1/f Frequency Noise Effects on Self-Heterodyne Linewidth Measurements

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***Abstract*—The effects of 1/f frequency noise on self-heterodyne detection are described and the results are applied to the problem of laser diode linewidth measurement. Laser diode linewidths determined by self-heterodyne methods are not adequate predictors of coherent communication system performance because these measurements often include significant broadening due to 1/f frequency noise. In this paper the self-heterodyne autocorrelation function and power spectrum are evaluated for both the white and the 1/f components of the frequency noise. From numerical analysis, the power spectrum resulting from the 1/f frequency noise is shown to be approximately Gaussian and an empirical expression is given for its linewidth. These results are applied to the problem of self-heterodyne linewidth measurements for coherent optical communications and the amount of broadening due to 1/f frequency noise is predicted. Two methods are then provided for estimating the portion of the measured self-heterodyne linewidth due to the white and 1/f components of the frequency noise spectrum.**

I. Introduction

LASER diode linewidths measured using delayed self-heterodyne detection often include significant broadening due to $1/f$ frequency noise. This complicates the evaluation of lasers for use in coherent communication systems because the linewidths determined by self-heterodyne methods are not adequate predictors of performance. Coherent system performance is strongly dependent on the Lorentzian component of the laser lineshape resulting from white frequency noise and is much less dependent on the component resulting from $1/f$ frequency noise. The performance of high data rate coherent communication systems is limited mostly by the white component of the frequency noise because the phase or frequency fluctuations resulting from $1/f$ frequency noise are very small over a symbol interval [1]. The $1/f$ frequency noise may impose a requirement for frequency tracking to keep the center frequency from drifting out of the receiver front end bandwidth; however, this requirement is met fairly easily with automatic frequency tracking circuits [2]. Homodyne receivers using phase-locked loops are somewhat more sensitive to $1/f$ noise; however, for typical levels of $1/f$ and white frequency noise the loop bandwidth requirements are still driven solely by the Lorentzian component of the linewidth. (This is true for $k < x\Delta\nu^2$, where $0.5 < x < 16$ depending on the type of phase-locked loop and the allowable level of phase-error variance [3]–[5]).

Given that the self-heterodyne linewidth measurement does not directly provide estimates of the frequency noise components which are needed, one might decide to use a direct measurement of frequency noise such as in [1]. The direct frequency noise measurement uses a Fabry–Perot resonator as a frequency discriminator. The disadvantages of this measurement are that it is difficult to calibrate and the laser is required to operate at fixed frequencies on the Fabry–Perot response curve. The self-heterodyne measurement is easier to set up, it is inherently calibrated, the laser may operate at any frequency and it is already in widespread use. The results given in this paper allow the self-heterodyne measurement to provide estimates of the frequency noise components without the difficulties associated with the frequency noise measurements.

Most treatments of delayed self-heterodyne detection have assumed laser phase noise solely of quantum origin leading to a Lorentzian lineshape [6]–[9]. Because the $1/f$ and white frequency noise components impact communication system performance differently, a good understanding of the effects of $1/f$ frequency noise on self-heterodyne linewidth measurements is needed so that this characterization technique can provide the information needed for accurate predictions of system performance. Recently, Kikuchi and Okoshi have noted the broadening effect of $1/f$ frequency noise on the laser diode lineshape as measured by the self-heterodyne technique [1], [10]–[12]. This paper provides a more complete analysis of the effects of $1/f$ frequency noise on self-heterodyne detection by developing the autocorrelation function and power spectrum of the photocurrent in terms of an arbitrary frequency noise spectrum and then evaluating them for both the white and the $1/f$ components of the frequency noise. These results are then applied to the problem of self-heterodyne linewidth measurement for high data rate heterodyne communications. Long delays are required for these measurements to provide adequate resolution; however, long delays result in considerable broadening of the self-heterodyne lineshape due to 1/frequency noise. To overcome this, two methods are presented for estimating the white and $1/f$ components of the frequency noise from the measured lineshape.

II. Frequency Noise Model

The starting point of the analysis presented here is based on previous work which treated only white frequency noise, so some details are omitted [6]. The optical field from a single longitudinal mode semiconductor laser can be modeled as a quasimonochromatic field with random phase fluctuations leading to broadening of the spectral line.

$$E(t) = E_o \exp j\left[\omega_o t + \phi(t)\right]. \tag{1}$$

The analysis presented here assumes the amplitude of the optical field is constant. The phase-noise spectrum S_ϕ is the power spectrum of the phase fluctuations and the frequency noise spectrum $S_{\dot\phi}$ is the power spectrum of the frequency fluctuations. The mean square phase fluctuation

$$\langle \Delta\phi^2(t)\rangle = \left\langle \left(\phi(t+\tau) - \phi(t)\right)^2\right\rangle \tag{2}$$

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is related to the frequency noise spectrum by [13]

$$\langle \Delta\phi^2(t) \rangle = \frac{2}{\pi} \int_{-\infty}^{\infty} \sin^2\left(\frac{\omega\tau}{2}\right) S_{\dot\phi}(\omega) \frac{d\omega}{\omega^2} \qquad (3)$$

where $\langle\ \rangle$ denotes a time average. The two-sided frequency noise spectrum can be modeled at frequencies below the relaxation oscillation frequency, as power-dependent white noise and power-independent 1/f noise

$$S_{\dot\phi}(\omega) = S_o + \frac{k}{|\omega|}. \qquad (4)$$

Many treatments include only the white noise even though a wide variety of semiconductor lasers have been shown to exhibit substantial levels of 1/f noise in addition to the white noise [14]–[20]. For all equations in this paper S_o and k are expressed in units of $(\mathrm{rd/s})^2/\mathrm{Hz}$ and $(\mathrm{rd/s})^3/\mathrm{Hz}$, respectively. S_o and k are usually measured single sided in units of $\mathrm{Hz}^2/\mathrm{Hz}$ and $\mathrm{Hz}^3/\mathrm{Hz}$ so values given in this paper are in these units. (To convert from measured values to the units used in the equations, the conversion factors are $(2\pi)^2/2$ and $(2\pi)^3/2$ for S_o and k, respectively.)

