$(\neg P) \wedge \neg Q \vdash \neg(P \vee Q)$ 증명

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg P \wedge \neg Q, P \vdash P}(Ax)}{\neg P \wedge \neg Q, P \vdash P \vee Q}(\vee_1)}{\neg P \wedge \neg Q, P \vdash \bot}(\neg_E) \quad \dfrac{\dfrac{}{\neg P \wedge \neg Q, Q \vdash Q}(Ax)}{\neg P \wedge \neg Q, Q \vdash P \vee Q}(\vee_2)}{\neg P \wedge \neg Q, P \vee Q \vdash \bot}(\neg_E)}{\neg P \wedge \neg Q \vdash \neg(P \vee Q)}(\neg_I)$$

$\neg P \vee \neg Q \vdash \neg(P \wedge Q)$ 증명

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg(P \wedge Q), P \vdash P}(Ax)}{\neg(P \wedge Q), P \vdash P \vee Q}(\vee_1) \quad \dfrac{\dfrac{}{\neg(P \wedge Q), Q \vdash Q}(Ax)}{\neg(P \wedge Q), Q \vdash P \vee Q}(\vee_2)}{\neg(P \wedge Q), P \vdash \bot}(\neg_E) \quad \dfrac{\dfrac{\dfrac{}{\neg(P \wedge Q), Q \vdash Q}(Ax) \quad \dfrac{}{\neg(P \wedge Q), Q \vdash P}(Ax)}{\neg(P \wedge Q), Q \vdash P \wedge Q}(\wedge_I)}{\neg(P \wedge Q), Q \vdash \bot}(\neg_E)}{\neg(P \wedge Q) \vdash \neg P \vee \neg Q}(\vee_E)}{\neg P \vee \neg Q \vdash \neg(P \wedge Q)}(\neg_I)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P \to U,\ P \wedge \neg U \vdash P \to U}(Ax) \quad \dfrac{}{P \to U,\ P \wedge \neg U \vdash P \wedge \neg U}(Ax)}{P \to U,\ P \wedge \neg U \vdash P}(e\to) \quad \dfrac{}{P \to U,\ P \wedge \neg U \vdash P}(Ax)}{P \to U,\ P \wedge \neg U \vdash U}(e\to) \quad \dfrac{\dfrac{}{P \to U,\ P \wedge \neg U \vdash P \wedge \neg U}(Ax)}{P \to U,\ P \wedge \neg U \vdash \neg U}(e\wedge) \quad (e\wedge)}{P \to U,\ P \wedge \neg U \vdash \bot}(i\neg)$$

4) $P \to U \vdash \neg(P \wedge \neg U)$

$$\dfrac{\dfrac{\dfrac{}{P \to U,\ \Pi \vdash \Pi}(Ax) \quad \dfrac{\dfrac{}{P \to U,\ P \vdash P \to U}(Ax) \quad \dfrac{}{P \to U,\ P \vdash P}(Ax)}{P \to U,\ P \vdash U}(e\to)}{P \to U,\ \Pi \vdash \Pi \vee U}(i\vee) \quad (e\vee)}{P \to U,\ \Pi \vee P \vdash \Pi \vee U}(e\vee)$$

3) $P \to U \vdash (\Pi \vee P) \to (\Pi \vee U)$ $(i\to)$

$\neg P \wedge \neg q, P \vdash \neg P \wedge \neg q$ (Ax)

$\neg P \wedge \neg q, P \vdash \neg P$ ($\wedge e_1$) $\quad \neg P \wedge \neg q, P \vdash P$ ($\wedge e_2$)

$\neg P \wedge \neg q, P \vee q \vdash P \vee q$ ($\vee e$) $\quad \neg P \wedge \neg q, q \vdash \bot$ ($\neg e$) $\quad (\neg P \wedge \neg q), P \vdash \bot$ ($\neg e$)

$(\neg P) \wedge (\neg q), P \vee q \vdash \bot$

7) $(\neg P) \wedge (\neg q) \vdash \neg (P \vee q)$

(Ax) $\quad$ (Ax)

$\neg(P \vee q), P \vdash \neg(P \vee q)$ ($\neg e$) $\quad \neg(P \vee q), P \vdash P$ ($\vee i$)

$\neg(P \vee q), P \vdash \bot$ ($\neg i$) $\quad \neg(P \vee q), P \vdash P \vee q$ ($\neg e$)

$\neg(P \vee q) \vdash \neg P$ ($\wedge i$)

6) $\neg(P \vee q) \vdash (\neg P) \wedge (\neg q)$

(Ax)

