

Top

Left side

TEST

Right side

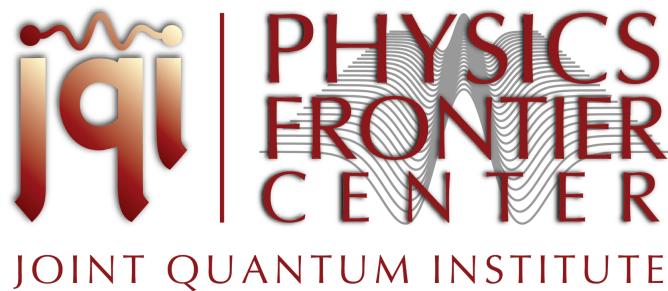
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# Funciones de correlación en óptica; ejemplos clásicos y cuánticos 3.

Postgrado, Física, UNAM, México, Marzo 2019

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[www.jqi.umd.edu](http://www.jqi.umd.edu)



El material del curso está disponible en:



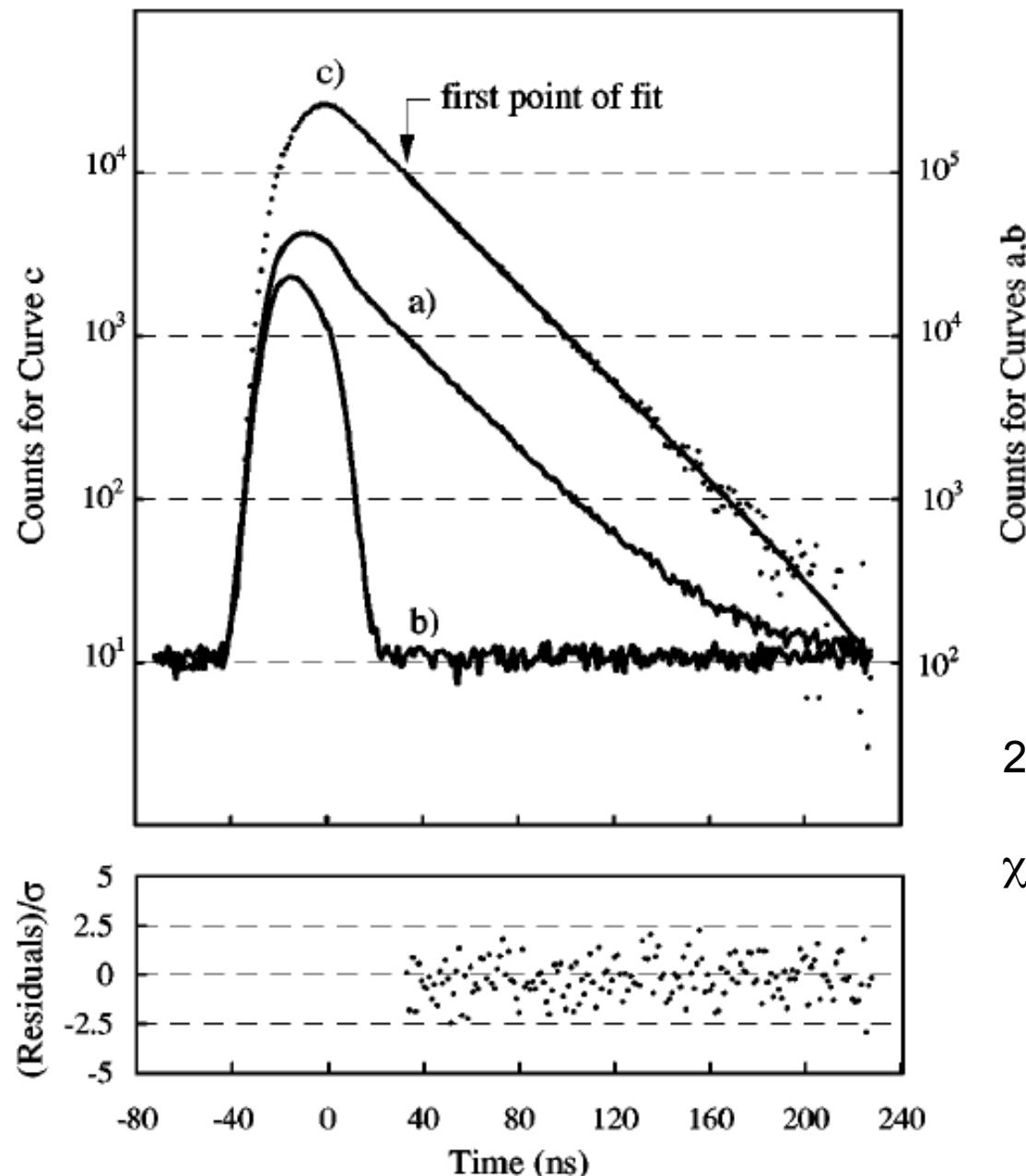
<http://www.physics.umd.edu/rgroups/amo/orozco/results/2019/Results19.htm>

# Notas sobre fluctuaciones, incertidumbre y ruido

# Una revisión rápida del método de $\chi^2$

- Al hacer un experimento los datos tienen una media y fluctuaciones alrededor de la media.
- Las fluctuaciones provienen de un proceso aleatorio (error estadístico) que debe estar distribuido como Gaussiana y posibles errores sistemáticos, que no desaparecen al hacer mas mediciones y pueden contribuir una distribución no Gaussiana alrededor de la media.

El método de “Maximum Likelihood” y  $\chi^2$  son una buena guía para recordar los errores.



J. E. Simsarian, L. A. Orozco, G. D. Sprouse, W. Z. Zhao, "Lifetime measurement of the 7p levels of francium," Phys. Rev. A **57**, 2448 (1998).

El vector de datos  $Y$  y el vector de valores esperados  $F(x)$  tienen  $n$  entradas.

Calcular la diferencia entre el valor medido  $y_i$  y el valor esperado  $f(x)_i$ , normalizando con el error de la medición.

$\chi^2$  es la suma de los cuadrados de las diferencias normalizadas

$$\chi^2 = \sum_i \left| \frac{y_i - f(x_i)}{\sigma_i} \right|^2$$

- Para errores solamente aleatorios la suma debe ser igual a los grados de libertad ( $n-m$ ).
- La distribucion de las diferencias normalizadas debe ser una Gaussiana
- Solamente un 67% de las barras de error deben tocar la linea esperada
- La transformada de Fourier de las diferencias normalizadas no debe contener ninguna frecuencia
- El estadístico  $\chi^2/(n-m)$ , llamado  $\chi^2$  reducida, tiene una media  $\sim 1$  y desviación standard de  $(2/(n-m))^{1/2}$

- Fluctuaciones en una medición deberían ser solamente estadísticas, pero en realidad muchas veces no lo son.
- La parte mas difícil de evaluar son las contribuciones sistemáticas.
- A veces se puede ver por problemas en la distribución no Gaussiana de los residuales normalizados.
- Si mas del 67% de las barras de error tocan la función esperada, el error está sobreestimado,  $\chi^2/(n-m)$  es muy pequeña.

Las fluctuaciones generalmente se caracterizan por una varianza y su correspondiente desviación estandard, pero eso puede no ser suficiente si las fluctuaciones no son Gaussianas.

La incertidumbre está relacionada con la varianza tanto estadística como sistemática.

Ruido se manifiesta como fluctuaciones con propiedades específicas (por ejemplo varianza) y densidades espectrales bien definidas como ejemplo el ruido de disparo,...

# Densidad espectral (de potencia)

- Dónde, en el espectro, está la potencia
- La potencia está relacionada con la varianza (fluctuaciones al cuadrado)
- La densidad espectral de potencia es la transformada de Fourier de la función de correlación de la varianza. (Teorema de Wiener, Khinchin, Kolmogorov)

# La transformada de Fourier de $f(t)$

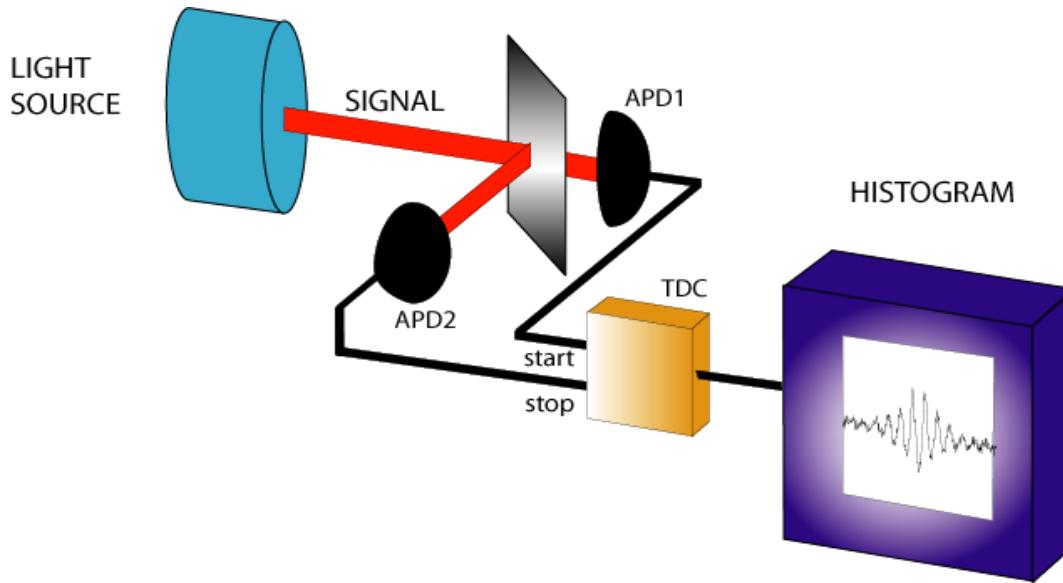
$$g(\omega) = \int_{-\infty}^{\infty} dt f(t) e^{i\omega t}$$

# La transformada de Fourier de $|g(\omega)|^2$

$$\begin{aligned} \int \frac{d\omega}{2\pi} |g(\omega)|^2 e^{-i\omega t} &= \int \frac{d\omega}{2\pi} g^*(\omega) e^{-i\omega t} \int dt' f(t') e^{i\omega t'} \\ &= \int dt' f(t') \int \frac{d\omega}{2\pi} g^*(\omega) e^{i\omega t'} e^{-i\omega t} \\ &= \int dt' f(t') \left[ \int \frac{d\omega}{2\pi} g(\omega) e^{-i\omega(t' - t)} \right]^* \\ &= \int dt' f(t') f(t' - t)^* \end{aligned}$$

# Ejemplo de medición de temperatura con nanofibras

# Hanbury-Brown and Twiss Intensidad-Intensidad



$$g^{(2)}(\tau) = \frac{\langle I(t)I(t + \tau) \rangle}{\langle I(t) \rangle^2}$$

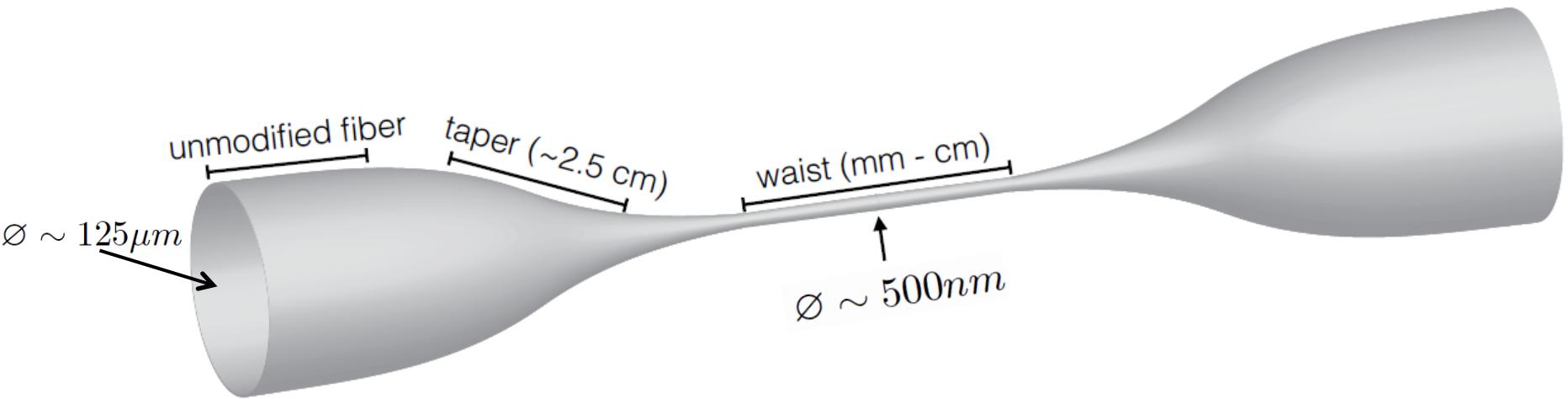
¿Podemos utilizar las fluctuaciones en la intensidad para medir la temperatura de unos átomos alrededor de una nanofibra?

# Correlaciones de la intensidad $\tau=0$

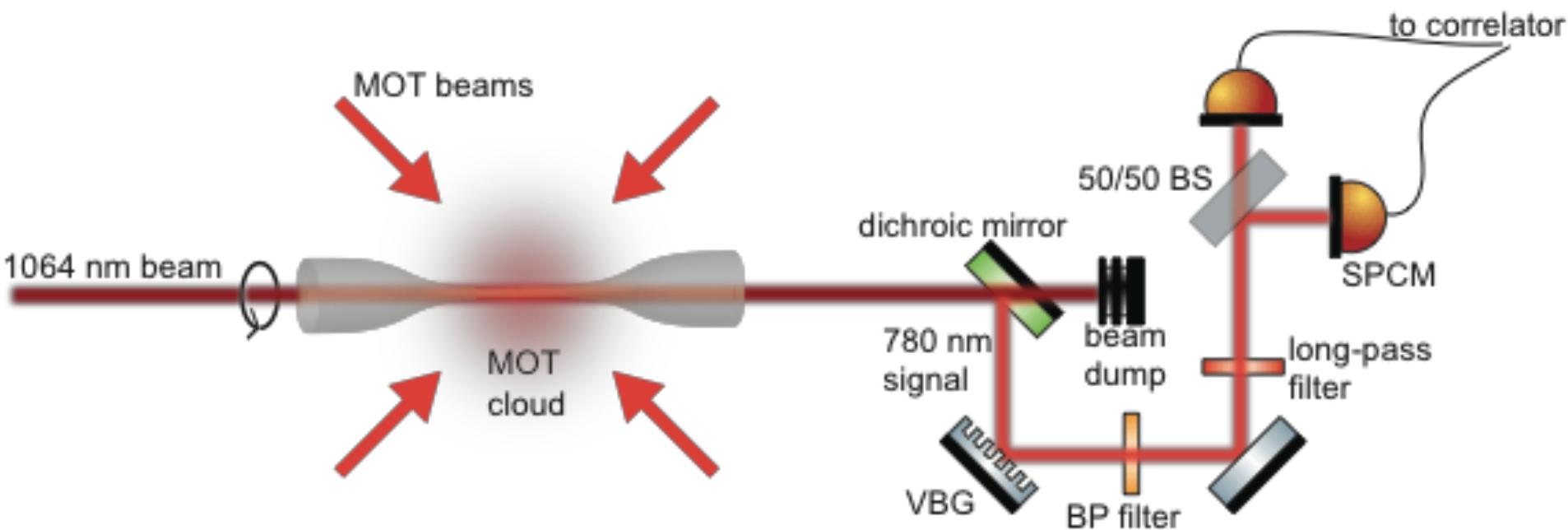
$$\begin{aligned}g^{(2)}(0) &= \frac{\langle I(t)^2 \rangle}{\langle I(t) \rangle^2} \\&= \frac{\langle (I_0 + \delta(t))^2 \rangle}{\langle I_0 + \delta(t) \rangle^2} \\&= 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}\end{aligned}$$

La varianza se debe a las velocidades

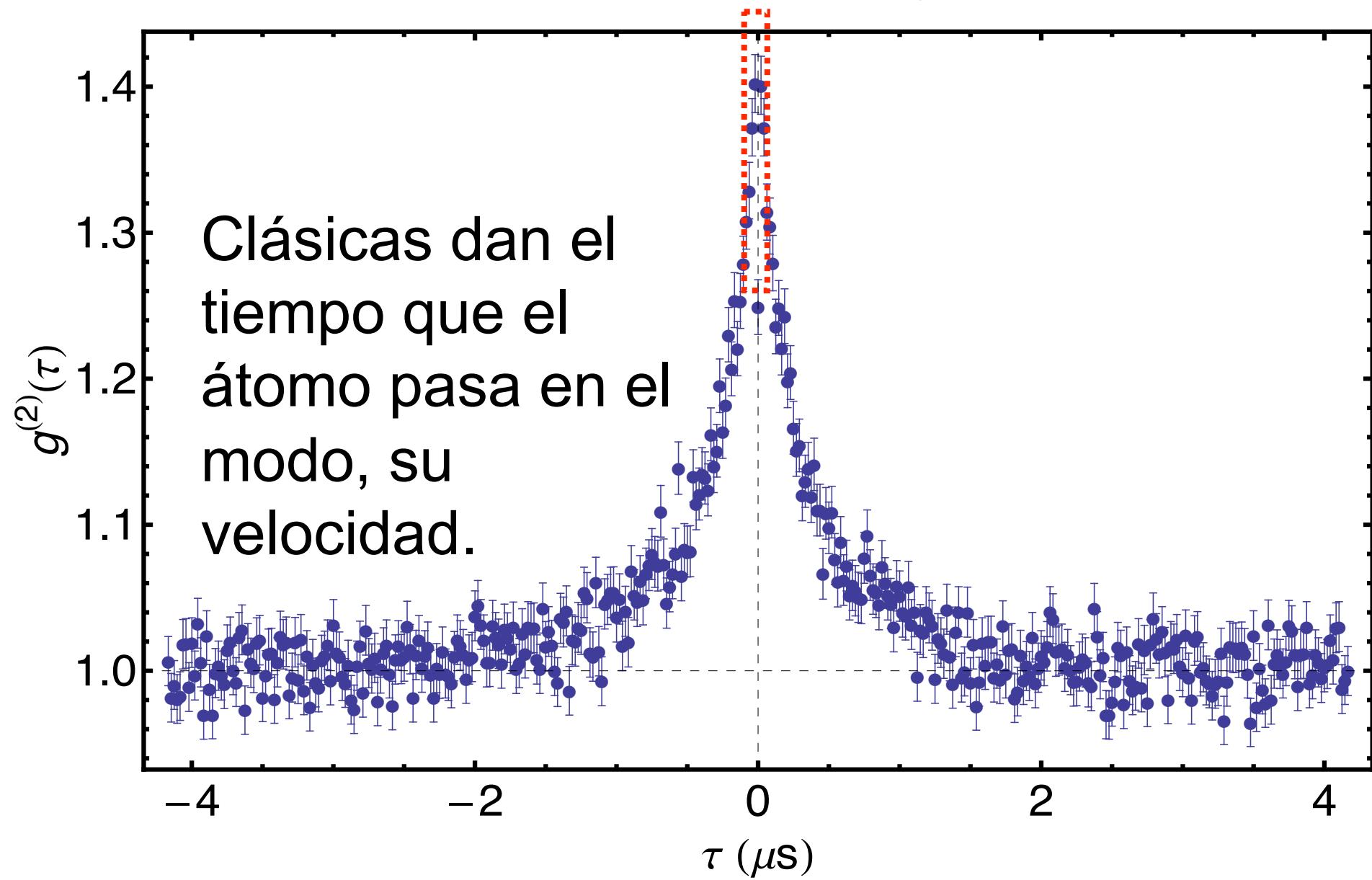
# Nanofibras ópticas



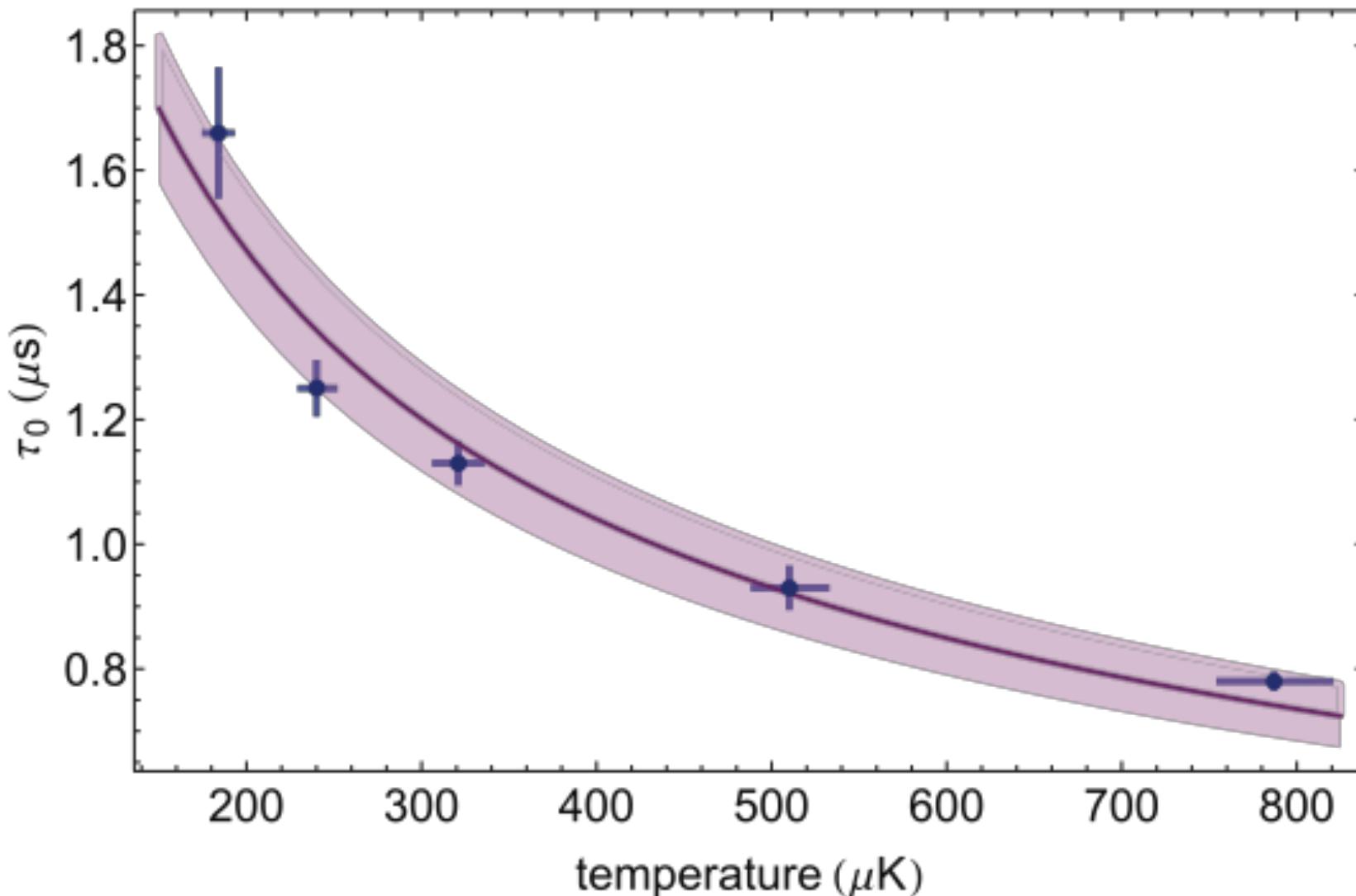
# Correlaciones con fotones emitidos por átomos fríos en el modo de la nanofibra



# Correlaciones clásicas y cuánticas



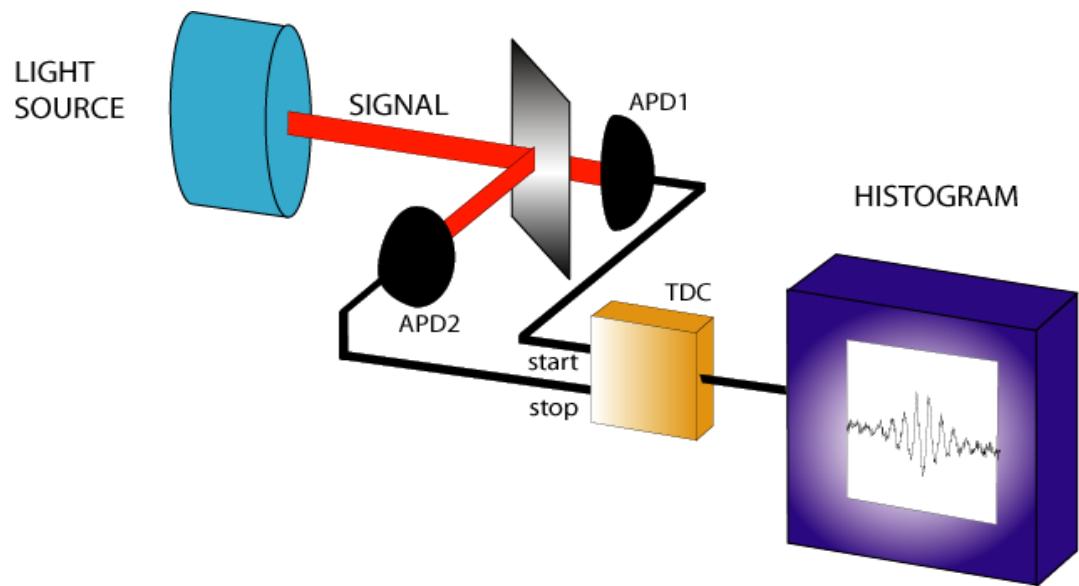
# Relación temperatura velocidad



¿Hay efectos cuánticos en Electrodinámica cuántica de cavidades?

Veamos las fluctuaciones y su varianza

# Hanbury-Brown and Twiss Intensidad-Intensidad



$$g^{(2)}(\tau) = \frac{\langle I(t)I(t + \tau) \rangle}{\langle I(t) \rangle^2}$$

# Optica cuántica

- El fotón es la mas pequeña fluctuación de la intensidad del campo electromagnético, de su varianza.
- El fotón es el cuanto de enegía del campo electromagnético. Con un campo de frecuencia  $\omega$  la energía  $\hbar\omega$ .

# Correlaciones de fotones (Glauber):

$$g^{(2)}(\tau) = \frac{\left\langle :\hat{I}(t)\hat{I}(t+\tau):\right\rangle}{\left\langle \hat{I}(t)\right\rangle^2} \quad g^{(2)}(\tau) = \frac{\left\langle :\hat{I}(\tau):\right\rangle_c}{\left\langle :\hat{I}:\right\rangle}$$

Si detectamos un fotón al tiempo  $t$  la  $g^{(2)}(\tau)$  da la probabilidad de que detecten un segundo fotón después de un tiempo  $\tau$ .

Mide las fluctuaciones, la varianza, la incertidumbre. Ordenamiento normal y temporal.

