Reference information about many standard Java classes appears at the end of the test. You might want to tear off those pages to make them easier to refer to while solving the programming problems.

Question 1. (8 points) (a) Draw the binary search tree that is created if the following numbers are inserted in the tree in the given order.

12 15 3 35 21 42 14

```
     12
    /  \
   3    15
       /  \
      14   35
          /  \
         21   42
```

(b) Draw a *balanced* binary search tree containing the same numbers given in part (a).

```
       15
      /   \
    12     35
   /  \   /  \
  3   14 21   42
```

Question 2. (3 points) Show that $3n^4 + 100\,n^2 + 42n$ is $O(n^4)$.

To show that $3n^4 + 100\,n^2 + 42n$ is $O(n^4)$ we need to pick some constant c such that

$$3n^4 + 100\,n^2 + 42n \le cn^4$$

for all but some finite (small) values of n. Picking $c = 4$ is sufficient, as is anything larger than 3.

(For full credit, answers were not required to show particular values of n for which the inequality holds, nor to give an argument that this holds for all larger values of n, but it was fine if that was included.)

Question 3. (2 points) Methods `hashCode()` and `equals()` are defined in class `Object` and are inherited by all Java classes. If a Java class overrides `equals()` it should normally also override `hashCode()` to be sure that `equals()` and `hashCode()` work together properly. Given two objects `o1` and `o2`, what should be true about the `hashCode()` function if `o1.equals(o2)`?

If `o1.equals(o2)` then `o1.hashCode()` should equal `o2.hashCode()`.

Question 4. (8 points) During the quarter we've looked at several different ways of implementing a collection of objects, in particular:

- Lists based on arrays
- Linked lists
- Trees, particularly binary search trees
- Hash tables

For each of the following operations, circle the expected time required for the operation. Your answers should assume that the data structure is performing well, i.e., binary search trees are reasonably well balanced and hash tables have a low load factor (i.e., your answer should be the *expected* time, not the *worst case* time if those two are different).

(a) ***contains***: return true or false depending on whether a particular object is a member of a collection.

Sorted list implemented with an array:

$O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

Sorted list implemented a with linked list:

$O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

Hash table: $O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

Binary search tree: $O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

(b) ***insert***: add a new item to the collection in the appropriate place

Unsorted list implemented with an array:

$O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

Sorted list implemented with an array:

$O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

Hash table: $O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

Binary search tree: $O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

The following class implements an *n* by *n* square 2-D array with several operations. It is used in the next two questions.

```
public class Grid {

   private double[][] grid;

   public Grid(int n) {
      grid = new double[n][n];   // array elements are all 0 here
   }                             // (this takes constant, O(1) time)

   public void set(int row, int col, double value) {
      grid[row][col] = value;
   }

   public double get(int row, int col) {
      return grid[row][col];
   }

   public void smash(int row, int col) {
      double sum = 0.0;
      for (int r = row-1; r <= row+1; r++) {
         for (int c = col-1; c <= col+1; c++) {
            sum = sum + grid[r][c];
         }
      }
      grid[row][col] = sum/9.0;
   }

   public void reverse(int row) {
      int L = 0;
      int R = grid.length-1;
      while (L < R) {
         double temp = grid[row][L];
         grid[row][L] = grid[row][R];
         grid[row][R] = temp;
         L++;
         R--;
      }
   }

   public void flip() {
      for (int r = 0; r < grid.length; r++) {
         reverse(r);
      }
   }

}
```

Question 5. (8 points) For each of the following code fragments, circle the smallest complexity class that gives the running time of the code as a function of the grid size n. You should assume that the parameter `n` is large enough so that no `IndexOutOfBounds` exceptions will occur.

(a)
```
public void parta(int n) {
   Grid g = new Grid(n);
   g.flip();
}
```

$O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

(b)
```
public void partb(int n) {
   Grid g = new Grid(n);
   for (int k = -10000; k <= 10000; k++) {
      g.smash(1, 1);
   }
}
```

$O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

(c)
```
public void partc(int n) {
   Grid g = new Grid(n);
   for (int r = 1; r < n; r = 2*r) {
      g.reverse(r);
   }
}
```

$O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

(d)
```
public void partd(int n) {
   Grid g = new Grid(n);
   for (int x = 1; x < n-1; x++) {
      g.smash(x,x);
   }
}
```

$O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

Question 7. (4 points) (a) What is the *expected* time needed by Quicksort to sort a list with n items? (circle)

$O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

(b) What is the *worst-case* time needed by Quicksort to sort a list with n items? (circle)

$O(1)$ $O(\log n)$ $O(n)$ $O(n \log n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$

