

# pruebas ar(1)

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Generamos la función para calcular la Corelación integral

```
# Definir la función para calcular la Corelación integral
correlation_integral <- function(data, r, d) {
  N <- length(data)
  count <- 0

  # Iterar sobre todos los pares (i, j) tal que 1 <= i < j <= N-d+1
  for (i in 1:(N-d)) {
    for (j in (i+1):(N-d+1)) {
      # Crear los vectores u_i^d y u_j^d
      u_i_d <- data[i:(i+d-1)]
      u_j_d <- data[j:(j+d-1)]

      # Calcular la distancia euclidiana entre u_i^d y u_j^d
      distance <- sqrt(sum((u_i_d - u_j_d)^2))

      # Verificar si la distancia es menor que r
      if (distance < r) {
        count <- count + 1
      }
    }
  }

  # Calcular C_N(r, d)
  C_N <- (2 / (N^2)) * count
  return(C_N)
}
```

Ahora generamos la función para el  $X_n(r,d)$

```
# Definir la función X_N(r, d)
X_N <- function(data, r, d) {
  C_N_d <- correlation_integral(data, r, d)
  C_N_d_minus_1 <- correlation_integral(data, r, d-1)
  C_N_d_plus_1 <- correlation_integral(data, r, d+1)

  X_N_value <- (C_N_d^2) / (C_N_d_minus_1 * C_N_d_plus_1)
  return(X_N_value)
}
```

```

# Caso especial para d = 1
X_N_d_1 <- function(data, r) {
  C_N_1 <- correlation_integral(data, r, 1)
  C_N_2 <- correlation_integral(data, r, 2)

  X_N_value <- (C_N_1^2) / C_N_2
  return(X_N_value)
}

```

pruebas con caso AR(1)

Generamos 1000 series de 1000 observaciones

```

set.seed(125) # Fijamos la semilla

# Definimos parámetros
n_series <- 1000 # Número de series a generar
n_obs <- 1000 # Número de observaciones por serie
phi <- 0.5 # Coeficiente AR(1)
sigma <- 1 # Desviación estándar del ruido

# Matriz para almacenar las series AR(1)
ar1_series <- matrix(0, nrow = n_obs, ncol = n_series)

# Generamos las series AR(1)
for (i in 1:n_series) {
  ar1_series[, i] <- arima.sim(n = n_obs, list(ar = phi), sd = sigma)
}

```

Obtenemos los valores de  $X_N(r, d)$  para los casos  $d=1$  hasta  $d=5$

```

# Parámetros
r <- 1 # Umbral para la norma
d_values <- 1:5 # Valores de d

# Inicializar listas para almacenar los resultados
X_N_results_ar <- matrix(0, nrow = n_series, ncol = length(d_values))
colnames(X_N_results_ar) <- paste("d =", d_values)

# Calcular  $X_{\{N\}}(r, d)$  para cada serie y para cada valor de d
for (i in 1:n_series) {
  series <- ar1_series[i, ]

  # Caso especial para d = 1
  X_N_results_ar[i, 1] <- X_N_d_1(series, r)

  # Calcular  $X_{\{N\}}(r, d)$  para d = 2, 3, 4, 5
  for (d in 2:length(d_values)) {
    X_N_results_ar[i, d] <- X_N(series, r, d_values[d])
  }
}

# Ver los resultados
head(X_N_results_ar)

```

```
##          d = 1      d = 2      d = 3      d = 4      d = 5
## [1,] 1.227202 1.165312 1.149121 1.093141 1.096733
## [2,] 1.229623 1.159305 1.136260 1.065084 1.091703
## [3,] 1.230952 1.193624 1.113393 1.085511 1.038592
## [4,] 1.233810 1.166031 1.095355 1.120412 1.079079
## [5,] 1.248054 1.173693 1.120600 1.066303 1.050376
## [6,] 1.251253 1.151497 1.130670 1.070827 1.077997
```