The actual frequency noise spectrum also has a resonance peak at the relaxation oscillation frequency and drops off above that frequency. The resonance frequency is typically well above 2 GHz so from (3) it is clear that neglecting these features only affects $\langle \Delta\phi^2(t) \rangle$ for τ in the vicinity of the reciprocal of the relaxation oscillation frequency and smaller. The effects of the relaxation oscillation on the frequency noise spectrum may be important for systems where the symbol rate approaches the relaxation oscillation frequency; however, these effects are neglected in this paper.

The component of the phase jitter, $\Delta\phi(\tau)$, due to the white frequency noise is commonly assumed to be a zero mean, stationary, Gaussian random process. The component of the phase jitter due to the 1/f frequency noise is also stationary and exhibits Gaussian statistics even though the phase and frequency noise, $\phi(\tau)$ and $\dot\phi(\tau)$, due to the 1/f frequency noise are not stationary [20]. Under these conditions

$$\left\langle \exp\left[\pm j\Delta\phi(t,\tau)\right]\right\rangle = \exp\left[-\tfrac{1}{2}\left\langle \Delta\phi^2(\tau)\right\rangle\right]. \qquad (5)$$

The laser field spectrum is found by Fourier transforming the field correlation function, $\langle E^*(t)E(t+\tau)\rangle$ where * indicates the complex conjugate. The resulting spectrum is the convolution of the Lorentzian spectrum associated with the white frequency noise and the approximately Gaussian spectrum arising from the 1/f noise. The result of convolving a Lorentzian spectrum and a Gaussian spectrum is known as a Voigt spectrum. The Voigt profile is used extensively in investigations of radiative transfer in the upper atmosphere. In this area of study the Voigt profile results from independent Lorentzian and Doppler broadening (Doppler broadening leads to a Gaussian lineshape). The Voigt function is not available in an analytic form; however, many useful approximations are available [21]–[25]. The algorithm provided in [24] uses a 12 term rational approximation to calculate the Voigt profile for any combination of Lorentzian and Gaussian linewidths. The relative error of this algorithm is less than $2 * 10^{-6}$. The accuracy of using a Voigt profile to model the laser diode lineshape is limited by the approximations used in modeling the spectrum due to 1/f frequency noise with a Gaussian.

The linewidth of the Lorentzian part of the lineshape taken by itself is $S_o/2\pi$ Hz FWHM (full width half maximum) and

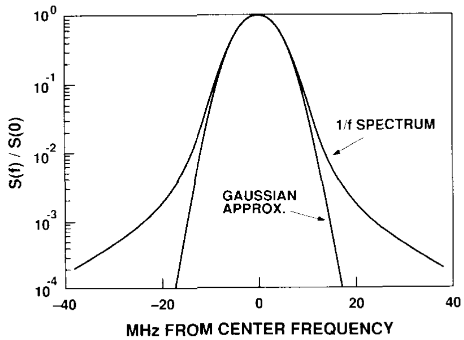


Fig. 1. Field spectrum due to 1/f frequency noise only compared to Gaussian approximation. $k = 2 * 10^{12}\ \mathrm{Hz}^2$ and the observation time is 1 ms.

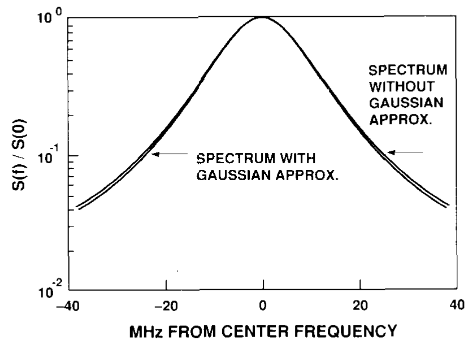


Fig. 2. Field spectrum for a laser with a 7-MHz natural linewidth and $k = 2 * 10^{12}\ \mathrm{Hz}^2$. The lower curve is the result obtained using a Gaussian approximation for the field spectrum due to 1/f frequency noise.

the linewidth of the approximately Gaussian part is roughly

$$\sqrt{4\ \ln(2)(k/2\pi^3)[1 + \ln(T_{\mathrm{obs}}\sqrt{k/2\pi^3})]}$$

Hz FWHM when observed for T_{obs} seconds and $T_{\mathrm{obs}}\sqrt{k/2\pi^3} >> 1$ [20] (k is the level of the 1/f noise as in (4)). Fig. 1 shows the lineshape due to the 1/f part of the frequency noise compared to the Gaussian approximation. The Gaussian approximation is only good near line center, however; about 90% of the power in the lineshape is contained within the region where the error in the Gaussian approximation is less than 10%. Hence, the lineshape which results from convolving the Gaussian approximation with a Lorentzian differs very little from the lineshape calculated without the approximation. The lower curve in Fig. 2 is the lineshape due to combined white and 1/f frequency noise components using the Gaussian approximation and the upper curve is the same field spectrum computed without the approximation. For this case with comparable linewidths due to 1/f and white frequency noise, the error resulting from the Gaussian approximation is less than 0.2 dB. For another case where the Lorentzian linewidth was about 1/10 the 1/f linewidth, the error was found to be as large as 2 dB. For typical lasers where the linewidth due to 1/f is less than or equal to the natural Lorentzian linewidth, the Gaussian approximation is a good approach to computing the complete lineshape. In unusual cases where the linewidth due to 1/f noise is much larger than the natural linewidth, the Gaussian approximation for the lineshape due to 1/f frequency noise is not ac-

