Nuclear Physics
Homework 4

Instructions. Solve each problem ***carefully*** and ***explain your procedure***.

1. Compton scattering

At the HERA collider ring the spins of the electrons going around the ring align themselves over time antiparallel to the magnetic guide fields (Sokolov-Ternov effect [So64]). This spin polarisation may be measured with the help of the spin dependence of Compton scattering. We solely consider the kinematics below.

a) Circularly polarised photons from an argon laser (514 nm) hit the electrons (26.67 GeV, straight flight path) head on. What energy does the incoming photon have in the rest frame of the electron?

b) Consider photon scattering through $90°$ and $180°$ in the electron rest frame. What energy does the scattered photon possess in each case? How large are the energies and scattering angles in the lab frame?

c) How good does the spatial resolution of a calorimeter have to be if it is 64 m away from the interaction vertex and should spatially distinguish between these photons?

2. Deep inelastic scattering

Derive the Callan-Gross relation

$$2xF_1(x) = F_2(x).$$

Which value for the mass of the target must be used?

3. Deep inelastic scattering

Deep inelastic electron-proton scattering is studied at the HERA collider. Electrons with 30 GeV are collided head on with 820 GeV protons.

a) Calculate the centre of mass energy of this reaction. What energy does an electron beam which hits a stationary proton target have to have to reproducethis centre of mass energy?

b) The relevant kinematical quantities in deep inelastic scattering are the square of the four momentum transfer Q^2 and the Bjorken scaling variable x. Q^2 may, e.g., be found from

$$q^2 = (p - p')^2 = 2m_e^2c^2 - 2(EE'/c^2 - |\vec{p}||\vec{p}'|cos(\theta)) \approx -\frac{4EE'}{c^2} sin^2\left(\frac{\theta}{2}\right).$$

Only the electron's kinematical variables (the beam energy E_e, the energy of the scattered electron E_e' and the scattering angle θ) appear here. In certain kinematical regions it is better to extract Q^2 from other variables since their experimental values give Q^2 with smaller errors. Find a formula for Q^2 where the scattering angles of the electron θ and of the scattered quark γ appear. The latter may be determined experimentally from measurements of the final state hadron energies and momenta. How?

c) What is the largest possible four momentum transfer Q^2 at HERA? What Q^2 values are attainable in experiments with stationary targets and 300 GeV beam energies? What spatial resolution of the proton does this value correspond to?

d) Find the kinematical region in Q^2 and x that can be reached with the ZEUS calorimeter which covers the angular region $7°$ to $178°$. The scattered electron needs to have at least 5 GeV energy to be resolved.

e) The electron-quark interaction can occur through neutral currents (γ, Z_0) or through charged ones ($W_\pm$). Estimate at which value of Q^2 the electromagnetic and weak interaction cross-sections are of the same size.

4. Spin polarisation

Muons are used to carry out deep inelastic scattering experiments at high beam energies. First a static target is bombarded with a proton beam. This produces charged pions which decay in flight into muons and neutrinos.

a) What is the energy range of the muons in the laboratory frame if magnetic fields are used to select a 350 GeV pion beam?

b) Why are the spins of such a monoenergetic muon beam polarised? How does the polarisation vary as a function of the muon energy?

5. Parton momentum fractions and x

Show that in the parton model of deep inelastic scattering, if we do not neglect the masses of the nucleon M and of the parton m, the momentum fraction ξ of the scattered parton in a nucleon with momentum P is given by

$$\xi = x\left[1 + \frac{m^2c^2 - M^2c^2x^2}{Q^2}\right].$$

In the deep inelastic domain

$$1 \gg \frac{x^2M^2c^2}{Q^2}, \quad 1 \gg \frac{m^2c^2}{Q^2}.$$

(Hint: for small ϵ, ϵ' we can approximate $\sqrt{1+\epsilon(1+\epsilon')} \approx 1 + \frac{\epsilon}{2}(1+\epsilon' - \frac{\epsilon}{4})$.)

6. Partons

Consider deep inelastic scattering of muons with energy 600 GeV off protons at rest. The data analysis is to be carried out at $Q^2 = 4\ GeV^2/c^2$.

a) What is the smallest value of x which can be attained under these circumstances? You may assume that the minimal scattering energy is $E' = 0$.

b) How many partons may be resolved with $x > 0.3$, $x > 0.03$ and in the full measurable range of x if we parameterise the parton distribution as follows:

$q_v(x) = A(1-x)^3/x$ for the valence quarks,
$q_s(x) = 0.4(1-x)^8/x$ for the sea quarks and
$g(x) = 4(1-x)^6/x$ for the gluons.

The role of the normalisation constant, A, is to take into account that there are 3 valence quarks.