SAMPLE-BASED ALLOCATION

***Risk and Asset Allocation* - Springer – `symmys.com`**

Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\} \tag{8.76}$$

$$\begin{aligned} \mathrm{CE}_{\boldsymbol{\mu}, \boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = {} & \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) \\ & -\frac{1}{2\zeta} \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T) \boldsymbol{\Sigma} \operatorname{diag}(\mathbf{p}_T) \boldsymbol{\alpha} \end{aligned} \tag{8.25}$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = [\operatorname{diag}(\mathbf{p}_T)]^{-1} \boldsymbol{\Sigma}^{-1} \left(\zeta \boldsymbol{\mu} + \frac{w_T - \zeta \mathbf{1}' \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}}{\mathbf{1}' \boldsymbol{\Sigma}^{-1} \mathbf{1}} \mathbf{1}\right) \tag{8.77}$$

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$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{ \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \qquad (8.76)$$

$$\widehat{\boldsymbol{\theta}}[i_T] \approx \boldsymbol{\theta}^{\mathrm{t}} \qquad (8.78)$$

$$i_T \equiv \{\mathrm{x}_1, \ldots, \mathrm{x}_T\}$$

$$\begin{aligned} \mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = {} & \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1} + \boldsymbol{\mu}) \\ & - \frac{1}{2\zeta} \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T) \boldsymbol{\Sigma} \operatorname{diag}(\mathbf{p}_T) \boldsymbol{\alpha} \end{aligned} \qquad (8.25)$$

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$$\widehat{\boldsymbol{\mu}}[i_T] \equiv \frac{1}{T} \sum_{t=1}^{T} \mathbf{l}_t \qquad (8.79)$$

$$\widehat{\boldsymbol{\Sigma}}[i_T] \equiv \frac{1}{T} \sum_{t=1}^{T} (\mathbf{l}_t - \widehat{\boldsymbol{\mu}})(\mathbf{l}_t - \widehat{\boldsymbol{\mu}})' \qquad (8.80)$$

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$$\widehat{\boldsymbol{\theta}}[i_T] \mapsto \widehat{\boldsymbol{\theta}}\left[I_T^{\boldsymbol{\theta}}\right] \quad (8.84)$$

$$I_T^{\boldsymbol{\theta}} \equiv \{\mathbf{X}_1^{\boldsymbol{\theta}}, \ldots, \mathbf{X}_T^{\boldsymbol{\theta}}\}$$

$$\widehat{\boldsymbol{\mu}}\left[I_T^{\boldsymbol{\mu},\boldsymbol{\Sigma}}\right] \sim \mathrm{N}\left(\boldsymbol{\mu}, \frac{\boldsymbol{\Sigma}}{T}\right) \quad (8.85)$$

$$T\widehat{\boldsymbol{\Sigma}}\left[I_T^{\boldsymbol{\mu},\boldsymbol{\Sigma}}\right] \sim \mathrm{W}(T-1, \boldsymbol{\Sigma}) \quad (8.86)$$

$$\Longleftrightarrow \quad {}_j\widehat{\boldsymbol{\mu}}^{\boldsymbol{\mu},\boldsymbol{\Sigma}}, \quad {}_j\widehat{\boldsymbol{\Sigma}}^{\boldsymbol{\mu},\boldsymbol{\Sigma}} \quad (8.88)$$

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$$\boldsymbol{\alpha}_s\left[I_T^{\boldsymbol{\theta}}\right] \equiv \boldsymbol{\alpha}\left(\widehat{\boldsymbol{\theta}}\left[I_T^{\boldsymbol{\theta}}\right]\right) \qquad (8.87)$$

$${}_j\boldsymbol{\alpha}_s^{\boldsymbol{\mu},\boldsymbol{\Sigma}} \equiv \zeta [\operatorname{diag}(\mathbf{p}_T)]^{-1} {}_j\widehat{\boldsymbol{\Sigma}}^{-1} {}_j\widehat{\boldsymbol{\mu}}. + \frac{w_T - \zeta \mathbf{1}' {}_j\widehat{\boldsymbol{\Sigma}}^{-1} {}_j\widehat{\boldsymbol{\mu}}}{\mathbf{1}' {}_j\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{1}} [\operatorname{diag}(\mathbf{p}_T)]^{-1} {}_j\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{1} \qquad (8.89)$$