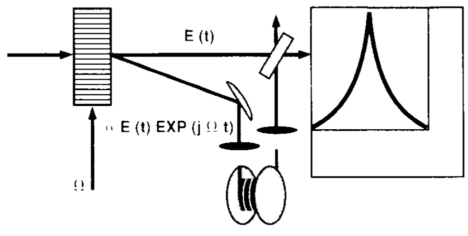


Fig. 3. Delayed self-heterodyne linewidth measurement setup.

curate and should be avoided. Thus, for common levels of $1/f$ noise, the complete laser diode lineshape is well approximated by a Voigt profile.

III. Detected Field Model and Photocurrent Autocorrelation Function

Following the development of [6], the detected field for the delayed self-heterodyne linewidth measurement setup as shown in Fig. 3 is the sum of a laser field and a time delayed and frequency shifted image of itself

$$E_T = E(t) + \alpha E(t + \tau_o) \exp j\Omega t \tag{6}$$

and α is a real factor which accounts for the amplitude ratio between the two fields. The time delay between the two fields is τ_0 and Ω is the mean frequency difference between the two fields. Assuming stationary fields, the autocorrelation of the photocurrent depends only on the intensity correlation function of the detected total field $G_{ET}^{(2)}(\tau)$

$$R_I(\tau) = e\eta G_{ET}^{(2)}(0)\delta(t) + \eta^2 G_{ET}^{(2)}(\tau) \tag{7}$$

where e is the electronic charge, η is the detector sensitivity, $\delta(t)$ is the Dirac delta function, and the optical intensity correlation function is

$$G_{ET}^{(2)}(\tau) = \langle E_T(t)E_T^*(t)E_T(t+\tau)E_T^*(t+\tau)\rangle. \tag{8}$$

The first term in (7) is the shot noise associated with the dc component of the photocurrent.

IV. Autocorrelation Function and Spectrum for White Frequency Noise

The autocorrelation function is found by substituting (6) into (8). This results in 16 terms, 10 of which average out to zero. Using (5) the result as given in [6] is

$$G_{ET}^{(2)}(\tau) = E_o^4\left[(1+\alpha^2)^2 + 2\alpha^2 \cos \Omega\tau \exp(A)\right]$$

where

$$A = -\langle \Delta\phi^2(\tau_o)\rangle - \langle \Delta\phi^2(\tau)\rangle + \tfrac{1}{2}\langle \Delta\phi^2(\tau - \tau_o)\rangle + \tfrac{1}{2}\langle \Delta\phi^2(\tau + \tau_o)\rangle. \tag{9}$$

By substituting (3) into (9) and combining the integrals, the intensity correlation function can be written in terms of the frequency noise spectrum

$$G_{ET}^{(2)}(\tau) = E_o^4\left[(1+\alpha^2)^2 + 2\alpha^2 \cos\Omega\tau \cdot \exp\left[-\frac{4}{\pi}\int_{-\infty}^{\infty} \sin^2\left(\frac{\omega\tau}{2}\right) \cdot \sin^2\left(\frac{\omega\tau_o}{2}\right)\frac{S_{\dot\phi}(\omega)}{\omega^2}\,d\omega\right]\right]. \tag{10}$$

This agrees with Kikuchi [12]. It is useful to observe at this point that the second term in this autocorrelation is similar to the field correlation for a single laser with the addition of a sine squared term which acts as a filter on the frequency noise spectrum. For small delay times this sine squared term effectively filters out most of the $1/f$ component of the frequency noise spectrum.

For the white part of the frequency noise spectrum, the integral in (10) is easily evaluated and yields

$$G_{ET}^{(2)}(\tau) = E_o^4\left[(1+\alpha^2)^2 + 2\alpha^2\cos\Omega\tau \cdot \exp\begin{Bmatrix} -S_o|\tau| & \text{for } |\tau| < \tau_o \\ -S_o\tau_o & \text{for } |\tau| > \tau_o \end{Bmatrix}\right] \quad \text{for } S_{\dot\phi} = S_o. \tag{11}$$

Using the Wiener–Khintchine theorem, the power spectral density resulting from the white component is the Fourier transform of (11), [6], [7]

$$S(\omega) = E_o^4\Bigg\{(1+\alpha^2)^2\delta(\omega) + 2\alpha^2\exp(-S_o\tau_o)\delta(\omega - \Omega) + 2\alpha^2 \cdot \frac{2S_o}{(S_o)^2 + (\omega-\Omega)^2} \cdot \left[1 - \exp(-S_o\tau_o)\left(\cos[(\omega-\Omega)\tau_o] + \frac{S_o}{\omega - \Omega}\sin[(\omega-\Omega)\tau_o]\right)\right]\Bigg\}. \tag{12}$$

In the limit of large delay times the component of the spectrum due to the white frequency noise becomes exactly Lorentzian with width equal to twice the individual laser linewidth. For delay times which are comparable to or shorter than the coherence time, the quasi-Lorentzian part is broadened and scalloped and power is shifted into the delta function at the modulation frequency. (Illustrations of this behavior are included in [6] and [7].) If the delay is much shorter than the coherence time, the spectrum consists solely of delta functions at the modulation frequency and at dc.

