Modelling the Wall Vibrations of Brass Wind Instruments

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Abstract: The vibration of the walls of brass wind instruments has been a subject of study in the field of musical acoustics throughout the last decades. The amplitude of such vibrations, stimulated by the oscillating air pressure inside the instrument bore, is very small compared to the dimensions of the instrument. However, it has been recently shown that at the flaring regions of the bell of brass instruments the internal air pressure can excite axi-symmetric vibrations of significantly large amplitude, which can affect the sound quality. Using both two- and three-dimensional finite element modelling, this paper presents a structural analysis of brass wind instrument bells. Furthermore, a multi-physics approach using an acoustic-structure interaction model studies the effect of the wall vibrations to the acoustic response of the instrument. The obtained results are comparable to those of experimental measurements and of simplified numerical methods.

Keywords: brass wind instruments, wall vibrations

1. Introduction

The effect of the wall vibrations of wind instruments to their sound and playability is a subject that has been lately approached by several researchers (e.g. [1, 2, 3, 4, 5, 6]). The significance, as well as the exact nature of this effect is still under investigation. Indeed recent experimental evidence [7] supports the claims of instrument makers and players that for certain types of brass wind instruments the vibrational behaviour of the walls should not be neglected any longer.

These experimental observations cannot be explained by the elliptical modes of vibration of the bell of an instrument. Such oscillations have a very high Q-factor, which can explain differences in a narrow frequency range, contrary to the wideband observations. Therefore it has been hypothesised that it is the axi-symmetrical resonances that are responsible for changes in the behaviour of an instrument that is free to vibrate. Even though such oscillations are smaller in amplitude than elliptical ones, their effect is magnified in the high-flaring regions of brass instrument bells, like those of trumpets, trombones and horns.

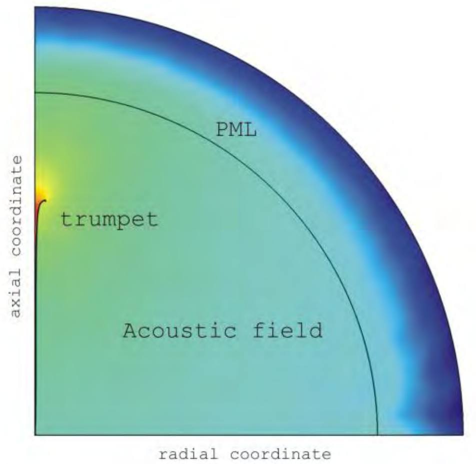

Figure 1. Acoustic pressure field of a trumpet, modelled using a two-dimensional axi-symmetric geometry.

An estimation of the effect of such axi-symmetric oscillations, using a simplified local treatment [7], showed differences in the input impedance and the transfer function of a trumpet similar to those observed in experiments. In this paper a more rigorous, global treatment is carried out using COMSOL. The attempt is to understand the nature of the structural behaviour of brass wind instruments and to analyse its effect on the radiated sound.

The modelling approach is divided in two parts. First, a purely structural simulation is implemented, using the structural mechanics module. This is carried out both in a two-dimensional axi-symmetric as well as in a three-dimensional environment. The former is limited to studying the axial resonances of a trumpet and the latter is used to identify the elliptical modes of vibration. Later, the effect of the wall vibrations to the air column is considered, using the acoustic-structure interaction and the thermoacoustics module. The input impedance and the transfer function of the instrument are calculated with the walls of the trumpet being fixed and compared to those of a free vibrating instrument.

2. Structural Mechanics

In all numerical experiments the borelist of a *Silver Flair* trumpet is used, similar to those used for experimental measurements in previous publications [7, 8]. The instrument is modelled as a linear elastic material with a constant loss factor. A purely structural study is first carried out in the frequency domain, by applying a unity pressure at the edge of the trumpet bell. The displacement of the bell is plotted over frequency in Figure 2.

