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***Risk and Asset Allocation* - Springer – `symmys.com`**

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Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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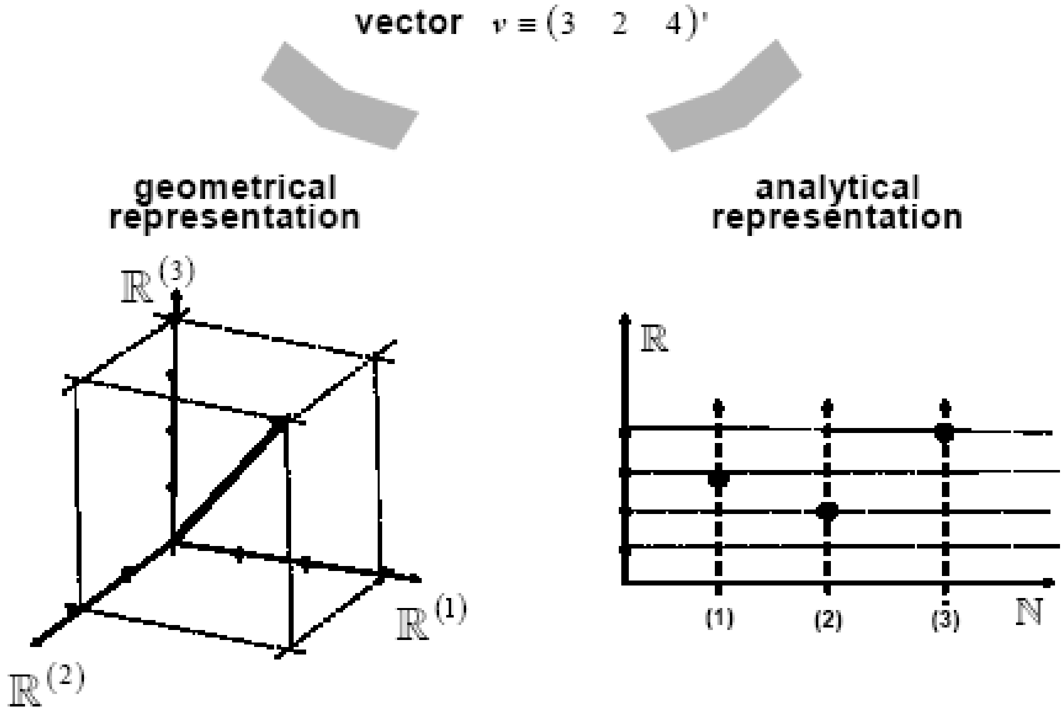

Fig. A.1. Representations of a vector

linear algebra

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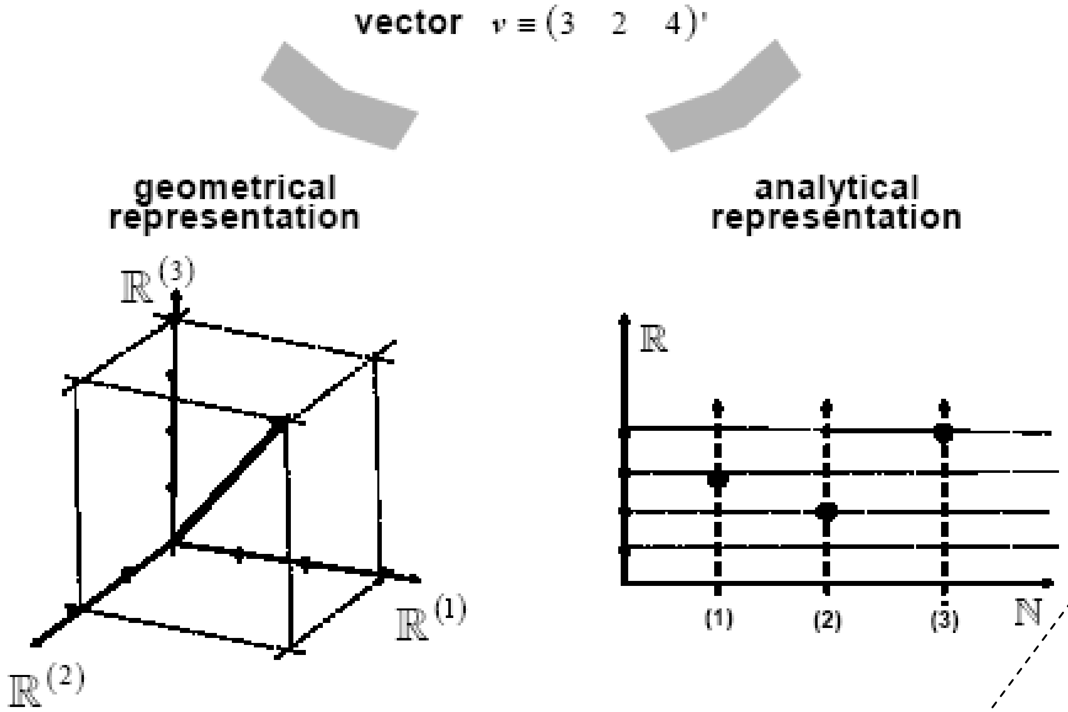

