Instituto Politécnico Nacional

Escuela Superior de Cómputo

TAREA 5

Materia:

Teoría de comunicaciones y señales

Grupo:

3CV14

Profesor:

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Expanda las siguientes funciones usando la serie trigonométrica de Fourier:

- $f(t) = e^{at}$, $(-\pi < t < \pi)$, donde $a \neq 0$ es una constante.

Para a_0, se tiene:

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\cdot dx$$

i.e;

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{ax}\cdot dx = \frac{1}{2\pi a} e^{ax}\Big|_{-\pi}^{\pi} = \frac{1}{2a\pi}\left(e^{a\pi} - e^{-a\pi}\right)$$

Para a_n, se tiene:

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right) dx$$

i.e;

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{ax}\cdot\cos\left(\frac{n\cancel{\pi}}{\cancel{\pi}}\cdot x\right) dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} e^{ax}\cdot\cos(n\cdot x)\cdot dx$$

$u = \cos(nx)$ | $dv = e^{ax}\cdot dx$

$du = -n\cdot \operatorname{sen}(nx)\,dx$ | $v = \frac{1}{a}\cdot e^{ax}$

$$\int a \cdot dv = a \cdot v - \int v \cdot da$$

$$\rightarrow a_n = \frac{1}{\pi}\left[\frac{1}{a} e^{ax} \cdot \cos(nx) + \int \frac{1}{a} e^{ax} \cdot n \cdot \operatorname{sen}(nx)\, dx\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{a} e^{ax} \cdot \cos(nx) + \frac{n}{a}\int e^{ax} \cdot \operatorname{sen}(nx)\, dx\right]$$

$$a = \operatorname{sen}(nx) \qquad dv = e^{ax}$$

$$da = n\cos(nx)\, dx \qquad v = \frac{1}{a} e^{ax}$$

$$a_n = \frac{1}{\pi}\left[\frac{1}{a} e^{ax} \cos(nx) + \frac{n}{a}\left[\operatorname{sen}(nx) \cdot \frac{1}{a} e^{ax} - \frac{1}{a}\int e^{ax} \cdot n \cdot \cos(nx)\, dx\right]\right]$$

$$\int e^{ax} \cdot \cos(nx)\, dx = \frac{1}{a} e^{ax} \cdot \cos(nx) + \frac{n}{a^2} \cdot \operatorname{sen}(nx)\, e^{ax} - \frac{n^2}{a^2}\int e^{ax} \operatorname{sen}(nx)$$

$$a_n = \frac{1}{\pi}\left[\frac{e^{ax}}{a^2+n^2}\left(a \cdot \cos(nx) + n\operatorname{sen}(nx)\right)\Big|_{-\pi}^{\pi}\right]$$

$$a_n = \frac{a}{\pi(a^2+n^2)}\left(e^{a\pi} \cdot (-1)^n - e^{-a\pi} \cdot (-1)^n\right)$$

Para el b_n, se tiene:

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot \operatorname{sen}\left(\frac{n\pi}{L}x\right)dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{ax}\cdot \operatorname{sen}(nx)\,dx$$

De igual forma es una integral cíclica por partes, por lo tanto:

$$b_n = \frac{1}{\pi}\left[\frac{e^{ax}}{a^2+n^2}\left(a\cdot \operatorname{sen}(nx) - n\cdot\cos(nx)\right)\right]_{-\pi}^{\pi}$$

$$b_n = \frac{1}{\pi(a^2+n^2)}\left[e^{a\pi}\left(-n(-1)^n\right) - e^{-a\pi}\left(-n(-1)^n\right)\right]$$

$$\therefore f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n\cdot\cos\left(\frac{n\pi}{2}x\right) + b_n\cdot \operatorname{sen}\left(\frac{n\pi}{2}x\right)\right]$$

$$f(t) = \frac{1}{2a\pi}\left(e^{a\pi}-e^{-a\pi}\right) + \sum_{n=1}^{\infty}\left[\frac{a(-1)^n\cdot\left(e^{a\pi}-e^{-a\pi}\right)}{\pi(a^2+n^2)}\ldots\right.$$

$$\left.\ldots\cos(nx) + \left(\frac{-n(-1)^n\left(e^{a\pi}-e^{-a\pi}\right)}{\pi(a^2+n^2)}\cdot \operatorname{sen}(nx)\right)\right]$$

- $f(t) = \cos(at)$, $(-\pi \leq t \leq \pi)$, donde $a \neq 0$ es una constante.

