Compactifying Fish

Principal Component Analysis and Dimension Reduction

Eli Griffiths

Simple Data is Not So Simple

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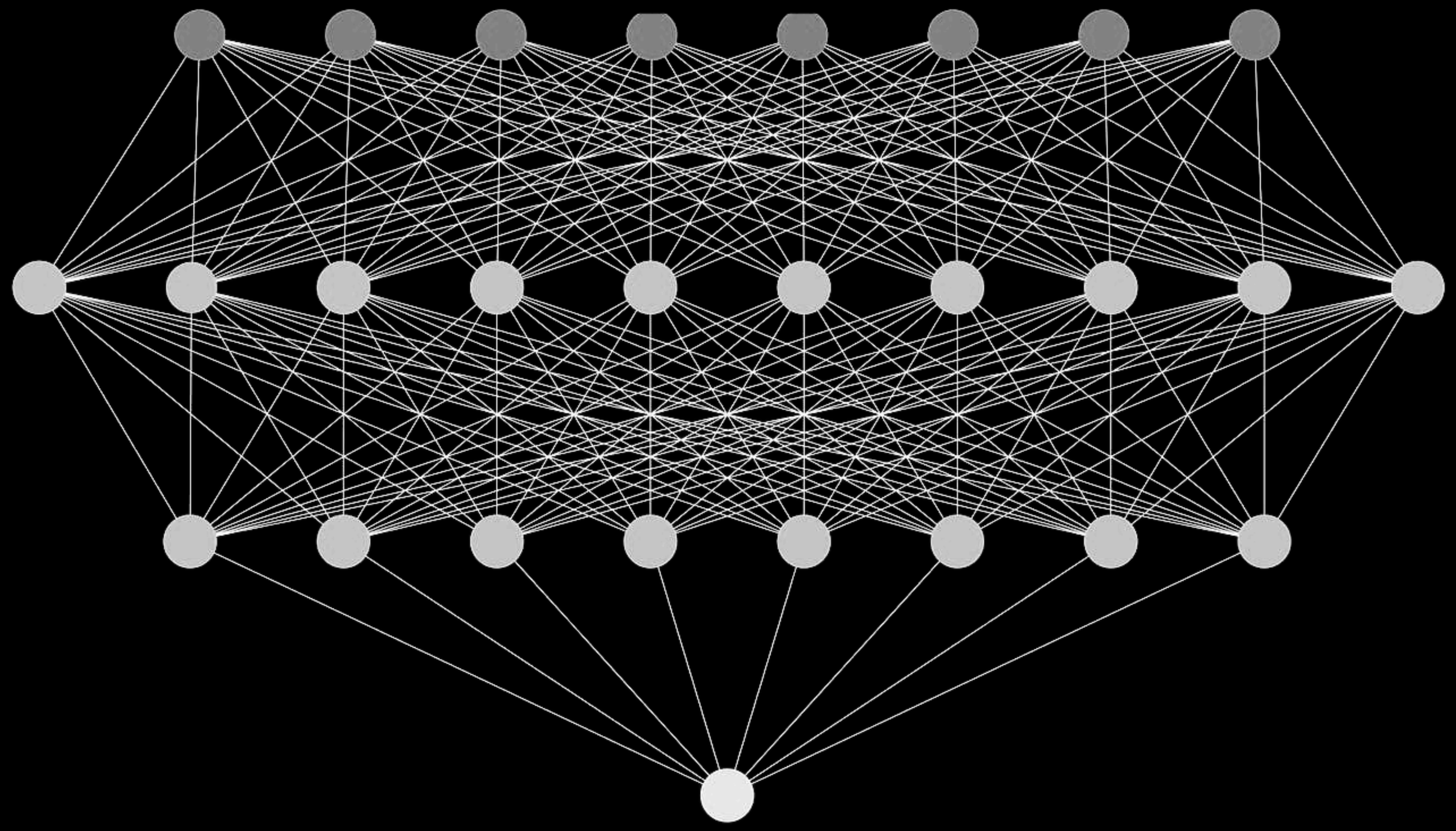
32

