Signals and Communication Theory

Discrete-time System Implementation

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Contents

Contents

Introduction

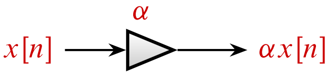
Now we focus on the implementation of discrete-time systems.

Hardware options for implementing discrete-time systems can be

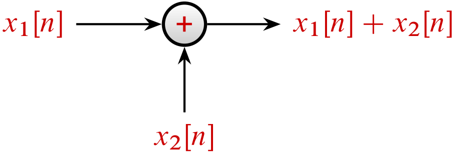
- Special-purpose (custom) chips such as application-specific integrated circuit (**ASICs**),
- Field-programmable gate arrays (**FPGAs**),
- General-purpose microprocessors or microcontrollers (μ**P**/μ**C**),
- Digital signal processors (**DSP**) with application-specific hardware (HW) accelerators.

Basic Building Blocks

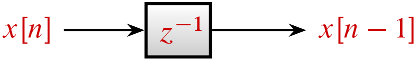
Multiplier



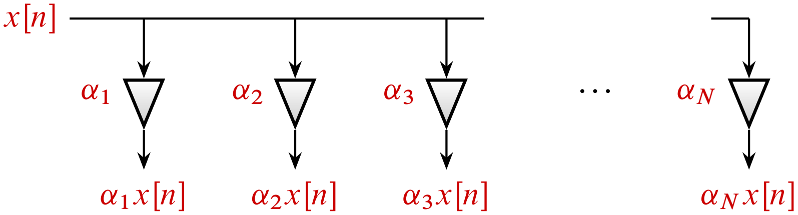
Adder



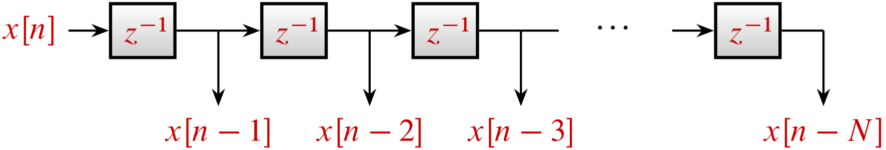
Delay



Basic Structures: **Multiple Constant Multiplication** (MCM)



