Computer Networks

Error Coding Overview (§3.2)

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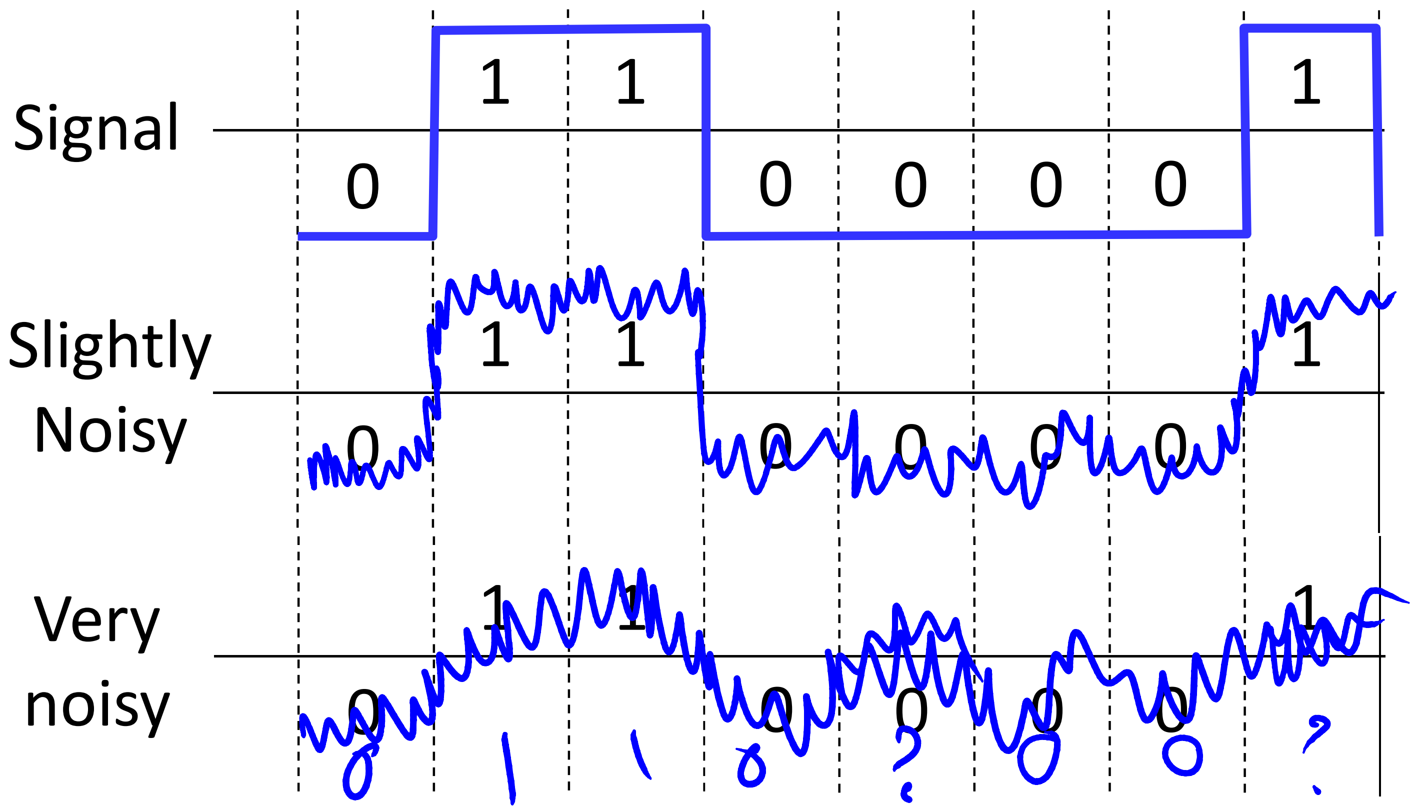
Topic

- Some bits will be received in error due to noise. What can we do?
  - Detect errors with codes »
  - Correct errors with codes »
  - Retransmit lost frames Later
- Reliability is a concern that cuts across the layers – we'll see it again

Problem – Noise may flip received bits



Approach – Add Redundancy

- Error detection codes
  - Add check bits to the message bits to let some errors be detected
- Error correction codes
  - Add more check bits to let some errors be corrected

- Key issue is now to structure the code to detect many errors with few check bits and modest computation

Motivating Example

- A simple code to handle errors:
  - Send two copies! Error if different.

0 1 ~~0~~ 0 1 ~~0~~

1 1

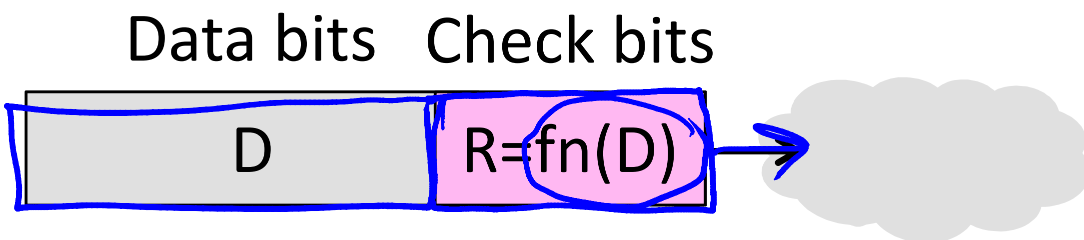
- How good is this code?
  - How many errors can it detect/correct?
  - How many errors will make ~~it fail?~~ 2

Motivating Example (2)

- We want to handle more errors with less overhead
  - Will look at better codes; they are applied mathematics
  - But, they can't handle all errors
  - And they focus on accidental errors (will look at secure hashes later)

