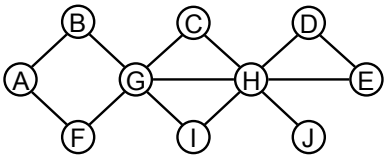
CSE 421
Introduction to Algorithms
Assignment #2
Due: Friday, 7/12/13

Please review the instructions found in Homework 1.

This assignment is part written, part programming. It all focuses on the articulation points algorithm presented in class, and the related problem of decomposing a graph into its biconnected components (defined below). I ***strongly*** recommend that you do problems 1–3 ***before*** you start the programming portion, and do them soon so you have time to ask questions before you get immersed in coding.

Recall that, a vertex in a connected undirected graph is an *articulation point* if removing it (and all edges incident to it, i.e., touching it) results in a non-connected graph. As important special cases, a graph having a single vertex has no articulation points, nor does a connected graph having just two vertices (which of course must be joined by an edge, otherwise it isn't connected).

1. [10 points] Simulate the algorithm presented in class for finding articulation points on the graph shown below.



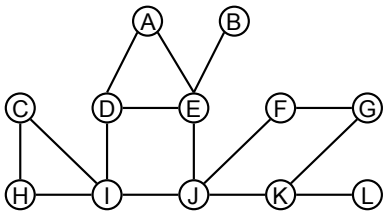
   Redraw the graph clearly showing (a) tree edges, (b) back edges, and, in a tidy table similar to ones shown in the sides, list (c) the DFS number and (d) LOW value assigned to each vertex, and (e) identify the articulation points. In addition, (f) list the edges in the order that they are explored (i.e., traversed for the 1st time) by the algorithm. For definiteness, start your DFS **at vertex C** and whenever the algorithm has a choice of which edge to follow next, pick the edge to the alphabetically first vertex among the choices. (Note that this is *not* a requirement of the algorithm; it's just to make Cyrus's life easier when grading them.)

2. [10 points] A connected graph is *biconnected* if it has no articulation points. A *biconnected component* of an undirected graph $G = (V, E)$ is a maximal subset B of the edges with the property that the graph $G_B = (V_B, B)$ is biconnected, where V_B is the subset of vertices incident to (touched by) edges in B. (I.e., G_B is B's *edge-induced subgraph*. *Maximal* means you can't enlarge B without destroying its biconnected property.)

   Note that a graph consisting of single edge is biconnected. Consequently, every edge in G is part of *some* biconnected component. In fact, every edge is part of *exactly one* biconnected components. (This really needs a proof, which you don't need to give, but basically it's true because if some edge were in two components, their union would also be biconnected, contradicting the "maximality" condition.) So, the biconnected components partition the edges. Another fact, just to help your intuition: two distinct edges lie on a common simple cycle if and only if they are in the same biconnected component. (This motivates the term "biconnected"—there are always two independent paths between places. Again, these statements need careful proof, but the idea is simple: if there weren't two paths, you could disconnect the graph by removing some vertex on the one path.)

   For example, the biconnected components of the graph below are the five sets of edges:

   - Component 1: $\{\{A, D\}, \{A, E\}, \{D, E\}, \{D, I\}, \{E, J\}, \{I, J\}\}$
   - Component 2: $\{\{B, E\}\}$
   - Component 3: $\{\{C, H\}, \{C, I\}, \{H, I\}\}$
   - Component 4: $\{\{F, G\}, \{F, J\}, \{G, K\}, \{J, K\}\}$, and
   - Component 5: $\{\{K, L\}\}$



   Find and list the biconnected components of the graph in problem 1.

3. [20 points] There is a very close relationship between biconnected components and articulation points. Namely, the articulation points are exactly the vertices at which two (or more) biconnected components are connected. Given this fact, it shouldn't be a big surprise that there is a linear time algorithm that identifies the biconnected components of a graph, and in fact this algorithm is quite similar to the articulation point algorithm.

   Give a modification of the articulation-points algorithm that finds biconnected components. Describe the algorithm (in English; it should only take a few sentences to describe the necessary modifications). Simulate it on the example in problem 1, showing enough of a trace of the algorithm's execution to demonstrate that it finds exactly the components you found above. [Hints: look carefully at the order in which the articulation points algorithm explores edges and discovers articulation points, and relate this to which edges are part of which biconnected components. Among other things, you might want to circle or otherwise mark the biconnected components on the edge list you produced as part of your solution to problem 1. Initially, focus on the first biconnected component to be completely explored by the depth-first search.]

4. [50 points] Implement, test, and time the algorithm you found in problem 3.

