Attilio Meucci

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Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

$$\boldsymbol{\alpha}^* \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \mathcal{S}\left(\boldsymbol{\alpha}\right) \right\} \qquad (8.1)$$

$$\mathrm{OC}\left(\boldsymbol{\alpha}\right) \equiv \mathcal{S}\left(\boldsymbol{\alpha}^*\right) - \mathcal{S}\left(\boldsymbol{\alpha}\right) \qquad (8.11)$$

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$$\boldsymbol{\alpha}^* \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}}{\operatorname{argmax}} \{\mathcal{S}(\boldsymbol{\alpha})\} \qquad (8.1)$$

$$\mathcal{C}^+(\boldsymbol{\alpha}) = \max\left\{0, \widetilde{s} - \widetilde{\mathcal{S}}(\boldsymbol{\alpha})\right\} \qquad (8.13)$$

$$\mathrm{OC}(\boldsymbol{\alpha}) \equiv \mathcal{S}(\boldsymbol{\alpha}^*) - \mathcal{S}(\boldsymbol{\alpha}) + \mathcal{C}^+(\boldsymbol{\alpha}) \qquad (8.16)$$

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$$\mathbf{L}_t^{\boldsymbol{\mu},\boldsymbol{\Sigma}} \sim \mathrm{N}(\boldsymbol{\mu},\boldsymbol{\Sigma}) \quad (8.19)$$

$$\boldsymbol{\theta} \overset{(3.64)}{\mapsto} \mathbf{X}_{T+\tau}^{\boldsymbol{\theta}}$$

$$\boldsymbol{\alpha}^* \equiv \underset{\boldsymbol{\alpha}\in\mathcal{C}}{\operatorname{argmax}} \left\{\mathcal{S}(\boldsymbol{\alpha})\right\} \qquad (8.1)$$

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$$\mathbf{L}_t^{\mu,\Sigma} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (8.19)$$

$$\mathbf{P}_{T+\tau}^{\mu,\Sigma} \sim \mathrm{N}(\boldsymbol{\xi}(\boldsymbol{\mu}), \boldsymbol{\Phi}(\boldsymbol{\Sigma})) \quad (8.20)$$

$$\boldsymbol{\xi}(\boldsymbol{\mu}) \equiv \mathrm{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) \quad (8.21) \quad \boldsymbol{\Phi}(\boldsymbol{\Sigma}) \equiv \mathrm{diag}(\mathbf{p}_T)\boldsymbol{\Sigma}\,\mathrm{diag}(\mathbf{p}_T)$$

$$\boldsymbol{\theta} \overset{(3.64)}{\mapsto} \mathbf{X}_{T+\tau}^{\boldsymbol{\theta}} \overset{(3.79)}{\mapsto} \mathbf{P}_{T+\tau}^{\boldsymbol{\theta}}. \quad (8.17)$$

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$$\Psi_{\boldsymbol{\alpha}}^{\boldsymbol{\mu},\boldsymbol{\Sigma}} \equiv \boldsymbol{\alpha}'\mathbf{P}_{T+\tau}^{\boldsymbol{\mu},\boldsymbol{\Sigma}} \quad (8.24)$$

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$$\left(\boldsymbol{\alpha},\mathbf{P}_{T+\tau}^{\boldsymbol{\theta}}\right) \overset{(5.10)\text{-}(5.15)}{\mapsto} \Psi_{\boldsymbol{\alpha}}^{\boldsymbol{\theta}}$$

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$$\begin{aligned}\mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = {} & \boldsymbol{\alpha}'\,\mathrm{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) \\ & -\frac{1}{2\zeta}\boldsymbol{\alpha}'\,\mathrm{diag}(\mathbf{p}_T)\,\boldsymbol{\Sigma}\,\mathrm{diag}(\mathbf{p}_T)\,\boldsymbol{\alpha}\end{aligned} \quad (8.25)$$

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$$\overline{\mathrm{CE}}(\boldsymbol{\mu},\boldsymbol{\Sigma}) = \frac{\zeta}{2}\left(C - \frac{B^2}{A}\right) + w_T\left(1 + \frac{B}{A} - \frac{w_T}{\zeta}\frac{1}{2A}\right) \quad (8.33)$$

$$A \equiv \mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}, \quad B \equiv \mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}, \quad C \equiv \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}. \quad (8.34)$$

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In our example the opportunity cost of a generic allocation $\boldsymbol{\alpha}$ that satisfies the budget constraint is the difference between the optimal level of satisfaction (8.33) and the satisfaction provided by the generic allocation (8.25).

