Taller 3

Irvin E. Zavala Roman 1270771

TALLER

$\mathcal{L}[(t-1)^2]$

$\mathcal{L}[t^3]$

$\mathcal{L}[e^{-3t} + 5\cos(t) - 4\cosh(2t)]$

1) $\mathcal{L}[(t-1)^2]$

$\int_0^\infty (t-1)^2 e^{-st}\,dt$

$= \int_0^\infty t^2 e^{-st}\,dt - 2\int_0^\infty t e^{-st}\,dt + \int_0^\infty e^{-st}\,dt$

$u = st$, $du = s\,dt$, $t = u/s$

$\int_0^\infty \left(\frac{u}{s}\right)^2 e^{-u}\left(\frac{du}{s}\right)$

$\frac{1}{s^3}\boxed{\int_0^\infty u^2 e^{-u}\,du} = 2!$

$u = st$, $du = s\,dt$

$\int_0^\infty \left(\frac{u}{s}\right) e^{-u}\left(\frac{du}{s}\right)$

$\frac{1}{s^2}\boxed{\int_0^\infty u e^{-u}\,du} = 1!$

$-\frac{1}{s}e^{-st}\Big|_{t=0}^{\infty}$

$-\frac{1}{s}e^{-\infty} - \left(-\frac{1}{s}e^{0}\right)$

$= \frac{2}{s^3} - \frac{2}{s^2} + \frac{1}{s}$

2) $\mathcal{L}[t^3]$ $\int_0^\infty t^3 e^{-st}\,dt$

$u = st$

$du = s\,dt$

$\int_0^\infty \left(\frac{u}{s}\right)^3 e^{-u}\left(\frac{du}{s}\right)$

$\frac{1}{s^4}\boxed{\int_0^\infty u^3 e^{-u}\,du} = 3!$

$= \frac{3!}{s^4} = \frac{6}{s^4}$

3) $\mathcal{L}[e^{-3t} + 5\cos(t) - 4\cosh(2t)]$

$\int_0^\infty e^{-3t} e^{-st}\,dt + 5\int_0^\infty \cos(t) e^{-st}\,dt - 4\int_0^\infty \cosh(2t) e^{-st}\,dt$

$\int_0^\infty e^{-3t-st}\,dt$

$\frac{1}{-3-s} e^{-3t-st}\Big|_{t=0}^{\infty}$

$\frac{1}{-3-s}e^{\infty} - \frac{1}{-3-s}e^{0}$

$-\frac{1}{-(3+s)} = \frac{1}{s+3}$

$\cos(at) = \frac{e^{iat} + e^{-iat}}{2}$

$\frac{1}{2}\int_0^\infty (e^{iat} + e^{-iat}) e^{-st}\,dt$

$\frac{1}{2}\int_0^\infty e^{iat-st} + e^{-iat-st}$

$\frac{1}{2}\left[\frac{e^{iat-st}}{ia-s} + \frac{e^{-iat-st}}{-ia-s}\right]_{t=0}^{\infty}$

$\frac{1}{2}\left[\frac{1}{ia-s} + \frac{1}{-ia-s}\right]$

$\frac{1}{2}\left[\frac{-ia-s+ia-s}{(ia-s)(-ia-s)}\right] = \frac{1}{2}\left[\frac{-2s}{(s^2+a^2)}\right]$

$\frac{s}{s^2+1}$

$\cosh(at) = \frac{e^{at} + e^{-at}}{2}$

$\frac{1}{2}\int_0^\infty (e^{2t} + e^{-2t}) e^{-st}\,dt$

$\frac{1}{2}\int_0^\infty (e^{(2-s)t} + e^{(-2-s)t})\,dt$

$\frac{1}{2}\left[\frac{e^{(2-s)t}}{2-s} + \frac{e^{(-2-s)t}}{-2-s}\right]_{t=0}^{\infty}$

$\frac{1}{2}\left[-\frac{1}{2-s} + \frac{1}{-2-s}\right] = \frac{1}{2}\left[-\frac{-2-s+2-s}{s^2-4}\right]$

$= \frac{1}{2}\left[\frac{2s}{s^2-4}\right] = \frac{s}{s^2-4}$

$\frac{1}{s+3} + \frac{5s}{s^2+1} - \frac{4}{s^2-4}$