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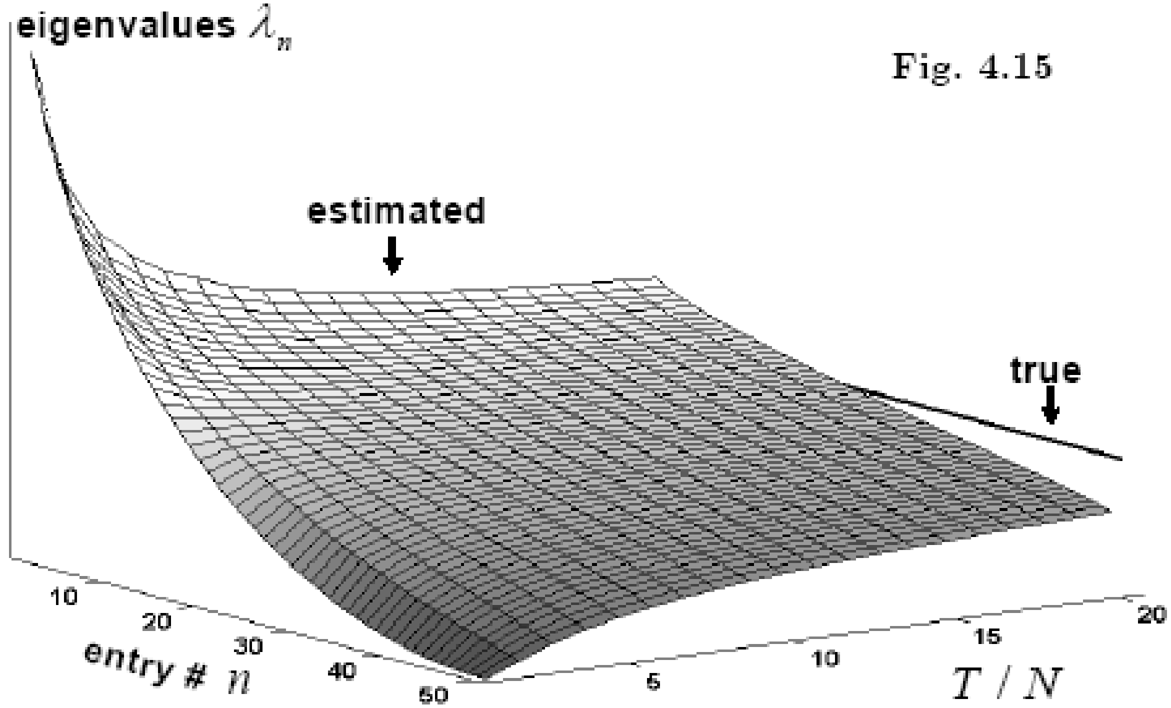

Fig. 4.15

$$ {}_j\boldsymbol{\alpha}_s^{\boldsymbol{\mu},\boldsymbol{\Sigma}} \equiv \zeta\,[\operatorname{diag}(\mathbf{p}_T)]^{-1}\,{}_j\widehat{\boldsymbol{\Sigma}}^{-1}\,{}_j\widehat{\boldsymbol{\mu}} + \frac{w_T - \zeta\mathbf{1}'\,{}_j\widehat{\boldsymbol{\Sigma}}^{-1}\,{}_j\widehat{\boldsymbol{\mu}}}{\mathbf{1}'\,{}_j\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{1}}[\operatorname{diag}(\mathbf{p}_T)]^{-1}\,{}_j\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{1} \quad (8.89)$$

$$ {}_j\widehat{\boldsymbol{\Sigma}}^{-1} = {}_j\widehat{\mathbf{E}}\operatorname{diag}\left(\frac{1}{{}_j\widehat{\lambda}_1}, \ldots, \frac{1}{{}_j\widehat{\lambda}_N}\right){}_j\widehat{\mathbf{E}}' \quad (8.109)$$

