

# Introduction to Management

## CSE 344

### Lectures 17: Design Theory

# Announcements

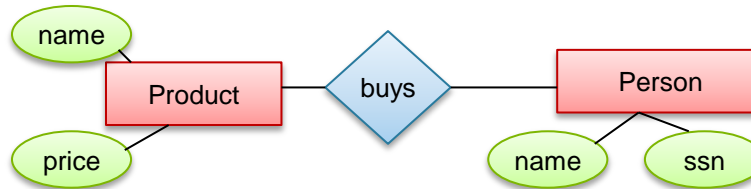
- No class/office hour on Monday
- Midterm on Wednesday (Feb 19) in class
- HW5 due next Thursday (Feb 20)
- No WQ next week (WQ6 due on Feb 25)
  - But try to finish early! (Lecture 15-17)

# Midterm

- **All material up to and including Lecture 15**
  - SQL, basic evaluation + indexes, RA, datalog-with-negation, RC, XML/XPath/XQuery, E/R diagram
- **Open books, open notes, no electronic devices**
  - Don't waste paper printing stuff. Normally, you shouldn't need any notes during the exam. My suggestion is to print, say, 5-6 selected slides from the lecture notes that you had trouble with, and to print your own homework, just in case you forget some cool solution you used there.
  - Make sure you understand all the concepts!

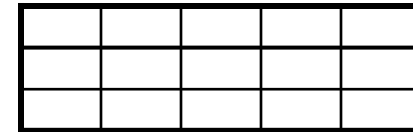
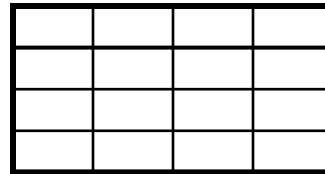
# Relational Schema Design

Conceptual Model:



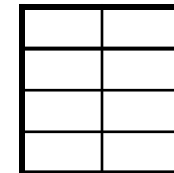
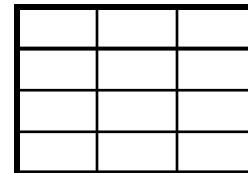
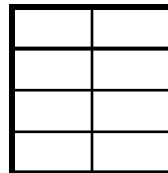
DONE

Relational Model:  
plus FD's



DONE

Normalization:  
Eliminates **anomalies**



Today

# Relational Schema Design

Name	<u>SSN</u>	<u>PhoneNumber</u>	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

One person may have multiple phones, but lives in only one city

Primary key is thus (SSN, PhoneNumber)

What is the problem with this schema?

# Relational Schema Design

Name	<u>SSN</u>	<u>PhoneNumber</u>	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

## Anomalies:

- Redundancy = repeat data
- Update anomalies = what if Fred moves to “Bellevue”?
- Deletion anomalies = what if Joe deletes his phone number?

Can you suggest a solution?

# Relation Decomposition

Break the relation into two:

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield

Name	<u>SSN</u>	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Westfield

<u>SSN</u>	<u>PhoneNumber</u>
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121

Anomalies have gone:

- No more repeated data
- Easy to move Fred to “Bellevue” (how ?)
- Easy to delete all Joe’s phone numbers (how ?)

# Relational Schema Design (or Logical Design)

How do we do this systematically?

- Start with some relational schema
- Find out its **functional dependencies** (FDs)
- Use FDs to **normalize** the relational schema



# Functional Dependencies (FDs)

## Definition

If two tuples agree on the attributes

$A_1, A_2, \dots, A_n$

then they must also agree on the attributes

$B_1, B_2, \dots, B_m$

Formally:

$A_1 \dots A_n$  **determines**  $B_1 \dots B_m$

$A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$

# Functional Dependencies (FDs)

**Definition**  $A_1, \dots, A_m \rightarrow B_1, \dots, B_n$  holds in R if:

$\forall t, t' \in R,$

$(t.A_1 = t'.A_1 \wedge \dots \wedge t.A_m = t'.A_m \Rightarrow t.B_1 = t'.B_1 \wedge \dots \wedge t.B_n = t'.B_n)$

R		$A_1$	...	$A_m$		$B_1$	...	$B_n$		
t										
t'										

if  $t, t'$  agree here then  $t, t'$  agree here

# Example

An FD holds, or does not hold on an instance:

<b>EmpID</b>	<b>Name</b>	<b>Phone</b>	<b>Position</b>
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

EmpID → Name, Phone, Position

Position → Phone

but not Phone → Position

# Example

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876 ←	Salesrep
E1111	Smith	9876 ←	Salesrep
E9999	Mary	1234	Lawyer

Position → Phone

# Example

EmpID	Name	Phone	Position
E0045	Smith	1234 →	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234 →	Lawyer

But not Phone → Position

# Example

name  $\rightarrow$  color  
category  $\rightarrow$  department  
color, category  $\rightarrow$  price

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Green	Toys	99

Do all the FDs hold on this instance?