X\_N\_results\_ar

```
##          d = 1      d = 2      d = 3      d = 4      d = 5
## [1,] 1.227202 1.165312 1.149121 1.093141 1.0967330
## [2,] 1.229623 1.159305 1.136260 1.065084 1.0917028
## [3,] 1.230952 1.193624 1.113393 1.085511 1.0385925
## [4,] 1.233810 1.166031 1.095355 1.120412 1.0790790
## [5,] 1.248054 1.173693 1.120600 1.066303 1.0503758
## [6,] 1.251253 1.151497 1.130670 1.070827 1.0779973
## [7,] 1.235518 1.156911 1.135782 1.107756 1.0624251
## [8,] 1.238010 1.167939 1.107932 1.113138 1.1438308
## [9,] 1.237938 1.154925 1.157509 1.075829 1.1217983
## [10,] 1.227096 1.163900 1.121680 1.087332 1.1180256
## [11,] 1.242101 1.167325 1.126097 1.082104 1.0680353
## [12,] 1.225548 1.177546 1.116398 1.096858 1.1215950
## [13,] 1.237889 1.144721 1.108313 1.095145 1.0260852
## [14,] 1.230976 1.177597 1.135601 1.099198 1.0649193
## [15,] 1.235276 1.175101 1.120012 1.125186 1.0155855
## [16,] 1.246237 1.174288 1.115356 1.104871 0.9261637
## [17,] 1.230391 1.154632 1.115929 1.118096 1.1322692
## [18,] 1.242929 1.165292 1.144558 1.115934 1.1562500
## [19,] 1.245662 1.165180 1.126609 1.101671 0.9957461
## [20,] 1.242276 1.162944 1.128033 1.128723 1.0093233
## [21,] 1.237143 1.155909 1.126877 1.140042 1.0818434
## [22,] 1.236965 1.159011 1.130771 1.128862 1.1143459
## [23,] 1.235219 1.164252 1.119856 1.125817 1.0179728
## [24,] 1.224111 1.148262 1.129876 1.086142 1.1314644
## [25,] 1.224493 1.171002 1.137010 1.109349 1.1077899
## [26,] 1.230034 1.155407 1.124942 1.120618 1.1196453
## [27,] 1.221787 1.156657 1.133583 1.103942 1.1335687
## [28,] 1.231697 1.163187 1.127522 1.127219 1.1668850
## [29,] 1.243750 1.181879 1.123013 1.090477 1.0588036
## [30,] 1.236520 1.178470 1.128649 1.109404 1.1779636
## [31,] 1.230895 1.161223 1.113862 1.107660 1.1347838
## [32,] 1.229816 1.177754 1.144422 1.126239 1.1586853
## [33,] 1.259318 1.149190 1.118940 1.104228 1.0117233
## [34,] 1.231623 1.160917 1.144201 1.121228 1.0923844
## [35,] 1.245891 1.170380 1.118161 1.105671 1.0696688
## [36,] 1.234344 1.154989 1.120510 1.072493 1.0867168
## [37,] 1.227432 1.157306 1.115125 1.111471 1.0297653
## [38,] 1.227973 1.151225 1.114032 1.121350 1.0680214
## [39,] 1.238595 1.138787 1.119076 1.083759 1.0754757
## [40,] 1.254039 1.167256 1.131297 1.112314 1.0516182
## [41,] 1.241383 1.174292 1.105207 1.108143 1.1018744
## [42,] 1.243463 1.159096 1.137510 1.122999 1.0509969
## [43,] 1.237980 1.149978 1.146976 1.116803 1.0741386
```

```

## [44,] 1.215651 1.143260 1.138030 1.094259 1.0596669
## [45,] 1.240338 1.189375 1.130124 1.117215 1.0964291
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## [48,] 1.245180 1.163402 1.119949 1.110351 1.1618783
## [49,] 1.232406 1.155671 1.116388 1.117812 0.9944239
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## [61,] 1.204714 1.181167 1.137983 1.131482 1.1146693
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## [66,] 1.239011 1.179278 1.121468 1.115284 1.0903141
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## [74,] 1.233532 1.160468 1.115881 1.106061 1.0968423
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## [97,] 1.250042 1.167597 1.122817 1.094906 1.0866268

```

```

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```

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## [200,] 1.228466 1.165761 1.132124 1.114870 1.1175649
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## [204,] 1.242845 1.171342 1.118721 1.080093 1.1327544
## [205,] 1.245613 1.170418 1.138029 1.110478 1.0832495

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## [206,] 1.235750 1.148086 1.117279 1.113630 1.0871174
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## [999,] 1.240807 1.156180 1.130819 1.112842 1.0337487
## [1000,] 1.237345 1.154270 1.121803 1.062194 1.1266228
```

Veamos como se comporta  $X_N(r, d)$  para este caso

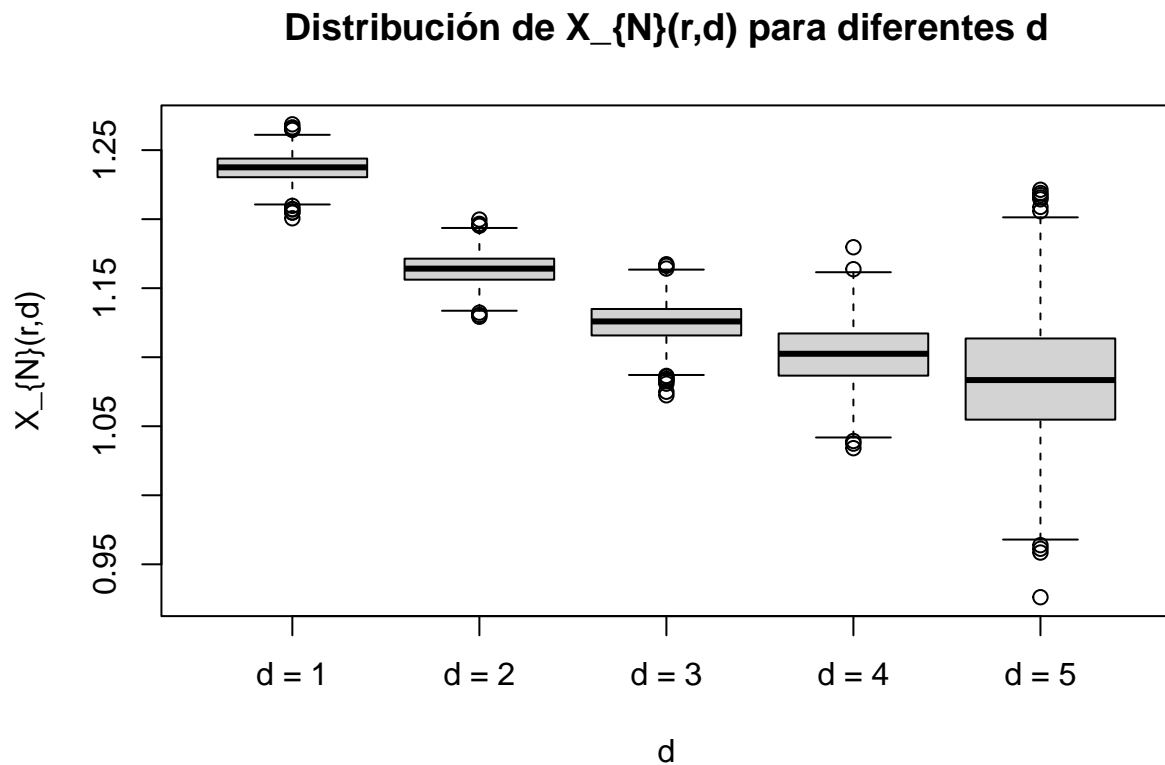
```
# Resumen de los resultados
summary(X_N_results_ar)
```

```
##      d = 1      d = 2      d = 3      d = 4
## Min.   :1.201   Min.   :1.129   Min.   :1.072   Min.   :1.034
## 1st Qu.:1.230   1st Qu.:1.156   1st Qu.:1.116   1st Qu.:1.087
## Median :1.238   Median :1.164   Median :1.126   Median :1.102
## Mean   :1.237   Mean   :1.164   Mean   :1.125   Mean   :1.102
## 3rd Qu.:1.244   3rd Qu.:1.171   3rd Qu.:1.135   3rd Qu.:1.117
## Max.   :1.269   Max.   :1.200   Max.   :1.167   Max.   :1.180
##      d = 5
```

```
## Min.    :0.9262
## 1st Qu.:1.0548
## Median :1.0834
## Mean   :1.0851
## 3rd Qu.:1.1136
## Max.    :1.2213
```

```
# Graficar los resultados
```

