



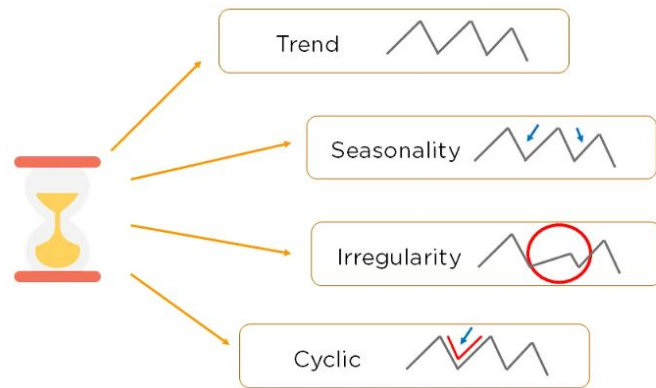
Minería de datos G.571

Pronóstico: ARFIMA

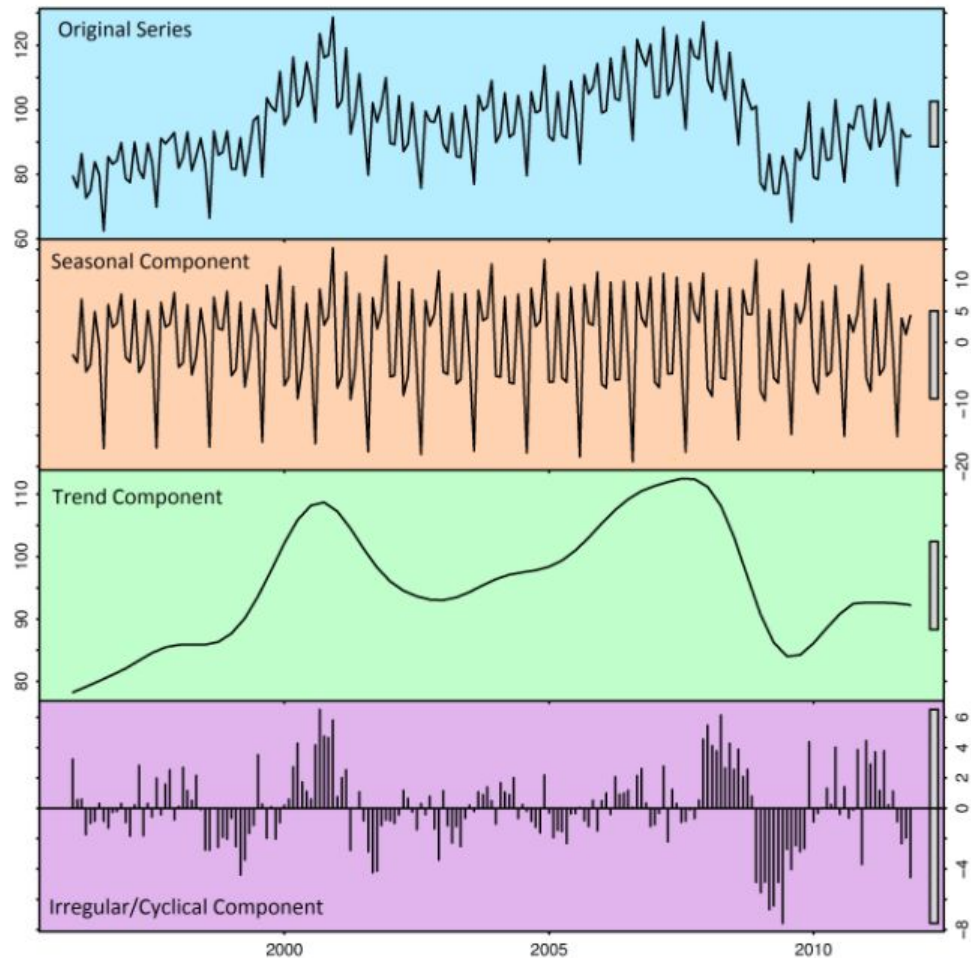
Minería de datos
Zavala Roman Irvin Eduardo 1270771

Series de tiempo

- **Tendencia:** Movimiento vertical de los datos en tiempos largos
- **Estacionalidad:** Indican comportamientos que suceden en lapsos regulares
- **Irregularidad:** Comportamiento que no pertenece a los 2 anteriores
- **Cíclico:** Oscilaciones que duran más de un año, puede no ser periodico
- **Estacionario:** Tiene las mismas propiedades estadísticas en todo el tiempo (promedio, varianza y covarianza constantes)



Series de tiempo



Autoregressive Model (AR)

Dice que los valores anteriores afectan los valores actuales.

$$\hat{y}_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \epsilon_t$$

$$y_t = a_0 + \sum_{n=1}^p a_n y_{t-n} + \epsilon_t$$

Moving Average Model (MA)

Dice que el valor de la variable dependiente actual depende de los términos de error de los días anteriores.

$$y_t = \varepsilon_t + \alpha_1 \varepsilon_{t-1} + \alpha_2 \varepsilon_{t-2} + \cdots + \alpha_q \varepsilon_{t-q}$$

$$r_t = b_0 + \sum_{n=1}^q b_n r_{t-n} + \epsilon_t$$

$$r_t = \hat{y}_t - y_t$$

Auto Regressive Moving Average (ARMA)

Combina los modelos AR y MA.

$$y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} + \cdots + \beta_p y_{t-p} + \varepsilon_t + \alpha_1 \varepsilon_{t-1} + \alpha_2 \varepsilon_{t-2} + \cdots + \alpha_q \varepsilon_{t-q}$$

Integrated (I)

Fuerza la estacionalidad de la serie de tiempo por medio de un proceso de “diferenciación”.

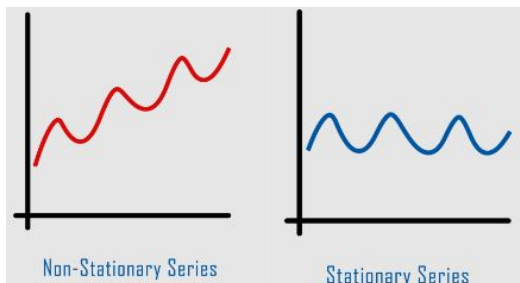
$By_t = y_{t-1}$ where B is called a backshift operator

Thus, a first order difference is written as

$$y'_t = y_t - y_{t-1} = (1 - B)y_t$$

In general, a d th-order difference can be written as

$$y'_t = (1 - B)^d y_t$$



```
dataset = pd.read_csv("MG_dataset.csv", on_bad_lines='skip')
dataset = dataset.shift(1)
dataset['1.2'][0] = 1.2
dataset = dataset.rename(columns={"1.2": "data"})
print(dataset)
print(dataset.diff())
```

```
      data
0  1.2000000000
1  1.0858000000
2  0.9824800000
3  0.8889800000
4  0.8043800000
...
1195 0.9211900000
1196 0.9514500000
1197 0.9716200000
1198 0.9828300000
1199 0.9864000000
```

```
[1200 rows x 1 columns]
      data
0      NaN
1 -0.1142000000
2 -0.1033200000
3 -0.0935000000
4 -0.0846000000
...
1195 0.0411800000
1196 0.0302600000
1197 0.0201700000
1198 0.0112100000
1199 0.0035700000
```

```
[1200 rows x 1 columns]
```

AutoRegressive Integrated Moving Average (ARIMA)

Junta todo lo anterior, generalmente tiene 3 parámetros:

p : Número de términos autorregresivos (AR)

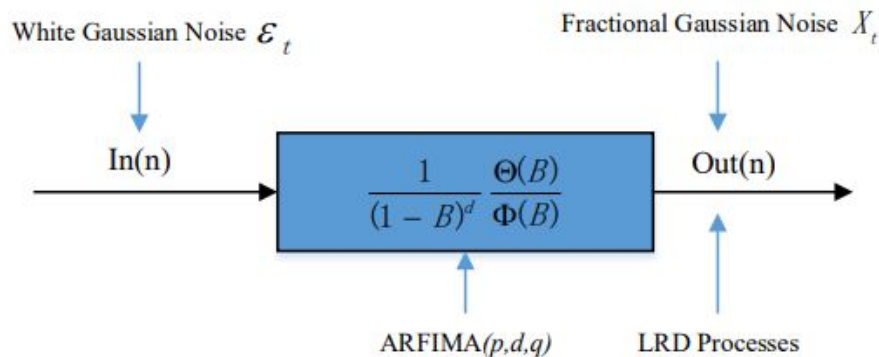
d : Número de diferencias no estacionarias (I)

q : Número de términos de moving-average (MA)

