INVESTOR'S OBJECTIVES

Risk and Asset Allocation **- Springer –** `symmys.com`

Attilio Meucci

www.symmys.com

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The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

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INVESTOR'S OBJECTIVES

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3.3 From invariants to market prices $\mathbf{P}_{T+\tau}$.

3.2 Projection of the invariants to the investment horizon

4 Estimating the distribution of the market invariants

3.1 The quest for invariance

- **Absolute wealth**

$$\Psi_{\boldsymbol{\alpha}} \equiv W_{T+\tau}\left(\boldsymbol{\alpha}\right)$$

$$\boxed{\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}' \mathbf{P}_{T+\tau}.} \quad (5.3)$$

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- **Net profits**

$$\Psi_{\boldsymbol{\alpha}} \equiv W_{T+\tau}(\boldsymbol{\alpha}) - w_T(\boldsymbol{\alpha}) \quad (5.8)$$

$$\boxed{\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}' \left(\mathbf{P}_{T+\tau} - \mathbf{p}_T\right)} \quad (5.9)$$

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- **Relative wealth**

$$\Psi_{\boldsymbol{\alpha}} \equiv W_{T+\tau}(\boldsymbol{\alpha}) - \gamma(\boldsymbol{\alpha}) W_{T+\tau}(\boldsymbol{\beta}) \quad (5.4)$$

$$\gamma(\boldsymbol{\alpha}) \equiv \frac{w_T(\boldsymbol{\alpha})}{w_T(\boldsymbol{\beta})} \quad (5.5)$$

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$$\mathbf{K} \equiv \mathbf{I}_N - \frac{\mathbf{p}_T \boldsymbol{\beta}'}{\boldsymbol{\beta}' \mathbf{p}_T}. \quad (5.7)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}.} \qquad (5.10)$$

$$\mathbf{M} \equiv \mathbf{a} + \mathbf{B}\mathbf{P}_{T+\tau} \qquad (5.11)$$

?

$$\boxed{\Psi_{\alpha} = \boldsymbol{\alpha}' \mathbf{M}.} \quad (5.10)$$

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$$\boxed{\Psi_{\alpha} = \alpha' \mathrm{M}} \quad (5.10)$$

strong dom.: $\Psi_{\alpha} \geq \Psi_{\beta}$ in all scenarios. (5.31)

Fig. 5.1

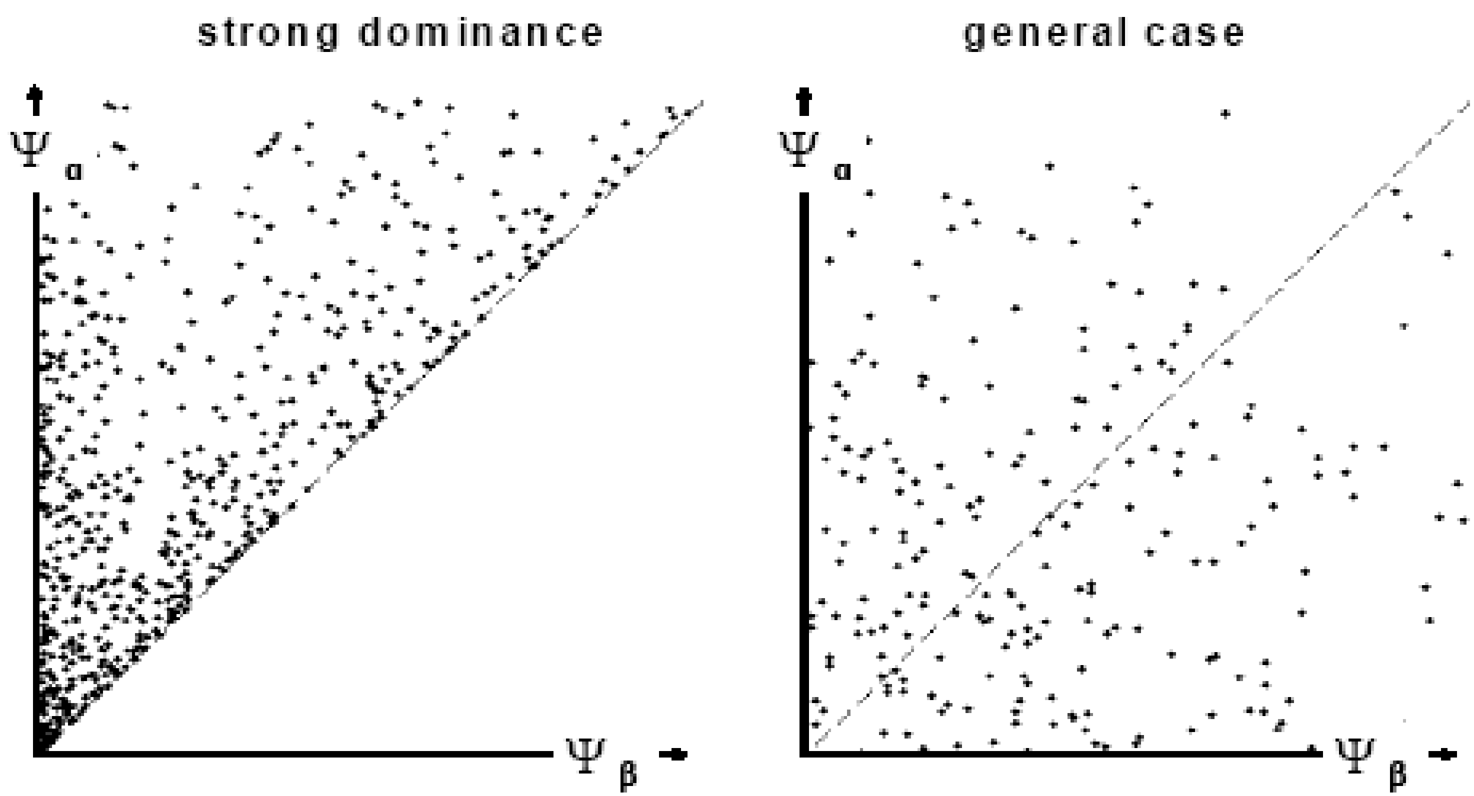

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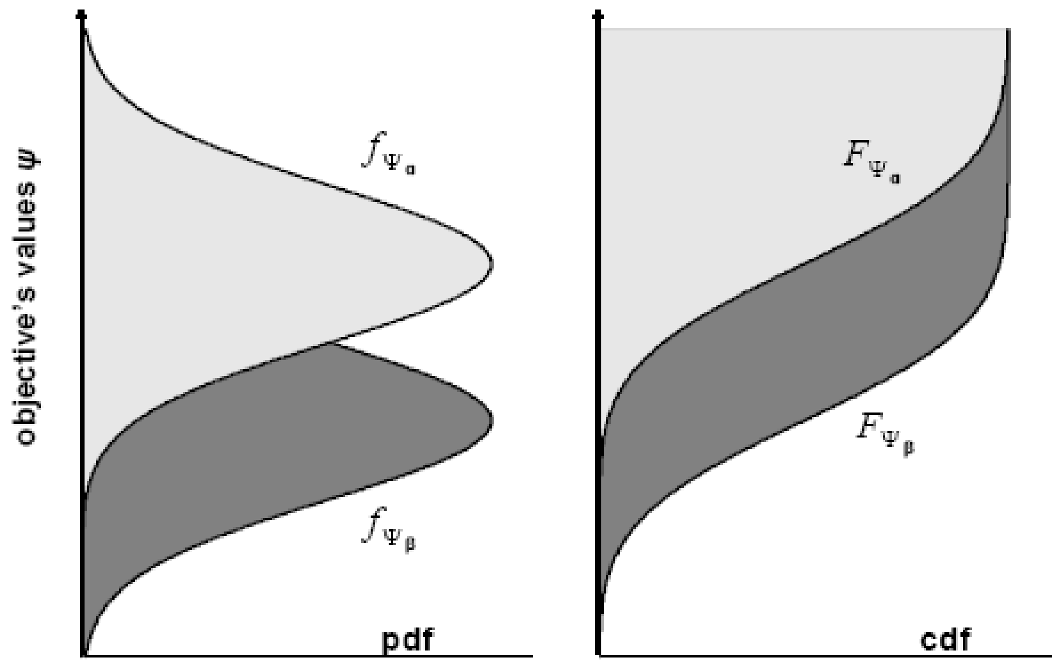
Fig. 5.2. Weak dominance



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SSD: $\mathrm{E}\left\{-\left(\Psi_{\boldsymbol{\alpha}} - \psi\right)^{-}\right\} \geq \mathrm{E}\left\{-\left(\Psi_{\boldsymbol{\beta}} - \psi\right)^{-}\right\}$ for all $\psi \in (-\infty, +\infty)$ (5.43)

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second-order stochastic dominance

$$\mathcal{I}^{2}\left[f_{\Psi_{\alpha}}\right](\psi) \leq \mathcal{I}^{2}\left[f_{\Psi_{\beta}}\right](\psi) \quad (5.44)$$

$$\mathcal{I}^{2}\left[f_{\Psi}\right](\psi) \equiv \mathcal{I}\left[F_{\Psi}\right](\psi) \equiv \int_{-\infty}^{\psi} F_{\Psi}(s)\, ds \quad (5.45)$$

SSD: $\mathrm{E}\left\{-\left(\Psi_{\alpha}-\psi\right)^{-}\right\} \geq \mathrm{E}\left\{-\left(\Psi_{\beta}-\psi\right)^{-}\right\}$ for all $\psi \in (-\infty,+\infty)$ (5.43)

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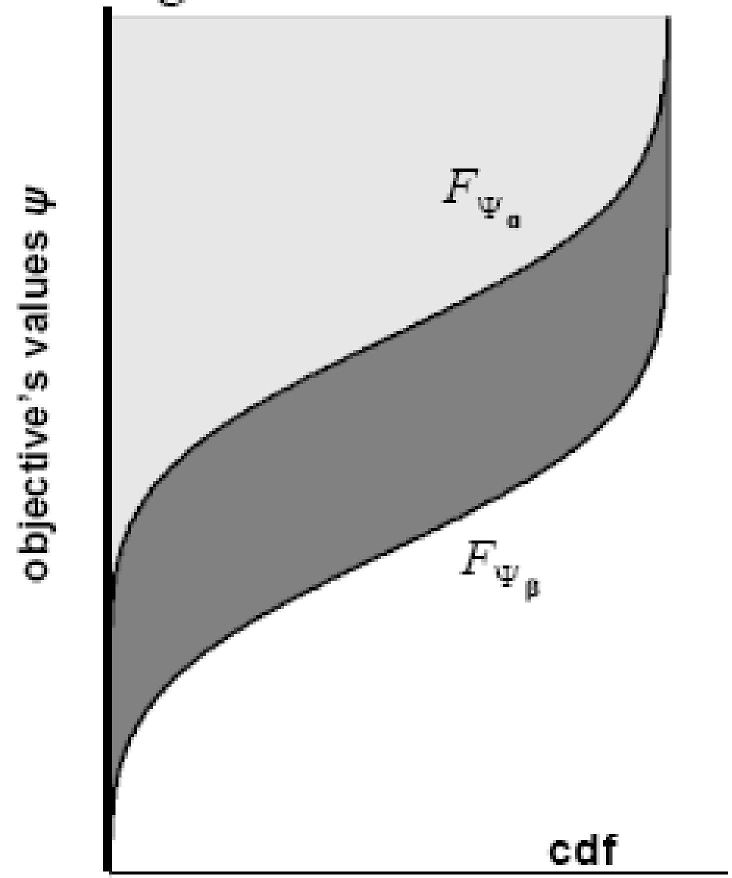

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order-q dominance.

$$q\text{-dom.: } \mathcal{I}^q\left[f_{\Psi_{\boldsymbol{\alpha}}}\right](\psi) \leq \mathcal{I}^q\left[f_{\Psi_{\boldsymbol{\beta}}}\right](\psi) \quad (5.46)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

strong dom.: $\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}}$ in all scenarios. (5.31)

$$\Downarrow$$

first-order dominance (weak)

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}' \mathbf{M}} \quad (5.10)$$

order zero dominance (strong)

$$\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios.} \quad (5.31)$$

$$\Downarrow$$

first-order dominance (weak)

$$F_{\Psi_{\boldsymbol{\alpha}}}(\psi) \leq F_{\Psi_{\boldsymbol{\beta}}}(\psi) \text{ for all } \psi \in (-\infty, +\infty) \quad (5.36)$$

$$\Downarrow$$

second-order stochastic dominance

$$\mathcal{I}^2 \left[f_{\Psi_{\boldsymbol{\alpha}}} \right] (\psi) \leq \mathcal{I}^2 \left[f_{\Psi_{\boldsymbol{\beta}}} \right] (\psi) \quad (5.44)$$

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order-q dominance.

$$q\text{-dom.: } \mathcal{I}^q \left[f_{\Psi_{\boldsymbol{\alpha}}} \right] (\psi) \leq \mathcal{I}^q \left[f_{\Psi_{\boldsymbol{\beta}}} \right] (\psi) \quad (5.46)$$

$$0\text{-dom.} \Rightarrow 1\text{-dom.} \Rightarrow \cdots \Rightarrow q\text{-dom.} \quad (5.47)$$

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Sharpe ratio

$$\mathrm{SR}(\boldsymbol{\alpha}) \equiv \frac{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\}}{\mathrm{Sd}\{\Psi_{\boldsymbol{\alpha}}\}} \quad (5.51)$$

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$$S\Omega_K(\alpha) \equiv \frac{\mathrm{E}\{\Psi_\alpha\} - K}{\widetilde{P}_K\{\Psi_\alpha\}} \Longleftrightarrow$$

omega $\Omega_K(\alpha) \equiv S\Omega_K(\alpha) - 1$

$$\widetilde{P}_K\{\Psi\} \equiv \mathrm{E}\{\max(K - \Psi, 0)\}$$

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Sortino ratio

$$So_K(\alpha) \equiv \frac{\mathrm{E}\{\Psi_\alpha\} - K}{\sqrt{\widetilde{P}^2_K\{\Psi_\alpha\}}}$$

$$\widetilde{P}^2_K\{\Psi\} \equiv \mathrm{E}\left\{\max(K - \Psi, 0)^2\right\}$$

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Kappa

$$\kappa_\lambda^n(\boldsymbol{\alpha}) \equiv \frac{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \lambda}{\left(\widetilde{P}_\lambda^n\{\Psi_{\boldsymbol{\alpha}}\}\right)^{\frac{1}{n}}}$$

$$\widetilde{P}_K^n\{\Psi\} \equiv \mathrm{E}\{\max(K - \Psi, 0)^n\}$$

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$$\boxed{\Psi_{\alpha} = \alpha' \mathbf{M}} \quad (5.10) \qquad\qquad \alpha \mapsto \mathcal{S}(\alpha) \quad (5.48)$$

- Money-equivalence

Sharpe ratio

$$\xcancel{\mathrm{SR}(\alpha)} \equiv \frac{\mathrm{E}\{\Psi_{\alpha}\}}{\mathrm{Sd}\{\Psi_{\alpha}\}} \qquad (5.51)$$

