaluation of complex problems, based on conflicting criteria. It is appropriate for different problems, since it depends on human interaction to lead the comparison process. Among the advantages of using AHP as a decision-analysis model, it makes possible to consider different importance levels between variables.



AHP METHOD IN THE WORLD

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The state of the art development of AHP (1979–2017): a literature review with a social network analysis

Ali Emrouznejad & Marianna Marra

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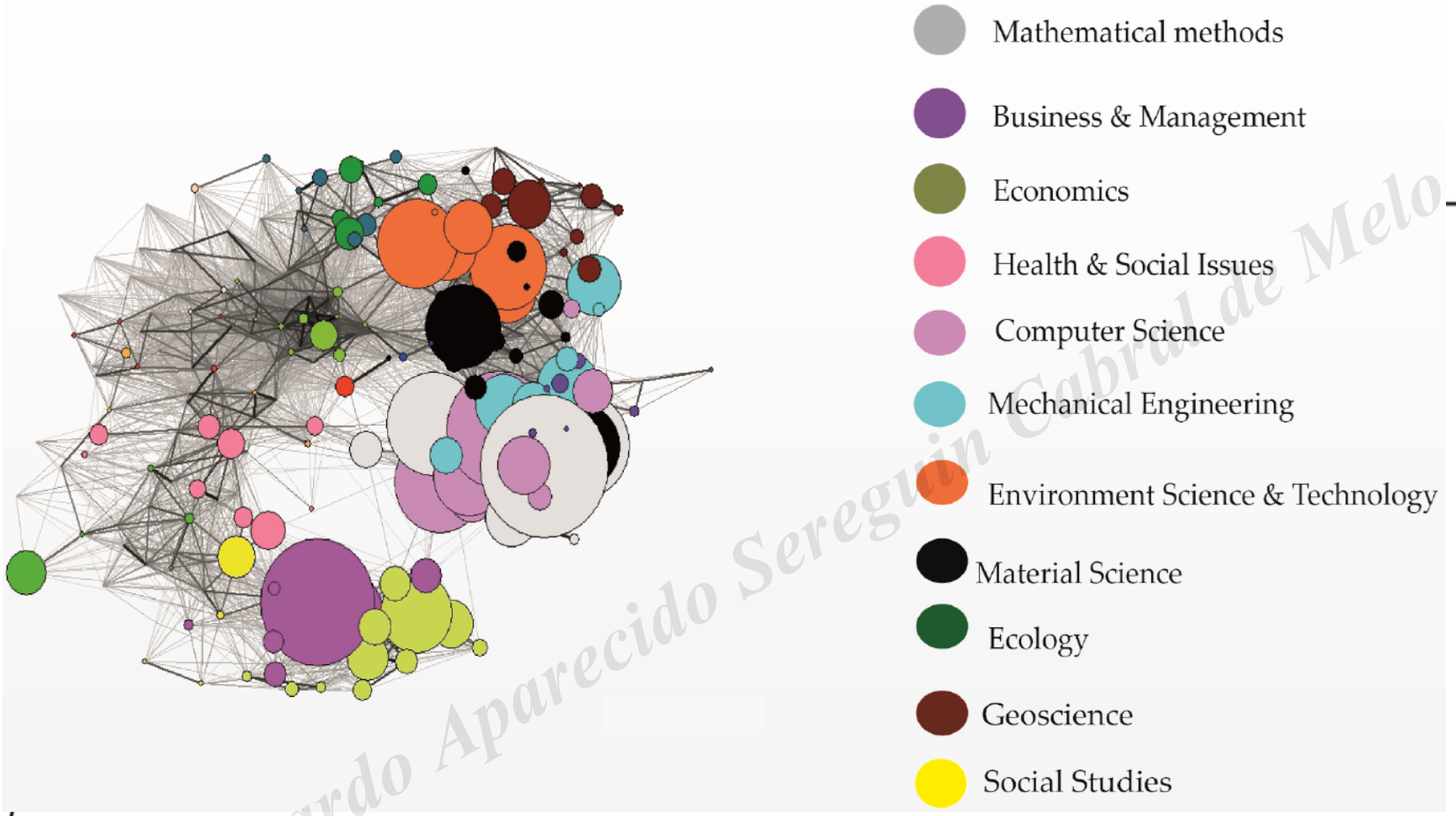
AHP METHOD IN THE WORLD

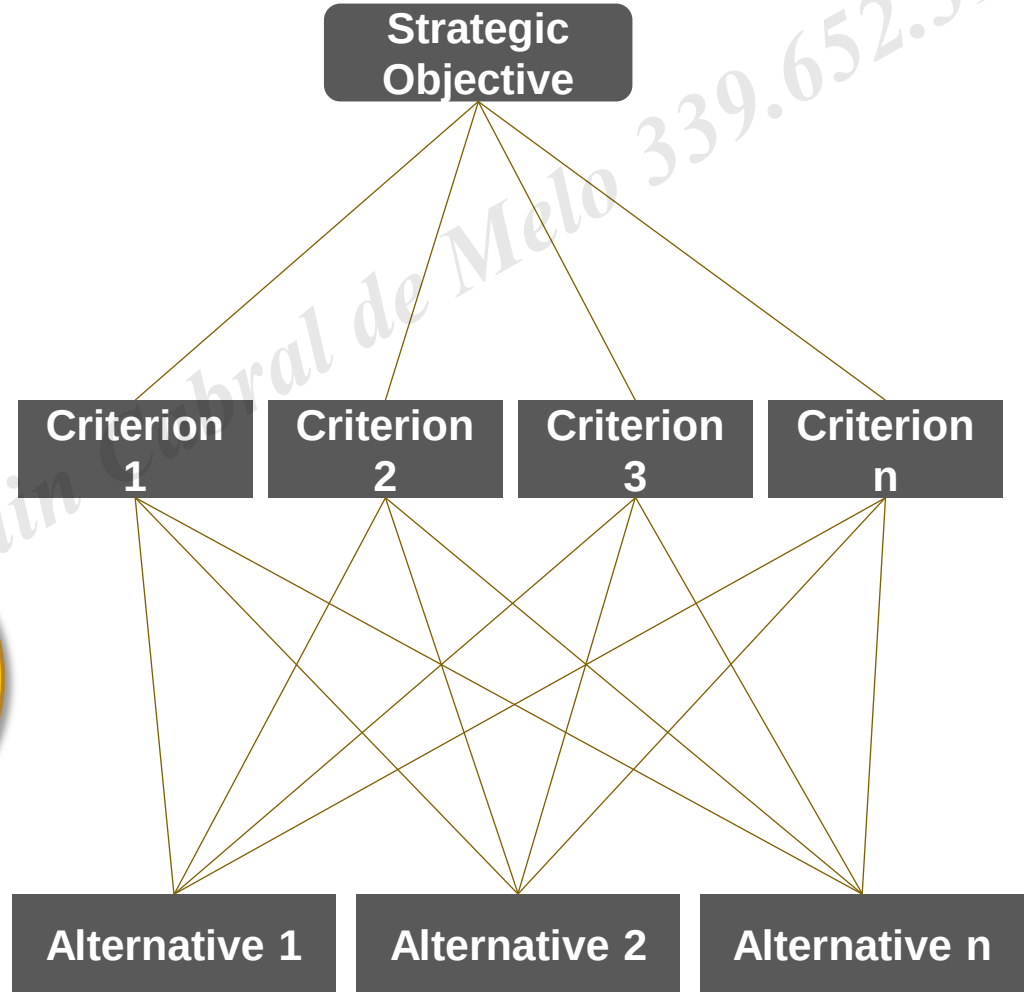


Ali Emrouznejad and Marianna Marra. The state of the art development of AHP (1979 - 2017): a literature review with a social network analysis. International Journal of Production Research, 2017.

AHP METHOD

Based on Multi-criteria Decision Support (MCDS), the AHP method is based on an hierarchical analysis of variables. In this model, a set of alternatives is evaluat[illegible]s of t[illegible]

In case of extremely complex problems, the model allows the use of sub-criteria.



AHP METHOD

One of the main characteristics of the method is to work with subjective issues regarding the performance attributions of the variables of the problem. Simplifying, in the application of the AHP method, it is not necessary to define a precise number regarding the performance of an alternative in a given criterion, in which a verbal attribution already satisfies the model. The attributions of the AHP method are performed based on a scale proposed by Thomas Saaty.