AutoRegressive Fractional Integrated Moving Average (ARFIMA)

Agrega diferenciación fraccional a para ser utilizado en series de tiempo donde se necesita predecir a largo plazo.

$$\Phi(B)(1 - B)^d X_t = \Theta(B)\varepsilon_t,$$



ARFIMA en Python

En Python no existe la implementación de ARFIMA con ninguna librería, pero lo que se puede hacer es aplicar la parte de “Fractional” de ARFIMA antes y teniendo el dataset diferenciado mandarlo al ARIMA de sklearn con el parámetro de diferenciación en 0.

```
[60] """Computes the weights for our fractionally differenced features up to a given threshold
      requirement for fixed-window fractional differencing.
      Args:
          d: A float representing the differencing factor
          length: An int length of series to be differenced
          threshold: A float representing the minimum threshold to include weights for
      Returns:
          A numpy array containing the weights to be applied to our time series
      """
      def findWeights_FFD(d, length, threshold):
```

ARFIMA en Python

```
"""Computes fractionally differenced series
    Args:
        d: A float representing the differencing factor (any positive fractional)
        series: A pandas dataframe with one or more columns of time-series values to be differenced
        threshold: Threshold value past which we ignore weights
                  (cutoff weight for window)
    Returns:
        diff_series: A numpy array of differenced series by d.
    """
def fracDiff(series, d, threshold = 1e-5):
```

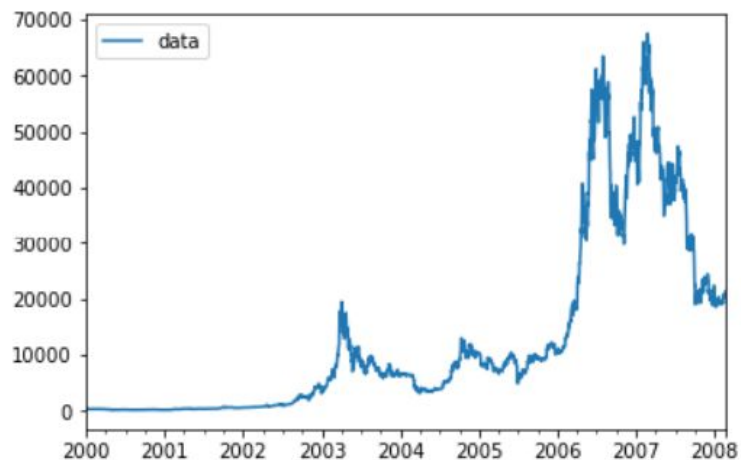
Ejemplo de uso:

```
df_result = fracDiff(test_series, 3.9, 1e-5)
model = ARIMA(df_train, order=(2,0,1))
```

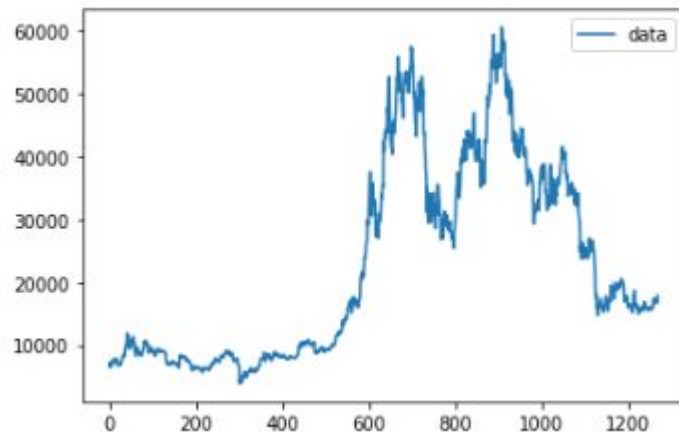
Créditos: [hongwai1920/Using-ARIMA-model-to-forecast-returns](https://hongwai1920.github.io/Using-ARIMA-model-to-forecast-returns)

Probando ARFIMA

Original dataset Bitcoin Price

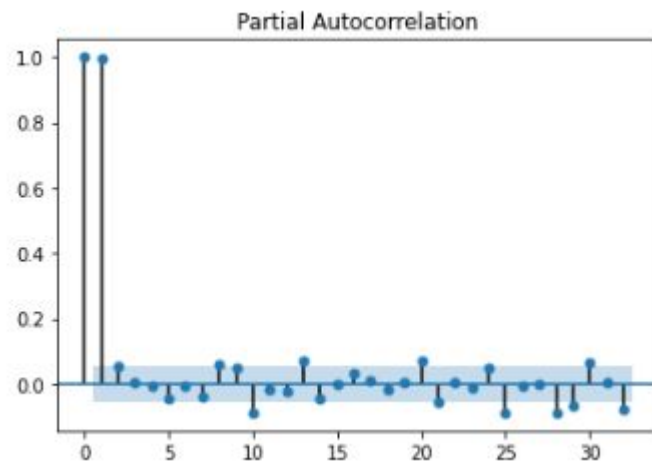
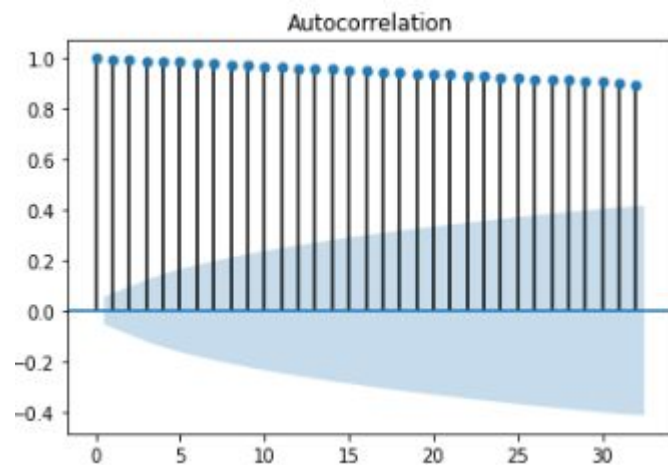


Después de diferenciación fraccional (0.02)



(Las fechas no corresponden al dataset)