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$$\widetilde{P}_{K}\{\Psi\} \equiv \mathrm{E}\{\max(K - \Psi, 0)\}$$

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$$\widetilde{P}_{K}^{2}\{\Psi\} \equiv \mathrm{E}\left\{\max(K - \Psi, 0)^{2}\right\}$$

Kappa

$$\xcancel{\kappa_{\lambda}^{n}(\alpha)} \equiv \frac{\mathrm{E}\{\Psi_{\alpha}\} - \lambda}{\left(\widetilde{P}_{\lambda}^{n}\{\Psi_{\alpha}\}\right)^{\frac{1}{n}}}$$

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- Money-equivalence

- Estimability

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto \left(f_{\Psi_{\boldsymbol{\alpha}}}, F_{\Psi_{\boldsymbol{\alpha}}}, \phi_{\Psi_{\boldsymbol{\alpha}}}\right) \mapsto \mathcal{S}(\boldsymbol{\alpha}) \quad (5.52)$$

$$f_{\psi} \mapsto \mathrm{E}\{\Psi\} \equiv \int_{\mathbb{R}} \psi f_{\psi}(\psi)\, d\psi. \quad (5.53)$$

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto f_{\Psi_{\boldsymbol{\alpha}}} \mapsto \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.54)$$

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- **Money-equivalence**

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- **Sensibility**

$$\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios } \Rightarrow \mathcal{S}(\boldsymbol{\alpha}) \geq \mathcal{S}(\boldsymbol{\beta}) \quad (5.55)$$

$$\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios} \quad (5.56)$$
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- **Consistence with stochastic dominance**

$$Q_{\Psi_{\boldsymbol{\alpha}}}(p) \geq Q_{\Psi_{\boldsymbol{\beta}}}(p) \text{ for all } p \in (0,1) \Rightarrow \mathcal{S}(\boldsymbol{\alpha}) \geq \mathcal{S}(\boldsymbol{\beta}) \quad (5.57)$$

$$\mathrm{E}\{\Psi\} \equiv \int_{-\infty}^{+\infty} \psi f_{\psi}(\psi)\, d\psi = \int_0^1 Q_{\Psi}(u)\, du. \quad (5.58)$$

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- **Constancy**

$$\Psi_{\mathbf{b}} \equiv \psi_{\mathbf{b}} \Rightarrow \mathcal{S}(\mathbf{b}) = \psi_{\mathbf{b}}. \quad (5.62)$$

$$\Psi_{\mathbf{b}} \equiv \psi_{\mathbf{b}} \Rightarrow \mathrm{E}\{\Psi_{\mathbf{b}}\} = \psi_{\mathbf{b}}. \quad (5.63)$$

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$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.49)$$

- **Positive homogeneity**

$$\Psi_{\lambda\boldsymbol{\alpha}} = \lambda\Psi_{\boldsymbol{\alpha}}, \quad \text{for all } \lambda \geq 0. \quad (5.64)$$

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- **Positive homogeneity**

$$\mathcal{S}(\lambda\boldsymbol{\alpha}) = \lambda\mathcal{S}(\boldsymbol{\alpha}), \quad \text{for all } \lambda \geq 0. \quad (5.65)$$

$$\mathrm{E}\{\Psi_{\lambda\boldsymbol{\alpha}}\} = \mathrm{E}\{\lambda\Psi_{\boldsymbol{\alpha}}\} = \lambda\,\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.66)$$

$$\Psi_{\lambda\boldsymbol{\alpha}} = \lambda\Psi_{\boldsymbol{\alpha}}, \quad \text{for all } \lambda \geq 0 \quad (5.64)$$

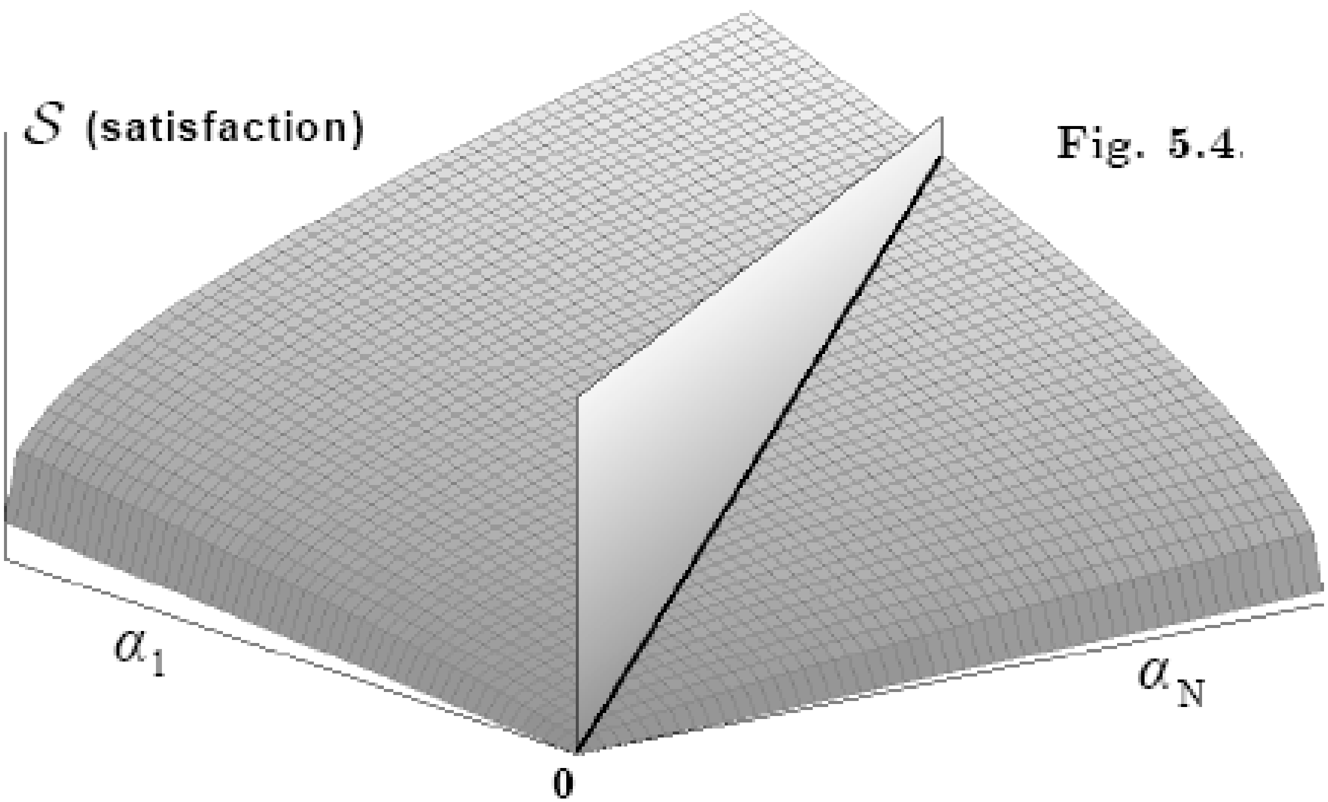

Fig. 5.4.

INVESTOR'S OBJECTIVES EVALUATION: SATISFACTION

Risk and Asset Allocation - Springer – `symmys.com`

$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \quad (5.48)$$

$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.49)$$

- **Positive homogeneity**

$$\mathcal{S}(\lambda\boldsymbol{\alpha}) = \lambda\mathcal{S}(\boldsymbol{\alpha}), \quad \text{for all } \lambda \geq 0. \quad (5.65)$$

$$\mathrm{E}\{\Psi_{\lambda\boldsymbol{\alpha}}\} = \mathrm{E}\{\lambda\Psi_{\boldsymbol{\alpha}}\} = \lambda\,\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.66)$$

$$\Downarrow$$

Euler:

$$\mathcal{S}(\boldsymbol{\alpha}) = \sum_{n=1}^{N} \alpha_n \frac{\partial \mathcal{S}(\boldsymbol{\alpha})}{\partial \alpha_n}. \quad (5.67)$$

$$\Downarrow$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'\mathbf{P}_{T+\tau}, \quad (5.68)$$

$$\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} = \sum_{n=1}^{N} \alpha_n\,\mathrm{E}\left\{P_{T+\tau}^{(n)}\right\} \quad (5.69)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

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$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.49)$$

- **Positive homogeneity**

$$\mathcal{S}(\lambda\boldsymbol{\alpha}) = \lambda\mathcal{S}(\boldsymbol{\alpha}), \quad \text{for all } \lambda \geq 0. \quad (5.65)$$

- **Translation invariance**

$$\Psi_{\boldsymbol{\alpha}+\boldsymbol{\beta}} = \Psi_{\boldsymbol{\alpha}} + \Psi_{\boldsymbol{\beta}}. \quad (5.70)$$

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$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \quad (5.48)$$

$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.49)$$

- **Positive homogeneity**

$$\mathcal{S}(\lambda\boldsymbol{\alpha}) = \lambda\mathcal{S}(\boldsymbol{\alpha}), \quad \text{for all } \lambda \geq 0. \quad (5.65)$$

- **Translation invariance**

$$\mathcal{S}(\boldsymbol{\alpha} + \mathbf{b}) = \mathcal{S}(\boldsymbol{\alpha}) + \psi_{\mathbf{b}} \quad (5.71)$$

$$\Psi_{\mathbf{b}} \equiv 1 \Rightarrow \mathrm{E}\{\Psi_{\boldsymbol{\alpha}+\lambda\mathbf{b}}\} = \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} + \lambda. \quad (5.73)$$

$$\Psi_{\boldsymbol{\alpha}+\boldsymbol{\beta}} = \Psi_{\boldsymbol{\alpha}} + \Psi_{\boldsymbol{\beta}}. \quad (5.70)$$

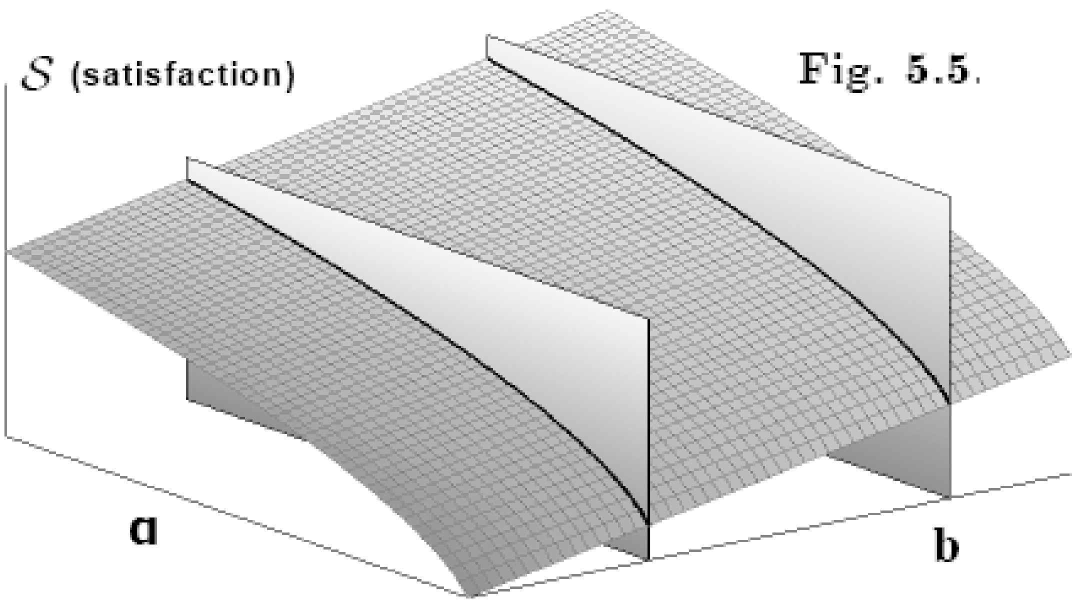

Fig. 5.5.

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

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- **super- additivity**

$$\mathcal{S}(\boldsymbol{\alpha} + \boldsymbol{\beta}) \geq \mathcal{S}(\boldsymbol{\alpha}) + \mathcal{S}(\boldsymbol{\beta}) \quad (5.75)$$

$$\mathrm{E}\{\Psi_{\boldsymbol{\alpha}+\boldsymbol{\beta}}\} = \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} + \mathrm{E}\{\Psi_{\boldsymbol{\beta}}\} \quad (5.77)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10) \qquad \boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \quad (5.48) \qquad \boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.49)$$

- **Positive homogeneity**

$$\mathcal{S}(\lambda\boldsymbol{\alpha}) = \lambda\mathcal{S}(\boldsymbol{\alpha}), \quad \text{for all } \lambda \geq 0. \quad (5.65)$$

- **Translation invariance**

$$\mathcal{S}(\boldsymbol{\alpha} + \mathbf{b}) = \mathcal{S}(\boldsymbol{\alpha}) + \psi_{\mathbf{b}} \quad (5.71)$$

- **super- additivity**

$$\mathcal{S}(\boldsymbol{\alpha} + \boldsymbol{\beta}) \geq \mathcal{S}(\boldsymbol{\alpha}) + \mathcal{S}(\boldsymbol{\beta}) \quad (5.75)$$

- **Co-monotonic additivity**

$$(\boldsymbol{\alpha}, \boldsymbol{\delta}) \text{ co-monotonic } \Rightarrow \mathcal{S}(\boldsymbol{\alpha} + \boldsymbol{\delta}) = \mathcal{S}(\boldsymbol{\alpha}) + \mathcal{S}(\boldsymbol{\delta}) \quad (5.80)$$

$$\mathrm{E}\{\Psi_{\boldsymbol{\alpha}+\boldsymbol{\beta}}\} = \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} + \mathrm{E}\{\Psi_{\boldsymbol{\beta}}\} \quad (5.77)$$

INVESTOR'S OBJECTIVES EVALUATION: SATISFACTION

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$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}}$ (5.10) $\quad \boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha})$ (5.48) $\quad \boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\}$ (5.49)

- **Positive homogeneity**

$$\mathcal{S}(\lambda\boldsymbol{\alpha}) = \lambda\mathcal{S}(\boldsymbol{\alpha}), \quad \text{for all } \lambda \geq 0. \qquad (5.65)$$

- **Translation invariance**

$$\mathcal{S}(\boldsymbol{\alpha} + \mathbf{b}) = \mathcal{S}(\boldsymbol{\alpha}) + \psi_{\mathbf{b}} \qquad (5.71)$$

- **super- additivity**

$$\mathcal{S}(\boldsymbol{\alpha} + \boldsymbol{\beta}) \geq \mathcal{S}(\boldsymbol{\alpha}) + \mathcal{S}(\boldsymbol{\beta}) \qquad (5.75)$$