# Example

name  $\rightarrow$  color  
category  $\rightarrow$  department  
color, category  $\rightarrow$  price

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Green	Toys	99

Do all the FDs hold on this instance?

No. color,category  $\rightarrow$  price does not hold

# Example

name → color  
category → department  
color, category → price

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Black	Toys	99
Gizmo	Stationary	Green	Office-supply.	59

What about this one ?



# Example

name → color  
category → department  
color, category → price

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Black	Toys	99
Gizmo	Stationary	Green	Office-suppl.	59

What about this one ?

# Terminology

- FD **holds** or **does not hold** on an instance
- If we can be sure that *every instance of  $R$*  will be one in which a given FD is true, then we say that  **$R$  satisfies the FD**
- If we say that  $R$  satisfies an FD  $F$ , we are **stating a constraint on  $R$**

# An Interesting Observation

If all these FDs are true:

$\text{name} \rightarrow \text{color}$   
 $\text{category} \rightarrow \text{department}$   
 $\text{color, category} \rightarrow \text{price}$

Then this FD also holds:

$\text{name, category} \rightarrow \text{price}$

If we find out from application domain that a relation satisfies some FDs, it doesn't mean that we found all the FDs that it satisfies! There could be more FDs implied by the ones we have.

# Closure of a set of Attributes

**Given** a set of attributes  $A_1, \dots, A_n$  (under FD set  $F$ )

The **closure**,  $\{A_1, \dots, A_n\}^+$  = the set of attributes  $B$  that  
(any reln that satisfies  $F$ ) satisfies  $A_1, \dots, A_n \rightarrow B$

Example:

1.  $\text{name} \rightarrow \text{color}$
2.  $\text{category} \rightarrow \text{department}$
3.  $\text{color}, \text{category} \rightarrow \text{price}$

Closures:

$$\text{name}^+ = \{\text{name}, \text{color}\}$$

$$\{\text{name}, \text{category}\}^+ = \{\text{name}, \text{category}, \text{color}, \text{department}, \text{price}\}$$

$$\text{color}^+ = \{\text{color}\}$$

# Closure Algorithm

$X = \{A_1, \dots, A_n\}$ .

**Repeat** until  $X$  doesn't change **do**:  
  **if**  $B_1, \dots, B_n \rightarrow C$  is a FD **and**  
     $B_1, \dots, B_n$  are all in  $X$   
  **then** add  $C$  to  $X$ .

Example:

1. name  $\rightarrow$  color
2. category  $\rightarrow$  department
3. color, category  $\rightarrow$  price

$\{\text{name, category}\}^+ =$   
   $\{\text{name, category, .....}\}$

# Closure Algorithm

$X = \{A_1, \dots, A_n\}$ .

**Repeat until X doesn't change do:**  
  **if**      $B_1, \dots, B_n \rightarrow C$  is a FD **and**  
           $B_1, \dots, B_n$  are all in X  
  **then** add C to X.

Example:

1. name  $\rightarrow$  color
2. category  $\rightarrow$  department
3. color, category  $\rightarrow$  price

$\{\text{name, category}\}^+ =$   
  { name, category, color, department, price }

Hence: name, category  $\rightarrow$  color, department, price

# Example

In class:

$R(A, B, C, D, E, F)$

A, B	→	C
A, D	→	E
B	→	D
A, F	→	B

Compute  $\{A, B\}^+$      $X = \{A, B, \dots\}$

Compute  $\{A, F\}^+$      $X = \{A, F, \dots\}$

# Example

In class:

$R(A, B, C, D, E, F)$

A, B	→	C
A, D	→	E
B	→	D
A, F	→	B

Compute  $\{A, B\}^+$      $X = \{A, B, C, D, E\}$

Compute  $\{A, F\}^+$      $X = \{A, F, \dots\}$



# Example

In class:

$R(A, B, C, D, E, F)$

A, B	→	C
A, D	→	E
B	→	D
A, F	→	B

Compute  $\{A, B\}^+$      $X = \{A, B, C, D, E\}$

Compute  $\{A, F\}^+$      $X = \{A, F, B, C, D, E\}$

# Example

In class:

$R(A, B, C, D, E, F)$

A, B	→	C
A, D	→	E
B	→	D
A, F	→	B

Compute  $\{A, B\}^+$      $X = \{A, B, C, D, E\}$

Compute  $\{A, F\}^+$      $X = \{A, F, B, C, D, E\}$

Can you see a “key” of R?