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \qquad (8.37)$$

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??

$$\left(\boldsymbol{\alpha},\mathbf{P}_{T+\tau}^{\boldsymbol{\theta}}\right) \overset{(5.10)\text{-}(5.15)}{\longmapsto} \Psi_{\boldsymbol{\alpha}}^{\boldsymbol{\theta}} \overset{(5.52)}{\longmapsto} \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \quad (8.23)$$

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$$\boldsymbol{\alpha}[\cdot]: \; i_T \mapsto \mathbb{R}^N \quad (8.38)$$

$$i_T \equiv \{\mathbf{x}_1, \ldots, \mathbf{x}_T\} \quad (8.40)$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = [\operatorname{diag}(\mathbf{p}_T)]^{-1} \boldsymbol{\Sigma}^{-1} \left(\zeta \boldsymbol{\mu} + \frac{w_T - \zeta \mathbf{1}' \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}}{\mathbf{1}' \boldsymbol{\Sigma}^{-1} \mathbf{1}} \mathbf{1} \right) (8.32)$$

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$$\boldsymbol{\alpha}[\cdot]: \; i_T \mapsto \mathbb{R}^N \quad (8.38)$$

$$i_T \equiv \{\mathbf{x}_1, \ldots, \mathbf{x}_T\} \quad (8.40)$$

true value of the market parameters

$$\boldsymbol{\alpha}[i_T] \equiv \boldsymbol{\alpha}\left(\boldsymbol{\theta}^t\right) \quad (8.39)$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = [\operatorname{diag}(\mathbf{p}_T)]^{-1} \boldsymbol{\Sigma}^{-1} \left(\zeta \boldsymbol{\mu} + \frac{w_T - \zeta \mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}} \mathbf{1}\right) \quad (8.32)$$

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$$\boldsymbol{\alpha}[i_T] \equiv \boldsymbol{\alpha}\cancel{\left(\boldsymbol{\theta}^{\mathrm{t}}\right)} \qquad (8.39)$$

$$\boldsymbol{\theta}^{\mathrm{t}} \notin i_T$$

$$\boldsymbol{\alpha}(\cancel{\boldsymbol{\mu}}, \cancel{\boldsymbol{\Sigma}}) = [\operatorname{diag}(\mathbf{p}_T)]^{-1} \boldsymbol{\Sigma}^{-1} \left(\zeta \boldsymbol{\mu} + \frac{w_T - \zeta \mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}} \mathbf{1} \right) \quad (8.32)$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{ \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \qquad (8.30)$$

$$\overline{\mathrm{CE}}(\cancel{\boldsymbol{\mu}}, \cancel{\boldsymbol{\Sigma}}) = \frac{\zeta}{2}\left(C - \frac{B^2}{A} \right) + w_T \left(1 + \frac{B}{A} - \frac{w_T}{\zeta} \frac{1}{2A} \right) \quad (8.33)$$

$$A \equiv \mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}, \quad B \equiv \mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}, \quad C \equiv \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}. \quad (8.34)$$

$$\overline{\mathcal{S}}(\boldsymbol{\theta}) \equiv \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}(\boldsymbol{\theta})) \equiv \max_{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}} \left\{ \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \qquad (8.31)$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \qquad (8.37)$$

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$$\boldsymbol{\alpha}[\cdot] : \; i_T \mapsto \mathbb{R}^N \quad (8.38)$$

$$i_T \equiv \{\mathbf{x}_1, \ldots, \mathbf{x}_T\} \quad (8.40)$$

best performer

$$b \equiv \underset{n \in \{1,\ldots,N\}}{\operatorname{argmax}} \{l_{T,n}\} \quad (8.43)$$

$$\boldsymbol{\alpha}[i_T] \equiv w_T \frac{\boldsymbol{\delta}^{(b)}}{p_T^{(b)}} \quad (8.42)$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\} \quad (8.30)$$

$$\overline{\mathcal{S}}(\boldsymbol{\theta}) \equiv \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}(\boldsymbol{\theta})) \equiv \max_{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}} \{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\} \quad (8.31)$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \quad (8.37)$$