	Variable	Variable	Variable	Variable
Variable				
Variable				
Variable				
Variable				

AHP METHOD

Saaty scale		
Importance relationship	**Level of Importance**	**Reciprocal**
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
More important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

AHP METHOD

For example ...

	A	B	C
A			
B			
C			

The evaluation is performed in pairs among the variables (criteria/alternatives)

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
More important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

AHP METHOD

For example ...

	A	B	C
A		5	
B			
C			

The evaluation is performed in pairs among the variables (criteria/alternatives)

In this case, **A** is more important than **B**

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
More important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

AHP METHOD

For example ...

	A	B	C
A		5	7
B			
C			

The evaluation is performed in pairs among the variables (criteria/alternatives)

In this case, **A** is much more important than **C**

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
More important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

AHP METHOD

For example ...

	A	B	C
A		5	7
B			3
C			

The evaluation is performed in pairs among the variables (criteria/alternatives)

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
More important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

In this case, **B** has moderate importance regarding **C**

AHP METHOD

For example ...

	A	B	C
A		5	7
B	1/5		3
C	1/7	1/3	

The evaluation is performed in pairs among the variables (criteria/alternatives)

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
More important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

AHP METHOD

(Consistency of Attributions)

AHP METHOD

	A	B	C
A		5	7
B	1/5		3
C	1/7	1/3	

Transitivity:

If **A > B** and **B > C**, then: **A > C**.

For ∀ A, B and C ∈ R.

In the AHP method, there is a small tolerance for inconsistences.

AHP METHOD

	A	B	C
A		5	7
B	1/5		3
C	1/7	1/3	

Transitivity:

If **A > B** and **B > C**, then: **A > C**.

For ∀ A, B and C ∈ R.

But how can I evaluate the attributions consistency?

AHP METHOD

	A	B	C
A		5	7
B	1/5		3
C	1/7	1/3	

Consistency test:

AHP allows to calculate a consistency ratio **(CR)** by comparing the Consistency Index **(CI)** of the the decision maker attributions of decision maker regarding a consistency index of a random matrix **(RI)**.

Saaty provides the RI values according to the indication below:

RI Values:

Order of Matrix	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

AHP METHOD

Where:

$$C.I. = \frac{(\lambda_{máx} - n)}{(n-1)}$$

$$C.R. = \frac{C.I.}{R.I.}$$

In order to make a set of attributions consistent, we need to obtain a $C.R. \leq 10\%$

Consistency test:

AHP allows to calculate a consistency ratio **(CR)** by comparing the Consistency Index **(CI)** of the the decision maker attributions of decision maker regarding a consistency index of a random matrix **(RI)**.

Saaty provides the RI values according to the indication below:

RI Values:

Order of Matrix	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

AHP METHOD

(General Process of the Model)

AXIOMATIC STRUCTURE

Definition of the set of criteria and alternatives

	Crit 1	Crit 2	...	Crit n
Alt 1	a_{11}	a_{12}	...	a_{1n}
Alt 2	a_{21}	a_{22}	...	a_{2n}
...	...	...	...	...
Alt n	a_{n1}	a_{n2}	...	a_{nn}

Construction of the evaluation matrix composed by the set of alternatives and criteria in evaluation

AXIOMATIC STRUCTURE

Definition of the set of criteria and alternatives

Evaluation of criteria

- Attribution of the judgments of importance among criteria;
- Consistency test of the attributions;
- Algebraic procedure to obtain the priority vector and the weights of the criteria (w_j).

Obtaining priorities of the criteria

AXIOMATIC STRUCTURE

Definition of the set of criteria and alternatives

↓

Evaluation of criteria

↓

Evaluation of alternatives in each criterion

- Attribution of judgments of importance among the alternatives in each criteria;
- Consistency test of the attributions;
- Algebraic process to obtain the priority vector for each alternative in each criterion (a_{ij}).

Obtaining the local priorities of alternatives in each criterion

AXIOMATIC STRUCTURE

Definition of the set of criteria and alternatives

↓

Evaluation of criteria

↓

Evaluation of alternatives in each criterion

↓

Process of preference aggregation

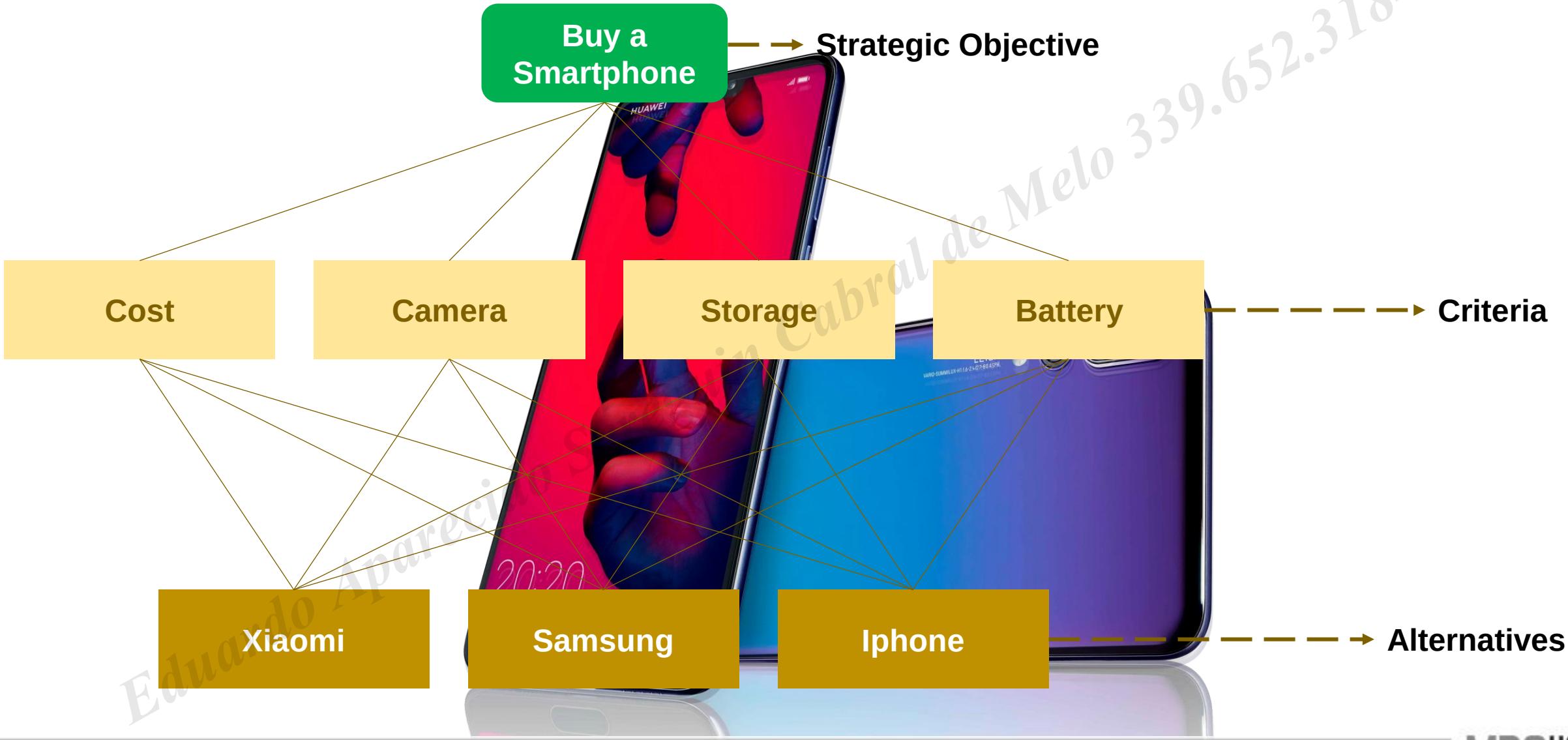
- Weighing of the priority vectors of the alternatives by weights of the respective criteria ($w_j a_{ij}$);
- Sum of priorities ($\sum w_j a_{ij}$).

Obtaining global priorities of the alternatives in the problematic.