V. Autocorrelation Function and Spectrum for $1/f$ Frequency Noise

For the $1/f$ part of the frequency noise spectrum, evaluation of the integral in (10) requires use of the identity

$$\frac{\sin \pi a x}{\pi x} = \int_{-(a/2)}^{(a/2)} \exp(\pm j2\pi x t)\,dt \tag{13}$$

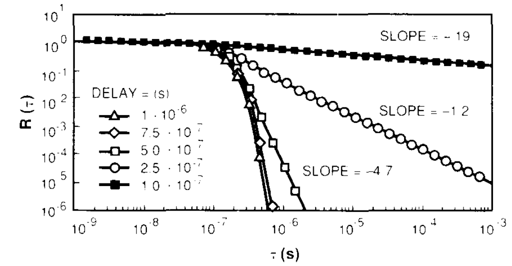


Fig. 4. Autocorrelation function for 1/f noise only. $k = 3 * 10^{12}$ Hz^2.

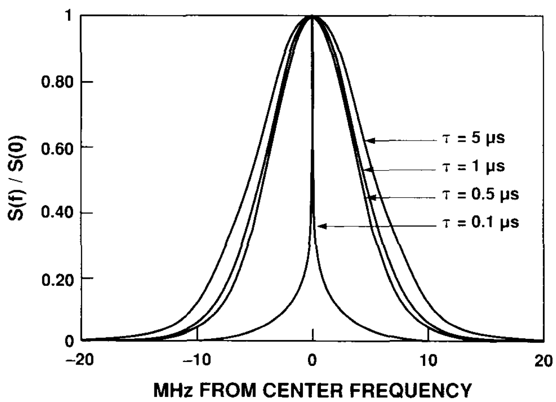


Fig. 5. Normalized self-heterodyne lineshape due to 1/f alone for $k = 2 * 10^{12}$ and delay times = 0.1, 0.5, 1, and 5 μs.

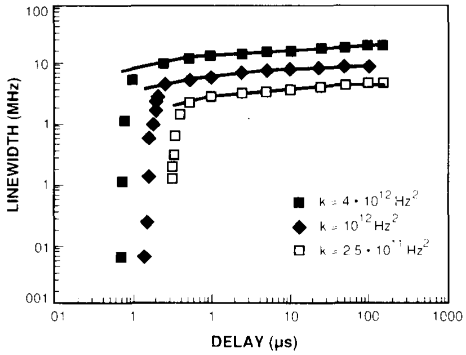


Fig. 6. Self-heterodyne linewidth due to 1/f noise only.

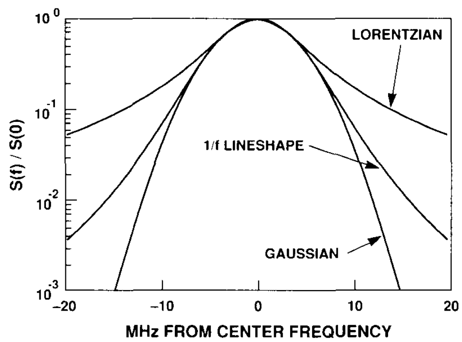


Fig. 7. Normalized self-heterodyne lineshape due to 1/f alone compared to Lorentzian and Gaussian curves with the same 3 dB width (shown for comparison) ($k = 2 * 10^{12}$ Hz^2, delay = 1 μs).

yielding [26]

$$G^{(2)}_{ET}(\tau) = E_o^4\Big\{(1+\alpha^2)^2 + 2\alpha^2\cos\Omega\tau \cdot \Big[(\tau+\tau_o)^{\{-k(\tau+\tau_o)^2/2\pi\}}\left(|\tau-\tau_o|\right)^{\{-k(\tau-\tau_o)^2/2\pi\}} \cdot \tau^{(k\tau^2/\pi)}\tau_o^{(k\tau_o^2/\pi)}\Big]\Big\} \quad \text{for } S_{\dot\phi} = \frac{k}{|\omega|}. \tag{14}$$

Fig. 4 is a plot of the bracketed expression in (14) for $k = 3 * 10^{12}$ Hz^2, showing the dependence of the autocorrelation, and thus the frequency spectrum, on the delay time. For times much greater than the delay time the slope of the log of the autocorrelation as plotted in Fig. 4 is $-k\tau_o^2/\pi$.

The power spectrum for the 1/f component has been obtained by fast Fourier transforming the autocorrelation function. The result is shown in Fig. 5 for several delay times and $k = 2 * 10^{12}$ Hz^2. (Note that this result neglects the dc component and the shot noise.)

The linewidth of the self-heterodyne spectrum due to 1/f noise alone is shown in Fig. 6 as a function of the delay for several reasonable values of k.

For the extreme case of short delays such that $k\tau_o^2/\pi \ll 1$, the autocorrelation function is composed of two parts; a narrow spike and a dc component. The narrow spike produces a broad base in the power spectrum. The dc component corresponds to a narrow spike in the power spectrum at the modulation frequency and as $k\tau_o^2/\pi$ becomes much less than 1, the power is shifted completely into the spike (which becomes a delta function in the limit of small $k\tau_o^2/\pi$).

