Instituto Politécnico Nacional

Escuela Superior de Cómputo

TAREA 8

Materia:

Teoría de comunicaciones y señales

Grupo:

3CV14

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6.79 Determine the DFT of the sequence:

$$x[n] = a^n \quad , \quad 0 \leq n \leq N-1$$

Solución:

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi}{N} \cdot nk} = \sum_{n=0}^{N-1} \left(a \cdot e^{-j\frac{2\pi}{N}k}\right)^n$$ mediante la serie geométrica

$$X[k] = \frac{1-a^N}{1-a \cdot e^{-j(2\pi/N)k}}$$

6.80 Evaluate the circular convolution:

where $y[n] = x[n] \otimes h[n]$

$$x[n] = u[n] - u[n-4] = \boxed{x_1[n]}$$

$$h[n] = u[n] - u[n-3] = \boxed{x_2[n]}$$

Solución:

$$x_1[n] = \begin{cases} 1, & \text{si } n = 0, 1, 2, 3 \\ 0, & \text{otro caso.} \end{cases}$$

$$x_2[n] = \begin{cases} 1, & \text{si } n = 0, 1, 2 \\ 0, & \text{otro caso.} \end{cases}$$

a) Assumming $N = 4$:

$$y[n] = \sum_{l=0}^{n-1} x_1[l] \cdot x_2[(n-l)N]$$

$$y[0] = \sum_{l=0}^{3} x_1[l] \cdot x_2[(-l)N] = x_1[0] \cdot x_2[0] + x_1[1] \cdot x_2[3] + \cdots$$

$x_1[2] \cdot x_2[2] + x_1[3] \cdot x_2[1] = 3$

$$y[1] = \sum_{l=0}^{3} x_1[l] \cdot x_2[(1-l)_N] = x_1[0] \cdot x_2[1] + x_1[1] \cdot x_2[0] + x_1[2] \cdot x_2[3] + x_1[3] \cdot x_2[2] = 3$$

$$y[2] = \sum_{l=0}^{3} x_1[l] \cdot x_2[(2-l)_N] = x_1[0] x_2[2] + x_1[1] x_2[1] + \cdots + x_1[2] x_2[0] + x_1[3] x_2[3] = 3$$

$$y[3] = \sum_{l=0}^{3} x_1[l] \cdot x_2[(3-l)_N] = x_1[0] \cdot x_2[3] + x_1[1] \cdot x_2[2] + \cdots + x_1[2] \cdot x_2[1] + x_1[3] \cdot x_2[0] = 3$$

$\therefore$

$y[n] = \{3, 3, 3, 3\}$ ✓

k) Assuming $N = 8$

$$y[n] = \sum_{l=0}^{N-1} x_1[l] \cdot x_2[(n-l)_N]$$

$$y[0] = \sum_{l=0}^{7} x_1[l] \cdot x_2[(-l)_N] = x_1[0] \cdot x_2[0] + x_1[1] \cdot x_2[7] + x_1[2] \cdot x_2[6] + x_1[3] \cdot x_2[5] + x_1[4] \cdot x_2[4] + x_1[5] \cdot x_2[3] + x_1[6] \cdot x_2[2] + x_1[7] \cdot x_2[1] = 1$$

$$y[1] = \sum_{l=0}^{7} x_1[l] \cdot x_2[(1-l)_N] = x_1[0] \cdot x_2[1] + x_1[1] \cdot x_2[0] + x_1[2] \cdot x_2[7] + x_2[3] \cdot x_2[6] + x_1[4] \cdot x_2[5] + x_2[5] \cdot x_2[4] + x_1[6] \cdot x_2[3] + x_1[7] \cdot x_2[2] = 2$$

$$y[2]=\sum_{\ell=0}^{7} x_1[\ell]\cdot x_2[(2-\ell)_N] = x_1[0]\cdot x_2[2]+x_1[1]\cdot x_2[1]+$$
$$x_1[2]\cdot x_2[0]+x_1[3]\cdot x_2[7]+$$
$$x_1[4]\cdot x_2[6]+x_1[5]\cdot x_2[5]+$$
$$x_1[6]\cdot x_2[4]+x_1[7]\cdot x_2[3]=3$$

$$y[3]=\sum_{\ell=0}^{7} x_1[\ell]\cdot x_2[(3-\ell)_N] = x_1[0]\cdot x_2[3]+x_1[1]\cdot x_2[2]+$$
$$x_1[2]\cdot x_2[1]+x_1[3]\cdot x_2[0]+$$
$$x_1[4]\cdot x_2[7]+x_1[5]\cdot x_2[6]+$$
$$x_1[6]\cdot x_2[5]+x_1[7]\cdot x_2[4]=3$$