Para a_0 se tiene:

$$a_0 = \frac{1}{2l} \int_{-l}^{l} f(x)\, dx = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(ax)\, dx$$

$$= \frac{1}{2a\pi} \cdot \operatorname{sen}(ax) \Big|_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi a} \left[\operatorname{sen}(a\pi) - \operatorname{sen}(a\pi) \right]$$, donde $a \notin \mathbb{Z}$

Para a_n, se tiene

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cdot \cos\left(\frac{n\pi}{l} x\right) dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} \cos(ax) \cos(nx)\, dx$$

Sabemos que:

$$\cos a \cdot \cos b = \frac{1}{2} \left[\cos(a+b) + \cos(a-b) \right]$$

$$\cos ax \cdot \cos bx = \frac{1}{2} \left[\cos(ax + nx) + \cos(ax - nx) \right]$$

$$a_n = \frac{1}{2\pi}\left[\int_{-\pi}^{\pi}\cos(ax+nx)\,dx + \int_{-\pi}^{\pi}\cos(ax-nx)\,dx\right]$$

$$a_n = \frac{1}{2\pi}\left[\frac{1}{a+n}\,\operatorname{sen}(ax+nx) + \frac{1}{a-n}\,\operatorname{sen}(ax-nx)\right]_{-\pi}^{\pi}$$

$$\cancel{a_n = \frac{1}{2\pi}\left[\frac{1}{a+n}\,\operatorname{sen}(ax+nx) + \frac{1}{a-n}\,\operatorname{sen}(ax-nx)\right]^{\pi}}$$

$$a_n = \frac{1}{2\pi}\left[\frac{1}{a+n}\,\operatorname{sen}(\pi(a+n)) + \frac{1}{a-n}\,\operatorname{sen}(\pi(a-n)) - \right.$$

$$\left. \frac{1}{a+n}\,\operatorname{sen}(-\pi(a+n)) - \frac{1}{a-n}\cdot\operatorname{sen}(-\pi(a-n))\right]$$

$$a_n = \frac{1}{2\pi}\left[\frac{(-1)^n\cdot 2\cdot\operatorname{sen}(a+n)}{a+n} \quad \frac{(-1)^n\cdot 2\cdot\operatorname{sen}(a-n)}{a-n}\right]$$

Para b_n, se tiene:

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot\operatorname{sen}\left(\frac{n\pi}{L}x\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}\cos(ax)\cdot\operatorname{sen}(nx)\,dx$$

Sabemos que:

$$\cos a\cdot\operatorname{sen} b = \frac{1}{2}\left(\operatorname{sen}(a+b) - \operatorname{sen}(a-b)\right)$$

$$\cos ax \cdot \operatorname{sen} nx = \frac{1}{2}\left(\operatorname{sen}(ax + nx) - \operatorname{sen}(ax - nx)\right)$$

$$\to b_n = \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} \operatorname{sen}(ax + nx)\,dx - \int_{-\pi}^{\pi} \operatorname{sen}(ax - nx)\,dx \right]$$

$$b_n = \frac{1}{2\pi}\left[-\frac{1}{a+n}\cos((a+n)x) + \frac{1}{a-n}\cos((a-n)x) \right]_{-\pi}^{\pi}$$

$$b_n = \frac{1}{2\pi}\left[-\frac{1}{a+n}\cdot\cos((a+n)\pi) + \frac{1}{a-n}\cos((a-n)\pi) + \dots \right.$$

$$\left. \dots \frac{1}{a+n}\cos((a+n)-\pi) - \frac{1}{a-n}\cos((a-n)-\pi)\right]$$

$$b_n = \frac{1}{2\pi}\left[-\frac{2\cos(a\pi + n\pi}{a+n} + \frac{2\cos(a\pi - n\pi)}{a-n}\right]$$

$$b_n = \frac{1}{2\pi}\left[\frac{2\cos(a\pi)(-1)^n}{a-n} - \frac{2\cos(a\pi)(-1)^n}{a+n}\right]$$