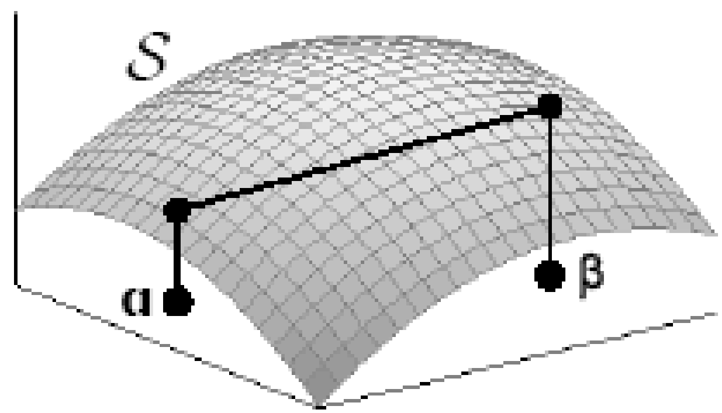
- **Co-monotonic additivity**

$$(\boldsymbol{\alpha}, \boldsymbol{\delta}) \text{ co-monotonic } \Rightarrow \mathcal{S}(\boldsymbol{\alpha} + \boldsymbol{\delta}) = \mathcal{S}(\boldsymbol{\alpha}) + \mathcal{S}(\boldsymbol{\delta}) \qquad (5.80)$$

- **Concavity**

$$\mathcal{S}(\lambda\boldsymbol{\alpha} + (1-\lambda)\boldsymbol{\beta}) \geq \lambda\mathcal{S}(\boldsymbol{\alpha}) + (1-\lambda)\mathcal{S}(\boldsymbol{\beta}) \qquad (5.81)$$

concave satisfaction Fig. 5.6



$$\mathrm{E}\{\Psi_{\boldsymbol{\alpha}+\boldsymbol{\beta}}\} = \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} + \mathrm{E}\{\Psi_{\boldsymbol{\beta}}\} \qquad (5.77)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \quad (5.48)$$

$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.49)$$

- **Risk aversion/propensity/neutrality**

$$\mathrm{RP}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \mathcal{S}(\boldsymbol{\alpha}) \quad (5.85)$$

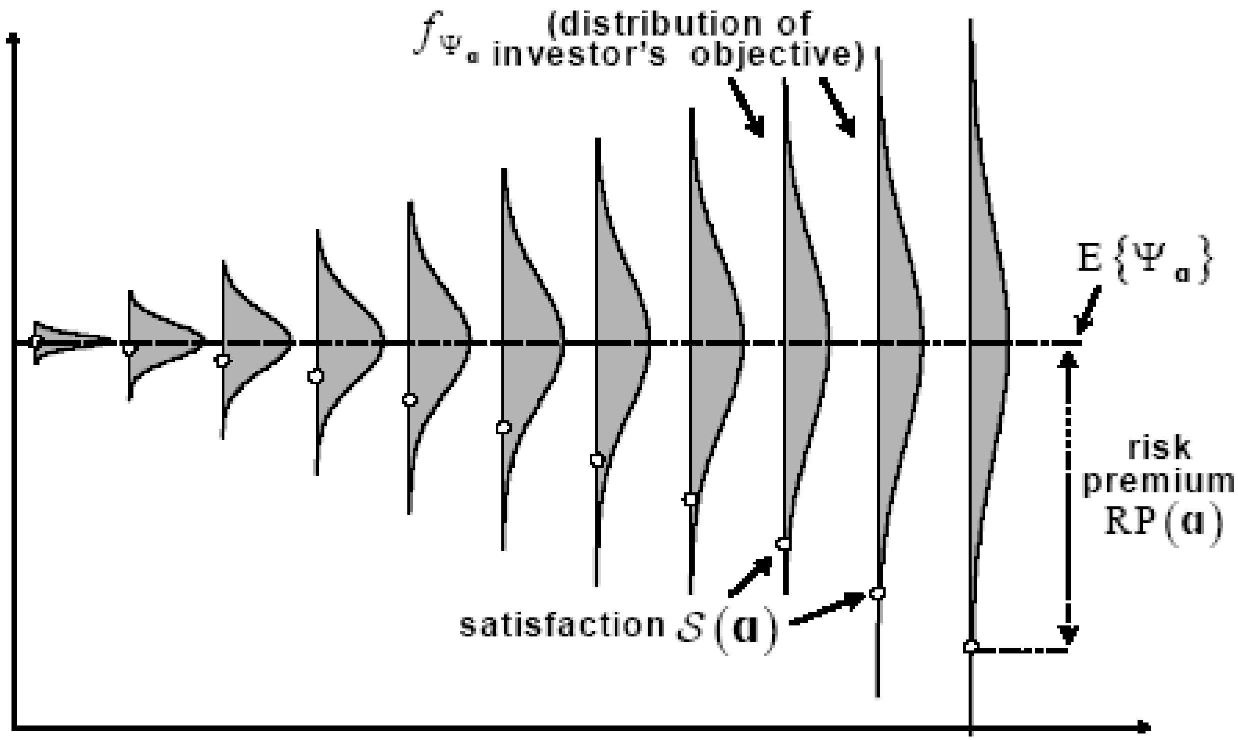

Fig. 5.7 ← allocations α with same expected value →

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \quad (5.48)$$

$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.49)$$

- **Risk aversion/propensity/neutrality**

$$\mathrm{RP}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \mathcal{S}(\boldsymbol{\alpha}) \quad (5.85)$$

risk aversion: $\mathrm{RP}(\boldsymbol{\alpha}) \geq 0$ (5.86)

risk propensity: $\mathrm{RP}(\boldsymbol{\alpha}) \leq 0$ (5.87)

risk neutrality: $\mathrm{RP}(\boldsymbol{\alpha}) \equiv 0.$ (5.88)

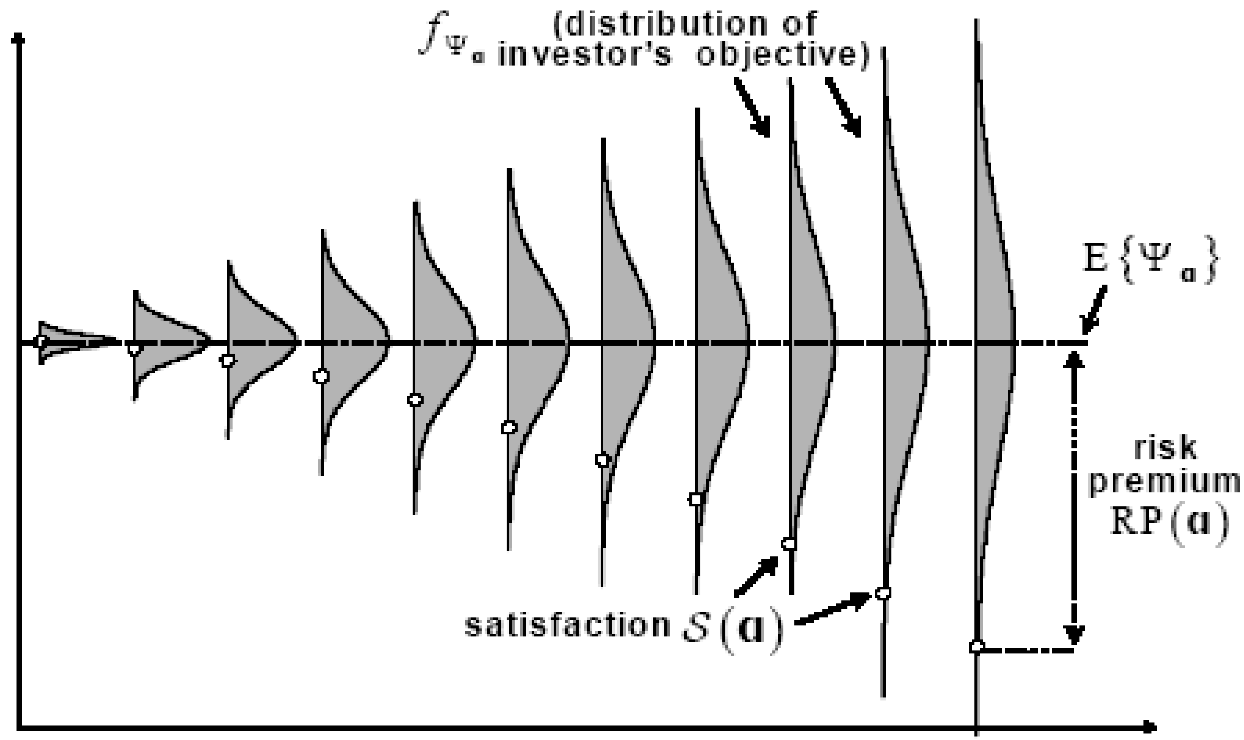

Fig. 5.7 ⟵ allocations $\boldsymbol{\alpha}$ with same expected value ⟶

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$$\boxed{\Psi_{\alpha} = \alpha' \mathrm{M}} \quad (5.10)$$

$$\alpha \mapsto \mathcal{S}(\alpha) \quad (5.48)$$

$$\alpha \mapsto \mathcal{S}(\alpha) \equiv \mathrm{E}\{\Psi_{\alpha}\} \quad (5.49)$$

- **Risk aversion/propensity/neutrality**

$$\mathrm{RP}(\alpha) \equiv \mathrm{E}\{\Psi_{\alpha}\} - \mathcal{S}(\alpha) \quad (5.85)$$

$$\mathrm{RP}(\alpha) \equiv \mathrm{E}\{\Psi_{\alpha}\} - \mathrm{E}\{\Psi_{\alpha}\} \equiv 0 \quad (5.89)$$

risk aversion: $\mathrm{RP}(\alpha) \geq 0$ (5.86)

risk propensity: $\mathrm{RP}(\alpha) \leq 0$ (5.87)

risk neutrality: $\mathrm{RP}(\alpha) \equiv 0.$ (5.88)

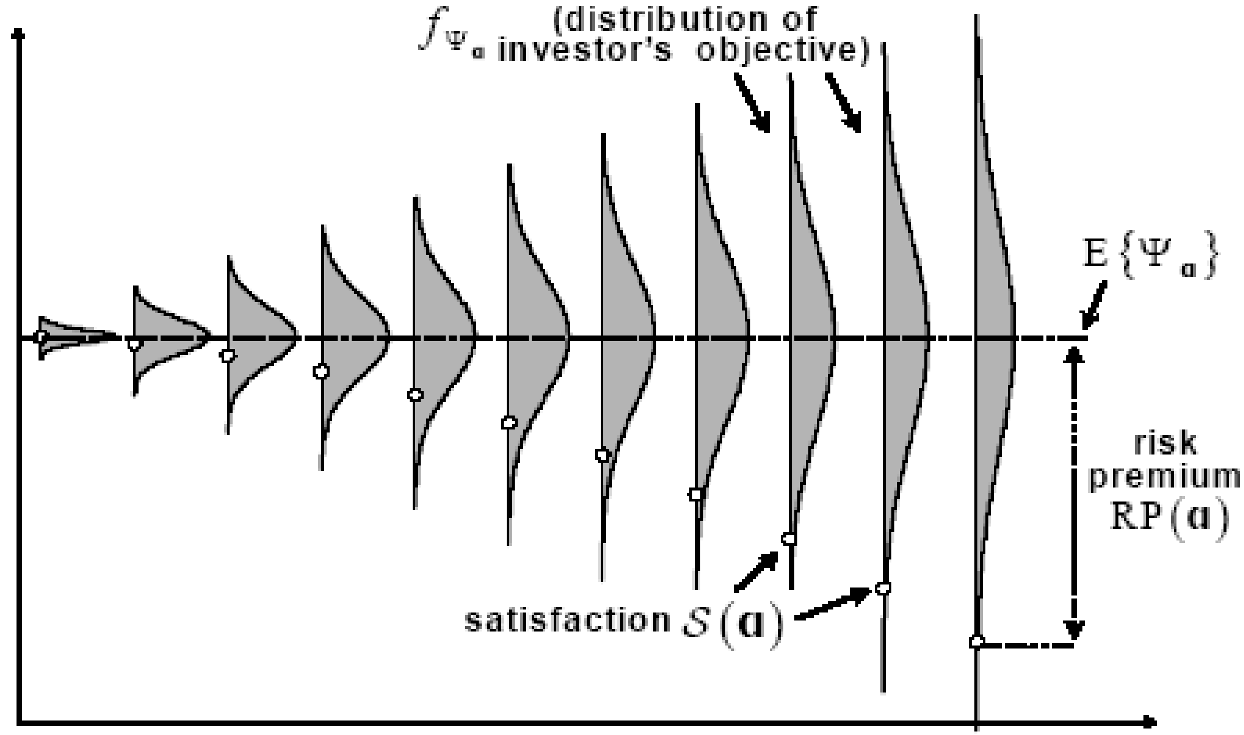

Fig. 5.7 ⟵ allocations α with same expected value ⟶

INVESTOR'S OBJECTIVES EVALUATION: SATISFACTION

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$$\boxed{\Psi_{\alpha} = \alpha' \mathbf{M}} \quad (5.10)$$

- **Money-equivalence**

- **Estimability**

$$\alpha \mapsto \Psi_{\alpha} \mapsto \left(f_{\Psi_{\alpha}}, F_{\Psi_{\alpha}}, \phi_{\Psi_{\alpha}} \right) \mapsto \mathcal{S}(\alpha) \quad (5.52)$$

- **Sensibility**

$$\Psi_{\alpha} \geq \Psi_{\beta} \text{ in all scenarios } \Rightarrow \mathcal{S}(\alpha) \geq \mathcal{S}(\beta) \quad (5.55)$$

- **Consistence with stochastic dominance**

$$Q_{\Psi_{\alpha}}(p) \geq Q_{\Psi_{\beta}}(p) \text{ for all } p \in (0,1) \Rightarrow \mathcal{S}(\alpha) \geq \mathcal{S}(\beta) \quad (5.57)$$

- **Constancy**

$$\Psi_{\mathbf{b}} \equiv \psi_{\mathbf{b}} \Rightarrow \mathcal{S}(\mathbf{b}) = \psi_{\mathbf{b}}. \quad (5.62)$$

$$\alpha \mapsto \mathcal{S}(\alpha) \quad (5.48)$$

- **Positive homogeneity**

$$\mathcal{S}(\lambda\alpha) = \lambda\mathcal{S}(\alpha), \quad \text{for all } \lambda \geq 0. \quad (5.65)$$

- **Translation invariance**

$$\mathcal{S}(\alpha + \mathbf{b}) = \mathcal{S}(\alpha) + \psi_{\mathbf{b}} \quad (5.71)$$

- **super- additivity**

$$\mathcal{S}(\alpha + \beta) \geq \mathcal{S}(\alpha) + \mathcal{S}(\beta) \quad (5.75)$$

- **Co-monotonic additivity**

$$(\alpha, \delta) \text{ co-monotonic} \Rightarrow \mathcal{S}(\alpha + \delta) = \mathcal{S}(\alpha) + \mathcal{S}(\delta) \quad (5.80)$$

- **Concavity**

$$\mathcal{S}(\lambda\alpha + (1-\lambda)\beta) \geq \lambda\mathcal{S}(\alpha) + (1-\lambda)\mathcal{S}(\beta) \quad (5.81)$$

- **Risk aversion/propensity/neutrality**

$$\mathrm{RP}(\alpha) \equiv \mathrm{E}\{\Psi_{\alpha}\} - \mathcal{S}(\alpha) \quad (5.85)$$