$g^{(2)}(0)=1$  Poissonian

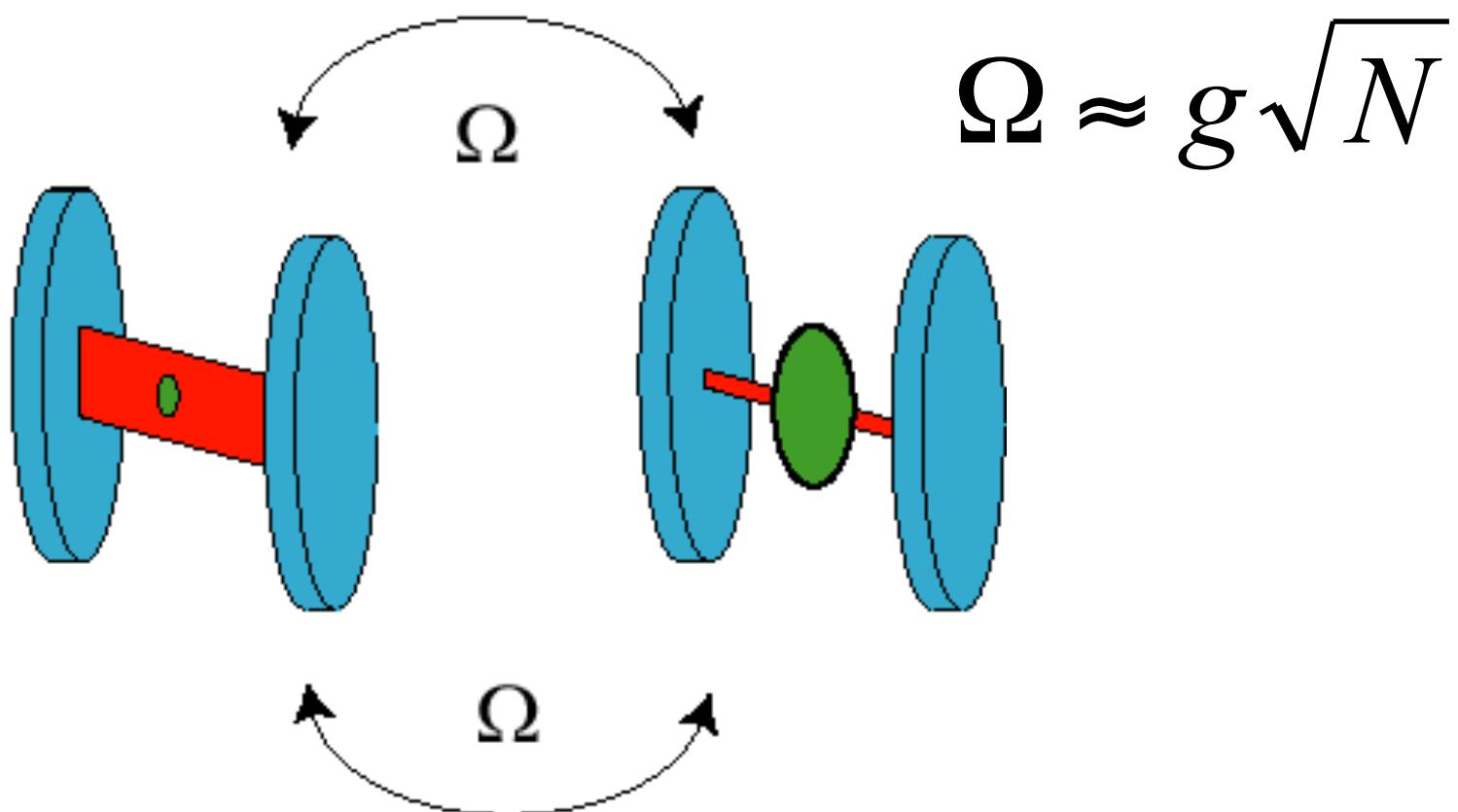
$g^{(2)}(0)>1$  Bunched

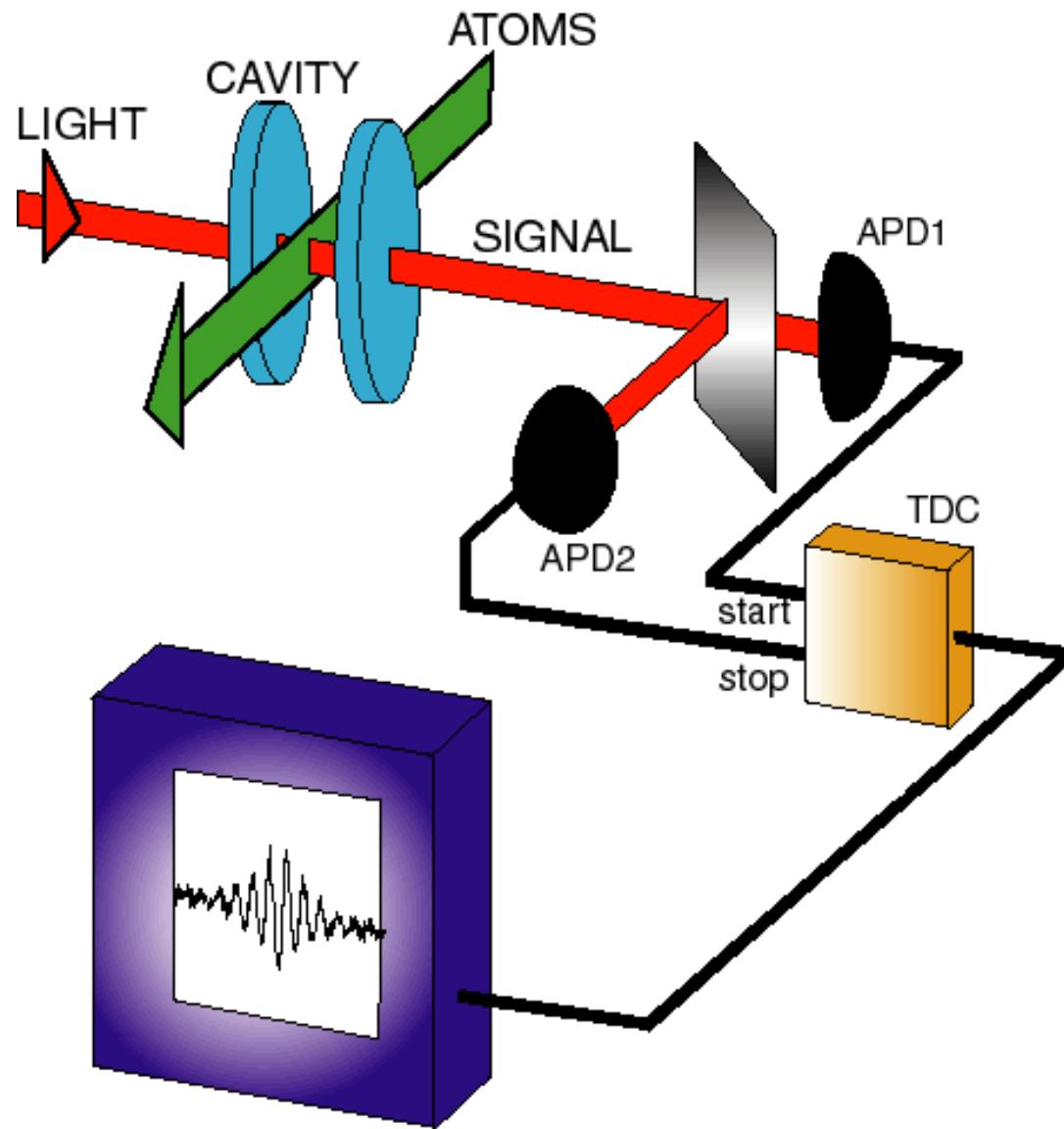
$g^{(2)}(0)<1$  Antibunched

# Oscilaciones, Modos Normales

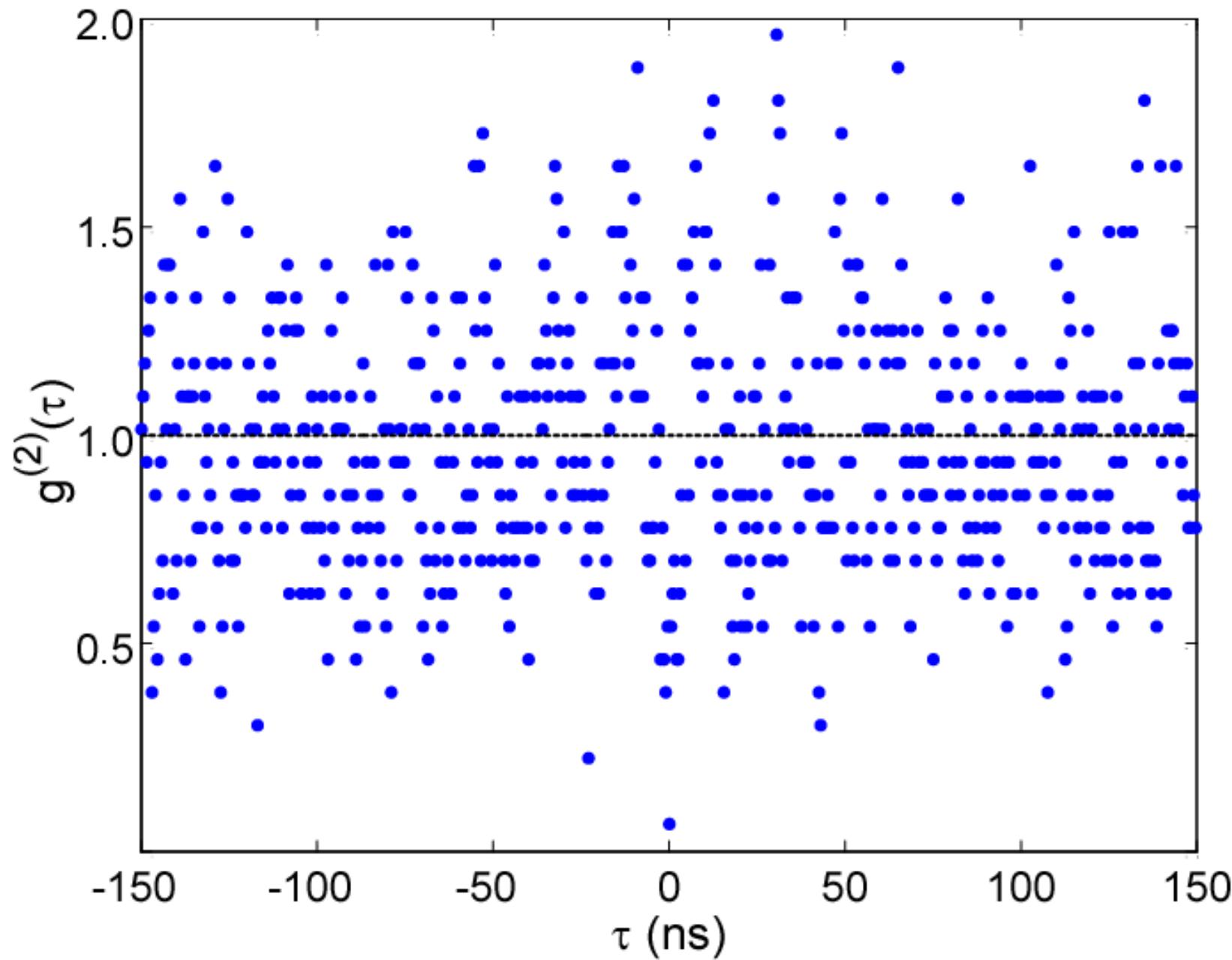
## Intercambio de excitación

Cavity Mode and Atoms

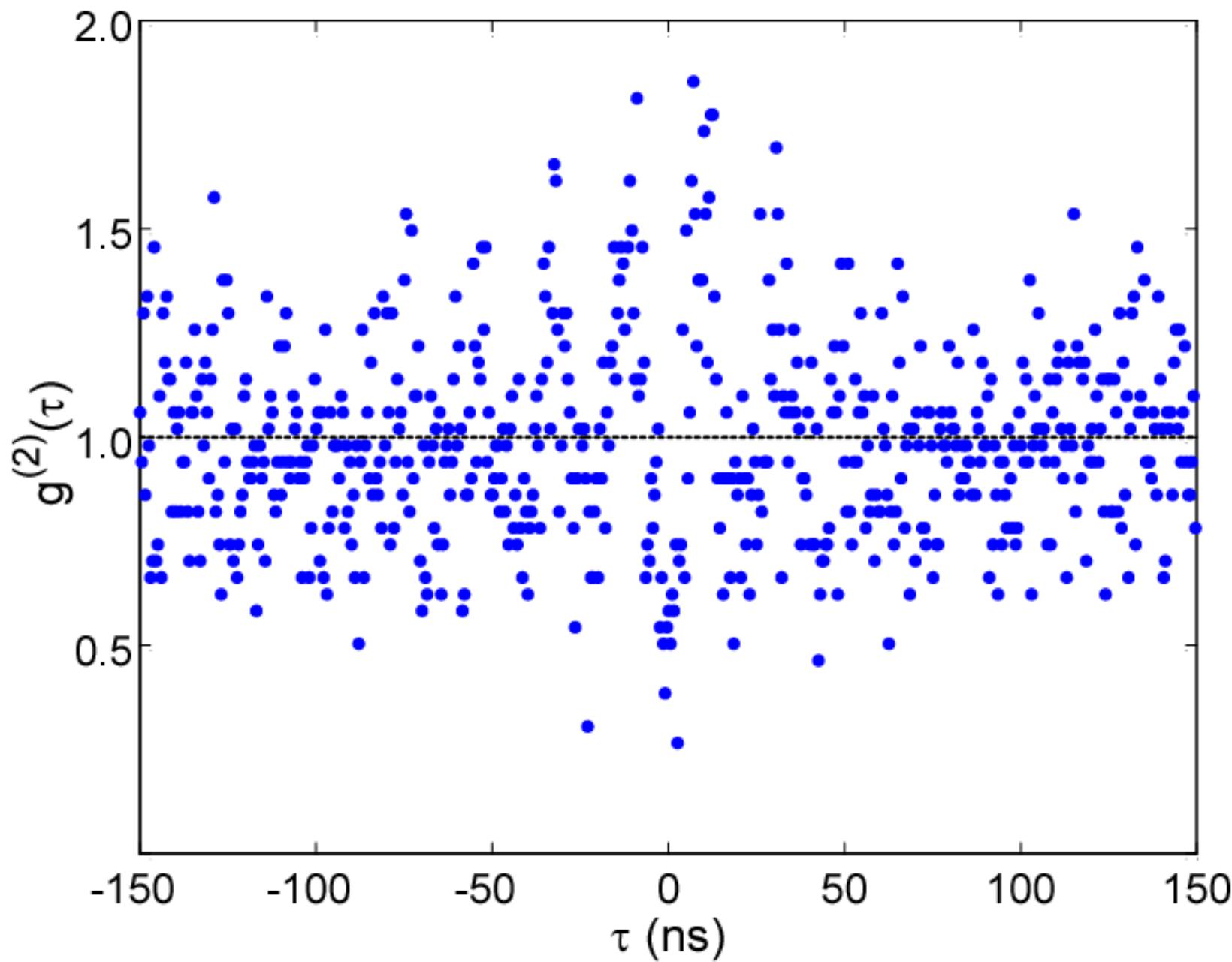




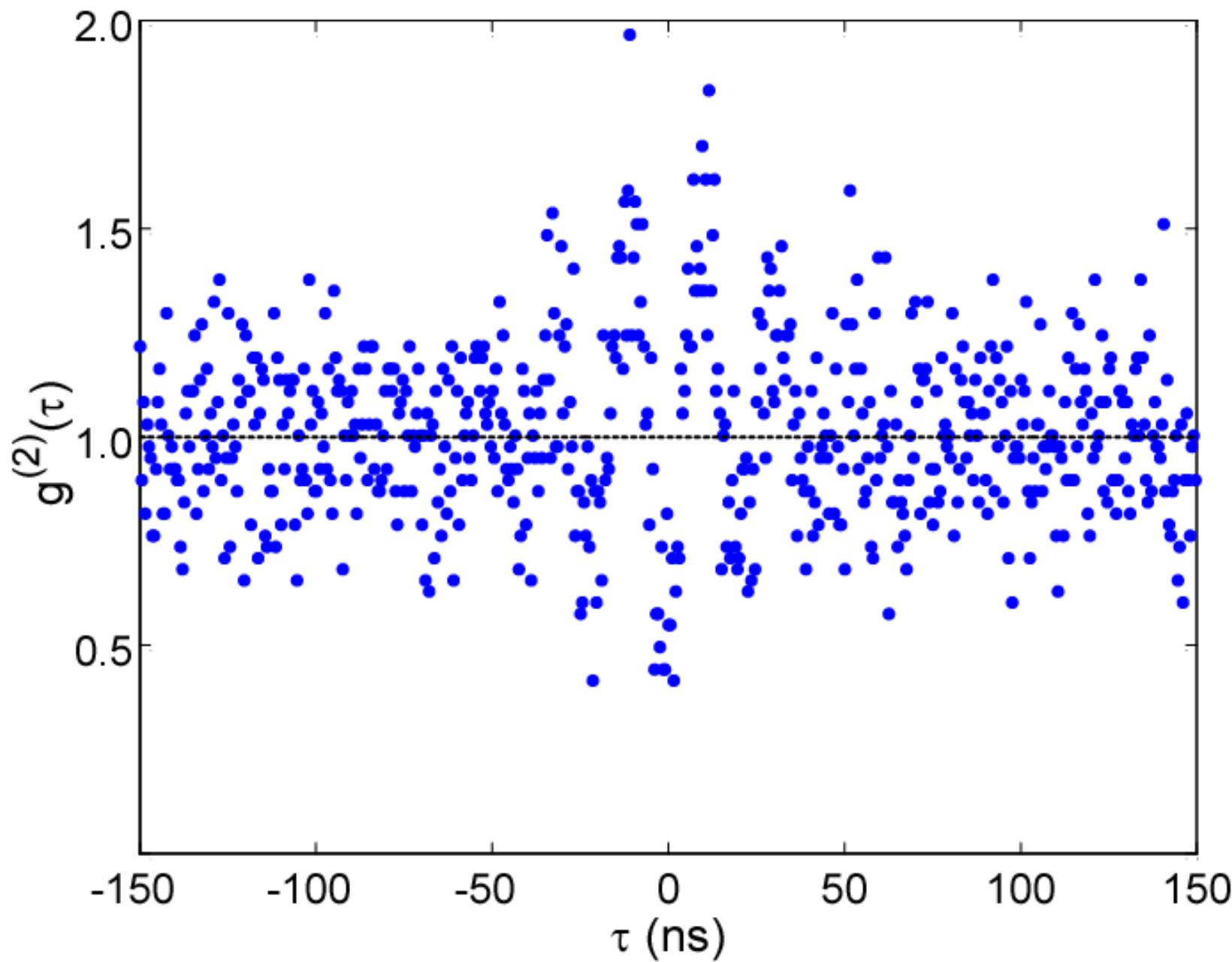
mean = 13



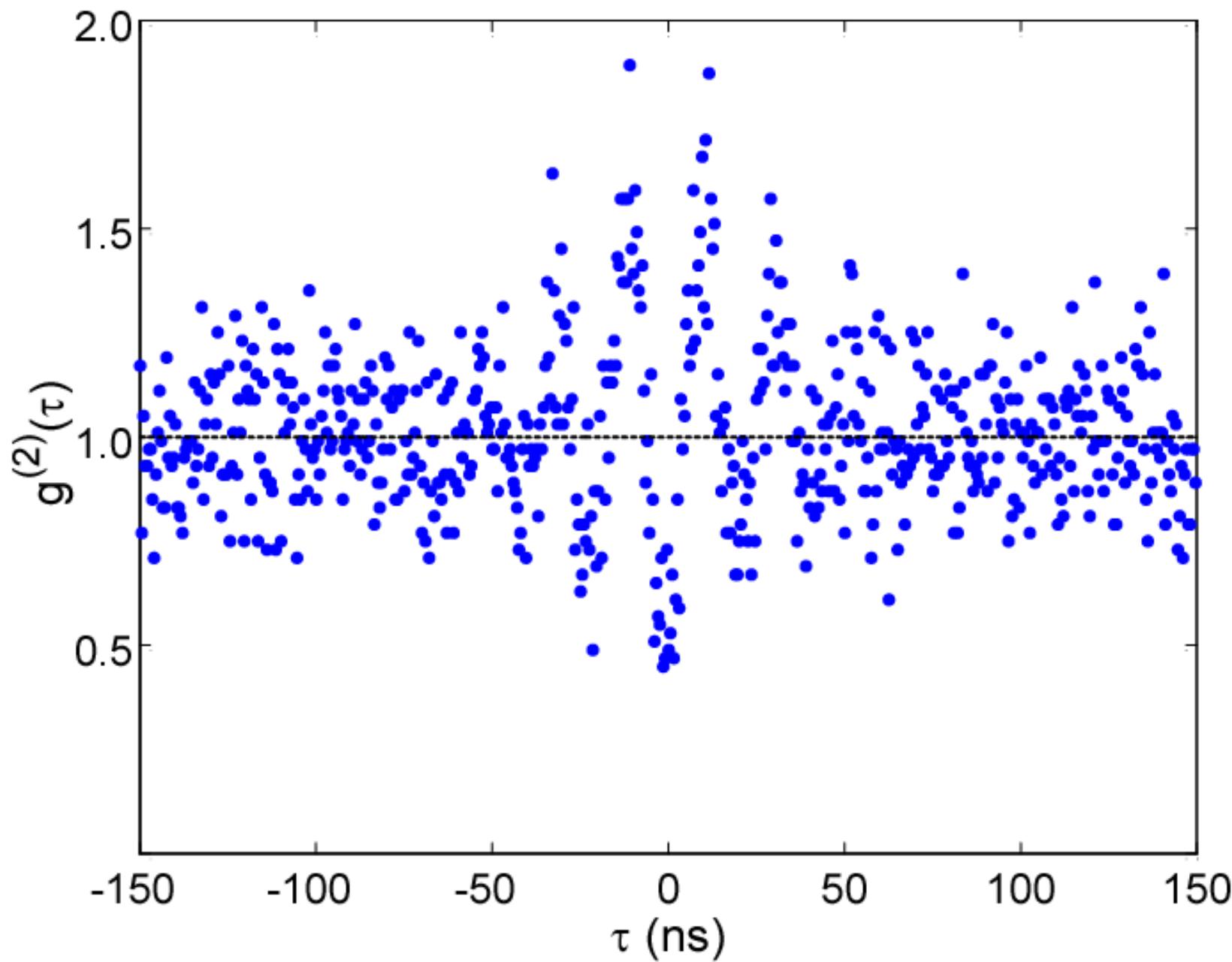
mean = 25



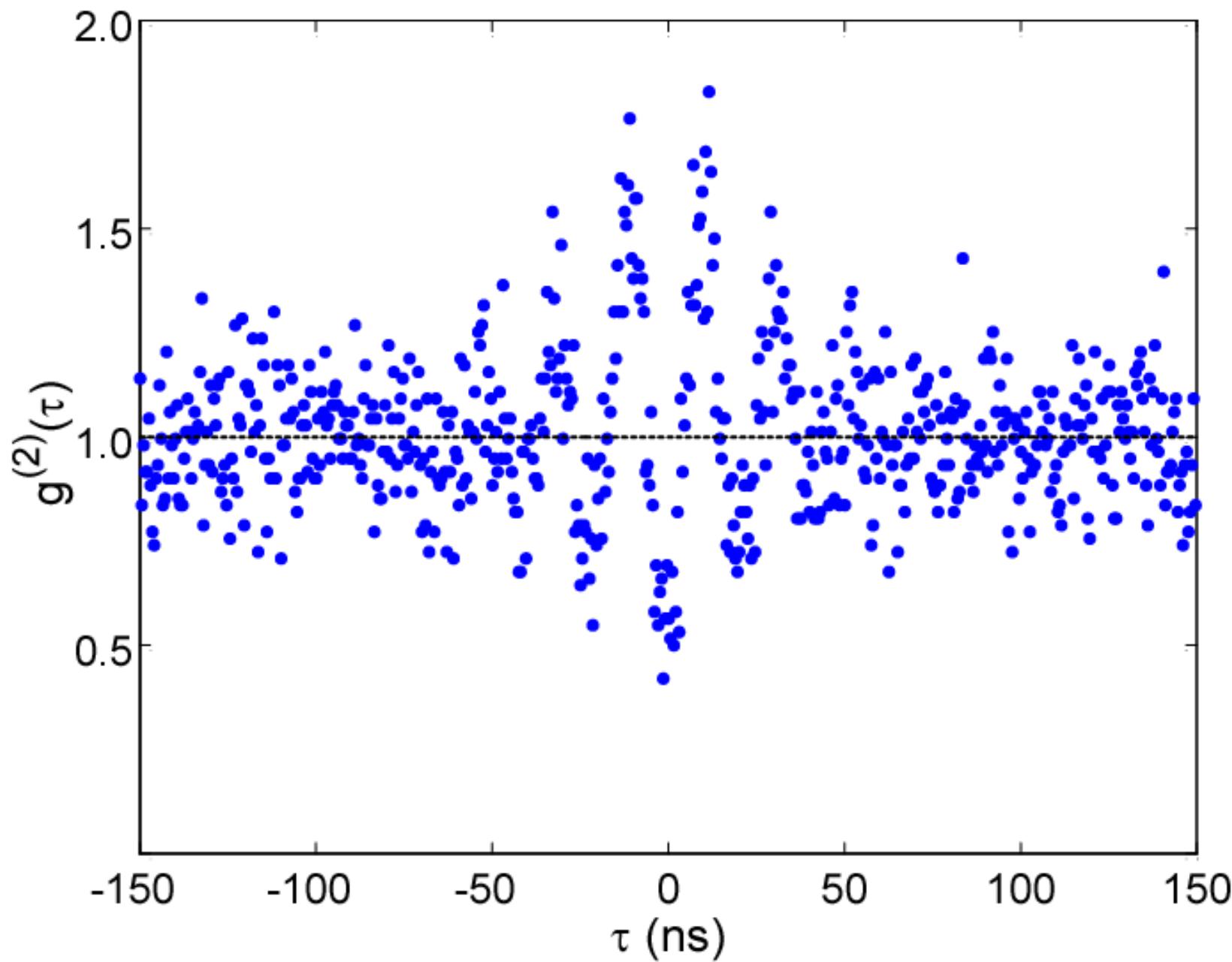
mean = 37



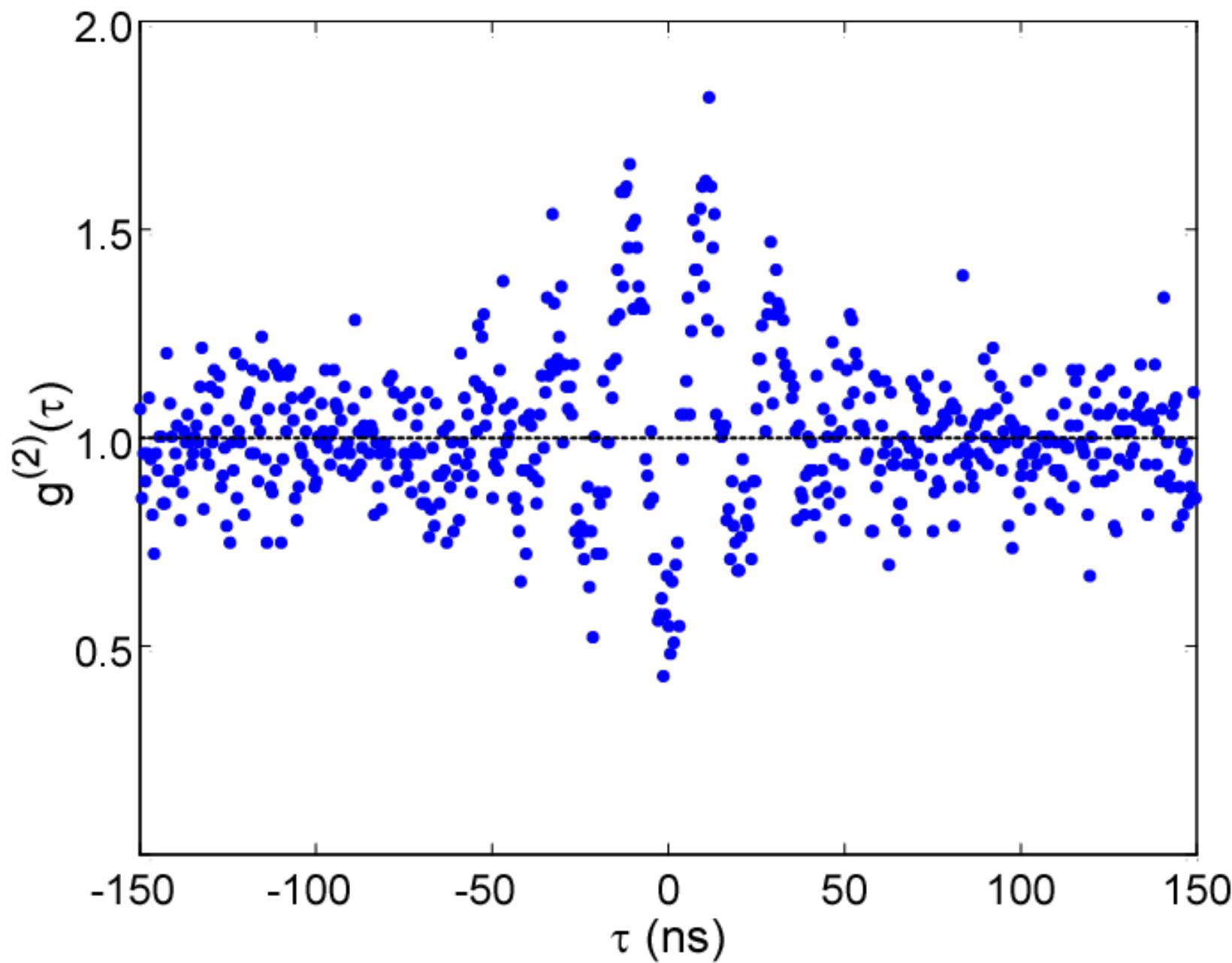
mean = 50



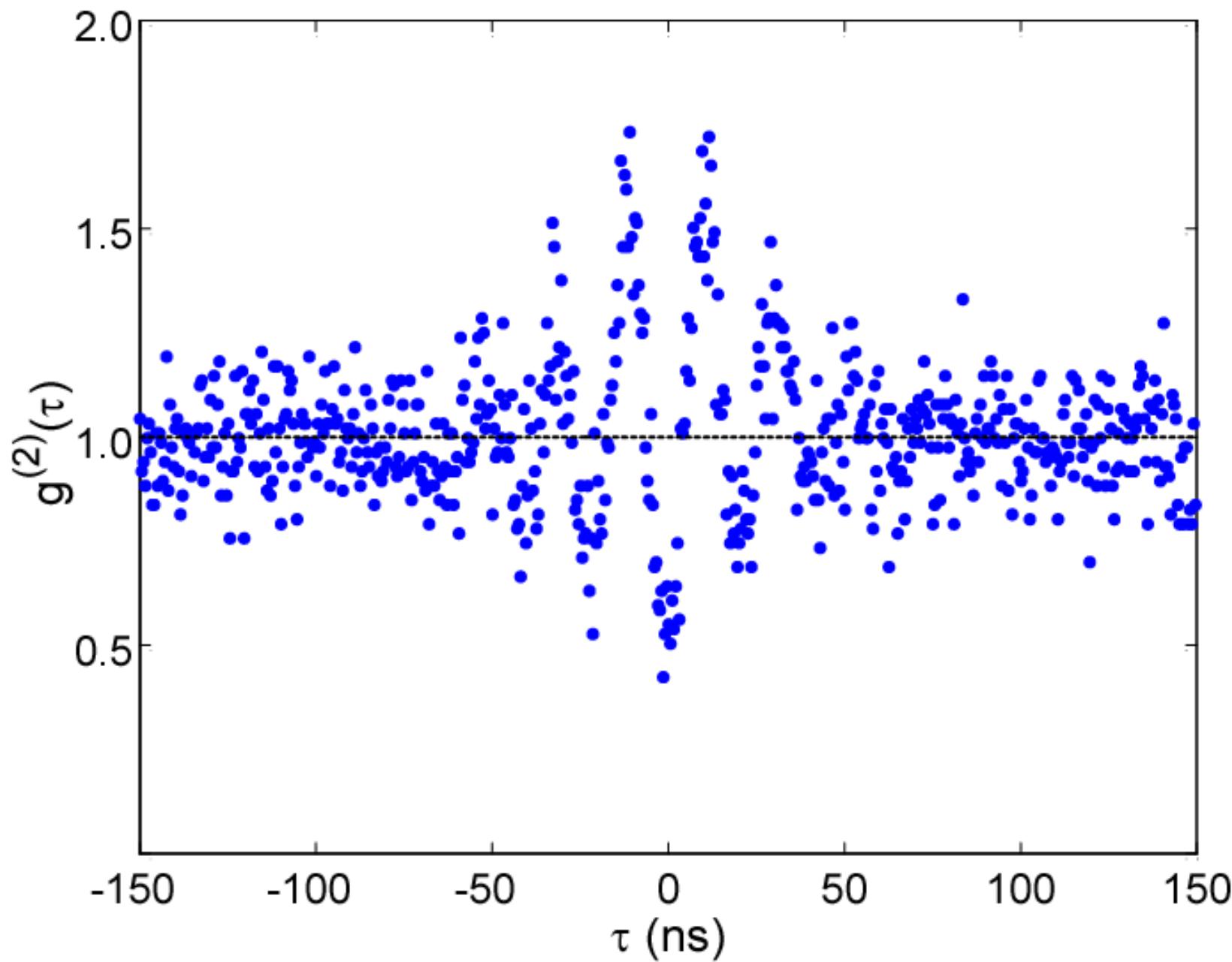
mean = 62



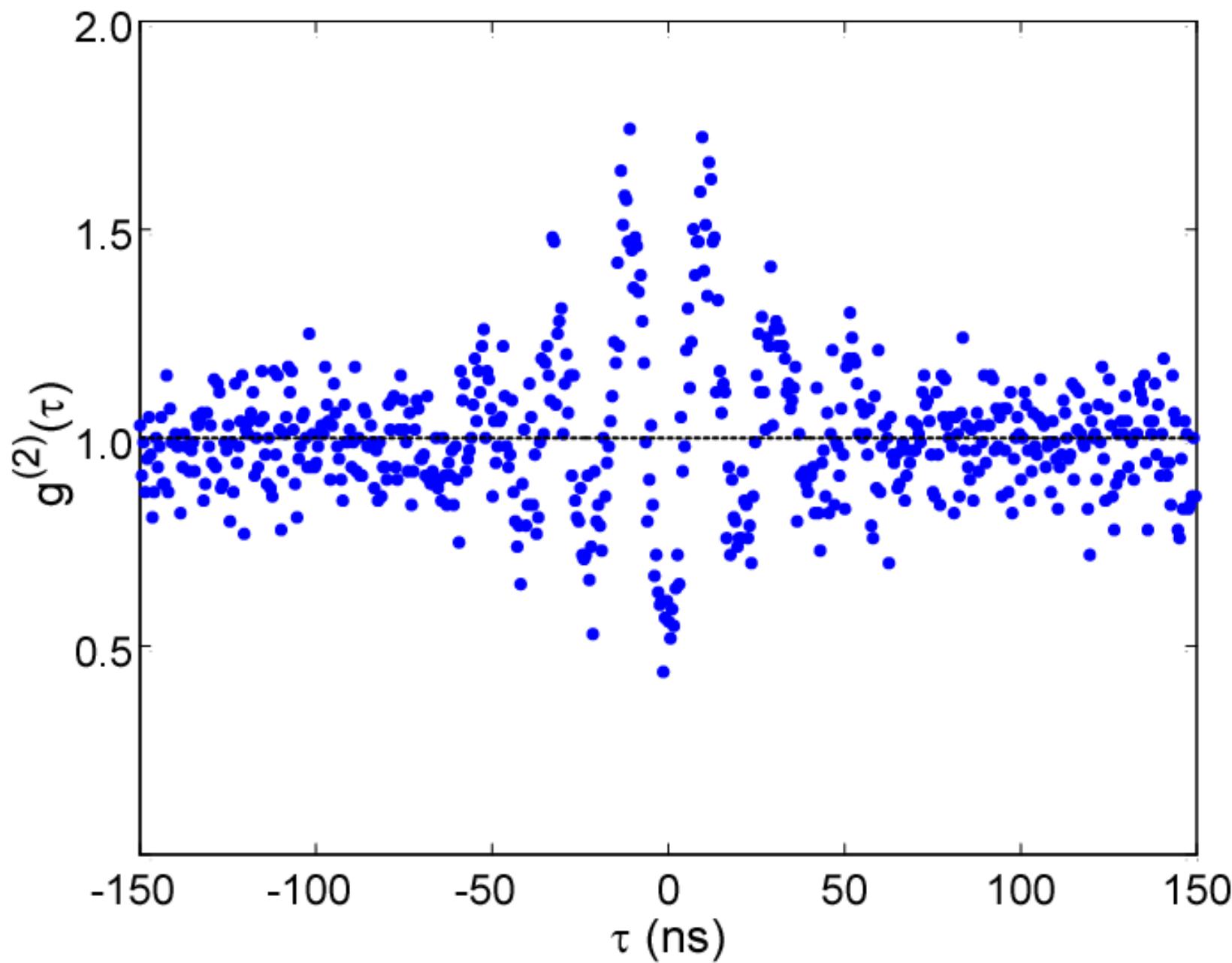
mean = 75



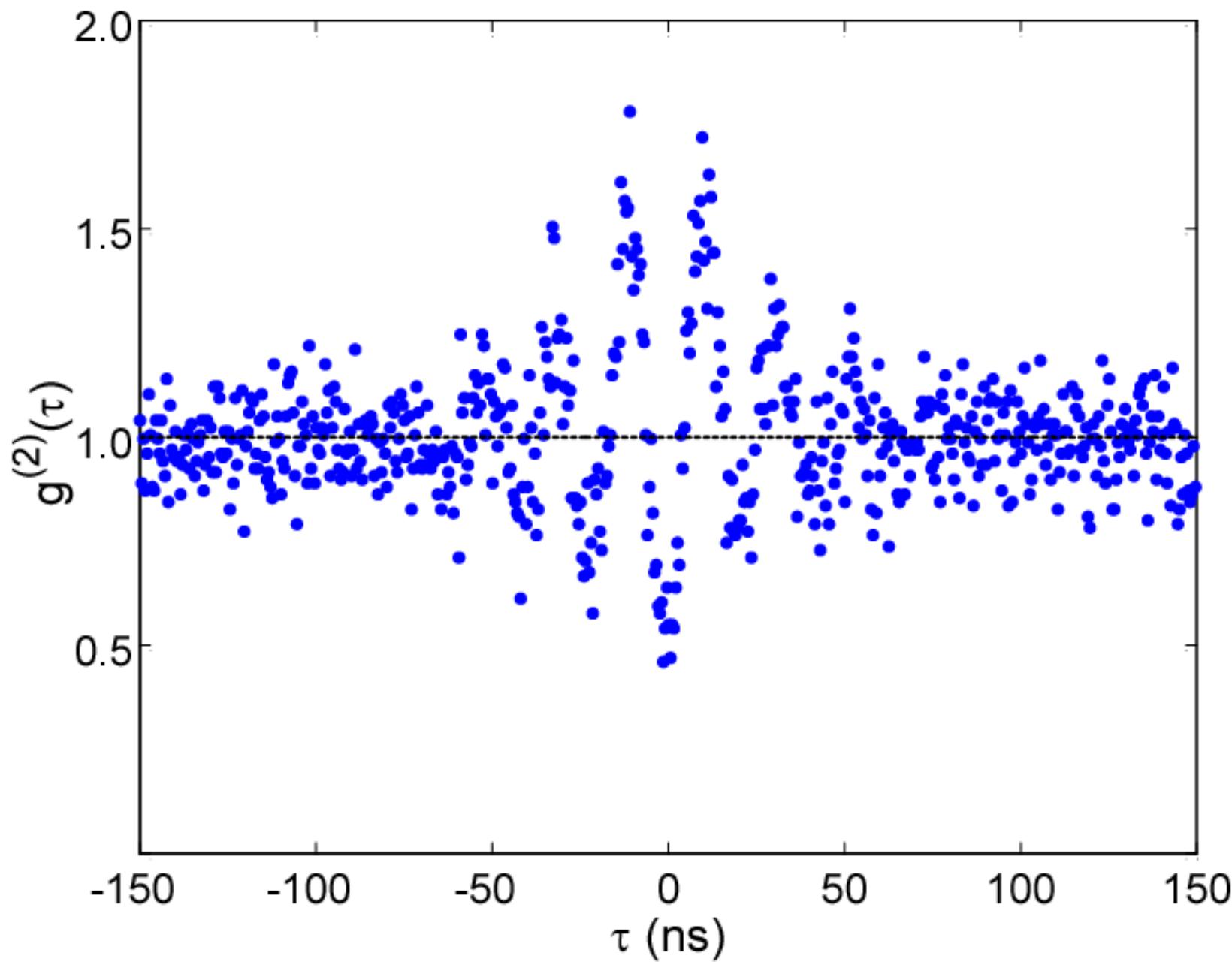
mean = 87



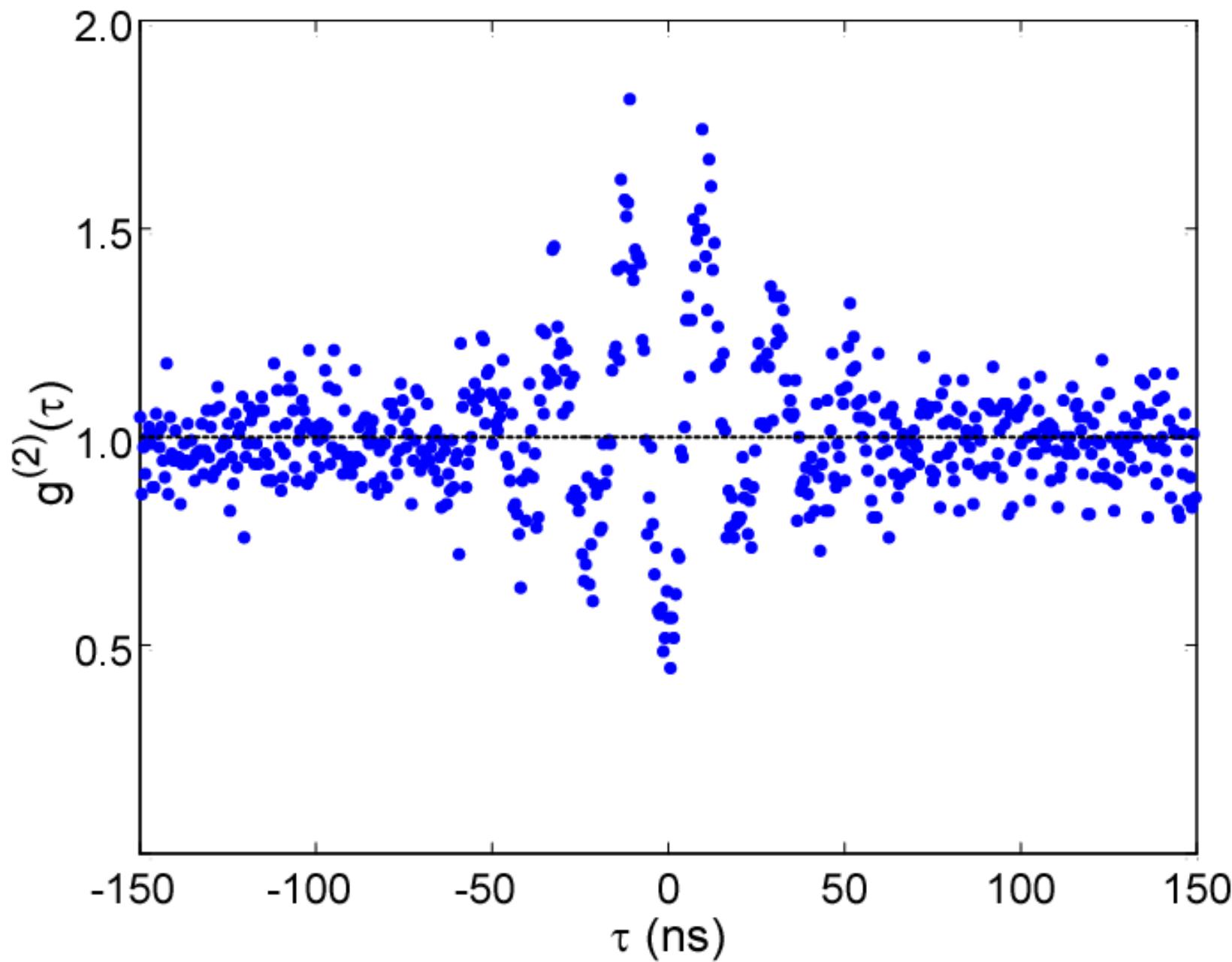
mean = 100



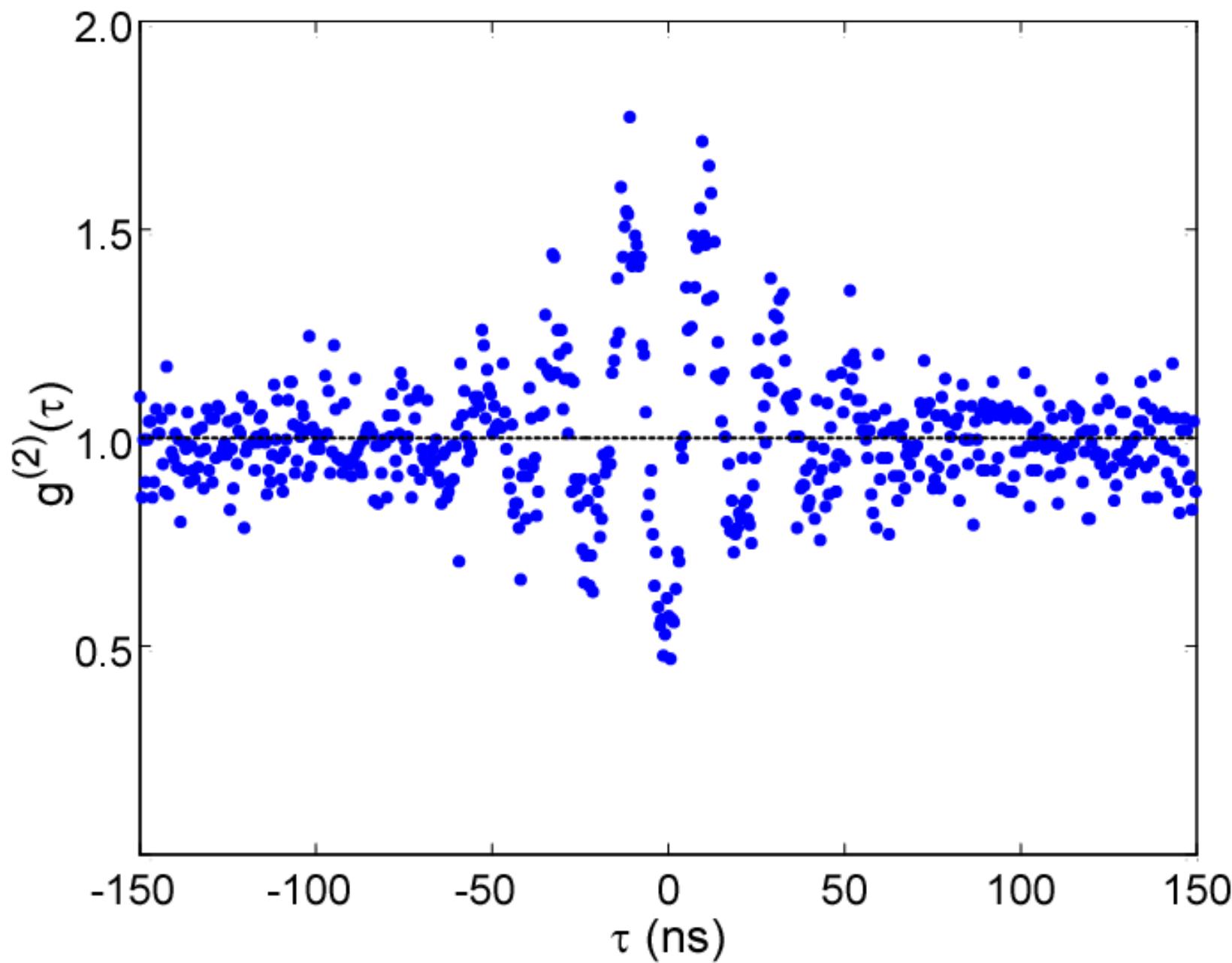
mean = 112



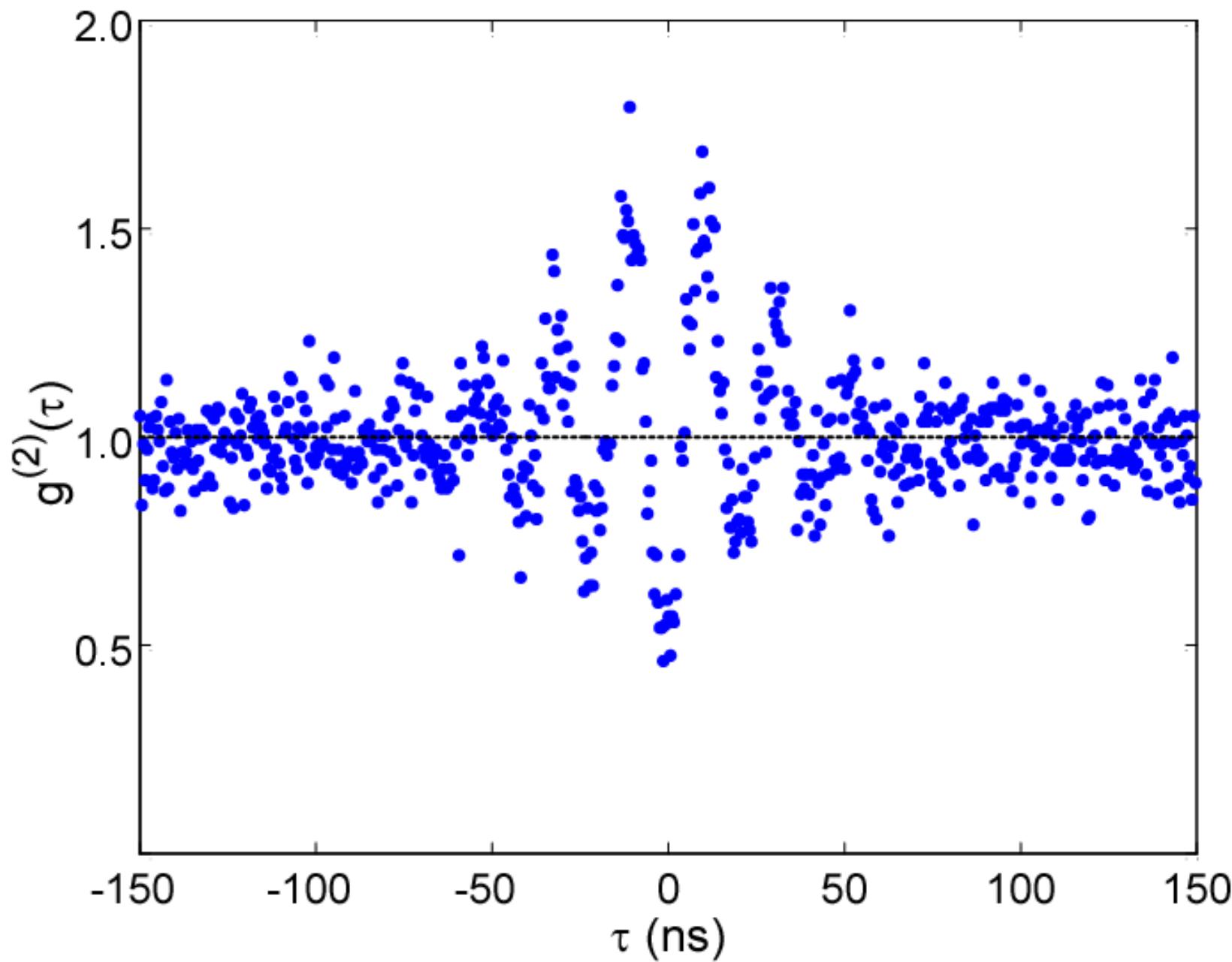
mean = 124



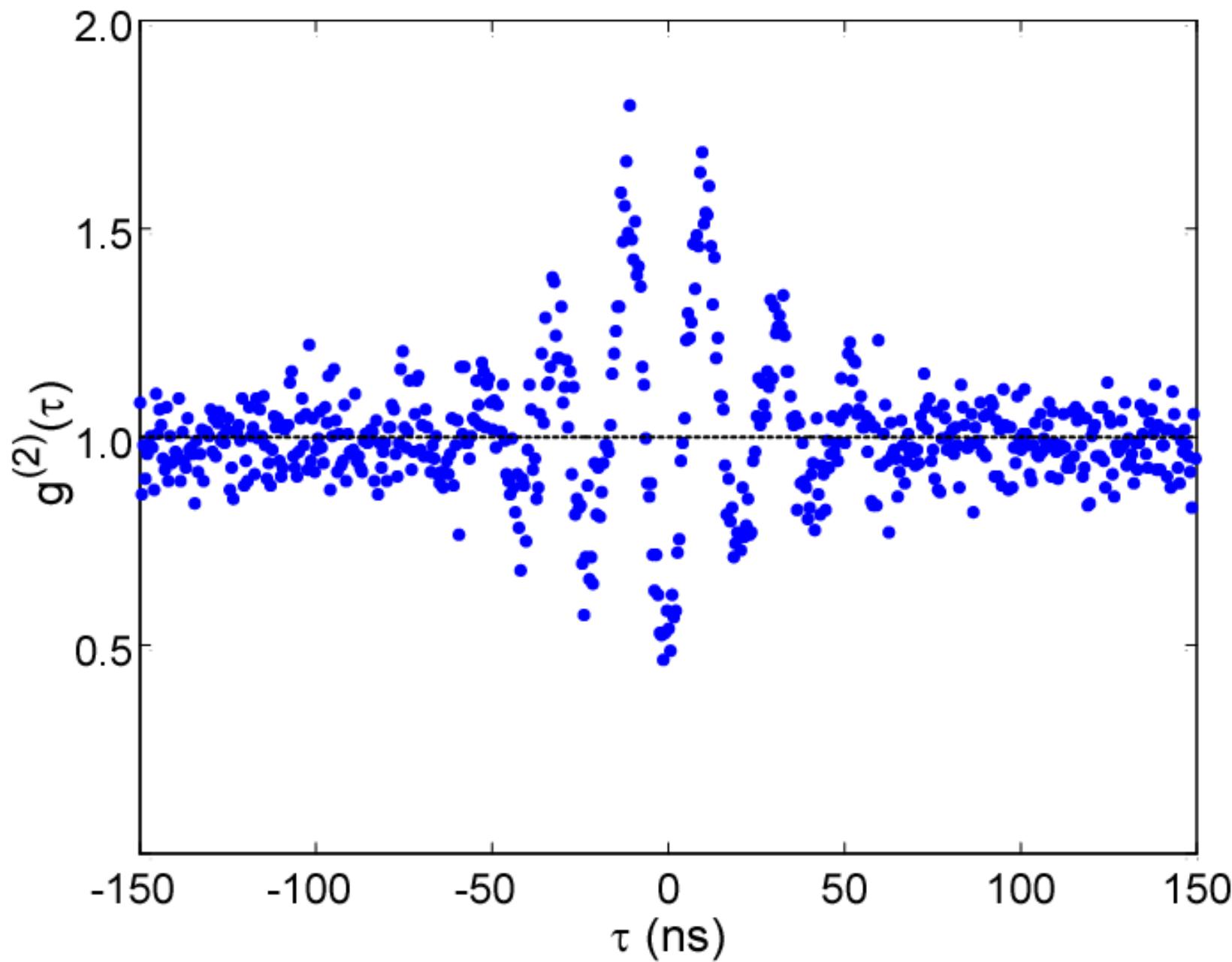
mean = 137



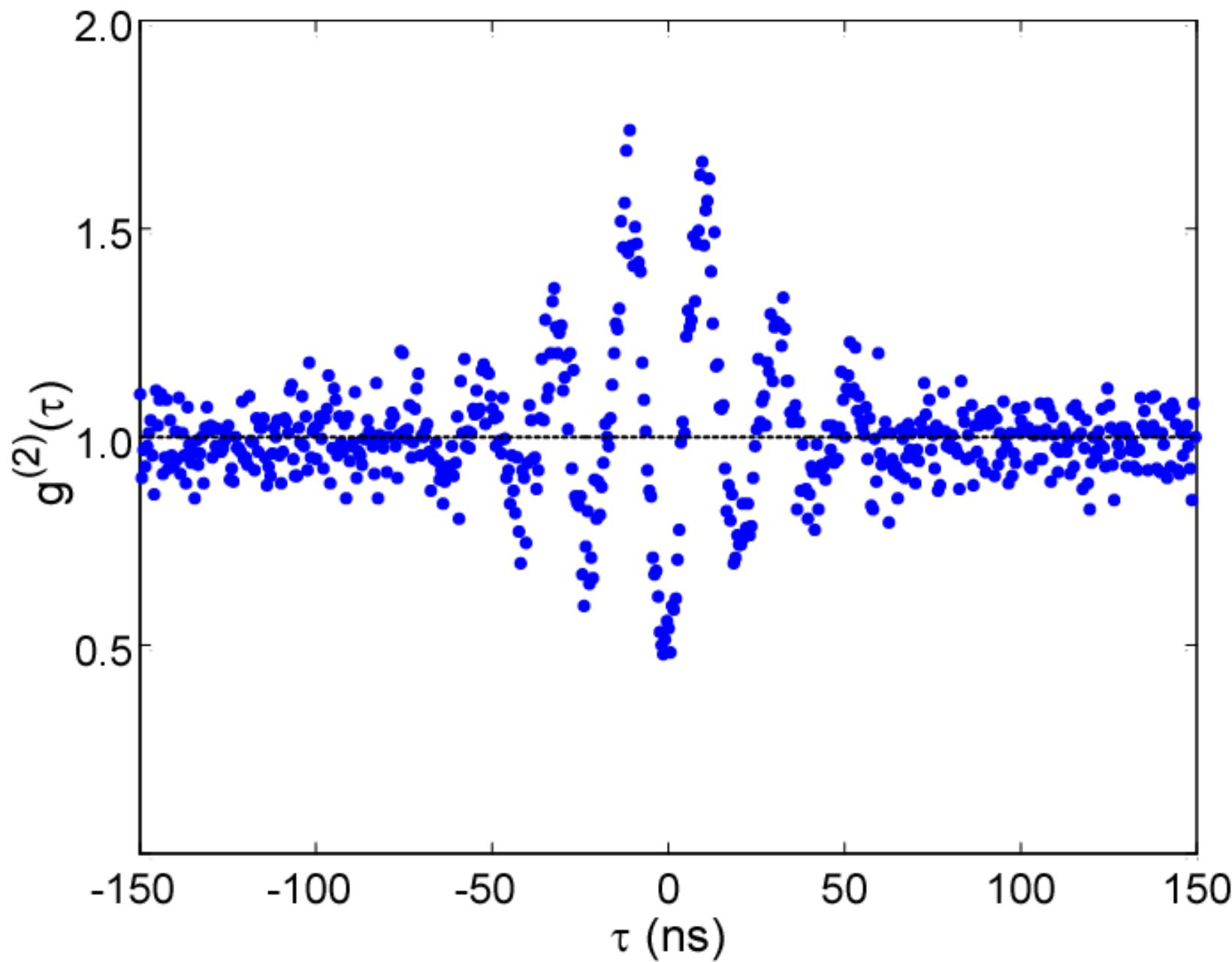
mean = 150



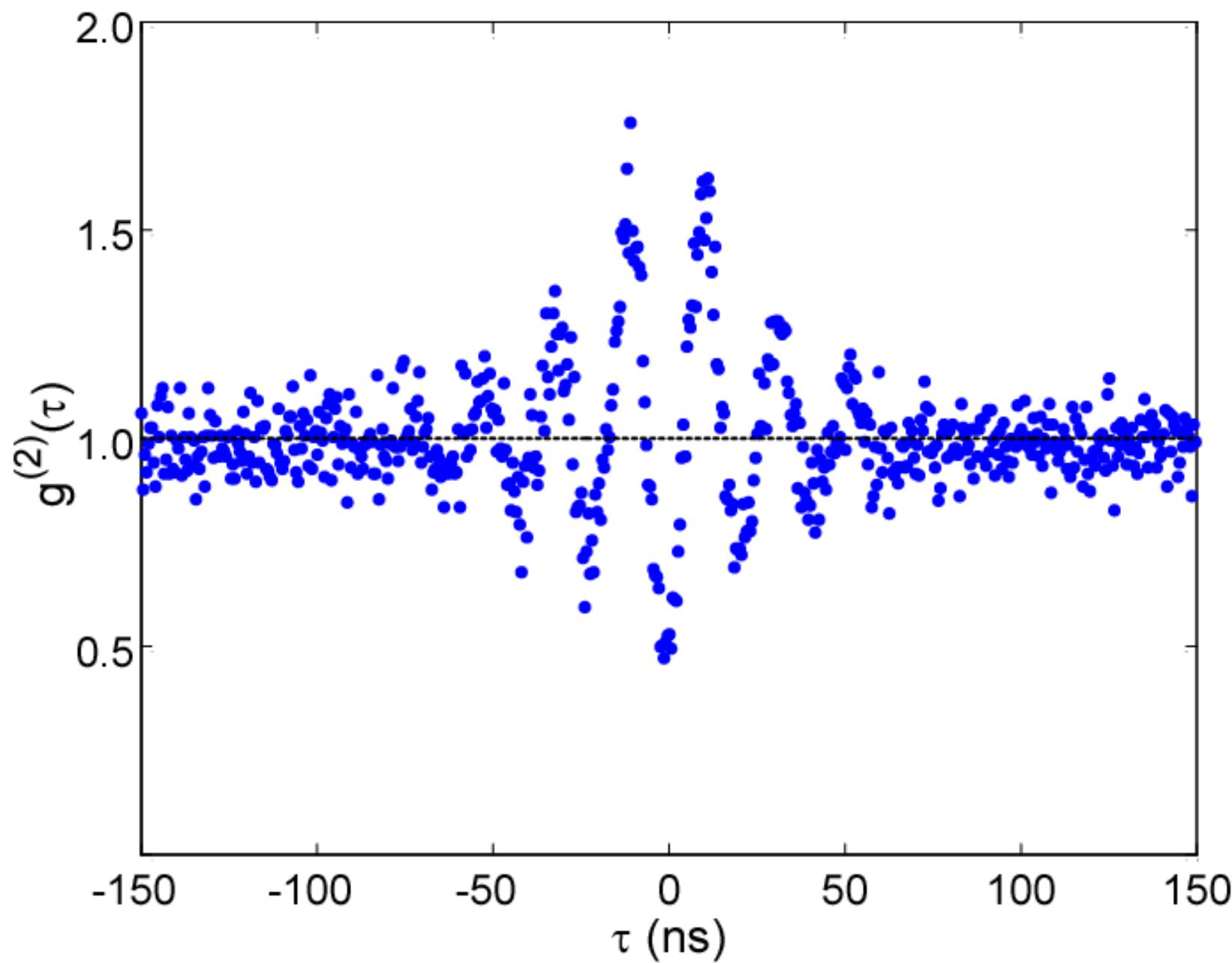
mean = 186



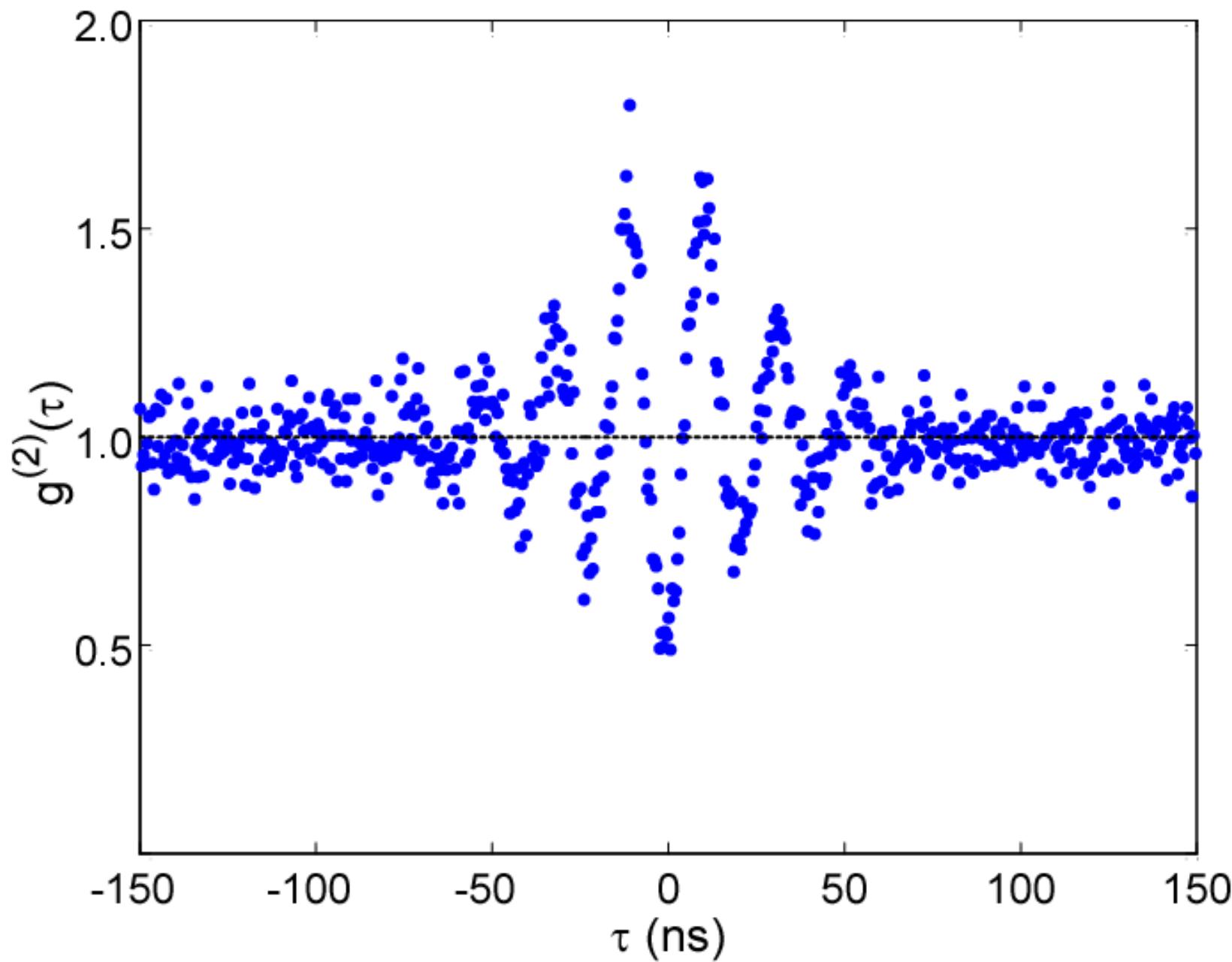
mean = 224



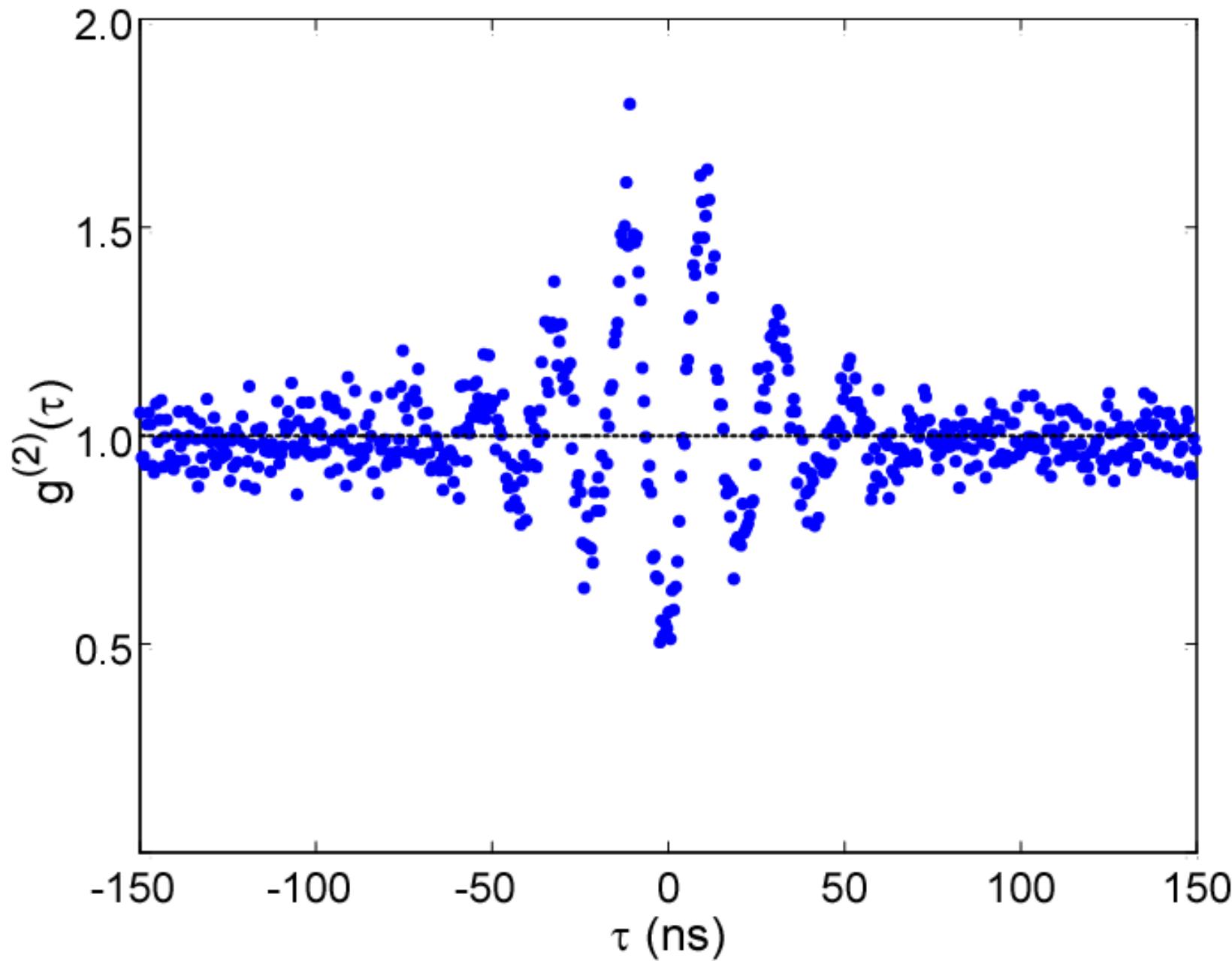
mean = 262



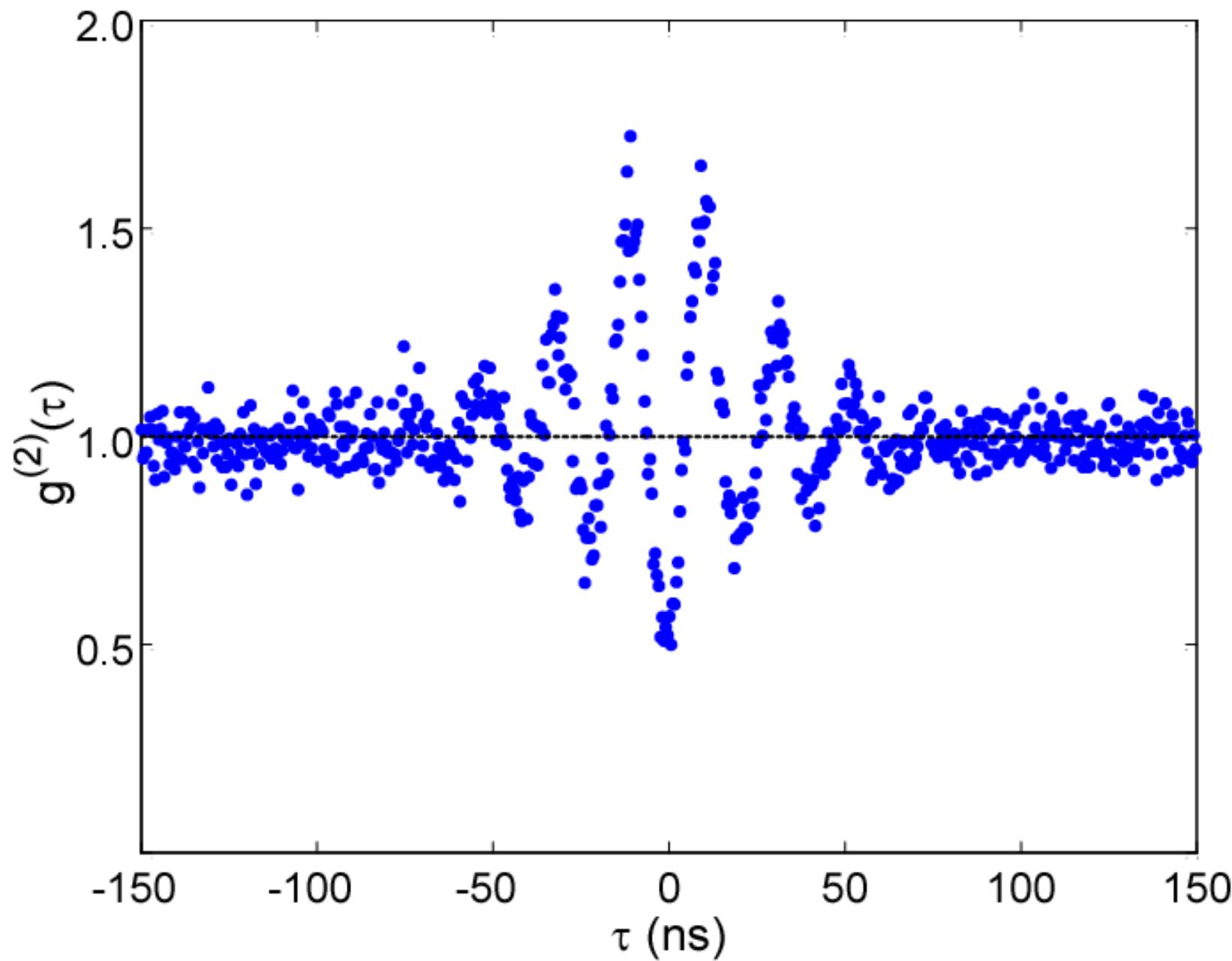
mean = 299



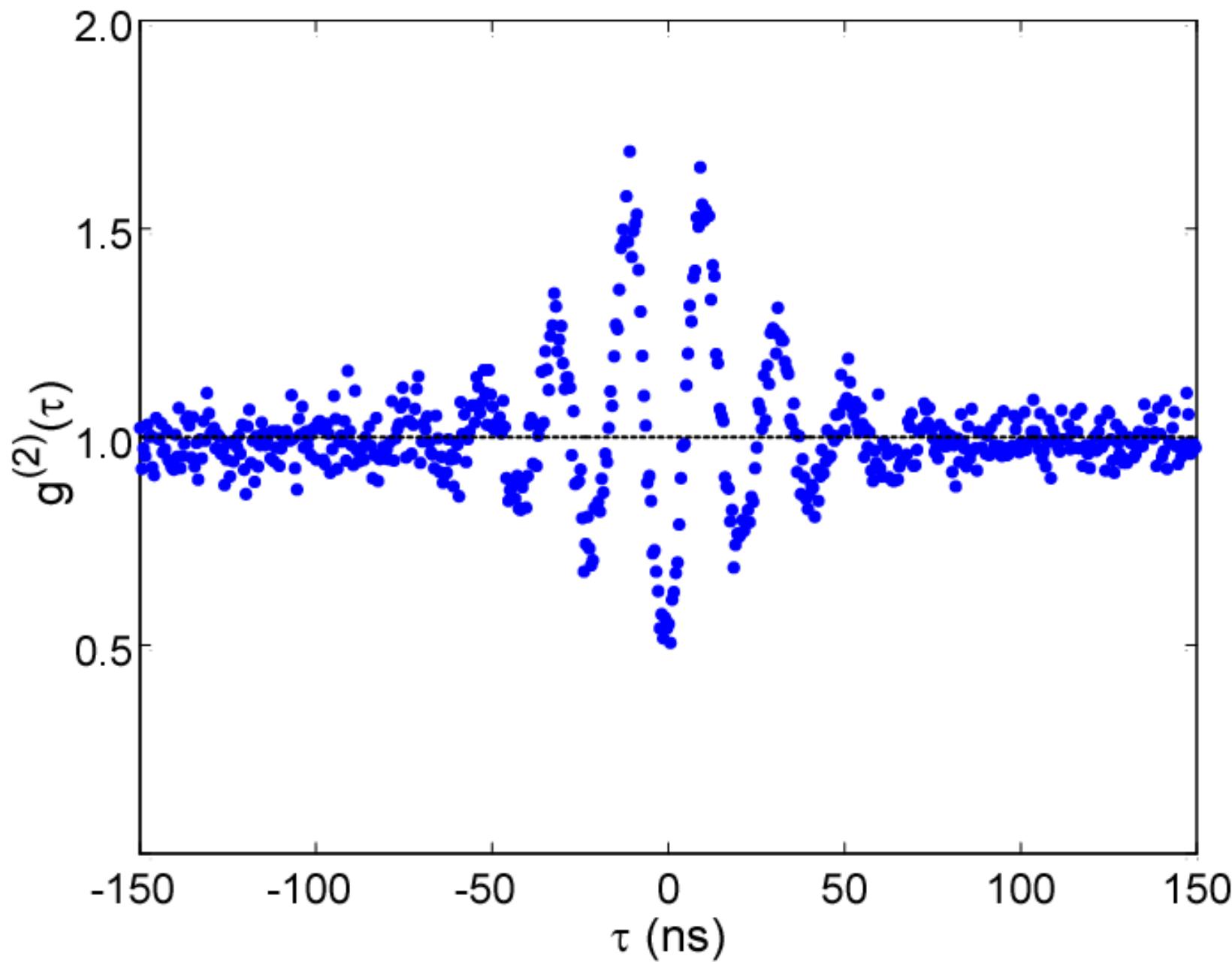
mean = 362



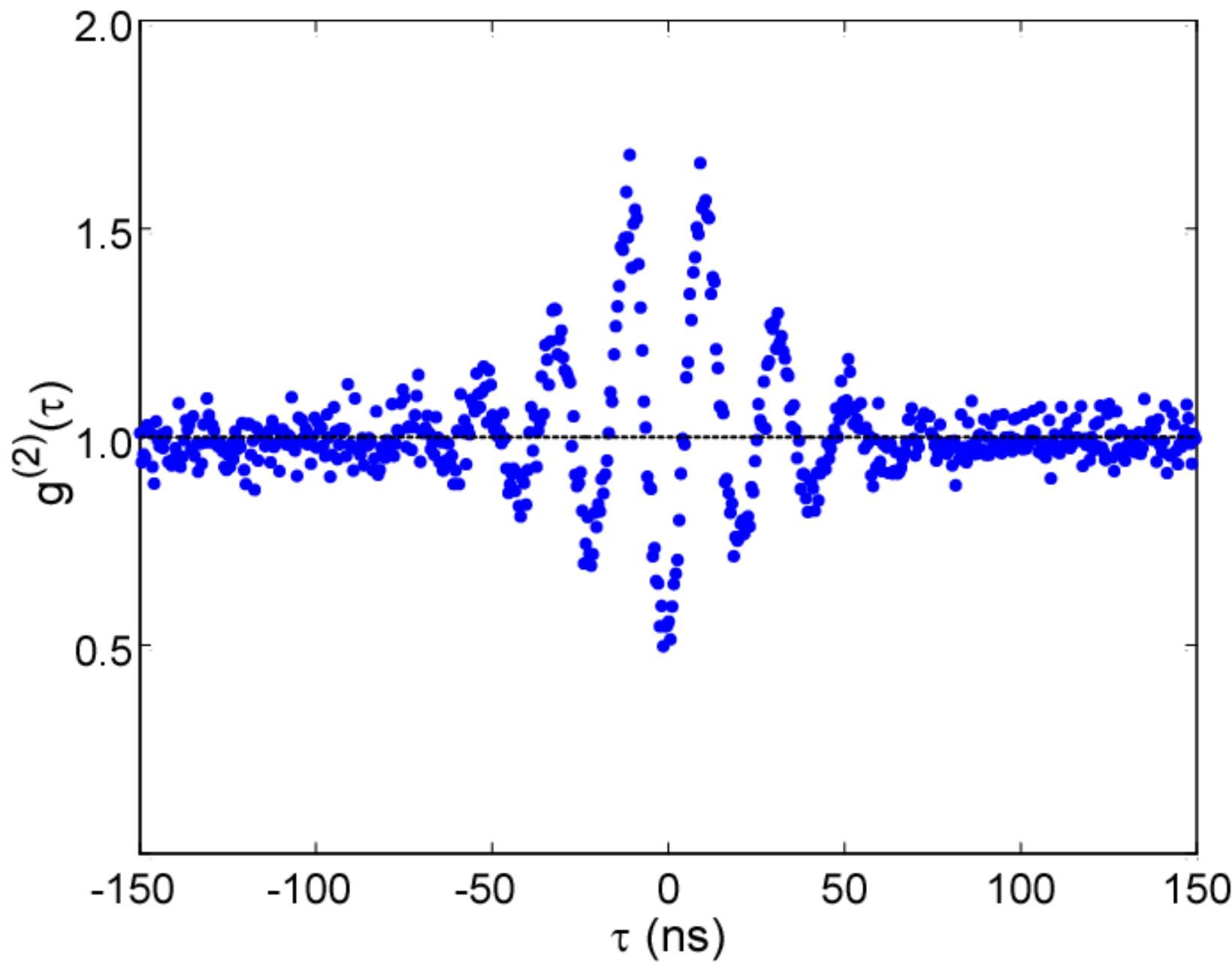
mean = 424



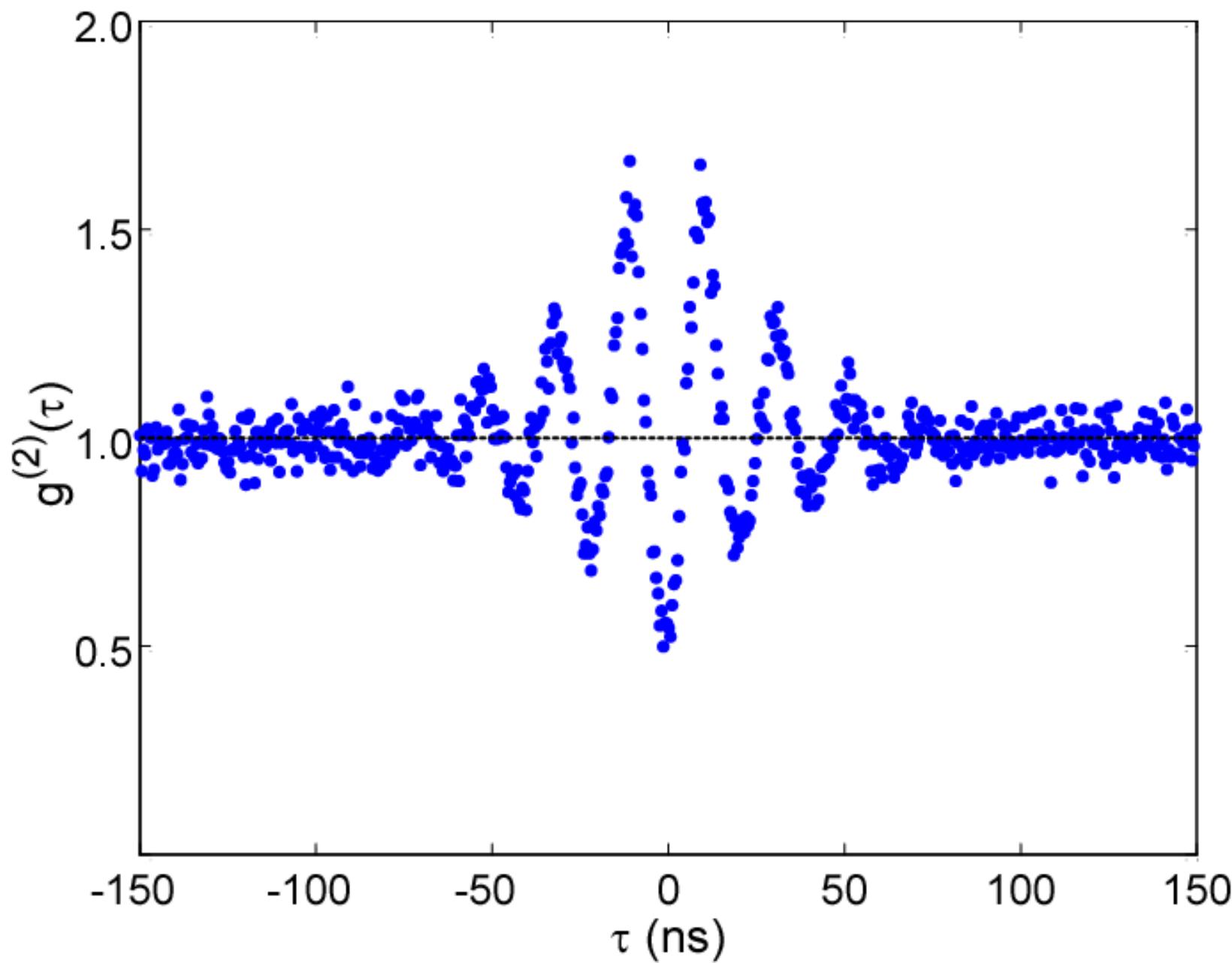
mean = 548



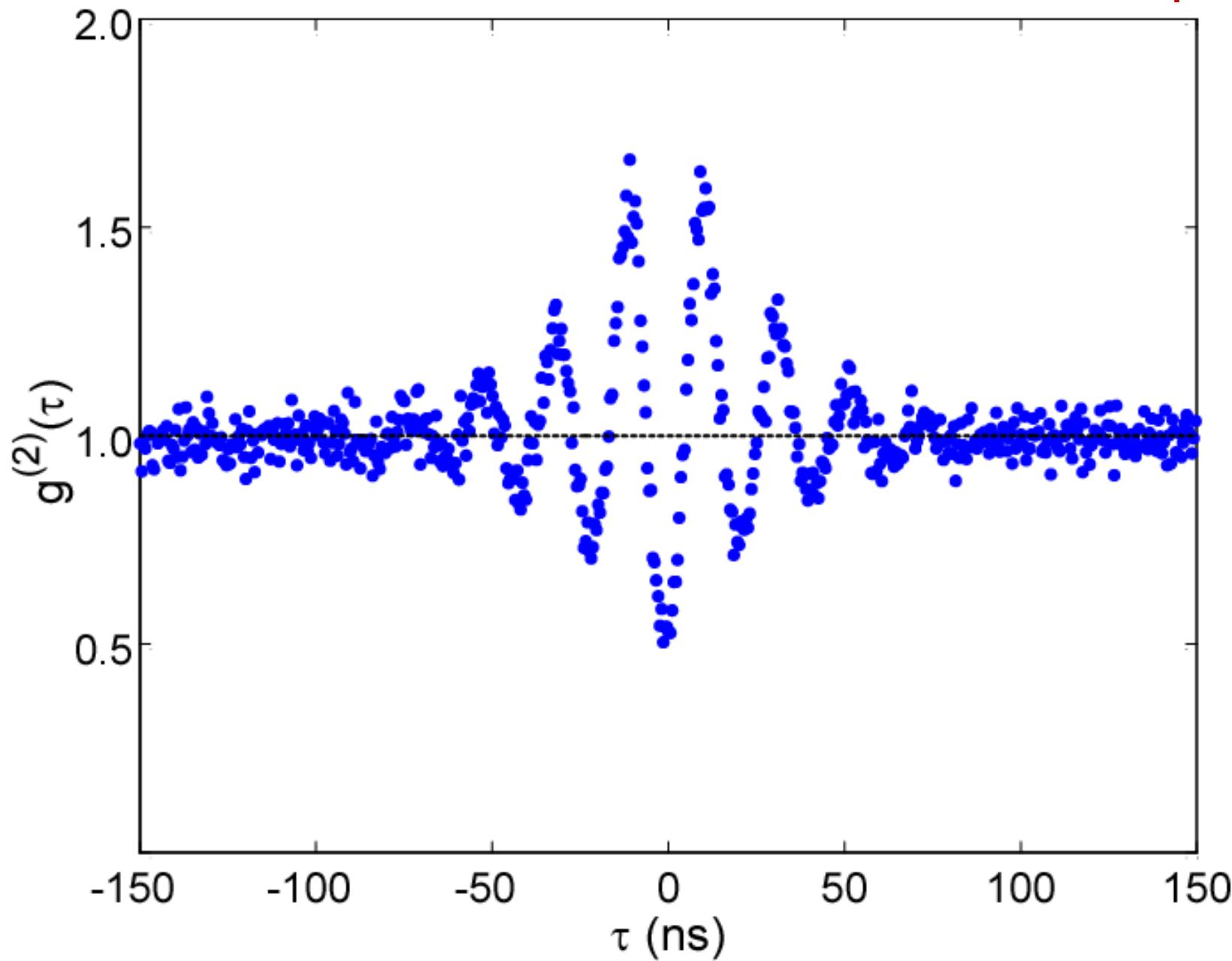
mean = 670

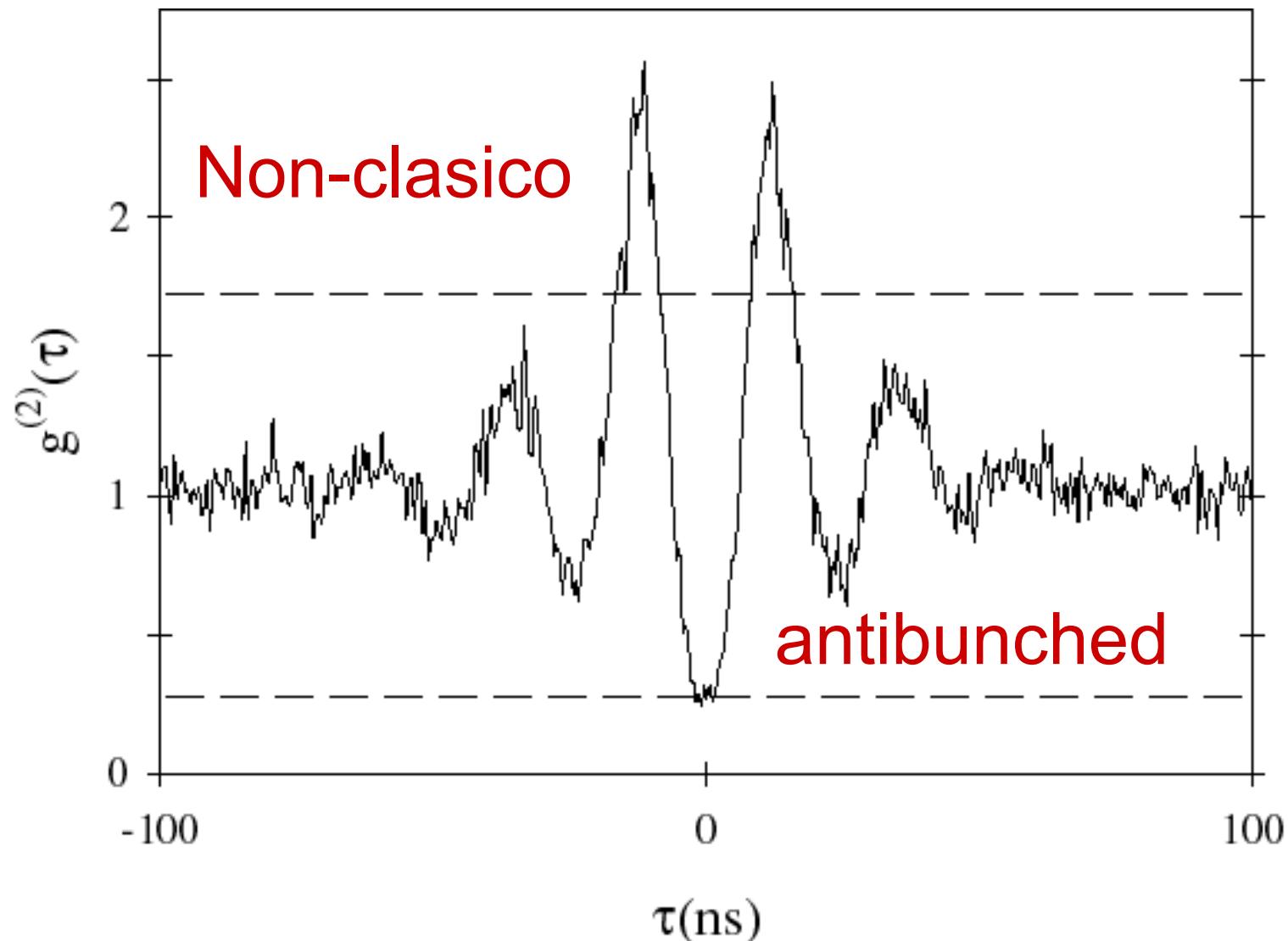


mean = 792



7 663 536 starts mean = 913 1 838 544 stops