# Practice at Home

Find all FD's implied by:

$A, B \rightarrow C$
$A, D \rightarrow B$
$B \rightarrow D$

Step 1: Compute  $X^+$ , for every  $X$ :

Step 2: Enumerate all FD's  $X \rightarrow Y$ , s.t.  $Y \subseteq X^+$  and  $X \cap Y = \emptyset$ :

# Practice at Home

Find all FD's implied by:

$$\begin{array}{lcl} A, B & \rightarrow & C \\ A, D & \rightarrow & B \\ B & \rightarrow & D \end{array}$$

Step 1: Compute  $X^+$ , for every  $X$ :

$$A^+ = A, \quad B^+ = BD, \quad C^+ = C, \quad D^+ = D$$

$$AB^+ = ABCD, \quad AC^+ = AC, \quad AD^+ = ABCD,$$

$$BC^+ = BCD, \quad BD^+ = BD, \quad CD^+ = CD$$

$$ABC^+ = ABD^+ = ACD^+ = ABCD \text{ (no need to compute— why ?)}$$

$$BCD^+ = BCD, \quad ABCD^+ = ABCD$$

# Practice at Home

Find all FD's implied by:

$A, B \rightarrow C$
$A, D \rightarrow B$
$B \rightarrow D$

Step 1: Compute  $X^+$ , for every  $X$ :

$A^+ = A, B^+ = BD, C^+ = C, D^+ = D$

$AB^+ = ABCD, AC^+ = AC, AD^+ = ABCD,$

$BC^+ = BCD, BD^+ = BD, CD^+ = CD$

$ABC^+ = ABD^+ = ACD^+ = ABCD$  (no need to compute— why ?)

$BCD^+ = BCD, ABCD^+ = ABCD$

Step 2: Enumerate all FD's  $X \rightarrow Y$ , s.t.  $Y \subseteq X^+$  and  $X \cap Y = \emptyset$ :

$AB \rightarrow CD, AD \rightarrow BC, ABC \rightarrow D, ABD \rightarrow C, ACD \rightarrow B$

# Keys

- A **superkey** is a set of attributes  $A_1, \dots, A_n$  s.t. for any other attribute  $B$ , we have  $A_1, \dots, A_n \rightarrow B$
- A **key** is a minimal superkey
  - A superkey and for which no subset is a superkey

# Computing (Super)Keys

- For all sets  $X$ , compute  $X^+$
- If  $X^+ = [\text{all attributes}]$ , then  $X$  is a superkey
- Try only the minimal  $X$ 's to get the keys
  - No subset of  $X$  is a superkey

# Example

Product(name, price, category, color)

name, category → price  
category → color

What is the key ?



# Example

Product(name, price, category, color)

name, category  $\rightarrow$  price  
category  $\rightarrow$  color

What is the key ?

$(\text{name, category}) + = \{ \text{name, category, price, color} \}$

Hence (name, category) is a key

# Key or Keys ?

Can we have more than one key ?

Given  $R(A,B,C)$  define FD's s.t. there are two or more keys

Try in class..

.....

# Key or Keys ?

Can we have more than one key ?

Given  $R(A,B,C)$  define FD's s.t. there are two or more keys

$$\begin{array}{l} A \rightarrow B \\ B \rightarrow C \\ C \rightarrow A \end{array}$$

or

$$\begin{array}{l} AB \rightarrow C \\ BC \rightarrow A \end{array}$$

or

$$\begin{array}{l} A \rightarrow BC \\ B \rightarrow AC \end{array}$$

what are the keys here ?

# Eliminating Anomalies

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

SSN → Name, City

What is the key?

Suggest a rule for decomposing the table to eliminate anomalies

# Eliminating Anomalies

Main idea:

- $X \rightarrow A$  is OK if  $X$  is a (super)key
- $X \rightarrow A$  is not OK otherwise
  - Need to decompose the table, but how?

# Boyce-Codd Normal Form

There are no  
“bad” FDs:

**Definition.** A relation  $R$  is in BCNF if:

Whenever  $X \rightarrow B$  is a non-trivial dependency,  
then  $X$  is a superkey.