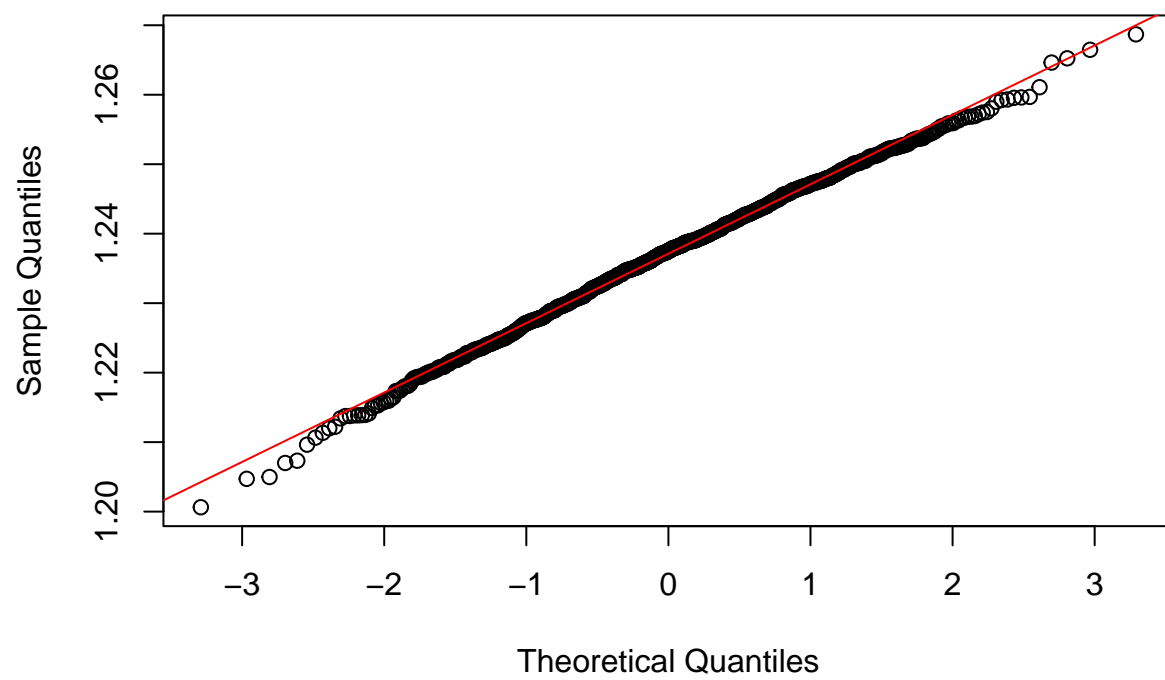
```
boxplot(X_N_results_ar, main="Distribución de  $X_{\{N\}}(r,d)$  para diferentes d", ylab=" $X_{\{N\}}(r,d)$ ", xlab="d")
```



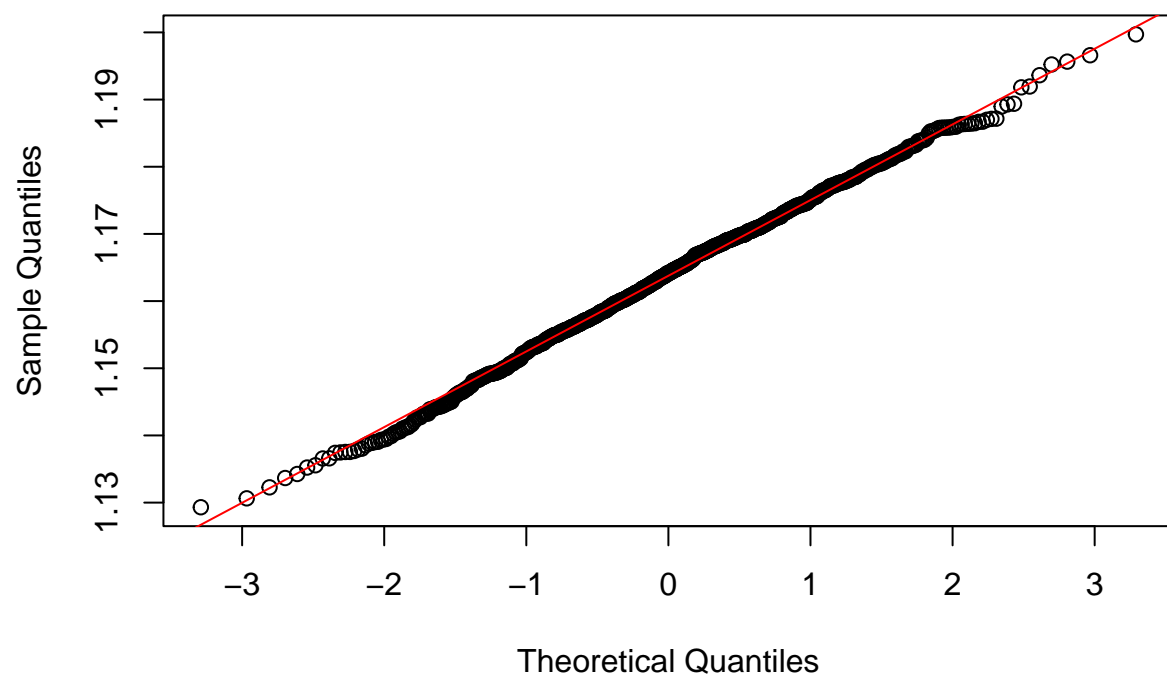
```
# Crear el QQ plot
```

```
for (i in 1:ncol(X_N_results_ar)) {
  qqnorm(X_N_results_ar[, i], main = paste("QQ Plot - Caso d=", i))
  qqline(X_N_results_ar[, i], col = "red")
}
```

