1)
$$M(N) = \begin{cases} 1 & \text{se } N = 0 \text{ ou } N = 1 \\ \min\limits_{0 \leq K \leq N-1} \{ M(K) + M(N-K-1) \} + N \end{cases}$$

Vamos verificar que $M(N) \geq \frac{1}{2}(N+1) \lg (N+1)$ p/ $N \geq 0$

Base:

$N = 0 \Rightarrow M(0) = 1 \geq \frac{1}{2}(0+1) \lg(0+1)$ //

$N = 1 \Rightarrow M(1) = 1 \geq \lg(2)$ //

Indução

$$M(N) = N + \min_{0 \leq K \leq N-1} \{ M(K) + M(N-K-1) \}$$

$$M(N) = N + \frac{1}{2} \min_{0 \leq K \leq N-1} \{ (K+1) \lg (K+1) + (N-K) \lg (N-K) \}$$

Para encontrar o mínimo, derivamos e igualamos à zero:

$$\frac{d}{dK}\left[(K+1)\lg(K+1) + (N-K)\lg(N-K)\right] = 0$$

$$\lg(K+1) + \frac{(K+1)}{(K+1)\ln(2)} - \lg(N-K) - \frac{(N-K)}{(N-K)\ln 2} = 0$$

$$\lg(K+1) - \lg(N-K) = 0$$

$$\lg\left(\frac{K+1}{N-K}\right) = 0 \iff \frac{K+1}{N-K} = 1 \iff K_{min} = \frac{N-1}{2} //$$

Então:

$$M(N) = N + \frac{1}{2} \min_{0 \leq K \leq N-1} \left\{(K+1)\lg(K+1) + (N-K)\lg(N-K)\right\}$$

$$M(N) \geq N + \frac{1}{2}\left[\left(\frac{N+1}{2}\right)\lg\left(\frac{N+1}{2}\right) + \left(\frac{N+1}{2}\right)\lg\left(\frac{N+1}{2}\right)\right]$$

$$= N + \left(\frac{N+1}{2}\right)\lg\left(\frac{N+1}{2}\right)$$

$$= \frac{N-1}{2} + \left(\frac{N+1}{2}\right)\lg(N+1)$$

$$\geq \left(\frac{N+1}{2}\right)\lg(N+1) //$$

7) Dado que soma é uma variável aleatória incrementada quando $X \in [1, 10]$ e X é par então usamos a variável binária:

$$X_i = \begin{cases} 1 & \text{se é par} \\ 0 & \text{c.c} \end{cases}$$

$$E[\text{soma}] = E\left[\sum_{i=1}^{N} X_i\right] = \sum_{i=1}^{N} E[X_i]$$

$$= \sum_{i=1}^{N} \frac{5}{10}\cdot 0 + \frac{1}{10}\cdot 2 + \frac{1}{10}\cdot 4 + \frac{1}{10}\cdot 6 + \frac{1}{10}\cdot 8 + \frac{1}{10}\cdot 10$$

$$= \frac{1}{10} \sum_{i=1}^{N} 5\cdot 0 + 2 + 4 + 6 + 8 + 10 = \frac{30N}{10} = 3N$$

```
eduardo@eduardo-340XAA-350XAA-550XAA:/tmp$ python3 teste.py
Tamanho: 10
Num reps: 30

As somas obtidas foram: [30, 34, 30, 22, 26, 26, 30, 34, 40, 44, 14, 32, 40, 54, 24, 46, 16, 3
6, 42, 34, 30, 18, 12, 36, 14, 26, 12, 32, 10, 42]

Tabela de frequencia
    (-0.001, 10.0]  (10.0, 20.0]  (20.0, 30.0]  ...  (70.0, 80.0]  (80.0, 90.0]  (90.0, 100.0]
0                0             0             1  ...             0             0              0
1                0             0             0  ...             0             0              0
2                0             0             1  ...             0             0              0
3                0             0             1  ...             0             0              0
4                0             0             1  ...             0             0              0
5                0             0             1  ...             0             0              0
6                0             0             1  ...             0             0              0
7                0             0             0  ...             0             0              0
8                0             0             0  ...             0             0              0
9                0             0             0  ...             0             0              0
10               0             1             0  ...             0             0              0
11               0             0             0  ...             0             0              0
12               0             0             0  ...             0             0              0
13               0             0             0  ...             0             0              0
14               0             0             1  ...             0             0              0
15               0             0             0  ...             0             0              0
16               0             1             0  ...             0             0              0
17               0             0             0  ...             0             0              0
18               0             0             0  ...             0             0              0
19               0             0             0  ...             0             0              0
20               0             0             1  ...             0             0              0
21               0             1             0  ...             0             0              0
22               0             1             0  ...             0             0              0
23               0             0             0  ...             0             0              0
24               0             1             0  ...             0             0              0
25               0             0             1  ...             0             0              0
26               0             1             0  ...             0             0              0
27               0             0             0  ...             0             0              0
28               1             0             0  ...             0             0              0
29               0             0             0  ...             0             0              0

[30 rows x 10 columns]
```