Equivalently:

**Definition.** A relation  $R$  is in BCNF if:

$\forall X$ , either  $X^+ = X$  or  $X^+ = [\text{all attributes}]$

Trivial FD

# BCNF Decomposition Algorithm

Normalize(R)

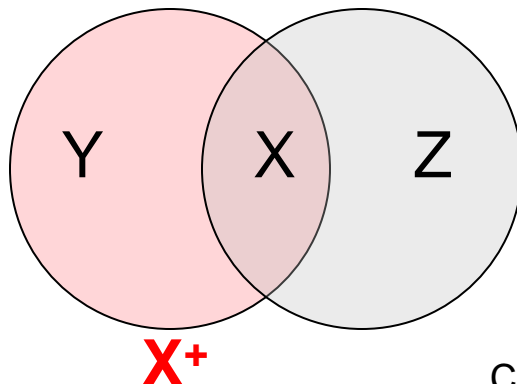
find  $X$  s.t.:  $X \neq X^+ \neq$  [all attributes]

**if** (not found) **then** “R is in BCNF”

**let**  $Y = X^+ - X$ ;  $Z =$  [all attributes] -  $X^+$

decompose R into  $R_1(X \cup Y)$  and  $R_2(X \cup Z)$

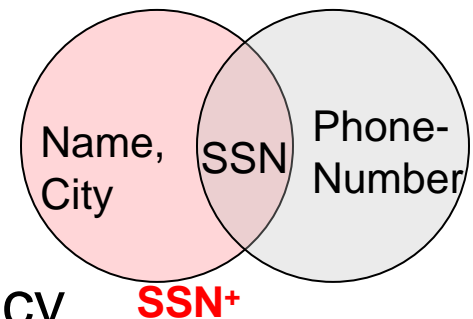
Normalize( $R_1$ ); Normalize( $R_2$ );



# Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

$SSN \rightarrow Name, City$



The only key is:  $\{SSN, PhoneNumber\}$

Hence  $SSN \rightarrow Name, City$  is a “bad” dependency

In other words:

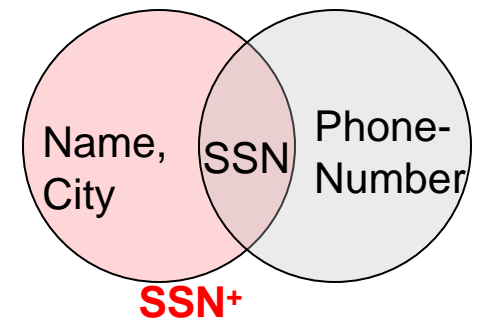
$SSN^+ = Name, City$  and is neither  $SSN$  nor  $All\ Attributes$



# Example BCNF Decomposition

Name	<u>SSN</u>	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Westfield

SSN  $\rightarrow$  Name, City



<u>SSN</u>	<u>PhoneNumber</u>
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121
987-65-4321	908-555-1234

Let's check anomalies:

- Redundancy ?
- Update Fred's city?
- Delete Joe's all ph?

Find  $X$  s.t.:  $X \neq X^+ \neq [\text{all attributes}]$

# Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN  $\rightarrow$  name, age

age  $\rightarrow$  hairColor



Find  $X$  s.t.:  $X \neq X^+ \neq [\text{all attributes}]$

# Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber, attr)

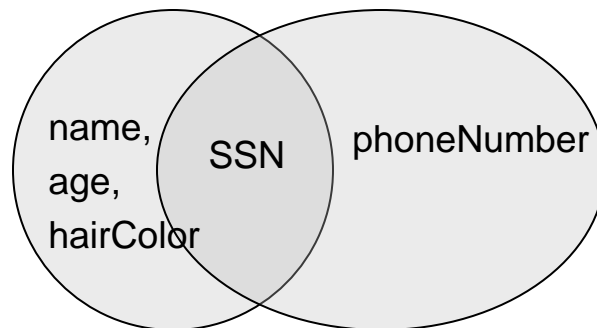
SSN  $\rightarrow$  name, age

age  $\rightarrow$  hairColor

Iteration 1: **Person**: SSN<sup>+</sup> = SSN, name, age, hairColor

Decompose into: **P**(SSN, name, age, hairColor)

**Phone**(SSN, phoneNumber, attr)



Find  $X$  s.t.:  $X \neq X^+ \neq [\text{all attributes}]$

# Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN  $\rightarrow$  name, age

age  $\rightarrow$  hairColor

What are  
the keys ?

Iteration 1: **Person**: SSN<sup>+</sup> = SSN, name, age, hairColor

Decompose into: **P**(SSN, name, age, hairColor)

**Phone**(SSN, phoneNumber)

Iteration 2: **P**: age<sup>+</sup> = age, hairColor

Decompose: **People**(SSN, name, age)

**Hair**(age, hairColor)

**Phone**(SSN, phoneNumber)

Find X s.t.:  $X \neq X^+ \neq [\text{all attributes}]$

# Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber)

SSN  $\rightarrow$  name, age

age  $\rightarrow$  hairColor

Note the keys!

