$P \to q \to r, q, P \vdash P$ (Ax)

$(P \to q \to r), q, P \vdash P \to q \to R$ (Ax)

$P \to (Q \to R), Q, P \vdash (Q \to R)$ (e→) $\quad P \to Q \to R, Q, P \vdash Q$ (Ax)

(→e)

$P \to Q \to R, Q, P \vdash R$ (→i)

$P \to Q \to R, Q \vdash P \to R$ (→i)

$P \to Q \to R \vdash Q \to P \to R$ (→i)

Ex5) $\vdash (P \to q \to r) \to (q \to p \to r)$

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$P \to Q \to R, P \to Q, P \vdash P$ (Ax)

$P \to (Q \to R), P \to Q, P \vdash P \to Q \to R$ (Ax) $\quad P \to q \to R, P \to q, P \vdash P \to Q$ (Ax) $\quad P \to q \to R, P \to Q, P \vdash P$ (Ax)

$P \to q \to R, P \to Q, P \vdash Q \to R \quad P \to Q \to R, P \to q, P \vdash Q$ (→e)

$P \to q \to R, P \to q, P \vdash R$ (→i)

$P \to q \to R, (P \to q) \vdash P \to R$ (→i)

$P \to q \to R \vdash (P \to q) \to P \to R$ (→i)

Ex4) $\vdash (p \to q \to R) \to (p \to q) \to p \to R$

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