Fig. A.1. Representations of a vector

functional analysis

linear algebra

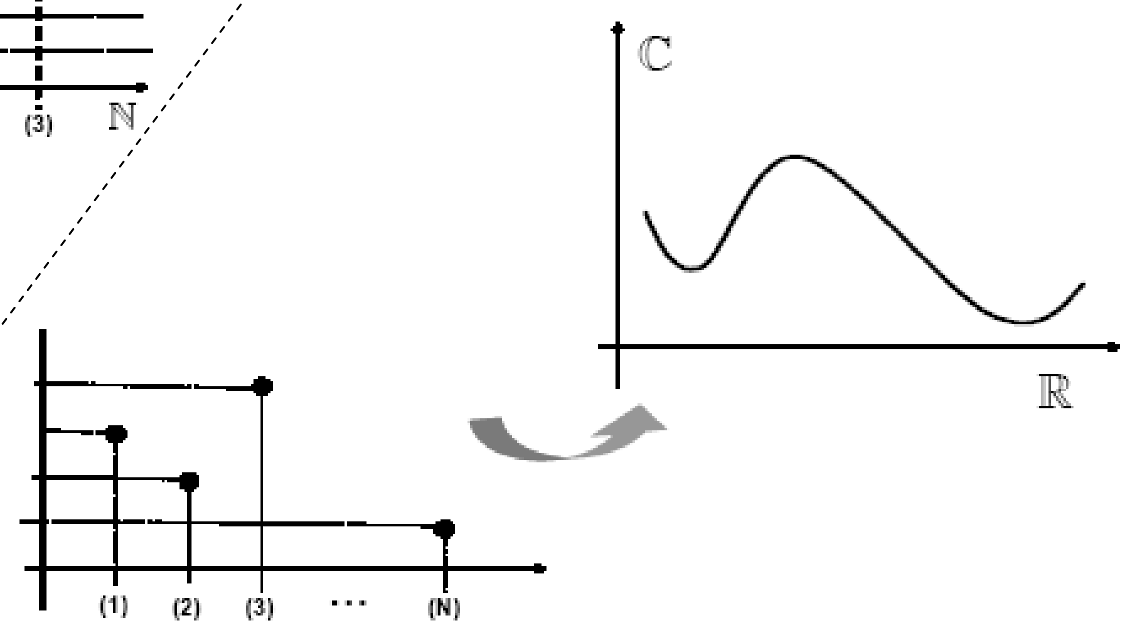

Fig. B.1. From linear algebra to functional analysis inner product $\langle \mathbf{u}, \mathbf{v} \rangle \equiv \sum_{n=1}^{N} u_n v_n .$ (A.5)

canonical basis

$$\left\langle \mathbf{v}, \boldsymbol{\delta}^{(n)} \right\rangle = v_n . \quad \text{(A.16)}$$

$$\begin{aligned} \boldsymbol{\delta}^{(1)} &\equiv (1, 0, \ldots, 0)' \\ &\vdots \\ \boldsymbol{\delta}^{(N)} &\equiv (0, 0, \ldots, 1)' . \end{aligned} \quad \text{(A.15)}$$

linear algebra

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functional analysis

inner product $$\langle u, v\rangle \equiv \int_{\mathbb{R}^N} u(\mathbf{x})\,\overline{v(\mathbf{x})}\,d\mathbf{x}, \quad \text{(B.5)}$$

