Attilio Meucci

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Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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	location	dispersion
affine equiv.	$$\operatorname{Loc}\{\mathbf{a}+\mathbf{B}\mathbf{X}\} = \mathbf{a} + \underset{\substack{\uparrow \\ \text{invertible matrix}}}{\mathbf{B}} \operatorname{Loc}\{\mathbf{X}\} \qquad (2.51)$$	

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"local" measure	$$\operatorname{Mod}\{\mathbf{X}\}\equiv\underset{\mathbf{x}\in\mathbb{R}^N}{\operatorname{argmax}}\{f_{\mathbf{X}}(\mathbf{x})\} \qquad (2.52)$$	

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	location	dispersion
affine equiv.	$\mathrm{Loc}\{\mathbf{a}+\mathbf{B}\mathbf{X}\} = \mathbf{a} + \underset{\uparrow \text{ invertible matrix}}{\mathbf{B}} \mathrm{Loc}\{\mathbf{X}\}$ (2.51)	
"local" measure	$\mathrm{Mod}\{\mathbf{X}\} \equiv \underset{\mathbf{x}\in\mathbb{R}^N}{\mathrm{argmax}}\{f_{\mathbf{X}}(\mathbf{x})\}$ (2.52)	
"semi-local" measure	median ???	

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affine equiv.	$\mathrm{Loc}\{\mathbf{a}+\mathbf{B}\mathbf{X}\}=\mathbf{a}+\underset{\substack{\uparrow\\ \text{invertible matrix.}}}{\mathbf{B}}\,\mathrm{Loc}\{\mathbf{X}\}$	(2.51)	
"local" measure	$\mathrm{Mod}\{\mathbf{X}\}\equiv\underset{\mathbf{x}\in\mathbb{R}^N}{\operatorname{argmax}}\{f_{\mathbf{X}}(\mathbf{x})\}$	(2.52)	
"semi-local" measure	median ???		
"global" measure	$\mathrm{E}\{\mathbf{X}\}\equiv(\mathrm{E}\{X_1\},\ldots,\mathrm{E}\{X_N\})'$	(2.54)	

$$|Z_X| \equiv \sqrt{(X - \mathrm{Loc}\{X\}) \frac{1}{\mathrm{Dis}\{X\}^2} (X - \mathrm{Loc}\{X\})}. \tag{2.59}$$

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$$\boxed{|Z_{a+bX}| = |Z_X|} \tag{2.60}$$

$$\mathrm{Dis}\{a + bX\} = |b| \, \mathrm{Dis}\{X\} \tag{1.32}$$

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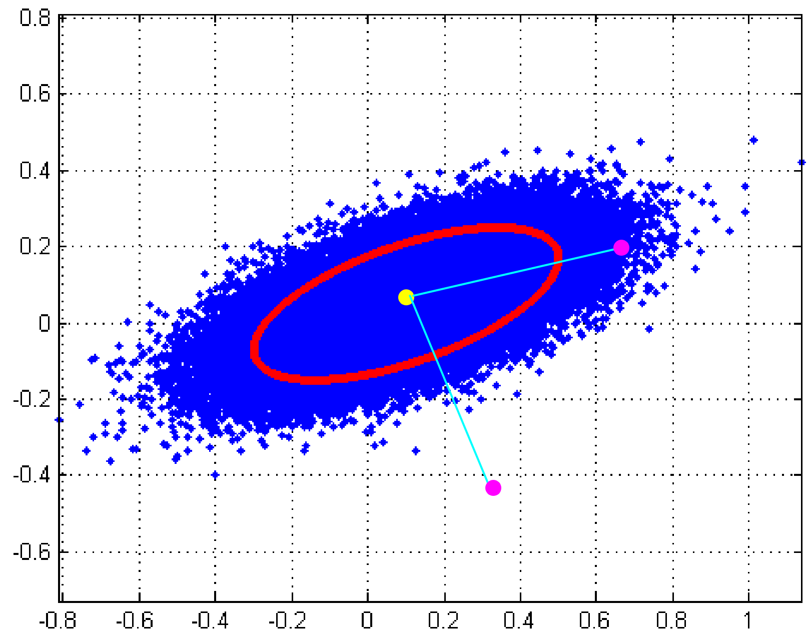
$$\boxed{|Z_{a+bX}| = |Z_X|} \quad (2.60)$$

$$\mathrm{Dis}\{a + bX\} = |b| \, \mathrm{Dis}\{X\} \quad (1.32)$$

$$\mathrm{Ma}_{\mathbf{X}} \equiv \mathrm{Ma}\left(\mathbf{X}, \mathrm{Loc}\{\mathbf{X}\}, \mathrm{DisSq}\{\mathbf{X}\}\right) \quad (2.62)$$

$$\mathrm{Ma}(\mathbf{x}, \boldsymbol{\mu}, \underset{\substack{\uparrow \\ \text{symmetric and positive}}}{\boldsymbol{\Sigma}}) \equiv \sqrt{(\mathbf{x} - \boldsymbol{\mu})' \, \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu})}, \quad (2.61)$$



