Appendix A. Illustrative Example

Consider an instance with the following characteristics. $L = 10$, $|T| = 3$ and $|P| = 3$. The capacity given by the vector is $C = (5, 5, 10)$. Table A.13 shows the item's length and demands.

Table A.13: Instance specifications.

Items	Demands		
	$t = 1$	$t = 2$	$t = 3$
$l_1 = 2$	6	3	13
$l_2 = 3$	5	5	7
$l_3 = 7$	1	6	7

The random-key vector, denoted as $\chi, \chi \in (0, 1)^{12}$, is considered as follows:

$$\chi = \Big(0.5, 0.8, 0.06, 0.12, 0.32, 0.93, 0.54, 0.02, 0.21, 0.68, 0.77, 0.48\Big). \tag{A.1}$$

The vector of residual capacity C^r is initialized as $(5, 5, 10)$. The matrices SL and R represent the sorting lists obtained for each period and the initialized residual demand of the instance, respectively.

$$SL = \begin{bmatrix} l_3 & l_2 & l_1 \\ l_3 & l_2 & l_1 \\ l_3 & l_2 & l_1 \end{bmatrix}, \quad R = \begin{bmatrix} 6 & 3 & 13 \\ 5 & 5 & 7 \\ 1 & 6 & 7 \end{bmatrix} \tag{A.2}$$

Starting with the initial random key $j = 1$, the value of ϕ_1 is determined using Equation (5.1), specifically, $\phi_1 = \lceil 5 \times 0.5 \rceil = 3$. Consequently, starting with the first element of the list in period 1 (corresponding to the first row of SL), the items are allocated to cutting pattern A_{11}. This process is outlined below:

- Since $l_3 \leq L$ and $X_{31} = 1 > 0$, line 5 in Algorithm 2 is satisfied, then item SL_{11} is assigned to the cutting pattern. As the second condition in line 8 is not valid since $\phi_1 a_{311} = 3 > R_{31} = 1$, the next item in the list is considered, i.e. $t = 1$ and $p = 2$ is considered, implying the second bigger item length will be checked;

- Item SL_{21} is assigned to the cutting pattern as line 5 in Algorithm 2 is satisfied. Since $l_3 + 2l_2 > 10$, line 8 is not valid. However, as the object reaches its capacity ($l_3 + l_2 = 10$), no more items are assigned.

- The generated cutting pattern is:

$$A_{11} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \tag{A.3}$$

- The suggested production for each item in this cutting pattern is 3. The values $X_{31} + X_{32} = 7$ and $X_{31} + X_{32} + X_{33} = 14$ serve as upper limits for the production of item type 3, while $X_{21} = 5$, $X_{21} + X_{22} = 10$, and $X_{21} + X_{22} + X_{23} = 17$ are upper limits for the production of item type 2. In accordance with Algorithm 3, the *demandsup* for each item are 7 and 5, respectively. Therefore, the cutting pattern frequency x_{11} is obtained via Equation (5.2) as:

$$max\left\{\left\lceil\frac{7}{1}\right\rceil, \left\lceil\frac{5}{1}\right\rceil\right\} = 7.$$

- Next, it is necessary to check if $x_{11} = 7$ is valid for the decoder. For this, the value of C_{11}^{inf} is calculated via Equation (5.3). It is noteworthy that item l_1 is the sole participant in this formula, given its exclusive positive demand in period 1, considering the production from the preceding cutting pattern. Therefore:

$$C_{11}^{inf} = \left\lceil\frac{X_{11}}{max_1}\right\rceil = \left\lceil\frac{6}{5}\right\rceil = 2.$$

  It is worth noting that C_{11}^{inf}, as calculated by Equation (5.4), would result in the same value, namely 2. This is because the only remaining cutting pattern to be generated, considering the demands of period 1, is the maximal homogeneous cutting pattern of item type 1.

- Given that $C_1^r = 5$, the maximum value for x_{11} is determined as $C_1^r - C_{11}^{inf} = 3$ (Line 3 of Algorithm 4). Hence, $x_{11} = 3$ and 3 units of items of type 2 and 3 are produced. The capacity, demands, and inventory variables are updated as follows: $C_1^r = 5 - 3 = 2$, $X_{21} = 5 - 3 = 2$, and $X_{31} = 1 - 3 = -2$ implying that $X_{31} = 0$, $s_{31} = 2$, and $X_{32} = 6 - 2 = 4$.

Proceeding to random key in position $j = 2$, the value of ϕ_2 is determined via Equation (5.1) and is given by $\phi_2 = \lceil 2 \times 0.8 \rceil = 2$. The process to generate cutting pattern A_{21} is described below.

- Item l_3 in position SL_{11} cannot be assigned since $X_{31} = 0$.

- Item l_2 in position SL_{21} is assigned once line 5 in Algorithm 2 is valid, but not line 8 ($a_{221}\phi_2 = 2 = X_{21}$).