QQ Plot – Caso d= 1

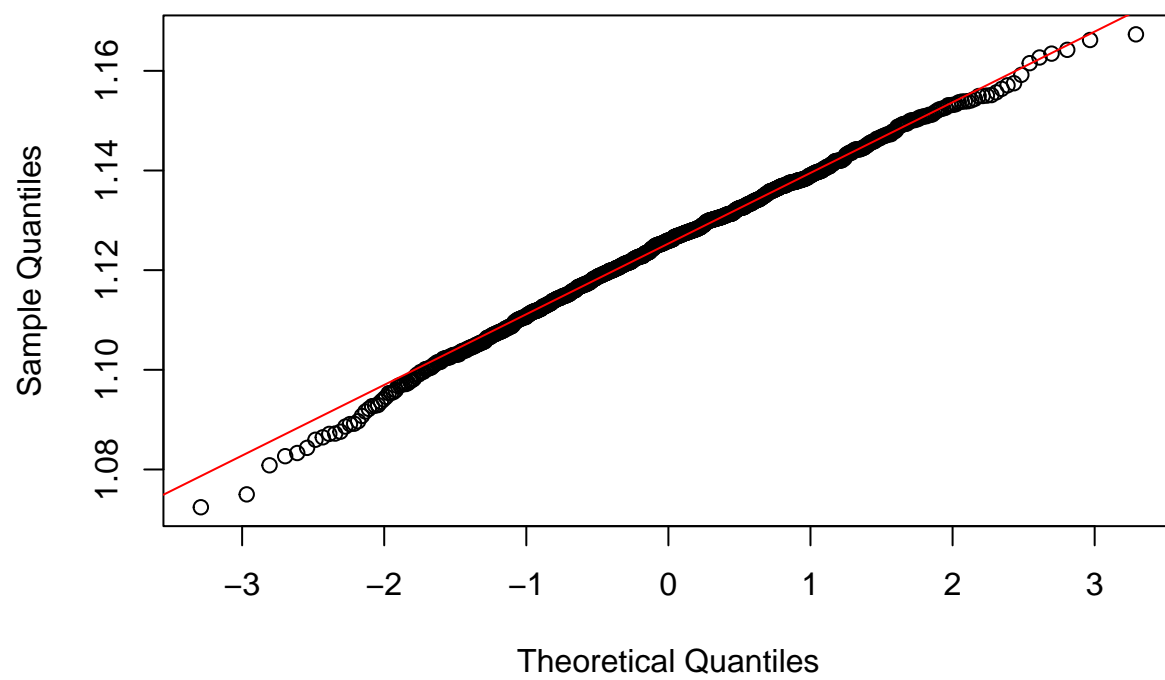


QQ Plot – Caso d= 2

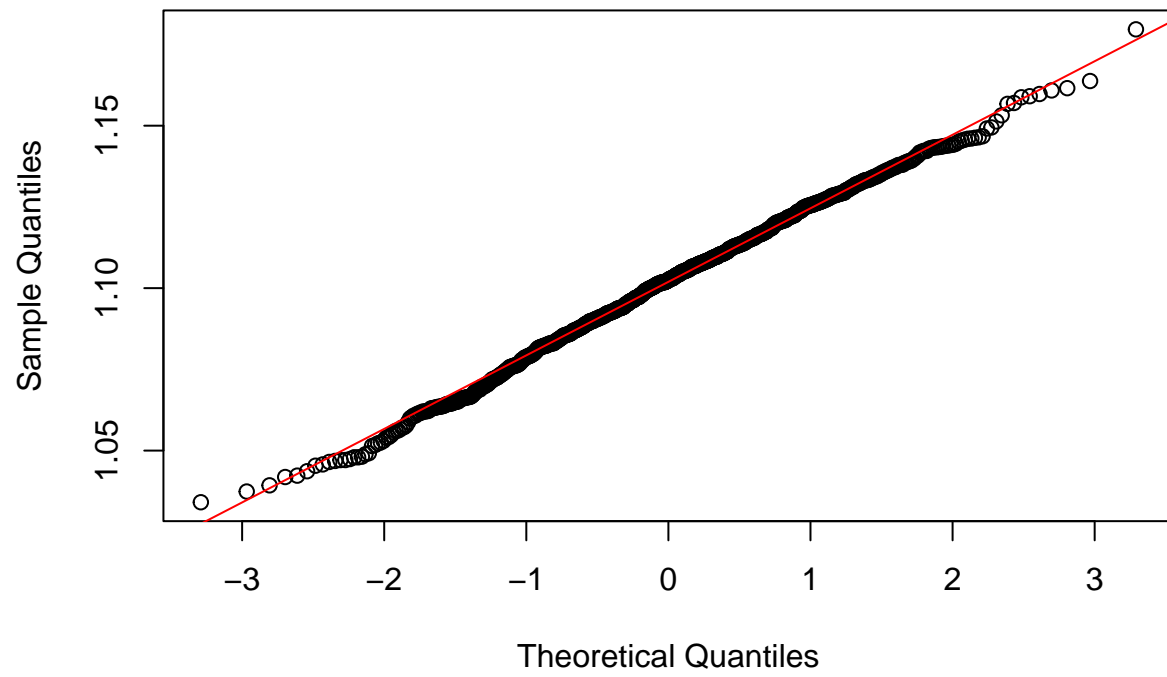




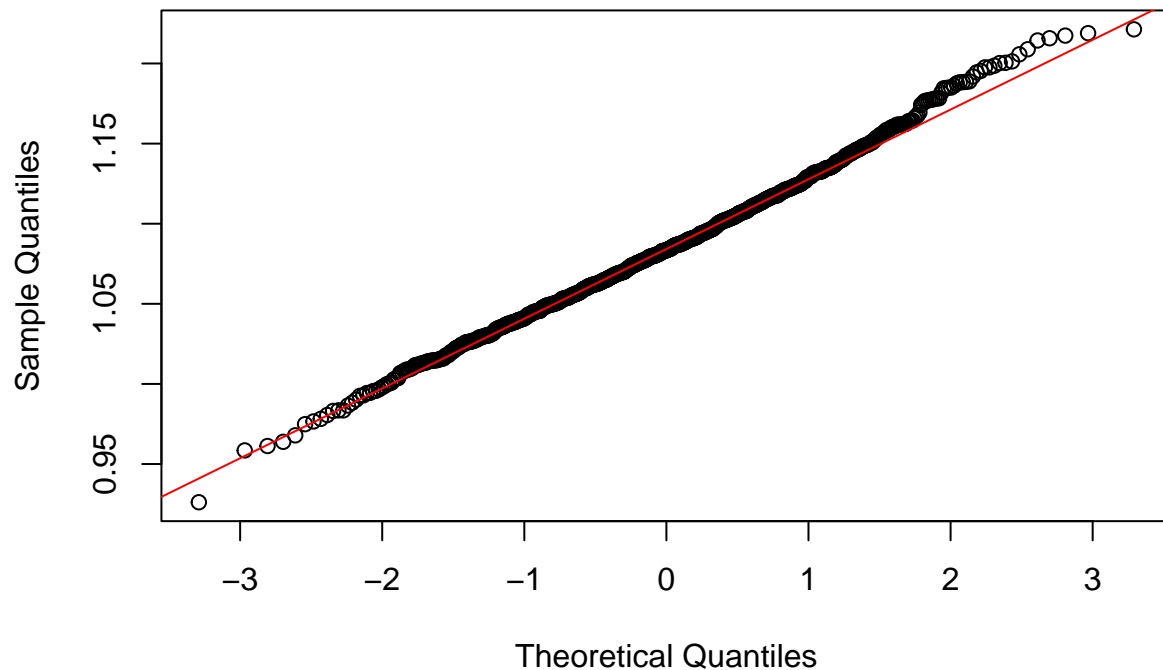
QQ Plot – Caso d= 3



QQ Plot – Caso d= 4



## QQ Plot – Caso d= 5



Se puede observar en los gráficos que se mantiene y hasta se ve de manera mas clara la tendencia.

```
# Longitud de cada serie
N <- 1000

# Calcular sqrt(N)
sqrt_N <- sqrt(N)

# Calcular sqrt(N) * ln(X_{N}(r,d)) para cada serie y cada valor de d
sqrt_N_ln_X_N_ar <- sqrt_N * log(X_N_results_ar)

# Ver los resultados
head(sqrt_N_ln_X_N_ar)
```

```
##           d = 1    d = 2    d = 3    d = 4    d = 5
## [1,] 6.474355 4.837946 4.395469 2.816168 2.919915
## [2,] 6.536680 4.674509 4.039552 1.993927 2.774539
## [3,] 6.570840 5.597032 3.396660 2.594679 1.197441
## [4,] 6.644166 4.857441 2.880144 3.595397 2.406744
## [5,] 7.007156 5.064557 3.600712 2.030113 1.554196
## [6,] 7.088098 4.460808 3.883607 2.163972 2.375028
```