Iteration 1: **Person**: SSN<sup>+</sup> = SSN, name, age, hairColor

Decompose into: P(SSN, name, age, hairColor)

Phone(SSN, phoneNumber)

Iteration 2: **P**: age<sup>+</sup> = age, hairColor

Decompose: People(SSN, name, age)

Hair(age, hairColor)

Phone(SSN, phoneNumber)

$R(A,B,C,D)$

$A \rightarrow B$   
 $B \rightarrow C$

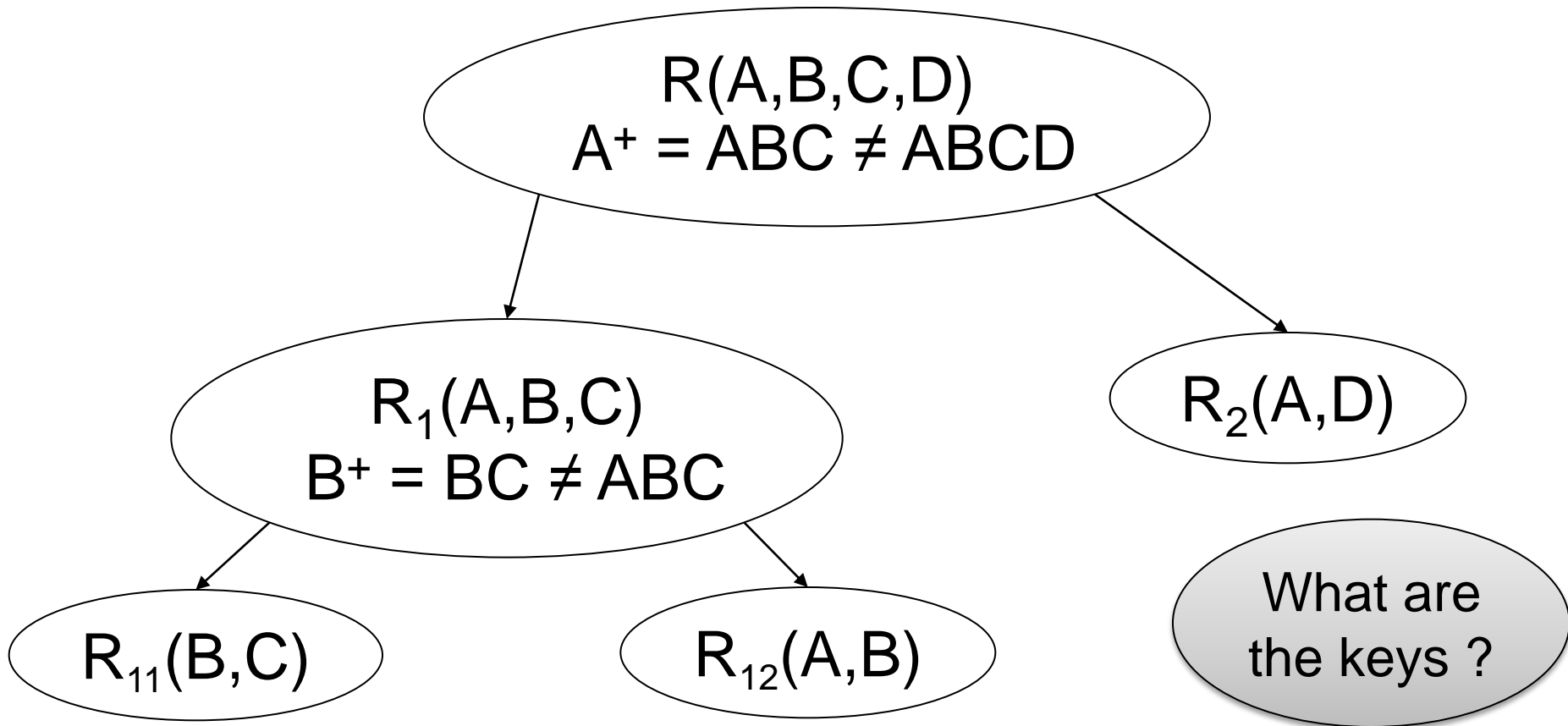
# Practice at Home

$R(A,B,C,D)$   
 $A^+ = ABC \neq ABCD$

$R(A,B,C,D)$

$A \rightarrow B$   
 $B \rightarrow C$

# Practice at Home



What happens if in  $R$  we first pick  $B^+$  ? Or  $AB^+$  ?