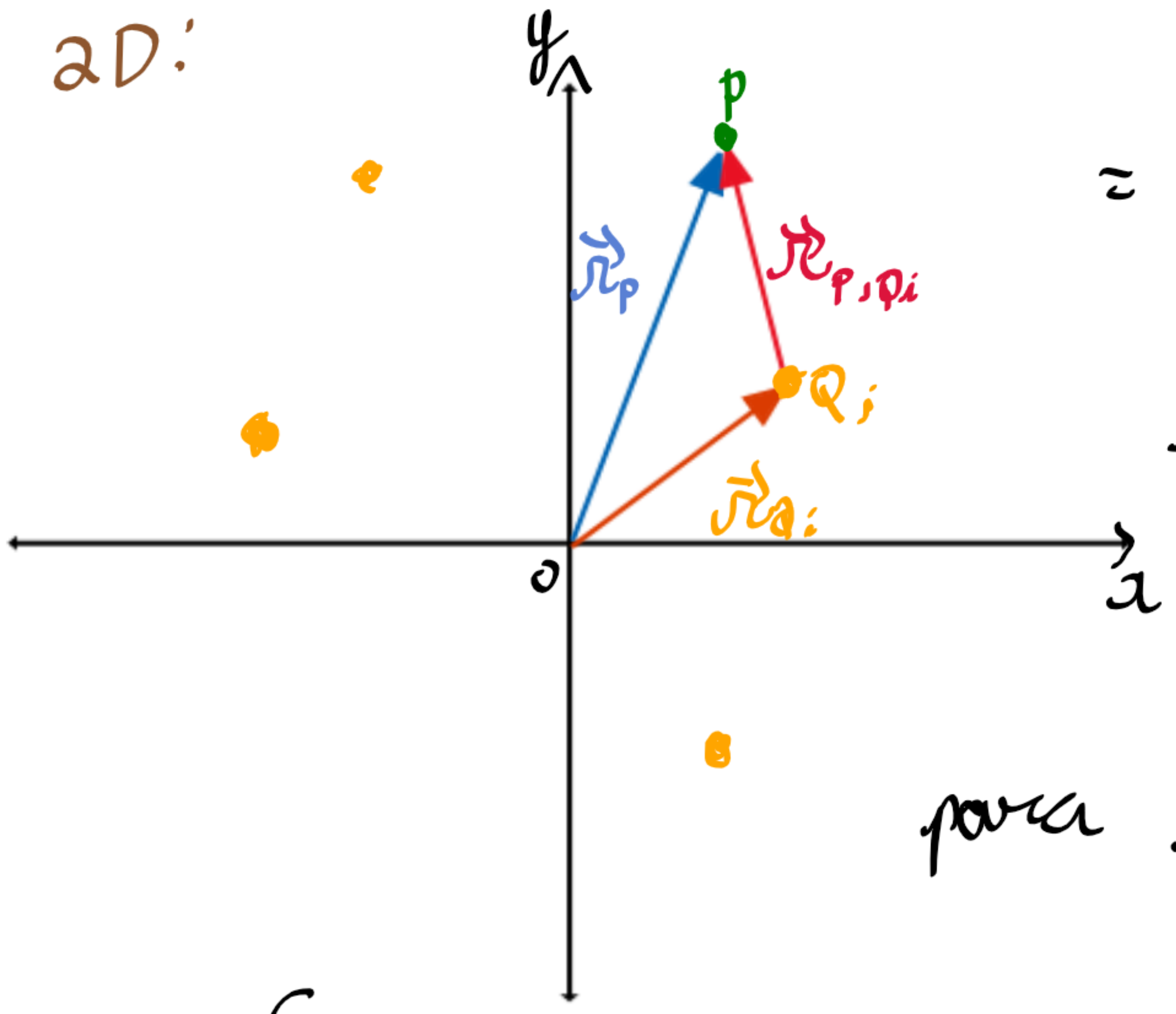
**Discreto 2D:**




$$\phi_{p,Q_i} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i}{\|\vec{r}_{p,Q_i}\|} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_i}{\|\vec{r}_p - \vec{r}_{Q_i}\|}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i}{\sqrt{(x_p - x_{Q_i})^2 + (y_p - y_{Q_i})^2}}$$