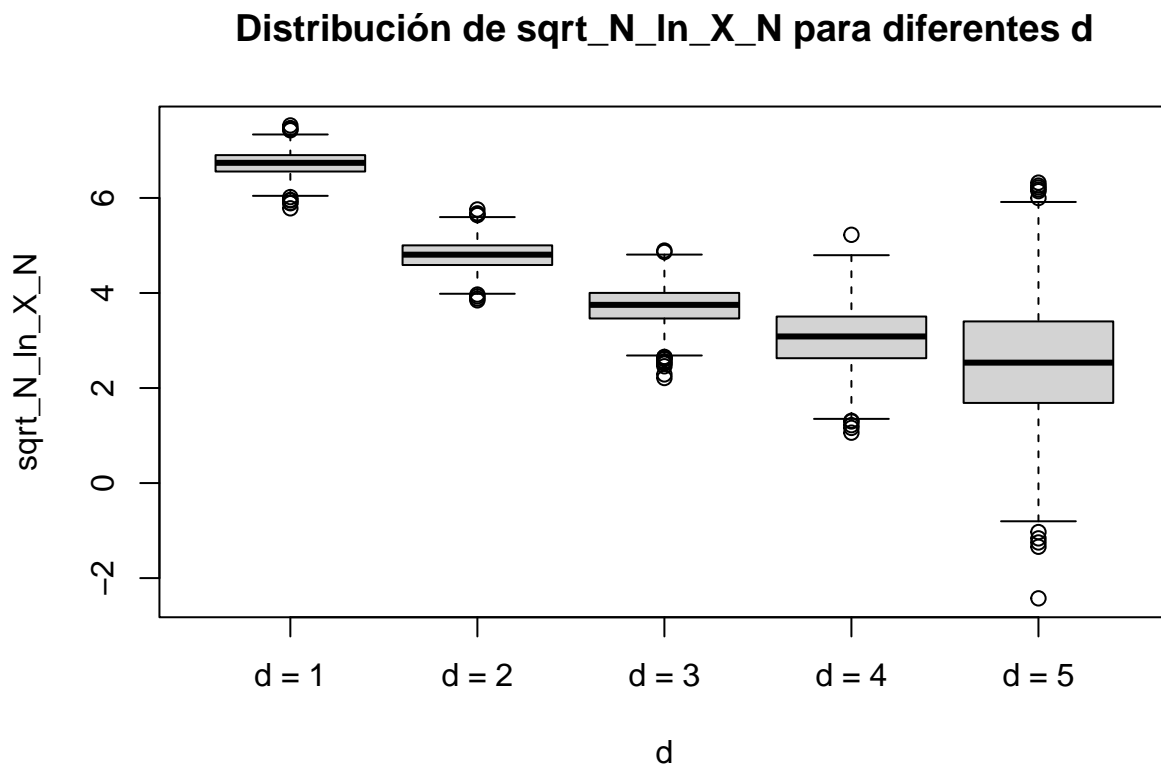
y vemos como se comporta:

```
# Resumen de los resultados
summary(sqrt_N_ln_X_N_ar)
```

```
##      d = 1      d = 2      d = 3      d = 4
## Min.   :5.782   Min.   :3.846   Min.   :2.211   Min.   :1.061
## 1st Qu.:6.556   1st Qu.:4.589   1st Qu.:3.464   1st Qu.:2.629
## Median :6.739   Median :4.807   Median :3.751   Median :3.086
## Mean   :6.726   Mean   :4.795   Mean   :3.726   Mean   :3.056
## 3rd Qu.:6.901   3rd Qu.:5.002   3rd Qu.:4.002   3rd Qu.:3.505
## Max.   :7.526   Max.   :5.758   Max.   :4.892   Max.   :5.225
##      d = 5
## Min.   : -2.426
## 1st Qu.: 1.688
## Median : 2.534
## Mean   : 2.557
## 3rd Qu.: 3.401
## Max.   : 6.322
```

```
# Graficar los resultados
```