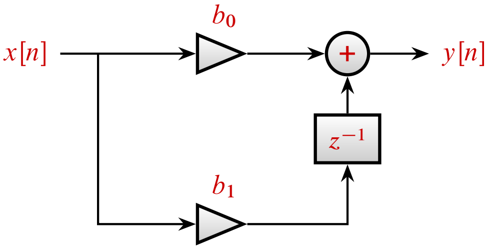
Basic Structures: **Delay Line**



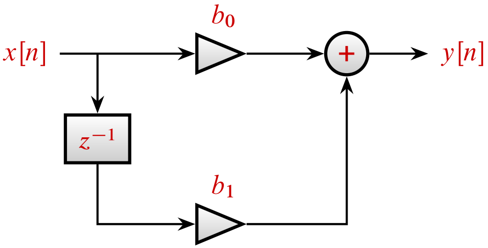
Contents

First Order FIR System

$$H(z) = b_0 + b_1 z^{-1}$$
$$Y(z) = b_0 X(z) + b_1 z^{-1} X(z)$$



Type F1A structure

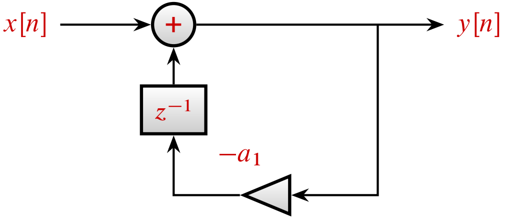


Type F1B structure

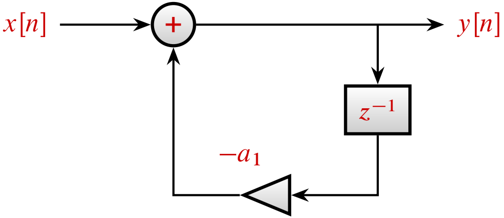
First Order Allpole IIR System

$$H(z) = \frac{1}{1 + a_1 z^{-1}}$$

$$Y(z) = -a_1 z^{-1} Y(z) + X(z)$$



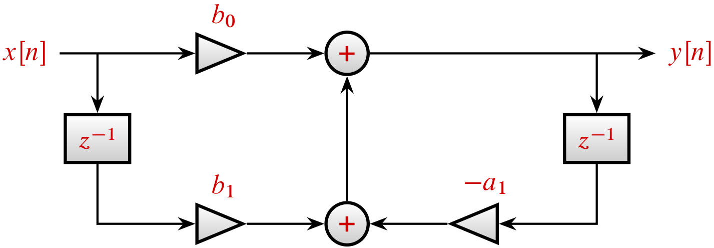
Type I1A structure



Type I1B structure

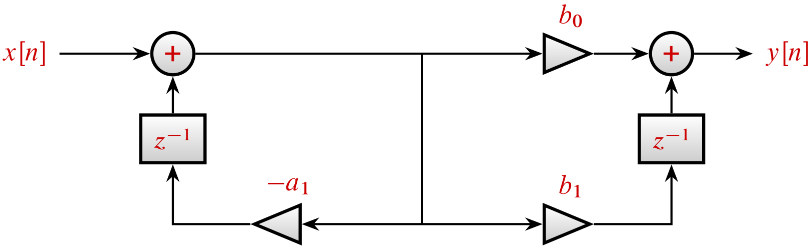
First Order IIR System: Direct Form I Structure

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}$$
$$= \underbrace{\left(b_0 + b_1 z^{-1}\right)}_{\text{Type F1B}} \underbrace{\frac{1}{1 + a_1 z^{-1}}}_{\text{Type I1B}}$$



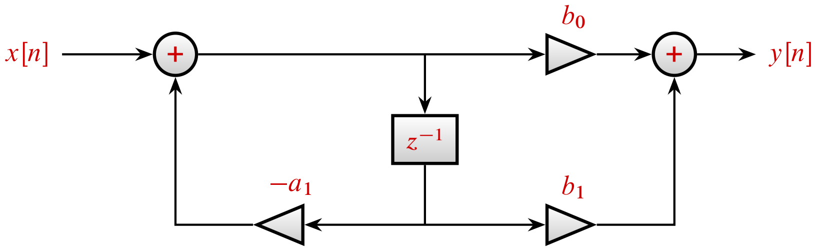
First Order IIR System: Direct Form I_t Structure

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}$$
$$= \underbrace{\frac{1}{1 + a_1 z^{-1}}}_{\text{Type I1A}} \underbrace{\left(b_0 + b_1 z^{-1}\right)}_{\text{Type F1A}}$$



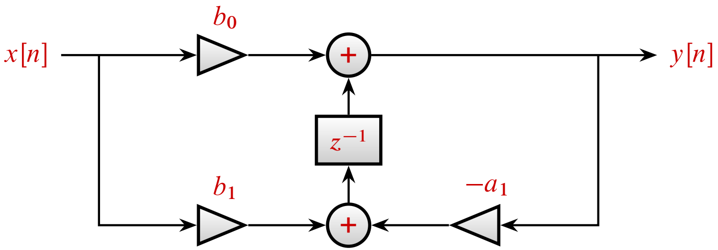
First Order IIR System: Direct Form II Structure

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}$$
$$= \underbrace{\frac{1}{1 + a_1 z^{-1}}}_{\text{Type I1B}} \underbrace{\left(b_0 + b_1 z^{-1}\right)}_{\text{Type F1B}}$$



First Order IIR System: Direct Form II$_t$ Structure

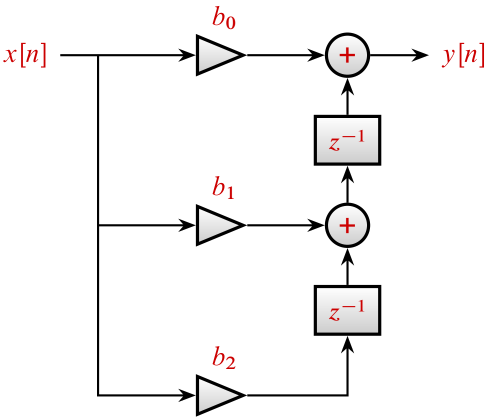
$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}} = \underbrace{\left(b_0 + b_1 z^{-1}\right)}_{\text{Type F1A}} \underbrace{\frac{1}{1 + a_1 z^{-1}}}_{\text{Type I1A}}$$



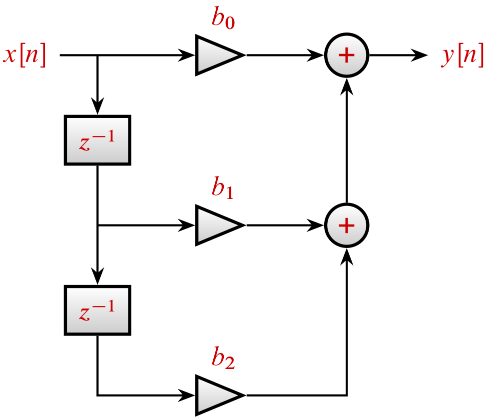
Contents

Second Order FIR System

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2}$$
$$Y(z) = b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z)$$



