① a) p = Roberto estava com ciúme de Irene ; q = ele não estava de bom humor.
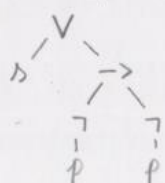
    $p \vee q$.

b) p = o barômetro indicar queda de pressão ; q = irá chover ; r = irá nevar.

    $p \to q \vee r$.

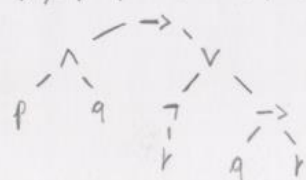c) p = Smith instalou o aquecimento central ; q = ele vendeu seu carro ; r = não pagou sua hipoteca.

    $p \to q \vee r$.

③ a) $(s \vee ((\neg p) \to (\neg p)))$ Subfórmulas.



    $s$ ;
    $p$ (2 vezes) ;
    $\neg p$ (2 vezes) ;
    $\neg p \to \neg p$ ;
    $s \vee (\neg p \to \neg p)$ ;

b) $(p \wedge q) \to (\neg r \vee (q \to r))$



Subfórmulas:

    $p$ ;
    $q$ (2x) ;
    $r$ (2x) ;
    $\neg r$ ;
    $p \wedge q$ ;
    $q \to r$ ;
    $\neg r \vee (q \to r)$ ;
    $(p \wedge q) \to (\neg r \vee (q \to r))$ ;

**② a)** $p \to (q \to r), p \to q \vdash p \to r$

1- $p \to (q \to r)$;

2- $p \to q$;

3- $\Gamma p$;

4- $\quad q \quad$ MP(3,2);

5- $\quad q \to r$ MP(3,1);

6- $L r \quad$ MP(4,5);

7- $p \to r \quad \to i(3\text{-}6)$;

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**b)** $p \to q \vdash \neg p \vee q$

1- $p \to q$

2- $p \vee \neg p$ LTE

3- $\Gamma p$

4- $| q \quad$ MP(3,1)

5- $| \neg p \vee q \quad V_i(q)$

6- $\Gamma \neg p$

7- $| \neg p \vee q \quad V_i(6)$

8- $\neg p \vee q \quad V_e(2, 3\text{-}5, 6\text{-}7)$

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**c)** $p \to (q \vee r), \neg q, \neg r \vdash \neg p$

1- $p \to (q \vee r)$;

2- $\neg q$;

3- $\neg r$;

4- $p \vee \neg p$ LTE;

5- $\Gamma p$;

6- $| q \vee r \quad$ MP(5,1);

7- $| \Gamma q$;

8- $| L \bot \quad \neg e(2,7)$;

9- $| \Gamma r$;

10- $| L \bot \quad \neg e(3,9)$;

11- $| \bot \quad V_e(6, 7\text{-}8, 9\text{-}10)$;

12- $| \neg p \quad \bot e(11)$;

13- $\Gamma \neg p$;

14- $\neg p \quad V_e(4, 5\text{-}12, 13)$;

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**e)** $p \wedge q \to r \vdash (p \to r) \vee (q \to r)$

1- $p \wedge q \to r$

2- $\Gamma p$

3- $1 \Gamma q$

4- $1 1 p \wedge q \quad \wedge i_2 (2,3)$

5- $1 L r \quad$ MP(4,1)

6- $L q \to r \quad \to i(3\text{-}5)$

7- $p \to (q \to r) \quad \to i(2\text{-}6)$

8- $p \vee \neg p \quad$ LTE

9- $\Gamma p$

10- $L q \to r \quad$ MP(9,7)

11- $\Gamma \neg p$

12- $1 \Gamma p$

13- $1 L \bot \quad \neg e(12,11)$

14- $L q \to r \quad \bot e(13)$

15- $q \to r \quad V_e(8, 9\text{-}10, 11\text{-}14)$

16- $(p \to r) \vee (q \to r) \quad V_{i_2}(15)$

d) $q \vdash (p \wedge q) \vee (\neg p \wedge q)$

1 - $q$;

2 - $p \vee \neg p$;

3 - $\ulcorner p$;

4 - $\lceil p \wedge q \quad \wedge_i (3,1)$;

5 - $\lfloor (p \wedge q) \vee (\neg p \wedge q) \quad \vee_{i_1} (4)$;

6 - $\ulcorner \neg p$;

7 - $\lceil \neg p \wedge q \quad \wedge_i (6,1)$;

8 $\lfloor (p \wedge q) \vee (\neg p \wedge q) \quad \vee_{i_2} (7)$;

9 - $(p \wedge q) \vee (\neg p \wedge q) \quad \vee_e (2, 3\text{-}5, 6\text{-}8)$;

g) $(s \rightarrow p) \vee (t \rightarrow q) \vdash (s \rightarrow q) \vee (t \rightarrow p)$.

1 - $(s \rightarrow p) \vee (t \rightarrow q)$

2 - $\ulcorner s \rightarrow p$

3 - $\mid \ulcorner s$

4 - $\mid \mid p \quad MP(3,2)$

5 - $\mid \mid \ulcorner \neg p$

6 - $\mid \mid \lfloor \bot \quad \neg e (4,5)$

7 - $\mid \lfloor q \quad \bot e (6)$

8 - $\lfloor s \rightarrow q \quad \rightarrow_i (3\text{-}7)$

9 - $\ulcorner t \rightarrow q$

10 - $\mid \ulcorner \neg (t \rightarrow q)$

11 - $\mid \lfloor \bot \quad \neg e (9,10)$

12 $\lfloor s \rightarrow q \quad \bot e (11)$

13 - $s \rightarrow q \quad \vee_e (1, 2\text{-}9, 9\text{-}12)$

14 - $(s \rightarrow q) \vee (t \rightarrow p) \quad \vee_{i_1} (13)$