2)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg P \to \bot, \neg P \vdash \neg P \to \bot}(Ax) \quad \dfrac{}{\neg P \to \bot, \neg P \vdash \neg P}(Ax)}{\neg P \to \bot, \neg P \vdash \bot}(e \to)}{\neg P \to \bot \vdash \neg\neg P}(i\neg)}{\neg P \to \bot \vdash P}(i\neg\neg)$$

1)

$$\dfrac{\dfrac{\dfrac{}{\neg\neg P, \neg P \vdash \neg\neg P}(e\neg) \quad \dfrac{}{\neg\neg P, \neg P \vdash \neg P}(e\neg)}{\neg\neg P, \neg P \vdash \bot}(PBC)}{\neg\neg P \vdash P}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{W,\neg P,P\vdash P}(ax)}{P,W,\neg P\vdash \neg P}(e\neg)}{P,W,\neg P\vdash \bot}(Ex)}{P,W,\neg P\vdash U}(i\to)}{W,\neg P\vdash P\to U}(e\to)$$

$$\dfrac{\dfrac{}{\neg P\,W\vdash W}(e\to)}{W,\neg P\vdash P}(e\neg)$$

$$\dfrac{\dfrac{W,\neg P\vdash \neg P(e\neg)}{W,\neg P\vdash \bot}(PBC)}{W\vdash P}(i\to^{W})$$

$$\vdash (P\to U)\to P)\to P$$

4)

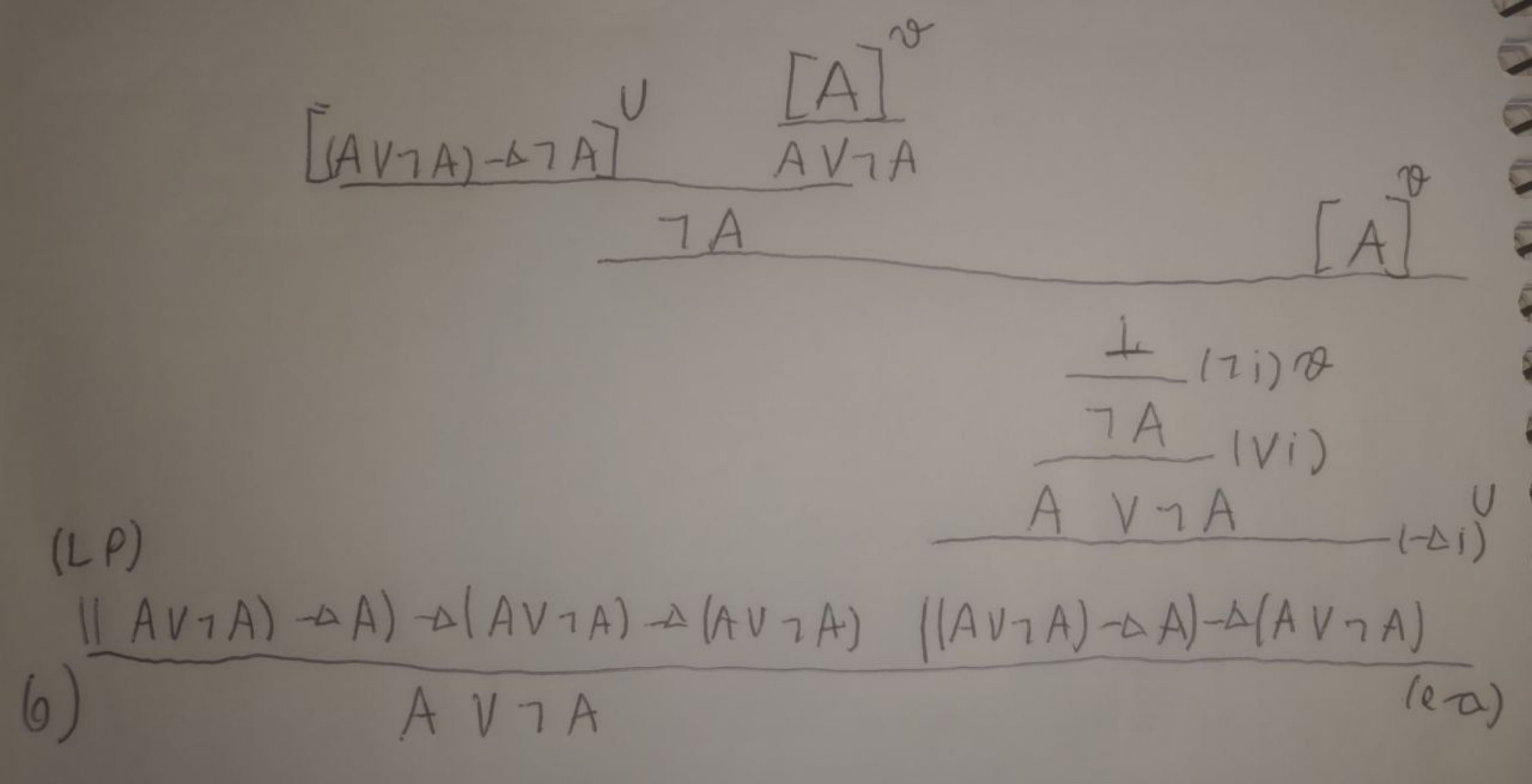
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P,W,\neg P\vdash P}(ax)}{P,W,\neg P\vdash \neg P}(ax)}{\vdash\bot}(e\neg)}{P,W,\neg P\vdash U}(Ex)}{W\vdash W}(i\to)\qquad \dfrac{}{W,\neg P\vdash P\to U}(e\to)$$

$$\dfrac{\dfrac{W,\neg P\vdash \neg P(e\neg)}{W,\neg P\vdash\bot}(e\neg)}{W\vdash P}(i\neg)$$

$$\dfrac{\dfrac{}{W\vdash \neg\neg P}(\neg\neg)}{(P\to U)\to P)\vdash P}(i\to^{W})$$

$$\vdash (P\to U)\to P)\to P$$

3)


(LP)
6)
((A∨¬A)→△A)→△(A∨¬A)
A∨¬A
(e→)
((A∨¬A)→△A)→△(A∨¬A)
A∨¬A
(→△i) U
⊥
¬A
(¬i) ϑ
(∨i)
[A] ϑ
[(A∨¬A)→△¬A] U
¬A
[A] ϑ
A∨¬A