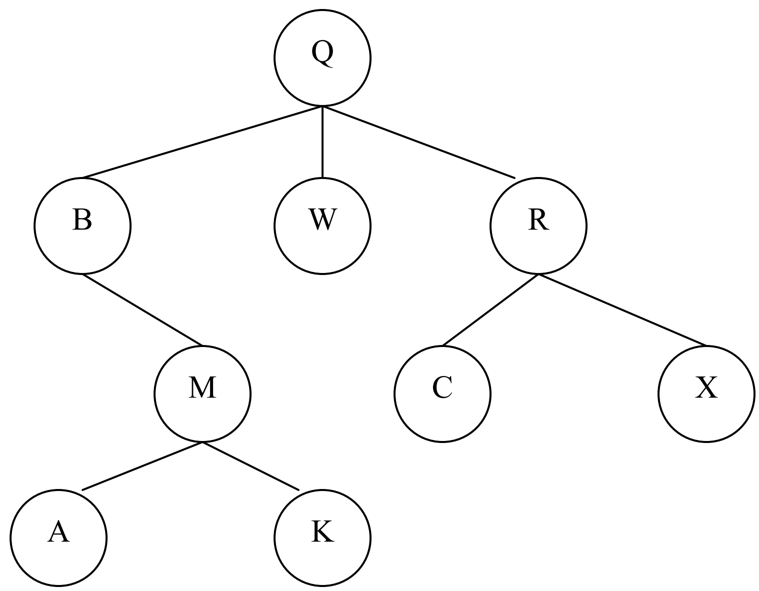
(c) What needs to be true about the implementation of Quicksort to ensure that the sort only requires the expected time instead of the worst-case time?

At each step of the sort, the partition function needs to divide the list into two roughly equal halves that can then be recursively sorted.

Question 8. (2 points) Two properties of a collection are its *size* and *capacity*. What is the difference between these two properties?

The *capacity* is the total space available in the collection. The *size* is the number of items currently stored in the collection.

Question 9. (7 points) Consider the following tree, which is not a binary tree.



(a) Which node(s) is(are) the roots of this tree?

Q

(b) Which node(s) is(are) the leaves of this tree?

A K W C X

(c) Write down the nodes in the order they are reached if we perform a *postorder* traversal of this tree starting with node Q.

A K M B W C X R Q

Question 10. (8 points) If we want to create a binary tree whose nodes contain integer values, we can represent the nodes using instances of the following Java class.

```
/** binary tree node with integer node values */
public class BTNode {
   public int value;           // value contained in this node
   public BTNode left;         // left subtree; null if empty
   public BTNode right;        // right subtree; null if empty
}
```

Complete the definition of the following method so it returns the sum of the values contained in all of the nodes of the binary tree with root `n`. Hint: Recursion *is* your friend.

```
/** Return the sum of the values in a binary tree with root n */
public int sum(BTNode n) {

   if (n == null) {
      return 0;
   } else {
      return n.value + sum(n.left) + sum(n.right);
   }

}
```

Question 12. (8 points) One common way to implement a list is a single-linked list containing a collection of nodes that refer to the data objects in the list. We can define the link nodes as follows:

```
/** One link in the list */
public class Link {
   public Object value;  // data value associated with this node
   public Link   next;   // next node in list or null if none

   /** Construct a new link with value v and next node n */
   public Link(Object v, Link n) {
      this.value = v;  this.next = n;
   }
}
```

A class using these links to implement a list would have an instance variable of type `Link` referring to the list data.

```
/** A simple list implemented with a linked list */
public class SimpleLinkedList {
   private Link head;    // first link in the list or null if
                         //    the list is empty
```

Complete the definition of method `addToEnd` below of class `SimpleLinkedList` so it adds a new value to the *end* of the list. You **may not** assume there are any additional instance variables in class `SimpleLinkedList`, and you **may not** add any.

```
/** Add value v to the end of this SimpleLinkedList */
public void addToEnd(Object v) {

   Link n = new Link(v, null);
   if (head == null) {
      head = n;
   } else {
      Link p = head;
      while (p.next != null) {
         p = p.next;
      }
      p.next = n;
   }

}
```

Question 13. (2 points) The infamous bouncing ball program contained code similar to the following to set up the buttons at the bottom of the window.

```
JButton stop = new JButton("stop");
JButton go = new JButton("go");
JPanel buttons = new JPanel();
buttons.add(stop);
buttons.add(go);
add(buttons, BorderLayout.SOUTH);
buttonListener = new SimButtonListener(world);
stop.addActionListener(buttonListener);
go.addActionListener(buttonListener);
```

How would the program's behavior change if the last two lines of code (the `addActionListener` method calls) were omitted?

Clicks on the buttons would be ignored. (Each button needs to be told to notify a listener object when an event occurs.)

Question 14. (4 points) A hash set or hash map can perform many operations like insert, delete, search (contains) and others very efficiently, this can only happen if the hash function and load factor have certain properties.

(a) What is a *hash function*? What must be true about the hash function for these operations to be very efficient?

A hash function converts a key value into an integer that is used to select the bucket (or slot) where that key should be stored. For hashing to be efficient, the hash function must distribute the keys among the buckets uniformly so no one bucket winds up with too many key values.

(b) What is the *load factor* of a hash table? What must be true about the load factor for operations to be very efficient?

The load factor is the ratio *n*/b of the number of items in the table (*n*) to the total number of buckets (*b*). For hashing to be efficient, this number needs to be small.