$P \wedge U, \neg U \vdash P \wedge U$ ($\wedge i$)

(Ax) $\quad P \wedge U, \neg U \vdash \neg U$ ($\neg e$) $\quad P \wedge U, \neg U \vdash U$ ($\neg e$)

(Ax) $\quad$ (Ax)

$P \wedge U, \neg P \vdash P \wedge U$ ($\wedge i$) $\quad P \wedge U, \neg P \vdash \neg P$ ($\neg e$)

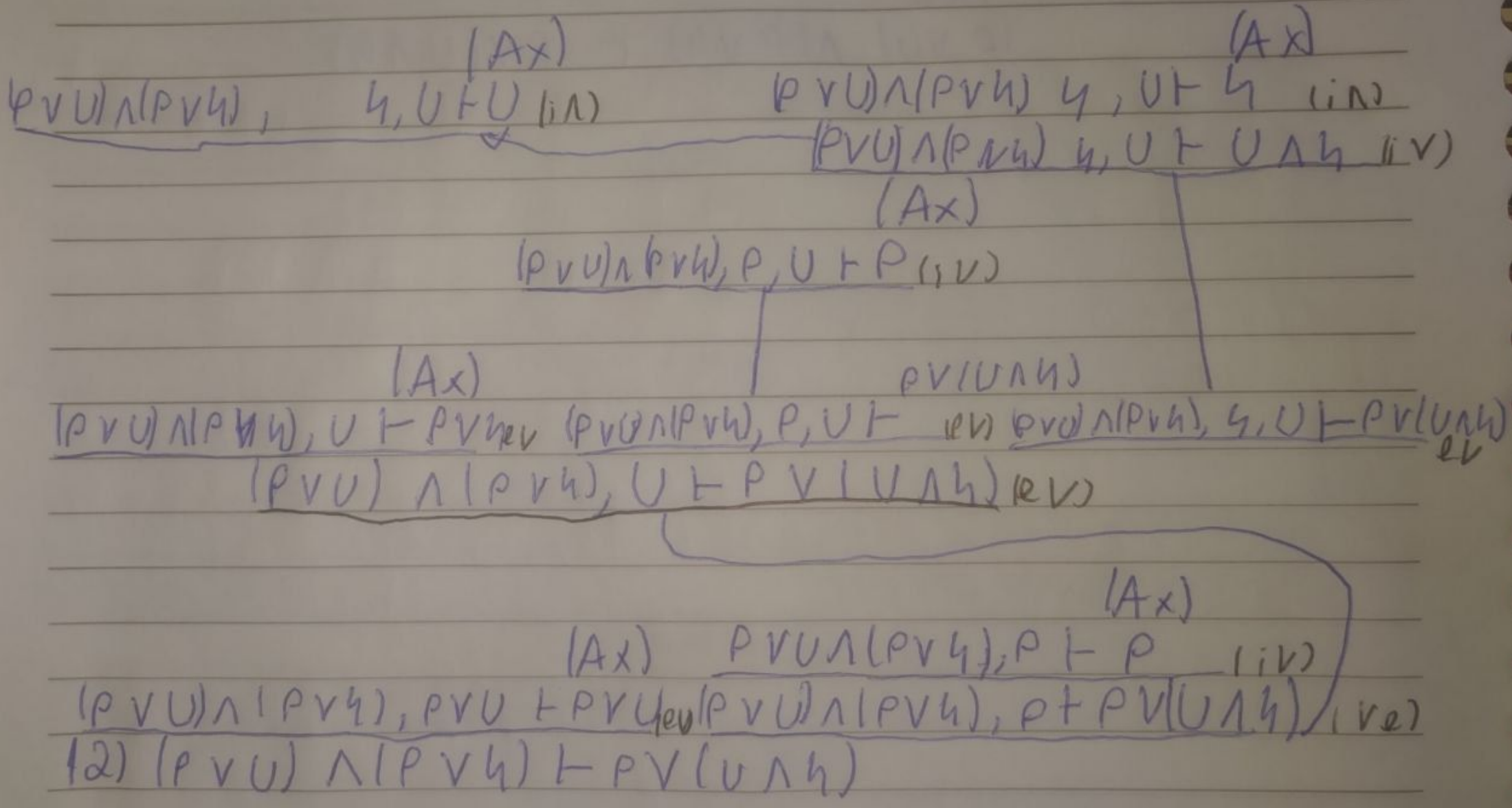
$P \wedge U, \neg P \vdash P$ ($\neg e$)

(Ax)