AHP EVALUATION

(Classic Model)

PROBLEM STRUCTURING



CRITERIA EVALUATION

Cost	Camera	Storage	Design

First, we will analyze the criteria, making possible obtaining their respective importance.

PAIRWISE EVALUATION OF THE CRITERIA

	Cost	Camera	Storage	Battery
Cost	1	ATTRIBUTION		
Camera	RECIPROCAL	1		
Storage			1	
Battery				1

Number of pairwise comparison will be always given by:

$$\frac{n(n-1)}{2}$$

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
Much more important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

PAIRWISE EVALUATION OF THE CRITERIA

	Cost	Camera	Storage	Battery
Cost				
Camera				
Storage				
Battery				

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
Much more important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

PAIRWISE EVALUATION OF THE CRITERIA

	Cost	Camera	Storage	Battery
Cost	1			
Camera		1		
Storage			1	
Battery				1

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
Much more important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

PAIRWISE EVALUATION OF THE CRITERIA

	Cost	Camera	Storage	Battery
Cost	1	3		
Camera	1/3	1		
Storage			1	
Battery				1

The Cost has moderate importance regarding the camera.

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
More important	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

PAIRWISE EVALUATION OF THE CRITERIA

	Cost	Camera	Storage	Battery
Cost	1	3	5	
Camera	1/3	1		
Storage	1/5		1	
Battery				1

The cost is more important than the storage.

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
Much more important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

PAIRWISE EVALUATION OF THE CRITERIA

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	1/3	1		
Storage	1/5		1	
Battery	1/7			1

The Cost is much more important than the design.

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
Much more important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

PAIRWISE EVALUATION OF THE CRITERIA

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	1/3	1	3	
Storage	1/5	1/3	1	
Battery	1/7			1

The Camera has moderate importance regarding the storage.

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
Much more important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

PAIRWISE EVALUATION OF THE CRITERIA

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	1/3	1	3	7
Storage	1/5	1/3	1	
Battery	1/7	1/7		1

The camera is much more important than the design.

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
More important	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

PAIRWISE EVALUATION OF THE CRITERIA

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	1/3	1	3	7
Storage	1/5	1/3	1	3
Battery	1/7	1/7	1/3	1

The storage has moderate importance regarding the design.

Importance relationship	Level of Importance	Reciprocal
Equality	1	1
Intermediate	2	1/2
Moderate importance	3	1/3
Intermediate	4	1/4
Much more important.	5	1/5
Intermediate	6	1/6
Much more important	7	1/7
Intermediate	8	1/8
Extremely more important.	9	1/9

NORMALIZATION OF THE JUDGEMENTS

Simplifying!

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	0.333	1	3	7
Storage	0.2	0.333	1	3
Battery	0.143	0.143	0.333	1
∑	1.676	4.476	9.333	18

	Cost	Camera	Storage	Battery
Cost				
Camera				
Storage				
Battery				

$$\frac{a_{ij}}{\sum a_{ij}}$$

NORMALIZATION OF THE JUDGEMENTS

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	0.333	1	3	7
Storage	0.2	0.333	1	3
Battery	0.143	0.143	0.333	1
∑	1.676	4.476	9.333	18

	Cost	Camera	Storage	Battery
Cost	0.597			
Camera				
Storage				
Battery				

1 / 1.676 = 0.597

NORMALIZATION OF THE JUDGEMENTS

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	0.333	1	3	7
Storage	0.2	0.333	1	3
Battery	0.143	0.143	0.333	1
∑	1.676	4.476	9.333	18

	Cost	Camera	Storage	Battery
Cost	0.597			
Camera	0.199			
Storage				
Battery				

0.333 / 1.676 = 0.199

NORMALIZATION OF THE JUDGEMENTS

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	0.333	1	3	7
Storage	0.2	0.333	1	3
Battery	0.143	0.143	0.333	1
∑	1.676	4.476	9.333	18

	Cost	Camera	Storage	Battery
Cost	0.597			
Camera	0.199			
Storage	0.119			
Battery				

0.2 / 1.676 = 0.119

NORMALIZATION OF THE JUDGEMENTS

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	0.333	1	3	7
Storage	0.2	0.333	1	3
Battery	0.143	0.143	0.333	1
∑	1.676	4.476	9.333	18

	Cost	Camera	Storage	Battery
Cost	0.597			
Camera	0.199			
Storage	0.119			
Battery	0.085			

0.143 / 1.676 = 0.085

NORMALIZATION OF THE JUDGEMENTS

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	0.333	1	3	7
Storage	0.2	0.333	1	3
Battery	0.143	0.143	0.333	1
∑	1.676	4.476	9.333	18

	Cost	Camera	Storage	Battery
Cost	0.597	0.670	0.536	0.389
Camera	0.199	0.223	0.321	0.389
Storage	0.119	0.075	0.107	0.167
Battery	0.085	0.032	0.036	0.055

NORMALIZATION OF THE JUDGEMENTS

	Cost	Camera	Storage	Battery	Priority Vector
Cost	0.597	0.670	0.535	0.389	0.548
Camera	0.199	0.223	0.321	0.389	0.283
Storage	0.119	0.075	0.107	0.167	0.117
Battery	0.085	0.032	0.035	0.055	0.052

$$Vector = \frac{\sum a_{ij}}{n}$$

But ... What about the consistency of the judgments?

AHP EVALUATION

(Consistency Calculation)

ANALYSIS OF CONSISTENCY

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	0.333	1	3	7
Storage	0.2	0.333	1	3
Battery	0.143	0.143	0.333	1

	Priority Vector
Cost	0.548
Camera	0.283
Storage	0.117
Battery	0.052

Using the priority vector, we will consider our judgment matrix.

ANALYSIS OF CONSISTENCY

	Priority Vector
Cost	0.548
Camera	0.283
Storage	0.117
Battery	0.052

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	0.333	1	3	7
Storage	0.2	0.333	1	3
Battery	0.143	0.143	0.333	1

	Cost	Camera	Storage	Battery
Cost	0.548			
Camera	0.182			
Storage	0.110			
Battery	0.078			

ANALYSIS OF CONSISTENCY

	Priority Vector
Cost	0.548
Camera	0.283
Storage	0.117
Battery	0.052

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	0.333	1	3	7
Storage	0.2	0.333	1	3
Battery	0.143	0.143	0.333	1

	Cost	Camera	Storage	Battery
Cost	0.548	0.849		
Camera	0.182	0.283		
Storage	0.110	0.094		
Battery	0.078	0.040		

ANALYSIS OF CONSISTENCY

	Priority Vector
Cost	0.548
Camera	0.283
Storage	0.117
Battery	0.052

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	0.333	1	3	7
Storage	0.2	0.333	1	3
Battery	0.143	0.143	0.333	1

	Cost	Camera	Storage	Battery
Cost	0.548	0.849	0.585	
Camera	0.182	0.283	0.351	
Storage	0.110	0.094	0.117	
Battery	0.078	0.040	0.039	

ANALYSIS OF CONSISTENCY

	Priority Vector
Cost	0.548
Camera	0.283
Storage	0.117
Battery	0.052

	Cost	Camera	Storage	Battery
Cost	1	3	5	7
Camera	0.333	1	3	7
Storage	0.2	0.333	1	3
Battery	0.143	0.143	0.333	1

	Cost	Camera	Storage	Battery
Cost	0.548	0.849	0.585	0.364
Camera	0.182	0.283	0.351	0.364
Storage	0.110	0.094	0.117	0.156
Battery	0.078	0.040	0.039	0.052

ANALYSIS OF CONSISTENCY

Obtaining lambda max:

$$\frac{\text{"Soma dos Pesos"}}{\text{Vetor Prioridade}}$$

	Cost	Camera	Storage	Battery
Cost	0.548	0.849	0.585	0.364
Camera	0.182	0.283	0.351	0.364
Storage	0.110	0.094	0.117	0.156
Battery	0.078	0.040	0.039	0.052

$\sum$

"Sum of Weights"
2.346
1.180
0.477
0.209

$\therefore$

2.346 / 0.548 = 4.281
1.180 / 0.283 = 4.166
0.477 / 0.117 = 4.077
0.209 / 0.052 = 4.038

	Priority Vector
Cost	0.548
Camera	0.283
Storage	0.117
Battery	0.052

$$\lambda_{máx} = \frac{\sum a_i}{n}$$

$$\sum = 16,562$$

$$\lambda_{máx} = \frac{16,562}{4}$$

$$\lambda_{máx} = 4,14$$

ANALYSIS OF CONSISTENCY

Values obtained until now:

$$\lambda_{máx} = 4{,}14$$

$R\,.I. = 0{,}9$ (Value indicated by Saaty for 4 variables)

$$C.I. = \frac{(4{,}14 - 4)}{(4 - 1)} = 0{,}046$$

Now, we need to obtain:

$$C.I. = \frac{(\lambda_{máx} - n)}{(n - 1)}$$

$$C.R. = \frac{C.I.}{R.I.}$$

$$C.R. = \frac{0{,}046}{0{,}9} = 0{,}052$$

The consistency ratio was under 10%, being favorable as indicated by Saaty.

