Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

SPECTRAL THEOREM - ***Risk and Asset Allocation*** - **Springer** – *`symmys.com`*

$\mathbf{m}$ is any fixed vector in $\mathbb{R}^N$

$$\mathbf{S} = \mathbf{E}\sqrt{\mathbf{\Lambda}}\sqrt{\mathbf{\Lambda}}\mathbf{E}' \quad \text{(A.70)}$$

$$\mathbf{\Lambda} \equiv \operatorname{diag}\left(\lambda_1, \ldots, \lambda_N\right) \quad \text{(A.65)}$$

$$\mathbf{E} \equiv \left(\mathbf{e}^{(1)}, \ldots, \mathbf{e}^{(N)}\right) \quad \text{(A.62)}$$

$$\mathbf{E}\mathbf{E}' = \mathbf{I}_N. \quad \text{(A.63)}$$

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$$\mathcal{E}_{\mathbf{m},\mathbf{S}} \equiv \left\{ \mathbf{x} \in \mathbb{R}^N \text{ such that } (\mathbf{x}-\mathbf{m})' \mathbf{S}^{-1} (\mathbf{x}-\mathbf{m}) \leq 1 \right\} \quad \text{(A.73)}$$

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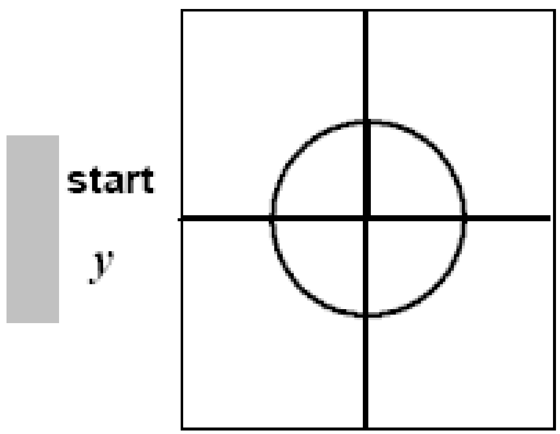

Fig. A.4

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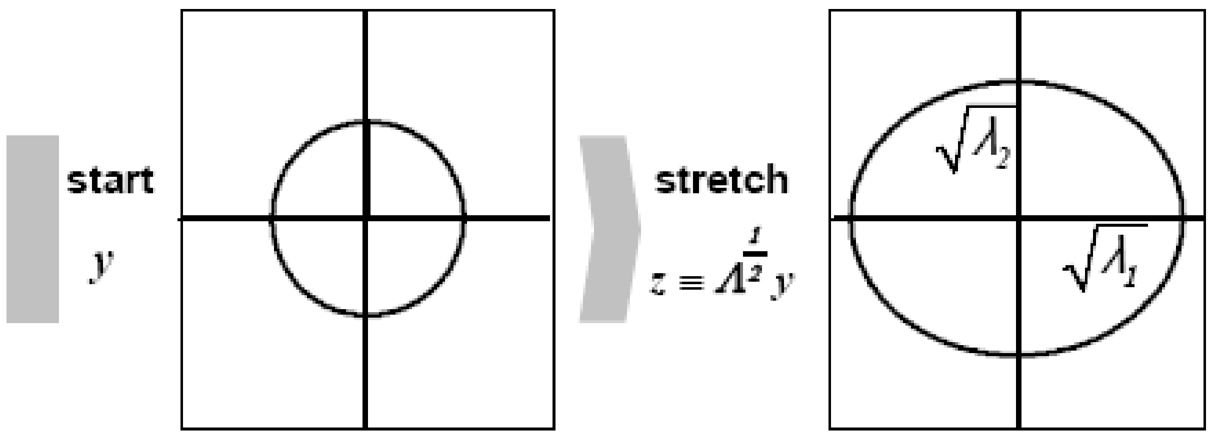

Fig. A.4

$$\text{Vol}\left\{ \mathcal{E}_{\mathbf{m},\mathbf{S}} \right\} = \gamma_N \sqrt{\lambda_1} \cdots \sqrt{\lambda_N} = \gamma_N \sqrt{|\boldsymbol{\Lambda}|} = \gamma_N \sqrt{|\mathbf{S}|} \quad \text{(A.77)}$$

