**Maximum likelihood questions**

1) The number of accidents on a one mile stretch of the Pennsylvania turnpike is thought to be independent and have the same mean from week to week.

1. The distribution of the number for one week would then follow
   1. An exponential distribution
   2. A Poisson distribution
   3. A discrete uniform distribution
   4. A geometric distribution

Hint: number of rare events in a process with independent increments

1. Data on the number of accidents is collected for each of five weeks and comes out to 2, 5, 2, 3, and 4 accidents. If you want to estimate the mean number, l, of accidents per week on this one mile stretch then the MLE would be the value of l that maximizes
   1. All of the above

Hint: how does rescaling or taking logarithms affect the maximization problem?

2) Twenty randomly selected adults over 50 years of age in Pennsylvania are asked if they have been vaccinated for shingles giving observations x1, x2, …, x20 (xi = 1 if yes and 0 if no) of the random variables X1,…,X20.

A) The random variable X7 would then follow a

a. Geometric distribution

b. Bernoulli distribution

c. Normal distribution

d. Poisson distribution

Hint: values can only be 0 or 1

B) These observations are to be used to estimate the probability, q, that a randomly selected Pennsylvanian over 50 has been vaccinated for shingles. To find the MLE of q, you would

a. maximize over the possible vectors (x1, x2, …, x20)

b. maximize over the possible values of q

c. maximize over the possible values of q & vectors (x1, x2, …, x20)

d. none of the above

Hint: what is the parameter and what are the data

3) Suppose Y = the time (hours) it takes a programmer to complete and debug a computer program has the pdf . Ten programs are assigned to be completed and the times to completion for each, Y1, Y2, …, Y10, are independent and identically distributed with the pdf, *f*, above. The MLE of a would be then be the

a. mean of Y1, Y2, …, Y10

b. median of Y1, Y2, …, Y10

c. minimum of Y1, Y2, …, Y10

d. maximum of Y1, Y2, …, Y10

Hint: The pdf has no probability below a.

4) In Pennsylvania, some roads have a mile-marker every tenth of a mile, while others only have mile-markers every mile. A car is driving east and breaks down. The driver decides to push his car to the next mile marker in that direction so it will be easy for a tow truck to find him when he calls for emergency service. A tow service gets a call that a driver is stranded after pushing his car 1/20 of a mile. What is the MLE of the type of mile-markers on that road (markers every tenth mile or markers every mile)?

a. every tenth of a mile

b. every mile

c. every 1/20th of a mile

Hint: there are only two possible values for the parameter – which has a higher likelihood?

5) True or False: Maximum Likelihood Estimator is unbiased.

Hint: As an example think about the MLE for q, based on data from a Uniform(0, q) distribution.

6) Which of the following is a consequence of the invariance property of maximum likelihood (where q is a parameter of the distribution of X)?

a. The MLE of log(q) is the log of the MLE of q using data from observing X.

b. Since q is still a parameter of the distribution of Y = log(X), then data based on observing Y will give the same MLE for q as when the data are based on observing X.

c. Since ln(q) is still a parameter of the distribution of Y = log(X), then the MLE of that parameter based on observing Y will give the log of the MLE of q as when the data are based on observing X.

d. The MLE of log(q) has the same variance as the MLE of q.

Hint: The invariance property relates to one-to-one functions of a parameter.

7) True or False: If X1, X2, …, Xn are i.i.d. with cdf , then the MLE for q can be found by finding the value of q that maximizes

Hint: remember that F is the cdf

8) In a mining operation, the proportion, X, of a mineral that is found in a sample piece of ore follows the Beta distribution with parameters a and 1 so its pdf is given by

Twenty sample ores are removed from the mine providing independent observations x1, x2, …,x20.

1. Which of the following will lead you to the maximum likelihood estimate of a?
   1. Maximizing the likelihood with respect to the average of the data
   2. Maximizing the likelihood with respect to a
   3. Maximizing the log of the likelihood with respect to the average of the data
   4. Maximizing the log of the likelihood respect to a
   5. Both a and c
   6. Both b and d

Hint: since log is a monotonic function, its maximum occurs at the same point.

1. In this case, the log likelihood function is given by

Hint: write the likelihood function as a product and take logs

1. The MLE of a in this problem would be

Hint: find a so *l’*= 0 and *l”* <0 where *l* is the log likelihood and the derivative is w.r.t. a

9). A polling company wants to know how many random calls it must make on average before reaching a person who is likely to vote in the next election and agrees to take their brief survey. To estimate this quantity they will use their normal procedure 20 times and collect data on X1, X2,…, X20 where Xi = how many calls it takes before the ith likely voter agrees to take the survey.

A) What type of distribution does Xi have?

a. Xi would be Bernoulli with an unknown probability, p, that Xi =1.

b. Xi would be Poisson with an unknown mean m.

c. Xi would be Binomial with n = 20 and an unknown probability, p, for the event in question.

d. Xi would be Geometric with an unknown probability, p, for the event in question.

Hint: we are modeling the number of trials until the first success

B) For observations Xi = xi for i = 1, …20, the log likelihood function would be

a.

b.

c.

d.

Hint: the Xi’s follow a geometric distribution

C) The MLE for p is then given by

a.

b.

c.

d.

Hint: find p so *l’*= 0 and *l”* <0 where *l* is the log likelihood and the derivative is w.r.t. p

1. Finally, to answer the polling company’s question about estimating how many random calls it must make on average per respondent using maximum likelihood, we would find the
   1. MLE of 1/p which is
   2. MLE of 1/p which is
   3. MLE of the median m which is *(the median of the x’s)*
   4. MLE of the derivative of the log likelihood

Hint: use the invariance property of the MLE

10) The time between earthquakes, X, depends on a parameter q that is associated with the sensitivity of the hardware used to detect them so that X follows the pdf

The times for three independent earthquakes are recorded and found (in days) to be 87, 565, and 122.

1. What is the likelihood function?

Hint: 5 i.i.d. observations give a product likelihood

1. What is , the MLE of q?
   1. 77.4
   2. 1/77.4
   3. 5,996,910/774
   4. 5,996,910/5!

Hint: find q so *l’*= 0 and *l”* <0 where *l* is the log likelihood and the derivative is w.r.t. q