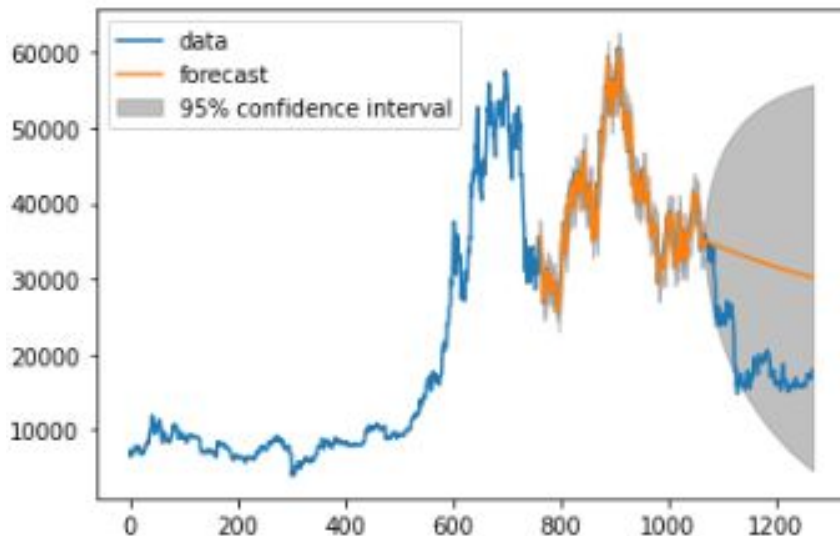
ACF y PACF



Predicción

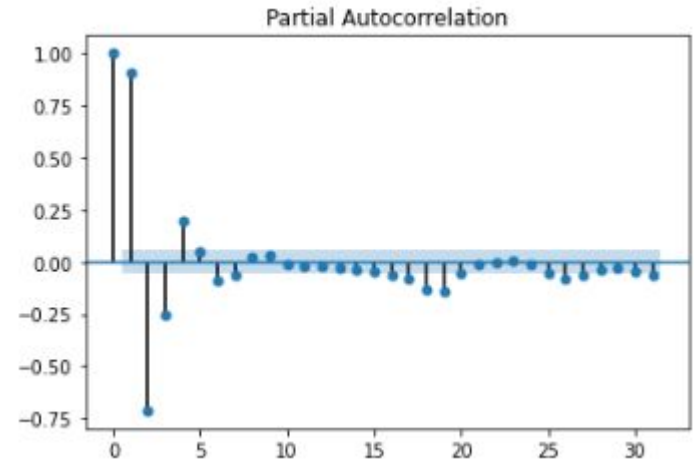
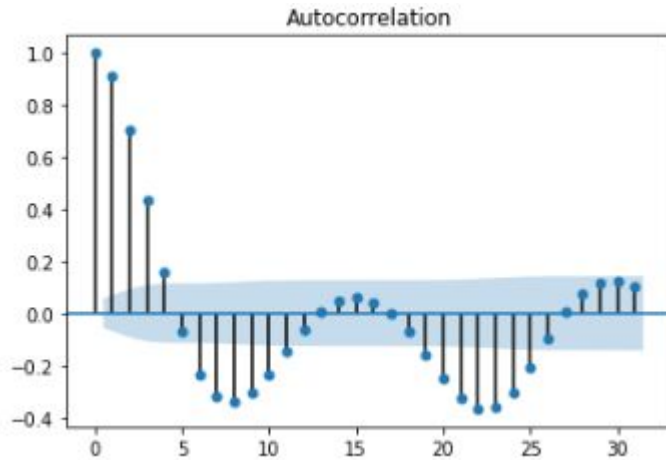
```
from statsmodels.tsa.arima.model import ARIMA
model = ARIMA(df_train, order=(1,0,1))
model_fit = model.fit()
print(model_fit.summary())
perc = 40
from statsmodels.graphics.tsaplots import plot_predict
fig, ax = plt.subplots()
s = pd.DataFrame(list(range(len(df))))
ax = df.loc[:].plot(ax=ax)

plot_predict(model_fit, len(df)-round(len(df) * perc/100), len(df), ax=ax)
plt.show()
```



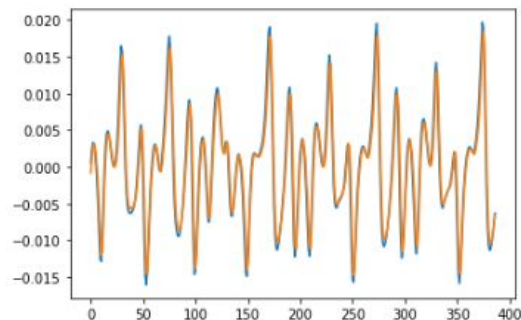
Nota! En ARIMA al dar argumento en d se aplica la diferenciación y luego se deshace para aplicar el modelo al dataset original, en este caso no supe cómo revertir este método :(

ARFIMA con MG dataset

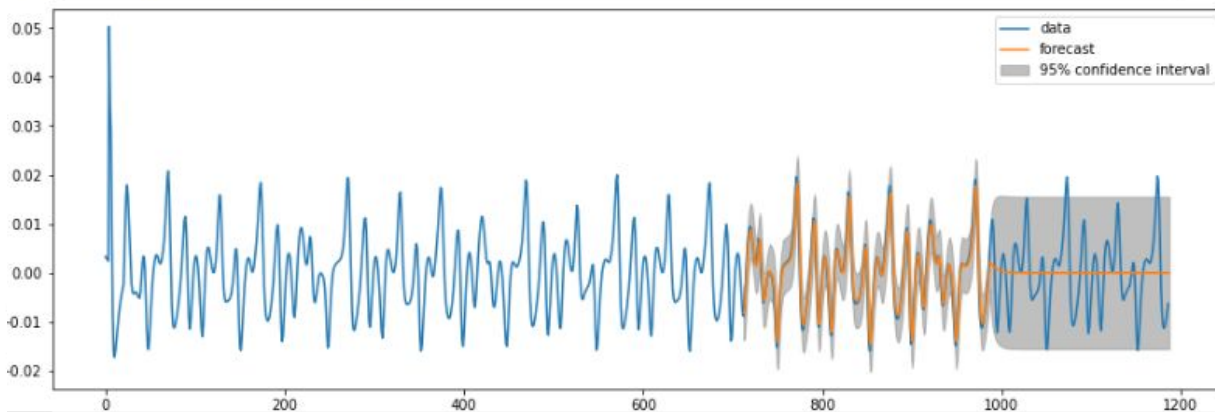


```
df_result = fracDiff(test_series, 1.99, 1e-5)
```

ARFIMA con MG dataset



<matplotlib.legend.Legend at 0x7f4aeeee2cbd0>



```
anteriores = list(df['data'][:800])
y_true = []
y_pred = []
print(anteriores)
for nuevo in df['data'][800:]:
    model = ARIMA(anteriores, order=(1,0,1))
    model_fit = model.fit()
    forecast = model_fit.forecast(alpha=0.05)[0]

    y_true.append(nuevo)
    y_pred.append(forecast)
    anteriores.append(nuevo)
```

SSE 0.0015276978127738456
MSE 3.9475395678910735e-06
RMSE 0.001986841606140528
MAE 0.0015451718698812444
R2 0.9289024206687991

Pronóstico ventanas

| TIEMPO | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 | T9 | ... | TN |
|-----------|-----|-----|-----|-----|-----|-----|-----|----|---------|-----|----|
| VENTANA 1 | T-3 | | T-1 | T+1 | | | | | | | |
| VENTANA 2 | T-5 | T-4 | | T-2 | T-1 | T+1 | | | Bitcoin | | |
| VENTANA 3 | T-3 | | T-1 | T+1 | T+2 | | | | | | |
| VENTANA 4 | T-5 | T-4 | | T-2 | T-1 | T+1 | T+2 | | | | |

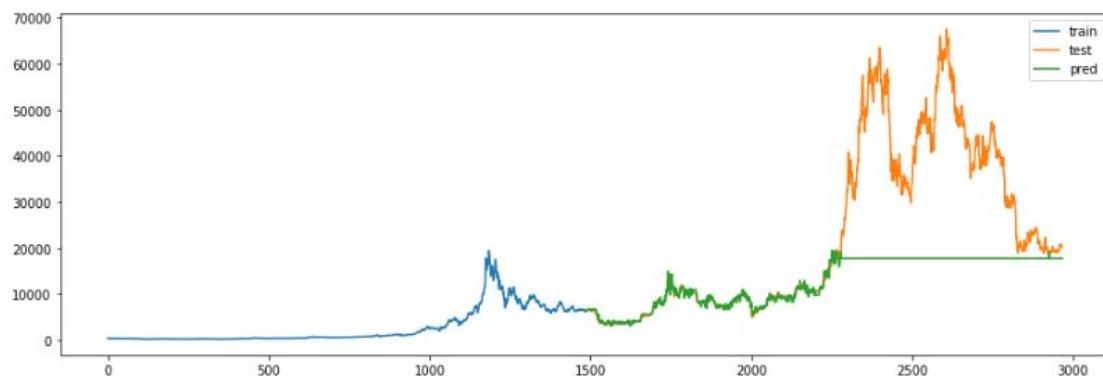
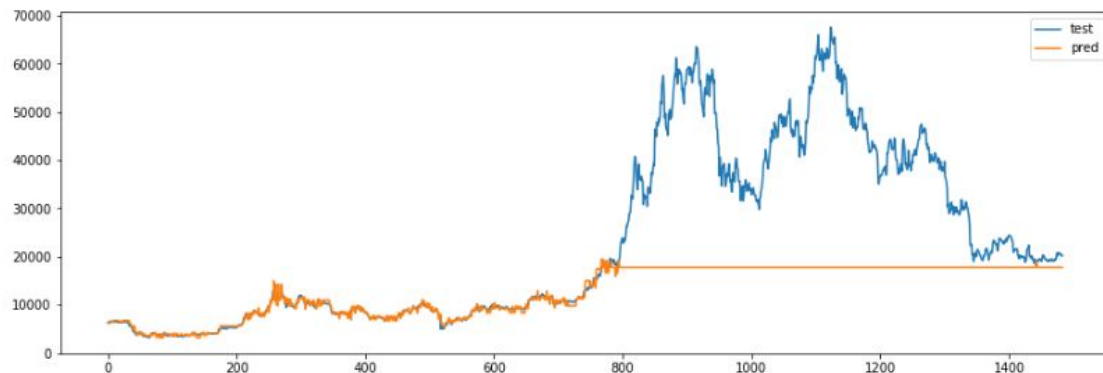
| TIEMPO | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 | T9 | ... | TN |
|-----------|-----|-----|-----|-----|-----|-----|-----|----|----|-----|----|
| VENTANA 1 | T-3 | T-2 | | T+1 | | | | | | | |
| VENTANA 2 | T-5 | | T-3 | T-2 | T-1 | T+1 | | | MG | | |
| VENTANA 3 | T-3 | T-2 | | T+1 | T+2 | | | | | | |
| VENTANA 4 | T-5 | | T-3 | T-2 | T-1 | T+1 | T+2 | | | | |