$$\Rightarrow \phi_p = \sum_{i=1}^{N_q}\phi_{p,Q_i} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N_q}\underbrace{\frac{q_i}{\sqrt{(x_p - x_{Q_i})^2 + (y_p - y_{Q_i})^2}}}_{=Q_i}$$

$$\Rightarrow \phi_p = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N_q} Q_i\text{, mas}$$

para $x_p = x_{Q_i}$ e $y_p = y_{Q_i} \Rightarrow Q_i = 0$. Assim,

$$Q_i = \begin{cases} 0, & x_p = x_{Q_i} \text{ e } y_p = y_{Q_i} \\ \dfrac{q_i}{\sqrt{(x_p - x_{Q_i})^2 + (y_p - y_{Q_i})^2}}, & \text{caso contrário} \end{cases}$$

x_arr = $(x_1, x_2, \dots, x_{N_x})$, y_arr = $(y_1, y_2, \dots, y_{N_y}) \Rightarrow$ xx, yy = meshgrid(x_arr, y_arr)

$$\Rightarrow xx = \begin{pmatrix} x_1, x_2, \dots, x_{N_x} \\ x_1, x_2, \dots, x_{N_x} \\ \vdots \quad \vdots \quad \ddots \quad \vdots \\ x_1, x_2, \dots, x_{N_x} \end{pmatrix},\quad yy = \begin{pmatrix} y_1 & y_1 & \cdots & y_1 \\ y_2 & y_2 & \cdots & y_2 \\ \vdots & \vdots & \ddots & \vdots \\ y_{N_y} & y_{N_y} & \cdots & y_{N_y} \end{pmatrix} \Rightarrow xx \cdot yy = \begin{pmatrix} x_1y_1 & x_2y_1 & \cdots & x_{N_x}y_1 \\ x_1y_2 & x_2y_2 & \cdots & x_{N_x}y_2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1y_{N_y} & x_2y_{N_y} & \cdots & x_{N_x}y_{N_y} \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \cdots & \phi_{1,N_y} \\ \phi_{21} & \phi_{22} & \phi_{23} & \cdots & \phi_{2,N_y} \\ \phi_{31} & \phi_{32} & \phi_{33} & \cdots & \phi_{3,N_y} \\ \vdots & & & \ddots & \vdots \\ \phi_{N_x,1} & \phi_{N_x,2} & \phi_{N_x,3} & \cdots & \phi_{N_x,N_y} \end{pmatrix} \Big\downarrow x$$