Classicamente  $g^{(2)}(0) > g^{(2)}(\tau)$  y  
tambien  $|g^{(2)}(0) - 1| > |g^{(2)}(\tau) - 1|$

## Cálculo con Mecánica Cuántica (time and normal order):

$$g^{(2)}(\tau) = \frac{\langle \mathcal{T} : \hat{I}(t)\hat{I}(t+\tau) : \rangle}{\langle \hat{I}(t) \rangle^2}$$

$$g^{(2)}(0) = \frac{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle^2}. \quad \hat{a}^\dagger \hat{a} = \hat{a} \hat{a}^\dagger - 1 \quad \hat{n} = \hat{a}^\dagger \hat{a}$$

$$\sigma^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 \quad g^{(2)}(0) = 1 + \frac{\sigma^2 - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2}.$$

La varianza  $\sigma^2$  del número de fotones es menor a la varianza de una distribución poissoniana.

- ¿Podemos decir algo sobre el máximo valor de la función de correlación  $g^{(2)}$ ?.
- No para la intensidad.
- Pero en ocasiones hay algo cuántico si es muy grande (OPO).
- Veremos otra función de correlación que si tiene una cota máxima, la correlación campo intensidad.

¿Qué es una imagen fantasmal (ghost imaging)?

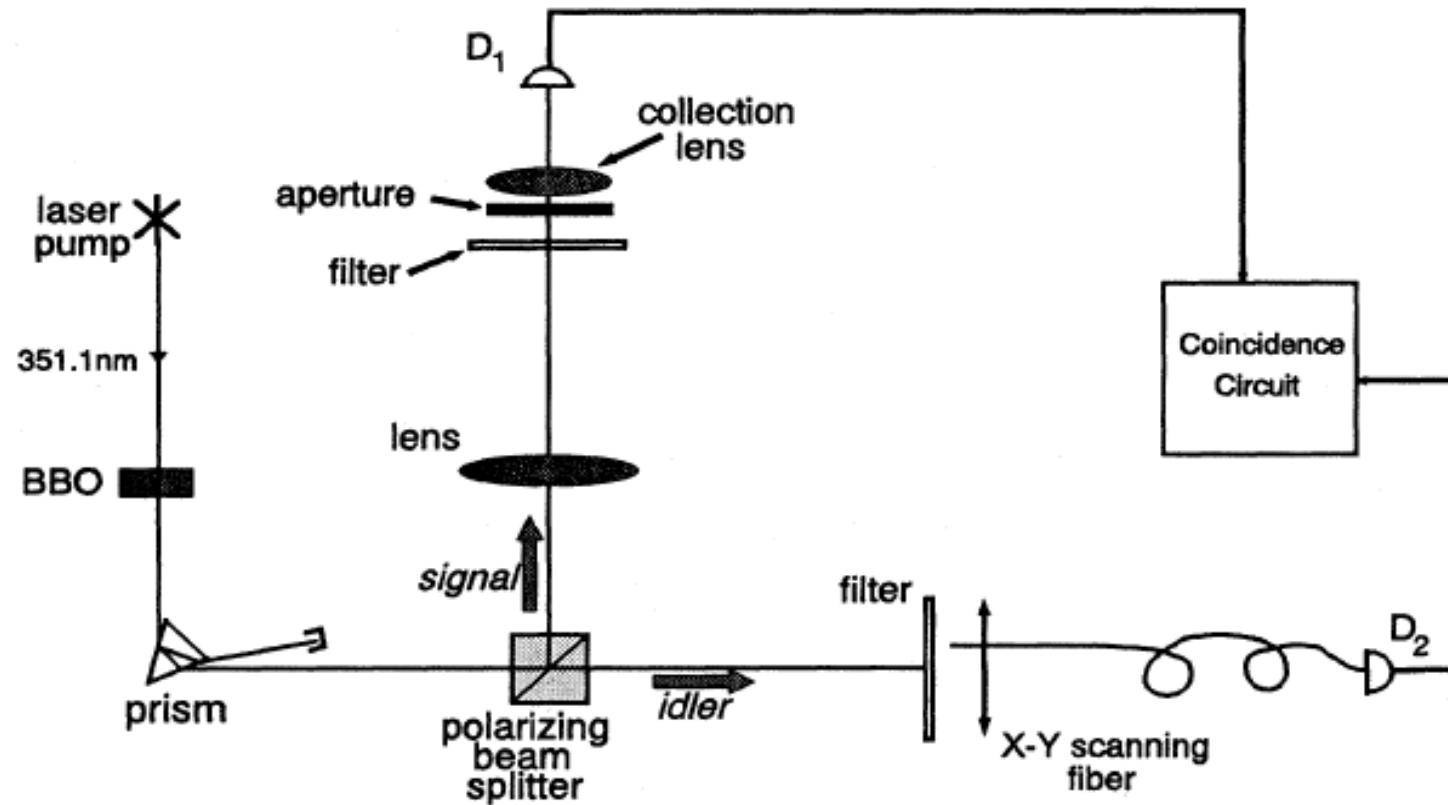
## Algunos artículos de revisión:

Baris I. Erkmen and Jeffrey H. Shapiro, “Ghost imaging: from quantum to classical to computational,” *Advances in Optics and Photonics* **2**, 405 (2010). doi:10.1364/AOP.2.000405.

Miles Padgett, Reuben Aspden, Graham Gibson, Matthew Edgar and Gabe Spalding, “Ghost Imaging,” *Optics and Photonics News*, p. 40, October 2016.

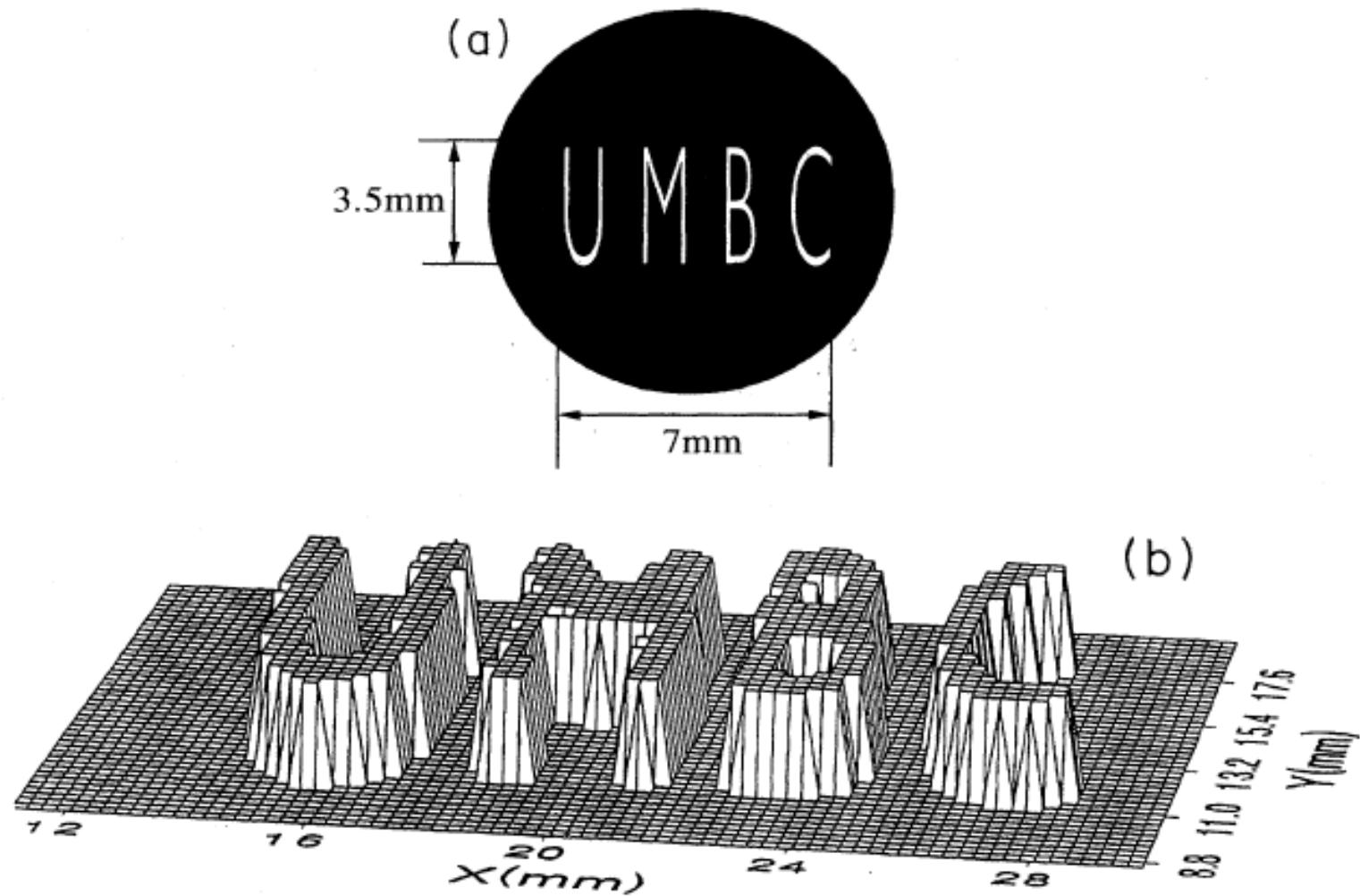