The contribution of the 1/f frequency noise to the laser field spectrum is approximately Gaussian, so it was expected that the contribution to the self-heterodyne spectrum would also be approximately Gaussian. Fig. 7 shows the 1/f lineshape and for comparison, a Gaussian and Lorentzian with the same width at half maximum. The 1/f lineshape is very close to the Gaussian near the center of the line, but is much stronger than the Gaussian farther away from the line center. This lineshape does not drop off as fast as a Gaussian, but it has much less power in the wings than a Lorentzian. The Gaussian component of the lineshape has nearly the same lineshape and $\sqrt{2}$ times the width of the field spectrum of the individual laser for cases where $k\tau_o^2/\pi \gg 1$.

Using (14) a Gaussian approximation for the autocorrelation was derived empirically. For $k\tau_o^2/\pi \gg 1$

$$G^{(2)}_{ET}(\tau) \approx E_o^4\Big\{(1+\alpha^2)^2 + 2\alpha^2\cos\Omega\tau \cdot \exp\left[-\tau^2\frac{k}{2\pi}\left(4.3 + \ln\frac{4.3\tau_o^{2.1}k}{\pi}\right)\right]\Big\} \quad \text{for } S_{\dot\phi} = \frac{k}{|\omega|}. \tag{15}$$

The power spectral density resulting from this approximation is also Gaussian. The linewidth predicted from this approximation is

$$G_{lw} = \frac{1}{\pi}\sqrt{\frac{2k\ln 2}{\pi}\left(4.3 + \ln\frac{4.3k\tau_o^{2.1}}{\pi}\right)}\ \text{Hz FWHM}. \qquad (16)$$

This approximation is plotted in Fig. 6 as a solid line. Good accuracy is obtained for large delays. For fixed delay τ_o the Gaussian linewidth is roughly proportional to $\sqrt{k}$. This $\sqrt{k}$ dependence leads to the $\sqrt{2}$ relationship between the Gaussian component of the field spectrum and the Gaussian component of the self-heterodyne lineshape.

Note that the linewidth due to the $1/f$ component is not independent of the delay even for large delays. This is due to the very low frequencies present in the $1/f$ noise. The analysis presented here is general in that it is valid for any delay. Results of any analysis which assumes mutual incoherence of the two optical fields (such as [27]), should be used with caution because of course the strong low frequency components of the $1/f$ noise are correlated even for very long delays.

VI. Autocorrelation Function and Spectrum for White and $1/f$ Frequency Noise Combined

For the combined white and $1/f$ frequency noise actually present in most semiconductor lasers the autocorrelation function is simply a combination of the results already given.

$$G_{ET}^{(2)}(\tau) = E_o^4 = \left\{(1+\alpha^2)^2 + 2\alpha\cos(\Omega\tau)LG\right\}$$

where

$$L = \exp\begin{Bmatrix} -S_o|\tau| & \text{for } |\tau| \le \tau_o \\ -S_o\tau_o & \text{for } |\tau| \ge \tau_o \end{Bmatrix}$$

and

$$G = (\tau+\tau_o)^{\{-k(\tau+\tau_o)^2/2\pi\}}\left(|\tau-\tau_o|\right)^{\{-k(\tau-\tau_o)^2/2\pi\}} \cdot \tau^{(k\tau^2/\pi)}\tau_o^{k\tau_o^2/\pi} \quad \text{for } S_{\dot\phi} = S_o + \frac{k}{|\omega|}. \qquad (17)$$

The power spectral density for the combined white and $1/f$ frequency noise can be evaluated by finding the FFT of (17). Fig. 8 shows plots of the linewidths which would be observed using self-heterodyne detection for lasers with natural linewidths of 10 and 2 MHz. The component of the linewidth due to the $1/f$ frequency noise by itself was shown in Fig. 6 for the same delays and k values used here.

The linewidth due to combined white and $1/f$ frequency noise can be approximated for long delay times using the Gaussian approximation given earlier. The relationship between the complete Voigt lineshape and the Gaussian and Lorentzian lineshapes is approximated by the expression

$$\alpha_V = \tfrac{1}{2}\left(1.0692\alpha_L + \sqrt{0.866639\alpha_L^2 + 4\alpha_G^2}\right) \qquad (18)$$

where α_V, α_G, and α_L are the Voigt, Gaussian, and Lorentzian linewidths, respectively. If the $1/f$ lineshape were exactly Gaussian, the error in (18) would be less than about 0.01 percent [28]. The linewidths predicted using these approximations are plotted as a solid line in Fig. 8. The error seen in this approximation is due to the slightly non-Gaussian lineshape resulting from the $1/f$ component of the frequency noise.

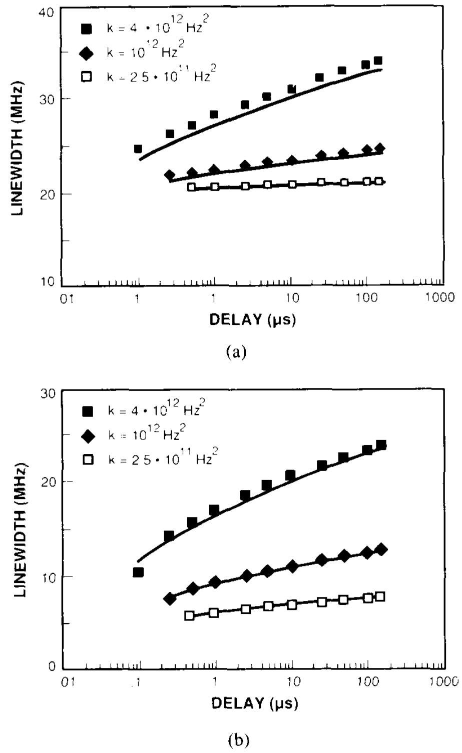


Fig. 8. Self-heterodyne linewidth versus delay. (a) $S_o/2\pi$ = 10 MHz. (b) $S_o/2\pi$ = 2 MHz.