Type F2A structure

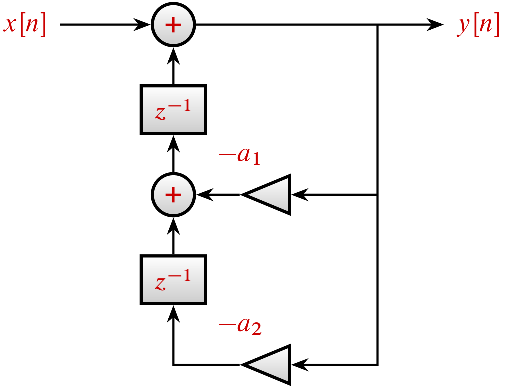


Type F2B structure

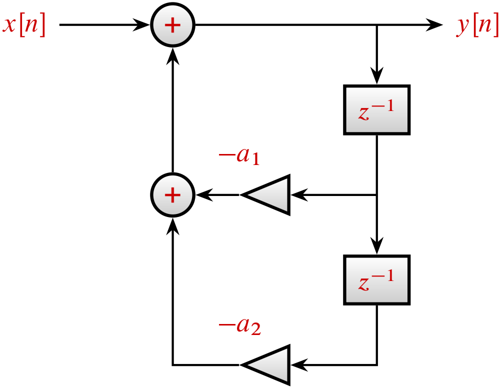
Second Order Allpole IIR System

$$H(z) = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$Y(z) = -a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) + X(z)$$



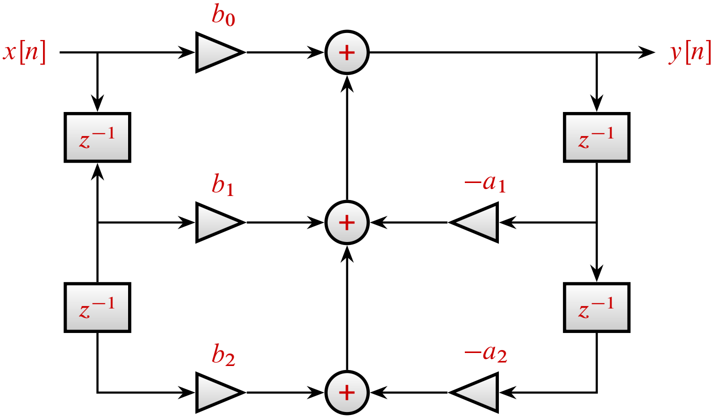
Type I2A structure



Type I2B structure

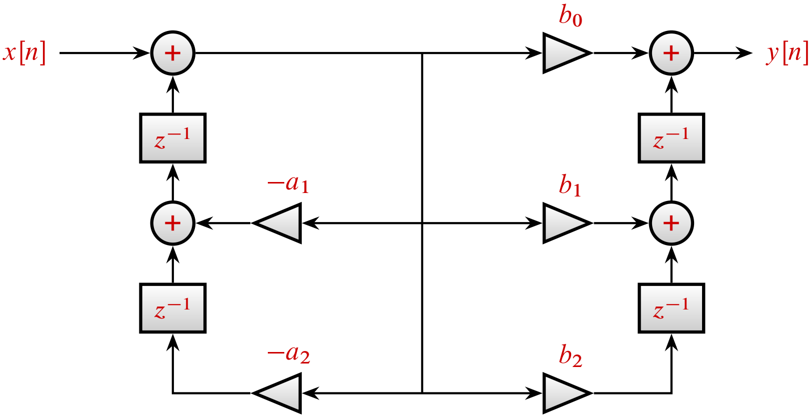
Second Order IIR System: Direct Form I Structure

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \underbrace{\left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right)}_{\text{Type F2B}} \underbrace{\frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}}_{\text{Type I2B}}$$