Dirac delta

$$\left\langle v, \delta^{(\mathbf{y})}\right\rangle \equiv v(\mathbf{y})\,. \quad \text{(B.16)}$$

$$\int_{\mathbb{R}^N} v(\mathbf{x})\,\overline{\delta^{(\mathbf{y})}(\mathbf{x})}\,d\mathbf{x} = v(\mathbf{y})\,. \quad \text{(B.17)}$$

linear algebra

inner product $$\langle \mathbf{u}, \mathbf{v}\rangle \equiv \sum_{n=1}^{N} u_n v_n\,. \quad \text{(A.5)}$$

canonical basis

$$\left\langle \mathbf{v}, \boldsymbol{\delta}^{(n)}\right\rangle = v_n\,. \quad \text{(A.16)}$$

$$\begin{aligned} \boldsymbol{\delta}^{(1)} &\equiv (1, 0, \ldots, 0)' \\ &\vdots \\ \boldsymbol{\delta}^{(N)} &\equiv (0, 0, \ldots, 1)'\,. \end{aligned} \quad \text{(A.15)}$$

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inner product $$\langle u, v\rangle \equiv \int_{\mathbb{R}^N} u(\mathbf{x})\overline{v(\mathbf{x})}d\mathbf{x}, \tag{B.5}$$

Dirac delta

$$\left\langle v, \delta^{(\mathbf{y})}\right\rangle \equiv v(\mathbf{y}). \tag{B.16}$$

$$\int_{\mathbb{R}^N} v(\mathbf{x})\overline{\delta^{(\mathbf{y})}(\mathbf{x})}d\mathbf{x} = v(\mathbf{y}). \tag{B.17}$$

approximate Dirac delta

$$\delta_\epsilon^{(\mathbf{y})}(\mathbf{x}) \equiv \frac{1}{(2\pi)^{\frac{N}{2}}\epsilon^N} e^{-\frac{1}{2\epsilon^2}(\mathbf{x}-\mathbf{y})'(\mathbf{x}-\mathbf{y})}. \tag{B.18}$$

$$\left\langle v, \delta_\epsilon^{(\mathbf{y})}\right\rangle \approx v(\mathbf{y}). \tag{B.19}$$

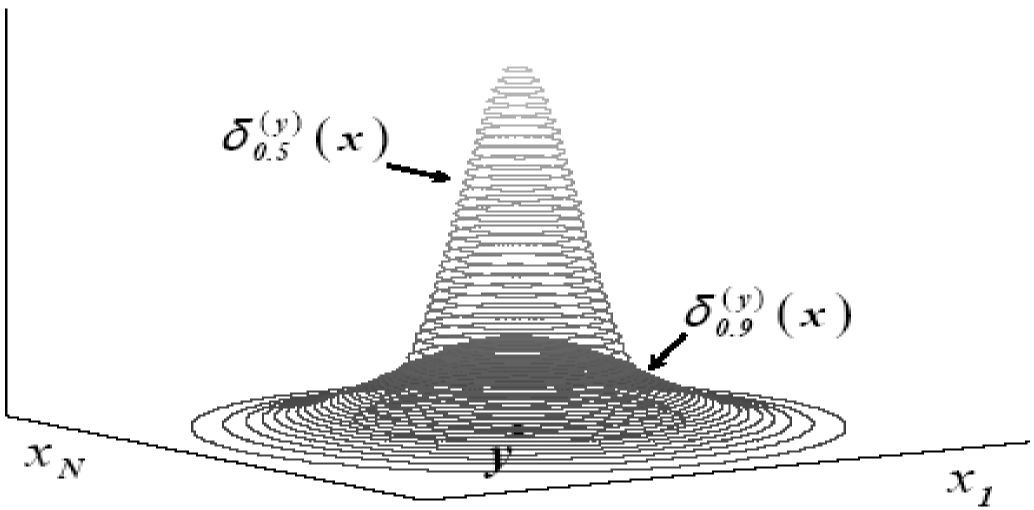

Fig. B.2. Approximation of the Dirac delta with Gaussian exponentials

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$$f_{i_T}(x) = \frac{1}{T}\sum_{t=1}^{T}\delta^{(x_t)}(x) \quad (1.120)$$

$$\approx$$

$$f_{i_T;\epsilon} \equiv f_{i_T} * \delta_{\epsilon}^{(0)} = \frac{1}{T}\sum_{t=1}^{T}\delta_{\epsilon}^{(x_t)} \quad (1.121)$$