Ghost imaging is also known as Single Pixel Imaging: (desarrollada en Rice University en los 1990's.) Ver por ejemplo: Marco F. Duarte, Mark A. Davenport, Dharmpal Takhar, Jason N. Laska, Ting Sun, Kevin F. Kelly, and Richard G. Baraniuk “Single-Pixel Imaging via Compressive Sampling” *IEEE Signal Processing Magazine* **25**, 85 (2008) DOI:10.1109/MSP.2007.914730

# La historia de un experimento:



1. T. B. Pittman, Y. H. Shih, D. V. Strekalov, and A. V. Sergienko, "Optical imaging by means of two-photon quantum entanglement," Phys. Rev. A **52**, R3429–R3432 (1995).

# Imagen medida



## Fotones correlacionados

- Fuente: conversión hacia abajo paramétrica esponánea (Spontaneous Parametric Down Conversion [SPDC]): proceso no lineal donde luz (bomba) a  $2\omega$  se convierte en dos haces a  $\omega$  (s y i) por sus siglas en inglés (signal and idler).
- La conservación de energía requiere  $2\omega = \omega_s + \omega_i$
- La conservación de momentum requiere:  $k_{2\omega} = k_s + k_i$
- La conservación del momento angular requiere: La polarización de la s y de la i tienen que sumar a la de la bomba.
- Los fotones pueden estar entrelazados

## ¿Qué es el entrelazamiento?

Propiedad donde dos ( o más) partículas están altamente correlacionados, tal correlación es independiente de la base donde se mide.

El estado (matriz de densidad) no puede ser escrito como un producto externo entre los dos estados originales.

Con dos polarizaciones: $|H\rangle_1, |V\rangle_1, |H\rangle_2, |V\rangle_2$

