Brannen and Ferguson[1] have reported experimental results which they believe to be incompatible with the observation by Hanbury Brown and Twiss[2] of correlation in the fluctuations of two photoelectric currents evoked by coherent beams of light. Brannen and Ferguson suggest that the existence of such a correlation would call for a revision of quantum theory. It is the purpose of this communication to show that the results of the two investigations are not in conflict, the upper limit set by Brannen and Ferguson being in fact vastly greater than the effect to be expected under the conditions of their experiment. Moreover, the Brown–Twiss effect, far from requiring a revision of quantum mechanics, is an instructive illustration of its elementary principles. There is nothing in the argument below that is not implicit in the discussion of Brown and Twiss, but perhaps I may clarify matters by taking a different approach.

Consider first an experiment which is simpler in concept than either of those that have been performed, but which contains the essence of the problem. Let *one* beam of light fall on *one* photomultiplier, and examine the statistical fluctuations in the counting-rate. Let the source be nearly monochromatic and arrange the optics so that, as in the experiments already mentioned, the difference in the length of the two light-paths from a point A in the photocathode to two points B and C in the source remains constant, to within a small fraction of a wave-length, as A is moved over the photocathode surface. (This difference need not be small, nor need the path-lengths themselves remain constant.) Now it will be found, even with the steadiest source possible, that the fluctuations in the counting-rate are slightly greater than one would expect in a random sequence of independent events occurring at the same average rate. There is a tendency for the counts to 'clump'. From the quantum point of view this is not surprising. It is typical of fluctuations in a system of bosons. I shall show presently that this extra fluctuation in the single-channel rate necessarily implies the cross-correlation found by Brown and Twiss. But first I propose to examine its origin and calculate its magnitude.

Let P denote the square of the electric field in the light at the cathode surface in one polarization, averaged over a few cycles. P is substantially constant over the photocathode at any instant, but as time goes on it fluctuates in a manner determined by the spectrum of the disturbance, that is, by the 'line shape'. Supposing that the light contains frequencies around ν_0, we describe the line shape by the normalized spectral density $g(\nu-\nu_0)$. The width of the distribution g, whether it be set by circumstances in the source itself or by a filter, determines the rate at which P fluctuates. For our purpose, the stochastic behaviour of P can be described by the correlation function $\overline{P(t)\,P(t+\tau)}$, which is related in turn to $g(\nu-\nu_0)$ by[3]

$$\overline{P(t)P(t+\tau)} = \overline{P}^2(1+|\rho|^2),$$

$$\text{where } \rho = \int_{-\infty}^{\infty} g(x)\,\exp 2\pi i\tau x\,\mathrm{d}x \qquad (1)$$

For the probability that a photoelectron will be ejected in time $\mathrm{d}t$, we must write $\alpha P\mathrm{d}t$, where α is constant throughout the experiment. It makes no difference whether we think of P as the square of an electric field-strength or as a photon probability density. (In this connexion the experiment of Forrester, Gudmundsen and Johnson[4] on the photoelectric mixing of incoherent light is interesting.) Assuming one polarization only, and one count for every photoelectron, we look at the number of counts n_T in a fixed interval T, and at the fluctuations in n_T over a sequence of such intervals. From the above relations, the following is readily derived:

$$\overline{n^2_T} - \overline{n}^2_T = \overline{n}_T(1 + \alpha\overline{P}\tau_0) \qquad (2)$$

$$\text{where } \tau_0 = \int_{-\infty}^{\infty} |\rho|^2\mathrm{d}\tau$$

and it has been assumed in deriving (2) that $T \gg \tau_0$. Now $\alpha\overline{P}$ is just the average counting-rate and τ_0, a correlation time determined by the light spectrum, is approximately the reciprocal of the spectral bandwidth $\Delta\nu$; in particular, if $\Delta\nu$ is the full width at half intensity of a Lorentzian density function, $\tau_0 = (\pi\Delta\nu)^{-1}$, while if $\Delta\nu$ is the width of a rectangular density function, $\tau_0 = \Delta\nu^{-1}$. We see that the fractional increase in mean-square fluctuation over the 'normal' amount is independent of T, and is about equal to the number of counts expected in an interval $1/\Delta\nu$. This number will ordinarily be very much smaller than one. The result, expressed in this way, does not depend on the counting efficiency.

If one insists on representing photons by wave packets and demands an explanation in those terms of the extra fluctuation, such an explanation can be given. But I shall have to use language which ought, as a rule, to be used warily. Think, then, of a stream of wave packets, each about $c/\Delta\nu$ long, in a random sequence. There is a certain probability that two such trains accidentally overlap. When this occurs they interfere and one may find (to speak rather loosely) four photons, or none, or something in between as a result. It is proper to speak of interference in this situation because the conditions of the experiment are just such as will ensure that these photons are in the same quantum state. To such interference one may ascribe the 'abnormal' density fluctuations in any assemblage of bosons.

Were we to carry out a similar experiment with a beam of electrons, we should, of course, find a slight suppression of the normal fluctuations instead of a slight enhancement; the accidentally overlapping wave trains are precisely the configurations excluded by the Pauli principle. Nor would we be entitled in that case to treat the wave function as a classical field.