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$$\mathcal{S}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}_s\left[I_T^{\boldsymbol{\theta}}\right]\right)$$

$$\mathrm{CE}({}_j\boldsymbol{\alpha}_s) = {}_je - \frac{{}_jv}{2\zeta} \quad (8.112)$$

$${}_jv \equiv {}_j\boldsymbol{\alpha}_s'\operatorname{diag}(\mathbf{p}_T)\boldsymbol{\Sigma}\operatorname{diag}(\mathbf{p}_T){}_j\boldsymbol{\alpha}_s \quad (8.110)$$

$${}_je \equiv {}_j\boldsymbol{\alpha}_s'\operatorname{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}). \quad (8.111)$$

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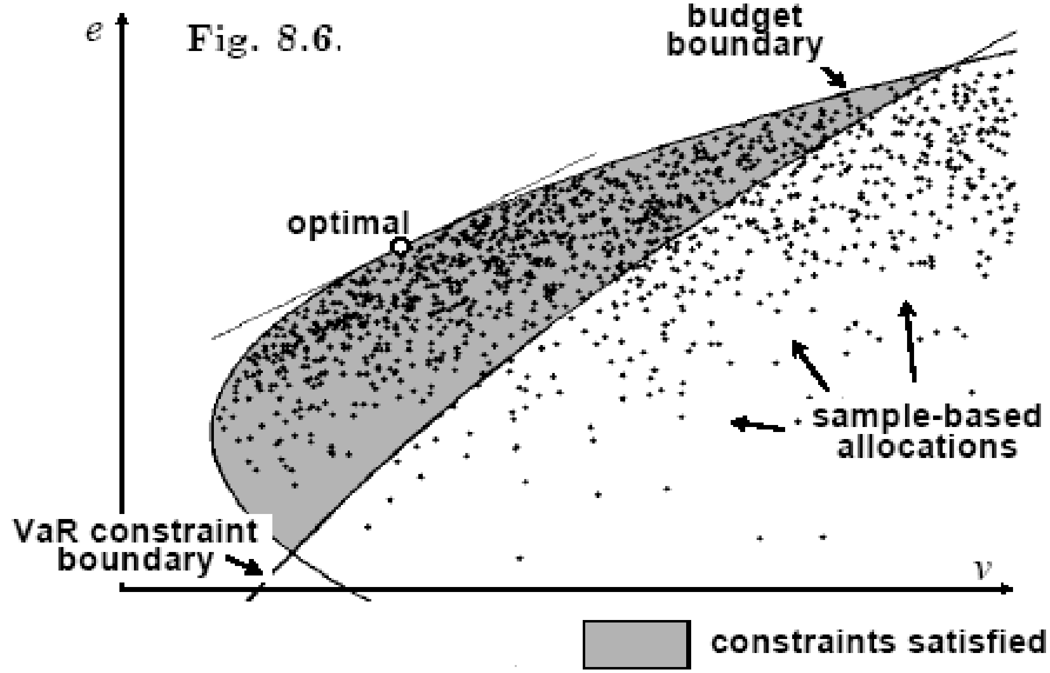