$$\Psi = \alpha |H\rangle_1 |H\rangle_2 + \beta |V\rangle_1 |V\rangle_2 + \gamma |H\rangle_1 |V\rangle_2 + \delta |V\rangle_1 |H\rangle_2$$

## Estados de Bell:

$$\Phi^\pm = (1/2)^{1/2}(|H\rangle_1|H\rangle_2 \pm |V\rangle_1|V\rangle_2)$$

$$\Psi^\pm = (1/2)^{1/2}(|H\rangle_1|V\rangle_2 \pm |V\rangle_1|H\rangle_2)$$

Si yo conozco la polarización de uno conozco perfectamente la polarización del otro

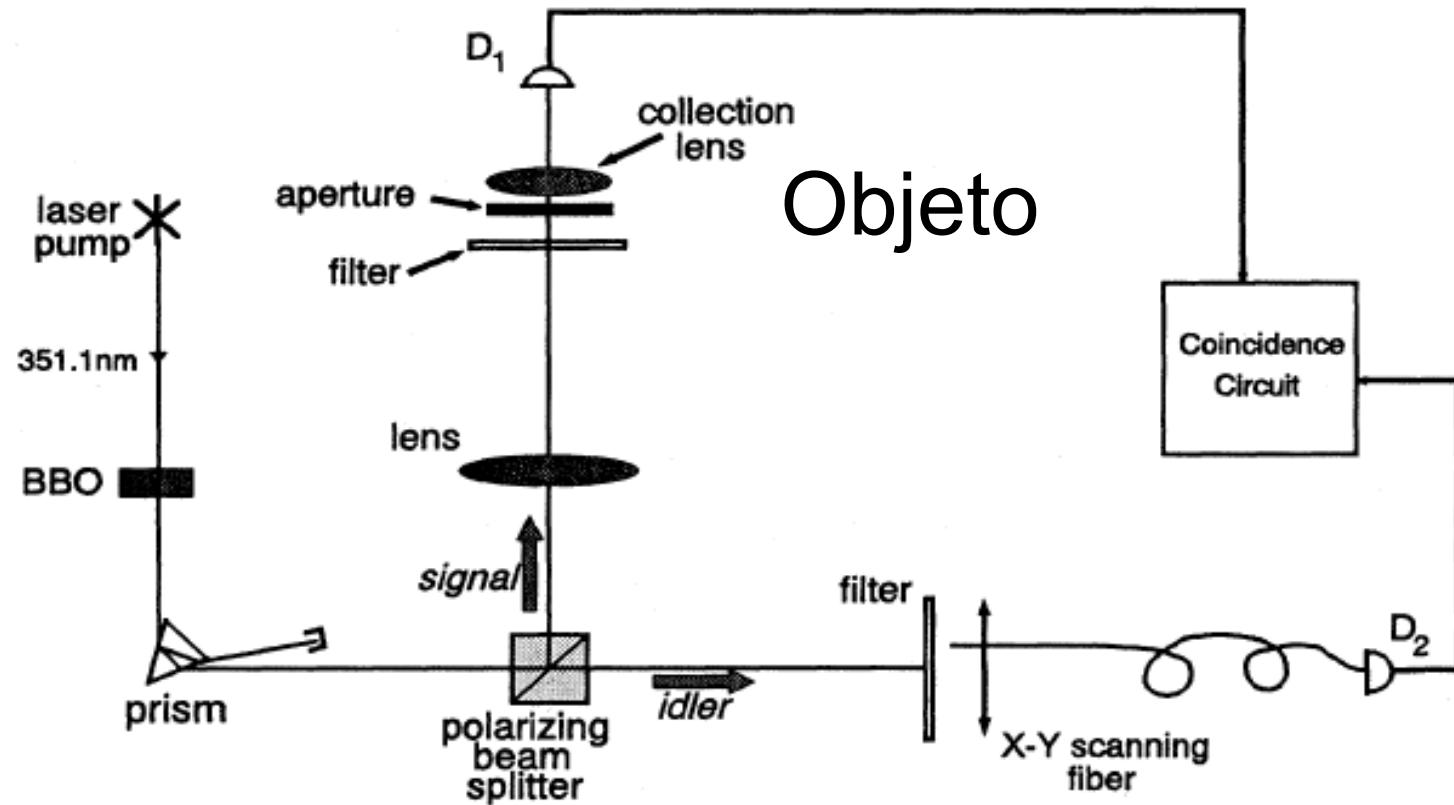
En la base circular  $|+\rangle, |-\rangle$  sucede lo mismo

La fuente de fotones está altamente correlacionada. Pero en cada realización el resultado individual es aleatorio.

Si condicionamos el detector de muchos pixeles a detectar uno en el de un pixel podemos reconstruir la imagen.

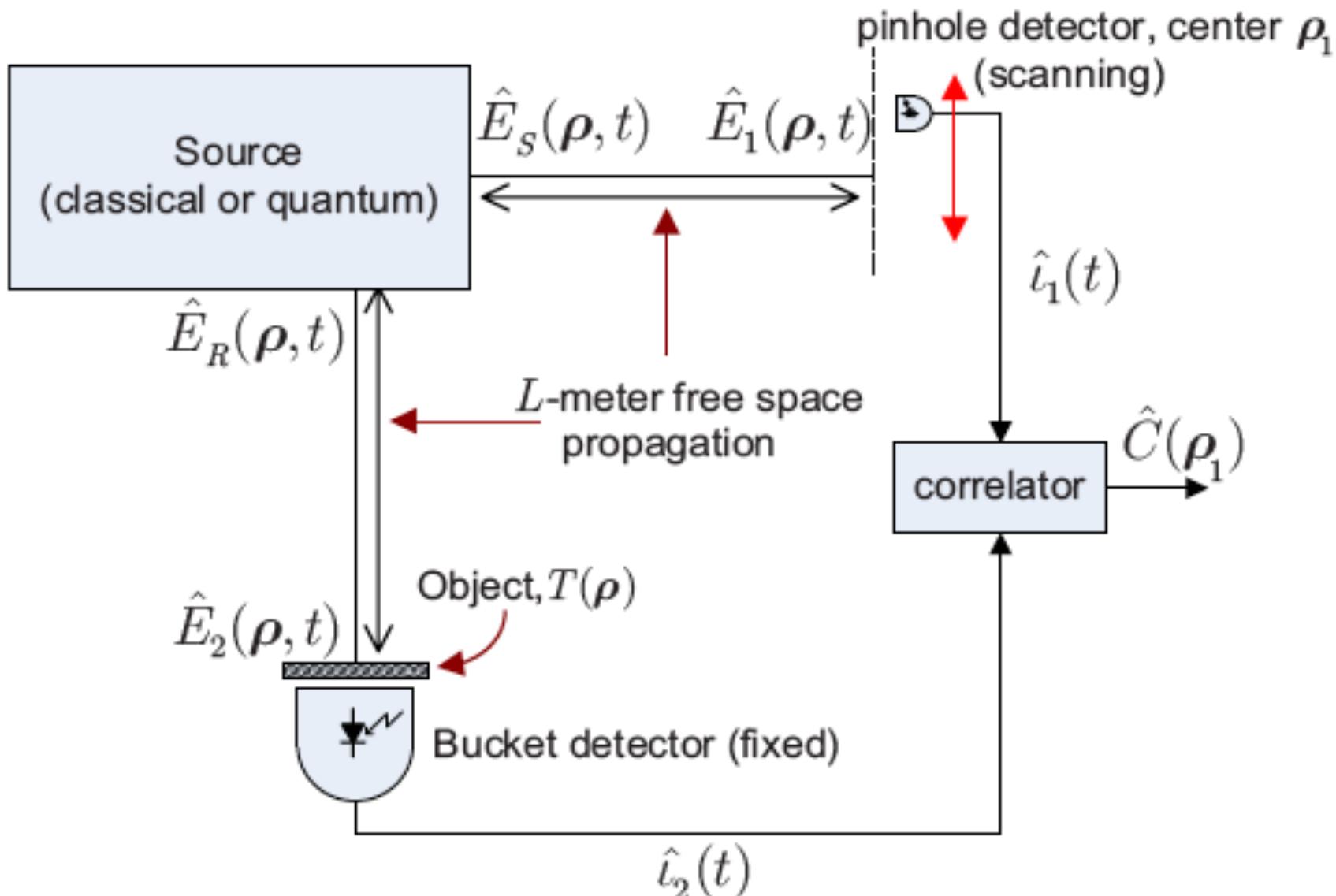
# Detector de un pixel (bucket)

Lentes (no son necesarias)



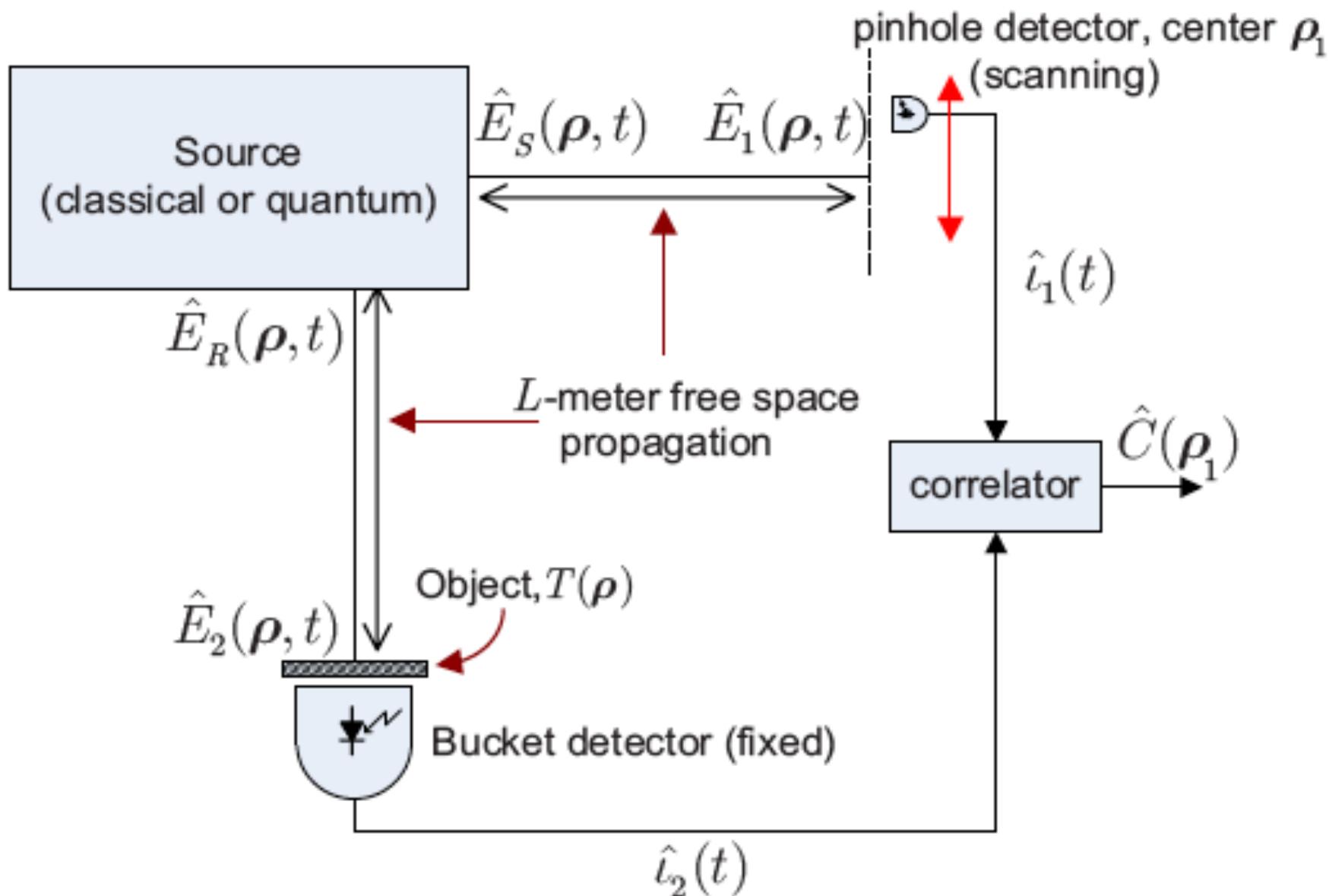
Detector multi pixel (barrido)  
Coincidencia (correlación, trigger)

Una generalización del aparato



Baris I. Erkmen and Jeffrey H. Shapiro, “Ghost imaging: from quantum to classical to computational,” *Advances in Optics and Photonics* **2**, 405–450 (2010) Baris I. Erkmen and Jeffrey H. Shapiro, “Ghost imaging: from quantum to classical to computational,” *Advances in Optics and Photonics* **2**, 405–450 (2010).

¿Es necesario tener fotones entrelazados?



Baris I. Erkmen and Jeffrey H. Shapiro, "Ghost imaging: from quantum to classical to computational," *Advances in Optics and Photonics* **2**, 405–450 (2010).