AHP EVALUATION

(Models with Quantitative Data)

EVALUATION WITH QUANTITATIVE DATA

Most of analyses based on multi-criteria models commonly have numerical data that already represent the relative importance of the alternatives in each criterion.

In this situation, it is also possible to use the AHP method!

EVALUATION WITH QUANTITATIVE DATA

	Cost
Xiaomi	R$ 1500.00
Samsung	R$1800.00
Iphone	R$ 5000.00

	Camera
Xiaomi	12 MP
Samsung	12 MP
Iphone	20 MP

	Storage
Xiaomi	64 Gb
Samsung	128 Gb
Iphone	128 Gb

	Battery Life
Xiaomi	24h
Samsung	18h
Iphone	10h

EVALUATION WITH QUANTITATIVE DATA

Normalization process

Cost monotonic, that is, the lower the better!

	Cost	Camera	Storage	Battery Life
Xiaomi	1500	12	64	24h
Samsung	1800	12	128	18h
Iphone	5000	20	128	10h

EVALUATION WITH QUANTITATIVE DATA

Normalization process

Cost monotonic, that is, the lower the better!

$$x = \frac{1}{a_{ij}}$$

$$\upsilon = \frac{a_{ij}}{\sum a_{ij}}$$

	Cost
Xiaomi	1500
Samsung	1800
Iphone	5000

	Cost
Xiaomi	**0.000667**
Samsung	**0.000556**
Iphone	**0.000200**
Σ	**0.001423**

	Cost
Xiaomi	**0.4687**
Samsung	**0.3906**
Iphone	**0.1406**

$$x_{Xiaomi} = \frac{1}{1500} = 0{,}000667$$

$$x_{Samsung} = \frac{1}{1800} = 0{,}000556$$

$$x_{Iphone} = \frac{1}{5000} = 0{,}000200$$

$$\upsilon_{Xiaomi} = \frac{0{,}000667}{0{,}001423} = 0{,}4687$$

$$\upsilon_{Samsung} = \frac{0{,}000556}{0{,}001423} = 0{,}3906$$

$$\upsilon_{Iphone} = \frac{0{,}000200}{0{,}001423} = 0{,}1406$$

EVALUATION WITH QUANTITATIVE DATA

Normalization process

Profit monotonic, that is, the greater the better!

	Cost	Camera	Storage	Battery Life
Xiaomi	0.468	12	64	24h
Samsung	0.390	12	128	18h
Iphone	0.140	20	128	10h

EVALUATION WITH QUANTITATIVE DATA

Normalization process

Profit monotonic, that is, the greater the better!

$$\upsilon = \frac{a_{ij}}{\sum a_{ij}}$$

	Camera
Xiaomi	12
Samsung	12
Iphone	20
Σ	**44**

	Camera
Xiaomi	**0.273**
Samsung	**0.273**
Iphone	**0.454**

$$\upsilon_{Xiaomi} = \frac{12}{44} = 0{,}273$$

$$\upsilon_{Samsung} = \frac{12}{44} = 0{,}273$$

$$\upsilon_{Iphone} = \frac{20}{44} = 0{,}454$$

EVALUATION WITH QUANTITATIVE DATA

Normalization process

Profit monotonic, that is, the greater the better!

	Cost	Camera	Storage	Battery Life
Xiaomi	0.468	0.273	64	24h
Samsung	0.390	0.273	128	18h
Iphone	0.140	0.454	128	10h

EVALUATION WITH QUANTITATIVE DATA

Normalization process

Profit monotonic, that is, the greater the better!

$$\upsilon = \frac{a_{ij}}{\sum a_{ij}}$$

	Storage
Xiaomi	64
Samsung	128
Iphone	128
Σ	**320**

	Storage
Xiaomi	**0.2**
Samsung	**0,4**
Iphone	**0,4**

$$\upsilon_{Xiaomi} = \frac{64}{320} = 0{,}2$$

$$\upsilon_{Samsung} = \frac{128}{320} = 0{,}4$$

$$\upsilon_{Iphone} = \frac{128}{320} = 0{,}4$$

EVALUATION WITH QUANTITATIVE DATA

Normalization process

Profit monotonic, that is, the greater the better!

	Cost	Camera	Storage	Battery Life
Xiaomi	0.468	0.273	0.2	24h
Samsung	0.390	0.273	0,4	18h
Iphone	0.140	0.454	0,4	10h

EVALUATION WITH QUANTITATIVE DATA

Normalization process

Profit monotonic, that is, the greater the better!

$$\upsilon = \frac{a_{ij}}{\sum a_{ij}}$$

	Battery Life
Xiaomi	24h
Samsung	18h
Iphone	10h
Σ	**52**

	Battery Life
Xiaomi	**0.462**
Samsung	**0.346**
Iphone	**0.192**

$$\upsilon_{Xiaomi} = \frac{24}{52} = 0{,}462$$

$$\upsilon_{Samsung} = \frac{18}{52} = 0{,}346$$

$$\upsilon_{Iphone} = \frac{10}{52} = 0{,}192$$

EVALUATION WITH QUANTITATIVE DATA

	Cost	Camera	Storage	Battery Life
Xiaomi	0.468	0.273	0.2	0.462
Samsung	0.390	0.273	0,4	0.346
Iphone	0.140	0.454	0,4	0.192

EVALUATION WITH QUANTITATIVE DATA

Aggregation process

Weights	0.548	0.283	0.117	0.052

	Cost	Camera	Storage	Battery Life
Xiaomi	0.468 (0.548)	0.273 (0.283)	0.200 . (0.117)	0.462 . (0.052)
Samsung	0.390 . (0.548)	0.273 (0.283)	0.400 . (0.117)	0.346 . (0.052)
Iphone	0.140 . (0.548)	0.454 . (0.283)	0.400 . (0.117)	0.192. (0.052)

	Cost	Camera	Storage	Battery Life
Xiaomi	0.256	0.077	0.023	0.024
Samsung	0.214	0.077	0.047	0.018
Iphone	0.077	0.128	0.047	0.010

EVALUATION WITH QUANTITATIVE DATA

Results

	Cost	Camera	Storage	Battery
Xiaomi	0.256	0.077	0.023	0.024
Samsung	0.214	0.077	0.047	0.018
Iphone	0.077	0.128	0.047	0.010