VII. Effects of $1/f$ on Linewidth Measurements

The broadening of the delayed self-heterodyne lineshape resulting from $1/f$ frequency noise has particular significance in the context of linewidth measurements for high data rate coherent communication systems. Since the performance of these systems is largely unaffected by $1/f$ noise, it is desirable to know the natural linewidth which would result from the white component of the frequency noise alone. In general, the linewidths estimated from self-heterodyne measurements are simply one-half of the full width at half-maximum of the self-heterodyne lineshape. From Fig. 8, it is obvious that this linewidth is often substantially larger than the natural Lorentzian component of the linewidth.

As first observed by Kikuchi and Okoshi, the broadening of the self-heterodyne linewidth by $1/f$ frequency noise can lead to a residual linewidth at high powers [1], [10]–[12]. The natural linewidth as predicted by the Schawlow–Townes formula with the broadening factors introduced by Henry [29], varies inversely with laser output power in the main longitudinal mode. The $1/f$ frequency noise is normally constant with power [20]. $1/f$ noise thus produces a residual linewidth at the high power limit and also causes a finite intercept when self-heterodyne linewidths are plotted against inverse power. For Fig. 9 the linewidth which would be measured by the self-heterodyne technique with a 5-μs delay is plotted for a constant level of $1/f$ noise ($k = 10^{12}$ Hz^2) and a varying natural linewidth. This simulates a plot of linewidth versus inverse power. For small natural linewidths a residual is observed and a line fitted to all of the data is seen to intercept the dependent axis at 3.0 MHz.

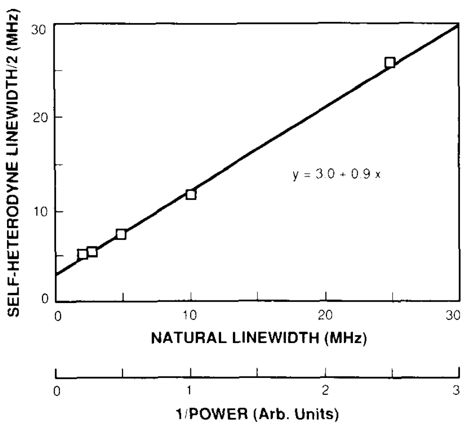


Fig. 9. Self-heterodyne linewidth/2 versus natural linewidth showing finite intercept due to $1/f$ noise ($k = 10^{12}$ Hz^2, delay = 5 μs).

The residual linewidth in the limit of high power is simply the $1/f$ self-heterodyne linewidth divided by 2. For the case in Fig. 9 the residual linewidth would be 3.9 MHz.

This procedure was repeated for a range of $1/f$ noise levels and several different delay times and the intercepts which resulted are plotted in Fig. 10. The line fits were produced using a range of five Lorentzian linewidths from 2 to 25 MHz. The intercepts are dependent on the range of points fitted and so should be seen as approximate values only.

In some circumstances direct measurement of the natural linewidth may be possible by tailoring the length of the delay used for the self-heterodyne measurement. For instance, if it were known from independent measurements that $k < 2.5 * 10^{11}$ Hz^2, then choosing a delay of 0.3 μs would make the $1/f$ contribution to the measured linewidth negligible as can be seen from Fig. 6. In this case the smallest Lorentzian linewidth which could be measured directly would be about 5 MHz [7]. This approach of tailoring the delay to reduce effects of $1/f$ noise has limited value because many semiconductor lasers have $k > 10^{13}$ Hz^2 leading to selection of a delay shorter than 0.15 μs. This delay would only allow direct measurement of Lorentzian linewidths greater than about 10 MHz.

The broadening effect of the $1/f$ frequency noise is most pronounced near the center of the self-heterodyne lineshape. If signal and noise levels permit, a more accurate estimate of the Lorentzian part of the linewidth of the laser diode under test can be obtained from the width 10 or 20 dB down from the maximum. Fig. 11 shows the linewidths determined from the width 20 dB down are much closer to the Lorentzian linewidth. The linewidths were determined from the 3, 10, and 20 dB widths of numerically generated lineshapes simply assuming a Lorentzian lineshape. (The 3-dB linewidth is the self-heterodyne linewidth divided by 2, the 10-dB linewidth is the self-heterodyne linewidth 10 dB down divided by 6, the 20-dB linewidth is the self-heterodyne linewidth 20 dB down divided by $2\sqrt{99}$.)

Good estimates of the Lorentzian and Gaussian components can be obtained by fitting a Voigt profile to the measured self-heterodyne lineshape. As noted earlier in developing the frequency noise model, efficient and accurate algorithms are available for estimating the Voigt lineshape for any combination of Lorentzian and Gaussian contributions [21]–[25]. Some of the approximations for the Voigt profile also provide derivatives