32

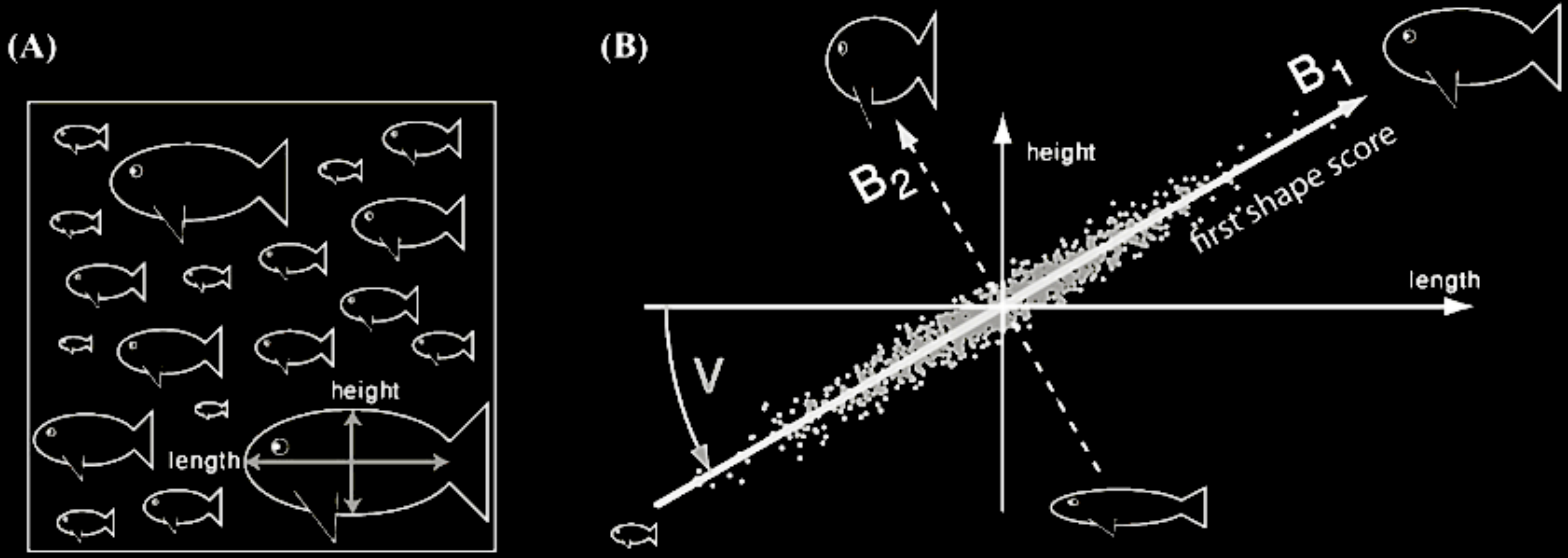
$$32 \times 32 \times (1 + 1 + 1) = 3,072$$

Why Dimension Reduction?

- **Data Science** – Analysis of high featured datasets
- **Machine Learning** – Dataset simplification
- **Neuroscience** – Neuron potentials and activation

