Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{ \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \qquad (8.76)$$

$$\begin{aligned} \mathrm{CE}_{\mu,\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = {} & \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) \qquad (8.25) \\ & -\frac{1}{2\zeta}\boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)\, \boldsymbol{\Sigma} \operatorname{diag}(\mathbf{p}_T)\, \boldsymbol{\alpha} \end{aligned}$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \left[\operatorname{diag}(\mathbf{p}_T)\right]^{-1} \boldsymbol{\Sigma}^{-1} \left(\zeta \boldsymbol{\mu} + \frac{w_T - \zeta \mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}} \mathbf{1} \right) \qquad (8.77)$$

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$$\boxed{\boldsymbol{\alpha}_{\mathrm{p}}\left[i_T\right] \equiv \boldsymbol{\alpha}} \qquad (8.64)$$

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$$[i_T] \mapsto \left[I_T^{\boldsymbol{\theta}}\right]$$

$$I_T^{\boldsymbol{\theta}} \equiv \{\mathbf{X}_1^{\boldsymbol{\theta}}, \ldots, \mathbf{X}_T^{\boldsymbol{\theta}}\}$$

$$\boldsymbol{\alpha}_{\mathrm{p}}\left[I_T^{\boldsymbol{\theta}}\right] \equiv \boldsymbol{\alpha}$$

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$$\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}_s[I_T^{\boldsymbol{\theta}}])$$

$$\mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}_{\mathrm{p}}) = w_T\left(1 + \frac{(\boldsymbol{\mu}'\mathbf{1})}{N}\right) - \frac{w_T^2}{2\zeta}\frac{\mathbf{1}'\boldsymbol{\Sigma}\mathbf{1}}{N^2} \quad (8.68)$$

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$$\boldsymbol{\alpha}_{\mathrm{p}}\left[I_T^{\boldsymbol{\theta}}\right] \equiv \boldsymbol{\alpha}$$

$$\mathcal{S}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}_s\left[I_T^{\boldsymbol{\theta}}\right]\right)$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}_{\mathrm{p}}) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}_{\mathrm{p}}) \qquad (8.67)$$

$$\begin{aligned} \mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = {} & \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) \qquad (8.25) \\ & -\frac{1}{2\zeta} \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)\, \boldsymbol{\Sigma} \operatorname{diag}(\mathbf{p}_T)\, \boldsymbol{\alpha} \end{aligned}$$

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$$\mathrm{OC}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}_{\mathrm{p}}) \equiv \overline{\mathrm{CE}}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) - \mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}_{\mathrm{p}}) + \mathcal{C}^{+}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}_{\mathrm{p}}) \qquad (8.70)$$

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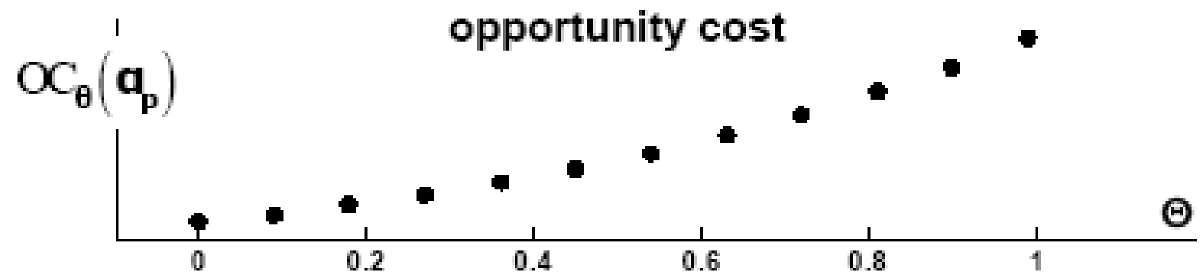
$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}_{\mathrm{p}}) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}_{\mathrm{p}}) \qquad (8.67)$$

$$\boldsymbol{\theta} \mapsto \mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}_{\mathrm{p}}), \quad \boldsymbol{\theta} \in \boldsymbol{\Theta} \qquad (8.71)$$

Fig. 8.3.



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$$\boldsymbol{\Xi}(\rho) \equiv \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & \ddots & & \vdots \\ \vdots & & \ddots & \rho \\ \rho & \cdots & \rho & 1 \end{pmatrix} \qquad (8.58)$$

$$\sqrt{\operatorname{diag}(\boldsymbol{\Sigma}(\rho))} \equiv (1 + \xi\rho)\,\mathbf{v}$$

$$\boldsymbol{\mu} \equiv p\sqrt{\operatorname{diag}(\boldsymbol{\Sigma}(\rho))} \qquad (8.59)$$