$$y[4]=\sum_{\ell=0}^{7} x_1[\ell]\cdot x_2[(4-\ell)_N] = x_1[0]\cdot x_2[4]+x_1[1]\cdot x_2[3]+$$
$$x_1[2]\cdot x_2[2]+x_1[3]\cdot x_2[1]+$$
$$x_1[4]\cdot x_2[0]+x_1[5]\cdot x_2[7]+$$
$$x_1[6]\cdot x_2[6]+x_1[7]\cdot x_2[5]=2$$

$$y[5]=\sum_{\ell=0}^{7} x_1[\ell]\cdot x_2[(5-\ell)_N] = x_1[0]\cdot x_2[5]+x_1[1]\, x_2[4]+$$
$$x_1[2]\cdot x_2[3]+x_1[3]\, x_2[2]+$$
$$x_1[4]\cdot x_2[1]+x_1[5]\, x_2[0]+$$
$$x_1[6]\cdot x_2[7]+x_1[7]\, x_2[6]=1$$

$$y[6]=\sum_{\ell=0}^{7} x_1[\ell]\cdot x_2[(6-\ell)_N] = x_1[0]\cdot x_2[6]+x_1[1]\, x_2[5]+$$
$$x_1[2]\cdot x_2[4]+x_1[3]\, x_2[3]+$$
$$x_1[4]\cdot x_2[2]+x_1[5]\, x_2[1]+$$
$$x_1[6]\cdot x_2[0]+x_1[7]\, x_2[7]=0$$

$$y[7]=\sum_{\ell=0}^{7} x_1[\ell]\cdot x_2[(7-\ell)_N] = x_1[0]\cdot x_2[7]+x_1[1]\, x_2[6]+$$
$$x_1[2]\cdot x_2[5]+x_1[3]\, x_2[4]+$$
$$x_1[4]\cdot x_2[3]+x_1[5]\, x_2[2]+$$
$$x_1[6]\cdot x_2[1]+x_1[7]\, x_2[0]=0$$

$$\therefore\ y[n]=\{1,2,3,3,2,1,0,0\}$$

(6.81) Consider the secuences $x[n]$ and $h[n]$ in Prob. 6.80.

a) Find the 4-point DFT of $x[n]$, $h[n]$ and $y[n]$

i) Para $x[n]$.

$$x[K] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}Kn}$$

$$x[K] = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi}{4}Kn}$$

$$x[K] = \sum_{n=0}^{3} x[n] e^{-j\frac{\pi}{2}Kn}$$

$$x[K] = 1\cdot e^{-j\frac{\pi}{2}K0} + 1\cdot e^{-j\frac{\pi}{2}K} + 1\cdot e^{-j\pi K} + 1\cdot e^{-j\frac{3\pi}{2}K}$$

$$x[K] = 1 + e^{-j\frac{\pi}{2}K} + e^{-j\pi K} + e^{-j\frac{3\pi}{2}K}$$

$$x[0] = 1 + e^{0} + e^{0} + e^{0} = 1 + 1 + 1 + 1 = 4$$

$$x[1] = 1 + e^{-j\frac{\pi}{2}} + e^{-j\pi} + e^{-j\frac{3\pi}{2}} = 1 - j - 1 + 1/j = 0$$

$$x[2] = 1 + e^{-j\pi} + e^{-j2\pi} + e^{-j3\pi} = 1 - 1 + 1 - 1 = 0$$

$$x[3] = 1 + e^{-j\frac{3\pi}{2}} + e^{-j3\pi} + e^{-j\frac{9\pi}{2}} = 1 - \frac{1}{j} - 1 + \frac{1}{j} = 0$$

ii) Para $h[n]$.

$$h[K] = \sum_{n=0}^{N-1} h[n] e^{-j\frac{2\pi}{N}Kn}$$

$$h[K] = \sum_{n=0}^{3} h[n]\, e^{-j\frac{2\pi}{4}Kn}$$

$$h[K] = \sum_{n=0}^{3} h[n]\, e^{-j\frac{\pi}{2}Kn}$$

$$h[K] = 1\cdot e^{-j\frac{\pi}{2}K0} + 1\cdot e^{-j\frac{\pi}{2}K} + 1\cdot e^{-j\pi K} + 0\cdot e^{-j\frac{3\pi}{2}K}$$