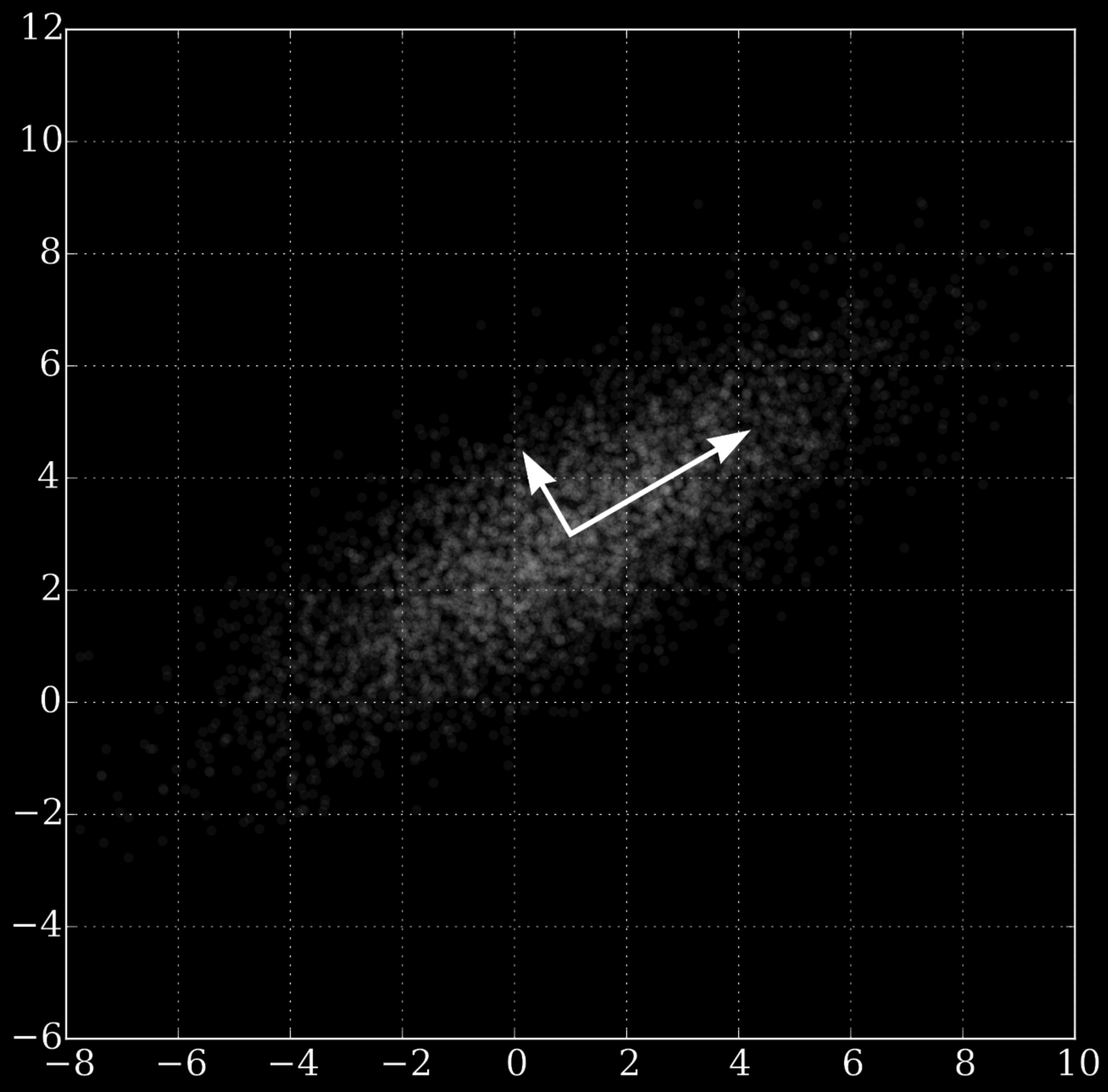


Reduction via Correlation



Principal Components

- Identify a set of correlations
- Pick the strongest ones to build ***axes***
- Project the data onto these axes



Capturing Correlation

$$\mathbf{X} = \begin{bmatrix} X_1 & X_2 & \cdots & X_n \end{bmatrix}^T$$

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$$\mathbf{K} = \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \cdots & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_{2,2} & & \sigma_{2,n} \\ \vdots & & \ddots & \vdots \\ \sigma_{n,1} & \sigma_{n,2} & \cdots & \sigma_{n,n} \end{bmatrix}$$

Symmetric Matrix

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Spectral Theorem

If A is symmetric, there exists an orthonormal $\boldsymbol{basis}$ of eigenvectors of A

The Projection

$$\mathbf{K}\, x_i = \lambda_i x_i \qquad \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_d \geq \ldots \geq \lambda_n$$

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$$\mathbf{P} = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_d \\ | & | & & | \end{bmatrix}$$

A Concrete Example

- MNIST Handwriting Dataset
- Comprised of 28 by 28 grayscale images
- Has 784 “features”

Original $d = 50$ $d = 100$ $d = 300$ $d = 500$

Thank You