# Correlacion en el desplazamiento del haz clásico

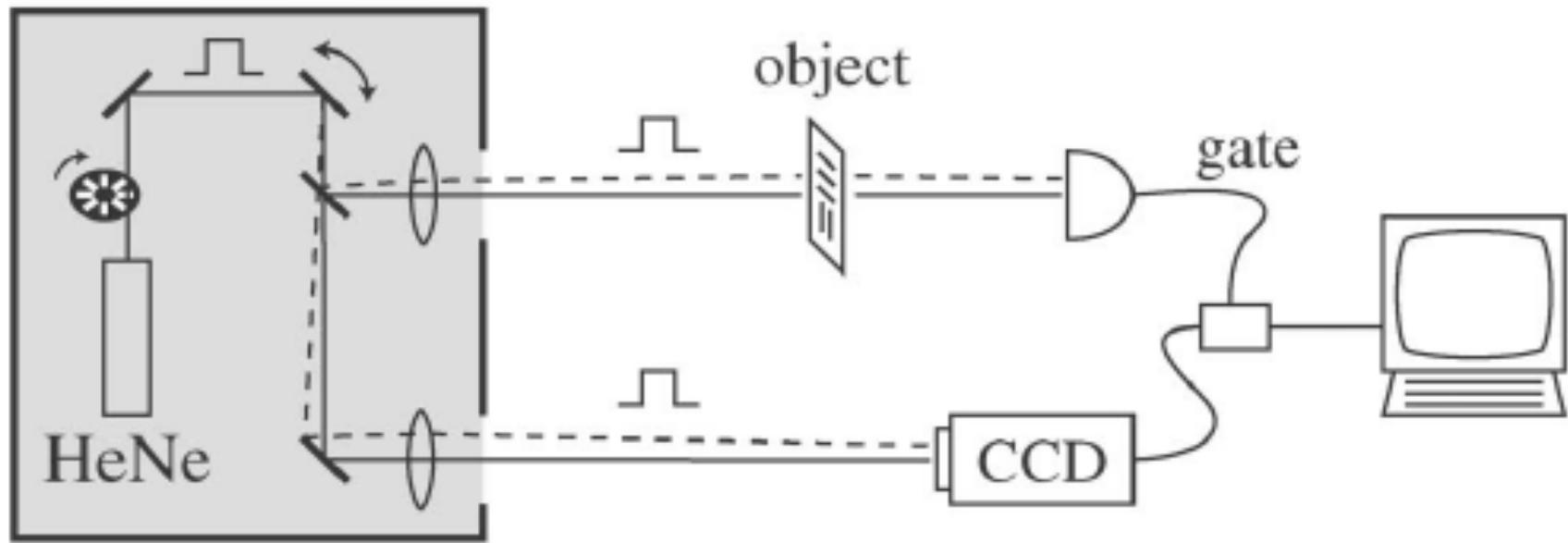


FIG. 2. The experimental setup used to perform coincidence imaging with a classically correlated source (shaded box).

R. S. Bennink, S. J. Bentley, and R. W. Boyd, “Two-photon” coincidence imaging with a classical source,” Phys. Rev. Lett. **89**, 113601 (2002).

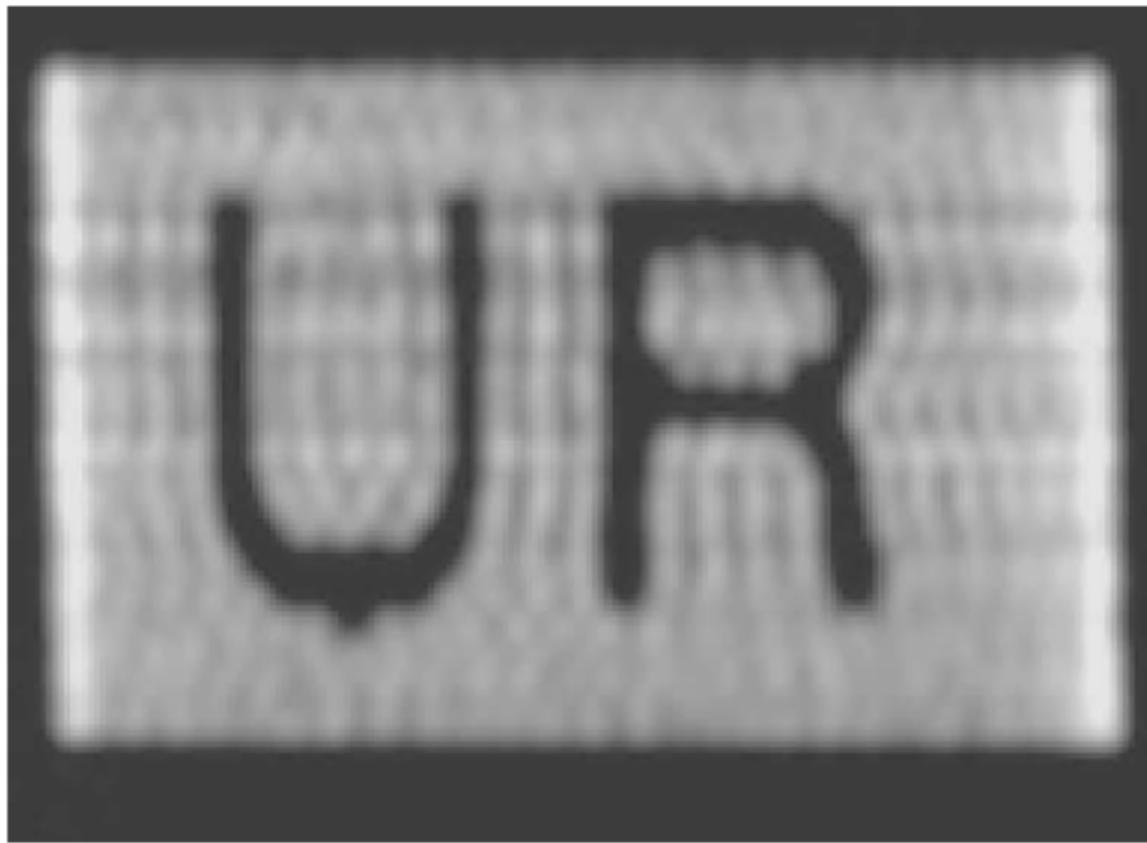


FIG. 3. The image formed in the reference arm when gated by the detector in the test arm. Such an image corresponds to the marginal probability distribution.

R. S. Bennink, S. J. Bentley, and R. W. Boyd, "Two-photon" coincidence imaging with a classical source," Phys. Rev. Lett. **89**, 113601 (2002).

# Generalización como un problema de probabilidades

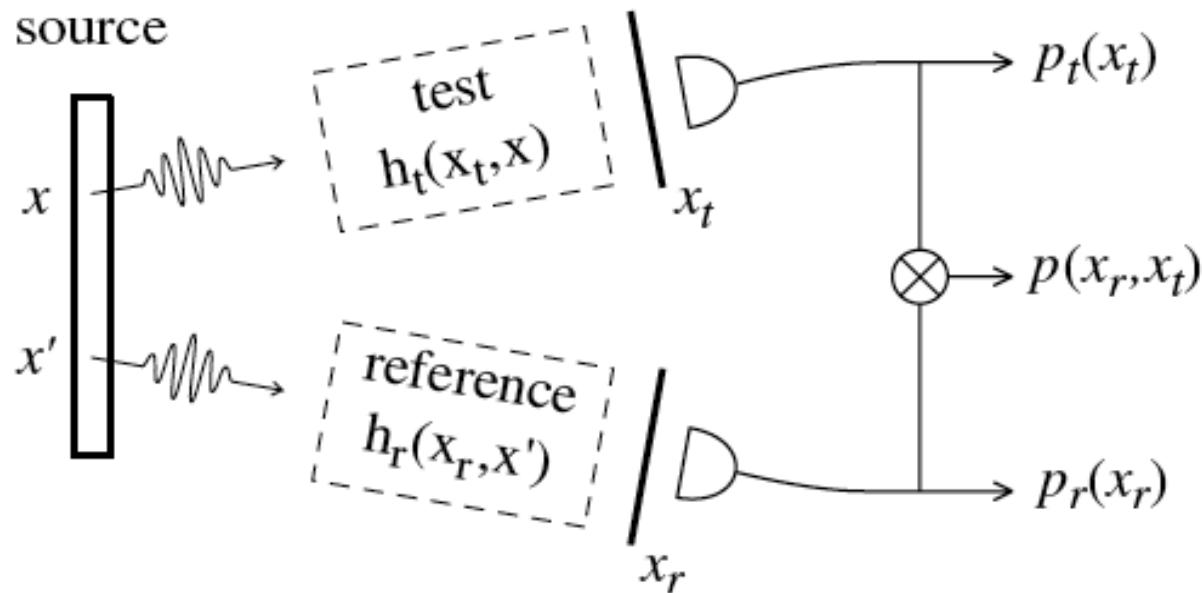


FIG. 1. (adapted from [12]) Two-photon coincidence imaging. The transfer function of the test system is to be obtained from the joint detection statistics using knowledge of the reference system.

R. S. Bennink, S. J. Bentley, and R. W. Boyd, “Two-photon” coincidence imaging with a classical source,” Phys. Rev. Lett. **89**, 113601 (2002).

Las probabilidades dada la distribución en la fuente

$$p_t(x_t) = \int dx' \left| \int dx h_t(x_t, x) \varphi(x, x') \right|^2,$$

$$p_r(x_r) = \int dx \left| \int dx' h_r(x_r, x') \varphi(x, x') \right|^2,$$

$$p(x_t, x_r) = \left| \int dx dx' h_t(x_t, x) h_r(x_r, x') \varphi(x, x') \right|^2,$$

$h$  es la función de transferencia adecuada  
es crucial que  $\varphi$  tenga correlaciones

Las funciones de correlación satisfacen la misma ecuación de onda (incluida la difracción) de la onda electromagnética apropiada.

- Las fuentes clásicas tienen (interferencia aleatoria) specke (correlaciones)



## Tabletop x-ray ghost imaging with ultra-low radiation

AI-XIN ZHANG,<sup>1,2,†</sup> YU-HANG HE,<sup>1,2,†</sup> LING-AN WU,<sup>1,2,4</sup> LI-MING CHEN,<sup>1,2,3,5</sup> AND BING-BING WANG<sup>1,2</sup>

<sup>1</sup>Institute of Physics, Chinese Academy of Sciences, Beijing 100191, China

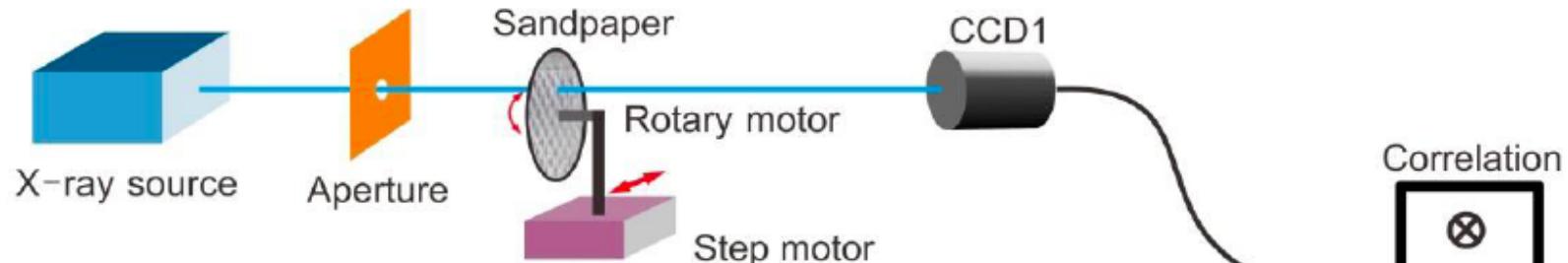
<sup>2</sup>University of Chinese Academy of Sciences, Beijing 100049, China

<sup>3</sup>CICIFSA and Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China

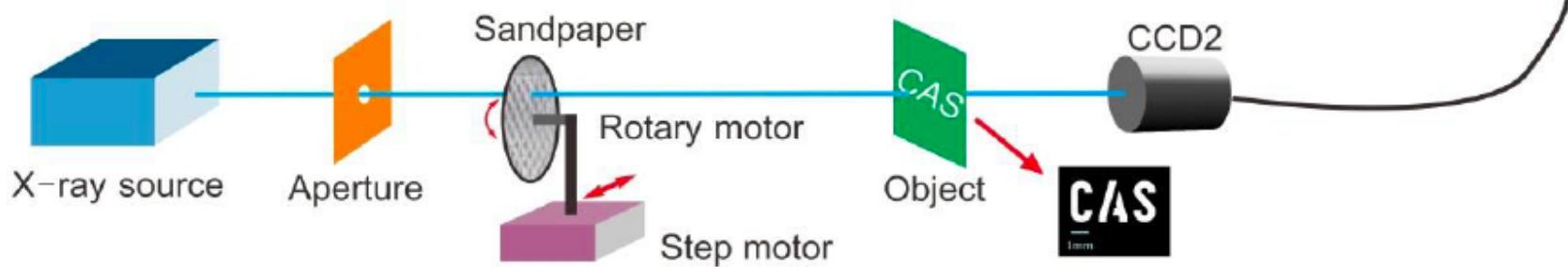
<sup>4</sup>e-mail: wula@aphy.iphy.ac.cn

<sup>5</sup>e-mail: lmchen@aphy.iphy.ac.cn

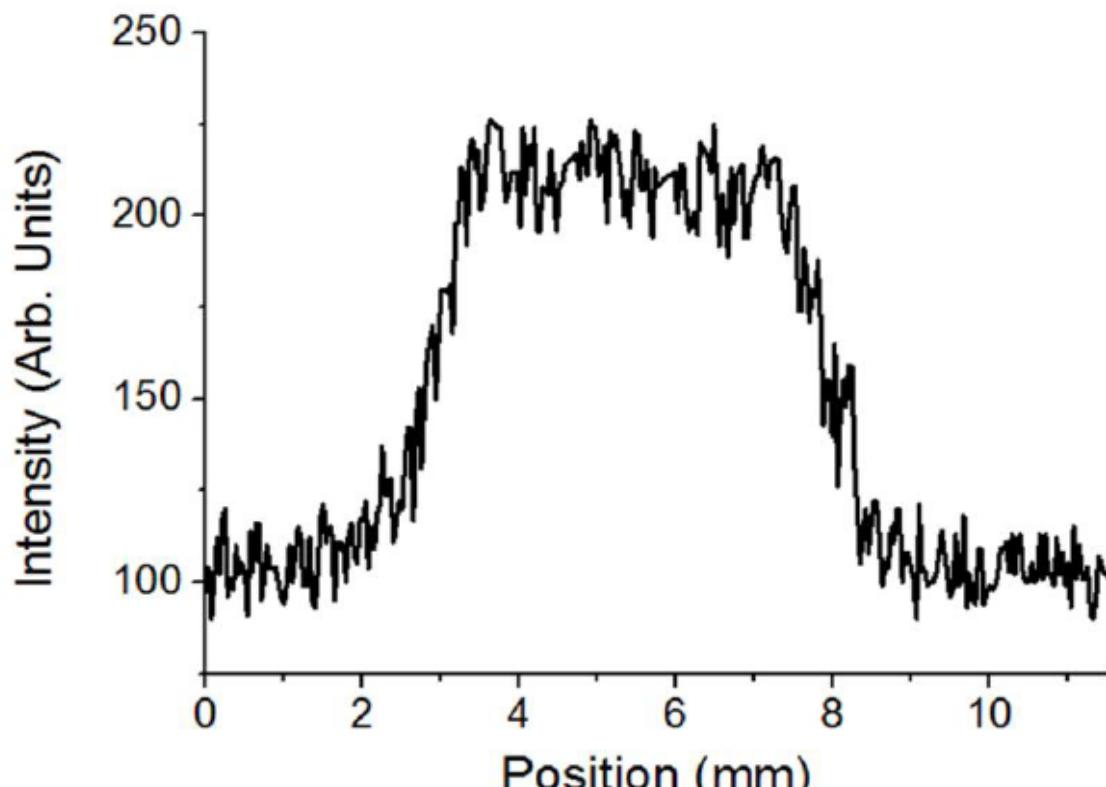
(a) Pre-recording of the reference signal (speckle patterns)



(b). Measurement of the bucket signal



Ai-Xin Zhang, Yu-Hang He, Ling-An Wu, Li-Ming Chen, Bing-Bing Wang, "Table-top X-ray Ghost Imaging with Ultra-Low Radiation" Optica, 5, 374 (2018).



(a)

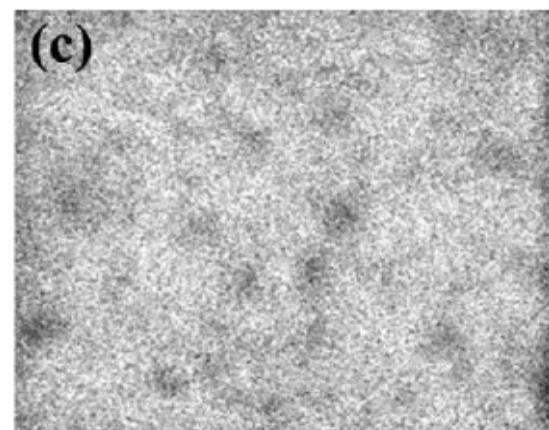
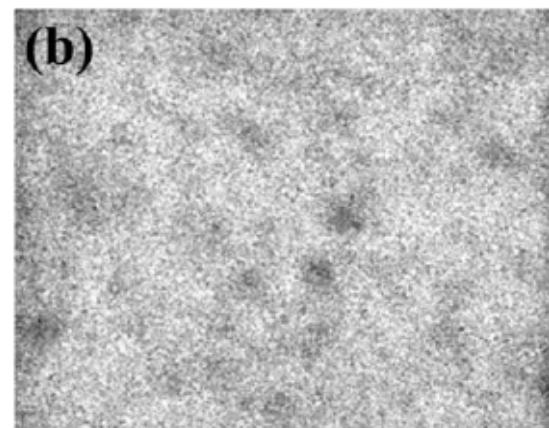
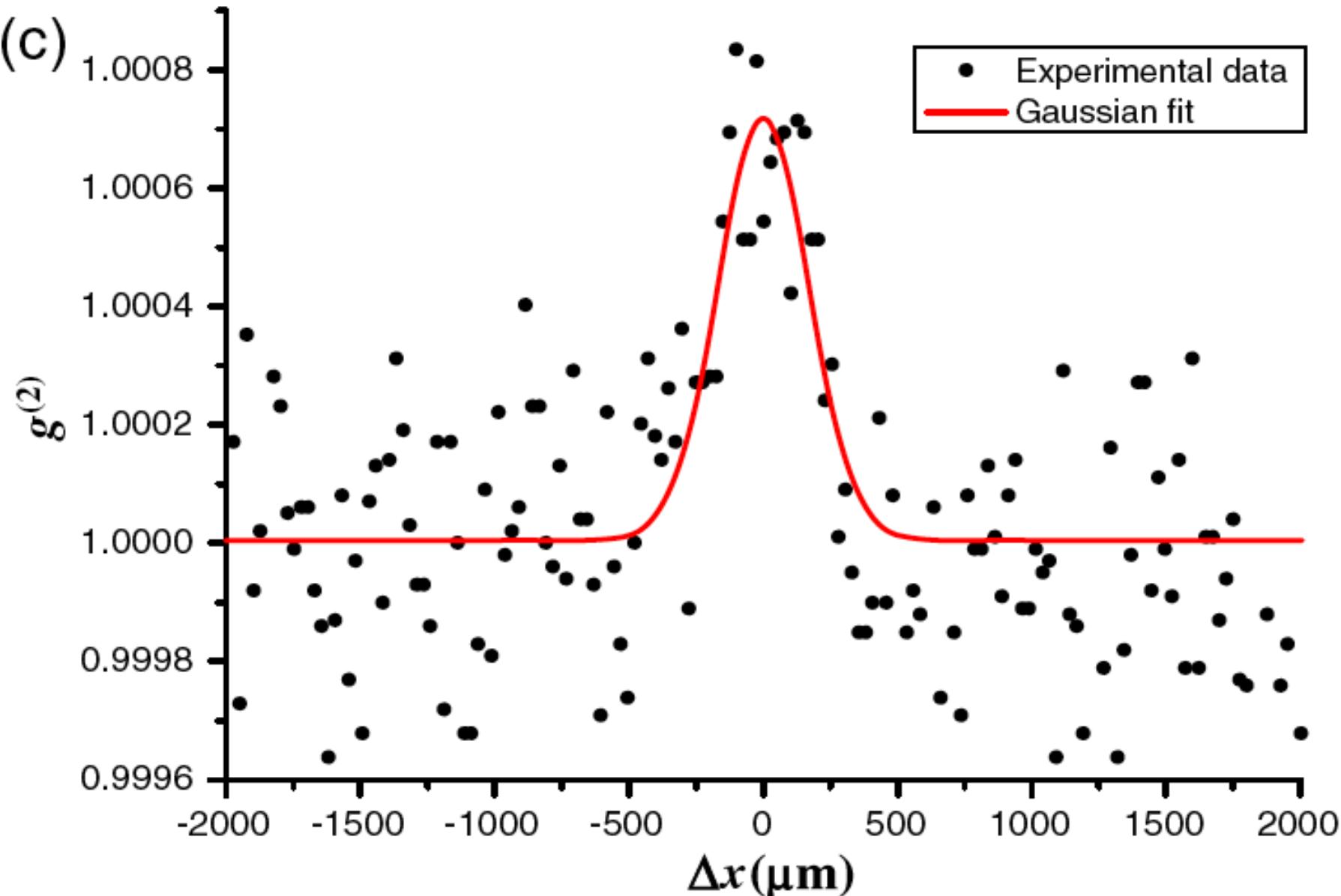
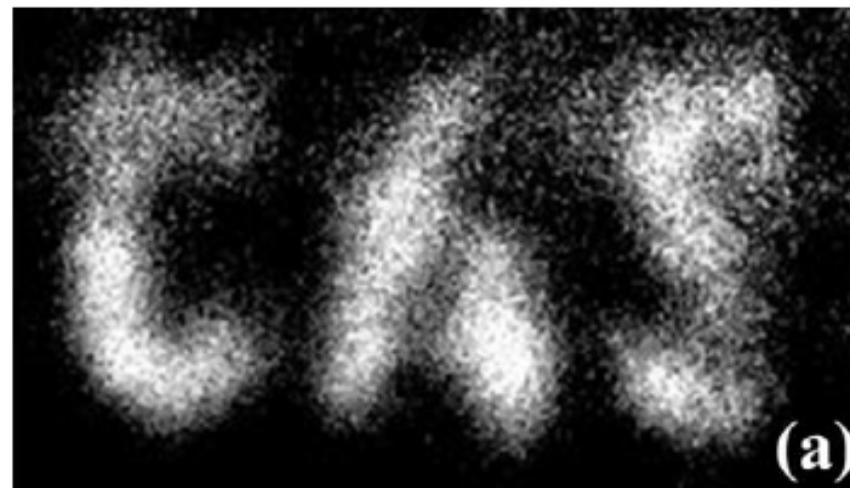


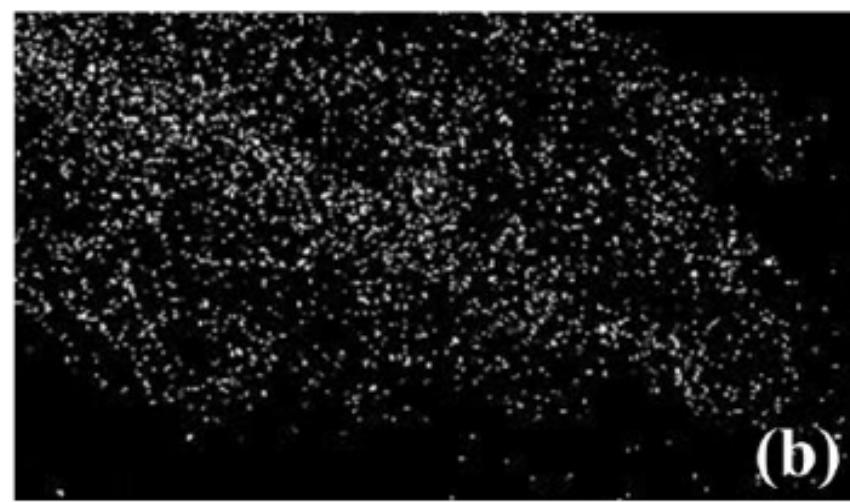
Fig. 2(a) Intensity profile of the direct x-ray beam; (b) Pre-recorded speckle pattern  $I_1$ ;  
(c) Speckle pattern  $I_1'$  in the second series of positions.



Ai-Xin Zhang, Yu-Hang He, Ling-An Wu, Li-Ming Chen, Bing-Bing Wang,  
"Table-top X-ray Ghost Imaging with Ultra-Low Radiation" Optica, 5, 374  
(2018).



(a)



(b)

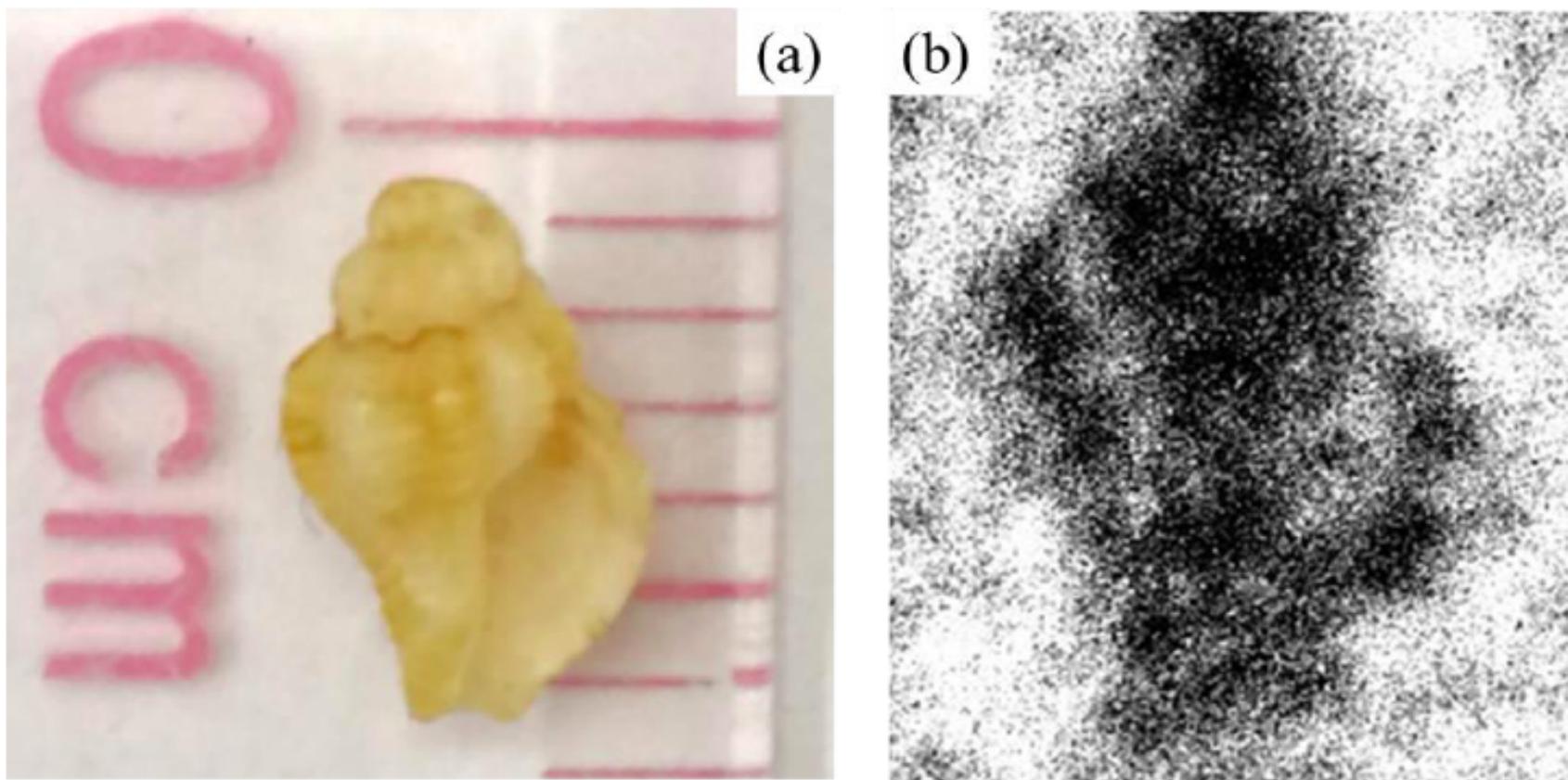
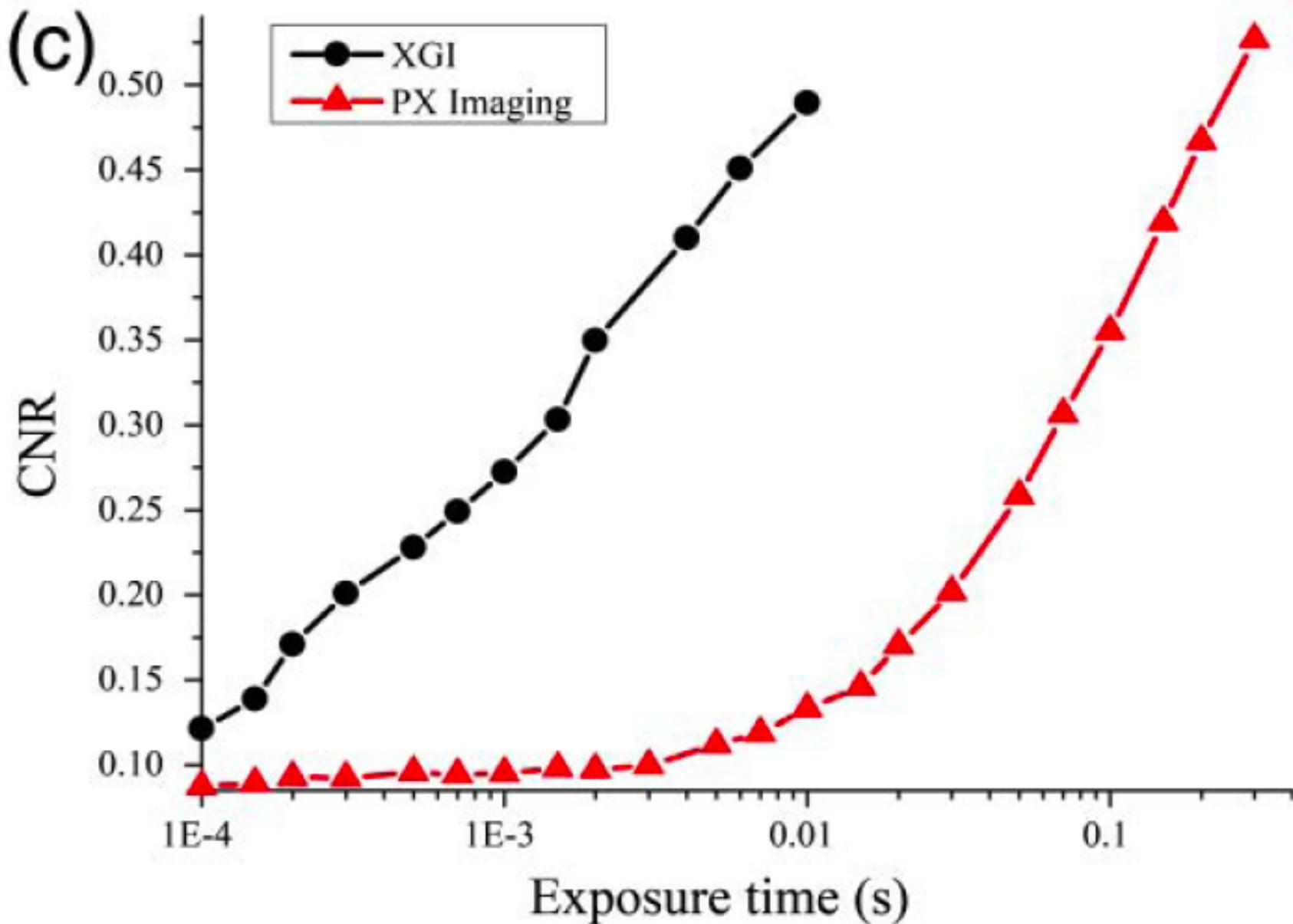
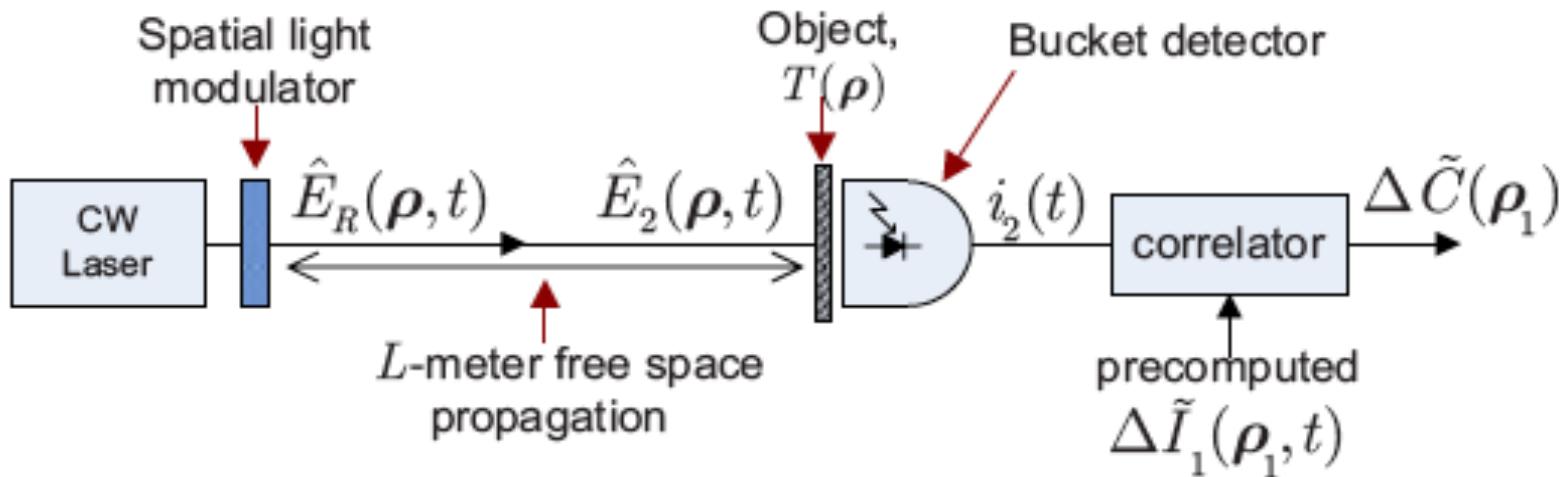