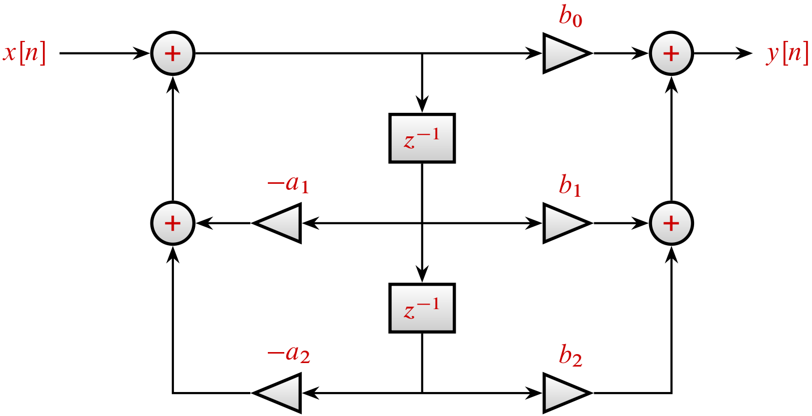
Second Order IIR System: Direct Form I_t Structure

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \underbrace{\frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}}_{\text{Type I2A}} \underbrace{\left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right)}_{\text{Type F2A}}$$



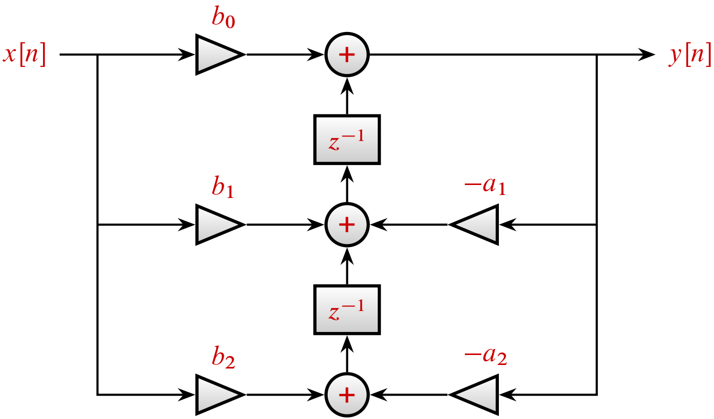
Second Order IIR System: Direct Form II Structure

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \underbrace{\frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}}_{\text{Type I2B}} \underbrace{\left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right)}_{\text{Type F2B}}$$



Second Order IIR System: Direct Form II$_t$ Structure

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \underbrace{\left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right)}_{\text{Type F2A}} \underbrace{\frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}}_{\text{Type I2A}}$$



Contents

Contents

Definition

A **cascade form** is obtained by expressing the numerator and denominator polynomials of the system function $H(z)$ as a product of polynomials of first and second degree, i.e.,

$$H(z) = G \frac{\prod_{k=1}^{K} P_k(z)}{\prod_{m=1}^{M} D_m(z)},$$

where G is a gain constant and

$$P_k(z) = 1 + \beta_{1,k} z^{-1} + \beta_{2,k} z^{-2},$$
$$D_m(z) = 1 + \alpha_{1,m} z^{-1} + \alpha_{2,m} z^{-2}.$$

Example 1

Consider the following system function

$$H(z) = \frac{P_1(z)P_2(z)P_3(z)}{D_1(z)D_2(z)D_3(z)}.$$

Different realizations can be obtained by different pole/zero polynomial pairings.

In practice due to the finite wordlength effects, each such cascade realization behaves differently from others.

Example 1: Different Cascade Realizations

$$\longrightarrow \boxed{\frac{P_1(z)}{D_1(z)}} \longrightarrow \boxed{\frac{P_2(z)}{D_2(z)}} \longrightarrow \boxed{\frac{P_3(z)}{D_3(z)}} \longrightarrow$$

$$\longrightarrow \boxed{\frac{P_1(z)}{D_2(z)}} \longrightarrow \boxed{\frac{P_2(z)}{D_3(z)}} \longrightarrow \boxed{\frac{P_3(z)}{D_1(z)}} \longrightarrow$$

$$\longrightarrow \boxed{\frac{P_1(z)}{D_3(z)}} \longrightarrow \boxed{\frac{P_2(z)}{D_1(z)}} \longrightarrow \boxed{\frac{P_3(z)}{D_2(z)}} \longrightarrow$$

$$\longrightarrow \boxed{\frac{P_2(z)}{D_2(z)}} \longrightarrow \boxed{\frac{P_3(z)}{D_3(z)}} \longrightarrow \boxed{\frac{P_1(z)}{D_1(z)}} \longrightarrow$$

$$\longrightarrow \boxed{\frac{P_3(z)}{D_3(z)}} \longrightarrow \boxed{\frac{P_1(z)}{D_1(z)}} \longrightarrow \boxed{\frac{P_2(z)}{D_2(z)}} \longrightarrow$$

$$\longrightarrow \boxed{\frac{P_3(z)}{D_3(z)}} \longrightarrow \boxed{\frac{P_2(z)}{D_2(z)}} \longrightarrow \boxed{\frac{P_1(z)}{D_1(z)}} \longrightarrow$$

Contents

Definition

A **parallel form** can be obtained by making use of the partial fraction expansion.