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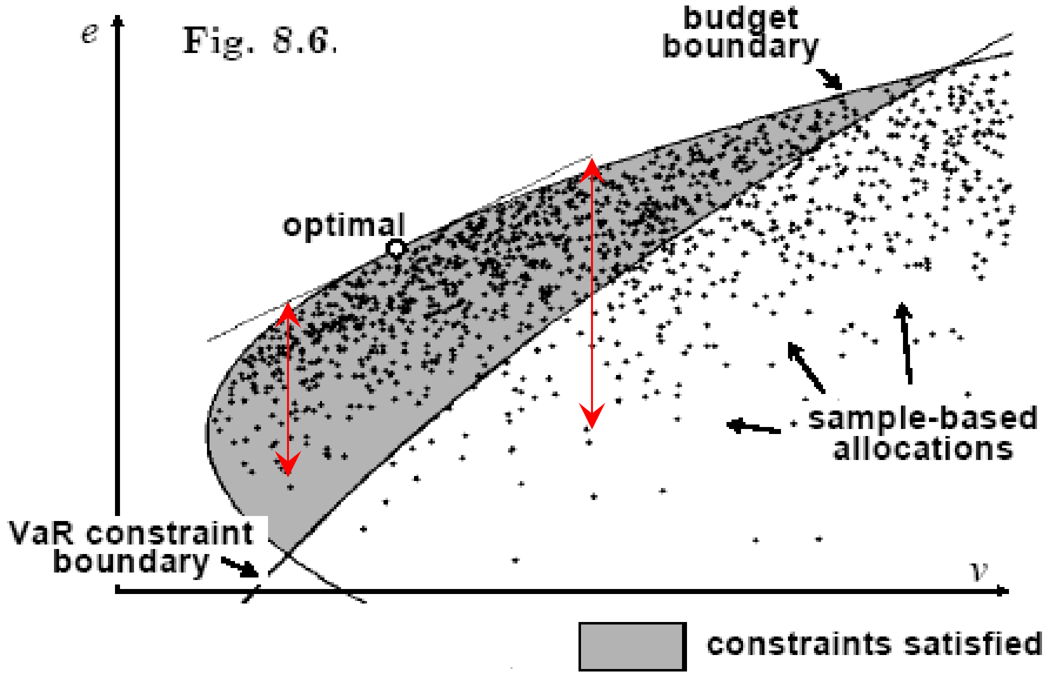
$$\mathcal{S}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}_s\left[I_T^{\boldsymbol{\theta}}\right]\right)$$

(8.92)

$$\mathrm{OC}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}_s\left[I_T^{\boldsymbol{\theta}}\right]\right) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}_s\left[I_T^{\boldsymbol{\theta}}\right]\right)$$



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$$\mathrm{OC}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}_s\left[I_T^{\boldsymbol{\theta}}\right]\right) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}_s\left[I_T^{\boldsymbol{\theta}}\right]\right) \quad (8.92)$$

Fig. 8.4.

$$\boldsymbol{\theta} \mapsto \mathrm{OC}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}_s\left[I_T^{\boldsymbol{\theta}}\right]\right), \quad \boldsymbol{\theta} \in \boldsymbol{\Theta}. \quad (8.93)$$

$$\boldsymbol{\Xi}(\rho) \equiv \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & \ddots & & \vdots \\ \vdots & & \ddots & \rho \\ \rho & \cdots & \rho & 1 \end{pmatrix} \quad (8.58)$$

$$\sqrt{\operatorname{diag}(\boldsymbol{\Sigma}(\rho))} \equiv (1 + \xi\rho)\,\mathbf{v} \qquad \boldsymbol{\mu} \equiv p\sqrt{\operatorname{diag}(\boldsymbol{\Sigma}(\rho))} \quad (8.59)$$

$$\rho \mapsto \mathrm{OC}_{\boldsymbol{\mu}(\rho),\boldsymbol{\Sigma}(\rho)}\left(\boldsymbol{\alpha}_s\left[I_T^{\boldsymbol{\mu}(\rho),\boldsymbol{\Sigma}(\rho)}\right]\right) \quad \rho \in \Theta \equiv [0,1) \quad (8.97)$$

$\mathcal{S}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}\right)$

$$\begin{aligned}\mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}\left(\boldsymbol{\alpha}\right) &= \boldsymbol{\alpha}'\operatorname{diag}\left(\mathbf{p}_T\right)\left(\mathbf{1}+\boldsymbol{\mu}\right) \\ &\quad -\frac{1}{2\zeta}\boldsymbol{\alpha}'\operatorname{diag}\left(\mathbf{p}_T\right)\boldsymbol{\Sigma}\operatorname{diag}\left(\mathbf{p}_T\right)\boldsymbol{\alpha}\end{aligned} \tag{8.25}$$