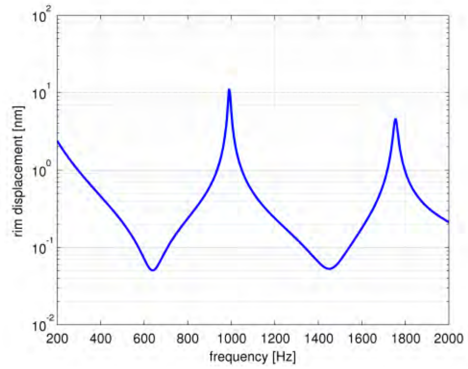

Figure 2. Rim displacement over frequency, caused by a unity pressure at the bell.

One element that strongly influences the vibrational behaviour of the instrument is the rim wire, around which the brass is folded at the end of the bell. The mass of this rim can significantly change the axial resonance frequencies of a trumpet. In order to estimate its radius a 3D eigenfrequency analysis has been carried out and the rim radius is fine-tuned, so that the (2,1) mode of the bell (corresponding to two nodal diameters and one nodal circle) appears at the same frequency as measured on an actual trumpet [8]. In the three-dimensional plots, shown in Figure 3, both elliptical and axial modes of vibration can be observed.

A further method used to study the vibrations of the instrument is to carry out a single-frequency study and to apply as boundary load on the interior of the trumpet a realistic pressure profile, calculated using BIAS [9]. The displacement along the walls of the instrument is plotted in Figure 4. It is compared to a more simplified formulation that uses a mass-spring model to represent the walls of the instrument, which is solved numerically using the finite difference method, implemented in MATLAB, as outlined in [10].

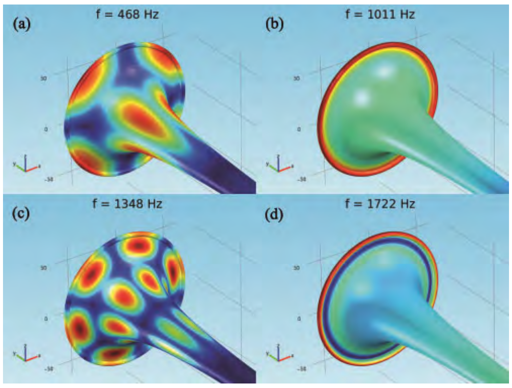

Figure 3. Modes of vibration of the trumpet bell, as calculated by a 3D simulation; (a) mode (2,1) at 468 Hz (b) first axial mode at 1011 Hz (c) mode (3,2) at 1348 Hz and (d) second axial mode at 1722 Hz.

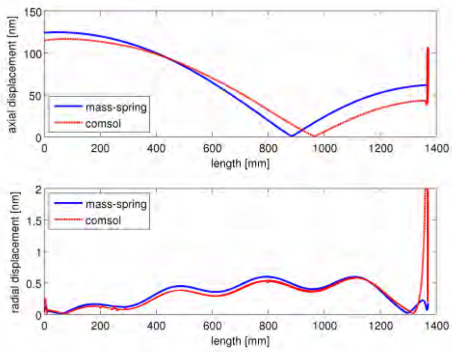

Figure 4. Axial (top) and radial (bottom) wall displacement as a function of position caused by a 250 Pa sinusoidal mouthpiece pressure at 1069 Hz, calculated using a finite difference (mass-spring) and a finite element (comsol) method.

Comparing the results of the finite difference and the finite element method, a noticeable difference can be observed in the calculated displacement near the rim of the bell. In the case of the COMSOL simulation the displacement maxima at the rim are caused by a rotational motion of the rim, especially at high frequencies. This motion is not captured by the mass-spring model but can cause a deformation of the structure at the end of the bell. Upon close inspection (as shown in Figure 5) it is revealed that the second axial resonance exhibits such a significant deformation, that cannot be predicted by the simplified mass-spring model.

It is worthwhile noticing that the calculated resonance frequencies vary for each method, as shown on Table 1.