Implementación ventanas

Solo devuelve
el dataset
creado
dependiendo
de la ventana

```
def predict_uno_2(dataset):  
    ventana = 3  
    predict_size = 1  
    df = pd.DataFrame(dataset)  
    cols = list()  
  
    ...  
    La tabla tiene (t-n) ... (t-1) | (t) ... (t+n)  
    ...  
    tabla = [list() for i in range(ventana-1+predict_size)]  
  
    for i in range(ventana, len(df)-predict_size+1):  
        tabla[0].append(df['data'][i-ventana])  
        tabla[1].append(df['data'][i-1])  
        tabla[2].append(df['data'][i])  
    for array in tabla:  
        cols.append(pd.Series(array))  
    df = pd.concat(cols, axis=1)  
  
    print(df)  
    return df
```

Predicción Bitcoin: 2 entradas/1 salida 50% train

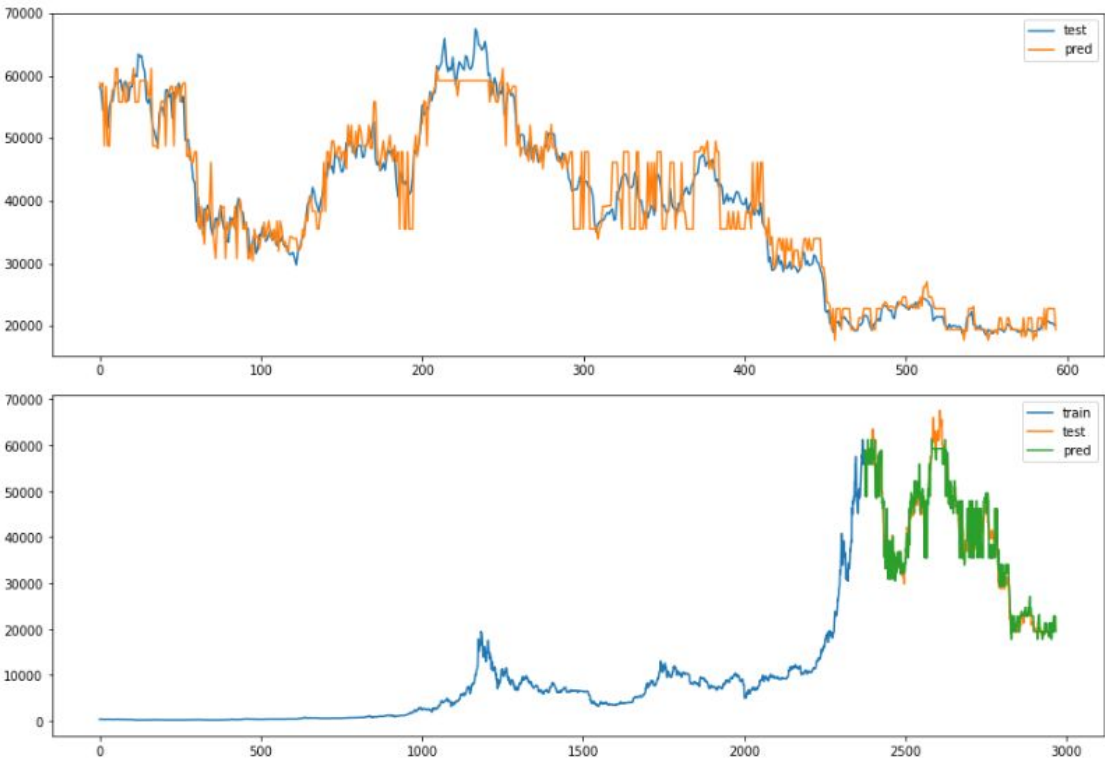


SSE [4.35365862e+11]
MSE [2.93373223e+08]
RMSE [17128.14124989]
MAE [10208.27335948]
R2 0.08315962115372233

| | 0 | 1 | 2 |
|------|--------------|--------------|--------------|
| 0 | 465.864014 | 424.102997 | 394.673004 |
| 1 | 456.859985 | 394.673004 | 408.084991 |
| 2 | 424.102997 | 408.084991 | 399.100006 |
| 3 | 394.673004 | 399.100006 | 402.092010 |
| 4 | 408.084991 | 402.092010 | 435.751007 |
| ... | ... | ... | ... |
| 2963 | 20287.957030 | 20817.982420 | 20633.695310 |
| 2964 | 20595.103520 | 20633.695310 | 20494.898440 |
| 2965 | 20817.982420 | 20494.898440 | 20482.958980 |
| 2966 | 20633.695310 | 20482.958980 | 20162.689450 |
| 2967 | 20494.898440 | 20162.689450 | 20208.769530 |

Mejor resultado para
bitcoin dataset!

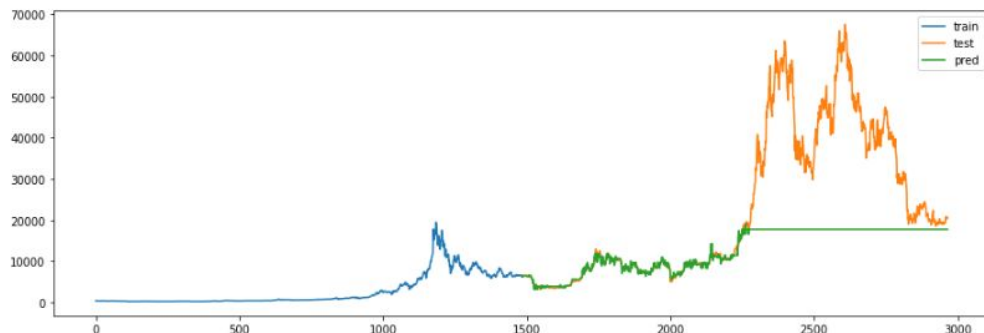
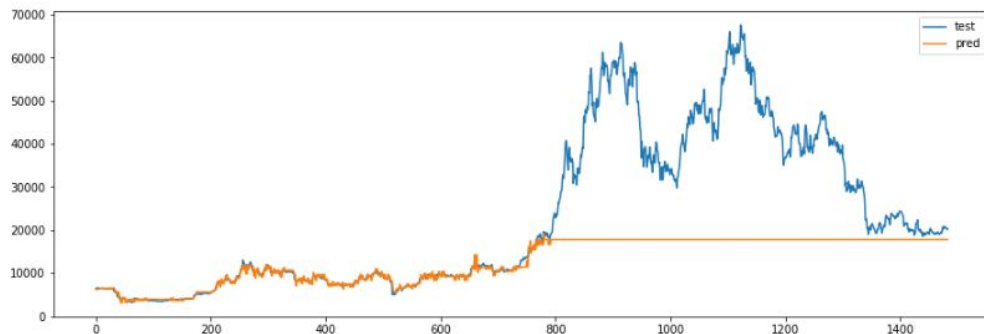
Predicción Bitcoin: 2 entradas/1 salida 80% train



SSE [6.63582433e+09]
MSE [11171421.43703903]
RMSE [3342.36763942]
MAE [2501.04320549]
R2 0.937012889532168

| | 0 | 1 | 2 |
|------|--------------|--------------|--------------|
| 0 | 465.864014 | 424.102997 | 394.673004 |
| 1 | 456.859985 | 394.673004 | 408.084991 |
| 2 | 424.102997 | 408.084991 | 399.100006 |
| 3 | 394.673004 | 399.100006 | 402.092010 |
| 4 | 408.084991 | 402.092010 | 435.751007 |
| ... | ... | ... | ... |
| 2963 | 20287.957030 | 20817.982420 | 20633.695310 |
| 2964 | 20595.103520 | 20633.695310 | 20494.898440 |
| 2965 | 20817.982420 | 20494.898440 | 20482.958980 |
| 2966 | 20633.695310 | 20482.958980 | 20162.689450 |
| 2967 | 20494.898440 | 20162.689450 | 20208.769530 |