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$$\boldsymbol{\alpha}[i_T] \equiv w_T \frac{\boldsymbol{\delta}^{(b)}}{p_T^{(b)}} \quad (8.42)$$

equally-weighted portfolio

$$\boldsymbol{\alpha}_{\mathrm{p}} \equiv \frac{w_T}{N} \operatorname{diag}(\mathbf{p}_T)^{-1} \mathbf{1}, \quad (8.65)$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\} \quad (8.30)$$

$$\overline{\mathcal{S}}(\boldsymbol{\theta}) \equiv \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}(\boldsymbol{\theta})) \equiv \max_{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}} \{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\} \quad (8.31)$$

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$$b \equiv \underset{n \in \{1,\ldots,N\}}{\operatorname{argmax}} \{l_{T,n}\} \quad (8.43)$$

$$\boldsymbol{\alpha}[i_T] \equiv w_T \frac{\boldsymbol{\delta}^{(b)}}{p_T^{(b)}} \quad (8.42)$$

equally-weighted portfolio

$$\boldsymbol{\alpha}_{\mathrm{p}} \equiv \frac{w_T}{N} \operatorname{diag}(\mathbf{p}_T)^{-1} \mathbf{1}, \quad (8.65)$$

sample-based allocation

$$\boldsymbol{\alpha}_{\mathrm{s}} = [\operatorname{diag}(\mathbf{p}_T)]^{-1} \widehat{\boldsymbol{\Sigma}}^{-1} \left(\zeta \widehat{\boldsymbol{\mu}} + \frac{w_T - \zeta \mathbf{1}' \widehat{\boldsymbol{\Sigma}}^{-1} \widehat{\boldsymbol{\mu}}}{\mathbf{1}' \widehat{\boldsymbol{\Sigma}}^{-1} \mathbf{1}} \mathbf{1} \right) \quad (8.82)$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\} \quad (8.30)$$

$$\overline{\mathcal{S}}(\boldsymbol{\theta}) \equiv \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}(\boldsymbol{\theta})) \equiv \max_{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}} \{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\} \quad (8.31)$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \quad (8.37)$$

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$$\boldsymbol{\alpha}[\cdot]: \; i_T \mapsto \mathbb{R}^N \quad (8.38)$$

$$i_T \equiv \{\mathbf{x}_1, \ldots, \mathbf{x}_T\} \quad (8.40)$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\} \quad (8.30)$$

$$\overline{\mathcal{S}}(\boldsymbol{\theta}) \equiv \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}(\boldsymbol{\theta})) \equiv \max_{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}} \{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\} \quad (8.31)$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \quad (8.37)$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}[i_T]) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\underline{\boldsymbol{\alpha}[i_T]}) \quad (8.44)$$

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$$\boldsymbol{\alpha}[\cdot]: \ i_T \mapsto \mathbb{R}^N \quad (8.38)$$

$$i_T \equiv \{\mathbf{x}_1, \ldots, \mathbf{x}_T\} \quad (8.40)$$

$$\boldsymbol{\alpha}[\cdot]: \ I_T^{\boldsymbol{\theta}} \mapsto \mathbb{R}^N \quad (8.49)$$

$$I_T^{\boldsymbol{\theta}} \equiv \left\{\mathbf{X}_1^{\boldsymbol{\theta}}, \ldots, \mathbf{X}_T^{\boldsymbol{\theta}}\right\} \quad (8.48)$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\} \quad (8.30)$$

$$\overline{\mathcal{S}}(\boldsymbol{\theta}) \equiv \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}(\boldsymbol{\theta})) \equiv \max_{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}} \{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\} \quad (8.31)$$

$$\begin{aligned} \operatorname{Loss}\left(\boldsymbol{\alpha}\left[I_T^{\boldsymbol{\theta}}\right], \boldsymbol{\alpha}(\boldsymbol{\theta})\right) &\equiv \mathrm{OC}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}\left[I_T^{\boldsymbol{\theta}}\right]\right) \quad (8.53) \\ &\equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}\left(\underline{\boldsymbol{\alpha}\left[I_T^{\boldsymbol{\theta}}\right]}\right) + \end{aligned}$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}[i_T]) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}[i_T]) \quad (8.44)$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \quad (8.37)$$

$$\boldsymbol{\alpha}[\cdot]: \ i_T \mapsto \mathbb{R}^N \quad (8.38)$$

$$i_T \equiv \{\mathbf{x}_1, \ldots, \mathbf{x}_T\} \quad (8.40)$$

$$\boldsymbol{\alpha}[\cdot]: \ I_T^{\boldsymbol{\theta}} \mapsto \mathbb{R}^N \quad (8.49)$$