Table 1: First and second axial resonance frequencies for a trumpet bell, as calculated using a 2D or 3D finite element method and a 2D finite difference method.

	FEM 2D	FEM 3D	FD
f_1	991 Hz	1011 Hz	1018 Hz
f_2	1754 Hz	1722 Hz	2413 Hz

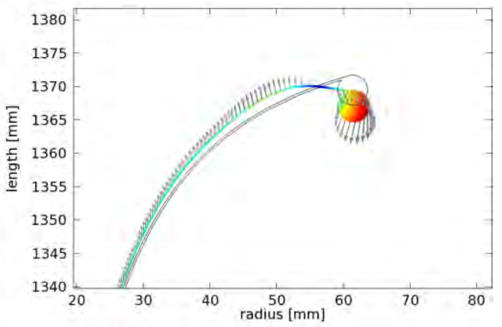

Figure 5. Second axial resonance deformation, at 1754 Hz, showing the existence of a node very close to the rim of the bell.

3. Boundary conditions

Another factor that influences the structural behaviour of the trumpet is the appearance of braces between the bell and the valve body, restricting the vibration of the trumpet at several points. So far it has been assumed that the whole instrument is free to vibrate; however, several simulations have been performed where parts of the instrument were constrained. This increases the axial resonance frequency of the bell depending on the position of the brace, such that the closer the brace is to the end of the bell the larger the increase in the frequency. However, there is an additional effect that shows a significant change in the vibrational behaviour of the instrument. A resonance appears at a frequency lower than the resonance frequency of the bell that exhibits maximum displacement at the mouthpiece, with a very small bell displacement. A comparison of resonance frequencies and corresponding displacements both at the bell and at the mouthpiece is shown on Table 2 for the case of a completely free trumpet and a trumpet with one or two braces, stimulated by a unity pressure at the bell. The first brace has been positioned 20 cm away from the rim and the second one 40 cm away. Figure 6 shows the displacement amplitude for both resonances in the case of two braces, when the trumpet is excited with a realistic internal pressure profile.

In reality the effect of the braces is more complex than simply restricting the vibration at certain points along the instrument. The work reported here only shows that it is important to consider the existence of the braces when modelling wall vibrations.

Table 2: Axial resonance frequencies for a trumpet with and without braces. f_x is the axial resonance of the bell and f_0 the one that corresponds to a maximal mouthpiece displacement. x_m and x_b are the amplitudes (in nm) of the mouthpiece and bell displacement.

free	f_x = 991 Hz x_m = 3184 x_b = 1817	
one brace	f_0 = 915 Hz x_m= 45 x_b= 4	f_x = 1154 x_m= 0 x_b= 2001
two braces	f_0 = 1094 Hz x_m= 434 x_b= 13	f_x = 1154 x_m= 0.4 x_b= 2001

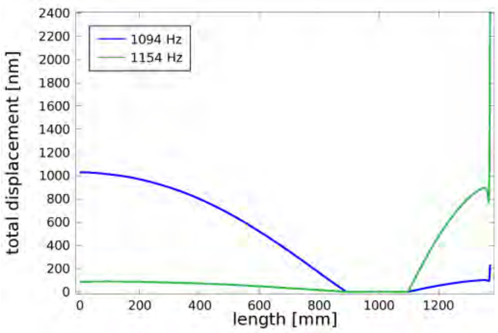

Figure 6. Displacement amplitude as a function of position for a trumpet with two braces positioned 20cm and 40cm away from the rim, stimulated by internal air pressure at 1094 and 1154 Hz.

4. Acoustic-Structure Interaction

Finally, in order to investigate the effect of wall vibrations to the behaviour of the instrument, acoustic simulations have been carried out in the frequency domain. Using a two-dimensional axi-symmetric geometry, the trumpet is surrounded by a quarter-circle of air, terminated in a perfectly matched layer, to absorb any reflections (see Figure 1). This is equivalent to an anechoic chamber, where usually acoustic measurements of radiated sound take place.