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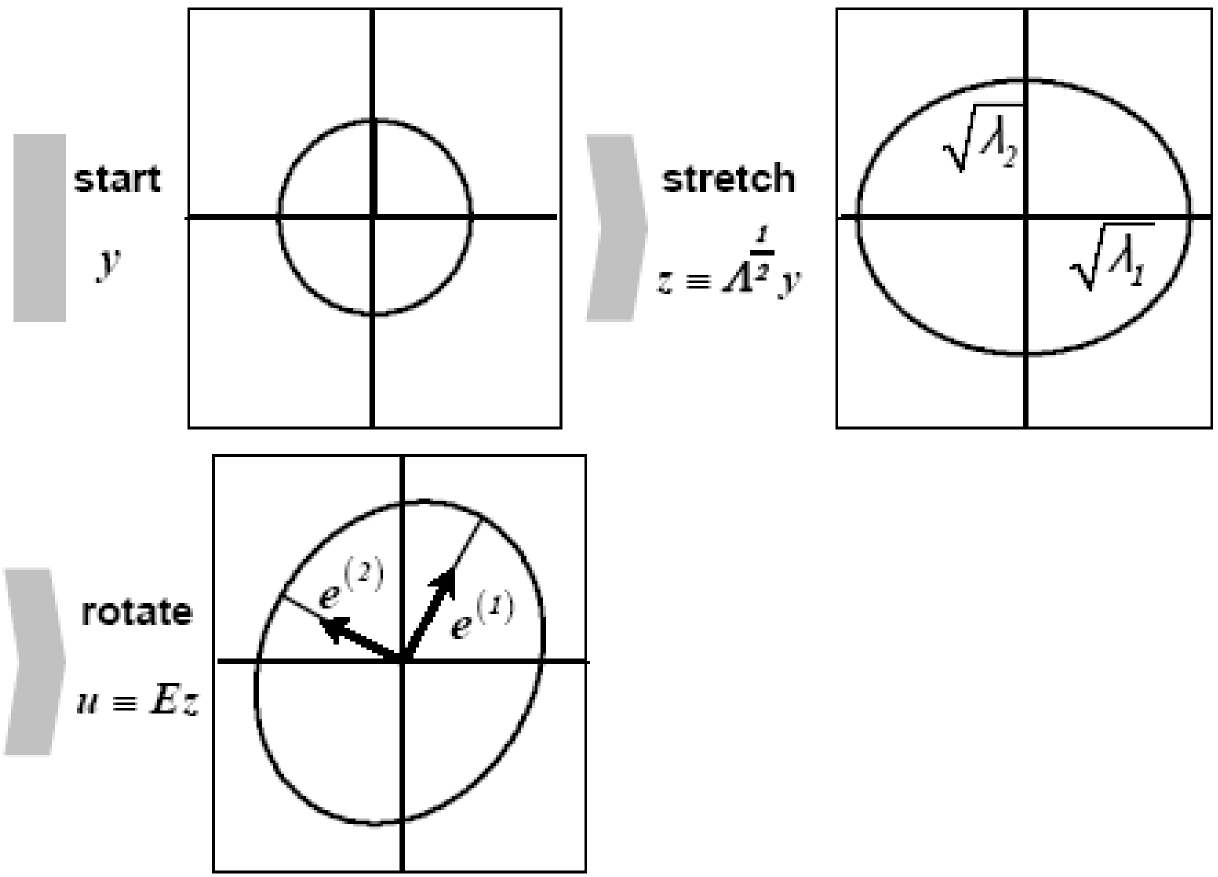

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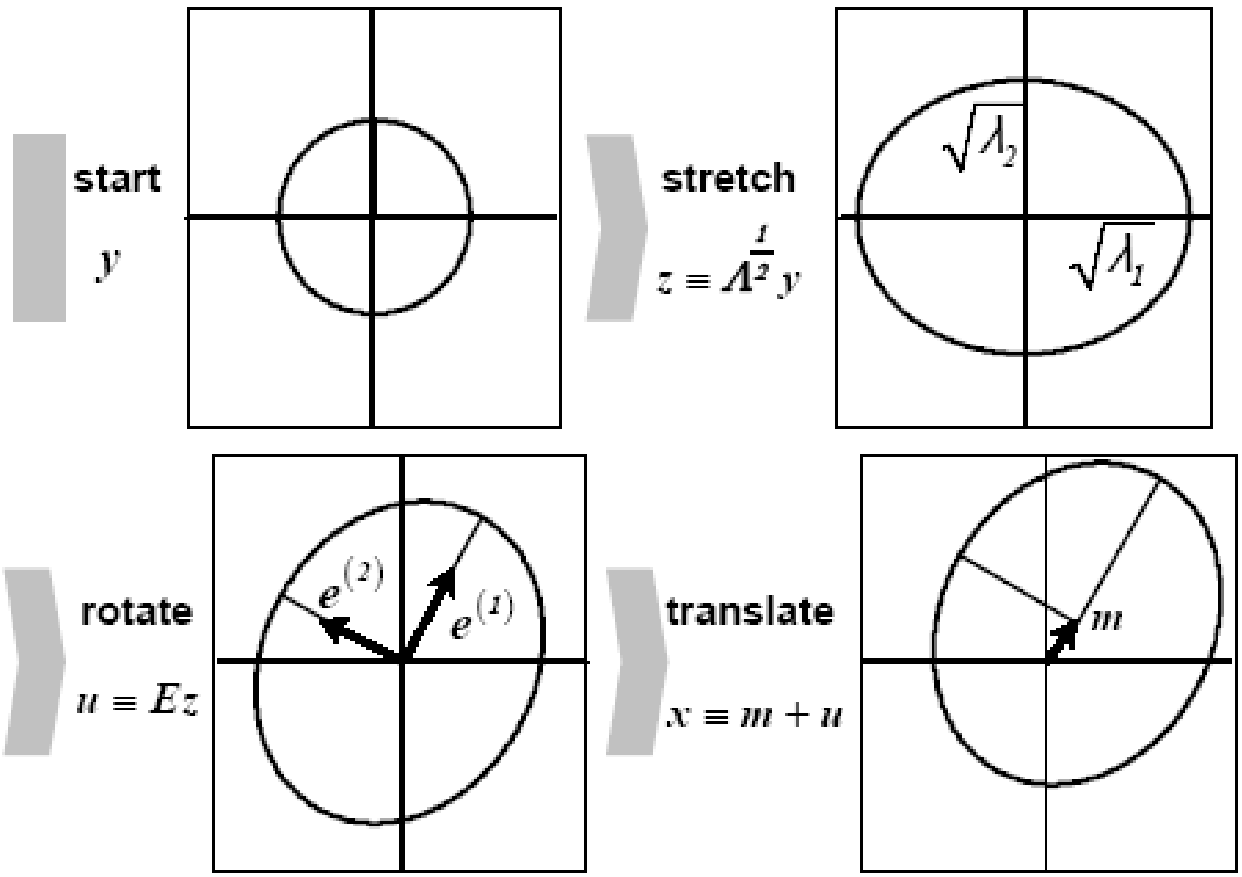

Fig. A.4