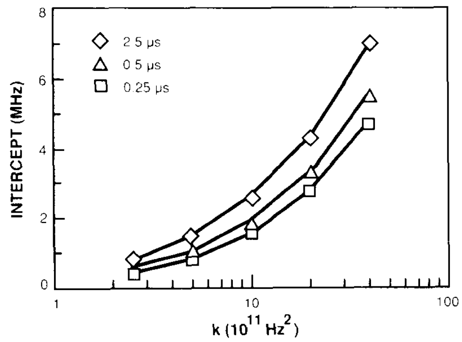


Fig. 10. Infinite power intercept due to $1/f$ frequency noise

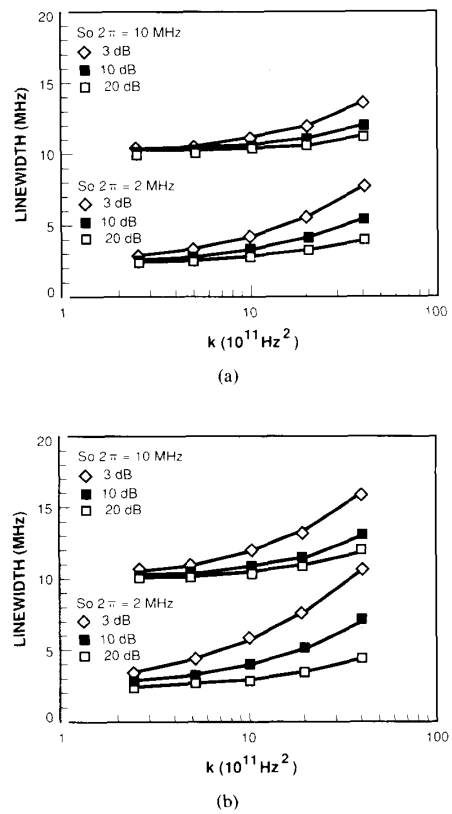
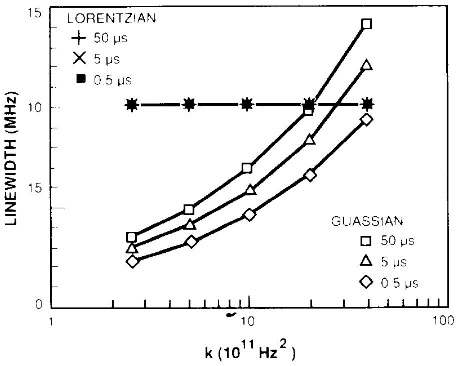


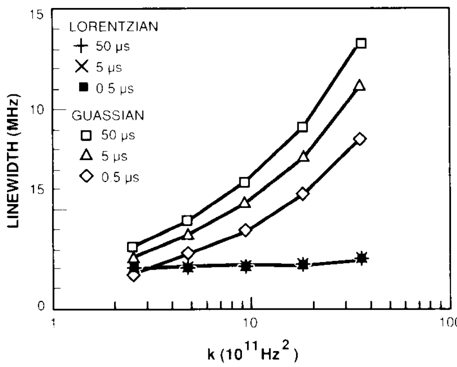
Fig. 11. Linewidth versus k determined from 3, 10, 20 dB widths of the self-heterodyne lineshape for 2- and 10-MHz Lorentzian linewidths, (a) delay = 0.5 μs, (b) delay = 50 μs.

with respect to all of the parameters allowing application of nonlinear least squares fitting procedures. These fitting procedures use the Voigt function and derivatives for each point in the lineshape and iterate until a minimum error is obtained.

An alternative method of using the Voigt approximations to estimate the Gaussian and Lorentzian parts of a measured self-heterodyne lineshape is given here. This approach requires fewer computations than the nonlinear least squares fitting procedures. The Lorentzian component is initially estimated from the 20-dB linewidth by simply assuming a Lorentzian lineshape. Then the Gaussian component is estimated by using the



(a)



(b)

Fig. 12. Lorentzian and Gaussian components estimated from Voigt fitting of self-heterodyne lineshape for delays of 0.5, 5, and 50 μs. (a) $S_o/2\pi$ = 10 MHz. (b) $S_o/2\pi$ = 2 MHz.

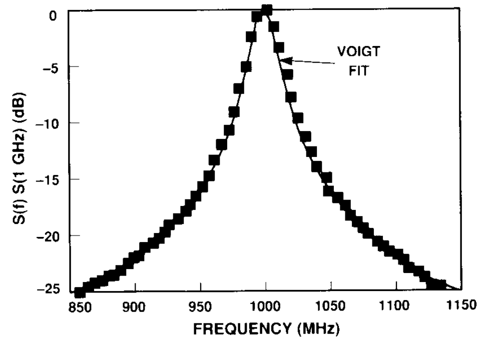
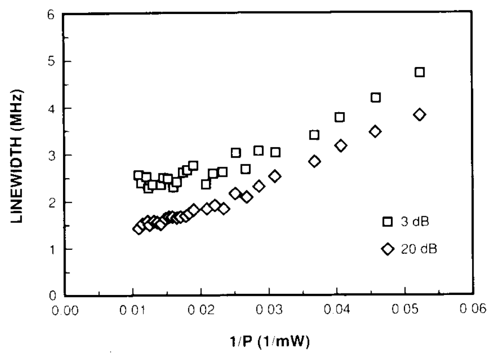
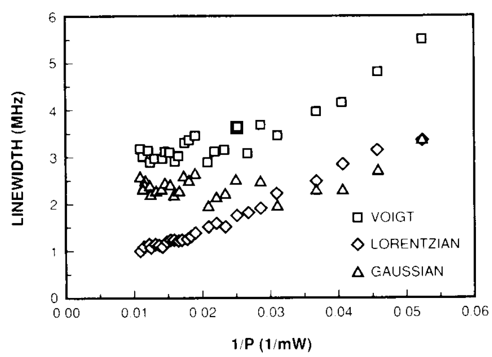


Fig. 13. Measured self-heterodyne lineshape and Voigt fit; 3-dB linewidth = 10.2 MHz, 20-dB linewidth = 8.0 MHz, Voigt linewidth = 12.1 MHz, Lorentzian linewidth = 7.2 MHz, Gaussian linewidth = 7.6 MHz.