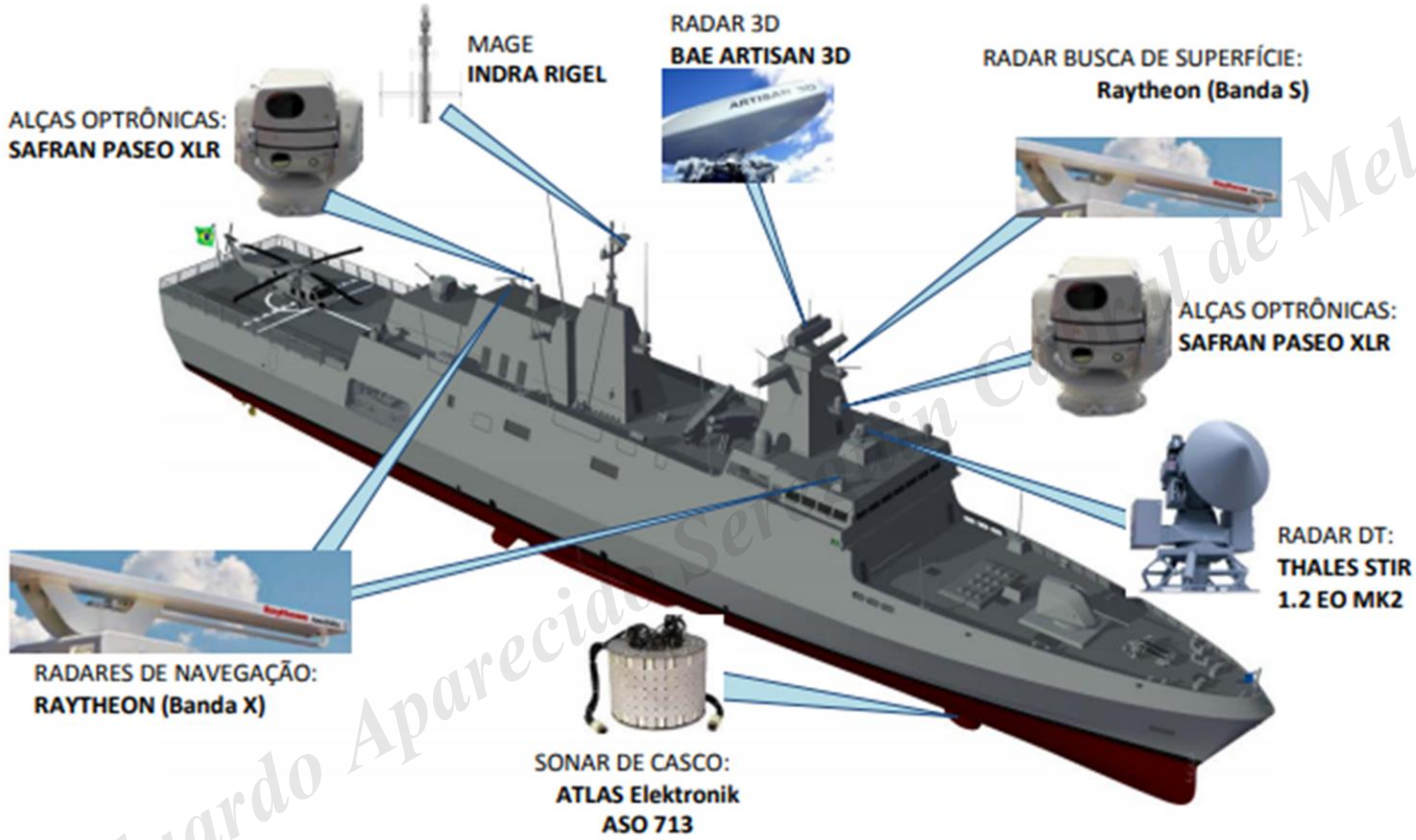
Σ

Final Priority	
0.381	1st
0.356	2nd
0.262	3rd

Method

AHP-GAUSSIAN

REAL PROBLEM OF THE BRAZILIAN NAVY



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Anais do XLVIII SBPO
Simpósio Brasileiro de Pesquisa Operacional
Vitória, ES, 27 a 30 de setembro de 2016.

UMA ABORDAGEM MULTICRITÉRIO PARA SELEÇÃO DE UM NAVIO DE GUERRA DE MÉDIO PORTE A SER CONSTRUÍDO NO BRASIL

Marcos dos Santos (Brazilian Navy & UFF)

Carlos Francisco Simões Gomes (Universidade Federal Fluminense - UFF)

Altina Silva Oliveira (Universidade Federal Fluminense - UFF)

Helder Gomes Costa (Universidade Federal Fluminense – UFF

RESUMO

O objetivo desse artigo é fundamentar a escolha de um navio de médio porte, no caso, 2.000 a 3.000 toneladas, a ser construído no Brasil, apresentando as opções de maneira hierarquizada. Dentre as inúmeras ferramentas do Apoio Multicritério à Decisão (AMD), será utilizado o método AHP. Os critérios serão elencados e os seus respectivos pesos serão atribuídos à luz da Estratégia Nacional de Defesa (END), do Programa Estratégico da Marinha e de entrevistas realizadas com Oficiais da Marinha do Brasil com mais de vinte anos de carreira. Para elencar os critérios foi utilizada a técnica do incidente crítico. A utilização do método AHP na escolha da unidade a ser construída apresenta-se como uma forma transparente e com viés claramente científico para que a sociedade brasileira tenha a percepção de que foi feita a melhor opção dentre os três modelos de navios apresentados.

Palavras-chave: Apoio Multicritério à Decisão (AMD); *Analytic Hierarchy Process* (AHP); Navios; Marinha do Brasil.

Tópicos: ADM – Apoio à Decisão Multicritério

REFERENCES

REFERENCES

MULTICRITERIA DECISION-MAKING IN THE SELECTION OF WARSHIPS: A NEW APPROACH TO THE AHP METHOD

Marcos dos Santos
Instituto Militar de Engenharia (IME)
Brazil

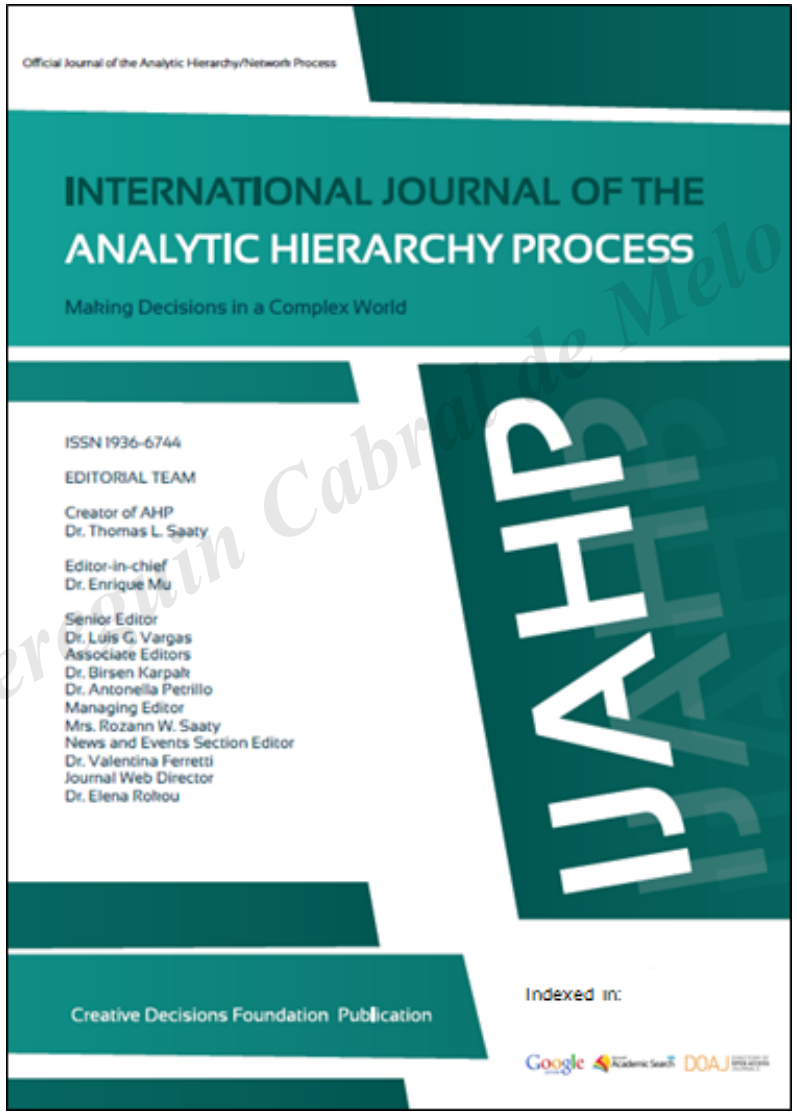
Igor Pinheiro de Araújo Costa
Universidade Federal Fluminense (UFF)
Brazil

Carlos Francisco Simões Gomes
Universidade Federal Fluminense (UFF)
Brazil

ABSTRACT

The budgetary constraints for the Brazilian Navy (BN) have caused several negative effects, resulting in an undersized fleet, decreasing the capacity to protect marine oil and natural gas fields, combat marine pollution from ships, and monitor other illegal activities at sea and inland waters. This paper aims to choose a medium-sized warship to be built by the BN, through the application of the Analytic Hierarchy Process (AHP) method. After a bibliometric study on Multiple-Criteria Decision-Making (MCDM), the AHP was chosen as the most appropriate method for the proposed case study. We analyzed three ship projects with regard to nine operational and economic criteria, taking into account the evaluations of BN officers with recognized experience and knowledge in military operations. We also introduced a sensitivity analysis based on the relationship between standard deviation and mean scores in order to verify and increase the reliability of the ranking. As a result, the methodology suggested that the best option is to build a brand-new ship with more significant modernizations to provide for the operational needs of the BN.

