Intersection between cylinder and ray

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1 Finding the Implicit function of a cylinder

We have that for a ray and a cylinder to coincide, the following equality must hold:

$$o + t\vec{d} = c + r \begin{pmatrix} \cos\theta \\ \sin\theta \\ z \end{pmatrix}$$

We will this have to solve for both t and θ. This becomes too complicated and we will have to find a better solution for it. We can use the implicit representation of a cylinder.

Let the axis of the cylinder be a line going through its center $\vec{x}_0 = (x_0, y_0, z_0)$ and in the direction of $\vec{v} = (a, b, c)$ with radius R. Then we can write the equation of the cylinder as the set of all points $\vec{x} = (x, y, z)$ satisfying:

$$|\vec{x} - \vec{x}_0|^2 = R^2 + [(\vec{x} - \vec{x}_0) \cdot \vec{v}]^2$$

where $z \in [z_0 - h, z_0 + h]$.

We have that our $\vec{x} = o + t\vec{d}$ such that

$$0 = R^2 + \left[\left(o + t\vec{d} - \vec{x}_0\right) \cdot \vec{v}\right]^2 - \left|o + t\vec{d} - \vec{x}_0\right|^2$$

Given that $|\vec{d}| = 1$, solving for t gives us the quadratic equation:

$$0 = ((\vec{d} \cdot \vec{v})^2 - 1)t^2 + 2\left[((o - \vec{x}_0) \cdot \vec{v})\,(\vec{d} \cdot \vec{v}) - (o - \vec{x}_0) \cdot \vec{d}\right] t + ((o - \vec{x}_0) \cdot \vec{v})^2 - |o - x_0|^2 + R^2$$

2 Normal derivation

2.1 Plane

For the plane intersection the normal is just the normal of the plane ($\vec{n}$), but we need to take the one that points towards the ray origin. So if the ray direction and the normal of the plane form an angle bigger than 90° ($\vec{n} \cdot \vec{d} > 0$), we take $-\vec{n}$ as the normal.

2.2 Cylinder

We have found the intersection point c where the ray intersects the cylinder, now the normal we want to find is the radial unit vector that is perpendicular to the cylinder axis. This vector will be perpendicular to the surface of the cylinder at the intersection point. Since it needs to point in the direction of the ray origin, if the intersection point was on the second face of the cylinder, we hit the cylinder from "inside", so we need to take $-\vec{n}$ as the normal.

To calculate it we take the vector $\vec{ic} = o + t\vec{d} - x_0$ pointing from the cylinder center to the intersection point, and subtract from it the components of this same $\vec{ic}$ vector along the axis of the cylinder, leaving us with only the vector perpendicular to the cylinder axis. To normalize we just have to divide the result by the cylinder radius.

$$\vec{n} = (\vec{ic} - \vec{v} \cdot \vec{ic})/R$$