$P \wedge U, \neg P \vee \neg U, \vdash \neg P \vee \neg U$ ($\vee e$) $\quad P \wedge U, \neg P \vdash \bot$ ($\vee e$) $\quad P \wedge U, \neg U \vdash \bot$ ($\vee e$)

$P \wedge U, \neg P \vee \neg U \vdash \bot$

5) $P \wedge U \vdash \neg(\neg P \vee \neg U)$



12) $(P \vee U) \wedge (P \vee H) \vdash P \vee (U \wedge H)$

$$\dfrac{\dfrac{\dfrac{}{\neg(P\wedge Q),\neg Q,P\wedge Q\vdash P\wedge Q}(Ax)}{\neg(P\wedge Q),\neg Q,P\wedge Q\vdash Q}(\wedge E_2)\qquad \dfrac{}{\neg(P\wedge Q),\neg Q,P\wedge Q\vdash \neg Q}(Ax)}{\neg(P\wedge Q),\neg Q,P\wedge Q\vdash \bot}(\neg E)$$

$$\dfrac{\neg(P\wedge Q),\neg Q\vdash \neg(P\wedge Q)}{\neg(P\wedge Q)\vdash Q\to\neg(P\wedge Q)}(\to I)$$

$$\dfrac{\dfrac{\dfrac{}{\neg(P\wedge Q),P\wedge Q\vdash P\wedge Q}(Ax)}{\neg(P\wedge Q),P\wedge Q\vdash P}(\wedge E_1)\qquad \dfrac{}{\neg(P\wedge Q),P\wedge Q\vdash \neg P}(Ax)}{\neg(P\wedge Q),P\wedge Q\vdash \bot}(\neg E)$$

$$\dfrac{\neg(P\wedge Q),P\wedge Q\vdash \bot}{\neg(P\wedge Q),P\vdash \neg(P\wedge Q)}(\to I)$$

$$\dfrac{\neg(P\wedge Q)\vdash P\to\neg(P\wedge Q)}{\neg(P\wedge Q)\vdash \neg P\vee\neg Q}(\vee I_2)$$

$$\dfrac{\dfrac{\dfrac{}{\neg(P\wedge Q),\neg Q,P\wedge Q\vdash P\wedge Q}(Ax)}{\neg(P\wedge Q),\neg Q,P\wedge Q\vdash Q}(\wedge E_2)\qquad \dfrac{}{\neg(P\wedge Q),\neg Q,P\wedge Q\vdash \neg Q}(Ax)}{\neg(P\wedge Q),\neg Q,P\wedge Q\vdash \bot}(\neg E)$$

$$\dfrac{\neg(P\wedge Q),\neg Q\vdash \neg(P\wedge Q)}{\neg(P\wedge Q)\vdash Q\to\neg(P\wedge Q)}(\to I)$$

$$\dfrac{\neg(P\wedge Q)\vdash P\to\neg(P\wedge Q)}{\neg(P\wedge Q)\vdash \neg P\vee\neg Q}(\vee I_1)$$

$$\dfrac{\dfrac{\dfrac{}{\neg P,P\wedge Q\vdash P\wedge Q}(Ax)}{\neg P,P\wedge Q\vdash P}(\wedge E_1)\qquad \dfrac{}{\neg P,P\wedge Q\vdash \neg P}(Ax)}{\neg P,P\wedge Q\vdash \bot}(\neg E)$$