(a)



(b)

Fig. 14. (a) Measured 3- and 20-dB linewidths versus 1/power. (b) Corresponding Voigt, Lorentzian and Gaussian linewidths versus 1/power, determined using the Voigt fitting procedure.

3-dB linewidth and (18). Using this estimate of the Gaussian component, a new estimate of the Lorentzian component can be found by using a Voigt approximation and iterating to find the Lorentzian value required to produce a 20 dB width equal to the measured width. The new Lorentzian value is then used to refine the estimate of the Gaussian component and these steps are repeated until the estimates converge. Convergence is fairly rapid because the 20-dB linewidth is dominated more by the Lorentzian contribution while the 3-dB linewidth is strongly affected by the Gaussian component. The Voigt approximation used here is the one found in [24].

To test its accuracy over a wide range of inputs, this Voigt lineshape fitting procedure was applied to self-heterodyne lineshapes computed numerically. Ideally, the fitting procedure would return exactly the magnitude of the Lorentzian component used to generate the lineshapes and the Gaussian component should agree with the linewidth predicted for 1/f alone. The results are given in Fig. 12. The estimates of the Lorentzian component are very close to the correct value with a small error appearing for the 2-MHz where the Gaussian component begins to be much larger than the Lorentzian component. The estimates of the Gaussian component were mostly within 2% of the predictions for the 2-MHz Lorentzian case (the maximum error of 14% occurred for τ_o = 0.5 μs and $k = 2.5 * 10^{11}$) and mostly within 10% for the 10-MHz Lorentzian case (the maximum error of 46% occurred for τ_o = 0.5 μs and $k = 2.5 * 10^{11}$).

This Voigt fitting procedure was also applied to actual self-heterodyne lineshapes. The experimental setup used a 500-m fiber delay line providing 2.5 μs of delay. Fig. 13 shows a measured lineshape and the Voigt lineshape resulting from the fit. The laser for this test was a 30-mW Hitachi HL8314E.

The results of applying the Voigt fitting procedure to a typical set of linewidth versus power measurements are shown in Fig. 14. The laser used for these tests was a Spectra Diode SDL-5410-C 100 mW laser diode. The data used to produce the fits were the 3- and 20-dB widths. The 3- and 20-dB widths were derived from the lineshape assuming a Lorentzian shape as discussed above. The linewidth components plotted in Fig. 14 are for the individual laser. The Lorentzian linewidth is one-half of the Lorentzian component of the self-heterodyne lineshape and the Gaussian component is $\sqrt{2}/2$ times the Gaussian component of the full self-heterodyne lineshape. The true width of the individual laser line is the Voigt linewidth calculated from the components using (18). Notice that the 3-dB linewidth, which is the commonly used result from this measurement, underestimates the full linewidth of the laser and overestimates the Lorentzian component which is the most important to coherent communications performance. It is important to note again that the Gaussian linewidth measured with the self-heterodyne setup is dependent on the length of the delay so any Gaussian linewidths quoted for self-heterodyne measurements must be accompanied by the value of the delay to be meaningful.

VIII. Conclusions

The autocorrelation function and power spectrum resulting from delayed self-heterodyne detection were developed in terms of an arbitrary frequency noise spectrum. These expressions were then evaluated for both the white and $1/f$ components of the laser frequency noise spectrum. The autocorrelation function for the $1/f$ frequency noise was given and numerical results were presented for the associated power spectrum. This power spectrum, due to $1/f$ frequency noise alone, was approximately Gaussian and an empirically derived expression was given for its width. Numerical results were also provided for the power spectrum resulting from the combined effect of $1/f$ and white frequency noise.

High data rate heterodyne communication system performance is limited by the white part of the frequency noise without much effect from the $1/f$ noise. For this reason the contribution of the $1/f$ noise to the self-heterodyne linewidth measurement is undesirable. The $1/f$ frequency noise leads to a residual linewidth at a high power and a finite intercept when linewidth measurements are plotted versus inverse power. This intercept could range from 0.5 to 7 MHz for reasonable conditions. For certain limited circumstances, it was shown that a careful choice of the self-heterodyne delay allows measurement of the natural Lorentzian lineshape without broadening due to $1/f$ noise.

For more general circumstances, two methods of estimating the Lorentzian component from the self-heterodyne lineshape were presented. The first was simply to deduce the laser linewidth from the width of the lineshape 10 or 20 dB down from the peak where the effects of the $1/f$ noise are not as great. This method performs well when the Gaussian contribution is much less than the Lorentzian contribution. The second method used approximations for the Voigt profile and searched for the Lorentzian and Gaussian components needed to match the 3 and 20 dB widths of the Voigt profile to the data. This method extracts the Lorentzian component well even for cases where the Gaussian component is large. More accurate extraction of the Voigt components may be possible by nonlinear least squares fitting procedures and this possibility deserves further investigation. Any Voigt fitting procedures will ultimately be limited by the slightly non-Gaussian character of the $1/f$ contribution to the self-heterodyne lineshape. The lineshape fitting techniques are also dependent on the assumption that the frequency noise spectrum is composed of only $1/f$ and white components.

The results presented show that the linewidth measured for a particular laser can depend substantially upon the delay used for the measurement and the level of the $1/f$ noise. It is suggested that all linewidths reported in the literature from self-heterodyne measurements should be accompanied by information on the length of the delay used and if possible an estimate of the $1/f$ frequency noise level of the laser.

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