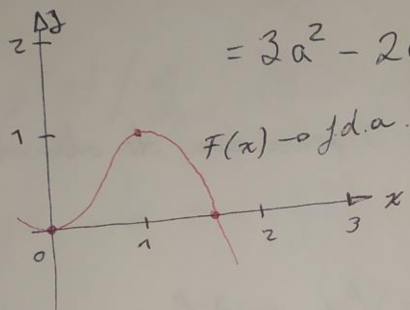
Mercredi, juin 2, 2021

1 a) $\int_0^1 6x(1-x)\,dx = \int_0^1 (6x-6x^2)\,dx$

$$= \frac{6x^2}{2} - \frac{6x^3}{3}\Big|_0^1 = 3 - 2 = 1$$

b) $F(x) = P(X \leq x) = \int_0^x 6a(1-a)\,da = \int_0^x (6a-6a^2)\,da$

$$= 3a^2 - 2a^3\Big|_0^x = 3x^2 - 2x^3$$



c) Aplicando probabilidad condicional, tenemos:

$$P\left(X \leq \tfrac{1}{2} \,\middle|\, \tfrac{1}{3} \leq X \leq \tfrac{2}{3}\right) = \frac{P\left(\frac{1}{3} \leq X \leq \frac{1}{2}\right)}{P\left(\frac{1}{3} \leq X \leq \frac{2}{3}\right)}$$

$$= \frac{F\left(\frac{1}{2}\right) - F\left(\frac{1}{3}\right)}{F\left(\frac{2}{3}\right) - F\left(\frac{1}{3}\right)}$$

$$= \frac{\left(3\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3\right) - \left(3\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right)^3\right)}{\left(3\left(\frac{2}{3}\right)^2 - 2\left(\frac{2}{3}\right)^3\right) - \left(3\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right)^3\right)}$$

$$= \frac{\left(\frac{3}{4} - \frac{2}{8}\right) - \left(\frac{3}{9} - \frac{2}{27}\right)}{\left(\frac{12}{9} - \frac{16}{27}\right) - \left(\frac{3}{9} - \frac{2}{27}\right)}$$

$$= \frac{\frac{3}{4} - \frac{1}{4} - \frac{1}{3} + \frac{2}{27}}{\frac{4}{3} - \frac{16}{27} - \frac{1}{3} + \frac{2}{27}} = \frac{\frac{2}{4} \quad \frac{-27+6}{81}}{\frac{3}{3} - \frac{14}{27}}$$

$$= \frac{\frac{1}{2} - \frac{21}{81}}{1 - \frac{14}{27}} = \frac{\frac{81-42}{162}}{\frac{27}{27} - \frac{14}{27}} = \frac{1}{2}$$

5 alumnos al azar

2% (cte.) aprobados

Prob. cuando menos 3 alumnos reprobados

Definimos la v.a:

$X(s)$ = Número de alumnos reprobados de los 5 seleccionados

$R_X = (0, 1, 2, 3, 4, 5)$

Deducimos que:

$p = 0.02$ cte. en las 5 selecciones de alumnos.

$n = 5$

Calculamos:

$P(X=k) = \binom{5}{k}(0.02)^k(0.98)^{5-k}, \; k = 0,1,2,3,4,5$ ← f.d.p.

Resolviendo:

$P(X \leq 3) = 1 - P(X \geq$

$= 1 - \left[\binom{5}{3}(0.02)^3(0.98)^{5-3}\right]$

$= 1 - \left[\frac{5!}{3!(5-3)!}(0.08)(0.98)^2\right]$

$= 1 - \left(\frac{5!}{3! \cdot 2!}(0.08)(0.9604)\right)$

$= 1 - \left(\frac{20}{2} \cdot (0.076832)\right)$

$= 0.99$

$0.98 \times 0.98 = 784$

3 a) $y = x^2 + 1$ es una función creciente de $0 < x < 1$, claramente diferenciable y por lo tanto continua en toda x, entonces:

$u = x^2 + 1$, implica $x = \sqrt{y-1}$

$$\frac{dx}{dy} = \frac{1}{2\sqrt{y-1}}$$

$$g(y) = f(x)\left|\frac{dx}{dy}\right| = (1)\left(\frac{1}{2\sqrt{y-1}}\right) = \frac{1}{2\sqrt{y-1}}, \quad 1 < y < 2$$

$$0 < x < 1 \rightarrow 0 < \sqrt{y-1} < 1 \rightarrow 1 < y < 2$$

b) $P(X \leq 0.29) = 0.75 \rightarrow \boxed{P(X > 0.29) = 0.25}$

$P(Y \leq k) = 0.25$

$P(1 - X \leq k) = 0.25$

$\boxed{P(X \geq 1 - k) = 0.25}$

Dicho lo anterior, comparamos

$1 - k = 0.29$

$k = 1 - 0.29 = 0.71$

(4) a) Definimos las v.a X y Y:

X = número de ases extraídos.

Y = número de reinas extraídas.

$R_x = (0, 1, 2, \cancel{3})$

$R_y = (0, 1, 2, \cancel{3})$

$$P(0,0) = \frac{\binom{4}{0}\binom{4}{0}\binom{44}{2}}{\binom{52}{2}} = \frac{473}{663}$$

$$P(0,1) = \frac{\binom{4}{0}\binom{4}{1}\binom{44}{1}}{\binom{52}{2}} = \frac{88}{663}$$

$$P(0,2) = \frac{\binom{4}{0}\binom{4}{2}}{\binom{52}{2}} = \frac{3}{663} \quad , \quad \cancel{P(0,3) =}$$

$$P(1,0) = \frac{\binom{4}{1}\binom{44}{1}}{\binom{52}{2}} = \frac{88}{663}$$

$$P(1,1) = \frac{\binom{4}{1}\binom{4}{1}}{\binom{52}{2}} = \frac{8}{663}$$

$P(1,2) = 0 \quad , \quad \cancel{P(1,3)}$

$$P(2,0) = \frac{\binom{4}{2}}{\binom{52}{2}} = \frac{3}{663}$$

$P(2,1) = 0$

$P(2,2) = 0$

b) Necesitamos la tabla de distribución de probabilidad conjunta:

y \ x	0	1	2	$P(Y=y)=g(y)$
0	$473/663$	$88/663$	$3/663$	$564/663$
1	$88/663$	$8/663$	0	$96/663$
2	$3/663$	0	0	$3/663$
$P(x)=P(X=x)$	$\frac{564}{663}$	$\frac{96}{663}$	$3/663$	1

La distribución marginal de X:

X	0	1	2
$P(x)=$	$564/663$	$96/663$	$3/663$

La distribución marginal de Y:

y	0	1	2
$g(y)$	$564/663$	$96/663$	$3/663$

(5) Es estrictamente creciente de $(0,\infty) \to Y = D^2$

$$y = d^2 \to d = \sqrt{y}$$

$$d' = \frac{1}{2\sqrt{y}}$$

$$g(y) = f(d)\,|d'| = e^{-\sqrt{y}} \cdot \frac{1}{2\sqrt{y}}$$

$$g(y) = \frac{1}{2\sqrt{y}} \cdot e^{-\sqrt{y}}$$

c es una v. a. distribuida uniformemente $(1,2)$

$$q(i) = \int_{-\infty}^{\infty} h(v \cdot i) \cdot g(v)\,|v|\,dv \leftarrow f.d.p$$

donde $i = c/y$

$v = y$

$c = v \cdot i$