INVESTOR'S OBJECTIVES EVALUATION: UTILITY

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Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

INVESTOR'S OBJECTIVES EVALUATION: UTILITY

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\} \equiv \int_{\mathbb{R}} u\left(\psi\right) f_{\Psi_{\boldsymbol{\alpha}}}\left(\psi\right) d\psi \quad (5.90)$$

$$\boldsymbol{\alpha} \mapsto \mathcal{S}\left(\boldsymbol{\alpha}\right) \quad (5.48)$$

$$\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\} = -\phi_{\Psi_{\boldsymbol{\alpha}}}\left(\frac{i}{\zeta}\right) \quad (5.92)$$

$$u\left(\psi\right) \equiv -e^{-\frac{1}{\zeta}\psi} \quad (5.91)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathrm{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathrm{CE}(\boldsymbol{\alpha}) \equiv u^{-1}\left(\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\}\right) \quad (5.93)$$

$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \quad (5.48)$$

$$\mathrm{CE}(\boldsymbol{\alpha}) \equiv -\zeta \ln\left(\phi_{\Psi_{\boldsymbol{\alpha}}}\left(\frac{i}{\zeta}\right)\right) \quad (5.94)$$

$$u(\psi) \equiv -e^{-\frac{1}{\zeta}\psi} \quad (5.92)$$

$$\boldsymbol{\alpha} \mapsto \mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\} \equiv \int_{\mathbb{R}} u(\psi)\, f_{\Psi_{\boldsymbol{\alpha}}}(\psi)\, d\psi \quad (5.90)$$

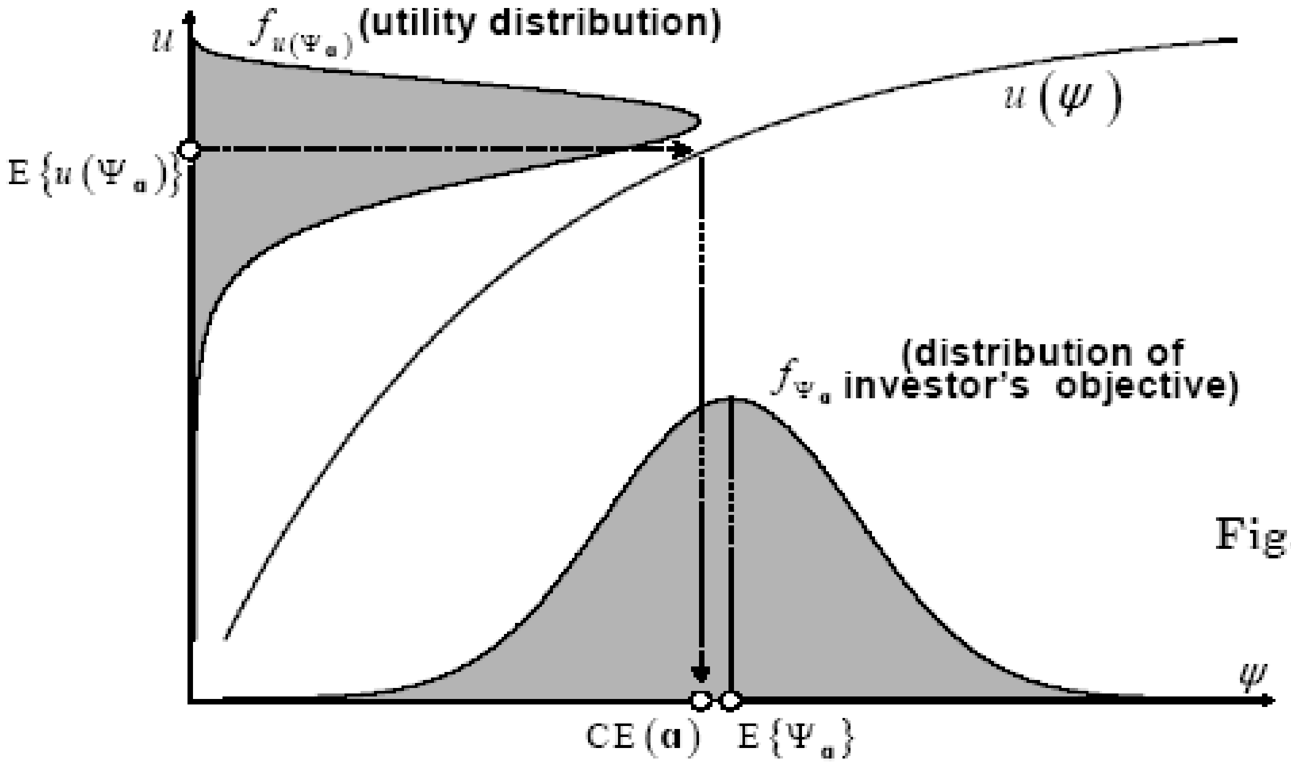

Fig. 5.8.

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

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$$\mathrm{CE}(\boldsymbol{\alpha}) \equiv -\zeta \ln\left(\phi_{\Psi_{\boldsymbol{\alpha}}}\left(\frac{i}{\zeta}\right)\right) \quad (5.94)$$

$$u(\psi) \equiv -e^{-\frac{1}{\zeta}\psi} \quad (5.92)$$

$$\mathrm{CE}(\boldsymbol{\alpha}) = \boldsymbol{\alpha}'\boldsymbol{\mu} - \frac{\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}}{2\zeta} \quad (5.144)$$

$$\mathbf{M} \equiv \mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.143)$$

$$\boldsymbol{\alpha} \mapsto \mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\} \equiv \int_{\mathbb{R}} u(\psi)\, f_{\Psi_{\boldsymbol{\alpha}}}(\psi)\, d\psi \quad (5.90)$$

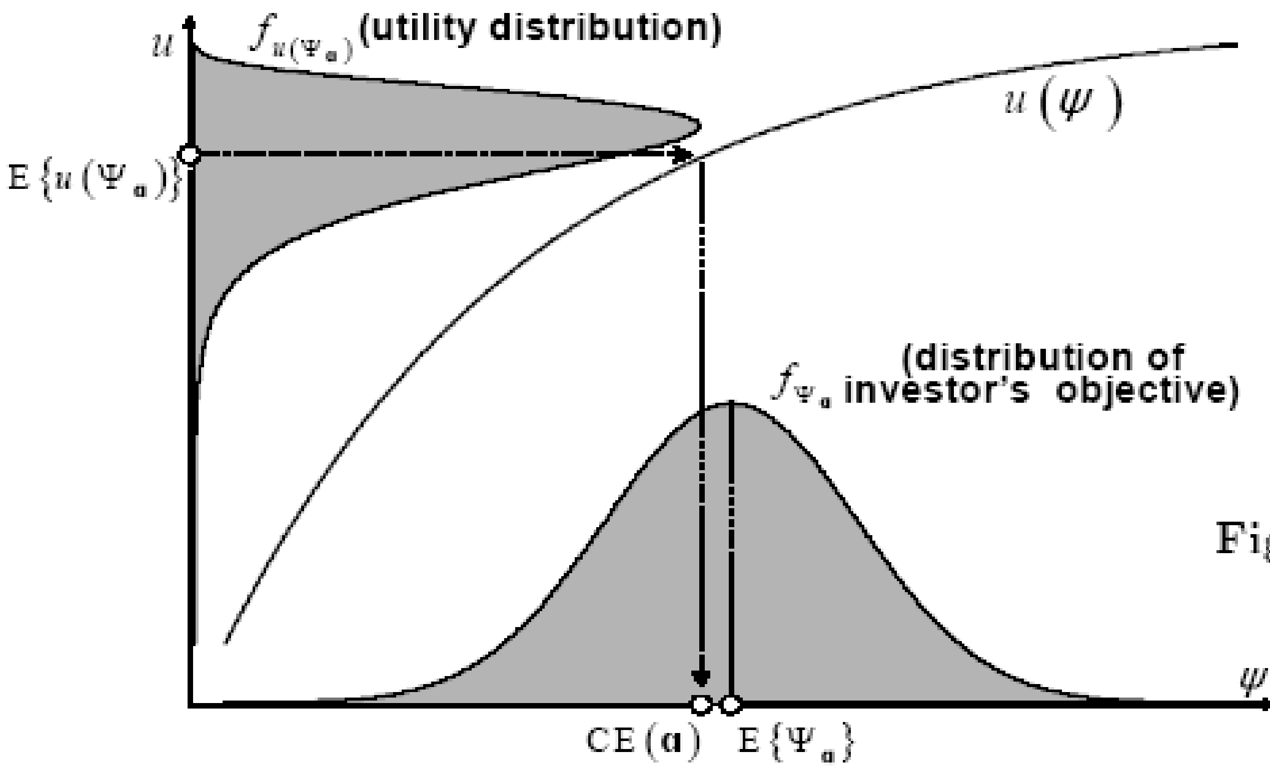

Fig. 5.8.

INVESTOR'S OBJECTIVES EVALUATION: UTILITY

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$$u(\psi) \equiv -e^{-\frac{1}{\zeta}\psi} \quad (5.92)$$

$$\mathbf{M} \equiv \mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.143)$$

- **Money-equivalence**

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$$\mathbf{M} \equiv \mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.143)$$

- **Money-equivalence**

- **Estimability**

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto f_{\Psi_{\boldsymbol{\alpha}}} \mapsto \mathrm{CE}(\boldsymbol{\alpha}) \quad (5.96)$$

INVESTOR'S OBJECTIVES EVALUATION: UTILITY

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$$u(\psi) \equiv -e^{-\frac{1}{\zeta}\psi} \quad (5.92)$$

$$\mathbf{M} \equiv \mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.143)$$

- **Money-equivalence**

- **Estimability**

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto f_{\Psi_{\boldsymbol{\alpha}}} \mapsto \mathrm{CE}(\boldsymbol{\alpha}) \quad (5.96)$$

- **Sensibility**

$$\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios } \Rightarrow \mathrm{CE}(\boldsymbol{\alpha}) \geq \mathrm{CE}(\boldsymbol{\beta}) \quad (5.100) \qquad \Longleftarrow \qquad \mathcal{D}u \geq 0, \quad (5.98)$$

INVESTOR'S OBJECTIVES EVALUATION: UTILITY

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10) \qquad \boldsymbol{\alpha} \mapsto \mathrm{CE}(\boldsymbol{\alpha}) \equiv u^{-1}\left(\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\}\right) \quad (5.93)$$

$$\mathrm{CE}(\boldsymbol{\alpha}) = \boldsymbol{\alpha}'\boldsymbol{\mu} - \frac{\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}}{2\zeta} \quad (5.144)$$

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- **Sensibility**

$$\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios } \Rightarrow \mathrm{CE}(\boldsymbol{\alpha}) \geq \mathrm{CE}(\boldsymbol{\beta}) \quad (5.100) \qquad \Longleftarrow \qquad \mathcal{D}u \geq 0, \quad (5.98)$$

- **Consistence with stochastic dominance**

$$Q_{\Psi_{\boldsymbol{\alpha}}}(p) \geq Q_{\Psi_{\boldsymbol{\beta}}}(p) \text{ for all } p \in (0,1) \Rightarrow \mathrm{CE}(\boldsymbol{\alpha}) \geq \mathrm{CE}(\boldsymbol{\beta}) \quad (5.109) \qquad \Longleftarrow \qquad \mathcal{D}u \geq 0, \quad (5.98)$$

$$\text{consistence with higher order dominance} \quad \Longleftarrow \quad (-1)^k \mathcal{D}^k u \leq 0, \; k = 1, 2, \ldots, q. \quad (5.111)$$

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- **Money-equivalence**

- **Estimability**

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- **Constancy**

$$\Psi_{\mathbf{b}} \equiv \psi_{\mathbf{b}} \Rightarrow \mathrm{CE}(\mathbf{b}) = \psi_{\mathbf{b}} \quad (5.112)$$

$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10) \qquad \boldsymbol{\alpha} \mapsto \mathrm{CE}(\boldsymbol{\alpha}) \equiv u^{-1}\left(\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\}\right) \quad (5.93)$$

- **Positive homogeneity**

$$\mathrm{CE}(\lambda\boldsymbol{\alpha}) = \lambda\,\mathrm{CE}(\boldsymbol{\alpha}) \quad (5.113) \qquad \Longleftarrow \qquad u(\psi) \equiv \psi^{1-\frac{1}{\gamma}} \quad (5.114)$$

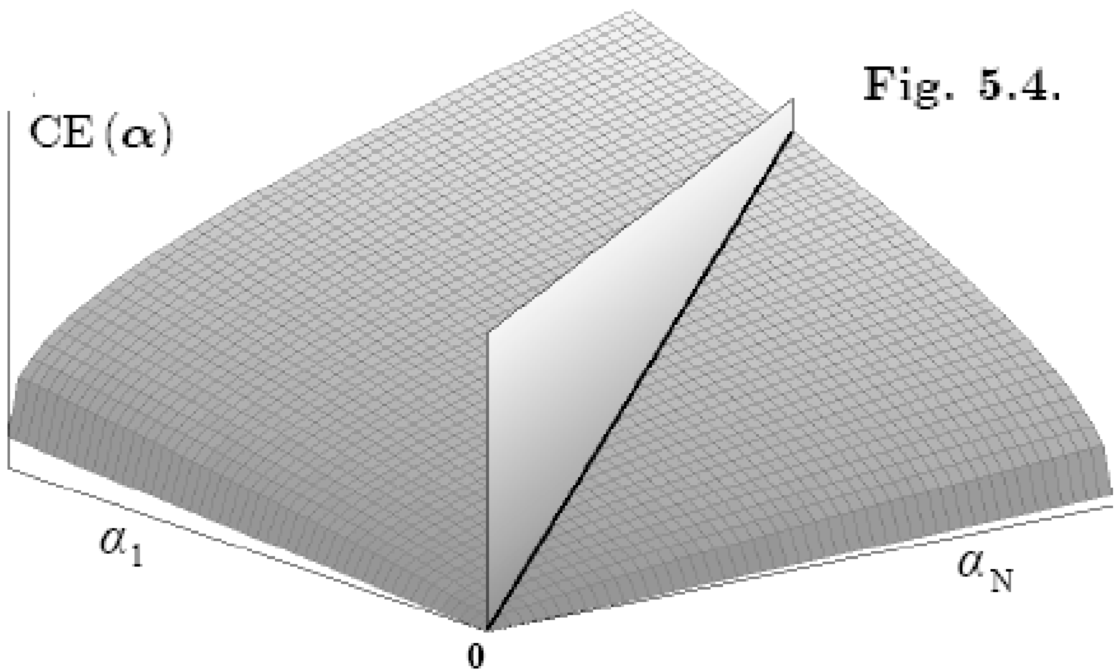

Fig. 5.4.