Keywords: Analytic Hierarchy Process; multi-criteria; warship



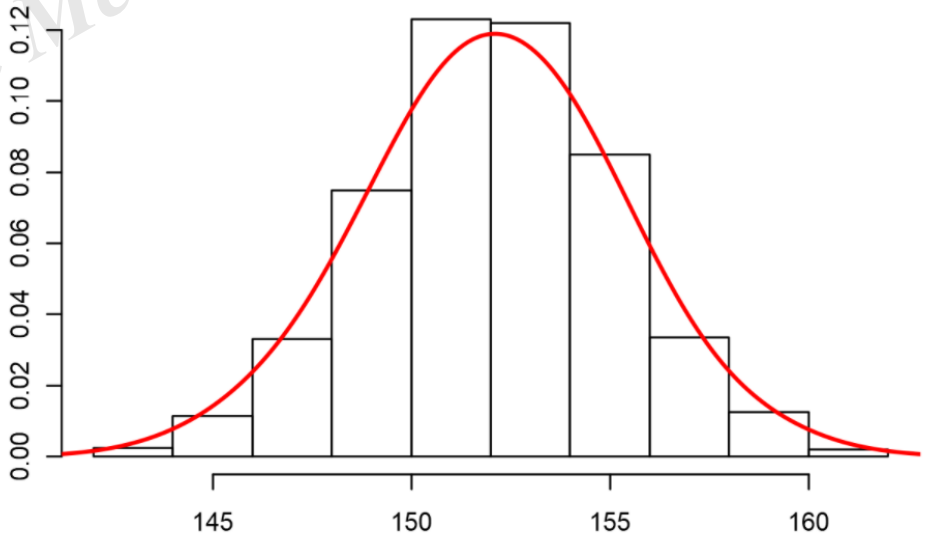
AHP-GAUSSIAN METHOD

What is different in AHP-Gaussian?

AHP-GAUSSIAN METHOD

The AHP-Gaussian method presents a new approach in relation to the original method, based on **an analysis of sensitivity from the gaussian factor**.

With this approach, it is possible to obtain the weights of the criteria through quantitative inputs of the alternatives in each criterion under analysis, that is, from data of the decision matrix.



AHP-GAUSSIAN METHOD

One of the points of the new model that stands out is related to the reduction of the cognitive effort by the decision maker, since it is not necessary the pairwise evaluation of the criteria to obtain their respective weights.

However, it is worth to highlight that the feasibility of the model is only satisfied in situation in which the alternatives have cardinal inputs under analysis.

... USING THE PREVIOUS EXAMPLE

	Cost
Xiaomi	R$ 1500.00
Samsung	R$1800.00
Iphone	R$ 5000.00

	Camera
Xiaomi	12 MP
Samsung	12 MP
Iphone	20 MP

	Storage
Xiaomi	64 Gb
Samsung	128 Gb
Iphone	128 Gb

	Battery Life
Xiaomi	24h
Samsung	18h
Iphone	10h

LET'S CONSIDER NORMALIZED VALUES

Step 1 – Determination of Decision Matrix

	Cost	Camera	Storage	Battery Life
Xiaomi	1200	12	64	24h
Samsung	1500	12	128	18h
Iphone	5000	20	128	10h

	Cost	Camera	Storage	Battery Life
Xiaomi	0.468	0.273	0.2	0.462
Samsung	0.390	0.273	0,4	0.346
Iphone	0.140	0.454	0,4	0.192

AHP-GAUSSIAN METHOD

Step 2 – Calculation of the average of alternatives in each criterion

	Xiaomi	Samsung	Iphone	Mean
Cost	0.468	0.390	0.140	**0.3333**
Camera	0.273	0.273	0.454	**0.3333**
Storage	0.2	0,4	0,4	**0.3333**
Battery	0.462	0.346	0.192	**0.3333**

AHP-GAUSSIAN METHOD

Step 3 – Calculation of the standard deviation of criteria based on the sample of the alternatives

	Xiaomi	Samsung	Iphone	Standard Deviation
Cost	0.468	0.390	0.140	**0.1714**
Camera	0.273	0.273	0.454	**0.1045**
Storage	0.2	0,4	0,4	**0.1155**
Battery	0.462	0.346	0.192	**0.1354**

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

Ps: observe the importance of Statistics in all areas of DS!

AHP-GAUSSIAN METHOD

Step 4 – Calculation of the gaussian factor for each criterion

	Mean	Standard Deviation	Gaussian Factor	Normalized Gaussian Factor
Cost	0.3333	0.1714	**0.5142**	0.3254
Camera	0.3333	0.1045	**0.3135**	0.1984
Storage	0.3333	0.1155	**0.3465**	0.2192
Battery	0.3333	0.1354	**0.4062**	0.2570

$$Fator\ Gaussiano = \frac{\sigma}{\bar{x}}$$

AHP-GAUSSIAN METHOD

Step 5 – Weighting of the decision matrix

Weights	0.3254	0.1984	0.2192	0.2570

	Cost	Camera	Storage	Battery Life
Xiaomi	0.468 (0.3254)	0.273 (0.1984)	0.200 . (0.2192)	0.462 . (0.2570)
Samsung	0.390 . (0.3254)	0.273 (0.1984)	0.400 . (0.2192)	0.346 . (0.2570)
Iphone	0.140 . (0.3254)	0.454 . (0.1984)	0.400 . (0.2192)	0.192 . (0.2570)

	Cost	Camera	Storage	Battery Life
Xiaomi	0.152	0.054	0.044	0.119
Samsung	0.127	0.054	0.088	0.089
Iphone	0.046	0.090	0.088	0.049

AHP-GAUSSIAN METHOD

Step 6 – Obtaining Ranking

	Cost	Camera	Storage	Battery Life	Sum of weighting
Xiaomi	0.152	0.054	0.044	0.119	**0.369**
Samsung	0.127	0.054	0.088	0.089	**0.358**
Iphone	0.046	0.090	0.088	0.049	**0.273**