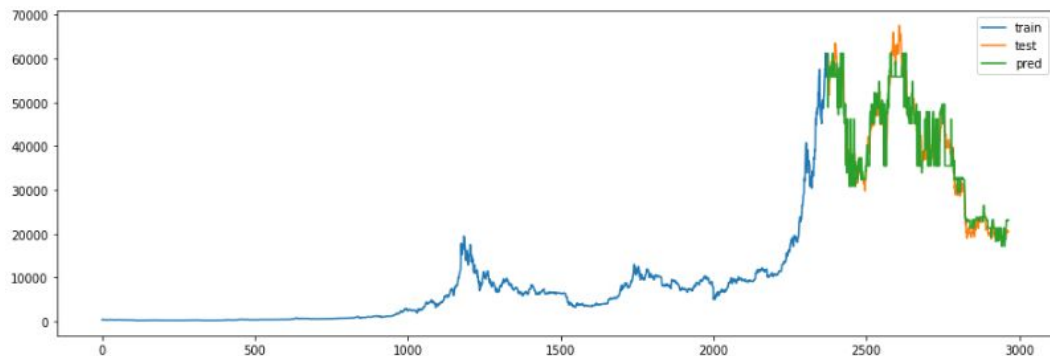
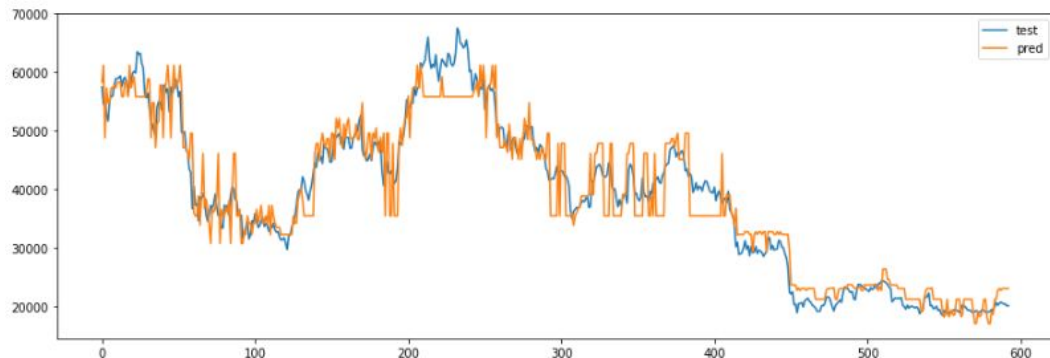
Predicción Bitcoin: 4 entradas/1 salida 50% train



SSE [4.35365794e+11]
 MSE [2.93571001e+08]
 RMSE [17133.9137552]
 MAE [10219.60313943]
 R2 0.08263389822819689

| | 0 | 1 | 2 | 3 | 4 |
|------|--------------|--------------|--------------|--------------|--------------|
| 0 | 465.864014 | 456.859985 | 394.673004 | 408.084991 | 399.100006 |
| 1 | 456.859985 | 424.102997 | 408.084991 | 399.100006 | 402.092010 |
| 2 | 424.102997 | 394.673004 | 399.100006 | 402.092010 | 435.751007 |
| 3 | 394.673004 | 408.084991 | 402.092010 | 435.751007 | 423.156006 |
| 4 | 408.084991 | 399.100006 | 435.751007 | 423.156006 | 411.428986 |
| ... | ... | ... | ... | ... | ... |
| 2961 | 20092.236330 | 20772.802730 | 20595.103520 | 20817.982420 | 20633.695310 |
| 2962 | 20772.802730 | 20287.957030 | 20817.982420 | 20633.695310 | 20494.898440 |
| 2963 | 20287.957030 | 20595.103520 | 20633.695310 | 20494.898440 | 20482.958980 |
| 2964 | 20595.103520 | 20817.982420 | 20494.898440 | 20482.958980 | 20162.689450 |
| 2965 | 20817.982420 | 20633.695310 | 20482.958980 | 20162.689450 | 20208.769530 |

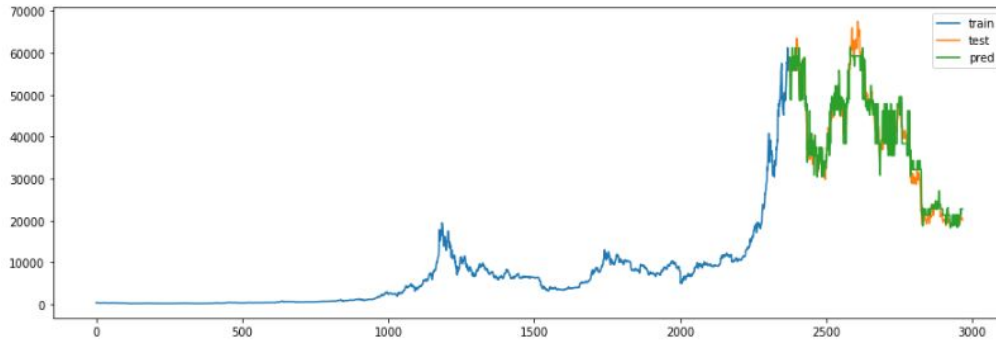
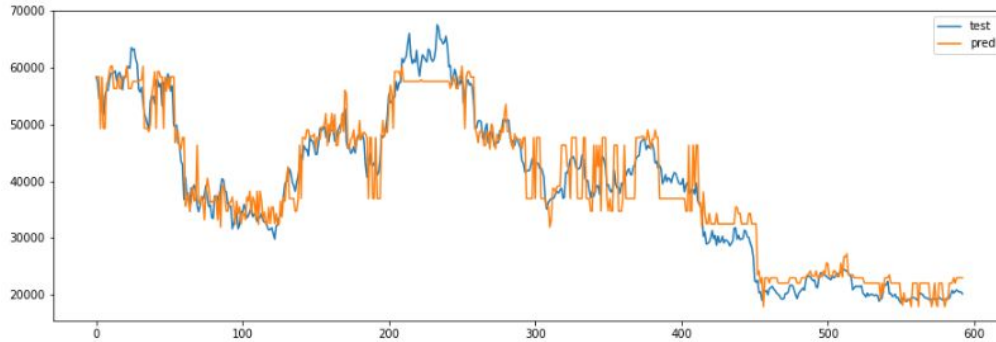
Predicción Bitcoin: 4 entradas/1 salida 80% train



SSE [8.17872952e+09]
MSE [13792123.97263254]
RMSE [3713.77489526]
MAE [2862.30439012]
R2 0.9220936294015181

| | 0 | 1 | 2 | 3 | 4 |
|------|--------------|--------------|--------------|--------------|--------------|
| 0 | 465.864014 | 456.859985 | 394.673004 | 408.084991 | 399.100006 |
| 1 | 456.859985 | 424.102997 | 408.084991 | 399.100006 | 402.092010 |
| 2 | 424.102997 | 394.673004 | 399.100006 | 402.092010 | 435.751007 |
| 3 | 394.673004 | 408.084991 | 402.092010 | 435.751007 | 423.156006 |
| 4 | 408.084991 | 399.100006 | 435.751007 | 423.156006 | 411.428986 |
| ... | ... | ... | ... | ... | ... |
| 2961 | 20092.236330 | 20772.802730 | 20595.103520 | 20817.982420 | 20633.695310 |
| 2962 | 20772.802730 | 20287.957030 | 20817.982420 | 20633.695310 | 20494.898440 |
| 2963 | 20287.957030 | 20595.103520 | 20633.695310 | 20494.898440 | 20482.958980 |
| 2964 | 20595.103520 | 20817.982420 | 20494.898440 | 20482.958980 | 20162.689450 |
| 2965 | 20817.982420 | 20633.695310 | 20482.958980 | 20162.689450 | 20208.769530 |