A partial fraction expansion of the system function in z^{-1} leads to the **Parallel Form I**, that is,

$$H(z) = \gamma_0 + \sum_k \frac{\gamma_{0,k} + \gamma_{1,k} z^{-1}}{1 + \alpha_{1,k} z^{-1} + \alpha_{2,k} z^{-2}}.$$

Similarly, a partial fraction expansion of the system function in z leads to the **Parallel Form II**, i.e.,

$$H(z) = \beta_0 + \sum_k \frac{\beta_{1,k} z^{-1} + \beta_{2,k} z^{-2}}{1 + \alpha_{1,k} z^{-1} + \alpha_{2,k} z^{-2}}.$$

Example 2

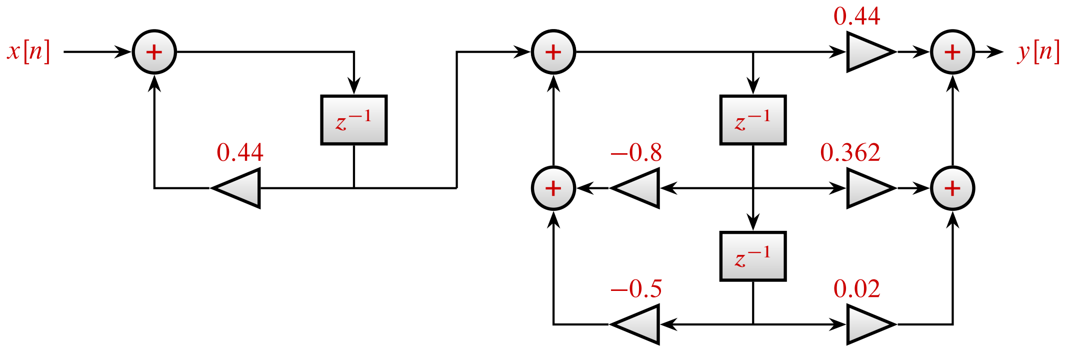
Assume that the system function is given by

$$H(z) = \frac{0.44z^{-1} + 0.36z^{-2} + 0.02z^{-2}}{1 + 0.4z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$
$$= \frac{0.44z^{2} + 0.36z^{1} + 0.02}{z^{3} + 0.4z^{2} + 0.18z - 0.2}$$

Develop the cascade and parallel forms.

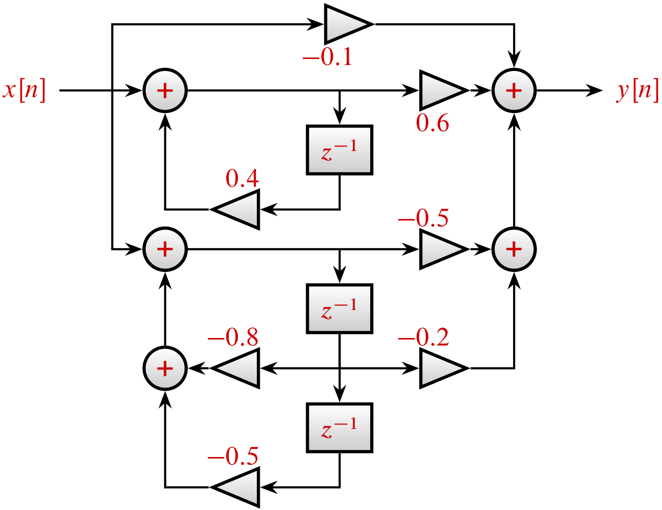
Answer to Example 2: Cascade Form

$$H(z) = \left(\frac{z^{-1}}{1 - 0.4z^{-1}}\right)\left(\frac{0.44 + 0.362z^{-1} + 0.02z^{-2}}{1 + 0.8z^{-1} + 0.5z^{-2}}\right)$$



Answer to Example 2: Parallel Form I

$$H(z) = -0.1 + \frac{0.6}{1 - 0.4z^{-1}} + \frac{-0.5 - 0.2z^{-1}}{1 + 0.8z^{-1} + 0.5z^{-2}}$$



Homework

- From [9], Projects 6.1, 6.2

[9] S. K. Mitra , **Digital Signal Processing Laboratory Using MATLAB**, 1999, McGraw Hill (`labmanual.pdf`)