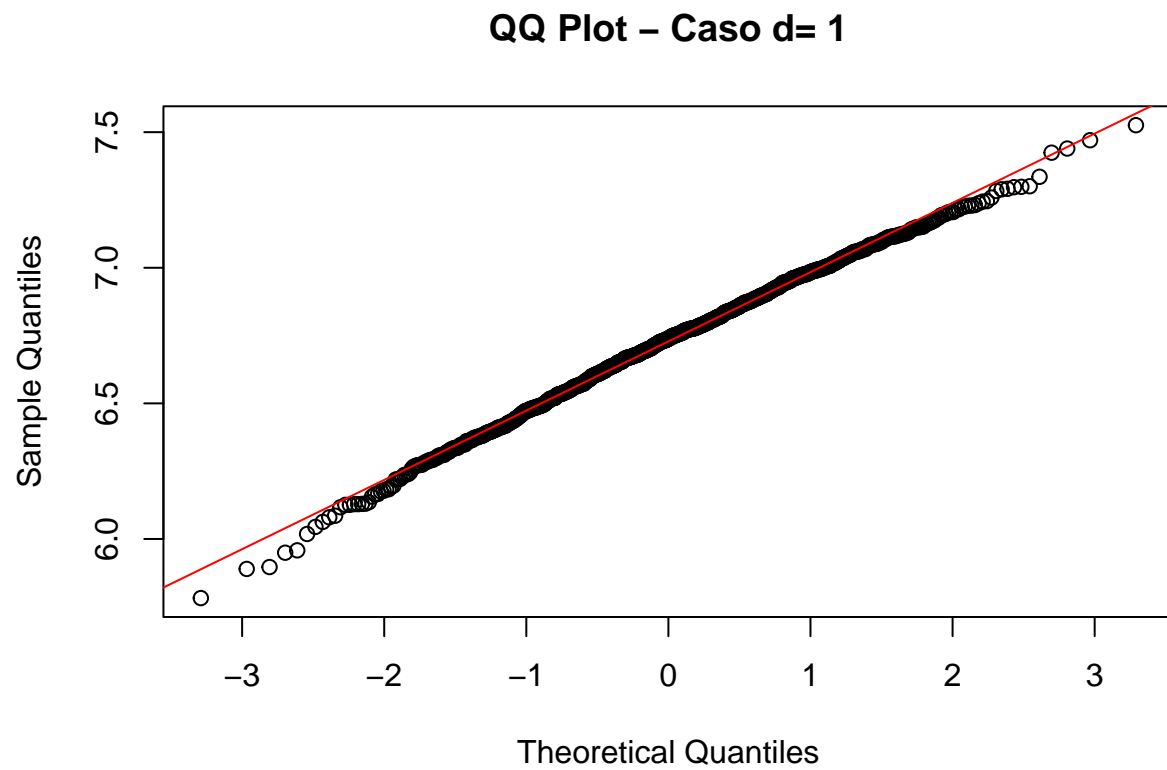
```
boxplot(sqrt_N_ln_X_N_ar, main="Distribución de sqrt_N_ln_X_N para diferentes d", ylab="sqrt_N_ln_X_N",
```