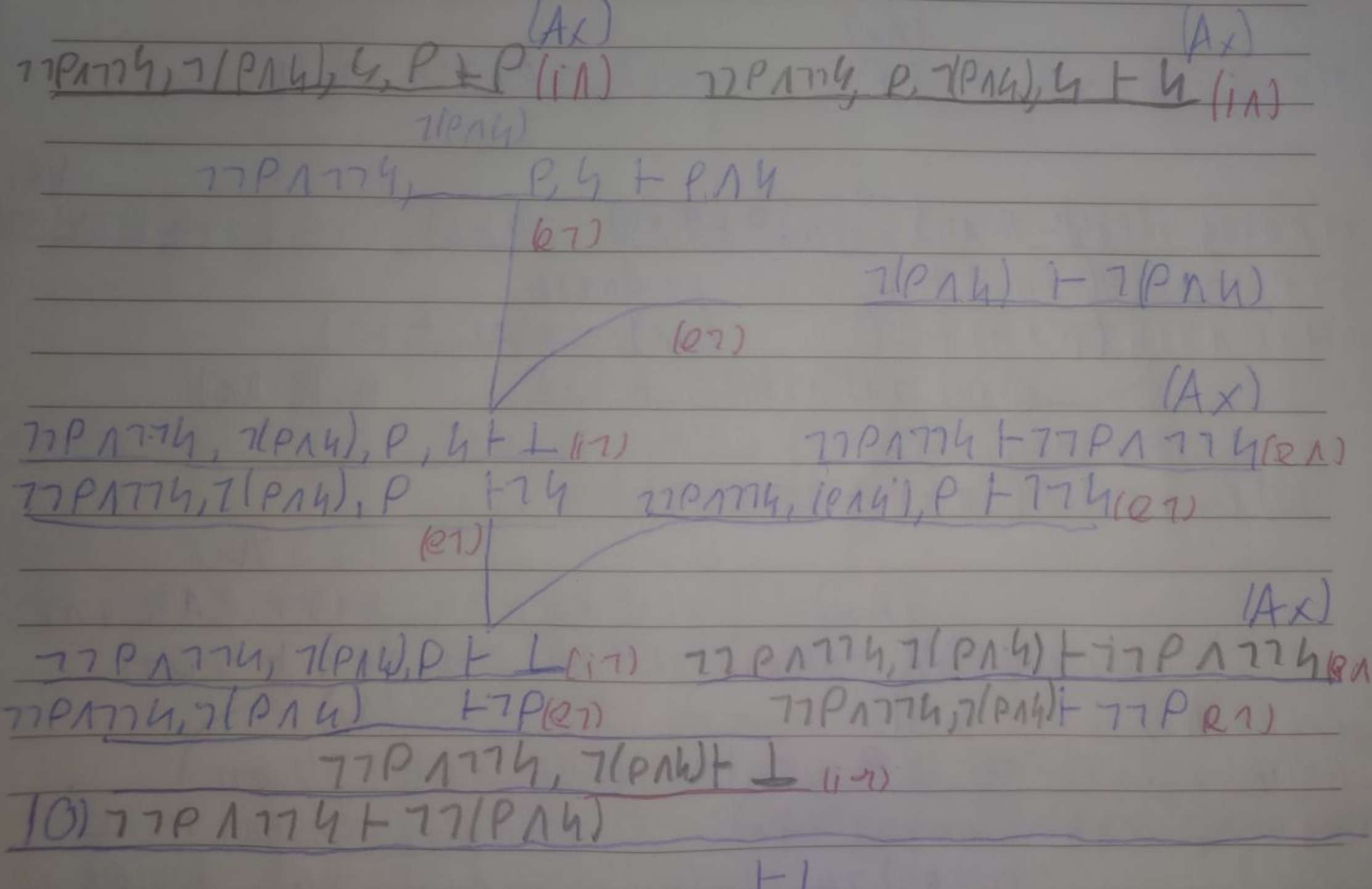
$$\dfrac{\neg P,P\wedge Q\vdash \bot}{\neg P,P\wedge Q\vdash \neg Q}(\bot E)$$

$$\dfrac{\dfrac{\dfrac{}{\neg Q,P\wedge Q\vdash P\wedge Q}(Ax)}{\neg Q,P\wedge Q\vdash Q}(\wedge E_2)\qquad \dfrac{}{\neg Q,P\wedge Q\vdash \neg Q}(Ax)}{\neg Q,P\wedge Q\vdash \bot}(\neg E)$$

$$\dfrac{\neg Q,P\wedge Q\vdash \bot}{\neg Q,P\wedge Q\vdash \neg P}(\bot E)$$

$$\dfrac{\neg P,P\wedge Q\vdash \neg P\qquad \neg Q,P\wedge Q\vdash \neg Q}{\neg P\vee\neg Q,P\wedge Q\vdash \neg P\vee\neg Q}(\vee E)$$

$$\dfrac{\neg P\vee\neg Q,P\wedge Q\vdash \bot}{\neg P\vee\neg Q\vdash \neg(P\wedge Q)}(\neg I)$$



(Ax)

$\upsilon \wedge \pi \vdash \upsilon \wedge \pi$

(e∧)

$\upsilon \wedge \pi \vdash \pi$

(i∨)

$\upsilon \wedge \pi \vdash \rho \vee \pi$

(Ax)

$\rho \vdash \rho$

(i∨)

$\rho \vdash \rho \vee \pi$

(Ax)

$\rho \vee (\upsilon \wedge \pi) \vdash \rho \vee (\upsilon \wedge \pi)$

(Ax)

$\upsilon \wedge \pi \vdash \upsilon \wedge \pi$ (e∧)

$\upsilon \wedge \pi \vdash \upsilon$ ~~HA~~

$\upsilon \wedge \pi \vdash \rho \vee \upsilon$ (i∨)

(Ax)