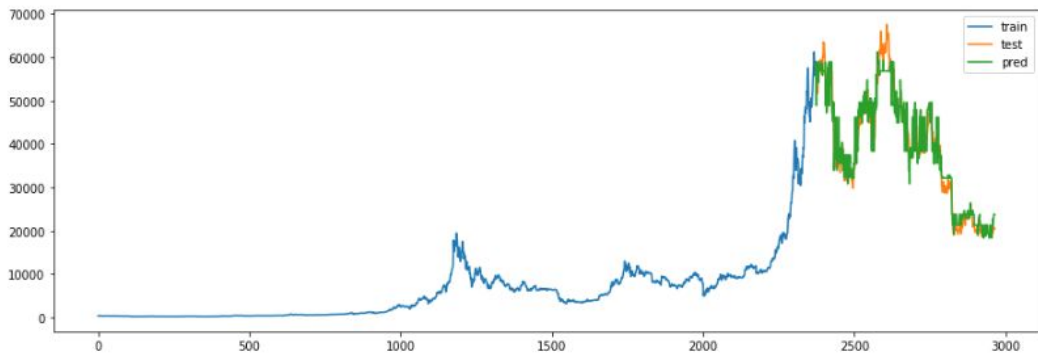
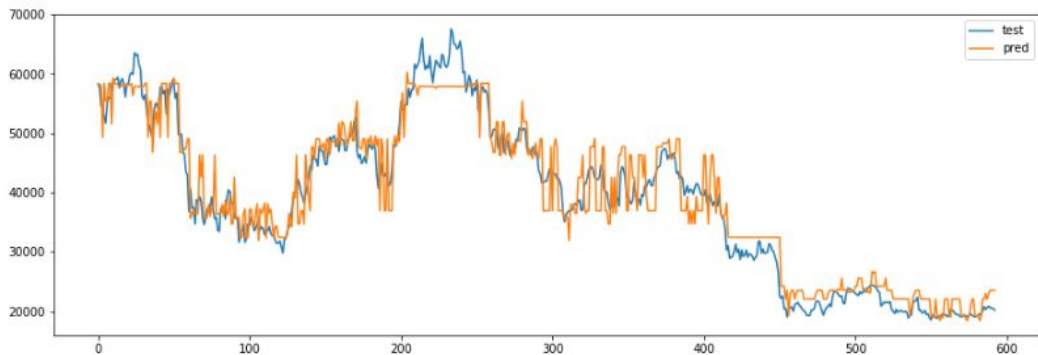
Predicción Bitcoin: 2 entradas/2 salidas 80% dataset



SSE 16823619228.14617
MSE 28370352.82992609
RMSE 5326.382715307462
MAE 5879.370046290042
R2 0.9198771904482907

| | 0 | 1 | 2 | 3 |
|------|--------------|--------------|--------------|--------------|
| 0 | 465.864014 | 424.102997 | 394.673004 | 408.084991 |
| 1 | 456.859985 | 394.673004 | 408.084991 | 399.100006 |
| 2 | 424.102997 | 408.084991 | 399.100006 | 402.092010 |
| 3 | 394.673004 | 399.100006 | 402.092010 | 435.751007 |
| 4 | 408.084991 | 402.092010 | 435.751007 | 423.156006 |
| ... | ... | ... | ... | ... |
| 2962 | 20772.802730 | 20595.103520 | 20817.982420 | 20633.695310 |
| 2963 | 20287.957030 | 20817.982420 | 20633.695310 | 20494.898440 |
| 2964 | 20595.103520 | 20633.695310 | 20494.898440 | 20482.958980 |
| 2965 | 20817.982420 | 20494.898440 | 20482.958980 | 20162.689450 |
| 2966 | 20633.695310 | 20482.958980 | 20162.689450 | 20208.769530 |

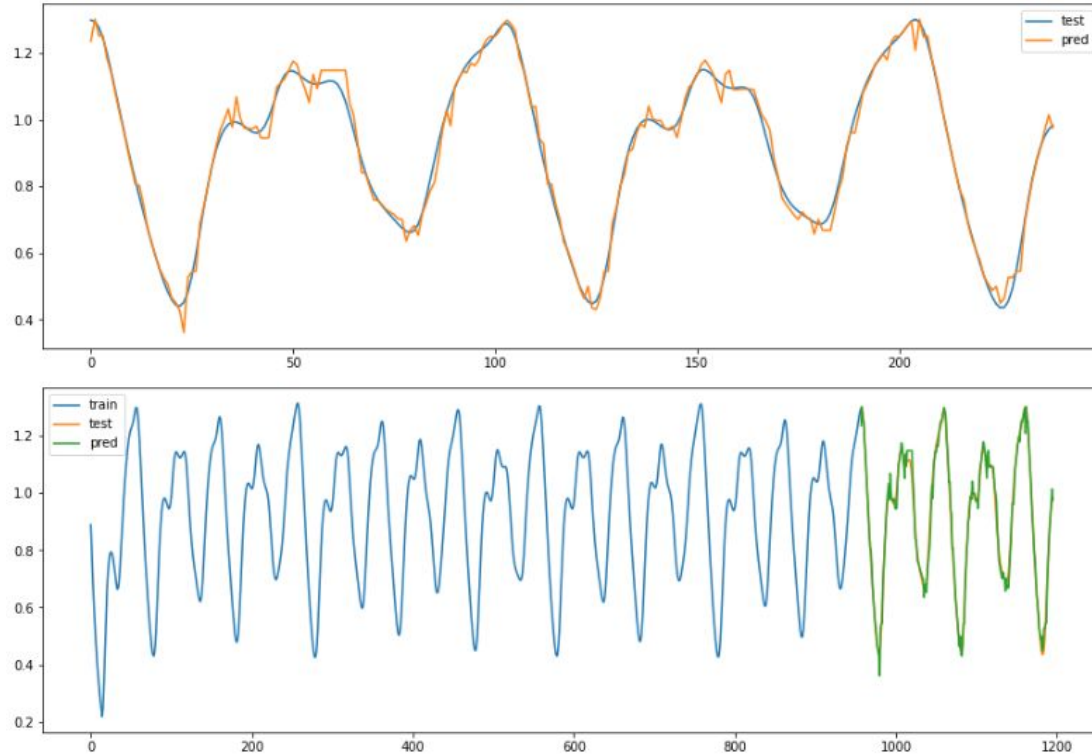
Predicción Bitcoin: 4 entradas/2 salidas 80% train



SSE 17019592511.656761
 MSE 28700830.54242287
 RMSE 5357.315609745507
 MAE 5906.161006104553
 R2 0.9189444614679174

| | 0 | 1 | 2 | 3 | 4 \ |
|------|--------------|--------------|--------------|--------------|--------------|
| 0 | 465.864014 | 456.859985 | 394.673004 | 408.084991 | 399.100006 |
| 1 | 456.859985 | 424.102997 | 408.084991 | 399.100006 | 402.092010 |
| 2 | 424.102997 | 394.673004 | 399.100006 | 402.092010 | 435.751007 |
| 3 | 394.673004 | 408.084991 | 402.092010 | 435.751007 | 423.156006 |
| 4 | 408.084991 | 399.100006 | 435.751007 | 423.156006 | 411.428986 |
| ... | ... | ... | ... | ... | ... |
| 2960 | 19344.964840 | 20092.236330 | 20287.957030 | 20595.103520 | 20817.982420 |
| 2961 | 20092.236330 | 20772.802730 | 20595.103520 | 20817.982420 | 20633.695310 |
| 2962 | 20772.802730 | 20287.957030 | 20817.982420 | 20633.695310 | 20494.898440 |
| 2963 | 20287.957030 | 20595.103520 | 20633.695310 | 20494.898440 | 20482.958980 |
| 2964 | 20595.103520 | 20817.982420 | 20494.898440 | 20482.958980 | 20162.689450 |
| | 5 | | | | |
| 0 | 402.092010 | | | | |
| 1 | 435.751007 | | | | |
| 2 | 423.156006 | | | | |
| 3 | 411.428986 | | | | |
| 4 | 403.556000 | | | | |
| ... | ... | | | | |
| 2960 | 20633.695310 | | | | |
| 2961 | 20494.898440 | | | | |
| 2962 | 20482.958980 | | | | |
| 2963 | 20162.689450 | | | | |
| 2964 | 20208.769530 | | | | |