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$$\mathrm{Ma}_{\mathbf{X}} \equiv \mathrm{Ma}(\mathbf{X}, \mathrm{Loc}\{\mathbf{X}\}, \mathrm{DisSq}\{\mathbf{X}\}) \quad (2.62)$$

$$\mathrm{Ma}(\mathrm{x}, \boldsymbol{\mu}, \underset{\substack{\uparrow \\ \text{symmetric and positive}}}{\boldsymbol{\Sigma}}) \equiv \sqrt{(\mathrm{x} - \boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\mathrm{x} - \boldsymbol{\mu})}, \quad (2.61)$$

$$\boxed{\mathrm{Ma}_{\mathbf{a}+\mathbf{BX}} = \mathrm{Ma}_{\mathbf{X}}} \quad (2.63)$$

$$\mathrm{DisSq}\{\mathbf{a} + \mathbf{BX}\} = \mathbf{B} \, \mathrm{DisSq}\{\mathbf{X}\} \, \mathbf{B}'. \quad (2.64)$$

symmetric and positive

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	location	dispersion
affine equiv.	$\mathrm{Loc}\{\mathbf{a}+\mathbf{B}\mathbf{X}\} = \mathbf{a} + \underset{\text{invertible matrix.}}{\underset{\uparrow}{\mathbf{B}}}\,\mathrm{Loc}\{\mathbf{X}\}$ (2.51)	$\mathrm{DisSq}\{\mathbf{a}+\underset{\text{invertible matrix.}}{\underset{\uparrow}{\mathbf{B}}}\mathbf{X}\} = \mathbf{B}\,\mathrm{DisSq}\{\mathbf{X}\}\,\mathbf{B}'$ (2.64)
"local" measure	$\mathrm{Mod}\{\mathbf{X}\} \equiv \underset{\mathbf{x}\in\mathbb{R}^N}{\mathrm{argmax}}\{f_{\mathbf{X}}(\mathbf{x})\}$ (2.52)	
"semi-local" measure	median ???	
"global" measure	$\mathrm{E}\{\mathbf{X}\} \equiv (\mathrm{E}\{X_1\},\ldots,\mathrm{E}\{X_N\})'$ (2.54)	

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	location	dispersion
affine equiv.	$$\operatorname{Loc}\{\mathbf{a}+\mathbf{B}\mathbf{X}\}=\mathbf{a}+\underset{\uparrow}{\mathbf{B}}\operatorname{Loc}\{\mathbf{X}\} \quad (2.51)$$ invertible matrix.	$$\operatorname{DisSq}\{\mathbf{a}+\mathbf{B}\mathbf{X}\}=\underset{\uparrow}{\mathbf{B}}\operatorname{DisSq}\{\mathbf{X}\}\mathbf{B}' \quad (2.64)$$ invertible matrix.
"local" measure	$$\operatorname{Mod}\{\mathbf{X}\}\equiv\underset{\mathbf{x}\in\mathbb{R}^N}{\operatorname{argmax}}\{f_{\mathbf{X}}(\mathbf{x})\} \quad (2.52)$$	$$\operatorname{MDis}\{\mathbf{X}\}\equiv-\left(\left.\frac{\partial^2\ln f_{\mathbf{X}}}{\partial\mathbf{x}\partial\mathbf{x}'}\right\vert_{\mathbf{x}=\operatorname{Mod}\{\mathbf{X}\}}\right)^{-1} \quad (2.65)$$
"semi-local" measure	median ???	
"global" measure	$$\operatorname{E}\{\mathbf{X}\}\equiv(\operatorname{E}\{X_1\},\ldots,\operatorname{E}\{X_N\})' \quad (2.54)$$	