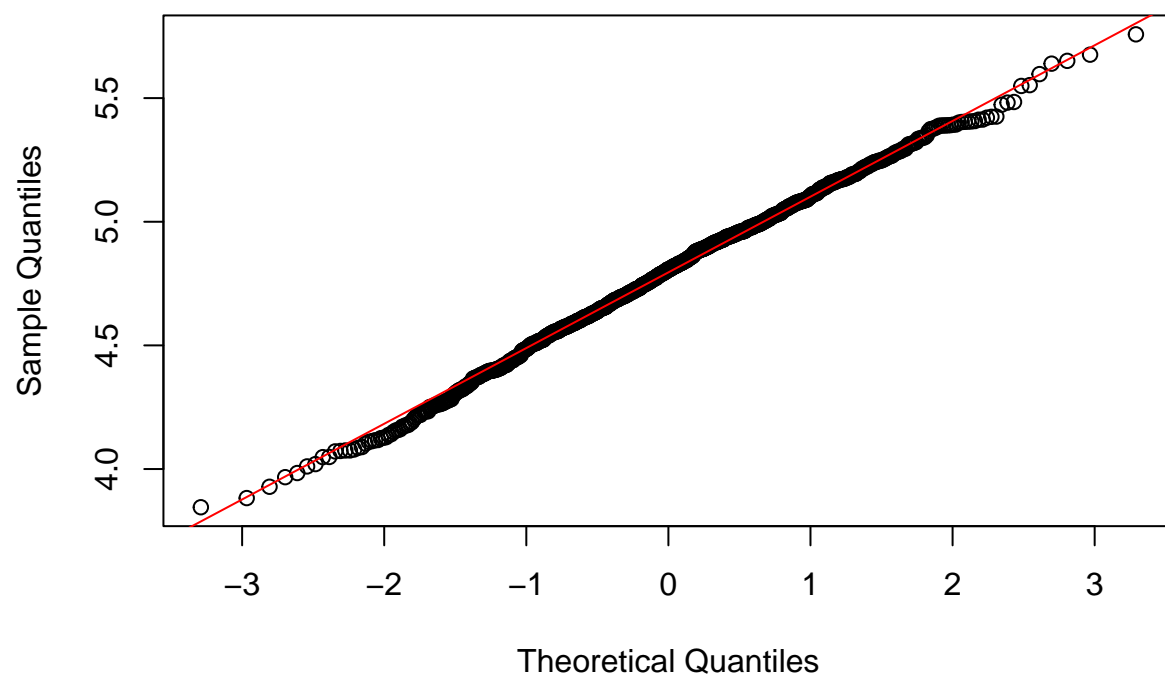
```
# Crear el QQ plot
```

```
for (i in 1:ncol(sqrt_N_ln_X_N_ar)) {
```

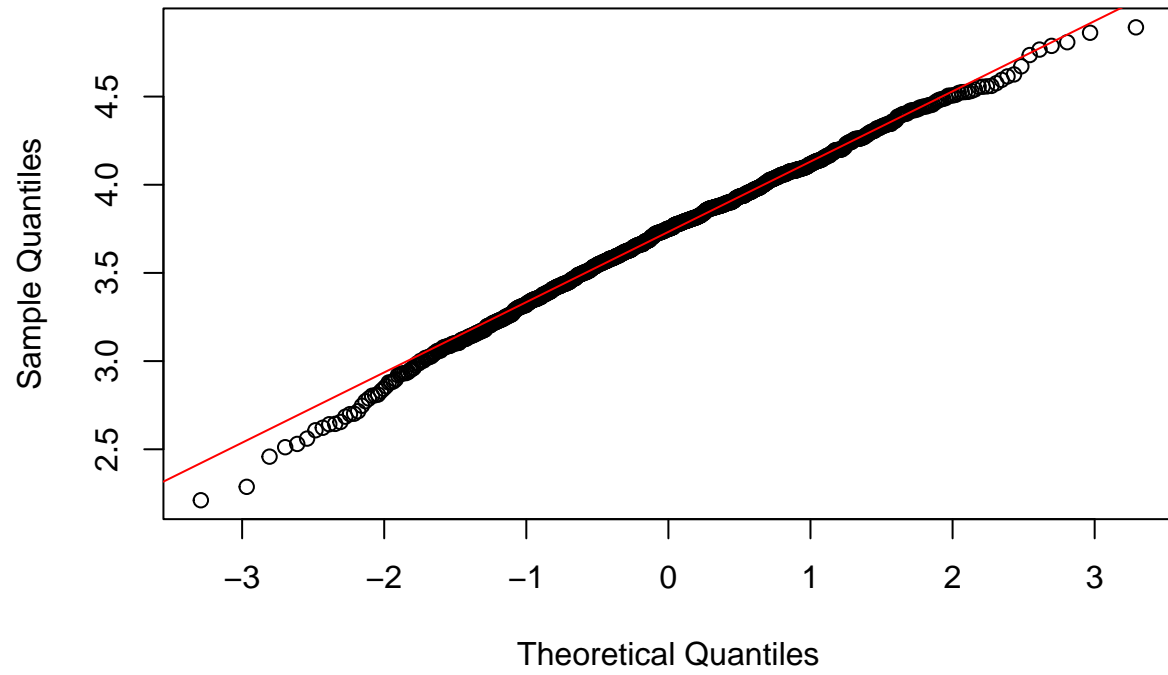
```
qqnorm(sqrt_N_ln_X_N_ar[, i], main = paste("QQ Plot - Caso d=", i))  
qqline(sqrt_N_ln_X_N_ar[, i], col = "red")  
}
```



