Nuclear Physics
Homework 1

Instructions. Solve each problem carefully and explain your procedure.

1. Isospin symmetry
One could naively imagine the three nucleons in the ^3H and ^3He nuclei as being rigid spheres. If one solely attributes the difference in the binding energies of these two nuclei to the electrostatic repulsion of the protons in ^3He, how large must the separation of the protons be? (The maximal energy of the electron in the β^--decay of ^3H is 18.6 keV.)

2. α decay
The α-decay of a ^{238}Pu ($\tau = 127\ yrs$) nuclide into a long lived ^{234}U ($\tau = 3.5 \times 10^5\ yrs$) daughter nucleus releases 5.49 MeV kinetic energy. The heat so produced can be converted into useful electricity by radio-thermal generators (RTG's). The *Voyager 2* space probe, which was launched on the 20.8.1977, flew past four planets, including Saturn which it reached on the 26.8.1981. Saturn's separation from the sun is 9.5 AU; 1 AU = *separation of the earth from the sun.*
a) How much plutonium would an RTG on *Voyager 2* with 5.5% efficiency have to carry so as to deliver at least 395 W electric power when the probe flies past Saturn?
b) How much electric power would then be available at Neptune (24.8.1989; 30.1 AU separation)?
c) To compare: the largest ever "solar paddles" used in space were those of the space laboratory *Skylab* which would have produced 10.5 kW from an area of 730 m^2 if they had not been damaged at launch. What area of solar cells would *Voyager 2* have needed?

3. Radioactivity
Naturally occuring uranium is a mixture of the ^{238}U (99.28%) and ^{235}U (0.72%) isotopes.
a) How old must the material of the solar system be if one assumes that at its creation both isotopes were present in equal quantities? How do you interpret this result? The lifetime of ^{235}U is $\tau = 1.015 \times 10^9\ yrs$. For the lifetime of ^{238}U use the data in Fig. 3.7 (Particle and Nuclei, Bogdan Povh et. al., 6th edition).
b) How much of the ^{238}U has decayed since the formation of the earth's crust $2.5 \times 10^9\ yrs$ ago?
c) How much energy per uranium nucleus is set free in the decay chain $^{238}U \rightarrow^{206} Pb$. A small proportion of ^{238}U spontaneously splits into, e.g., $^{142}_{54}Xe$ and $^{96}_{38}Sr$.

4. Radon activity
After a lecture theatre whose walls, floor and ceiling are made of concrete ($10 \times 10 \times 4\ m^3$) has not been aired for several days, a specific activity A from ^{222}Rn of 100 Bq/m^3 is measured.
a) Calculate the activity of ^{222}Rn as a function of the lifetimes of the parent and daughter nuclei.
b) How high is the concentration of ^{238}U in the concrete if the effective thickness from which the ^{222}Rn decay product can diffuse is 1.5 cm?

5. Mass formula
Isaac Asimov in his novel *The Gods Themselves* describes a universe where the stablest nuclide with $A = 186$ is not $^{186}_{74}W$ but rather $^{186}_{94}Pu$. This is claimed to be a consequence of the ratio of the strengths of the strong and electromagnetic interactions being different to that in our universe. Assume that only the electromagnetic coupling constant α differs and that both the strong interaction and the nucleon masses are unchanged. How large must α be in order that $^{186}_{82}Pb$, $^{186}_{88}Ra$ and $^{186}_{94}Pu$ are stable?

6. α decay again
The binding energy of an α particle is 28.3 MeV. Estimate, using the mass formula (2.8) (Particle and Nuclei, Bogdan Povh et. al., 6th edition), from which mass number A onwards α-decay is energetically allowed for all nuclei.

7. *Cross-section*

Deuterons with an energy $E_{kin} = 5\ MeV$ are perpendicularly incident upon a tritium target, which has a mass occupation density $\mu_t = 0.2\ mg/cm^2$, so as to investigate the reaction ${}^3H(d,n)^4He$.

a) How many neutrons per second pass through a detector with a reception area of $A = 20\ cm^2$ which is at a distance $R = 3\ m$ from the target and an angle $\theta = 30°$ to the deuteron beam direction, if the differential cross-section $d\sigma/d\Omega$ at this angle is 13 mb/sr and the deuteron current applied to the target is $I_d = 2\ \mu A$?

b) How many neutrons per second does the detector receive if the target is tilted so that the same deuteron current now approaches it at 80° instead of 90° ?

8. *Absorption length*

A particle beam is incident upon a thick layer of an absorbing material (with n absorbing particles per unit volume). How large is the absorption length, i.e., the distance over which the intensity of the beam is reduced by a factor of $1/e$ for the following examples?

a) Thermal neutrons ($E \approx 25\ meV$) in cadmium ($\rho = 8.6\ g/cm^3$, $\sigma = 24506\ barn$).

b) $E_\gamma = 2\ MeV$ photons in lead ($\rho = 11.3\ g/cm^3$, $\sigma = 15.7\ barn/atom$).

c) Antineutrinos from a reactor in earth ($\rho = 5\ g/cm^3$, $\sigma \approx 10^{-19}\ barn/electron$; interactions with nuclei may be neglected; $Z/A \approx 0.5$).