$\xrightarrow{\quad y \quad}$

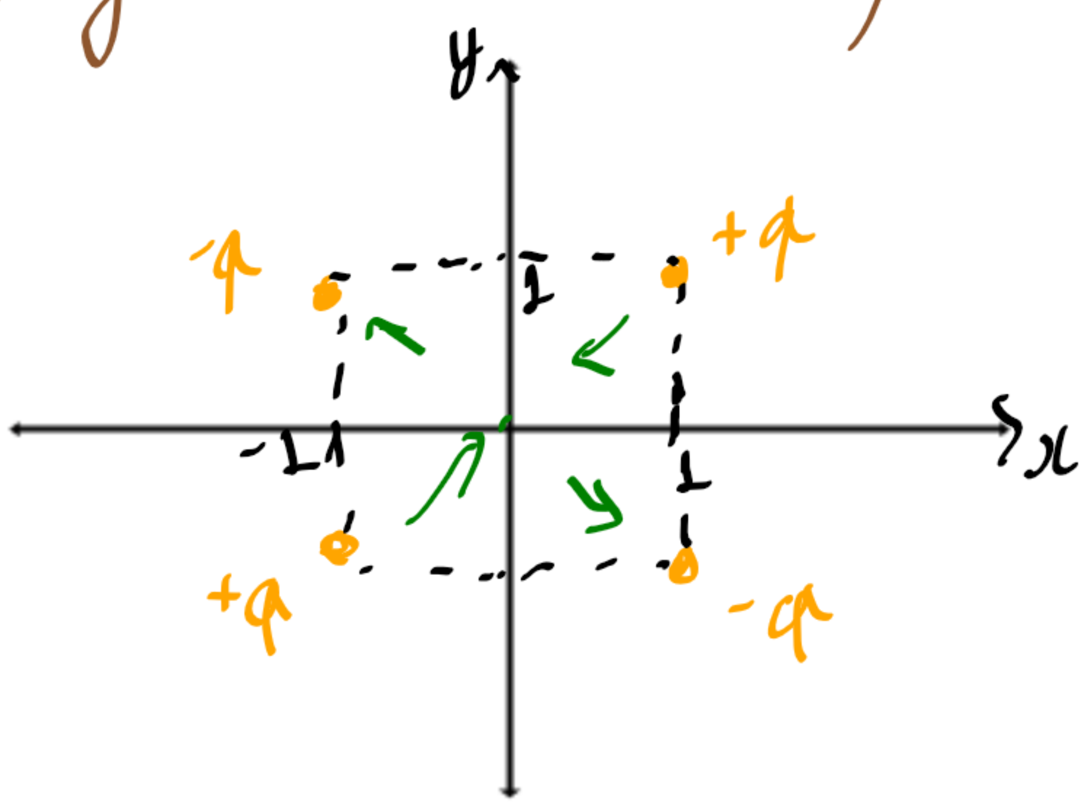
$$\frac{\partial\phi_{ij}}{\partial x_i} = \frac{\phi(x_i + \Delta x) - \phi(x_i - \Delta x)}{2\Delta x} = \frac{\phi_{i+1,j} - \phi_{i-1,j}}{2\Delta x}$$

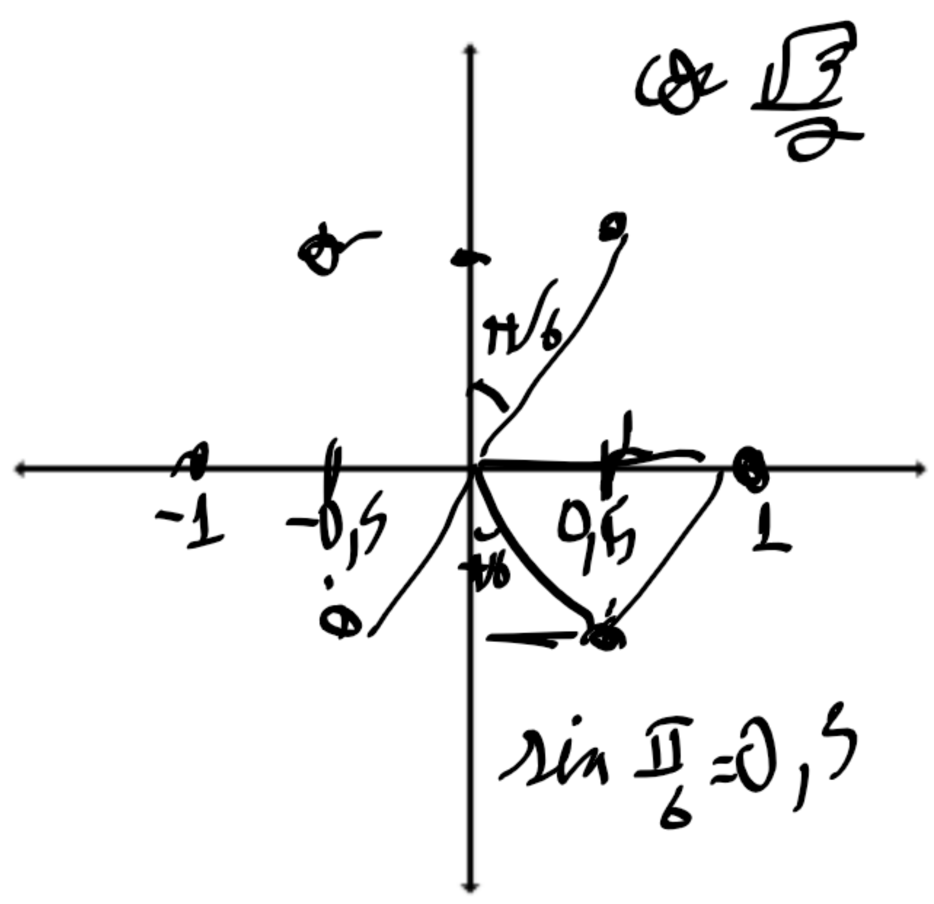
$$\frac{\partial\phi_{ij}}{\partial x_j} = \frac{\phi(y_j + \Delta y) - \phi(y_j - \Delta y)}{2\Delta y} = \frac{\phi_{i,j+1} - \phi_{i,j-1}}{2\Delta y}$$

