

Determinantes - Matriz de Ordem 1, 2 e 3 - Tarefa Básica

Tarefa Básica

01.

a) $\begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} \Rightarrow \det A: 10 - 3 = \underline{7}$ b) $\begin{vmatrix} -2 & -4 \\ 3 & 6 \end{vmatrix} \Rightarrow -2 \cdot 6 - 3 \cdot 4 = -12 - 12 = \underline{0}$

c) $\begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 4 & -2 \end{vmatrix} \Rightarrow 3 - (-7) = 10$
 $-6 + 1 + 8 = 3$
 $-3 + 3 + 16 = 16$

d) $\begin{vmatrix} 3 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{vmatrix} \Rightarrow 36 - 16 = \underline{20}$
 $36 + 2 - 2 = 36$

02.

$A_{3 \times 3} = a_{ij} = \begin{cases} -3, & \text{se } i=j \\ 0, & \text{se } i \neq j \end{cases} \Rightarrow \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

$a_{11} = i=j = -3$
 $a_{22} = i=j = -3$
 $a_{33} = i=j = -3$
 $\text{resto } i \neq j = 0$

$\det A = \begin{vmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{vmatrix} \Rightarrow -27 - 0 = \underline{-27} = (A)$
 $0 + 0 + 0 = 0$
 $-27 + 0 + 0 = -27$

03. $x^2 + 12x + 9$

$$\begin{array}{c} \begin{array}{c} \diagup \quad \diagdown \\ \begin{array}{ccccc} x & 1 & x & 1 & x \\ 3 & x & 4 & 3 & x \\ 1 & 3 & 3 & 1 & 3 \end{array} \\ \diagdown \quad \diagup \end{array} \end{array} = 3x^2 + 4 + 9x - (x^2 + 12x + 9) = \textcircled{-3}$$

$$3x^2 + 4 + 9x - x^2 - 12x - 9$$

$$3x^2 + 4 + 9x$$

$$2x^2 - 3x - 5 = -3$$

$$2x^2 - 3x - 2 \Rightarrow A=2, B=-3, C=-2$$

$$\Delta = b^2 - 4 \cdot a \cdot c \Rightarrow (-3)^2 - 4 \cdot 2 \cdot -2$$

$$\Delta = 9 + 16 = \underline{\underline{25}}$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{25}}{2 \cdot 2} \Rightarrow x = \frac{3 \pm 5}{4}$$

$$\left. \begin{array}{l} x_1 = \frac{3+5}{4} = \frac{8}{4} = \underline{\underline{2}} \\ x_2 = \frac{3-5}{4} = \frac{-2}{4} = \underline{\underline{-\frac{1}{2}}} \end{array} \right\} (E)$$

04.

$$2 \cdot 0 + x - 1 + 0 = x - 1$$

$$\begin{vmatrix} x-1 & -1 & 0 \\ 0 & x+1 & -1 \\ 2 & -1 & x+1 \end{vmatrix} \begin{vmatrix} x-1 & -1 \\ 0 & x+1 \\ 2 & -1 \end{vmatrix} \Rightarrow \begin{aligned} x^3 + x^2 - x + 1 - (x-1) &= 2 \\ x^3 + x^2 - x + 1 - x + 1 &= 2 \\ x^3 + x^2 - 2x + 2 &= 2 \end{aligned}$$

$$x^3 + x^2 - x - 1 + 2 + 0 = x^3 + x^2 - x + 1$$

$$x^3 + x^2 - 2x = 0$$

$$x(x^2 + x - 2) = 0 \Rightarrow A=1, B=1, C=-2$$

$$\Delta = b^2 - 4 \cdot a \cdot c \Rightarrow 1^2 - 4 \cdot 1 \cdot -2 \Rightarrow 1 + 8 = 9$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2 \cdot a} \Rightarrow \frac{-1 \pm \sqrt{9}}{2 \cdot 1} \Rightarrow \frac{-1 \pm 3}{2}$$

$$x_1 = \frac{-1+3}{2} = \frac{2}{2} = 1$$

$$x_2 = \frac{-1-3}{2} = \frac{-4}{2} = -2$$

$$x_1 + x_2 = -2 + 1 = -1$$

$$R: -1 = (C)$$

05.

$$A_{3 \times 2} = a_{ij} = 2i - 3j \quad A_{3 \times 2} \cdot B_{2 \times 3} = I_{3 \times 3} = AB$$

$$B_{2 \times 3} = b_{jk} = k - j$$

$$A = \begin{bmatrix} \frac{-1}{11} & \frac{-4}{12} \\ \frac{1}{21} & \frac{-2}{22} \\ \frac{3}{31} & \frac{0}{32} \end{bmatrix} \begin{cases} a_{11} = 2 \cdot 1 - 3 \cdot 1 = -1 \\ a_{12} = 2 \cdot 1 - 3 \cdot 2 = -4 \\ a_{21} = 2 \cdot 2 - 3 \cdot 1 = 1 \\ a_{22} = 2 \cdot 2 - 3 \cdot 2 = -2 \\ a_{31} = 2 \cdot 3 - 3 \cdot 1 = 3 \\ a_{32} = 2 \cdot 3 - 3 \cdot 2 = 0 \end{cases}$$

$$B = \begin{bmatrix} \frac{0}{11} & \frac{1}{12} & \frac{2}{13} \\ \frac{-1}{21} & \frac{0}{22} & \frac{1}{23} \end{bmatrix} \begin{cases} b_{11} = 1 - 1 = 0 // b_{21} = 1 - 2 = -1 \\ b_{12} = 2 - 1 = 1 // b_{22} = 2 - 2 = 0 \\ b_{13} = 3 - 1 = 2 // b_{23} = 3 - 2 = 1 \end{cases}$$

Continuação do 5. $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} -1 & -4 \\ 1 & -2 \\ 3 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} +4 & -1 & -6 \\ 2 & 1 & 0 \\ 0 & 3 & 6 \end{bmatrix}$$

$0+0-12=-12$

$$\det AB = \begin{vmatrix} +4 & -1 & -6 \\ 2 & 1 & 0 \\ 0 & 3 & 6 \end{vmatrix} \Rightarrow -12 - (-12) = 0 = (C)$$

$24+0-36=-12$

06.

$$A_{2 \times 3} \cdot B_{3 \times 2} = \exists = AB_{2 \times 2} = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2+0+0=2}{11} & \frac{-2+0-2=-4}{12} \\ \frac{-1-1+0=-2}{21} & \frac{1+1+0=2}{22} \end{bmatrix}$$

$$\det AB = \begin{vmatrix} 2 & -4 \\ -2 & 2 \end{vmatrix} = 4 - 8 = -4 = (D)$$

