$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{U \vdash P \to Q}(Ax) \quad \dfrac{}{U \vdash P}(Ax)}{U \vdash Q}(\to e) \quad \dfrac{}{U \vdash Q \to \Pi \to T}(Ax)}{Q \to \Pi \to T, P \to Q, P \vdash \Pi \to T}(\to e) \; Q \to \Pi \to T, P \to Q, P \simeq U}{Q \to \Pi \to T, P \to Q \vdash P \to \Pi \to T}(i \to)}{Q \to R \to T \vdash (P \to Q) \to P \to \Pi \to T}(i \to)}{3) \vdash (Q \to \Pi \to T) \to (P \to Q) \to P \to \Pi \to T}(i \to)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P \to Q, P \vdash P \to Q}(Ax)}{P \to Q \vdash P \to P \to Q}(i \to)}{2) \quad \vdash (P \to Q) \to (P \to P \to Q)}(i \to)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P \to P \to Q, P \vdash P \to P \to Q}(Ax)^e \quad \dfrac{}{P \to P \to Q, P \vdash P}(Ax)^e}{P \to P \to Q, P \vdash Q}(\to e) \quad \dfrac{}{P \to P \to Q, P \vdash P}(Ax)^e}{P \to P \to Q, P \vdash P \to Q}(\to e)}{1) \quad \vdash (P \to P \to Q) \to P \to Q}(i \to)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{(P\to Q\to R),\ Q,\ P\vdash P\to Q\to R}(Ax)\qquad \dfrac{}{\Delta\ P\to\Delta Q\to\Delta\Gamma,Q,\ P\vdash P}(Ax)}{P\to\Delta(Q\to\Delta R),\ Q,\ P\vdash Q\to\Delta R}(e\to\Delta)}{P\to\Delta(Q\to\Delta R),\ Q,\ P\vdash R}(\to\Delta e)}{P\to\Delta Q\to\Delta R,\ Q,\ P\vdash R}(\to\Delta i)}{P\to\Delta Q\to\Delta R,\ Q\vdash P\to R}(\to\Delta i)}{P\to\Delta Q\to\Delta R\vdash Q\to P\to R}(\to\Delta i)}{\vdash(P\to\Delta Q\to\Delta R)\to\Delta(Q\to P\to\Delta R)}(\to\Delta i)$$

Ex 5)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Delta\ P\to\Delta Q\to\Delta R,\ P\to\Delta Q,\ P\vdash P}(Ax)\qquad \dfrac{}{\Delta\ P\to\Delta Q\to\Delta R,\ P\to\Delta Q,\ P\vdash Q}(Ax)}{P\to\Delta Q\to\Delta R,\ P\to\Delta Q,\ P\vdash Q}(\to\Delta e)}{P\to\Delta Q\to\Delta R,\ P\to\Delta Q\vdash P\to Q}(\to\Delta i)}{P\to\Delta Q\to\Delta R\vdash(P\to\Delta Q)\to\Delta P\to R}(\to\Delta i)}{\vdash(P\to\Delta Q\to\Delta R)\to\Delta(P\to\Delta Q)\to\Delta P\to R}(\to\Delta i)$$

Ex 4)