em que $\Delta x$ = x_arr[1] − x_arr[0] e $\Delta y$ = y_arr[1] − y_arr[0]

$$\vec{E} = -\nabla\Phi \Rightarrow E_x = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ -\frac{\partial\Phi_{21}}{\partial x_2} & -\frac{\partial\Phi_{22}}{\partial x_2} & -\frac{\partial\Phi_{23}}{\partial x_2} & \cdots & -\frac{\partial\Phi_{2,N_y}}{\partial x_2} \\ -\frac{\partial\phi_{31}}{\partial x_3} & -\frac{\partial\phi_{32}}{\partial x_3} & -\frac{\partial\phi_{33}}{\partial x_3} & \cdots & -\frac{\partial\phi_{3,N_y}}{\partial x_3} \\ \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix},\quad E_y = \begin{pmatrix} 0 & -\frac{\partial\phi_{12}}{\partial y_2} & -\frac{\partial\phi_{13}}{\partial y_3} & & 0 \\ 0 & -\frac{\partial\Phi_{22}}{\partial y_2} & -\frac{\partial\Phi_{23}}{\partial y_3} & \cdots & 0 \\ 0 & -\frac{\partial\phi_{32}}{\partial y_2} & -\frac{\partial\phi_{33}}{\partial y_3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -\frac{\partial\phi_{N_x2}}{\partial y_2} & -\frac{\partial\phi_{N_x3}}{\partial y_3} & \cdots & 0 \end{pmatrix}$$