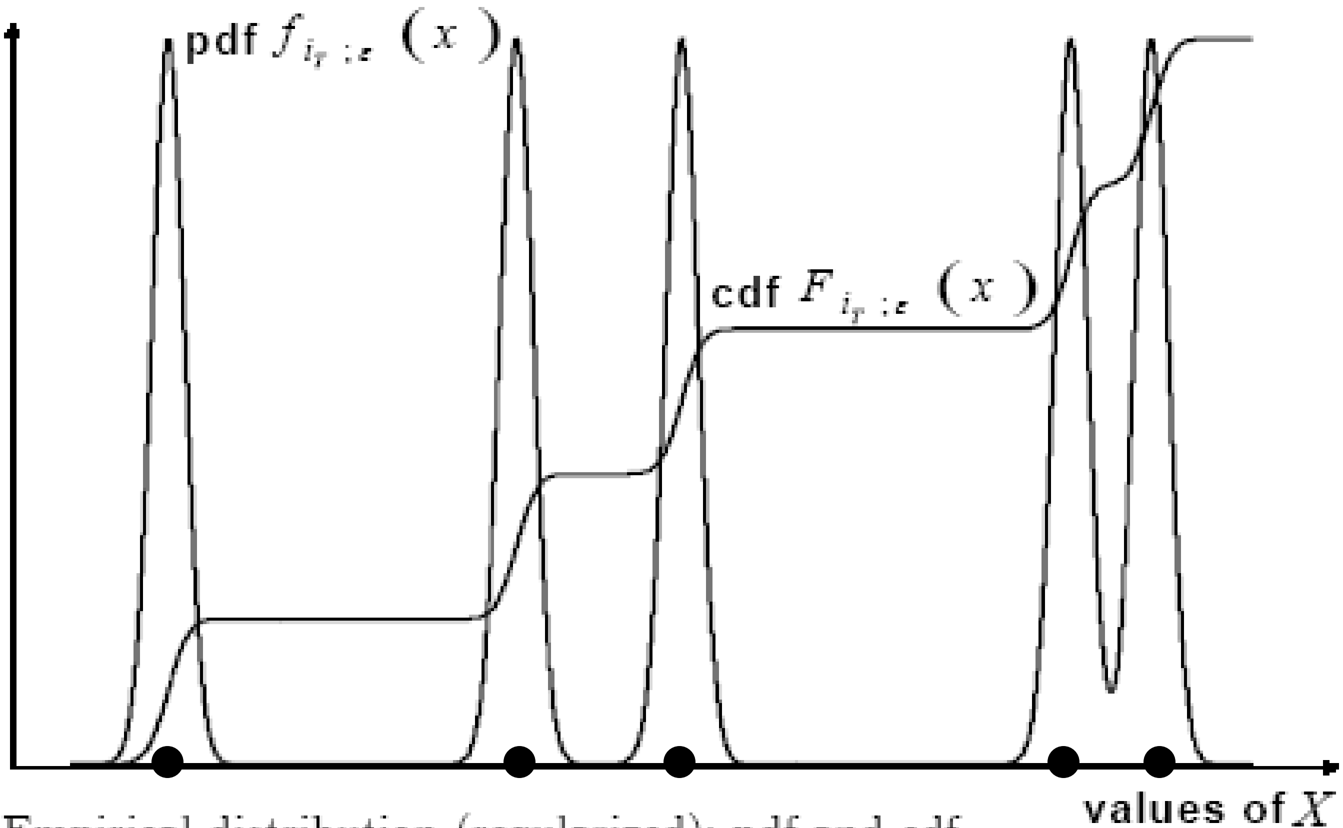

Fig. 1.12. Empirical distribution (regularized): pdf and cdf