   Input format: To keep things simple, assume the input will consist of a positive integer "N" followed by some number of pairs of integers in the range 0 to $N-1$. "N" represents the number of vertices in the graph, and each pair u, v represents an edge between vertices u and v. Although a good program really should check for the following malformed inputs, you may simply assume that you never get: negative numbers, numbers outside the range 0 to $N-1$, an even number of integers in total (i.e., the last pair is unfinished), or a pair u, v where $u = v$, or where either (u, v) or (v, u) duplicates a previous pair in the list.

   Output Format: Print out the number of nodes, number of edges, number of biconnected components, number of articulation points, list the articulation points, list the edges in each biconnected component, and the algorithm's run time (excluding input/output; see below).

   Implementation: I don't care much what programming language you use; C/C++/C#, Java, Perl, Python, R, Ruby all seem reasonable (ask otherwise). You should use some flavor of edge-list representation so that your algorithm is faster on sparse graphs, but I recommend keeping this as simple as possible. In particular, you may use built-in or standard library packages for hash tables, lists, dynamic arrays, etc., for the edge lists so that you don't need to spend a lot of time duplicating all that fun linked list code you wrote in your data structures course. (However, depending on the language you chose, you may need to be careful to avoid hidden $\Omega(n)$ operations such as copying large data structures, which may increase your algorithm's asymptotic run time.) Since the time spent reading the graph is excluded from your timing study (below), it's OK if the data structures are somewhat slow to construct, just so you can efficiently traverse all the edges as needed by the main algorithm. Also, you do *not* have to obey anything like the "order edges alphabetically" convention I use on by-hand examples—just traverse them in whatever order comes naturally for your data structure.

   Testing: run it on a specific sample graph that I will provide later and **turn in a printout of your result**. (You should of course do more extensive testing as well, both much simpler and much more complex examples, but you only need to turn in this one case.)

   Timing: Also run it on a variety of graphs of different sizes and different densities (average number of edges per vertex) and measure its run time on each.

   To simplify your job doing the timing measurements, this zip archive contains a number of random graphs of various sizes in the specified input format (in the "tests" folder). Each should be connected and have 1 to 10 biconnected components. If you are interested in viewing them, the smaller of those graphs are also provided as ".dot" files for the wonderful Graphviz program (in the "dots" folder). Finally, if you want to generate additional test cases, or are curious, the "generator" folder contains the C and R code I used to generate these examples. You are not *required* to use any of this.

   Two hints on timing measurements: Record separately or exclude from your timing measurements the time spent reading inputs, since this may dominate the interesting part. Similarly, except for the summary parameters giving numbers of vertices, edges and components, you may want to disable output formatting and printing during you timing runs. See also the FAQ page for other timing tips.

   Results: Write a brief report (2-3 pages, say) summarizing your measurements. Include a graph (old fashioned scatter plot) of run time versus problem size. You might plot time versus vertices or time vs edges or time divided by vertices vs vertices plus edges, etc. Which do you find most informative? Explain why. Fit a "trend

line" to the data (e.g., with Excel). Compare to the theoretical big-O bounds for your sample graphs. Are your observations in line with the dogma we spout about the utility of big-O analysis? Are there discrepancies? Can you explain them? Does number of edges versus number of vertices have any bearing on performance? Also, if possible, give us a quick summary of the kind of processor/memory system on which you ran your timing tests. E.g., "1.2 Ghz Intelerola Kryptonite dodeca-core processor with 64 kb cache @ 5Ghz and 96 Gb DDR7 DRAM @ 2Ghz."

Turn In: Turn in your code and output for the required test case via the Catalyst link on the course home page. The rest of the deliverables may be turned in on paper or via Catalyst, as usual. Bundle all your files into a single tar or zip archive, submitted to the Catalyst link on the course home page.

Please put both your **name** and **UW netid (email)** on all parts so that we can easily correlate them.

5. **Extra Credit:** One or more of:

   (a) Also describe and implement any other algorithm you like for solving the problem, e.g., this simple but slow one:

   As mentioned, two (different) edges are in the same biconnected component if and only there is a simple cycle in G that includes both. So, you could do something like this: for every pair of edges, try to find a path from one to the other (maybe by DFS or BFS), and then try to find a path back, excluding the edges in the path already taken.

   (b) Give a big-O analysis of the running time for the algorithm you implemented in part a.

   (c) Run this algorithm on the same set of graphs as you did the linear time algorithm (or at least the smaller graphs; it may be too slow on large ones).

   (d) Extend your report to include the run times for this algorithm in comparison to both the linear time algorithm and to the general big-O dogma.