Intersection between cylinder and ray

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1 Finding the Implicit function of a cylinder

We have that for a ray and a cylinder to coincide, the following equality must hold:

$$o + t\vec{d} = c + r \begin{pmatrix} \cos\theta \\ \sin\theta \\ z \end{pmatrix}$$

We will this have to solve for both t and θ. This becomes too complicated and we will have to find a better solution for it. We can use the implicit representation of a cylinder.
Let the axis of the cylinder be a line going through it's center $\vec{x}_0 = (x_0, y_0, z_0)$ and in the direction of $\vec{v} = (a, b, c)$ with radius R, we can write the equation of the cylinder as the set of all points $\vec{x} = (x, y, z)$ satisfying:

$$|\vec{x} - \vec{x}_0|^2 = R^2 + [(\vec{x} - \vec{x}_0) \cdot \vec{v}]^2$$

where z $\in [z_0 - h, z_0 + h]$
We have that our $\vec{x} = o + t\vec{d}$ such that

$$0 = R^2 + \left[\left(o + t\vec{d} - \vec{x}_0\right) \cdot \vec{v}\right]^2 - \left|o + t\vec{d} - \vec{x}_0\right|^2$$

Solving for t gives us the quadratic equation:

$$0 = ((\vec{d} \cdot \vec{v})^2 - 1)t^2 + 2\left((o - x_0) \cdot \vec{v} \cdot (\vec{d} \cdot \vec{v}) - (o - x_0)\,\vec{d}\right)t + ((o - x_0) \cdot \vec{v})^2 - (o - x_0)^2$$