$$h[K] = 1 + e^{-j\frac{\pi}{2}K} + e^{-j\pi K}$$

$$H[0] = 1 + e^{0} + e^{0} \cancel{+e} = 1+1+1 = 3$$

$$H[1] = 1 + e^{-j\frac{\pi}{2}} + e^{-j\pi} = 1 - j - 1 = -j$$

$$H[2] = 1 + e^{-j\pi} + e^{-j2\pi} = 1 - 1 + 1 = 1$$

$$H[3] = 1 + e^{-j\frac{3}{2}\pi} + e^{-j3\pi} = 1 + j - 1 = j$$

iii) Para $y[n]$:

$$y[K] = \sum_{n=0}^{N-1} y[n]\, e^{-j\frac{2\pi}{N}Kn}$$

$$y[K] = \sum_{n=0}^{3} y[n]\cdot e^{\frac{-2j\pi}{4}Kn}$$

$$y[K] = \sum_{n=0}^{3} y[n]\cdot e^{-j\frac{\pi}{2}Kn}$$

$$y[K] = 3\cdot e^{-j\frac{\pi}{2}K\cancel{0}} + 3\cdot e^{-j\frac{\pi}{2}K} + 3\cdot e^{-j\pi K} + 3\cdot e^{-j\frac{3}{2}K\pi}$$

$$y[K] = 3 + 3e^{-j\frac{\pi}{2}K} + 3e^{-j\pi K} + 3e^{-j\frac{3\pi}{2}K}$$

$$y[0] = 3 + 3e^{0} + 3e^{0} + 3e^{0} = 3+3+3+3 = 12$$

$$y[1] = 3 + 3e^{-j\pi/2} + 3e^{-j\pi} + 3e^{-j\frac{3}{2}\pi} = 3 - 3j - 3 + 3/j = 0$$

$$y[2] = 3 + 3e^{-j\pi} + 3e^{-j2\pi} + 3e^{-j\frac{6}{2}\pi} = 3 - 3 + 3 - 3 = 0$$

$$y[3] = 3 + 3e^{-j\frac{3\pi}{2}} + 3e^{-j3\pi} + 3e^{-j\frac{9}{2}\pi} = 3 - \frac{3}{j} - 3 + \frac{3}{j} = 0$$

$\therefore$

$x[0] = 4$, $x[1] = 0$, $x[2] = 0$, $x[3] = 0$

$H[0] = 3$, $H[1] = -j$, $H[2] = 1$, $H[3] = j$

$y[0] = 12$, $y[1] = 0$, $y[2] = 0$, $y[3] = 0$

b) Find $y[n]$ by takying the IDFT of $y[K]$.

$$y[n] = \frac{1}{N} \sum_{n=0}^{N-1} y[K] e^{j\frac{2\pi}{N}nK}$$

$$y[n] = \frac{1}{4} \sum_{n=0}^{3} y[K] e^{j\frac{\pi}{2}nK}$$

$$y[n] = \frac{1}{4} \left[3e^{j\frac{\pi}{2}0K} + 3e^{j\frac{\pi}{2}K} + 3e^{j\pi K} + 3e^{j\frac{3\pi}{2}K} \right]$$

Para $K = 0$:

$$y[n] = \frac{1}{4} [3 + 3 + 3 + 3]$$

$$y[n] = \frac{4}{4} [3 + 3 + 3 + 3]$$

$$y[n] = \{3, 3, 3, 3\}$$

(6.82) Consider a continuous-time signal $x(t)$ that has been prefiltered by a low-pass filter with a cutoff frecuency of 10 KHz. The spectrum of $x(t)$ is estimated by use of the N-point DFT. The desided frecuency resolution is 0.1 Hz. Determine the required value of N (assuming a power of 2) and the necessary data length T_1.

$f_c = 10\ KHz$
$f_r = 0.1\ Hz$

Solución:

Proponiendo un valor para N asumiendo una potencia de 2 y usando la fórmula siguiente:

$$f_r = \frac{f_s}{N} \quad \therefore \quad N = \frac{f_s}{f_r}$$

y la frecuencia de muestreo tomando en cuenta el teorema de Nyquist:

$$f_s = \frac{1}{T} > 2 f_N ,$$ donde f_N es la frecuencia Nyquist y $2 f_N$ es la taza de Nyquist y f_s es la frecuencia de muestreo, por lo tanto tomando en cuenta lo anterior la f_s será:

$f_s = 20\ KHz$, calculando N:

$$N = \frac{f_s}{f_r} = \frac{20\ KHz}{0.1\ Hz} = 200\ KHz \longrightarrow N = 2^{18}$$

Para calcular T_1 se usa la fórmula siguiente:

$$T_1 = \frac{N}{f_s}$$

Sustituyendo valores se obtiene:

$$T_1 = \frac{2^{18}}{20\,KHz} = 13.1072$$

$\therefore$

$$T_1 = 13.1072$$
$$N = 2^{18}$$