$\rho \vdash \rho$

$\rho \vdash \rho \vee \upsilon$ (i∨)

(Ax)

$\rho \vee (\upsilon \wedge \pi) \vdash \rho \vee (\upsilon \wedge \pi)$

$\rho \vee (\upsilon \wedge \pi) \vdash (\rho \vee \upsilon)$ (i∧) $\rho \vee (\upsilon \wedge \pi) \vdash (\rho \vee \pi)$ (e∧)

11) $\rho \vee (\upsilon \wedge \pi) \vdash (\rho \vee \upsilon) \wedge (\rho \vee \pi)$

(Ax) $\qquad\qquad$ (Ax)

$p \wedge (u \vee \psi) \vdash p \wedge (u \vee \psi)$ (e∧) $\qquad$ $p \wedge (u \vee \psi) \vdash p \wedge (u \vee \psi)$ (e∧)

$p \wedge (u \vee \psi) \vdash p$ ~~(i∧)~~ (∨i) $\qquad$ $p \wedge (u \vee \psi) \vdash p$ ~~(i∧)~~ (∨i)

$p \wedge (u \vee \psi) \vdash p \vee u$ (i∧) $\qquad$ $p \wedge (u \vee \psi) \vdash p \vee \psi$ (i∧)

13) $p \wedge (u \vee \psi) \vdash (p \vee u) \wedge (p \vee \psi)$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P\wedge Q\vdash P\wedge Q}(Ax)}{P\wedge Q\vdash P\wedge Q}(e\wedge)}{P\wedge Q\vdash Q}(i\vee)}{P\wedge Q\vdash Q\vee\Pi}(e\vee)}{(P\wedge Q)\vee(P\wedge\Pi)\vdash Q\vee\Pi}(i\wedge)}{(P\wedge Q)\vee(P\wedge\Pi)\vdash P\wedge(Q\vee\Pi)}(i\wedge)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P\wedge\Pi\vdash P\wedge\Pi}(Ax)}{P\wedge\Pi\vdash\Pi}(e\wedge)}{P\wedge\Pi\vdash Q\vee\Pi}(i\vee)}{(P\wedge Q)\vee(P\wedge\Pi)\vdash Q\vee\Pi}(e\vee)$$

$$\dfrac{\dfrac{\dfrac{}{P\wedge Q\vdash P\wedge Q}(Ax)}{P\wedge Q\vdash Q}(e\wedge)}{P\wedge Q\vdash Q\vee\Pi}(i\vee)$$

$$\dfrac{\dfrac{\dfrac{}{P\wedge Q\vdash P\wedge Q}(Ax)}{P\wedge Q\vdash P}(e\wedge)}{P\wedge Q\vdash P}(e\vee)$$

$$\dfrac{\dfrac{\dfrac{}{P\wedge\Pi\vdash P\wedge\Pi}(Ax)}{P\wedge\Pi\vdash\Pi}(e\wedge)}{P\wedge\Pi\vdash P}(e\vee)$$

$$\dfrac{\dfrac{\dfrac{}{(P\wedge Q)\vee(P\wedge\Pi)\vdash(P\wedge Q)\vee(P\wedge\Pi)}(Ax)}{(P\wedge Q)\vee(P\wedge\Pi)\vdash P}(e\vee)}{(P\wedge Q)\vee(P\wedge\Pi)\vdash P}(i\wedge)$$

$$(P\wedge Q)\vee(P\wedge\Pi)\vdash P\wedge(Q\vee\Pi)$$

$\vdash p \land \neg p$ (e∧) $\qquad \vdash p \land \neg p$ (e∧)

$p \land \neg p \vdash \neg p$ (e¬) $\qquad p \land \neg p \vdash p$ (e¬)

$p \land \neg p \vdash \bot$ (i¬)

16) $\vdash \neg(p \land \neg p)$

(Ax)

$\neg(p \lor \neg p), p \vdash p$ (eV) $\qquad$ (Ax)

$\neg(p \lor \neg p), p \vdash p \lor \neg p$ ~~(e¬)~~ $\qquad \neg(p \lor \neg p), p \vdash \neg(p \lor \neg p)$ (e¬)

$\neg(p \lor \neg p), p \vdash \bot$ (e¬)

$\neg(p \lor \neg p) \vdash \neg p$ (i¬)

$\neg(p \lor \neg p) \vdash \neg(p \lor \neg p)$ (e¬) $\qquad \neg(p \lor \neg p) \vdash p \lor \neg p$ (e¬)

$\neg(p \lor \neg p) \vdash \bot$ (i¬)

~~17)~~ 15) $\vdash \neg\neg(p \lor \neg p)$