It is also necessary to consider that due to the relatively small dimensions of a wind instrument bore, viscous and thermal losses at the walls of the tube affect the wave propagation and, hence, the acoustic response of the instrument [11]. The acoustic-structure interaction module of COMSOL does not take such losses into account. However it is now possible to include this effect by introducing a thin layer (in this case of thickness 1 mm) next to the wall interior. Within this domain the thermoacoustics module is used, which can include the viscosity and thermal conduction effects. Coupled to the pressure acoustics in the remaining of the tube, the model can simulate the pressure and flow propagation inside the instrument bore. The resulting boundary layer, where the flow is retarded due to frictional forces, can be observed in Figure 7 to have the expected thickness of around 50µm [8].

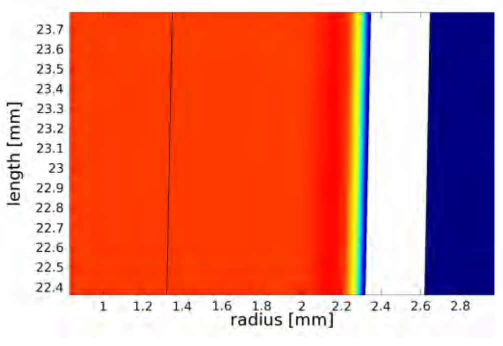

Figure 7. Flow velocity in a region close to the trumpet wall, with red being the maximum and blue the minimum value. The formation of a thin boundary layer next to the wall can be observed.

Since in this case there is no predefined thermoacoustic-structure interaction module, it is necessary to manually specify the coupling conditions. Thus, the pressure inside the instrument has to be applied as a boundary load to the wall interior and the air velocity at the boundary needs to be equated to the velocity of the wall due to its oscillations, fulfilling a no-slip boundary condition. The formation of the boundary layer follows from the thermal conduction and viscous losses modelled in the thermoacoustics domain. Note that for the exterior of the tube such a formulation is not necessary, since the thermal and viscous effects there are negligible.

In order to evaluate the acoustic properties of the instrument a frequency sweep is carried out, where the trumpet is excited by a normal acceleration at the mouthpiece end, the magnitude of which is set to

$$a = \frac{2f}{r^2}, \tag{1}$$

where f is the frequency and r the input radius of the trumpet mouthpiece. Hence the input impedance Z of the instrument, which is defined as the ratio between pressure p and volume flow

$$U = \pi r^2 u, \tag{2}$$

u being the particle velocity, is equal to the input pressure, since

$$\begin{aligned} Z &= \frac{p}{U} = \frac{p}{\pi r^2 u} = \frac{p}{\pi r^2}\frac{2\pi f}{a} \\ &= \frac{1}{a}\frac{2fp}{r^2} = p. \end{aligned} \tag{3}$$

This quantity is very important for the characterisation of a wind instrument, since it shows at which frequencies the instrument can be played [12]. Namely, only at the locations of the impedance maxima is it possible for the player to sustain a standing wave inside the instrument bore.

The frequency sweep has been performed twice. Once with the walls of the instrument being completely fixed and once being free to vibrate. During the second run and in order to avoid a full-body motion of the instrument at low frequencies a fixed constraint was still imposed to the first 40 cm of the trumpet. Figure 8 shows the input impedance of the trumpet as calculated in each case. In the lower plot the difference between the two curves is plotted, with circles marking the locations of the impedance peaks.

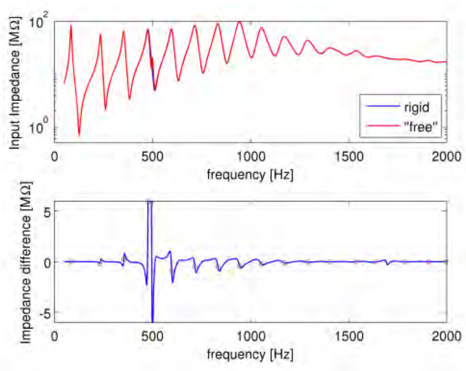

Figure 8. Input impedance of a trumpet with rigid and free walls (top) and the corresponding difference (bottom) as a function of frequency.