$$I_T^{\boldsymbol{\theta}} \equiv \left\{\mathbf{X}_1^{\boldsymbol{\theta}}, \ldots, \mathbf{X}_T^{\boldsymbol{\theta}}\right\} \quad (8.48)$$

stress-test

$$\boldsymbol{\theta} \mapsto \mathrm{OC}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}\left[I_T^{\boldsymbol{\theta}}\right]\right), \quad \boldsymbol{\theta} \in \Theta. \quad (8.57)$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\right\} \quad (8.30)$$

$$\overline{\mathcal{S}}(\boldsymbol{\theta}) \equiv \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}(\boldsymbol{\theta})) \equiv \max_{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}} \left\{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\right\} \quad (8.31)$$

$$\begin{aligned} \mathrm{Loss}\left(\boldsymbol{\alpha}\left[I_T^{\boldsymbol{\theta}}\right], \boldsymbol{\alpha}(\boldsymbol{\theta})\right) &\equiv \mathrm{OC}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}\left[I_T^{\boldsymbol{\theta}}\right]\right) \quad (8.53) \\ &\equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}\left[I_T^{\boldsymbol{\theta}}\right]\right) + \end{aligned}$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}[i_T]) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}[i_T]) \quad (8.44)$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \quad (8.37)$$

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$$\boldsymbol{\alpha}[\cdot]: \; i_T \mapsto \mathbb{R}^N \qquad (8.38)$$

$$i_T \equiv \{\mathbf{x}_1, \ldots, \mathbf{x}_T\} \qquad (8.40)$$

$$\boldsymbol{\alpha}[\cdot]: \; I_T^{\boldsymbol{\theta}} \mapsto \mathbb{R}^N \qquad (8.49)$$

$$I_T^{\boldsymbol{\theta}} \equiv \left\{\mathbf{X}_1^{\boldsymbol{\theta}}, \ldots, \mathbf{X}_T^{\boldsymbol{\theta}}\right\} \qquad (8.48)$$

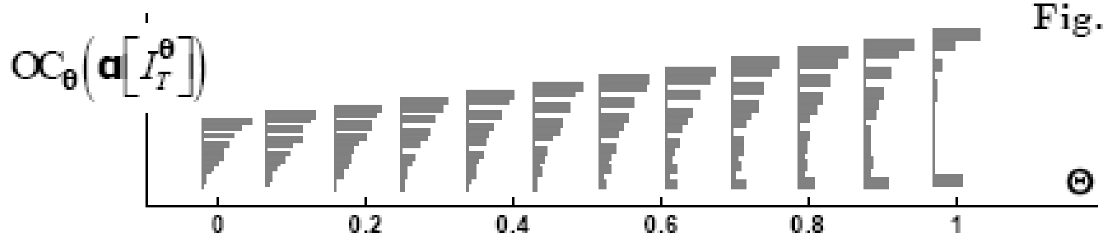

Fig. 8.2

$$\boldsymbol{\Xi}(\rho) \equiv \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & \ddots & & \vdots \\ \vdots & & \ddots & \rho \\ \rho & \cdots & \rho & 1 \end{pmatrix} \qquad (8.58)$$

$$\sqrt{\operatorname{diag}(\boldsymbol{\Sigma}(\rho))} \equiv (1 + \xi\rho)\,\mathbf{v}$$

$$\boldsymbol{\mu} \equiv p\sqrt{\operatorname{diag}(\boldsymbol{\Sigma}(\rho))} \qquad (8.59)$$

stress-test

$$\boldsymbol{\theta} \mapsto \mathrm{OC}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}\left[I_T^{\boldsymbol{\theta}}\right]\right), \quad \boldsymbol{\theta} \in \Theta. \qquad (8.57)$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\right\} \qquad (8.30)$$

$$\overline{\mathcal{S}}(\boldsymbol{\theta}) \equiv \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}(\boldsymbol{\theta})) \equiv \max_{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}} \left\{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\right\} \qquad (8.31)$$

$$\begin{aligned} \operatorname{Loss}\left(\boldsymbol{\alpha}\left[I_T^{\boldsymbol{\theta}}\right], \boldsymbol{\alpha}(\boldsymbol{\theta})\right) &\equiv \mathrm{OC}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}\left[I_T^{\boldsymbol{\theta}}\right]\right) \qquad (8.53) \\ &\equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}\left(\boldsymbol{\alpha}\left[I_T^{\boldsymbol{\theta}}\right]\right) + \end{aligned}$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}[i_T]) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}[i_T]) \qquad (8.44)$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \equiv \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \qquad (8.37)$$