Predicción MG: 2 entradas/1 salida

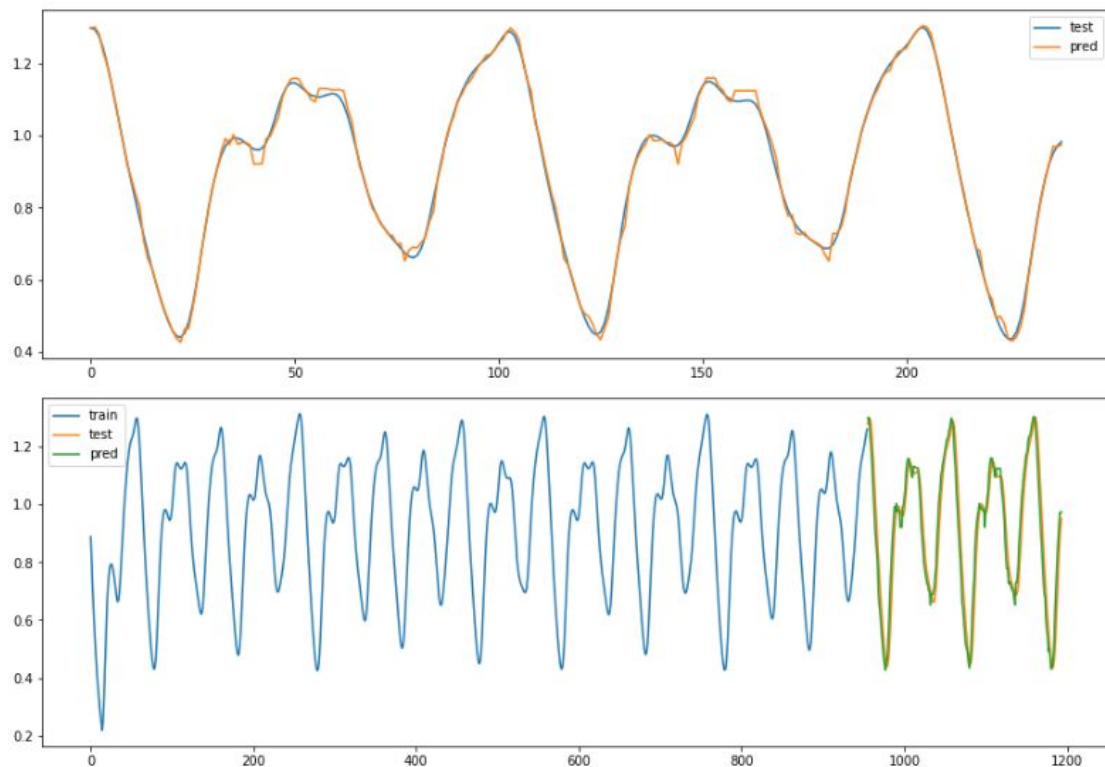


SSE [0.17424055]
MSE [0.00072904]
RMSE [0.02700074]
MAE [0.01921778]
R2 0.9872111979967401

| | 0 | 1 | 2 |
|------|---------|---------|---------|
| 0 | 1.20000 | 1.08580 | 0.88898 |
| 1 | 1.08580 | 0.98248 | 0.80438 |
| 2 | 0.98248 | 0.88898 | 0.72784 |
| 3 | 0.88898 | 0.80438 | 0.65857 |
| 4 | 0.80438 | 0.72784 | 0.59590 |
| ... | ... | ... | ... |
| 1191 | 0.69324 | 0.76448 | 0.88001 |
| 1192 | 0.76448 | 0.82756 | 0.92119 |
| 1193 | 0.82756 | 0.88001 | 0.95145 |
| 1194 | 0.88001 | 0.92119 | 0.97162 |
| 1195 | 0.92119 | 0.95145 | 0.98283 |

Predicción MG: 4 entradas/1 salida

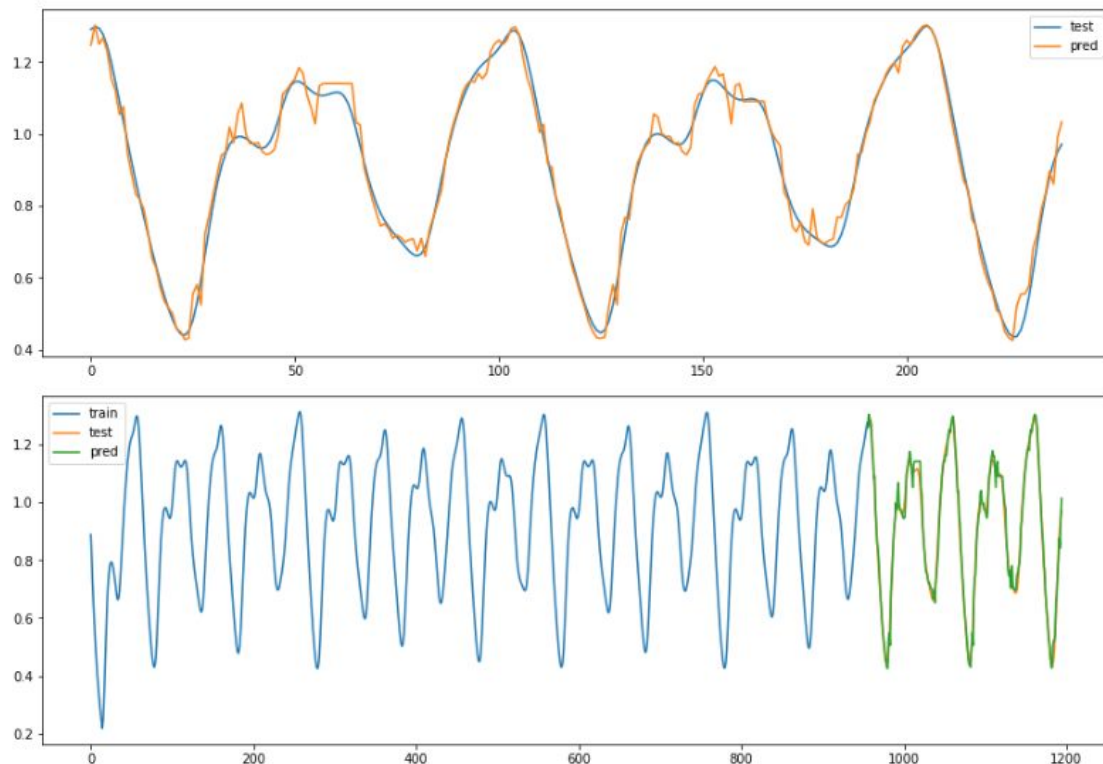
Mejor resultado para MG dataset!



SSE [0.05452576]
MSE [0.00022814]
RMSE [0.01510435]
MAE [0.01091619]
R2 0.9959979516151715

| | 0 | 1 | 2 | 3 | 4 |
|------|---------|---------|---------|---------|---------|
| 0 | 1.20000 | 0.98248 | 0.88898 | 0.80438 | 0.72784 |
| 1 | 1.08580 | 0.88898 | 0.80438 | 0.72784 | 0.65857 |
| 2 | 0.98248 | 0.80438 | 0.72784 | 0.65857 | 0.59590 |
| 3 | 0.88898 | 0.72784 | 0.65857 | 0.59590 | 0.53919 |
| 4 | 0.80438 | 0.65857 | 0.59590 | 0.53919 | 0.48788 |
| ... | ... | ... | ... | ... | ... |
| 1189 | 0.54945 | 0.69324 | 0.76448 | 0.82756 | 0.88001 |
| 1190 | 0.61897 | 0.76448 | 0.82756 | 0.88001 | 0.92119 |
| 1191 | 0.69324 | 0.82756 | 0.88001 | 0.92119 | 0.95145 |
| 1192 | 0.76448 | 0.88001 | 0.92119 | 0.95145 | 0.97162 |
| 1193 | 0.82756 | 0.92119 | 0.95145 | 0.97162 | 0.98283 |