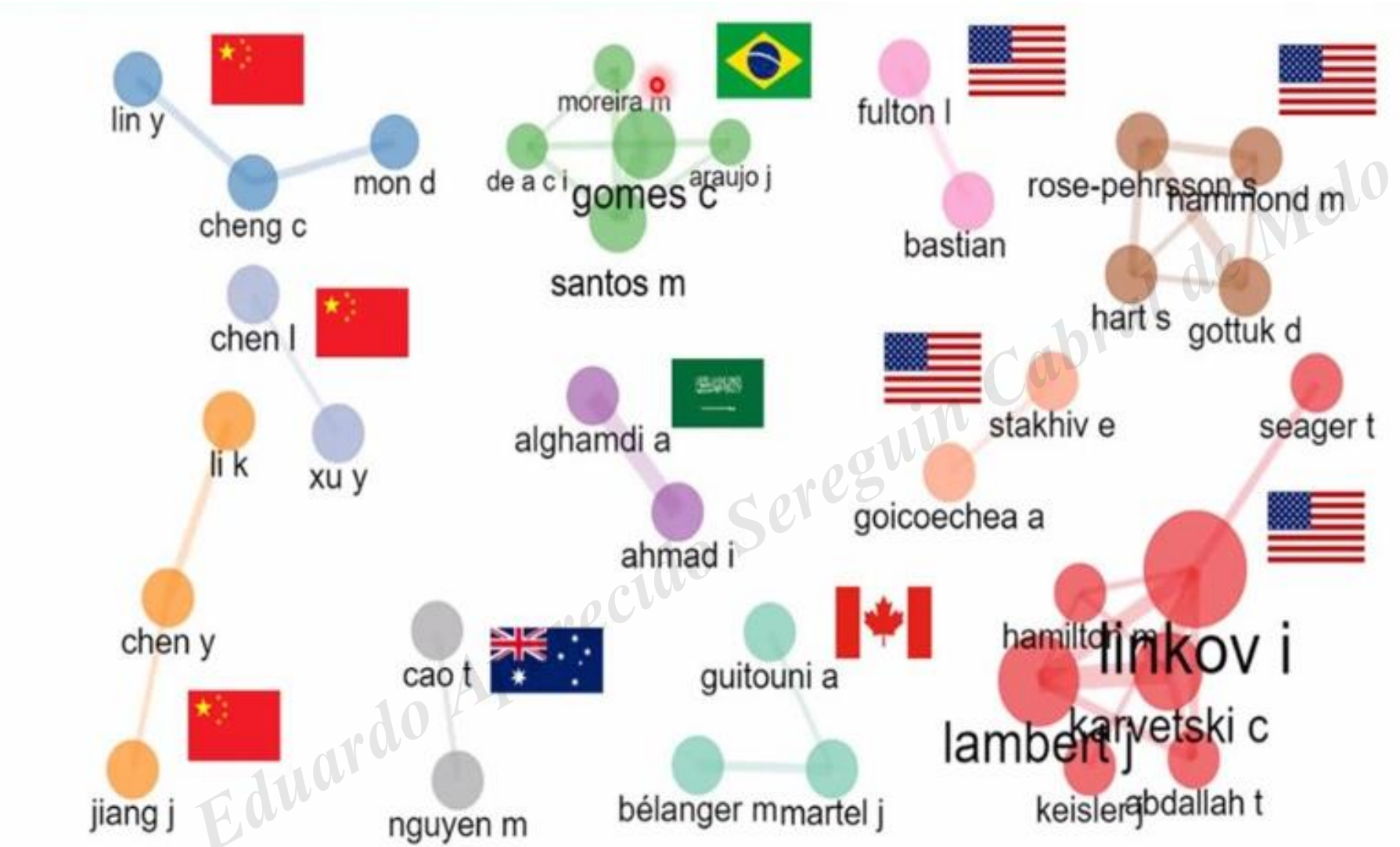
AHP-GAUSSIAN METHOD

Step 7 – Obtaining new ranking

	AHP-Gaussian	Ranking
Xiaomi	0.369	1st
Samsung	0.358	2nd
Iphone	0.273	3rd

AHP
0.381
0.356
0.262

THE METHOD IS BEING USED IN BRAZIL AND IN THE WORLD



Scopus

THE METHOD IS BEING USED IN BRAZIL AND IN THE WORLD

electronics

MDPI

Review

A Systematic Review of the Applications of Multi-Criteria Decision Aid Methods (1977–2022)

Marcio Pereira Basílio [1,2,*], Valdecy Pereira [2], Helder Gomes Costa [2], Marcos Santos [3] and Amartya Ghosh [4]

1 Military Police of the Rio de Janeiro, Rio de Janeiro 21941-901, Brazil
2 Department of Production Engineering, Federal Fluminense University (UFF), Niteroi 24210-240, Brazil;
3 Military Institute of Engineering (IME), Rio de Janeiro 21941-901, Brazil;
4 Symbiosis Institute of Business Management (SIBM), Hyderabad 509217, India;

Abstract: Multicriteria methods have gained traction in academia and industry practices for effective decision-making. This systematic review investigates and presents an overview of multi-criteria

APPLICATION OF THE METHOD IN REAL PROBLEMS

Quadro para Tomada de Decisões estratégicas e o método AHP Gaussiano: uma abordagem multi-metodológica na seleção de um modelo de aeronave cargueira de grande porte para a Força Aérea Brasileira

Leandro de Mattos Bento Soares

IPQM

Marcos dos Santos

IME

APPLICATION OF THE METHOD IN REAL PROBLEMS

XXVIII SIMPEP
Simpósio de Engenharia de Produção

AVALIAÇÃO DE AQUISIÇÕES HOSPITALARES A PARTIR DO MÉTODO MULTICRITÉRIO AHP-GAUSSIANO

LANA PRISCILA CAVADAS DA SILVA
UNIVERSIDADE FEDERAL FLUMINENSE - UFF

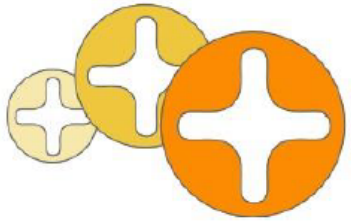
CARLOS FRANCISCO SIMÕES GOMES
UNIVERSIDADE FEDERAL FLUMINENSE - UFF

MARCOS DOS SANTOS
INSTITUTO MILITAR DE ENGENHARIA - IME

MBA USP ESALQ

APPLICATION OF THE METHOD IN REAL PROBLEMS

XXIV ENMC e XII ECTM
13 a 15 de Outubro de 2021



XXIV **ENMC**
Encontro Nacional de Modelagem Computacional
XII **ECTM**
Encontro de Ciência e Tecnologia de Materiais

ESTRATÉGIA DE SELEÇÃO DE EXECUTIVOS PARA UMA MULTINACIONAL: UMA ANÁLISE A PARTIR DOS MÉTODOS AHP-GAUSSIANO E PROPPAGA

David de Oliveira Costa[1]
Marcos dos Santos[2]
Carlos Francisco Simões Gomes[3]
[1] Universidade Federal Fluminense – Niterói, RJ, Brasil
[2] Instituto Militar de Engenharia – Niterói, RJ, Brasil
[3] Universidade Federal Fluminense – Niterói, RJ, Brasil

Resumo. *Esta pesquisa tem o objetivo de estabelecer uma estratégia assertiva de contratação de executivos para multinacionais. Visto que é fundamental entender que processo de seleção do profissional que, futuramente, possa desempenhar um papel de liderança e as suas atribuições, em uma organização passe por validar se o respectivo profissional está alinhado*

APPLICATION OF THE METHOD IN REAL PROBLEMS

REVISTA SIMEP

PROPOSTA DE FABRICAÇÃO DE ARMAÇÃO DE ÓCULOS ATRAVÉS DA IMPRESSÃO 3D: UMA ABORDAGEM A PARTIR DO MÉTODO AHP-GAUSSIANO

Brayan de Almeida Lima (UNIGRANRIO)
Marcos dos Santos (IME)
Daniel Augusto de Moura Pereira (UFCG)
Rubens Lopes de Oliveira (UFRJ)

Resumo

A utilização da tecnologia de manufatura aditiva, popularmente chamada impressão 3D, está cada vez mais presente na vida das pessoas. O objetivo deste trabalho é propor uma análise multicritério, através de um método de apoio à tomada de decisão, na seleção de um material

APPLICATION OF THE METHOD IN REAL PROBLEMS

104 *Modern Management based on Big Data II and Machine Learning and Intelligent Systems III*
A.J. Tallón-Ballesteros (Ed.)

doi:10.3233/FAIA210237

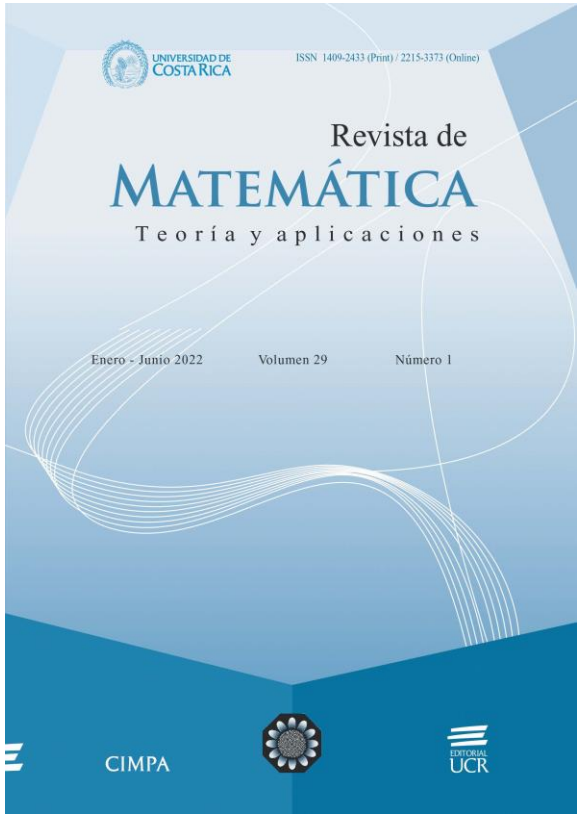
Ordering of Warships for the Brazilian Navy Using the New Method: AHP-Gaussian with Pearson's Correlation