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathrm{M}} \quad (5.10) \qquad \boldsymbol{\alpha} \mapsto \mathrm{CE}(\boldsymbol{\alpha}) \equiv u^{-1}\left(\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\}\right) \quad (5.93)$$

- **Positive homogeneity**

$$\mathrm{CE}(\lambda\boldsymbol{\alpha}) = \lambda\,\mathrm{CE}(\boldsymbol{\alpha}) \quad (5.113) \qquad \Longleftarrow \qquad u(\psi) \equiv \psi^{1-\frac{1}{\gamma}} \quad (5.114)$$

$$\Downarrow$$

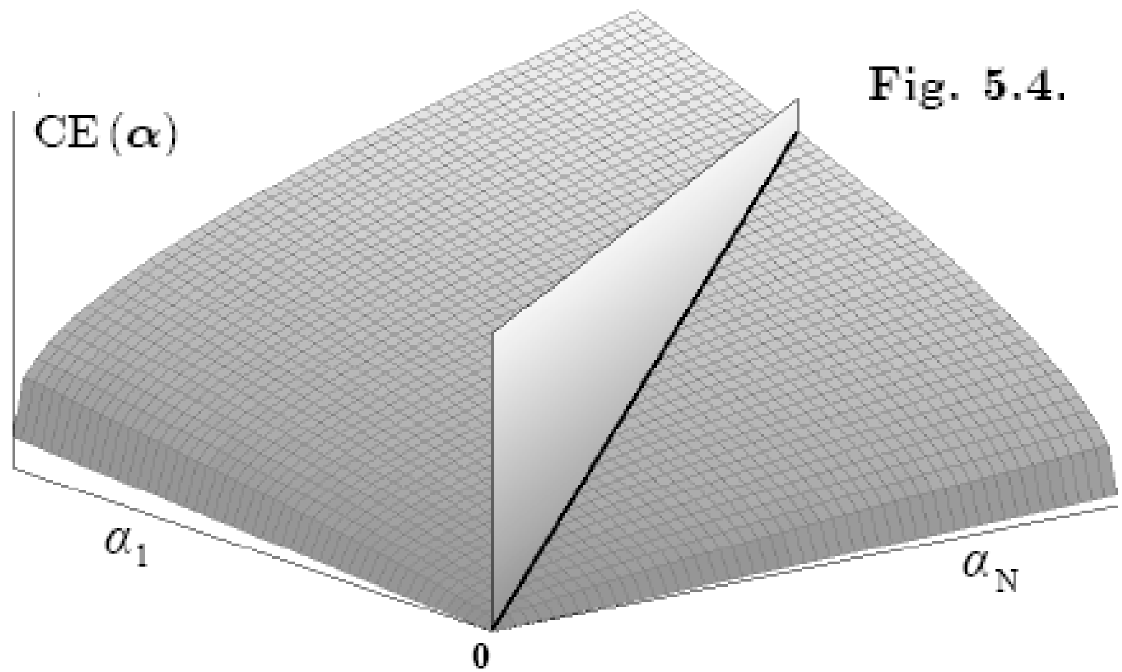

Fig. 5.4.

Euler:

$$\mathrm{CE}(\boldsymbol{\alpha}) = \sum_{n=1}^{N} \alpha_n \left[\mathrm{E}\left\{M_n\left(\boldsymbol{\alpha}'\mathrm{M}\right)^{-\frac{1}{\gamma}}\right\}\left(\mathrm{CE}(\boldsymbol{\alpha})\right)^{\frac{1}{\gamma}}\right] \quad (5.152)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10) \qquad \boldsymbol{\alpha} \mapsto \mathrm{CE}(\boldsymbol{\alpha}) \equiv u^{-1}\left(\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\}\right) \quad (5.93)$$

- **Positive homogeneity**

$$\mathrm{CE}(\lambda\boldsymbol{\alpha}) = \lambda\,\mathrm{CE}(\boldsymbol{\alpha}) \quad (5.113) \qquad \Longleftarrow \qquad u(\psi) \equiv \psi^{1-\frac{1}{\gamma}} \quad (5.114)$$

- **Translation invariance**

$$\Psi_{\mathbf{b}} \equiv 1 \Rightarrow \mathrm{CE}(\boldsymbol{\alpha} + \lambda\mathbf{b}) = \mathrm{CE}(\boldsymbol{\alpha}) + \lambda \quad (5.115) \qquad \Longleftarrow \qquad u(\psi) \equiv -e^{-\frac{1}{\zeta}\psi} \quad (5.91)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10) \qquad \boldsymbol{\alpha} \mapsto \mathrm{CE}(\boldsymbol{\alpha}) \equiv u^{-1}\left(\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\}\right) \quad (5.93)$$

- **Positive homogeneity**

$$\mathrm{CE}(\lambda\boldsymbol{\alpha}) = \lambda\,\mathrm{CE}(\boldsymbol{\alpha}) \quad (5.113) \qquad \Longleftarrow \qquad u(\psi) \equiv \psi^{1-\frac{1}{\gamma}} \quad (5.114)$$

- **Translation invariance**

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- **Concavity**

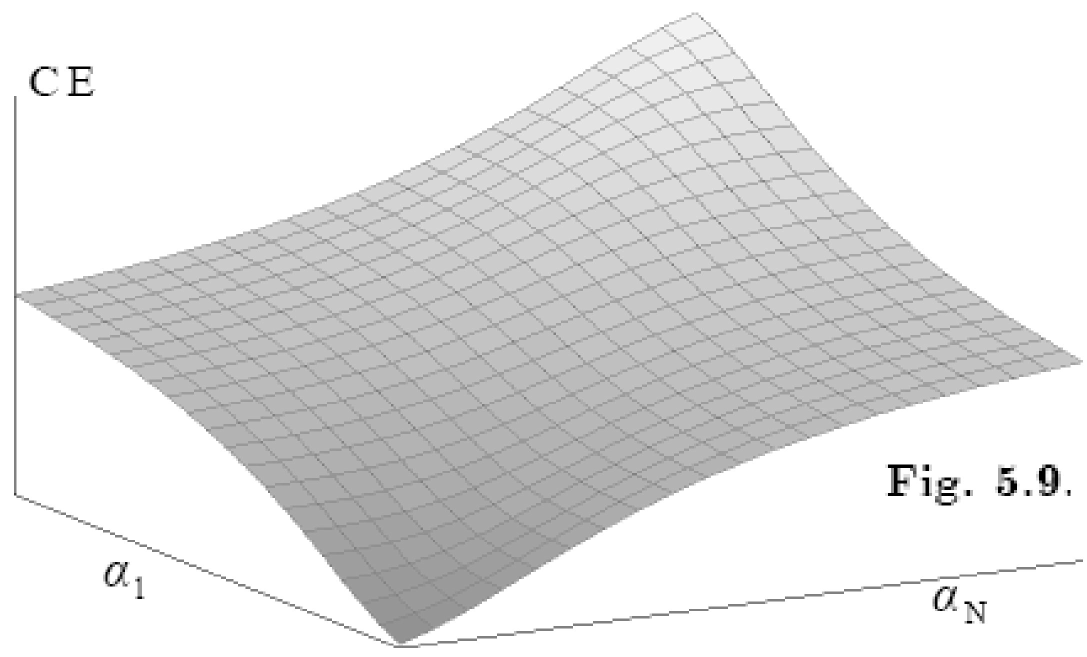

Fig. 5.9.

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10) \qquad \boldsymbol{\alpha} \mapsto \mathrm{CE}(\boldsymbol{\alpha}) \equiv u^{-1}\left(\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\}\right) \quad (5.93)$$

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$$\mathrm{CE}(\lambda\boldsymbol{\alpha}) = \lambda\,\mathrm{CE}(\boldsymbol{\alpha}) \quad (5.113) \qquad \Longleftarrow \qquad u(\psi) \equiv \psi^{1-\frac{1}{\gamma}} \quad (5.114)$$

- **Translation invariance**

$$\Psi_{\mathbf{b}} \equiv 1 \Rightarrow \mathrm{CE}(\boldsymbol{\alpha} + \lambda\mathbf{b}) = \mathrm{CE}(\boldsymbol{\alpha}) + \lambda \quad (5.115) \qquad \Longleftarrow \qquad u(\psi) \equiv -e^{-\frac{1}{\zeta}\psi} \quad (5.91)$$

- ~~**super- additivity**~~

- ~~**Co-monotonic additivity**~~

- ~~**Concavity**~~

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$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}}$ (5.10) $\quad \boldsymbol{\alpha} \mapsto \mathrm{CE}(\boldsymbol{\alpha}) \equiv u^{-1}\left(\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\}\right)$ (5.93)

- Positive homogeneity

  $\mathrm{CE}(\lambda\boldsymbol{\alpha}) = \lambda\,\mathrm{CE}(\boldsymbol{\alpha})$ (5.113) $\quad \Longleftarrow \quad u(\psi) \equiv \psi^{1-\frac{1}{\gamma}}$ (5.114)

- Translation invariance

  $\Psi_{\mathbf{b}} \equiv 1 \Rightarrow \mathrm{CE}(\boldsymbol{\alpha} + \lambda\mathbf{b}) = \mathrm{CE}(\boldsymbol{\alpha}) + \lambda$ (5.115) $\quad \Longleftarrow \quad u(\psi) \equiv -e^{-\frac{1}{\zeta}\psi}$ (5.91)

- ~~super- additivity~~

- ~~Co-monotonic additivity~~

- ~~Concavity~~

- Risk aversion/propensity/neutrality

  $\mathrm{RP}(\boldsymbol{\alpha}) = \mathrm{E}\left\{\Psi_{\boldsymbol{\alpha}}\right\} - \mathrm{CE}(\boldsymbol{\alpha})$ (5.119) $\quad u \text{ concave} \Leftrightarrow \mathrm{RP}(\boldsymbol{\alpha}) \geq 0$ (5.120)

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10) \qquad \boldsymbol{\alpha} \mapsto \mathrm{CE}(\boldsymbol{\alpha}) \equiv u^{-1}\left(\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\}\right) \quad (5.93)$$

$$\mathrm{RP}(\boldsymbol{\alpha}) = \mathrm{E}\left\{\Psi_{\boldsymbol{\alpha}}\right\} - \mathrm{CE}(\boldsymbol{\alpha}) \quad (5.119) \qquad \Longrightarrow \qquad \mathrm{RP}(\boldsymbol{\alpha}) \approx \frac{1}{2}\mathrm{A}\left(\mathrm{E}\left\{\Psi_{\boldsymbol{\alpha}}\right\}\right)\mathrm{Var}\left\{\Psi_{\boldsymbol{\alpha}}\right\} \quad (5.122)$$

$$\mathrm{A}(\psi) \equiv -\frac{\mathcal{D}^2 u(\psi)}{\mathcal{D}u(\psi)} \quad (5.121)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10) \qquad \boldsymbol{\alpha} \mapsto \mathrm{CE}(\boldsymbol{\alpha}) \equiv u^{-1}\left(\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\}\right) \quad (5.93)$$

$$\mathrm{RP}(\boldsymbol{\alpha}) = \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \mathrm{CE}(\boldsymbol{\alpha}) \quad (5.119) \qquad \Longrightarrow \qquad \mathrm{RP}(\boldsymbol{\alpha}) \approx \frac{1}{2}\mathrm{A}\left(\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\}\right)\mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.122)$$

$$\mathrm{A}(\psi) \equiv -\frac{\mathcal{D}^2 u(\psi)}{\mathcal{D}u(\psi)} \quad (5.121)$$

$$\mathrm{A}(\psi) \equiv \frac{\psi}{\gamma\psi^2 + \zeta\psi + \eta}. \quad (5.132) \quad \begin{cases} \begin{matrix}\eta \equiv 0 \\ \text{HARA}\end{matrix} \begin{cases} \zeta > 0 \text{ and } \gamma \equiv 0 & u(\psi) = -e^{-\frac{1}{\zeta}\psi} \quad (5.133) \\ \zeta > 0 \text{ and } \gamma \equiv -1 & u(\psi) = \psi - \dfrac{1}{2\zeta}\psi^2. \quad (5.134) \\ \zeta \equiv 0 \begin{cases} \gamma \geq 1 & u(\psi) \equiv \psi^{1-\frac{1}{\gamma}} \quad (5.135) \\ \gamma \to 1 & u(\psi) \equiv \ln\psi. \quad (5.136) \\ \gamma \to \infty & u(\psi) \equiv \psi. \quad (5.137) \end{cases} \end{cases} \\ \gamma \equiv 0 \qquad u(\psi) \equiv \mathrm{erf}\left(\dfrac{\psi}{\sqrt{2\eta}}\right) \quad (5.138) \end{cases}$$

INVESTOR'S OBJECTIVES EVALUATION: QUANTILE & VaR

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Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

$$\mathbb{P}\left\{w_T - W_{T+\tau} < L_{\max}\right\} \geq c. \quad (5.155)$$

$$\mathrm{VaR}_c\left(\boldsymbol{\alpha}\right) \quad (5.158)$$

$$\Psi_{\alpha} \equiv W_{T+\tau}(\alpha) - w_T \quad (5.156)$$

$$Q_{\Psi_{\alpha}}(1-c) \geq -L_{\max}. \quad (5.157)$$

$$\Updownarrow$$

$$\mathbb{P}\{w_T - W_{T+\tau} < L_{\max}\} \geq c. \quad (5.155)$$

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$$\boldsymbol{\alpha} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \equiv Q_{\Psi_{\boldsymbol{\alpha}}}(1-c) \quad (5.159)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv W_{T+\tau}(\boldsymbol{\alpha}) - w_T \quad (5.156)$$

$$Q_{\Psi_{\boldsymbol{\alpha}}}(1-c) \geq -L_{\max} \quad (5.157)$$

$$\Updownarrow$$

$$\mathbb{P}\{w_T - W_{T+\tau} < L_{\max}\} \geq c. \quad (5.155)$$

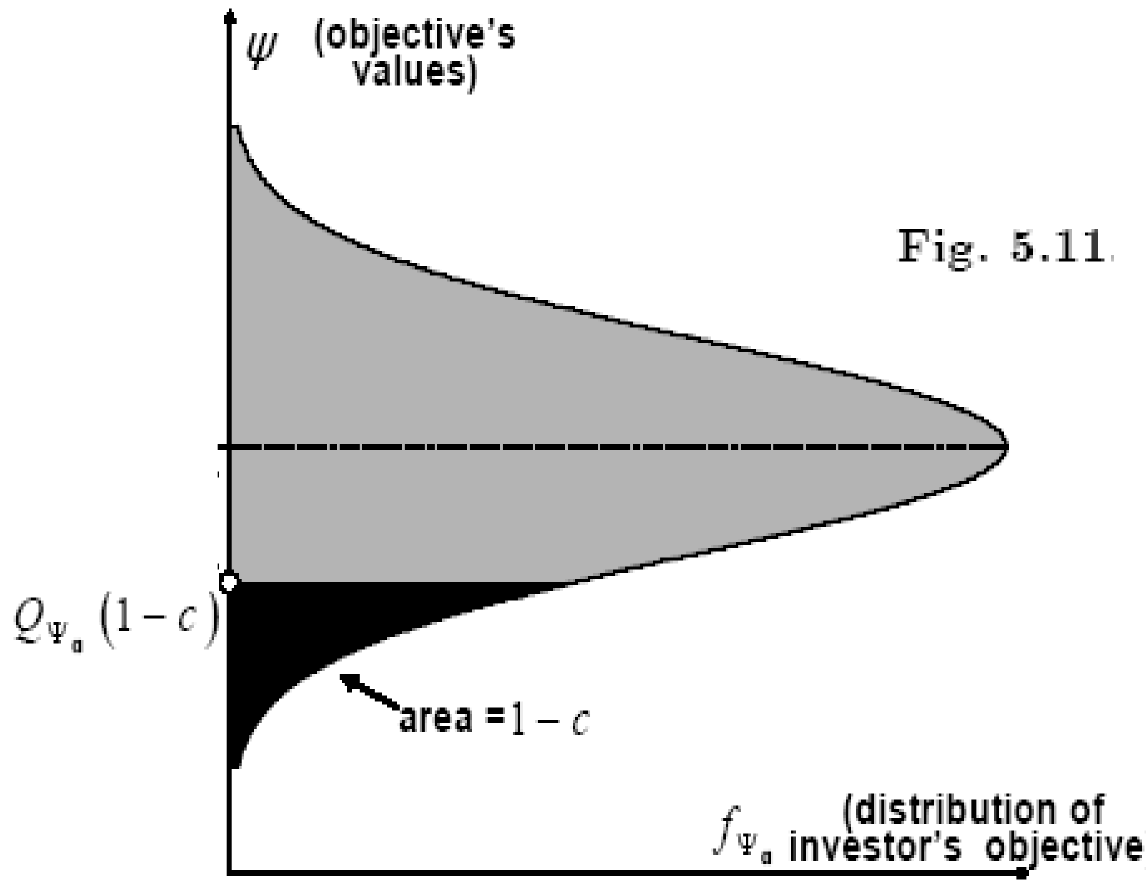

Fig. 5.11

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \equiv Q_{\Psi_{\boldsymbol{\alpha}}}(1-c) \quad (5.159)$$