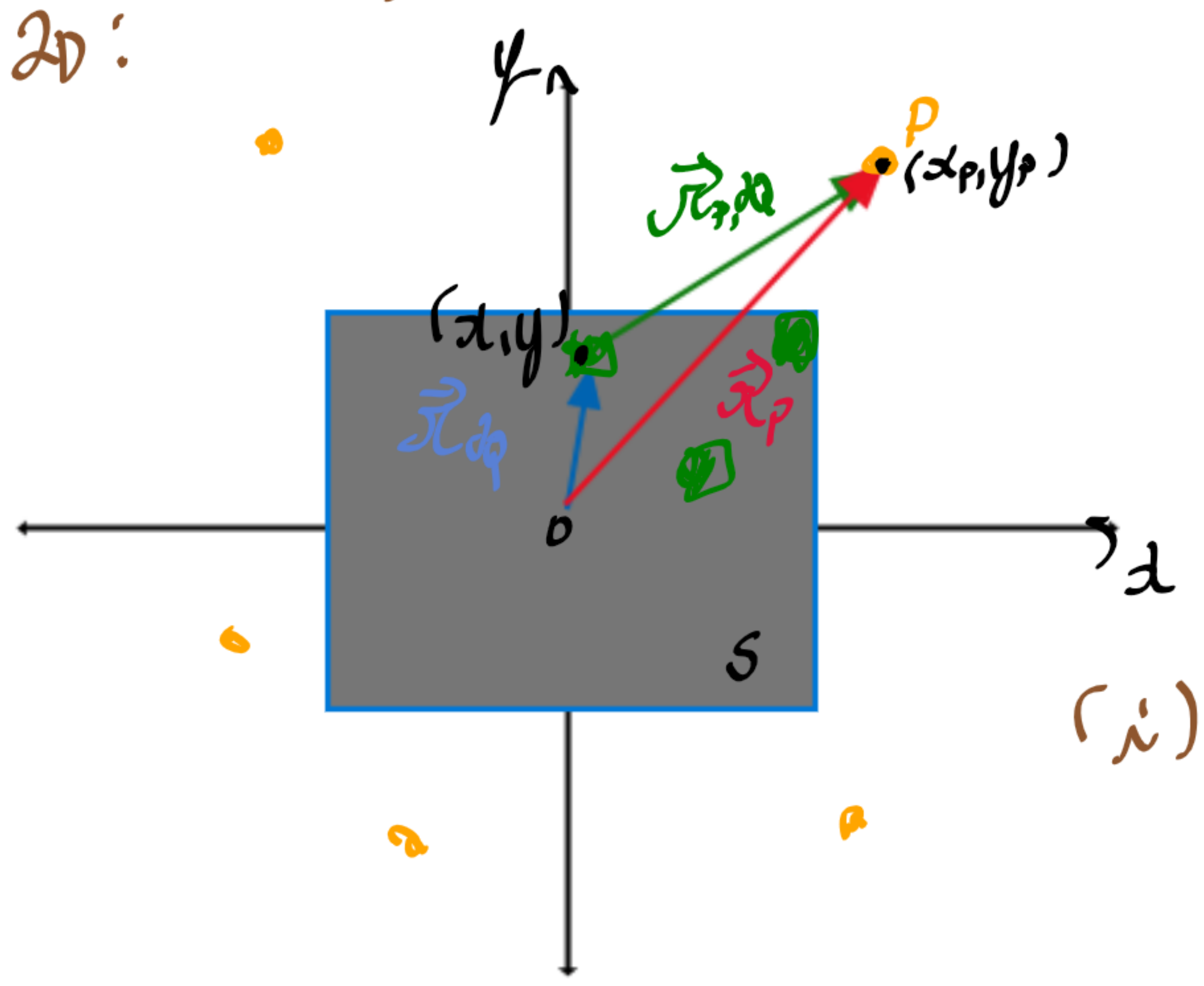
**Algumas distribuições:**



Distribuição contínua de cargas:

2D:




$$\phi_p = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma(x,y)}{\|\vec{r}_p - \vec{r}_\varphi\|} dA = \frac{1}{4\pi\varepsilon_0}\iint_S \frac{\sigma(x,y)}{\sqrt{(x-x_p)^2+(y-y_p)^2}} dx\,dy$$

Para retângulo: $S = \{(x,y) \in \mathbb{R}^2 : -\frac{L_x}{2} \leq x \leq \frac{L_x}{2}, -\frac{L_y}{2} \leq y \leq \frac{L_y}{2}\}$

$$\Rightarrow \phi_p = \frac{1}{4\pi\varepsilon_0}\int_{-\frac{L_x}{2}}^{\frac{L_x}{2}}\int_{-\frac{L_y}{2}}^{\frac{L_y}{2}} \frac{\sigma(x,y)}{\sqrt{(x-x_p)^2+(y-y_p)^2}} dx\,dy$$

(i) $\sigma(x,y) = q_0 \sin\left(\frac{2\pi}{L_x}x\right)\sin\left(\frac{2\pi}{L_y}y\right)$

$$\Rightarrow \phi_p = \frac{q_0}{4\pi\varepsilon_0}\int_{-\frac{L_x}{2}}^{\frac{L_x}{2}}\int_{-\frac{L_y}{2}}^{\frac{L_y}{2}} \frac{\sin\left(\frac{2\pi}{L_x}x\right)\sin\left(\frac{2\pi}{L_y}y\right)}{\sqrt{(x-x_p)^2+(y-y_p)^2}} dy\,dx$$

(ii) $\sigma(x,y) = q_0(x+y)$:

$$\phi_p = \frac{q_0}{4\pi\varepsilon_0}\int_{-\frac{L_x}{2}}^{\frac{L_x}{2}}\int_{-\frac{L_y}{2}}^{\frac{L_y}{2}} \frac{x+y}{\sqrt{(x-x_p)^2+(y-y_p)^2}} dy\,dx$$