Turning now to the split-beam experiment, let n_1 be the number of counts of one photomultiplier in an interval T, and let n_2 be the number of counts in the other in the same interval. As regards the fluctuations in n_1 alone, from interval to interval, we face the situation already analysed, except that we shall now assume both polarizations present. The fluctuations in orthogonal polarizations are independent, and we have, instead of (2),

$$\overline{\Delta n_1^2} = \overline{n_1^2} - \overline{n}_1^2 = \overline{n}_1(1 + \tfrac{1}{2}\overline{n}_1\tau_0/T) \qquad (3)$$

where n_1/T has been written for the average counting-rate in channel 1. A similar relation holds for n_2. Now if we should connect the two photomultiplier outputs together, we would clearly revert to a single-channel experiment with a count $n = n_1 + n_2$. We must then find:

$$\overline{\Delta n^2} = \bar{n}\,(1 + \tfrac{1}{2}\bar{n}\tau_0/T) \quad (4)$$

But $\overline{\Delta n^2} = \overline{(\Delta n_1 + \Delta n_2)^2}$

$$= \bar{n}_1(1+\tfrac{1}{2}\bar{n}_1\tau_0/T) + \bar{n}_2(1+\tfrac{1}{2}\bar{n}_2\tau_0/T) + \overline{2\Delta n_1 \Delta n_2} \quad (5)$$

From (4) and (5) it follows that:

$$\overline{\Delta n_1 \Delta n_2} = \tfrac{1}{2}\bar{n}_1{}^2\tau_0/T \quad (6)$$

This is the positive cross-correlation effect of Brown and Twiss, although they express it in a slightly different way. It is merely another consequence of the 'clumping' of the photons. Note that if we had separated the branches by a polarizing filter, rather than a half-silvered mirror, the factor 1/2 would be lacking in (4), and (5) would have led to $\overline{\Delta n_1 \Delta n_2} = 0$, which is as it should be.

If we were to split a beam of electrons by a non-polarizing mirror, allowing the beams to fall on separate electron multipliers, the outputs of the latter would show a negative cross-correlation. A split beam of classical particles would, of course, show zero cross-correlation. As usual in fluctuation phenomena, the behaviour of fermions and the behaviour of bosons deviate in opposite directions from that of classical particles. The Brown–Twiss effect is thus, from a *particle* point of view, a characteristic quantum effect.

It remains to show why Brannen and Ferguson did not find the effect. They looked for an increase in coincidence-rate over the 'normal' accidental rate, the latter being established by inserting a delay in one channel. Their single-channel rate was 5×10^4 counts per sec., their accidental coincidence rate about 20 per sec., and their resolving time about 10^{-8} sec. To analyse their experiment one may conveniently take the duration T of an interval of observation to be equal to the resolving time. One then finds that the coincidence-rate should be enhanced, in consequence of the cross-correlation, by the factor $(1 + \tau_0/2T)$. Unfortunately, Brannen and Ferguson do not specify their optical band-width; but it seems unlikely, judging from their description of their source, that it was much less than 10^{11} cycles/sec., which corresponds to a spread in wavelength of rather less than 1 A. at 4358 A. Adopting this figure for illustration, we have $\tau_0 = 10^{-11}$ sec., so that the expected fractional change in coincidence-rate is 0·0005. This is much less than the statistical uncertainty in the coincidence-rate in the Brannen and Ferguson experiment, which was about 0·01. Brown and Twiss did not count individual photoelectrons and coincidences, and were able to work with a primary photoelectric current some 10^4 times greater than that of Brannen and Ferguson. It ought to be possible to detect the correlation effect by the method of Brannen and Ferguson. Setting counting efficiency aside, the observing time required is proportional to the resolving time and inversely proportional to the square of the light flux per unit optical band-width. Without a substantial increase in the latter quantity, counting periods of the order of years would be needed to demonstrate the effect with the apparatus of Brannen and Ferguson. This only adds lustre to the notable achievement of Brown and Twiss.

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A CYTOCHEMICAL REACTION FOR NUCLEOPROTEIN

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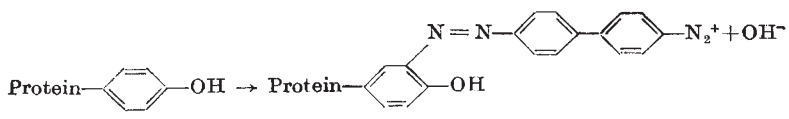
We have recently completed a study of a cytochemical reaction for protein which appears to be specific for protein in a special form of combination with nucleic acid.

When a diazonium hydroxide is applied to a tissue section, the following components may react: (1) tyrosine and histidine and perhaps tryptophan side-chains in proteins, for example:



(2) some aliphatic —NH_2 and —NH— groups to give triazenes; (3) phenols, certain bases and similar compounds of low molecular weight; (4) some doubt has existed in the past as to whether the nucleic acid bases might couple. They do not appear to do so *in vitro*; but the possibility exists that they might do so in the peculiar physico-chemical conditions prevailing in a tissue section.

If, before the diazonium hydroxide is applied, the section is treated with benzoyl chloride, all components capable of reacting with diazonium hydroxides should react with the benzoyl chloride, so that subsequent reaction with diazonium hydroxide should be blocked. Consequently, pretreatment of sections with benzoyl chloride should entirely eliminate the formation of chromophores. In fact, it does not: after benzoylation there is a residual (mainly nuclear) reaction[1].

The nature of this tissue reactant which is able to react with diazonium hydroxide after benzoylation has been the object of our present study. This reaction sequence was first tried by Mitchell[2], who