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \mu_{\boldsymbol{\alpha}} + \sigma_{\boldsymbol{\alpha}} \operatorname{erf}^{-1}(1-2c) \quad (5.175)$$

$$\Psi_{\boldsymbol{\alpha}} \sim \mathrm{N}\left(\mu_{\boldsymbol{\alpha}}, \sigma^2_{\boldsymbol{\alpha}}\right) \quad (5.173)$$

$$\left\{ \begin{aligned} \mu_{\boldsymbol{\alpha}} &\equiv \boldsymbol{\alpha}'(\boldsymbol{\mu} - \mathbf{P}_T) \\ \sigma^2_{\boldsymbol{\alpha}} &\equiv \boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha} \end{aligned} \right. \quad (5.174)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.172)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \equiv Q_{\Psi_{\boldsymbol{\alpha}}}(1-c) \quad (5.159)$$

- **Money-equivalence**

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \mu_{\boldsymbol{\alpha}} + \sigma_{\boldsymbol{\alpha}}\,\mathrm{erf}^{-1}(1-2c) \quad (5.175)$$

$$\Psi_{\boldsymbol{\alpha}} \sim \mathrm{N}\left(\mu_{\boldsymbol{\alpha}}, \sigma^2_{\boldsymbol{\alpha}}\right) \quad (5.173)$$

$$\begin{cases} \mu_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\boldsymbol{\mu} - \mathbf{P}_T) \\ \sigma^2_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha} \end{cases} \quad (5.174)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.172)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \equiv Q_{\Psi_{\boldsymbol{\alpha}}}(1-c) \quad (5.159)$$

- **Money-equivalence**

- **Estimability**

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto F_{\Psi_{\boldsymbol{\alpha}}} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.160)$$

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \mu_{\boldsymbol{\alpha}} + \sigma_{\boldsymbol{\alpha}}\,\mathrm{erf}^{-1}(1-2c) \quad (5.175)$$

$$\Psi_{\boldsymbol{\alpha}} \sim \mathrm{N}\left(\mu_{\boldsymbol{\alpha}}, \sigma^2_{\boldsymbol{\alpha}}\right) \quad (5.173)$$

$$\left\{\begin{array}{l} \mu_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\boldsymbol{\mu} - \mathbf{P}_T) \\ \sigma^2_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha} \end{array}\right. \quad (5.174)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.172)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \equiv Q_{\Psi_{\boldsymbol{\alpha}}}(1-c) \quad (5.159)$$

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \mu_{\boldsymbol{\alpha}} + \sigma_{\boldsymbol{\alpha}}\,\mathrm{erf}^{-1}(1-2c) \quad (5.175)$$

$$\Psi_{\boldsymbol{\alpha}} \sim \mathrm{N}\left(\mu_{\boldsymbol{\alpha}}, \sigma^2_{\boldsymbol{\alpha}}\right) \quad (5.173)$$

$$\left\{\begin{array}{l} \mu_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\boldsymbol{\mu} - \mathbf{P}_T) \\ \sigma^2_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha} \end{array}\right. \quad (5.174)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.172)$$

- **Money-equivalence**

- **Estimability**

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto F_{\Psi_{\boldsymbol{\alpha}}} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.160)$$

- **Consistence with stochastic dominance**

$$Q_{\Psi_{\boldsymbol{\alpha}}}(p) \geq Q_{\Psi_{\boldsymbol{\beta}}}(p) \text{ for all } p \in (0,1) \Rightarrow \mathrm{Q}_c(\boldsymbol{\alpha}) \geq \mathrm{Q}_c(\boldsymbol{\beta}) \quad (5.161)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathrm{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \equiv Q_{\Psi_{\boldsymbol{\alpha}}}(1-c) \quad (5.159)$$

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \mu_{\boldsymbol{\alpha}} + \sigma_{\boldsymbol{\alpha}}\,\mathrm{erf}^{-1}(1-2c) \quad (5.175)$$

$$\Psi_{\boldsymbol{\alpha}} \sim \mathrm{N}\left(\mu_{\boldsymbol{\alpha}}, \sigma^2_{\boldsymbol{\alpha}}\right) \quad (5.173)$$

$$\left\{\begin{array}{l} \mu_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\boldsymbol{\mu} - \mathbf{P}_T) \\ \sigma^2_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha} \end{array}\right. \quad (5.174)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.172)$$

- **Money-equivalence**

- **Estimability**

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto F_{\Psi_{\boldsymbol{\alpha}}} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.160)$$

- **Sensibility**

$$\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios } \Rightarrow \mathrm{Q}_c(\boldsymbol{\alpha}) \geq \mathrm{Q}_c(\boldsymbol{\beta}) \quad (5.162)$$

- **Consistence with stochastic dominance**

$$Q_{\Psi_{\boldsymbol{\alpha}}}(p) \geq Q_{\Psi_{\boldsymbol{\beta}}}(p) \text{ for all } p \in (0,1) \Rightarrow \mathrm{Q}_c(\boldsymbol{\alpha}) \geq \mathrm{Q}_c(\boldsymbol{\beta}) \quad (5.161)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

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$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \mu_{\boldsymbol{\alpha}} + \sigma_{\boldsymbol{\alpha}} \operatorname{erf}^{-1}(1-2c) \quad (5.175)$$

$$\Psi_{\boldsymbol{\alpha}} \sim \mathrm{N}\left(\mu_{\boldsymbol{\alpha}}, \sigma^2_{\boldsymbol{\alpha}}\right) \quad (5.173)$$

$$\left\{\begin{array}{l} \mu_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\boldsymbol{\mu} - \mathbf{P}_T) \\ \sigma^2_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha} \end{array}\right. \quad (5.174)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.172)$$

- **Money-equivalence**
- **Estimability**

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto F_{\Psi_{\boldsymbol{\alpha}}} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.160)$$

- **Sensibility**

$$\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios } \Rightarrow \mathrm{Q}_c(\boldsymbol{\alpha}) \geq \mathrm{Q}_c(\boldsymbol{\beta}) \quad (5.162)$$

- **Consistence with stochastic dominance**

$$Q_{\Psi_{\boldsymbol{\alpha}}}(p) \geq Q_{\Psi_{\boldsymbol{\beta}}}(p) \text{ for all } p \in (0,1) \Rightarrow \mathrm{Q}_c(\boldsymbol{\alpha}) \geq \mathrm{Q}_c(\boldsymbol{\beta}) \quad (5.161)$$

- **Constancy**

$$\Psi_{\mathbf{b}} = \psi_{\mathbf{b}} \Rightarrow \mathrm{Q}_c(\mathbf{b}) = \psi_{\mathbf{b}} \quad (5.163)$$

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

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- **Positive homogeneity**

$$\mathrm{Q}_c(\lambda\boldsymbol{\alpha}) = \lambda\,\mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.164)$$

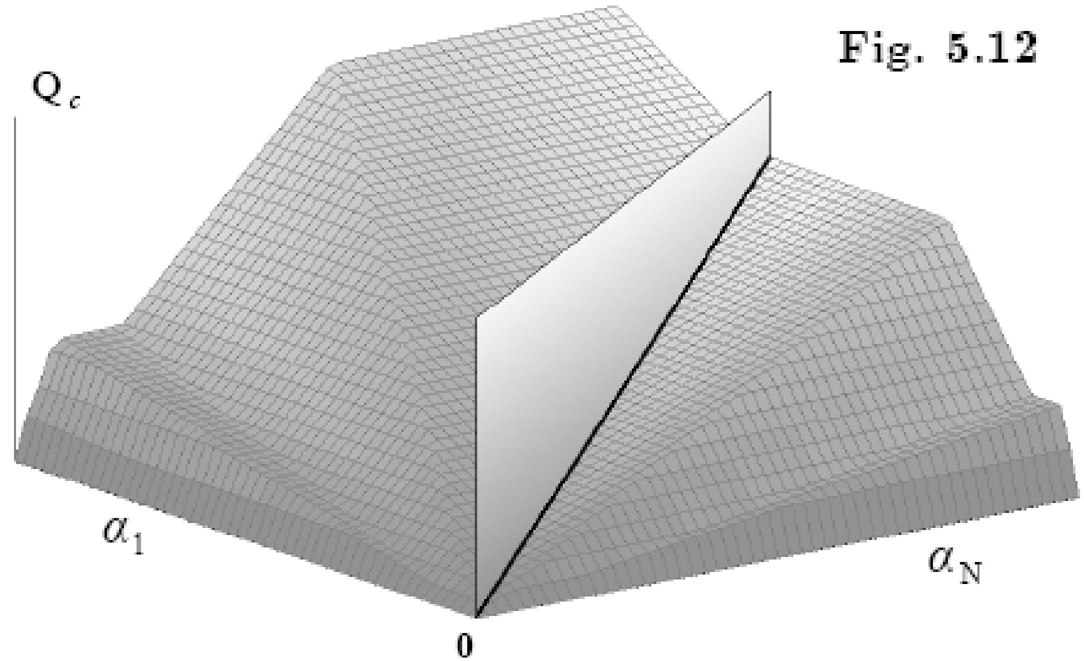

Fig. 5.12

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$$\mathrm{Q}_c(\lambda\boldsymbol{\alpha}) = \lambda\,\mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.164)$$

$$\Downarrow$$

Euler:

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \boldsymbol{\alpha}'\frac{\partial\,\mathrm{Q}_c(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}} \quad (5.188)$$

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \mu_{\boldsymbol{\alpha}} + \sigma_{\boldsymbol{\alpha}}\,\mathrm{erf}^{-1}(1-2c) \quad (5.175)$$

$$\Psi_{\boldsymbol{\alpha}} \sim \mathrm{N}\left(\mu_{\boldsymbol{\alpha}}, \sigma^2_{\boldsymbol{\alpha}}\right) \quad (5.173)$$

$$\begin{cases} \mu_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\boldsymbol{\mu} - \mathbf{P}_T) \\ \sigma^2_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha} \end{cases} \quad (5.174)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.172)$$

$$\frac{\partial\,\mathrm{Q}_c(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}} = \boldsymbol{\mu} - \mathbf{p}_T + \frac{\boldsymbol{\Sigma}\boldsymbol{\alpha}}{\sqrt{\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}}}\sqrt{2}\,\mathrm{erf}^{-1}(1-2c) \quad (5.189)$$

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$$\left\{\begin{array}{l} \mu_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\boldsymbol{\mu} - \mathbf{P}_T) \\ \sigma^2_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha} \end{array}\right. \quad (5.174)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.172)$$

- **Positive homogeneity**

$$\mathrm{Q}_c(\lambda\boldsymbol{\alpha}) = \lambda\,\mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.164)$$

$$\Downarrow$$

Euler:

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \boldsymbol{\alpha}'\frac{\partial\,\mathrm{Q}_c(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}} \quad (5.188)$$

$$\frac{\partial\,\mathrm{Q}_c(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}} = \boldsymbol{\mu} - \mathbf{p}_T + \frac{\boldsymbol{\Sigma}\boldsymbol{\alpha}}{\sqrt{\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}}}\sqrt{2}\,\mathrm{erf}^{-1}(1-2c) \quad (5.189)$$

$$\boxed{\frac{\partial\,\mathrm{Q}_c(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}} = \mathrm{E}\left\{\mathrm{M}\,|\,\boldsymbol{\alpha}'\mathrm{M} = \mathrm{Q}_c(\boldsymbol{\alpha})\right\} \quad (5.190)}$$

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$$\Psi_{\boldsymbol{\alpha}} \sim \mathrm{N}\left(\mu_{\boldsymbol{\alpha}}, \sigma^2_{\boldsymbol{\alpha}}\right) \quad (5.173)$$

$$\left\{\begin{array}{l} \mu_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\boldsymbol{\mu} - \mathbf{P}_T) \\ \sigma^2_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha} \end{array}\right. \quad (5.174)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.172)$$

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$$\boldsymbol{\alpha} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \equiv Q_{\Psi_{\boldsymbol{\alpha}}}(1-c) \quad (5.159)$$

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \mu_{\boldsymbol{\alpha}} + \sigma_{\boldsymbol{\alpha}}\,\mathrm{erf}^{-1}(1-2c) \quad (5.175)$$

$$\Psi_{\boldsymbol{\alpha}} \sim \mathrm{N}\left(\mu_{\boldsymbol{\alpha}}, \sigma^2_{\boldsymbol{\alpha}}\right) \quad (5.173)$$

$$\left\{\begin{array}{l} \mu_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\boldsymbol{\mu} - \mathbf{P}_T) \\ \sigma^2_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha} \end{array}\right. \quad (5.174)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.172)$$

- **Positive homogeneity**

$$\mathrm{Q}_c(\lambda\boldsymbol{\alpha}) = \lambda\,\mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.164)$$

- **Translation invariance**

$$\Psi_{\mathbf{b}} \equiv 1 \Rightarrow \mathrm{Q}_c(\boldsymbol{\alpha} + \lambda\mathbf{b}) = \mathrm{Q}_c(\boldsymbol{\alpha}) + \lambda \quad (5.165)$$