$\therefore$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cdot \cos\left(\frac{n\pi}{2}x\right) + b_n \cdot \operatorname{sen}\left(\frac{n\pi}{2}\cdot x\right)\right]$$

$$f(t) = \frac{\operatorname{sen}(a\pi)}{\pi a} + \sum_{n=1}^{\infty}\left[\frac{1}{2\pi}\left(\frac{(-1)^n \cdot 2\cdot \operatorname{sen}(a\pi)}{a+n} + \frac{(-1)^n \cdot 2\cdot \operatorname{sen}(a\cdot)}{a-n}\right)\right.$$

$$\cos(nt) + \frac{1}{2\pi}\left[\frac{2\cos(a\pi)(-1)^n}{a-n} + \frac{2\cos(a\pi)(-1)^n}{a+n}\right]\operatorname{sen}(nt)\Bigg]$$

- ~~sea~~ $f(t) = \cancel{\cos}\ \sin(at)$, $(-\pi < t < \pi)$ donde a no es entera.

Calculando a_0, se tiene:

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} \operatorname{sen}(ax)\,dx$$

$$= \frac{1}{2\pi}\left[-\frac{1}{a}\cos(ax)\right]_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi}\left[-\frac{1}{a}\cos(a\pi) + \frac{1}{a}\cos(-a\pi)\right]$$

$$a_0 = \frac{1}{2\pi}\left[-\frac{2\cos(a\pi)}{a}\right] = \frac{\cos(a\pi)}{a\pi}$$

Para a_n, se tiene:

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} \operatorname{sen}(ax)\cdot\cos(nx)\,dx$$

Sabemos que:

$$\operatorname{sen} a \cdot \cos b = \frac{1}{2}\left(\operatorname{sen}(a+b) + \operatorname{sen}(a-b)\right)$$

$$\operatorname{sen}(ax)\cdot\cos(nx) = \frac{1}{2}\left[\operatorname{sen}(ax+nx) + \operatorname{sen}(ax-nx)\right]$$

i.e;

$$a_n = \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} \operatorname{sen}(ax+nx)\,dx + \int_{-\pi}^{\pi} \operatorname{sen}(ax-nx)\,dx\right]$$

$$a_n = \frac{1}{2\pi}\left[\frac{-\cos(ax+nx)}{a+n} - \frac{\cos(ax-nx)}{a-n}\right]_{-\pi}^{\pi}$$

$$a_n = \frac{1}{2\pi}\left[\frac{-\cos(a\pi+n\pi)}{a+n} - \frac{\cos(a\pi-n\pi)}{a-n} + \frac{\cos(a+n)-\pi}{a+n} + \frac{\cos(a-n)+\pi}{a-n}\right]$$

$$a_n = \frac{1}{2\pi}\left[\frac{-2\cos((a+n)\pi)}{a+n} - \frac{2\cos((a-n)\pi)}{a-n}\right]$$

Para b_n, se tiene:

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\operatorname{sen}\left(\frac{n\pi}{L}\right)dx = \frac{1}{\pi}\int_{-\pi}^{\pi}\operatorname{sen}(ax)\cdot\operatorname{sen}(nx)\,dx$$

Sabemos que

$$\operatorname{sen} a \cdot \operatorname{sen} b = \frac{1}{2}\left(\cos(a-b) - \cos(a+b)\right)$$

$$\operatorname{sen}(ax)\cdot\operatorname{sen}(nx) = \frac{1}{2}\left(\cos((a-n)x) - \cos((a+n)x)\right)$$

i.e;

$$b_n = \frac{1}{2\pi}\left[\int_{-\pi}^{\pi}\cos((a-n)x)\,dx - \int_{-\pi}^{\pi}\cos((a+n)x)\,dx\right]$$

$$b_n = \frac{1}{2\pi}\left[\frac{\text{sen}((a-n)x)}{a-n} - \frac{\text{sen}((a+n)x)}{a+n}\right]_{-\pi}^{\pi}$$