Fig. 4. (a) Ordinary photo of the shell; (b) XGI gray-scale transmission image.



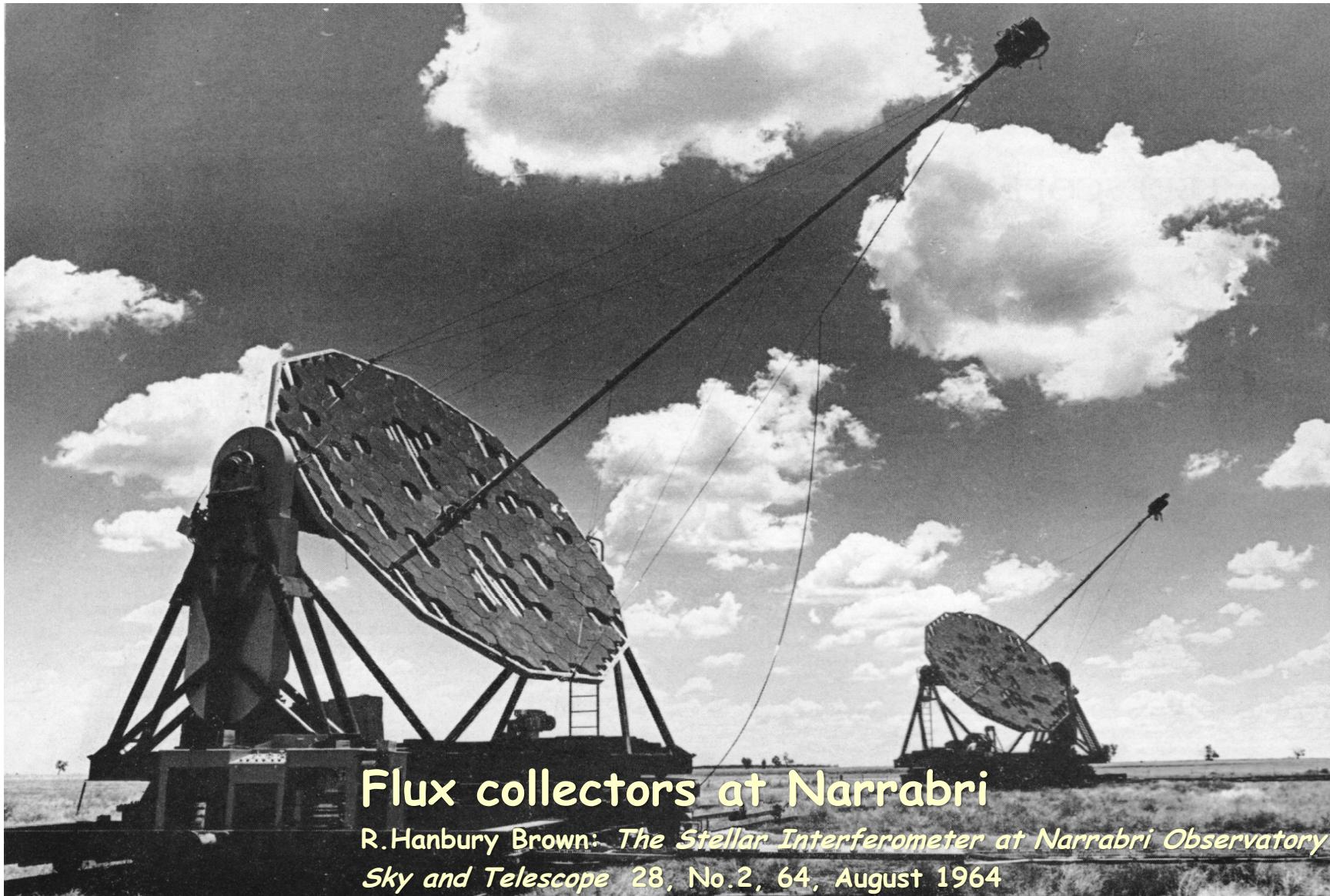


Computational ghost-imaging setup.

Noten que no hay lente y solo hay una fuente

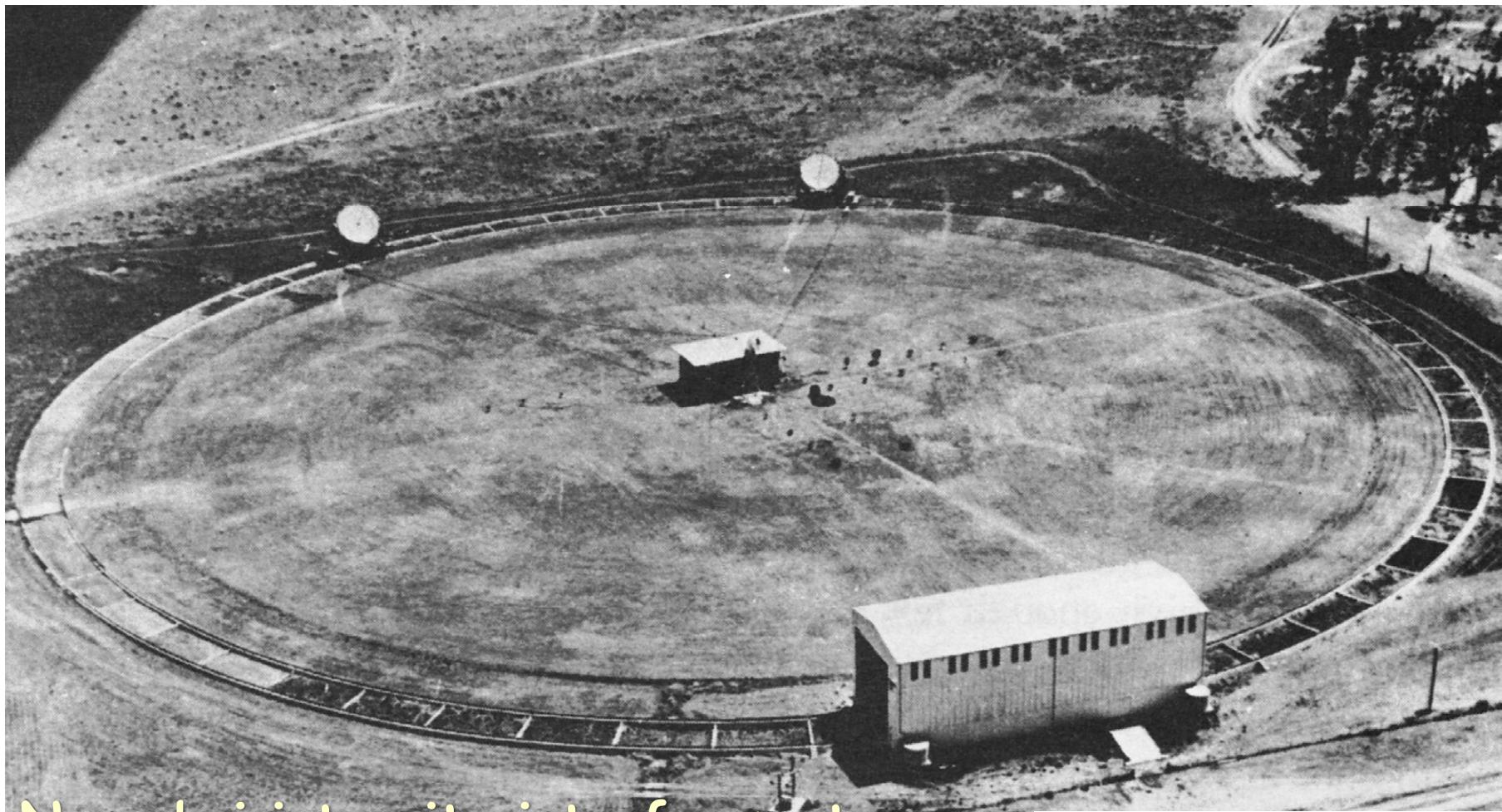
Baris I. Erkmen and Jeffrey H. Shapiro, “Ghost imaging: from quantum to classical to computational,” *Advances in Optics and Photonics* **2**, 405–450 (2010) Baris I. Erkmen and Jeffrey H. Shapiro, “Ghost imaging: from quantum to classical to computational,” *Advances in Optics and Photonics* **2**, 405–450 (2010).

Handbury Brown y Twiss



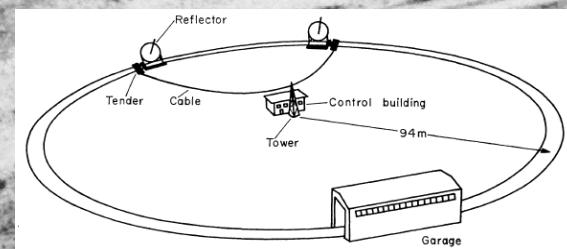
## Flux collectors at Narrabri

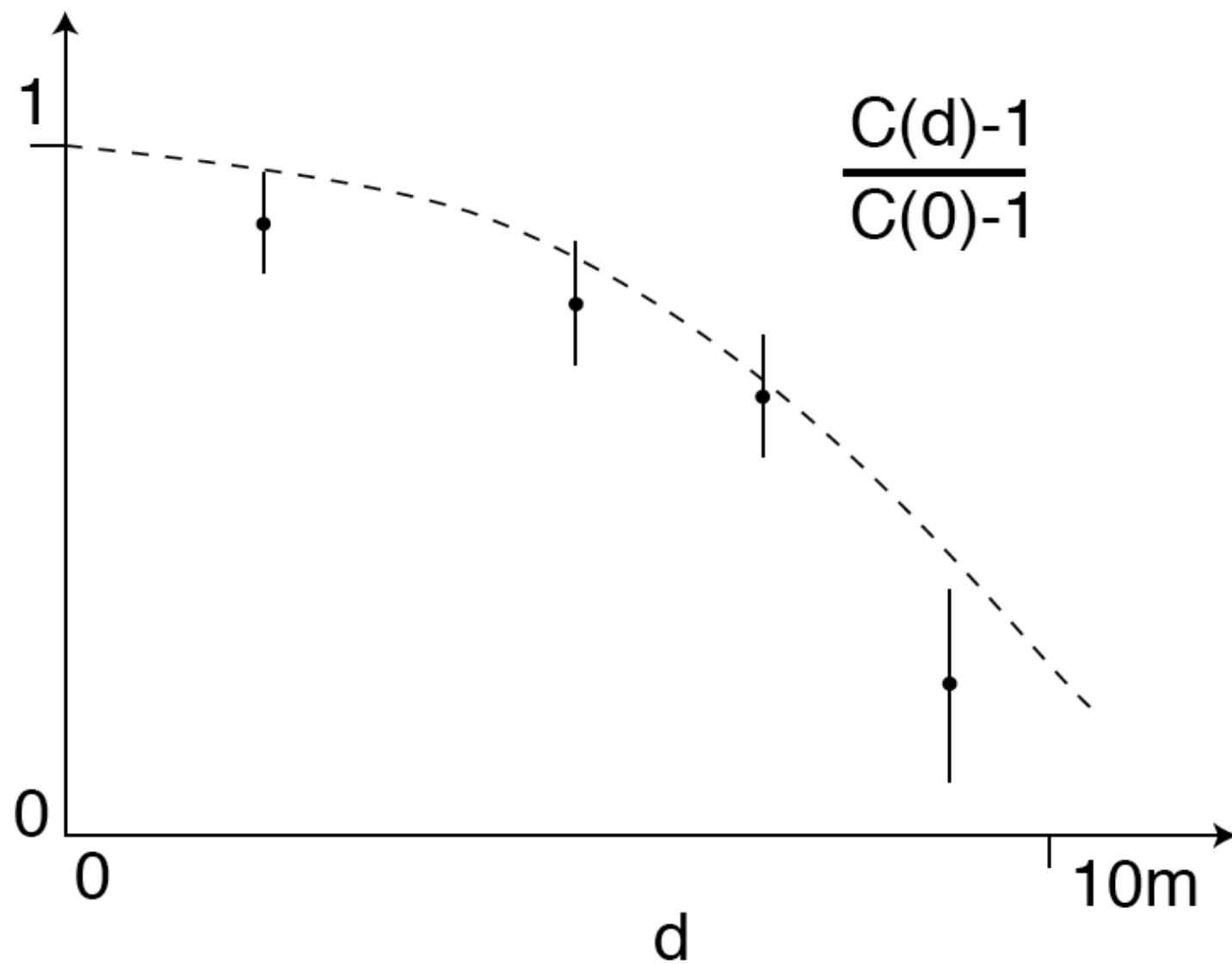
R. Hanbury Brown: *The Stellar Interferometer at Narrabri Observatory*  
*Sky and Telescope* 28, No. 2, 64, August 1964



## Narrabri intensity interferometer with its circular railway track

R. Hanbury Brown: *BOFFIN. A Personal Story of the Early Days of Radar, Radio Astronomy and Quantum Optics* (1991)





Medición del diámetro angular de Sirio por HBT en Australia.

Una solución a la pregunta si es clásico o cuántico

## **Hanbury Brown–Twiss effect and thermal light ghost imaging: A unified approach**

Li-Gang Wang,<sup>1,2</sup> Sajid Qamar,<sup>3</sup> Shi-Yao Zhu,<sup>1,2</sup> and M. Suhail Zubairy<sup>3,4</sup>

<sup>1</sup>*Department of Physics, Zhejiang University, 310027 Hangzhou, China*

<sup>2</sup>*Department of Physics, Chinese University of Hong Kong, Shatin, N. T., Hong Kong*

<sup>3</sup>*Center for Quantum Physics, COMSATS Institute of Information Technology, Islamabad, Pakistan*

<sup>4</sup>*Department of Physics and Institute for Quantum Studies, Texas A&M University, College Station, Texas 77843-4242, USA*

(Received 14 September 2008; revised manuscript received 1 January 2009; published 24 March 2009)

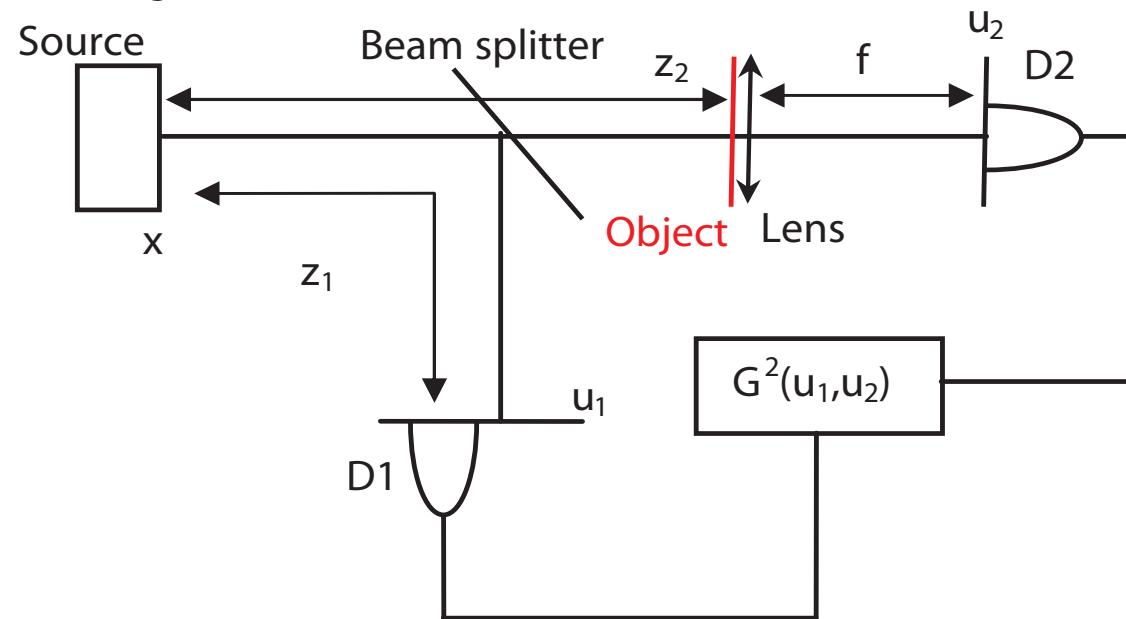
We compare the Hanbury Brown–Twiss (HBT) and the thermal light ghost imaging schemes in both near and far fields. Both effects arise as a result of the intensity fluctuations of the thermal light and we find that the essential physics behind the two effects is the same. The difference however is that, in the ghost imaging, large number of bits information of an object needs to be treated together, whereas, in the HBT, there is only one bit information required to be obtained. In the HBT experiment far field is used for the purpose of easy detection, while in the ghost image experiment near (or not far) field is used for good quality image.

DOI: [10.1103/PhysRevA.79.033835](https://doi.org/10.1103/PhysRevA.79.033835)

PACS number(s): 42.50.Ar, 42.50.Dv, 42.50.St

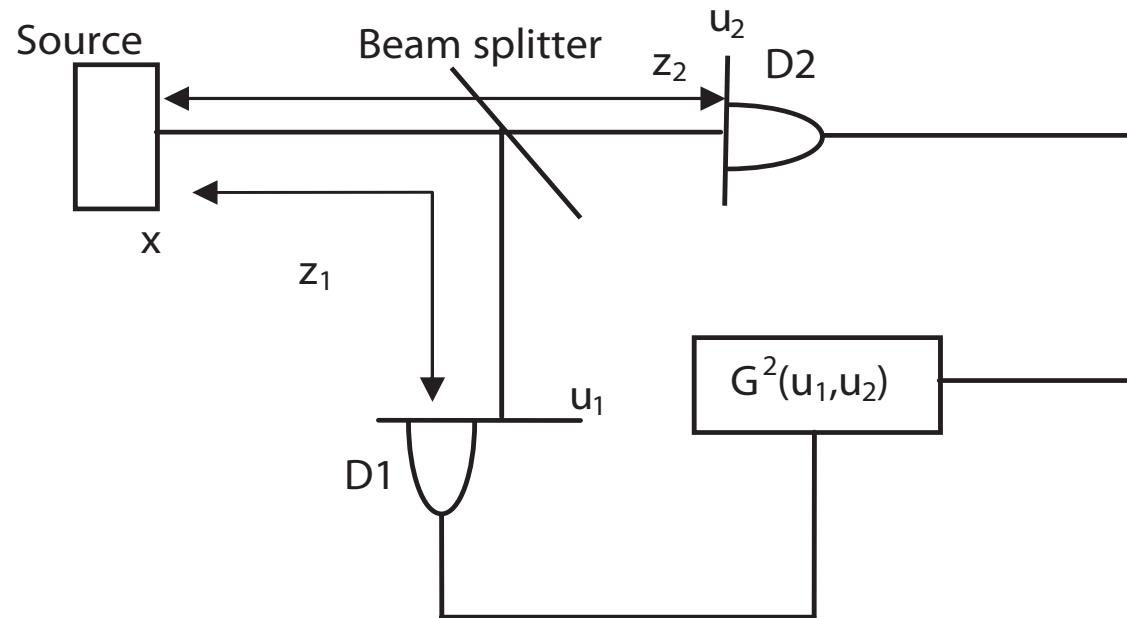
# Arreglo de Imágen Fantasma

Thermal light



# Arreglo HBT

Thermal light



## Parecen iguales:

- Hay un lente y el detector está en el foco  
there is a lens and the detector is at the focal point.
- Hanbury Brown and Twiss is far field, the detector located where the object was.
- We have to propagate the correlations appropriately.
- We demand only one bit of information on HBT and many bits of information on GI to have a good image

# The intensity correlation of thermal light:

The result depends only on the first order correlation, the coherence (linewidth) of the source only.

$$G^2(u_1, u_2) = \langle I_1 I_2 \rangle = \langle I(u_1) \rangle \langle I(u_2) \rangle + |\Gamma(u_1, u_2)|^2$$

$$\Gamma(u_1, u_2) = \langle E^+(u_1) E(u_2) \rangle$$

Gaussian Schell model,  $\sigma_i$  intensity deviation,  $\sigma_g$  correlation length.

$$\langle E_s^+(x_1) E_s(x_2) \rangle = G_0 \exp \left[ -\frac{x_1^2 + x_2^2}{4\sigma_I^2} - \frac{(x_1 - x_2)^2}{2\sigma_g^2} \right]$$

# Propagation of correlations:

$$\Gamma(u_1, u_2) = \langle E^+(u_1) E(u_2) \rangle$$

$$= \int \int \langle E_s^+(x_1) E_s(x_2) \rangle h_1^*(x_1, u_1) h_2(x_2, u_2) dx_1 dx_2,$$

Where  $h_i(x_i, u_i)$  is the appropriate propagation functions of the correlation from the source to the detectors along path I which depend on the optical elements of the paths. given the setup.

$h_1$  is the same for both setups

$$h_1^{\text{H,G}}(x, u_1) = \left( -\frac{i}{\lambda z_1} \right)^{1/2} \exp \left[ -\frac{i\pi}{\lambda z_1} (x^2 - 2xu_1 + u_1^2) \right]$$

$h_2$  is different for both setups

$$h_2^H(x, u_2) = \left(-\frac{i}{\lambda z_2}\right)^{1/2} \exp\left[-\frac{i\pi}{\lambda z_2}(x^2 - 2xu_2 + u_2^2)\right],$$

$$h_2^G(x, u_2) = \left(-\frac{i}{\lambda f}\right)^{1/2} \left(-\frac{i}{\lambda z_2}\right)^{1/2} \int dv H(v) \\ \times \exp\left[-\frac{i\pi}{\lambda z_2}(x^2 - 2xv + v^2) - \frac{i\pi}{\lambda f}(-2vu_2 + u_2^2)\right],$$

$u_1 \neq 0$  and  $u_2 = 0$  (transverse HBT)

$$\text{HBT}(\bar{u}_1, 0) = \exp\left(-\frac{\bar{u}_1^2}{\bar{\sigma}_g^2 + \frac{\bar{z}^2}{4\pi^2 \bar{\sigma}_I^2}}\right)$$

$\bar{u}_1^2$ ,  $\bar{z}$ , and  $\bar{\sigma}_{I,g}$  are in units of  $\lambda$

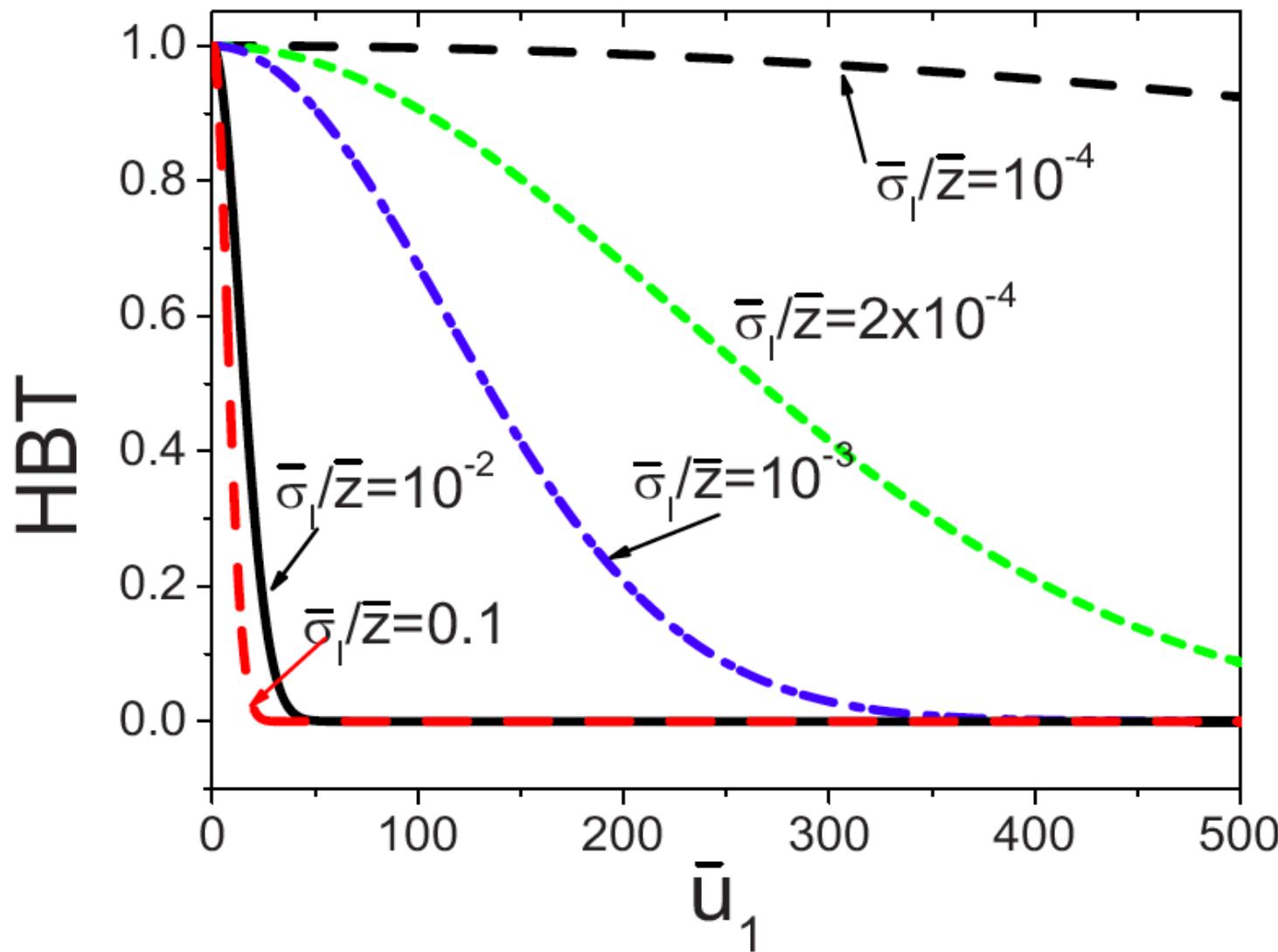


FIG. 2. (Color online) HBT effect for different  $\bar{\sigma}_I/\bar{z}=10^{-4}$ ,  $2\times 10^{-4}$ ,  $10^{-3}$ ,  $10^{-2}$ , and  $0.1$  (from top to below), with  $\bar{\sigma}_g=10$  and  $\bar{z}=10^5$ .