- ~~**Concavity**~~

$$\frac{\partial^2 \mathrm{Q}_c(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}'\partial\boldsymbol{\alpha}} = -\left.\frac{\partial \ln f_{\Psi_{\boldsymbol{\alpha}}}(\psi)}{\partial\psi}\right|_{\psi=\mathrm{Q}_c(\boldsymbol{\alpha})} \mathrm{Cov}\{\mathbf{M}|\Psi_{\boldsymbol{\alpha}} = \mathrm{Q}_c(\boldsymbol{\alpha})\} - \left.\frac{\partial\,\mathrm{Cov}\{\mathbf{M}|\Psi_{\boldsymbol{\alpha}} = \psi\}}{\partial\psi}\right|_{\psi=\mathrm{Q}_c(\boldsymbol{\alpha})} \quad (5.191)$$

INVESTOR'S OBJECTIVES EVALUATION: QUANTILE & VaR

Risk and Asset Allocation **- Springer –** `symmys.com`

$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \equiv Q_{\Psi_{\boldsymbol{\alpha}}}(1-c) \quad (5.159)$$

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \mu_{\boldsymbol{\alpha}} + \sigma_{\boldsymbol{\alpha}}\,\mathrm{erf}^{-1}(1-2c) \quad (5.175)$$

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$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

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- ~~super- additivity~~

- ~~Concavity~~

- ~~Risk aversion/propensity/neutrality~~

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}' \mathrm{M}} \quad (5.10)$$

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$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.172)$$

- Positive homogeneity

$$\mathrm{Q}_c(\lambda\boldsymbol{\alpha}) = \lambda \mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.164)$$

- Translation invariance

$$\Psi_{\mathbf{b}} \equiv 1 \Rightarrow \mathrm{Q}_c(\boldsymbol{\alpha} + \lambda\mathbf{b}) = \mathrm{Q}_c(\boldsymbol{\alpha}) + \lambda \quad (5.165)$$

- super- additivity

- Co-monotonic additivity

$$(\boldsymbol{\alpha}, \boldsymbol{\delta}) \text{ co-monotonic } \Rightarrow \mathrm{Q}_c(\boldsymbol{\alpha} + \boldsymbol{\delta}) = \mathrm{Q}_c(\boldsymbol{\alpha}) + \mathrm{Q}_c(\boldsymbol{\delta}) \quad (5.167)$$

- Concavity

- Risk aversion/propensity/neutrality

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The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

conditional excess function

$$L_{\widetilde{\psi}}(z) \equiv \mathbb{P}\left\{X \leq \widetilde{\psi} - z \mid X \leq \widetilde{\psi}\right\} = \frac{F_X\left(\widetilde{\psi} - z\right)}{F_X\left(\widetilde{\psi}\right)} \tag{5.182}$$

EXTREME VALUE THEORY

Risk and Asset Allocation **- Springer –** `symmys.com`

conditional excess function

$$L_{\widetilde{\psi}}(z) \equiv \mathbb{P}\left\{X \leq \widetilde{\psi} - z \mid X \leq \widetilde{\psi}\right\} = \frac{F_X\left(\widetilde{\psi} - z\right)}{F_X\left(\widetilde{\psi}\right)} \tag{5.182}$$

generalized Pareto cumulative distribution function

$$G_{\xi,v}(z) \equiv 1 - \left(1 + \frac{\xi}{v} z\right)^{-1/\xi} \tag{5.183}$$

EXTREME VALUE THEORY

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conditional excess function

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Pickands (1975) and Balkema and De Haan (1974)

$$\boxed{1 - L_{\widetilde{\psi}}(z) \approx G_{\xi,v}(z)} \quad (5.184)$$

EXTREME VALUE THEORY

conditional excess function

$$L_{\widetilde{\psi}}(z) \equiv \mathbb{P}\left\{X \leq \widetilde{\psi} - z \mid X \leq \widetilde{\psi}\right\} = \frac{F_X\left(\widetilde{\psi} - z\right)}{F_X\left(\widetilde{\psi}\right)} \qquad (5.182)$$

generalized Pareto cumulative distribution function

$$G_{\xi,v}(z) \equiv 1 - \left(1 + \frac{\xi}{v} z\right)^{-1/\xi} \qquad (5.183)$$

Pickands (1975) and Balkema and De Haan (1974)

$$\boxed{1 - L_{\widetilde{\psi}}(z) \approx G_{\xi,v}(z)} \qquad (5.184)$$

$$\mathrm{Q}_c(\boldsymbol{\alpha}) \approx \widetilde{\psi} + \frac{v(\boldsymbol{\alpha})}{\xi(\boldsymbol{\alpha})}\left[1 - \left(\frac{1-c}{F_{\Psi_{\boldsymbol{\alpha}}}\left(\widetilde{\psi}\right)}\right)^{-\xi(\boldsymbol{\alpha})}\right] \qquad (5.186)$$

CORNISH-FISHER EXPANSION

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CORNISH-FISHER EXPANSION

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$$Q_X(p) = \mathrm{E}\{X\} + \mathrm{Sd}\{X\}\left[z(p) + \frac{1}{6}\left(z^2(p) - 1\right)\mathrm{Sk}\{X\}\right] + \cdots \tag{5.179}$$

quantile of <u>generic</u> distribution

CORNISH-FISHER EXPANSION

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$$Q_X(p) = \mathrm{E}\{X\} + \mathrm{Sd}\{X\}\left[z(p) + \frac{1}{6}\left(z^2(p) - 1\right)\mathrm{Sk}\{X\}\right] + \cdots \qquad (5.179)$$

quantile of <u>generic</u> distribution

powers of quantile of standard normal distribution

$$z(p) \equiv \sqrt{2}\,\mathrm{erf}^{-1}(2p-1) \qquad (5.178)$$

CORNISH-FISHER EXPANSION

***Risk and Asset Allocation* - Springer – `symmys.com`**

$$Q_X(p) = \boxed{\mathrm{E}\{X\}} + \boxed{\mathrm{Sd}\{X\}}\left[z(p) + \frac{1}{6}\left(z^2(p) - 1\right)\boxed{\mathrm{Sk}\{X\}}\right] + \cdots \qquad (5.179)$$

quantile of <u>generic</u> distribution

powers of quantile of standard normal distribution

$$z(p) \equiv \sqrt{2}\,\mathrm{erf}^{-1}(2p-1) \qquad (5.178)$$

$$\boxed{\mathrm{CM}^{\mathbf{a}+\mathbf{BX}}_{m_1\cdots m_k}} = \sum_{n_1,\ldots,n_k=1}^{N} B_{m_1,n_1}\cdots B_{m_k,n_k}\,\mathrm{CM}^{\mathbf{X}}_{n_1\cdots n_k} \qquad (2.93)$$

CORNISH-FISHER EXPANSION

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$$Q_X(p) = \boxed{\mathrm{E}\{X\}} + \boxed{\mathrm{Sd}\{X\}}\left[z(p) + \frac{1}{6}\left(z^2(p) - 1\right)\boxed{\mathrm{Sk}\{X\}}\right] + \cdots \quad (5.179)$$

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examples

$$k=3: \quad \mathrm{Sk}\{X_l, X_m, X_n\} \equiv [\mathrm{Sk}\{\mathbf{X}\}]_{lmn} \equiv \frac{\mathrm{CM}^{\mathbf{X}}_{lmn}}{\mathrm{Sd}\{X_l\}\,\mathrm{Sd}\{X_m\}\,\mathrm{Sd}\{X_n\}} \quad (2.95)$$

$$k=4: \quad \mathrm{Ku}\{X_l, X_m, X_n, X_p\} \equiv [\mathrm{Ku}\{\mathbf{X}\}]_{lmnp} \equiv \frac{\mathrm{CM}^{\mathbf{X}}_{lmnp}}{\mathrm{Sd}\{X_l\}\,\mathrm{Sd}\{X_m\}\,\mathrm{Sd}\{X_n\}\,\mathrm{Sd}\{X_p\}} \quad (2.96)$$

INVESTOR'S OBJECTIVES EVALUATION: COHERENT MEASURES

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INVESTOR'S OBJECTIVES EVALUATION: COHERENT MEASURES

Risk and Asset Allocation - Springer – `symmys.com`

$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \tag{5.10}$$

$$\mathrm{Coh}\left(\boldsymbol{\alpha}\right) \equiv \mathrm{E}\left\{\Psi_{\boldsymbol{\alpha}}\right\} - \gamma \left\|\min\left(0, \Psi_{\boldsymbol{\alpha}} - \mathrm{E}\left\{\Psi_{\boldsymbol{\alpha}}\right\}\right)\right\|_{\mathbf{M};p} . \tag{5.198}$$

- **Sensibility**

$$\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios} \Rightarrow \mathrm{Coh}\left(\boldsymbol{\alpha}\right) \geq \mathrm{Coh}\left(\boldsymbol{\beta}\right) \tag{5.193}$$

- **Positive homogeneity**

$$\mathrm{Coh}\left(\lambda\boldsymbol{\alpha}\right) = \lambda\,\mathrm{Coh}\left(\boldsymbol{\alpha}\right) \tag{5.194}$$

- **Translation invariance**

$$\Psi_{\mathbf{b}} \equiv 1 \Rightarrow \mathrm{Coh}\left(\boldsymbol{\alpha} + \lambda\mathbf{b}\right) = \mathrm{Coh}\left(\boldsymbol{\alpha}\right) + \lambda. \tag{5.195}$$

- **super- additivity**

$$\mathrm{Coh}\left(\boldsymbol{\alpha} + \boldsymbol{\beta}\right) \geq \mathrm{Coh}\left(\boldsymbol{\alpha}\right) + \mathrm{Coh}\left(\boldsymbol{\beta}\right) \tag{5.197}$$

INVESTOR'S OBJECTIVES EVALUATION: COHERENT MEASURES

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\operatorname{Coh}(\boldsymbol{\alpha}) \equiv \operatorname{E}\{\Psi_{\boldsymbol{\alpha}}\} - \gamma \left\| \min\left(0, \Psi_{\boldsymbol{\alpha}} - \operatorname{E}\{\Psi_{\boldsymbol{\alpha}}\}\right) \right\|_{\mathbf{M};p} \quad (5.198)$$

- Money-equivalence

- **Sensibility**

$$\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios} \Rightarrow \operatorname{Coh}(\boldsymbol{\alpha}) \geq \operatorname{Coh}(\boldsymbol{\beta}) \quad (5.193)$$

- **Positive homogeneity**

$$\operatorname{Coh}(\lambda\boldsymbol{\alpha}) = \lambda \operatorname{Coh}(\boldsymbol{\alpha}) \quad (5.194)$$

- **Translation invariance**

$$\Psi_{\mathbf{b}} \equiv 1 \Rightarrow \operatorname{Coh}(\boldsymbol{\alpha} + \lambda\mathbf{b}) = \operatorname{Coh}(\boldsymbol{\alpha}) + \lambda. \quad (5.195)$$

- **super- additivity**

$$\operatorname{Coh}(\boldsymbol{\alpha} + \boldsymbol{\beta}) \geq \operatorname{Coh}(\boldsymbol{\alpha}) + \operatorname{Coh}(\boldsymbol{\beta}) \quad (5.197)$$

- Concavity

$$\operatorname{Coh}(\lambda\boldsymbol{\alpha} + (1-\lambda)\boldsymbol{\beta}) \geq \lambda \operatorname{Coh}(\boldsymbol{\alpha}) + (1-\lambda)\operatorname{Coh}(\boldsymbol{\beta}) \quad (5.200)$$

INVESTOR'S OBJECTIVES EVALUATION: SPECTRAL MEASURES

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\operatorname{Spc}(\boldsymbol{\alpha}) \equiv \operatorname{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.203)$$

- Money-equivalence

- Estimability

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto \left(f_{\Psi_{\boldsymbol{\alpha}}}, F_{\Psi_{\boldsymbol{\alpha}}}, \phi_{\Psi_{\boldsymbol{\alpha}}}\right) \mapsto \operatorname{Spc}(\boldsymbol{\alpha}) \quad (5.201)$$

- Sensibility

$$\begin{aligned} &\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios} \\ &\quad \Rightarrow \operatorname{Coh}(\boldsymbol{\alpha}) \geq \operatorname{Coh}(\boldsymbol{\beta}) \end{aligned} \quad (5.193)$$

- Positive homogeneity

$$\operatorname{Coh}(\lambda\boldsymbol{\alpha}) = \lambda \operatorname{Coh}(\boldsymbol{\alpha}) \quad (5.194)$$

- Translation invariance

$$\Psi_{\mathbf{b}} \equiv 1 \Rightarrow \operatorname{Coh}(\boldsymbol{\alpha} + \lambda\mathbf{b}) = \operatorname{Coh}(\boldsymbol{\alpha}) + \lambda. \quad (5.195)$$

- super- additivity

$$\operatorname{Coh}(\boldsymbol{\alpha} + \boldsymbol{\beta}) \geq \operatorname{Coh}(\boldsymbol{\alpha}) + \operatorname{Coh}(\boldsymbol{\beta}) \quad (5.197)$$

- Co-monotonic additivity

$$\begin{aligned} &(\boldsymbol{\alpha}, \boldsymbol{\delta}) \text{ co-monotonic} \\ &\quad \Rightarrow \operatorname{Spc}(\boldsymbol{\alpha} + \boldsymbol{\delta}) = \operatorname{Spc}(\boldsymbol{\alpha}) + \operatorname{Spc}(\boldsymbol{\delta}) \end{aligned} \quad (5.202)$$

- Concavity

$$\begin{aligned} &\operatorname{Coh}(\lambda\boldsymbol{\alpha} + (1-\lambda)\boldsymbol{\beta}) \geq \\ &\quad \lambda \operatorname{Coh}(\boldsymbol{\alpha}) + (1-\lambda)\operatorname{Coh}(\boldsymbol{\beta}) \end{aligned} \quad (5.200)$$

INVESTOR'S OBJECTIVES EVALUATION: SPECTRAL MEASURES

Risk and Asset Allocation* - Springer – *symmys.com

$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

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- **Money-equivalence**

- **Estimability**

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto \left(f_{\Psi_{\boldsymbol{\alpha}}}, F_{\Psi_{\boldsymbol{\alpha}}}, \phi_{\Psi_{\boldsymbol{\alpha}}}\right) \mapsto \mathrm{Spc}(\boldsymbol{\alpha}) \quad (5.201)$$

- **Sensibility**

$$\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios} \Rightarrow \mathrm{Coh}(\boldsymbol{\alpha}) \geq \mathrm{Coh}(\boldsymbol{\beta}) \quad (5.193)$$

- **Consistence with weak stochastic dominance**

$$Q_{\Psi_{\boldsymbol{\alpha}}}(p) \geq Q_{\Psi_{\boldsymbol{\beta}}}(p) \text{ for all } p \in (0,1) \Rightarrow \mathrm{Spc}(\boldsymbol{\alpha}) \geq \mathrm{Spc}(\boldsymbol{\beta}) \quad (5.204)$$

- **Constancy**

$$\Psi_{\mathbf{b}} \equiv \psi_{\mathbf{b}} \Rightarrow \mathrm{Spc}(\mathbf{b}) = \psi_{\mathbf{b}}. \quad (5.205)$$