The comparison of the transfer function of the instrument is shown on the upper plot of Figure 9. The transfer function of the instrument is here defined as the frequency domain ratio between the pressure at the open end and the pressure in the mouthpiece entry. The lower plot shows the difference of the two curves, again marking the location of the impedance maxima, where the instrument is likely to be played.

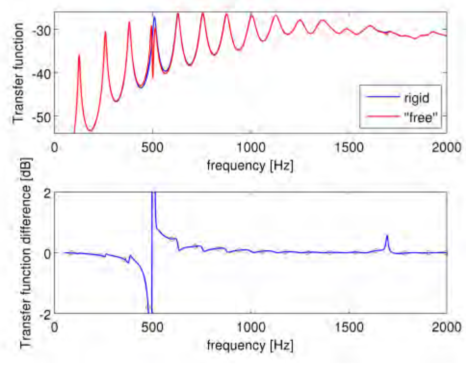

Figure 9. Transfer function of a trumpet with rigid and free walls (top) and the corresponding difference (bottom) as a function of frequency.

The order of magnitude of the calculated differences is comparable to previously published experimental measurements [7]. Furthermore, it can be observed that at the structural resonances the sign of the difference in the transfer function changes. Unfortunately it is not easy to qualitatively compare these numerical results with experimental measurements; the exact effect of the braces and the bends in a real trumpet is still unknown [13]. Therefore, future comparisons will involve completely normal, straight trumpet bells with a rim, but without any bends or braces. The behaviour of such custom bells will be much easier to be captured by a model.

Finally, the displacement of the rim of the bell is depicted in Figure 10. The dashed line represents the displacement for a purely structural simulation (as in Section 2) using the same fixed constraint of 40 cm. Even though the structural resonances exhibit a high Q-factor, their interaction with the air column resonances results in a broad-band region of relatively large displacements. Since this region contains more than one impedance peaks, a significant change in the behaviour of the instrument might occur.

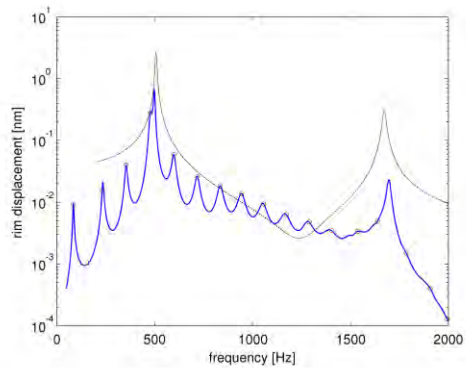

Figure 10. Displacement of the rim of the bell over frequency, stimulated with a unity volume flow at the mouthpiece entry. The dashed-black curve shows the structural axial resonances of a trumpet with the same boundary conditions.

5. Conclusions

In this paper it has been demonstrated how COMSOL multiphysics can be used to model the behaviour of a brass wind instrument. In particular, a structural analysis of a trumpet has been carried out, showing the various modes of vibration of the instrument. Then this structural mechanics model has been coupled to a pressure acoustics one, with the use of the thermocaoustics module, in order to capture the formation of the boundary layer.

The effect of the wall vibration on the input impedance and transfer function of the instrument has been shown to be of the same order of magnitude as measured in experiments and also predicted by simpler numerical formulations. However, this more rigorous treatment gives further insight into the way that the mechanical vibrations are coupled to the air-column inside the instrument.

Future research will include both comparisons of the model output to measurements carried out on custom-made straight bells, as well as full three-dimensional simulations, aiming at incorporating the effect of the bends and braces in the performance of the instrument. Finally, a more exhaustive investigation on the material parameters of the instruments should be performed in order to establish how small changes in the manufacture of an instrument can alter its acoustic properties.

6. References

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