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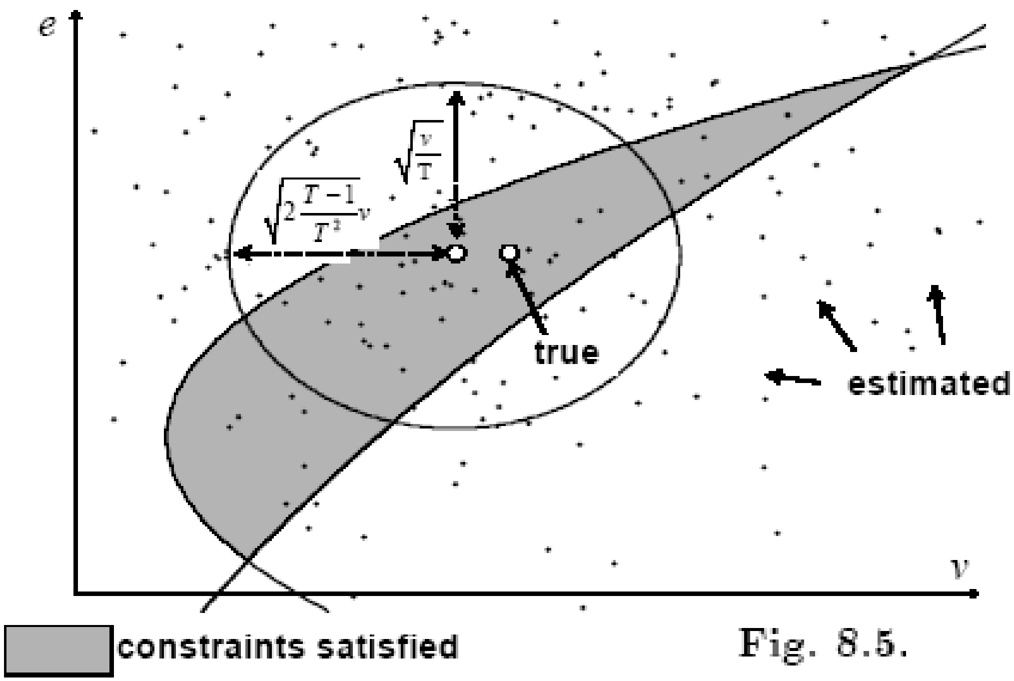

Fig. 8.5.

$$T\widehat{v} \sim \mathrm{Ga}\,(T-1, v) \qquad \widehat{e} \sim \mathrm{N}\left(e, \frac{v}{T}\right) \qquad (8.102)$$

$$\mathrm{E}\{\widehat{v}\} = \frac{T-1}{T}v, \qquad \mathrm{E}\{\widehat{e}\} = e. \qquad (8.103)$$

$$\mathrm{Sd}\{\widehat{v}\} = \sqrt{2\frac{T-1}{T^2}}v, \qquad \mathrm{Sd}\{\widehat{e}\} = \sqrt{\frac{v}{T}}. \qquad (8.104)$$

$\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})$

$$\mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = \boldsymbol{\alpha}'\,\mathrm{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) - \frac{1}{2\zeta}\boldsymbol{\alpha}'\,\mathrm{diag}(\mathbf{p}_T)\,\boldsymbol{\Sigma}\,\mathrm{diag}(\mathbf{p}_T)\,\boldsymbol{\alpha} \qquad (8.25)$$

$\mathcal{S}_{\widehat{\boldsymbol{\theta}}}(\boldsymbol{\alpha})$

$$\mathcal{S}_{\widehat{\boldsymbol{\mu}},\widehat{\boldsymbol{\Sigma}}} \equiv \widehat{e} - \frac{\widehat{v}}{2\zeta} \qquad (8.105)$$

$$\widehat{v} \equiv \boldsymbol{\alpha}'\,\mathrm{diag}(\mathbf{p}_T)\,\widehat{\boldsymbol{\Sigma}}\,\mathrm{diag}(\mathbf{p}_T)\,\boldsymbol{\alpha} \qquad (8.100)$$

$$\widehat{e} \equiv \boldsymbol{\alpha}'\,\mathrm{diag}(\mathbf{p}_T)(\mathbf{1}+\widehat{\boldsymbol{\mu}}). \qquad (8.101)$$