- Item l_1 in position SL_{31} is assigned 3 times to the cutting pattern ($l_2 + 3l_1 = 9 < 10$). This allocation satisfies line 5 and two loop in line 8, since $a_{121}\phi_2 = 3 \times 2 = 6 = X_{11}$ but $(a_{121} + 1)\phi_2 > X_{11}$.

- Nonetheless, since no items can fit into the remaining space of the object, which is 1, the assignment process will not change the current cutting pattern. The cutting pattern A_{21} is then presented as follows: $A_{21} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$.

- The suggested production for items l_1 and l_2, considering the value ϕ_2, are 6 and 2, respectively. Given that these production quantities precisely align with the corresponding residual demands X_{11} and X_{21}, the *demandsup* values according to Algorithm 3 are 6 and 2, respectively. Hence, in accordance with formula (5.2) the frequency x_{21} is determined as the maximum value between $\lceil \frac{6}{3} \rceil = 2$ and $\lceil \frac{2}{2} \rceil = 1$, resulting in $x_{21} = 2$.

- The validity of $x_{21} = 2$ for the decoder is established by the absence of remaining demands in period 1 ($C_{21}^{inf} = 0$), coupled with an available capacity of 2. Then, the demand variables are updated as follows: $X_{21} = 0$ and $X_{11} = 0$

Next, random key in position $j = 3$ will be used. The updated residual demand matrix, R, is given by:

$$R = \begin{bmatrix} 0 & 3 & 13 \\ 0 & 5 & 7 \\ 0 & 4 & 7 \end{bmatrix}. \tag{A.4}$$

Since no positive demands exist in period 1, the assignment procedure commences with list 2 of matrix SL. The capacity value used in formula (5.1) is now given by $C_2^r = 5$. The value ϕ_3 is calculated by $\phi_3 = \lceil 5 \times 0.06 \rceil = 1$. The construction process for cutting pattern A_{32} unfolds as follows:

- Following the sequence of list SL_2, item type 3 is assigned to the object. Since no more assignments for this item are allowed due to the object's capacity, the next item on the list will be visited.

- The next item is of type 2. However, since $l_3 + l_2 = 10 = L$, no more items can fit into the object.

- The generated cutting pattern is: $A_{31} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

- The suggested production for each item in this cutting pattern is 1. The values $X_{22} = 5$ and $X_{22} + X_{23} = 12$ serve as upper limits for the production of item type 2, while $X_{32} = 4$ and $X_{32} + X_{33} = 11$ are upper limits for the production of item type 3. In accordance with Algorithm 3, the *demandsup* values for each item are 5 and 4. Therefore, the cutting pattern frequency x_{32} is obtained via Equation (5.2) as:

$$max \left\{ \left\lceil \frac{5}{1} \right\rceil, \left\lceil \frac{4}{1} \right\rceil \right\} = 5.$$

- Next, it is necessary to check if $x_{32} = 5$ is valid for the decoder. For this, the value of C_{32}^{inf} is calculated via Equation (5.3). It is noteworthy that item l_1 is the sole participant in this formula, given its exclusive positive demand in period 1, considering the production from the preceding cutting pattern. Therefore:

$$C_{32}^{inf} = \left\lceil \frac{X_{21}}{max_1} \right\rceil = \left\lceil \frac{3}{5} \right\rceil = 1.$$

- Given that $C_2^r = 5$, the maximum value for x_{32} is determined as $C_2^r - C_{32}^{inf} = 4$. Hence, $x_{11} = 4$ and 4 units of items of type 2 and 3 are produced. The capacity, demands, and inventory variables are updated as follows: $C_2^r = 5 - 4 = 1$, $X_{22} = 5 - 4 = 1$, and $X_{32} = 4 - 4 = 0$.

Proceeding to random key in position $j = 4$, the value of ϕ_4 is determined via Equation (5.1) and is given by $\phi_4 = \lceil 1 \times 0.12 \rceil = 1$. The process for generating cutting pattern A_{42} is identical to that for cutting pattern A_{21}. Since the cutting pattern is the same, the description will focus solely on computing the frequency x_{42}.

- The suggested production for items l_1 and l_2, considering the value $\phi_4 = 1$, are 3 and 1, respectively. Given that these production quantities precisely align with the corresponding residual demands X_{12} and X_{22}, the assigned values for x_4^1 and x_4^2, which means the *demandsup* values are 3 and 1, respectively. Hence, the frequency x_{41} is obtained via formula (5.2) as the maximum between $\lceil \frac{3}{3} \rceil = 1$ and $\lceil \frac{1}{1} \rceil = 1$, which is 1.

- The validity of $x_{41} = 1$ for the decoder is established by the absence of remaining demands in period 2 ($C_{41}^{inf} = 0$), coupled with an available capacity of 1. Then, the demands of period 2 are all met, and no inventory is generated.