$$b_n = \frac{1}{2\pi}\left[\frac{\text{sen}((a-n)\pi)}{a-n} - \frac{\text{sen}((a+n)\pi)}{a+n} - \frac{\text{sen}((a-n)-\pi)}{a-n} + \frac{\text{sen}((a+n)-\pi)}{a+n}\right]$$

$$b_n = \frac{1}{2\pi}\left[\frac{2\,\text{sen}((a-n)\pi)}{a-n} - \frac{2\,\text{sen}((a+n)\pi)}{a+n}\right]$$

$$\therefore\; f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cdot \cos\left(\frac{n\pi}{2}x\right) + b_n \cdot \text{sen}\left(\frac{n\pi}{2}x\right)\right]$$

$$f(t) = \frac{-\cos(a\pi)}{a\pi} + \sum_{n=1}^{\infty}\left[\frac{1}{2\pi}\left[-\frac{2\cos((a+n)\pi)}{a+n} - \dots\right.\right.$$

$$\dots \left.\frac{2\cos((a-n)\pi)}{a-n}\right]\cos(nt) + \dots$$

$$\left.\dots \frac{1}{2\pi}\left[\frac{2\,\text{sen}((a-n)\pi)}{a-n} - \frac{2\,\text{sen}((a+n)\pi)}{a+n}\right]\text{sen}(nt)\right]$$

- $f(t) = \begin{cases} 0 & , \text{ para } -\pi < t < 0 \\ t & , \text{ para } 0 < t < \pi \end{cases}$

Para a_0, se tiene:

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\, dx = \frac{1}{2\pi} \int_0^{\pi} x\, dx = \frac{1}{2\pi} \left[\frac{x^2}{2} \right]_0^{\pi} = \frac{\pi^2}{4\pi}$$

Para a_n, se tiene:

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cdot \cos\left(\frac{n\pi}{L} x\right) dx = \frac{1}{\pi} \int_0^{\pi} x \cdot \cos(nx)\, dx$$

$$u = x \qquad dv = \cos(nx)\, dx$$
$$du = dx \qquad v = \frac{1}{n} \operatorname{sen}(nx)$$

$$a_n = \frac{1}{\pi} \left[\frac{x \cdot \operatorname{sen} x}{n} - \frac{1}{n} \int_{-\pi}^{\pi} \operatorname{sen}(nx)\, dx \right]$$

$$= \frac{1}{\pi} \left[\frac{x \cdot \operatorname{sen} x}{n} + \frac{\cos(nx)}{n^2} \right]_0^{\pi} = \frac{1}{\pi} \left[\frac{\pi(-1)^n}{n} + \frac{(-1)^n}{n^2} - \frac{1}{n^2} \right]$$

$$= \frac{1}{\pi} \left[\frac{(-1)^n}{n} \left(\pi + \frac{1}{n} \right) - \frac{1}{n^2} \right]$$

Para b_n, se tiene:

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \cdot \operatorname{sen}\left(\frac{n\pi}{2} x\right) dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot \text{sen}(nx)\,dx$$

$$u = x \qquad dv = \text{sen}(nx)\,dx$$
$$du = dx \qquad v = -\frac{1}{n}\cos(nx)$$

$$b_n = \frac{1}{\pi}\left[-\frac{x\cdot\cos(nx)}{n} + \frac{1}{n}\int_0^{\pi}\cos(nx)\,dx\right]$$

$$b_n = \frac{1}{n\pi}\left[-x\cdot\cos(nx) + \frac{\text{sen}(nx)}{n}\right]_0^{\pi}$$

$$b_n = \frac{1}{n\pi}\left[-(-1)^n\pi + \frac{(-1)^n}{n}\right]$$

$\therefore$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n\cdot\cos\left(\frac{n\pi}{2}\cdot x\right) + b_n\cdot\text{sen}\left(\frac{n\pi}{2}\cdot x\right)\right]$$

$$f(t) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{1}{\pi}\left[\frac{(-1)^n}{n}\left(\pi+\frac{1}{n}\right) - \frac{1}{n}\right]\cdot\cos(nx) + \cdots\right.$$

$$\left.\cdots\frac{1}{n\pi}\left[\frac{(-1)^n}{n} - (-1)^n\pi\right]\cdot\text{sen}(nx)\right]$$