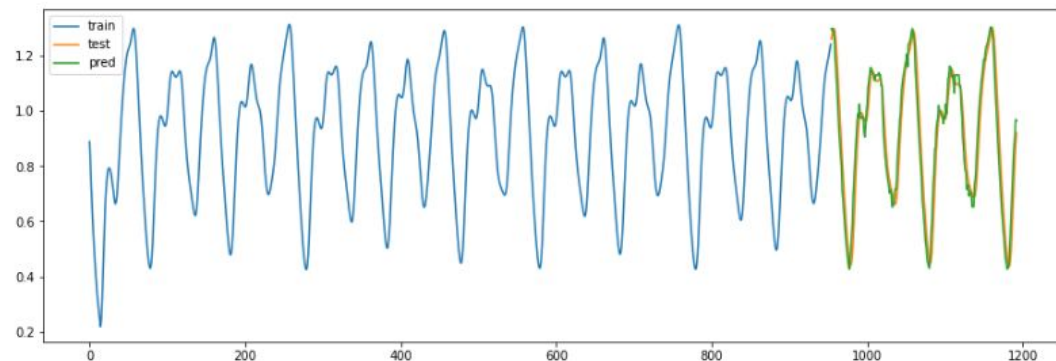
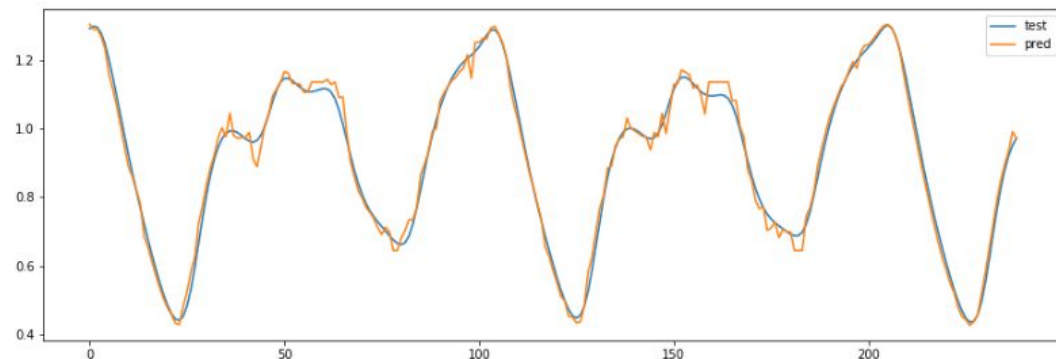
Predicción MG: 2 entradas/2 salidas



SSE 0.5242577047999997
MSE 0.0021935468820083667
RMSE 0.046835316610527644
MAE 0.046925439330543973
R2 0.9808206975185996

| | 0 | 1 | 2 | 3 |
|------|---------|---------|---------|---------|
| 0 | 1.20000 | 1.08580 | 0.88898 | 0.80438 |
| 1 | 1.08580 | 0.98248 | 0.80438 | 0.72784 |
| 2 | 0.98248 | 0.88898 | 0.72784 | 0.65857 |
| 3 | 0.88898 | 0.80438 | 0.65857 | 0.59590 |
| 4 | 0.80438 | 0.72784 | 0.59590 | 0.53919 |
| ... | ... | ... | ... | ... |
| 1190 | 0.61897 | 0.69324 | 0.82756 | 0.88001 |
| 1191 | 0.69324 | 0.76448 | 0.88001 | 0.92119 |
| 1192 | 0.76448 | 0.82756 | 0.92119 | 0.95145 |
| 1193 | 0.82756 | 0.88001 | 0.95145 | 0.97162 |
| 1194 | 0.88001 | 0.92119 | 0.97162 | 0.98283 |

Predicción MG: 4 entradas/2 salidas



SSE 0.2717791786
MSE 0.0011371513748953976
RMSE 0.03372167514960367
MAE 0.0314919665271967
R2 0.9900516731271898

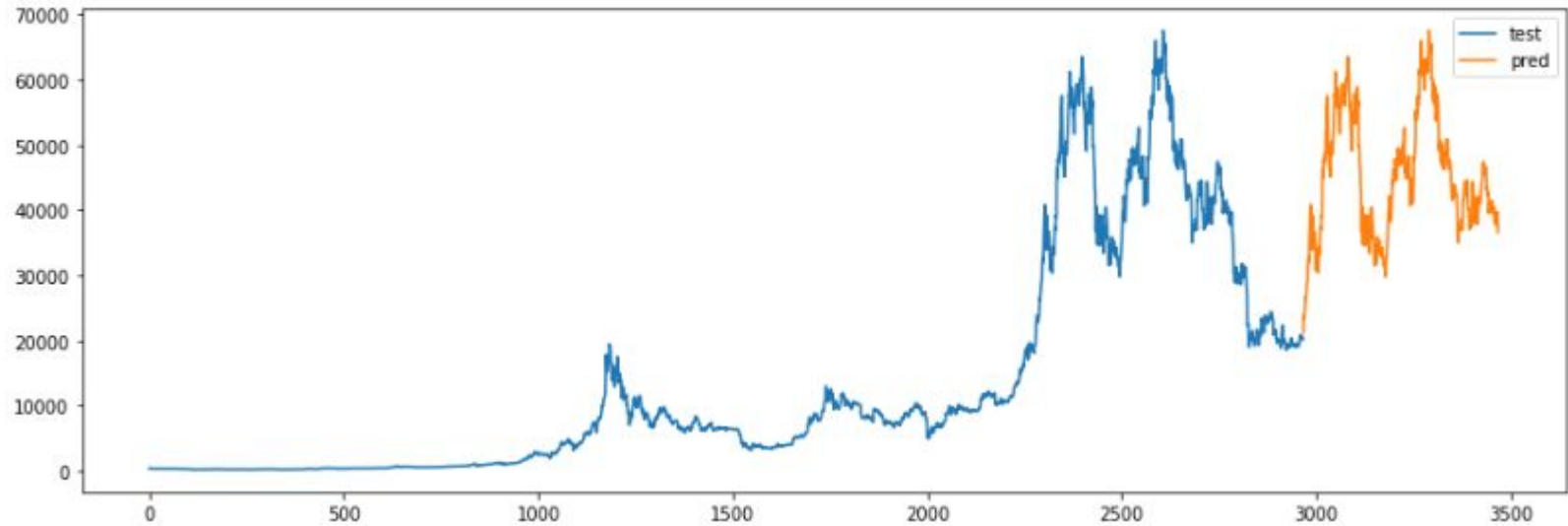
| | 0 | 1 | 2 | 3 | 4 | 5 |
|------|---------|---------|---------|---------|---------|---------|
| 0 | 1.20000 | 0.98248 | 0.88898 | 0.80438 | 0.72784 | 0.65857 |
| 1 | 1.08580 | 0.88898 | 0.80438 | 0.72784 | 0.65857 | 0.59590 |
| 2 | 0.98248 | 0.80438 | 0.72784 | 0.65857 | 0.59590 | 0.53919 |
| 3 | 0.88898 | 0.72784 | 0.65857 | 0.59590 | 0.53919 | 0.48788 |
| 4 | 0.80438 | 0.65857 | 0.59590 | 0.53919 | 0.48788 | 0.44145 |
| ... | ... | ... | ... | ... | ... | ... |
| 1188 | 0.49305 | 0.61897 | 0.69324 | 0.76448 | 0.82756 | 0.88001 |
| 1189 | 0.54945 | 0.69324 | 0.76448 | 0.82756 | 0.88001 | 0.92119 |
| 1190 | 0.61897 | 0.76448 | 0.82756 | 0.88001 | 0.92119 | 0.95145 |
| 1191 | 0.69324 | 0.82756 | 0.88001 | 0.92119 | 0.95145 | 0.97162 |
| 1192 | 0.76448 | 0.88001 | 0.92119 | 0.95145 | 0.97162 | 0.98283 |

Predicciones después del dataset

Con las configuraciones de 2 y 4 entradas no es suficiente para hacer un modelo que prediga más allá, por lo que para predecir a futuro se amplió la ventana en ambos dataset.



Predicciones después del dataset



Bitcoin dataset con 5 entradas y 1 salida

Referencias

Bora, N. (2021). *Understanding ARIMA models for machine learning*. Capital One.

<https://www.capitalone.com/tech/machine-learning/understanding-arima-models/>

Fernandez, J. (2022). *Creating an ARIMA model for time series forecasting*. Towards Data Science.

<https://towardsdatascience.com/creating-an-arima-model-for-time-series-forecasting-ff3b619b848d>

Liu, K., Chen, Y., & Zhang, X. (2017). An evaluation of ARFIMA (autoregressive fractional integral moving average) programs. *Axioms*, 6(4), 16. <https://doi.org/10.3390/axioms6020016>

Maklin, C. (2019). *ARIMA model Python example — time series forecasting*. Towards Data Science.

<https://towardsdatascience.com/machine-learning-part-19-time-series-and-autoregressive-integrated-moving-average-model-arima-c1005347b0d7>

Simplilearn. (2021). *Understanding time series analysis in python*. Simplilearn.com; Simplilearn.

<https://www.simplilearn.com/tutorials/python-tutorial/time-series-analysis-in-python>

Dataset bitcoin:

<https://finance.yahoo.com/quote/BTC-USD/history?period1=1410825600&period2=1667692800&interval=1d&filter=history&frequency=1d&includeAdjustedClose=true>