Using Error Codes

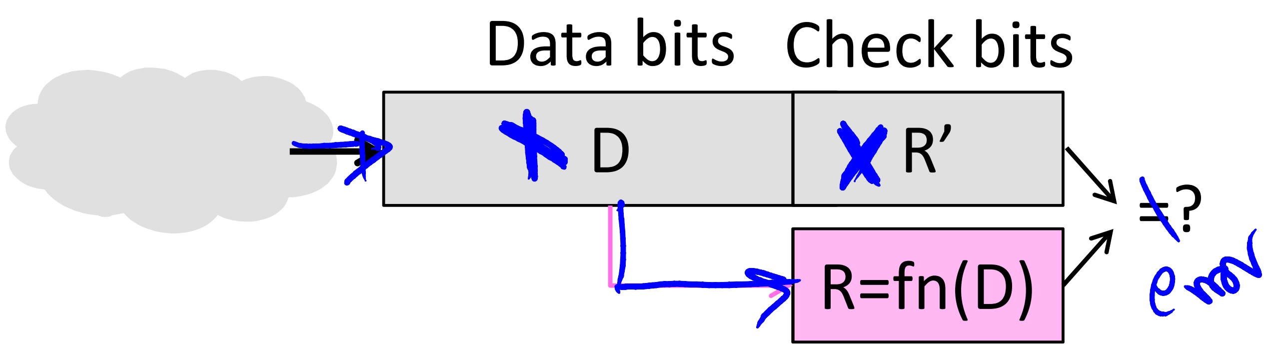
- Codeword consists of D data plus R check bits (=systematic block code)



- Sender:
  - Compute R check bits based on the D data bits; send the codeword of D+R bits

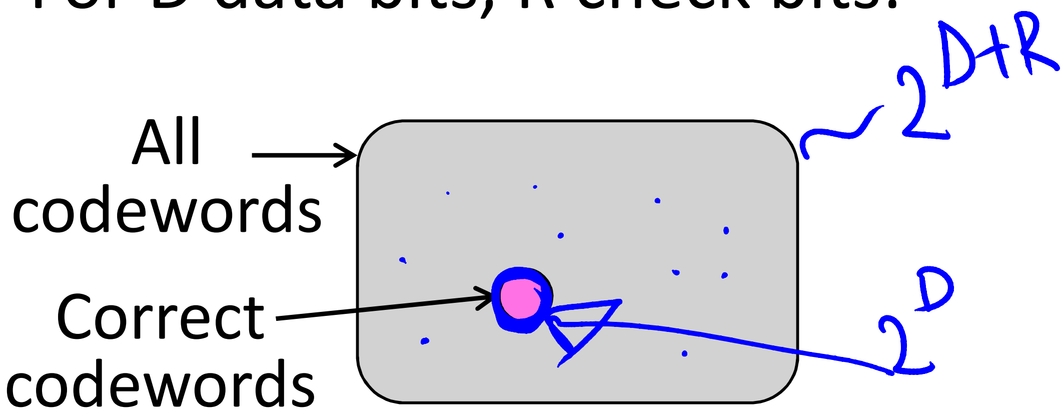
Using Error Codes (2)

- Receiver:
  - Receive D+R bits with unknown errors
  - Recompute R check bits based on the D data bits; error if R doesn't match R'



Intuition for Error Codes

- For D data bits, R check bits:

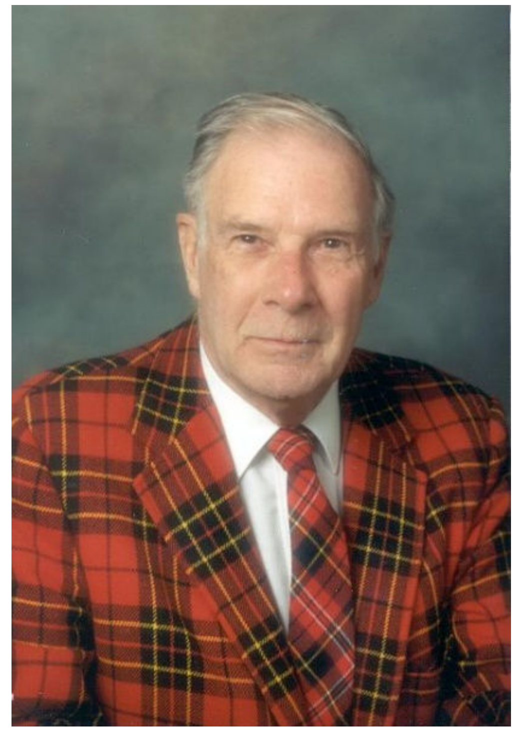


- Randomly chosen codeword is unlikely to be correct; overhead is low

$\approx \frac{1}{2^R}$

R.W. Hamming (1915-1998)

- Much early work on codes:
  - "Error Detecting and Error Correcting Codes", BSTJ, 1950
- See also:
  - "You and Your Research", 1986



Source: IEEE GHN, © 2009 IEEE

Hamming Distance

- Distance is the number of bit flips needed to change D_1^{xR} to D_2^{xR}

  $1 \rightarrow 111$, $0 \rightarrow 000$ distance $= 3$

- Hamming distance of a code is the minimum distance between any pair of codewords $HD = 3$

Hamming Distance (2)

- Error detection:
  - For a code of distance d+1, up to d errors will always be detected

$d+1=3 \Rightarrow d=2$

000 111

001 010
100 011
101 110

Hamming Distance (3)

- Error correction:
  - For a code of distance 2d+1, up to d errors can always be corrected by mapping to the closest codeword

HD=3 2d+1=3 010 110

d=1 000 111

END