Carlos Francisco Simões GOMES [a,1], Marcus Vinícius Gonçalves RODRIGUES [a], Igor Pinheiro de Araújo COSTA [a] and Marcos dos SANTOS [b]
[a] *Federal Fluminense University – UFF, Niterói, RJ, Brazil*
[b] *Military Engineering Institute – IME, Rio de Janeiro, RJ, Brazil.*

APPLICATION OF THE METHOD IN REAL PROBLEMS

© Revista de Matemática: Teoría y Aplicaciones 2021 **28**(1) : 1–10
CIMPA – UCR ISSN: 1409-2433 (Print), 2215-3373 (Online)
DOI: https://doi.org/10.15517/rmta.v28i1.00000 preliminary

AHP-GAUSSIAN AND VFT FOR SUPPLIER SELECTION IN BRAZILIAN AIRFORCE LOAD PLANE PURCHASING



Get the hands

COMPUTATIONAL TOOL

	Cost	Camera	Storage	Battery Life
Xiaomi	1200	12	64	24h
Samsung	1500	12	128	18h
Iphone	5000	20	128	10h

PROBLEM: SELECT AN INSTRUCTOR

	Tempo de Instrutoria (em dias)	Antiguidade na Turma	Tempo de Experiência na área (em anos)	Média das Avaliações
João	345	36/200 = 0,18	23	9,5
Júlio	720	71/187 = 0,38	28	8,7
Juca	980	110/192 = 0,57	19	9,2
Jorge	110	12/213 = 0,06	12	10
José	504	159/192 = 0,83	25	9,3

COMPUTATIONAL TOOL

AHP - GAUSSSIANO

Número de Alternativas	5
Número de Critérios	4

Limpar Tudo

GERAR BASE
AHP-Gaussiano

GERAR BASE
AHP-Gaussiano + AHP

PROCESSAR
AHP-Gaussiano

PROCESSAR
AHP-Gaussiano + AHP

Tipo	MAX	MAX	MAX	MAX
	C1	C2	C3	C4
A1				
A2				
A3				
A4				
A5				

COMPUTATIONAL TOOL

Tipo	MAX	MIN	MAX	MAX
	Instrutoria	Antiguidade	Experiência	Avaliações
João	345	0,18	23	9,5
Júlio	720	0,38	28	8,7
Juca	980	0,57	19	9,2
Jorge	110	0,06	12	10
José	504	0,83	25	9,3

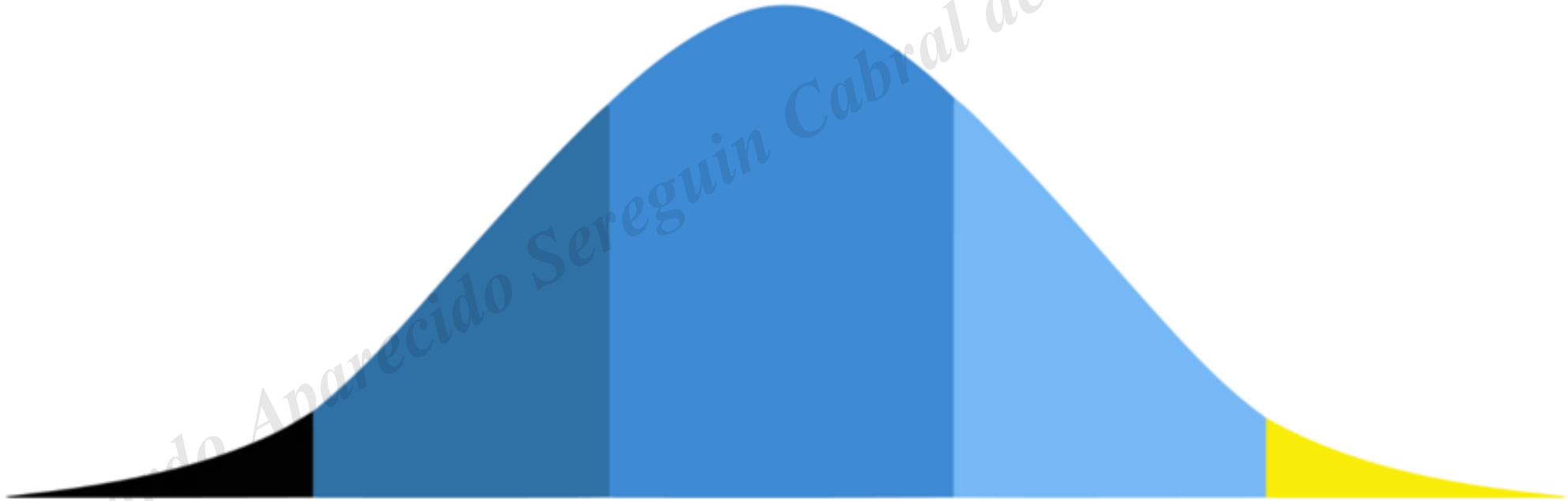
	C1	C2	C3	C4	AHP-G	RANK
A1	0,129748026	0,199746675	0,214953271	0,203426124	0,181142	2
A2	0,270778488	0,094616846	0,261682243	0,186295503	0,171765	4
A3	0,368559609	0,063077897	0,177570093	0,197002141	0,172434	3
A4	0,041368936	0,599240025	0,112149533	0,214132762	0,358383	1
A5	0,189544942	0,043318556	0,23364486	0,199143469	0,116277	5

Média	0,2	0,2	0,2	0,2
Desvio Padrão	0,126125333	0,231189861	0,057838312	0,010111965
Fator Gaussiano	0,630626667	1,155949307	0,28919156	0,050559824

Fator G. Norma.	0,296580235	0,543636568	0,136005192	0,023778006

COMPUTATIONAL TOOL

To cite the software: MOREIRA, Miguel Ângelo Lellis; SANTOS, Marcos dos; GOMES, Carlos Francisco Simões. Gaussian AHP Software Web (v.1). 2021.



https://marcosdossantos.shinyapps.io/gaussian_ahp/

MODELING A PROBLEM FROM ZERO

APPLICATIONS OF OPERATIONAL RESEARCH

Air New Zealand	Alocação	11.2	6,7 milhões
Taco Bell	Program	11.5	13 milhões
Waste Management	Desenv cole	11.7	100 milhões
Bank Hapoalim Group	Dese p		31 milhões mais receitas
Sears	Prog atendimen		42 milhões
Conoco-Phillips	Avaliação de projetos de		Não estimada
Workers' Compensation	Gestão de pedidos de benefícios por alto risco		4 milhões
Westinghouse	Avaliar projetos de pesquisa e desenvolvimento		Não estimada
Merrill Lynch	Gestão de riscos de liquidez para linha	.2	4 bilhões mais liquidez
PSA Peugeot Citroën	Orientar o processo de veículos efic	6.8	130 milhões mais lucros
KeyCorp	A	.6	20 milhões
General Motors		9	90 milhões
Deere & Company	C		1 bilhão menos estoque
Time Inc.	G		3,5 milhões mais lucros
Bank One Corporation	Ge c	19.2	75 milhões mais lucros
Merrill Lynch	Anál se	para o fornecimento de 20.2	50 milhões mais receitas

STRENGTHS OF THE METHOD

1. New method (state of the art);
2. Easy to understand (differently from the majority);
3. Easy to use (just fulfill the Decision Matrix);
4. Possibility of working with a large volume of data (many alternatives and many criteria);
5. Require less cognitive effort by the decision maker (does not work with parity comparison);
6. Computational tool that performs all calculations of the method;
7. Applicable in numerous management problems of everyday life of high, medium and low complexity.

I learned the AHP Method by Thomas Saaty.

I learned the AHP-Gaussian Method.

I learned AHP and AHP-Gaussian Methods Using Excel and R.

MBA USP ESALQ

Thank you.

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