- **Positive homogeneity**

$$\mathrm{Coh}(\lambda\boldsymbol{\alpha}) = \lambda\,\mathrm{Coh}(\boldsymbol{\alpha}) \quad (5.194)$$

- **Translation invariance**

$$\Psi_{\mathbf{b}} \equiv 1 \Rightarrow \mathrm{Coh}(\boldsymbol{\alpha} + \lambda\mathbf{b}) = \mathrm{Coh}(\boldsymbol{\alpha}) + \lambda. \quad (5.195)$$

- **super- additivity**

$$\mathrm{Coh}(\boldsymbol{\alpha} + \boldsymbol{\beta}) \geq \mathrm{Coh}(\boldsymbol{\alpha}) + \mathrm{Coh}(\boldsymbol{\beta}) \quad (5.197)$$

- **Co-monotonic additivity**

$$(\boldsymbol{\alpha}, \boldsymbol{\delta}) \text{ co-monotonic} \Rightarrow \mathrm{Spc}(\boldsymbol{\alpha} + \boldsymbol{\delta}) = \mathrm{Spc}(\boldsymbol{\alpha}) + \mathrm{Spc}(\boldsymbol{\delta}) \quad (5.202)$$

- **Concavity**

$$\mathrm{Coh}(\lambda\boldsymbol{\alpha} + (1-\lambda)\boldsymbol{\beta}) \geq \lambda\,\mathrm{Coh}(\boldsymbol{\alpha}) + (1-\lambda)\,\mathrm{Coh}(\boldsymbol{\beta}) \quad (5.200)$$

- **Risk aversion**,

$$\mathrm{RP}(\boldsymbol{\alpha}) \geq 0.$$

INVESTOR'S OBJECTIVES EVALUATION: EXPECTED SHORTFALL

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}' \mathbf{M}} \tag{5.10}$$

$$\boxed{\mathrm{E}\left\{\Psi_{\boldsymbol{\alpha}}\right\} = \int_{\mathbb{R}} \psi f_{\Psi_{\boldsymbol{\alpha}}}\left(\psi\right) d\psi \;\; = \int_0^1 Q_{\Psi_{\boldsymbol{\alpha}}}\left(s\right) ds} \tag{5.206}$$

INVESTOR'S OBJECTIVES EVALUATION: EXPECTED SHORTFALL

Risk and Asset Allocation **- Springer –** `symmys.com`

$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boxed{\mathrm{E}\left\{\Psi_{\boldsymbol{\alpha}}\right\} = \int_{\mathbb{R}} \psi f_{\Psi_{\boldsymbol{\alpha}}}(\psi)\, d\psi = \int_0^1 Q_{\Psi_{\boldsymbol{\alpha}}}(s)\, ds} \quad (5.206)$$

$$\boxed{\mathrm{ES}_c(\boldsymbol{\alpha}) \equiv \frac{1}{1-c}\int_0^{1-c} Q_{\Psi_{\boldsymbol{\alpha}}}(s)\, ds.} \quad (5.207)$$

$$\Updownarrow$$

$$\mathrm{ES}_c(\boldsymbol{\alpha}) = \mathrm{TCE}_c(\boldsymbol{\alpha}) = \mathrm{CVaR}_c \equiv \mathrm{E}\left\{\Psi_{\boldsymbol{\alpha}} \middle| \Psi_{\boldsymbol{\alpha}} \leq \mathrm{Q}_c(\boldsymbol{\alpha})\right\} \quad (5.208)$$

INVESTOR'S OBJECTIVES EVALUATION: EXPECTED SHORTFALL

Risk and Asset Allocation **- Springer –** `symmys.com`

$$\boxed{\Psi_{\alpha} = \alpha' \mathrm{M}} \quad (5.10)$$

$$\mathrm{ES}_c(\alpha) \equiv \frac{1}{1-c} \int_0^{1-c} Q_{\Psi_{\alpha}}(s)\, ds. \quad (5.207)$$

- Money-equivalence
- $\boxed{\text{Estimability}}$
- $\boxed{\text{Sensibility}}$
- Consistence with weak stochastic dominance
- Constancy
- $\boxed{\text{Positive homogeneity}}$
- $\boxed{\text{Translation invariance}}$
- $\boxed{\text{super- additivity}}$
- $\boxed{\text{Co-monotonic additivity}}$
- Concavity
- Risk aversion

INVESTOR'S OBJECTIVES EVALUATION: EXPECTED SHORTFALL

Risk and Asset Allocation - Springer – symmys.com

$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\mathrm{ES}_c(\boldsymbol{\alpha}) \equiv \frac{1}{1-c}\int_0^{1-c} Q_{\Psi_{\boldsymbol{\alpha}}}(s)\,ds \quad (5.207)$$

- **Positive homogeneity**

$$\mathrm{ES}_c(\lambda\boldsymbol{\alpha}) = \lambda\,\mathrm{ES}_c(\boldsymbol{\alpha}) \quad (5.210)$$

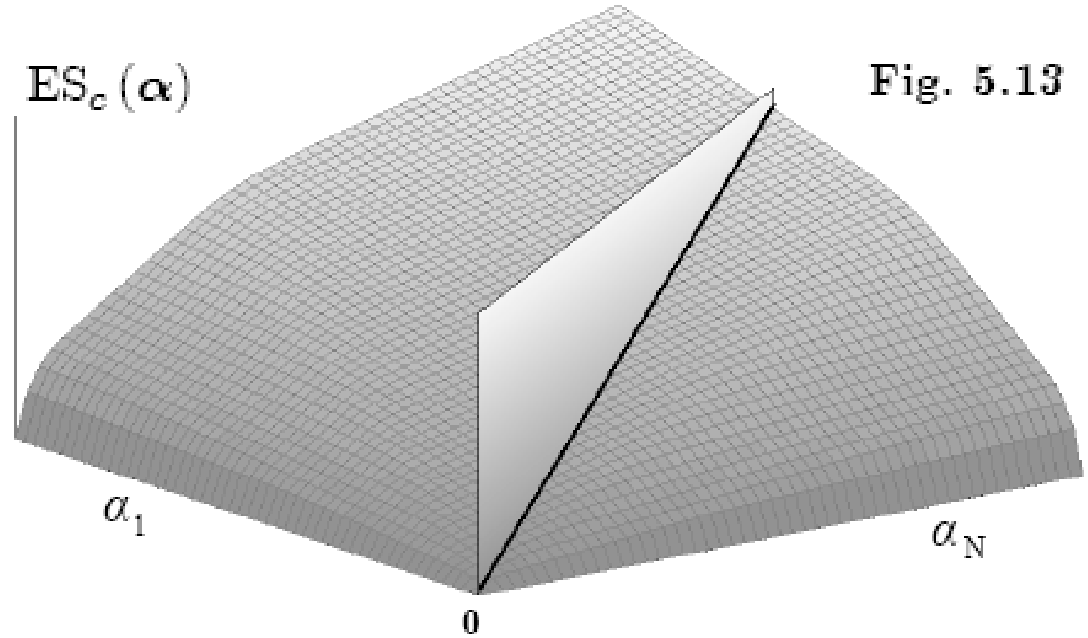

Fig. 5.13

INVESTOR'S OBJECTIVES EVALUATION: EXPECTED SHORTFALL

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$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}' \mathbf{M}}$ (5.10)

$$\mathrm{ES}_c(\boldsymbol{\alpha}) \equiv \frac{1}{1-c} \int_0^{1-c} Q_{\Psi_{\boldsymbol{\alpha}}}(s)\, ds, \quad (5.207)$$

- **Positive homogeneity**

$$\mathrm{ES}_c(\lambda \boldsymbol{\alpha}) = \lambda\, \mathrm{ES}_c(\boldsymbol{\alpha}) \quad (5.210)$$

$$\Downarrow$$

Euler:

$$\mathrm{ES}_c(\boldsymbol{\alpha}) = \boldsymbol{\alpha}' \frac{\partial\, \mathrm{ES}_c}{\partial \boldsymbol{\alpha}} \quad (5.239)$$

$$\boxed{\frac{\partial\, \mathrm{ES}_c}{\partial \boldsymbol{\alpha}} = \mathrm{E}\left\{\mathbf{M} \mid \boldsymbol{\alpha}' \mathbf{M} \leq \mathrm{Q}_c(\boldsymbol{\alpha})\right\} \quad (5.238)}$$

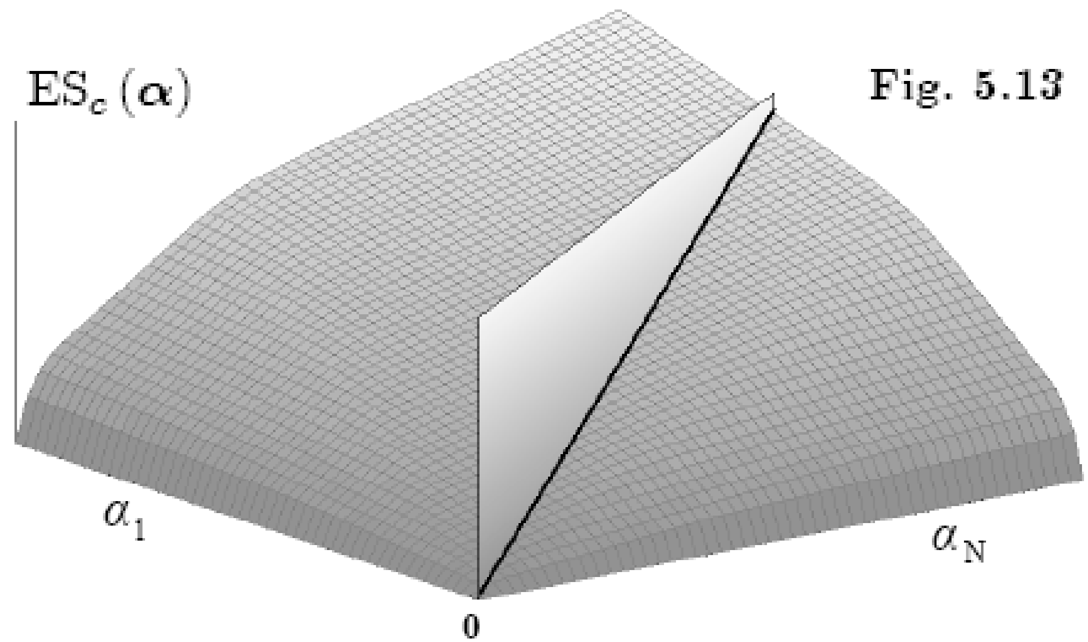

Fig. 5.13

INVESTOR'S OBJECTIVES EVALUATION: ES & SPECTRAL MEASURES

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boxed{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} = \int_{\mathbb{R}} \psi f_{\Psi_{\boldsymbol{\alpha}}}(\psi)\, d\psi = \int_0^1 Q_{\Psi_{\boldsymbol{\alpha}}}(s)\, ds} \quad (5.206)$$

$$\boxed{\mathrm{ES}_c(\boldsymbol{\alpha}) \equiv \frac{1}{1-c}\int_0^{1-c} Q_{\Psi_{\boldsymbol{\alpha}}}(s)\, ds} \quad (5.207)$$

$$\Updownarrow$$

$$\mathrm{ES}_c(\boldsymbol{\alpha}) = \mathrm{TCE}_c(\boldsymbol{\alpha}) = \mathrm{CVaR}_c \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}} | \Psi_{\boldsymbol{\alpha}} \leq \mathrm{Q}_c(\boldsymbol{\alpha})\} \quad (5.208)$$

$$\boxed{\mathrm{Spc}_{\varphi}(\boldsymbol{\alpha}) \equiv \int_0^1 \varphi(p)\, Q_{\Psi_{\boldsymbol{\alpha}}}(p)\, dp,} \quad (5.216)$$

φ decreasing

$$\varphi(1) \equiv 0 \quad (5.217)$$

$$\int_0^1 \varphi(p)\, dp \equiv 1$$

INVESTOR'S OBJECTIVES EVALUATION: ES & SPECTRAL MEASURES

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boxed{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} = \int_{\mathbb{R}} \psi f_{\Psi_{\boldsymbol{\alpha}}}(\psi)\, d\psi = \int_0^1 Q_{\Psi_{\boldsymbol{\alpha}}}(s)\, ds} \quad (5.206)$$

$$\boxed{\mathrm{ES}_c(\boldsymbol{\alpha}) \equiv \frac{1}{1-c}\int_0^{1-c} Q_{\Psi_{\boldsymbol{\alpha}}}(s)\, ds} \quad (5.207)$$

$$\Updownarrow$$

$$\mathrm{ES}_c(\boldsymbol{\alpha}) = \mathrm{TCE}_c(\boldsymbol{\alpha}) = \mathrm{CVaR}_c \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}} | \Psi_{\boldsymbol{\alpha}} \leq \mathrm{Q}_c(\boldsymbol{\alpha})\} \quad (5.208)$$

$$\boxed{\mathrm{Spc}_{\varphi}(\boldsymbol{\alpha}) \equiv \int_0^1 \varphi(p)\, Q_{\Psi_{\boldsymbol{\alpha}}}(p)\, dp,} \quad (5.216)$$

$$\varphi \text{ decreasing}$$
$$\varphi(1) \equiv 0 \quad (5.217)$$
$$\int_0^1 \varphi(p)\, dp \equiv 1$$

$$\varphi_{\mathrm{ES}_c}(p) \equiv \frac{H^{(c-1)}(-p)}{1-c} \quad (5.218)$$

INVESTOR'S OBJECTIVES EVALUATION: VaR & SPECTRAL MEASURES

Risk and Asset Allocation - Springer – `symmys.com`

$$\boxed{\Psi_{\alpha} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\mathrm{E}\{\Psi_{\alpha}\} = \int_{\mathbb{R}} \psi f_{\Psi_{\alpha}}(\psi)\, d\psi = \int_0^1 Q_{\Psi_{\alpha}}(s)\, ds \quad (5.206)$$

$$\mathrm{ES}_c(\boldsymbol{\alpha}) \equiv \frac{1}{1-c}\int_0^{1-c} Q_{\Psi_{\alpha}}(s)\, ds. \quad (5.207)$$

$$\Updownarrow$$

$$\mathrm{ES}_c(\boldsymbol{\alpha}) = \mathrm{TCE}_c(\boldsymbol{\alpha}) = \mathrm{CVaR}_c \equiv \mathrm{E}\{\Psi_{\alpha} | \Psi_{\alpha} \leq \mathrm{Q}_c(\boldsymbol{\alpha})\} \quad (5.208)$$

$$\mathrm{Spc}_{\varphi}(\boldsymbol{\alpha}) \equiv \int_0^1 \varphi(p)\, Q_{\Psi_{\alpha}}(p)\, dp, \quad (5.216)$$

φ decreasing,

$$\varphi(1) \equiv 0, \quad (5.217)$$

$$\int_0^1 \varphi(p)\, dp \equiv 1.$$

$$\varphi_{\mathrm{ES}_c}(p) \equiv \frac{H^{(c-1)}(-p)}{1-c} \quad (5.218)$$

$$\varphi_{\mathrm{Q}_c} \equiv \delta^{(1-c)} \quad (5.219)$$