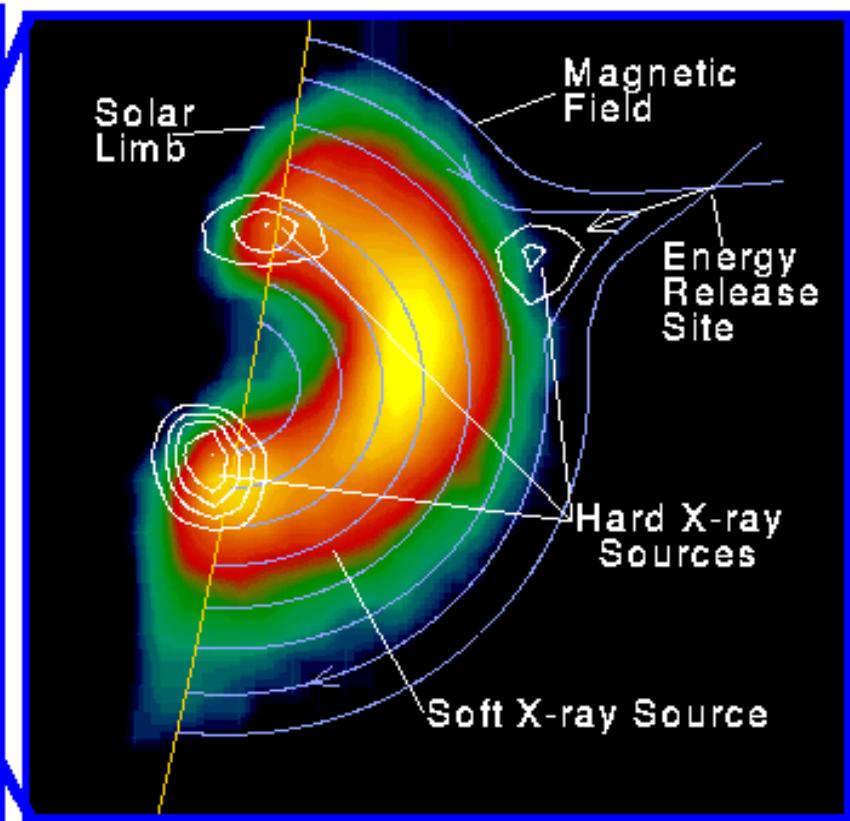
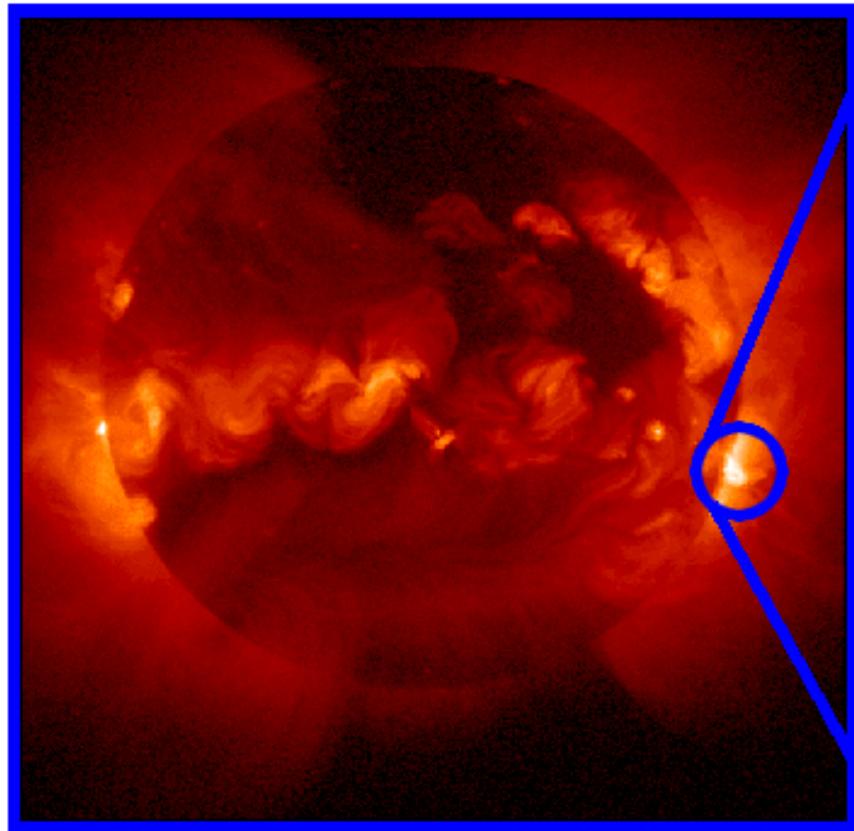
Misión RHESSI telescopio.

¿Cómo crear una imagen con un detector de un pixel?

Rotational Modulation Collimators

1975 Minoru Oda

Tomar un conteo con una apertura para conocer la amplitud en una componente de Fourier,  
Continuar con muchos filtros diferentes hasta hacer la imagen.



Yohkoh X-ray Image of a Solar Flare, Combined Image in Soft X-rays (left) and Soft X-rays with Hard X-ray Contours (right). Jan 13, 1992.

## Preguntas:

- ¿Cuál es el límite de la resolución con este tipo de imágenes?
- ¿Cómo escala con la correlación espacial de la fuente? ¿Existen correlaciones mas favorables?
- ¿Se podría utilizar la correlación  $h_\theta(\tau)$  (Campo-Intensidad) para obtener una imagen tanto de la intensidad como de la fase?
- ¿Qué se puede hacer de análisis espectral con este tipo de imagen? Si, ver Phys. Rev. Applied 9, 021001 (2018).
- ¿Cómo utilizarla en telescopios?

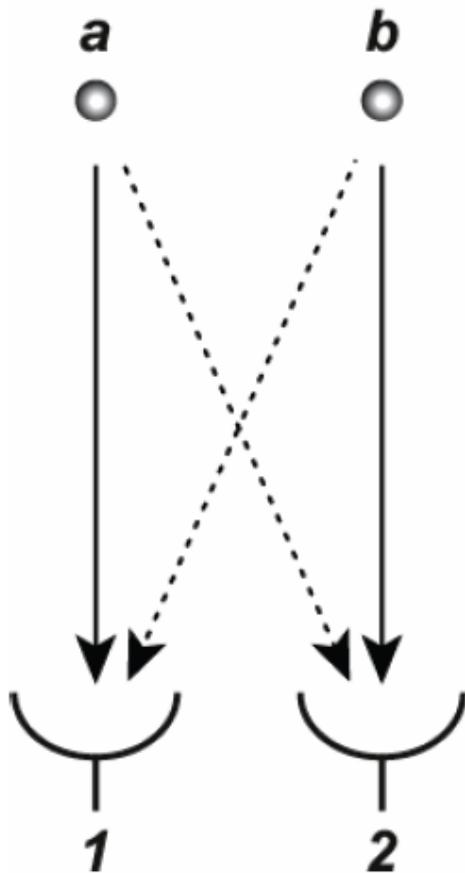
¡Usen correlaciones!

Gracias

# Symmetry of 2 particle wave-function

QM view:

- two identical particles incident on two detectors
- consider two-particle wavefunction
- probability amplitude for particle detection in 1 and 2:



$$\langle 1|a\rangle = \langle 2|a\rangle \equiv a, \quad \langle 1|b\rangle = \langle 2|b\rangle \equiv b,$$

probability  $P$  to detect two particles simultaneously in each detector (coincidences):

**Classical particles:**

$$P = |\langle 1|a\rangle\langle 2|b\rangle|^2 + |\langle 1|b\rangle\langle 2|a\rangle|^2 = 2|a|^2|b|^2$$

**$C = 0$**

**Bosons:**

$$P = |\langle 1|a\rangle\langle 2|b\rangle + \langle 1|b\rangle\langle 2|a\rangle|^2 = 4|a|^2|b|^2$$

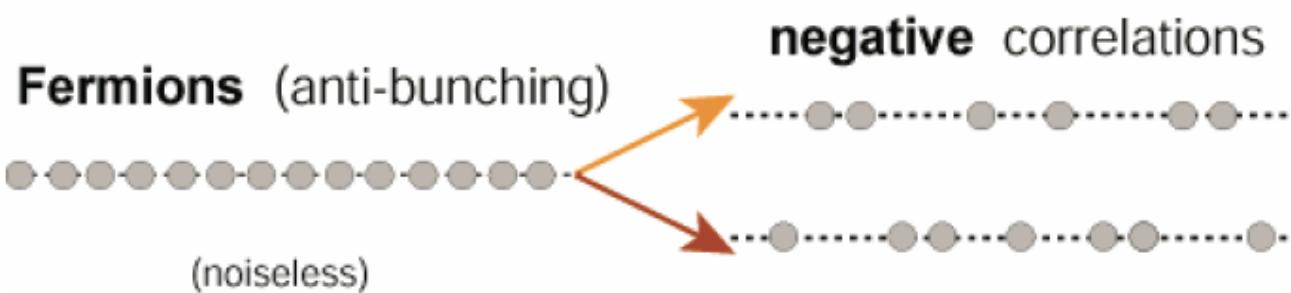
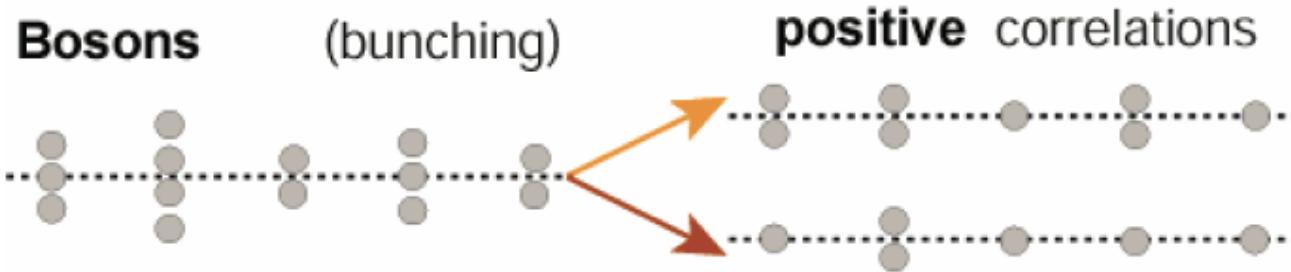
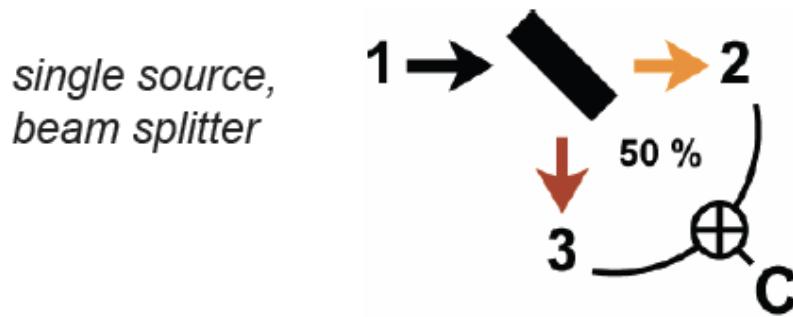
**bunching**  
 **$C > 0$**

**Fermions:**

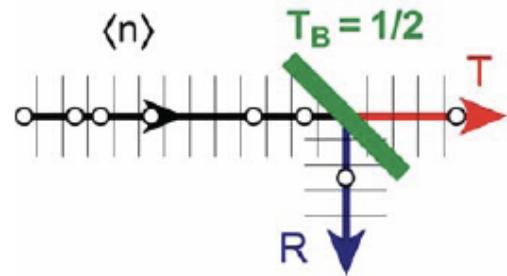
$$P = |\langle 1|a\rangle\langle 2|b\rangle - \langle 1|b\rangle\langle 2|a\rangle|^2 = 0$$

**anti-bunching**  
 **$C < 0$**

## Probing the statistics: illustration



# HBT experiments and particle statistics



$$C \sim \langle \Delta n_T \Delta n_T \rangle \sim [\langle \Delta n^2 \rangle - \langle n \rangle]$$

*fluctuations*                                   *particle nature*

*sub-Poissonian*

$$\mathbf{C} < 0$$

$$\langle \Delta n^2 \rangle < \langle n \rangle$$

*Poissonian*

$$\mathbf{C} = 0$$

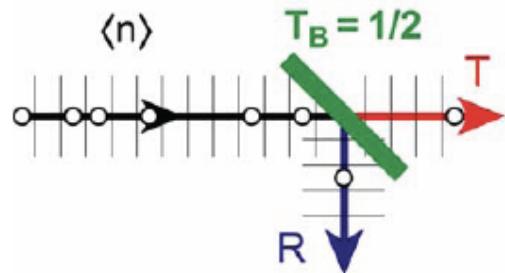
$$\langle \Delta n^2 \rangle = \langle n \rangle$$

*super-Poissonian*

$$\mathbf{C} > 0$$

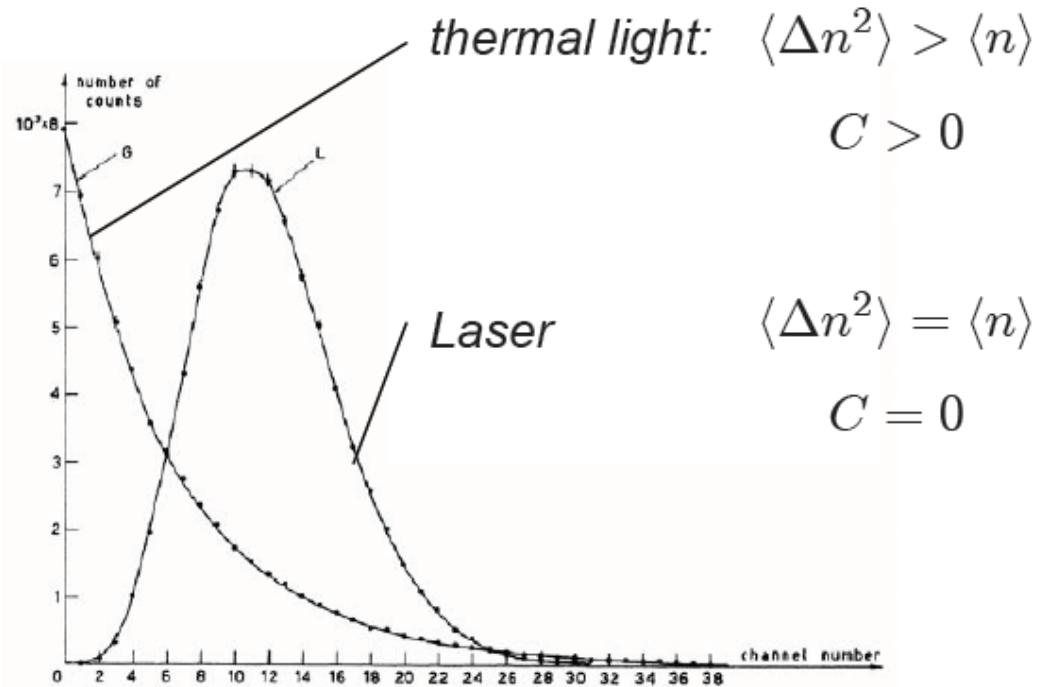
$$\langle \Delta n^2 \rangle > \langle n \rangle$$

# classical photons (1965)

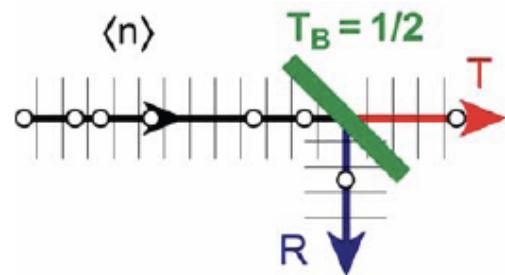


$$C \sim \langle \Delta n_T \Delta n_T \rangle \sim [\langle \Delta n^2 \rangle - \langle n \rangle]$$

*fluctuations*                                   *particle nature*



# anti-bunching of photons (1977)

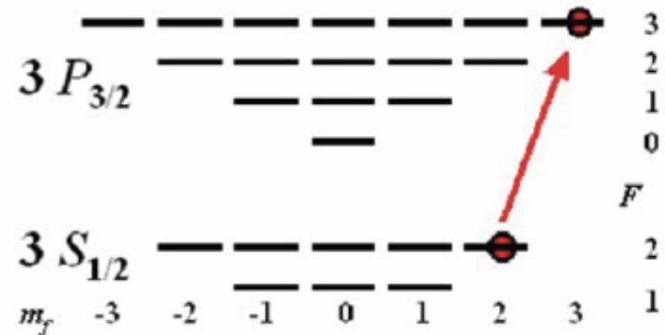
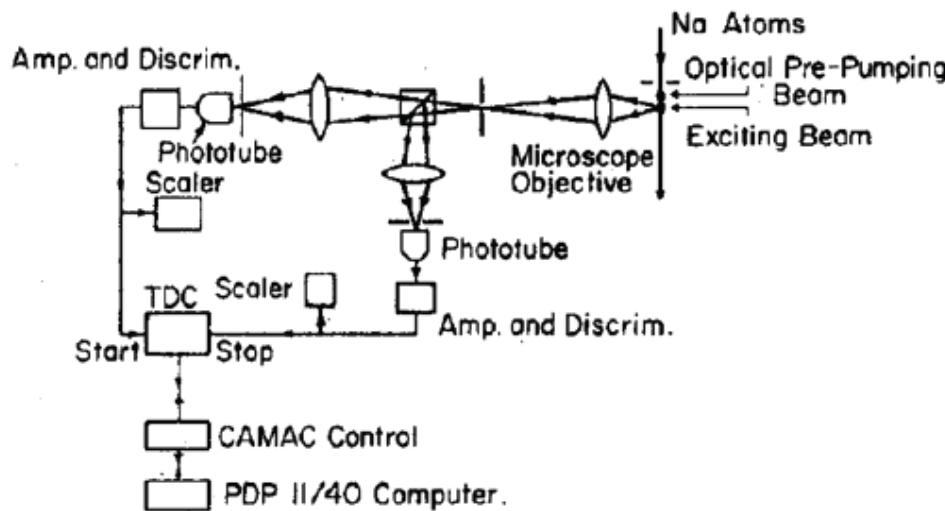


$$C \sim \langle \Delta n_T \Delta n_T \rangle \sim [\langle \Delta n^2 \rangle - \langle n \rangle]$$

↑ *fluctuations*      ↑ *particle nature*

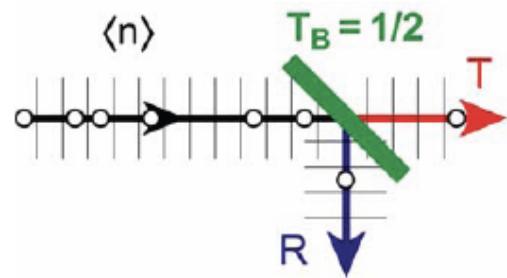
single photon source: (single atom, qu.dot)

$$\langle \Delta n^2 \rangle < \langle n \rangle \quad C < 0$$



beam of sodium atoms is first prepumped into the  $3S1/2, F = 2, m_F = 2$  state  
 transition driven by an exciting beam  
 fluorescence light detected  
 finite lifetime of 16 ns → single atom cannot  
 emit two photons in a shorter time

# anti-bunching of photons (1977)

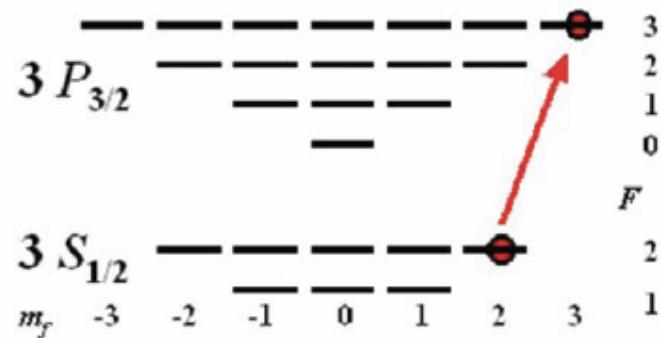
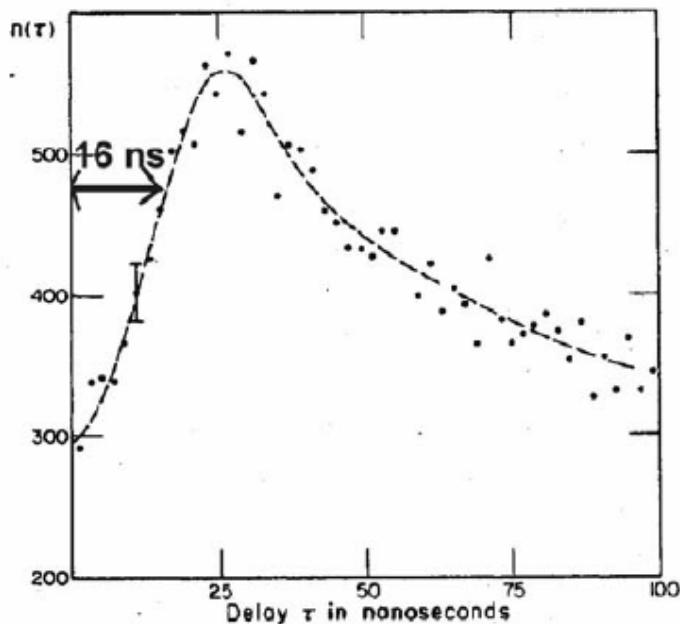


$$C \sim \langle \Delta n_T \Delta n_T \rangle \sim [\langle \Delta n^2 \rangle - \langle n \rangle]$$

↑ *fluctuations*      ↑ *particle nature*

single photon source: (single atom)

$$\langle \Delta n^2 \rangle < \langle n \rangle \quad C < 0$$



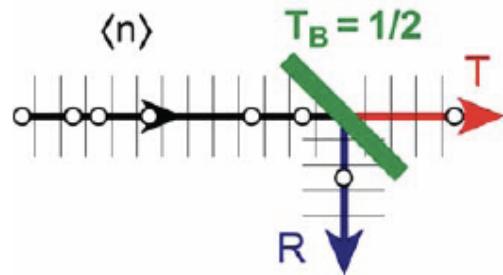
beam of sodium atoms is first prepumped into the  $3S1/2, F = 2, m_F = 2$  state

transition driven by an exciting beam

fluorescence light detected

finite lifetime of 16 ns → single atom cannot emit two photons in a shorter time

## anti-bunching of photons



$$C \sim \langle \Delta n_T \Delta n_T \rangle \sim [\langle \Delta n^2 \rangle - \langle n \rangle]$$

*fluctuations*      *particle nature*

single photon source: (single qu.dot)

$$\langle \Delta n^2 \rangle < \langle n \rangle \quad C < 0$$

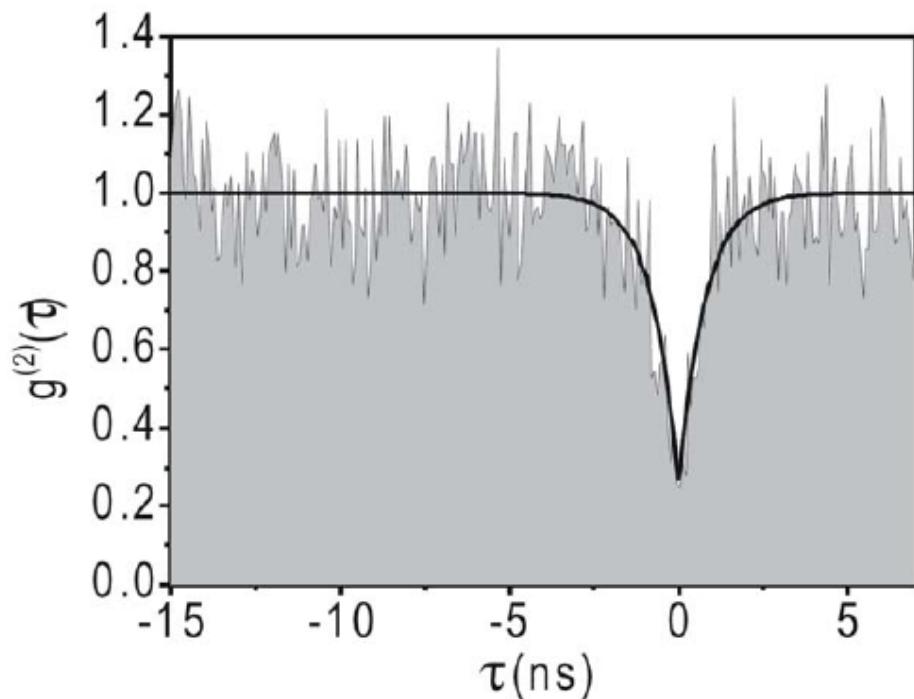
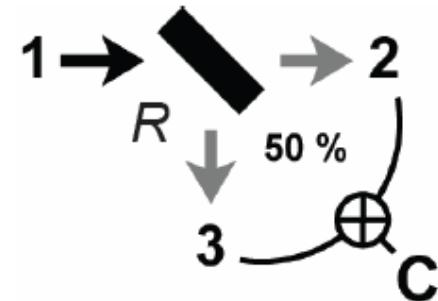


Figure from Kapon-group, EPFL Lausanne

# Classical derivation for a stream of hard core particles

$$C \sim \langle \Delta n_T \Delta n_T \rangle \sim [\langle \Delta n^2 \rangle - \langle n \rangle]$$

*fluctuations*      *particle nature*



*stream of hard core particles*

*n particles per unit time arriving at the beam splitter:*

on average:  $\langle n \rangle = \sum_{n=0}^{\infty} \rho_n n$       *probability distribution for n particles*       $\langle n_T \rangle = T \langle n \rangle$        $\langle n_R \rangle = R \langle n \rangle$

*probability that i particles are transmitted, while n-i are reflected:*

$$T^i (1 - T)^{n-i}$$

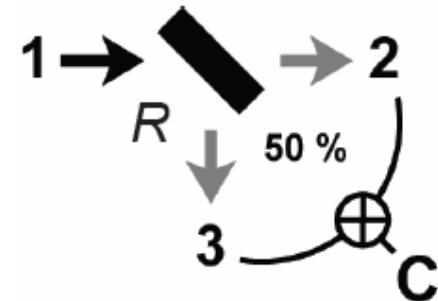
*choose i out of n indistinguishable particles:*

$$n!/[i!(n - i)!] \equiv \binom{n}{i}$$

# Classical derivation for a stream of hard core particles

$$C \sim \langle \Delta n_T \Delta n_T \rangle \sim [\langle \Delta n^2 \rangle - \langle n \rangle]$$

$\uparrow$  *fluctuations*       $\uparrow$  *particle nature*



→ *average of the product of number of transmitted and reflected particles*

$$\begin{aligned}
 \langle n_t n_r \rangle &= \sum_n \rho_n \left\{ \sum_{i=0}^n \binom{n}{i} T^i (T-t)^{n-i} \cdot i(n-i) \right\} \\
 &= -T \langle n \rangle - T^2 \langle n(n-1) \rangle + \underbrace{\sum_n \rho_n n \left\{ \sum_{i=0}^n \binom{n}{i} T^i (1-T)^{n-i} \cdot i \right\}}_{= nT} \\
 &= -T \langle n \rangle - T^2 \langle n(n-1) \rangle + T \langle n^2 \rangle \\
 &= T(1-T) \{ \langle n^2 \rangle - \langle n \rangle \}
 \end{aligned}$$