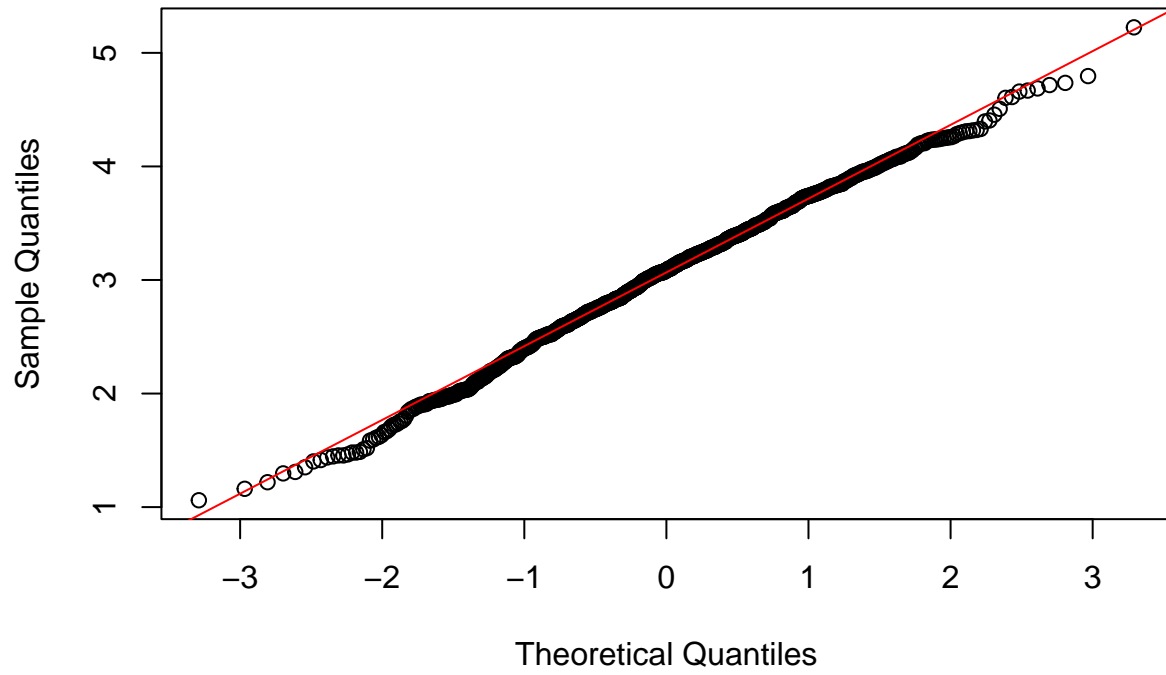
QQ Plot – Caso d= 2



QQ Plot – Caso d= 3

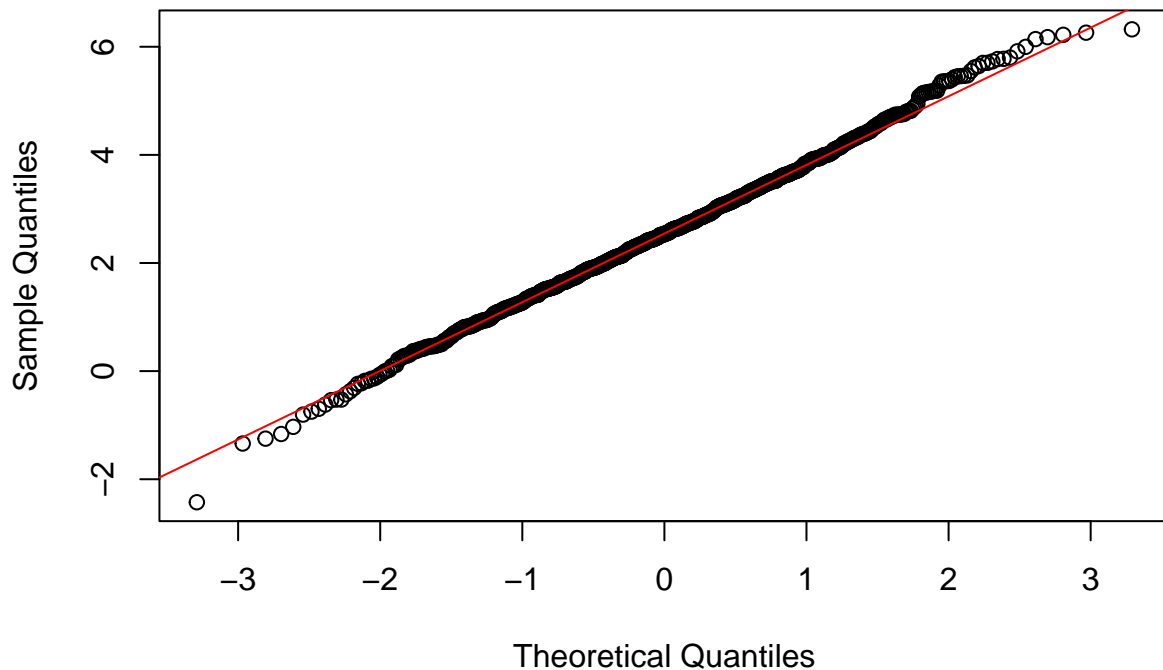


QQ Plot – Caso d= 4





## QQ Plot – Caso d= 5



Para el caso  $AR(1)$  también se pudo observar que  $\sqrt{N} \ln(X_N(r, d))$  mantenía una tendencia a normalidad asintótica en menor medida, teniendo problemas mas tempranos con los valores de  $d$  en contrate con el caso IID, también al aumentar de 100 a 500 y a 1000 series se pudo observar aun mas el aumento de esta tendencia. Durante las pruebas con 500 series para el caso  $d=5$  algunas veces obtenían correlaciones integrales 0, por lo que valores indefinidos para  $X_N(r, d)$ , esto no se dio en la prueba con 1000 series  $AR(1)$ .

```
library(MASS)
```

test por aplicar:

```
# Función Shapiro-Wilk test
shapiro_test <- function(series) {
  # Shapiro-Wilk test
  shapiro.test(series)
}

# Aplicar el Shapiro-Wilk test a cada columna
shapiro_results <- apply(sqrt_N_ln_X_N_ar, 2, function(column) {
  # Quitar valores NA
  column <- na.omit(column)
  shapiro_test(column)
})

# p-values
shapiro_p_values <- sapply(shapiro_results, function(x) x$p.value)
```

```
shapiro_p_values
```

```
##          d = 1          d = 2          d = 3          d = 4          d = 5
## 0.043424692 0.157500112 0.008344508 0.024657969 0.301237807
```

```
# Función del Kolmogorov-Smirnov test
ks_test <- function(series) {
  # K-S test comparando la serie a una distribución normal
  ks.test(series, "pnorm", mean = mean(series), sd = sd(series))
}

# Aplicar el K-S test a cada columna
ks_results <- apply(sqrt_N_ln_X_N_ar, 2, function(column) {
  # Remove NA values if they exist
  column <- na.omit(column)
  ks_test(column)
})

# p-values
ks_p_values <- sapply(ks_results, function(x) x$p.value)

ks_p_values
```

```
##          d = 1          d = 2          d = 3          d = 4          d = 5
## 0.4708963 0.2835113 0.2613561 0.4574208 0.8759034
```