

1) Dada a sequência de dimensões $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$ e a recorrência:

$$m[i, j] = \begin{cases} 0 & \text{se } i \geq j \\ \min_{i \leq k < j} \{ m[i, k] + p[i-1] \cdot p[k] \cdot p[j] + m[k+1, j] \} & \text{C.C.} \end{cases}$$

1 2

$$m[1, 1] + m[2, 2] + P_0 \cdot P_1 \cdot P_2 = 150 \checkmark$$

2 3

$$m[2, 2] + m[3, 3] + P_1 \cdot P_2 \cdot P_3 = 360 \checkmark \quad K=1$$

3 4

$$m[3, 3] + m[4, 4] + P_2 \cdot P_3 \cdot P_4 = 180 \checkmark \quad K=2$$

4 5

$$m[4, 4] + m[5, 5] + P_3 \cdot P_4 \cdot P_5 = 3000 \checkmark \quad K=3$$

5 6

$$m[5, 5] + m[6, 6] + P_4 \cdot P_5 \cdot P_6 = 1500 \checkmark \quad K=4$$

			1	2	3	4	5	6
5	P_0	1	0	150				
10	P_1	2	0	0	360			
3	P_2	3	0	0	0	180		
12	P_3	4	0	0	0	0	3K	
5	P_4	5	0	0	0	0	0	1,5K
50	P_5	6	0	0	0	0	0	0
6	P_6	7	0	0	0	0	0	0

1 3

$$m[1, 1] + m[2, 3] + P_0 \cdot P_1 \cdot P_3 = 960$$

$$m[1, 2] + m[3, 3] + P_0 \cdot P_2 \cdot P_3 = 330 \checkmark$$

2 4

$$m[2, 2] + m[3, 4] + P_1 \cdot P_2 \cdot P_4 = 330 \checkmark \quad K=2$$

$$m[2, 3] + m[4, 4] + P_1 \cdot P_3 \cdot P_4 = 960$$

3 5

$$m[3, 3] + m[4, 5] + P_2 \cdot P_3 \cdot P_5 = 4800$$

$$m[3, 4] + m[5, 5] + P_2 \cdot P_4 \cdot P_5 = 930 \checkmark$$

4 6

$$m[4, 4] + m[5, 6] + P_3 \cdot P_4 \cdot P_6 = 1860 \checkmark \quad K=4$$

$$m[4, 5] + m[6, 6] + P_3 \cdot P_5 \cdot P_6 = 6600$$

			1	2	3	4	5	6
5	P_0	1	0	150	330			
10	P_1	2	0	0	360	330		
3	P_2	3	0	0	0	180	930	
12	P_3	4	0	0	0	0	3K	1860
5	P_4	5	0	0	0	0	0	1,5K
50	P_5	6	0	0	0	0	0	0
6	P_6	7	0	0	0	0	0	0

$K=4$

1 4

$$m[1, 1] + m[2, 4] + P_0 * P_1 * P_4 = 580$$

$$m[1, 2] + m[3, 4] + P_0 * P_2 * P_4 = 405 \checkmark$$

$$m[1, 3] + m[4, 4] + P_0 * P_3 * P_4 = 630$$

2 5

$$m[2, 2] + m[3, 5] + P_1 * P_2 * P_5 = 2930 \checkmark$$

$$m[2, 3] + m[4, 5] + P_1 * P_3 * P_5 = 9360$$

$$m[2, 4] + m[5, 5] + P_1 * P_4 * P_5 = 2830$$

3 6

$$m[3, 3] + m[4, 6] + P_2 * P_3 * P_6 = 2076$$

$$m[3, 4] + m[5, 6] + P_2 * P_4 * P_6 = 1770 \checkmark$$

$$m[3, 5] + m[6, 6] + P_2 * P_5 * P_6 = 1830$$

$K = 4$

5	P_0
10	P_1
3	P_2
12	P_3
5	P_4
50	P_5
6	P_6

	1	2	3	4	5	6
1	0	150	330	405		
2	0	0	360	330	2430	
3	0	0	0	180	930	1770
4	0	0	0	0	3K	1860
5	0	0	0	0	0	1,5K
6	0	0	0	0	0	0
7	0	0	0	0	0	0

1 5

$$m[1, 1] + m[2, 5] + P_0 * P_1 * P_5 = 4930$$

$$m[1, 2] + m[3, 5] + P_0 * P_2 * P_5 = 1830$$

$$m[1, 3] + m[4, 5] + P_0 * P_3 * P_5 = 6330$$

$$m[1, 4] + m[5, 5] + P_0 * P_4 * P_5 = 1655 \checkmark$$

2 6

$$m[2, 2] + m[3, 6] + P_1 * P_2 * P_6 = 1950 \checkmark$$

$$m[2, 3] + m[4, 6] + P_1 * P_3 * P_6 = 2940$$

$$m[2, 4] + m[5, 6] + P_1 * P_4 * P_6 = 2130$$

$$m[2, 5] + m[6, 6] + P_1 * P_5 * P_6 = 5430$$

$K = 2$

5	P_0
10	P_1
3	P_2
12	P_3
5	P_4
50	P_5
6	P_6

	1	2	3	4	5	6
1	0	150	330	405	1655	
2	0	0	360	330	2430	1950
3	0	0	0	180	930	1770
4	0	0	0	0	3K	1860
5	0	0	0	0	0	1,5K
6	0	0	0	0	0	0
7	0	0	0	0	0	0

1 6

$$m[1, 1] + m[2, 6] + P_0 * P_1 * P_6 = 2250$$

$$m[1, 2] + m[3, 6] + P_0 * P_2 * P_6 = 2010 \checkmark$$

$$m[1, 3] + m[4, 6] + P_0 * P_3 * P_6 = 2550$$

$$m[1, 4] + m[5, 6] + P_0 * P_4 * P_6 = 2055$$

$$m[1, 5] + m[6, 6] + P_0 * P_5 * P_6 = 3155$$

$K = 2$

5	P_0
10	P_1
3	P_2
12	P_3
5	P_4
50	P_5
6	P_6

	1	2	3	4	5	6
1	0	150	330	405	1655	2010
2	0	0	360	330	2430	1950
3	0	0	0	180	930	1770
4	0	0	0	0	3K	1860
5	0	0	0	0	0	1,5K
6	0	0	0	0	0	0
7	0	0	0	0	0	0

O valor de K destacado representa o índice, no vetor de dimensões, do menor valor obtido ao calcularmos as recorrências. Os itens:

	1	2	3	4	5	6
1	0	1	2	2	4	2
2	0	0	2	2	2	2
3	0	0	0	3	4	4
4	0	0	0	0	4	4
5	0	0	0	0	0	5
6	0	0	0	0	0	0
7	0	0	0	0	0	0

5	p_0
10	p_1
3	p_2
12	p_3
5	p_4
50	p_5
6	p_6

Com essa matriz, vemos que o grupo final da parentização tem $K=2$, que equivale a $p_K=3$, então a última multiplicação é $5 \times 3 \cdot 3 \times 6$, então:

$$\begin{aligned}
 5 \times 6 &= 5 \times 3 \cdot 3 \times 6 \\
 &= (5 \times 10 \cdot 10 \times 3) \cdot (3 \times 5 \cdot 5 \times 6) \\
 &= ((5 \times 10 \cdot 10 \times 3) \cdot ((3 \times 12 \cdot 12 \times 5) \cdot (5 \times 50 \cdot 50 \cdot 6))) //
 \end{aligned}$$

Achamos a parentização ótima.