Next, the random key in position $j = 5$ will be used. The updated residual demand matrix, R, is given by:

$$R = \begin{bmatrix} 0 & 0 & 13 \\ 0 & 0 & 7 \\ 0 & 0 & 7 \end{bmatrix}. \tag{A.5}$$

Since no positive demands exist in period 2, the assignment procedure commences with list 3 of matrix SL. The capacity value used in formula (5.1) is now given by $C_3^r = 10$. The value ϕ_5 is calculated by $\phi_5 = \lceil 10 \times 0.32 \rceil = 4$. Because of the instance characteristics, the construction process for cutting pattern A_{53} unfolds in the same manner as cutting patterns A_{11} and A_{32}. Therefore, the description will concentrate solely on computing the frequency x_{53}.

- The suggested production for each item in this cutting pattern is 4. Since $X_{23} = 7$ and $X_{33} = 7$ are upper limits for the production of item type 2 and 3, respectively, the cutting pattern frequency x_{53} is obtained via Equation (5.2) as:

$$max \left\{ \left\lceil \frac{7}{1} \right\rceil, \left\lceil \frac{7}{1} \right\rceil \right\} = 7.$$

- Next, it is necessary to check if $x_{53} = 7$ is valid for the decoder. For this, the value of C_{53}^{inf} is calculated via Equation (5.3).

$$C_{53}^{inf} = \left\lceil \frac{X_{13}}{max_1} \right\rceil = \left\lceil \frac{10}{5} \right\rceil = 2.$$

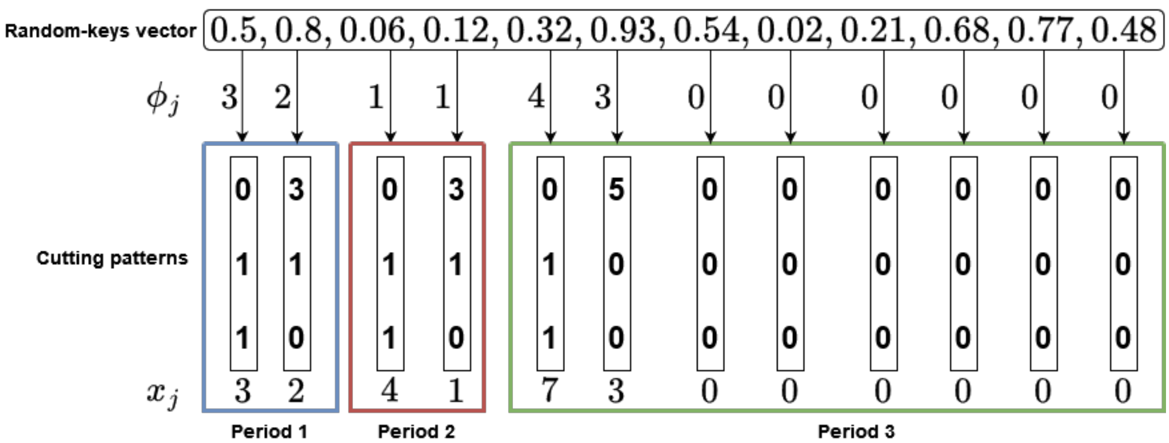
- Given that $C_3^r = 10$, the maximum value for x_{53} is determined as $C_3^r - C_{53}^{inf} = 8$. Hence, $x_{53} = 7$ is valid for the decoder. As 7 units of items of type 2 and 3 are produced, the capacity and demands variables are updated as follows: $C_3^r = 10-7 = 3$, $X_{23} = 7-7 = 0$, and $X_{33} = 7-7 = 0$.

Proceeding, random key in position $j = 6$ is used. The value of ϕ_6 is determined by $\lceil 3 \times 0.93 \rceil = 3$. Cutting pattern A_{63} uses only items of length l_1, as it is the only item with a positive demand in period 3. Since $4 \times 3 = 12 < X_{13}$, but $5 \times 3 = 15 > X_{13}$, line 8 of Algorithm 2 is not satisfied when $a_{163} = 5$. Therefore, A_{63} is the maximal homogeneous cutting pattern of item type 1, given by: $A_{63} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$.

The frequency x_{63} is then obtained as $\lceil \frac{13}{5} \rceil = 3$. Since $C_3^r = 3$ and $C_{63}^{inf} = 0$, the value 3 is a valid frequency for the decoder. Hence, $X_{13} = 13 - 15 = -2$, which implies $X_{13} = 0$ and $s_{13} = 2$.

Figure A.10 represents chromosome χ considering the presented decoder. Note that even if ϕ_j takes non-zero values for $j = 7, ..., 12$, the generated cutting pattern would still be empty since there are no positive demands to fulfill.

Figure A.10: Chromosome χ representation according to the decoder.



In conclusion, chromosome χ yields a feasible solution that uses three different cutting patterns, six setups, and requires 20 stock objects.