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	location	dispersion
affine equiv.	$$\mathrm{Loc}\{\mathbf{a}+\mathbf{B}\mathbf{X}\}=\mathbf{a}+\underset{\substack{\uparrow \\ \text{invertible matrix.}}}{\mathbf{B}}\,\mathrm{Loc}\{\mathbf{X}\} \quad (2.51)$$	$$\mathrm{DisSq}\{\mathbf{a}+\mathbf{B}\mathbf{X}\}=\underset{\substack{\uparrow \\ \text{invertible matrix.}}}{\mathbf{B}}\,\mathrm{DisSq}\{\mathbf{X}\}\,\mathbf{B}' \quad (2.64)$$
“local” measure	$$\mathrm{Mod}\{\mathbf{X}\}\equiv\underset{\mathbf{x}\in\mathbb{R}^N}{\operatorname{argmax}}\{f_{\mathbf{X}}(\mathbf{x})\} \quad (2.52)$$	$$\mathrm{MDis}\{\mathbf{X}\}\equiv-\left(\left.\frac{\partial^2\ln f_{\mathbf{X}}}{\partial\mathbf{x}\partial\mathbf{x}'}\right\|_{\mathbf{x}=\mathrm{Mod}\{\mathbf{X}\}}\right)^{-1} \quad (2.65)$$
“semi-local” measure	median ???	range ???
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	location	dispersion
affine equiv.	$\mathrm{Loc}\{\mathbf{a}+\mathbf{B}\mathbf{X}\}=\mathbf{a}+\mathbf{B}\,\mathrm{Loc}\{\mathbf{X}\}$ (2.51) ↑ invertible matrix.	$\mathrm{DisSq}\{\mathbf{a}+\mathbf{B}\mathbf{X}\}=\mathbf{B}\,\mathrm{DisSq}\{\mathbf{X}\}\,\mathbf{B}'$ (2.64) ↑ invertible matrix.
"local" measure	$\mathrm{Mod}\{\mathbf{X}\}\equiv\underset{\mathbf{x}\in\mathbb{R}^N}{\mathrm{argmax}}\{f_{\mathbf{X}}(\mathbf{x})\}$ (2.52)	$\mathrm{MDis}\{\mathbf{X}\}\equiv-\left(\left.\frac{\partial^2\ln f_{\mathbf{X}}}{\partial\mathbf{x}\partial\mathbf{x}'}\right\|_{\mathbf{x}=\mathrm{Mod}\{\mathbf{X}\}}\right)^{-1}$ (2.65)
"semi-local" measure	median ???	range ???
"global" measure	$\mathrm{E}\{\mathbf{X}\}\equiv(\mathrm{E}\{X_1\},\ldots,\mathrm{E}\{X_N\})'$ (2.54)	$\mathrm{Cov}\{X_m,X_n\}\equiv[\mathrm{Cov}\{\mathbf{X}\}]_{mn}$ (2.68) $\equiv\mathrm{E}\{(X_m-\mathrm{E}\{X_m\})(X_n-\mathrm{E}\{X_n\})\}$

$$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \qquad (2.155)$$

$$\mathrm{E}\{\mathbf{X}\} = \mathrm{Mod}\{\mathbf{X}\} = \boldsymbol{\mu}. \qquad (2.158)$$

$$\mathrm{Cov}\{\mathbf{X}\} = \mathrm{MDis}\{\mathbf{X}\} = \boldsymbol{\Sigma}. \qquad (2.159)$$

$$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \qquad (2.155)$$

$$\mathrm{E}\{\mathbf{X}\} = \mathrm{Mod}\{\mathbf{X}\} = \boldsymbol{\mu}. \qquad (2.158)$$

$$\mathrm{Cov}\{\mathbf{X}\} = \mathrm{MDis}\{\mathbf{X}\} = \boldsymbol{\Sigma}. \qquad (2.159)$$

$$\mathbf{X} \sim \mathrm{LogN}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \qquad (2.217)$$

$$\mathrm{E}\{X_n\} = e^{\mu_n + \frac{\Sigma_{nn}}{2}} \qquad (2.219)$$

$$\mathrm{Cov}\{X_m, X_n\} = e^{\mu_m + \mu_n + \frac{\Sigma_{mm}}{2} + \frac{\Sigma_{nn}}{2}} \left(e^{\Sigma_{mn}} - 1\right) \qquad (2.220)$$

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$$\mathrm{Cor}\{X_m, X_n\} \equiv [\mathrm{Cor}\{\mathbf{X}\}]_{mn} \equiv \frac{\mathrm{Cov}\{X_m, X_n\}}{\mathrm{Sd}\{X_m\}\,\mathrm{Sd}\{X_n\}} \tag{2.133}$$

$$\mathrm{Cor}\left\{X_m, X_n\right\} \equiv \left[\mathrm{Cor}\left\{\mathbf{X}\right\}\right]_{mn} \equiv \frac{\mathrm{Cov}\left\{X_m, X_n\right\}}{\mathrm{Sd}\left\{X_m\right\}\mathrm{Sd}\left\{X_n\right\}} \qquad (2.133)$$

$$-1 \leq \mathrm{Cor}\left\{X_m, X_n\right\} \leq 1 \qquad (2.135)$$

$$(X_m, X_n) \text{ independent} \Rightarrow \mathrm{Cor}\left\{X_m, X_n\right\} = 0. \qquad (2.136)$$

$$\begin{cases} X_m = a + bX_n \Leftrightarrow \mathrm{Cor}\left\{X_m, X_n\right\} = 1 & (2.137) \\ X_m = a - bX_n \Leftrightarrow \mathrm{Cor}\left\{X_m, X_n\right\} = -1, & (2.138) \\ \left.\begin{array}{c} Y_m \equiv a + bX_m \\ Y_n \equiv c + dX_n \end{array}\right\} \Rightarrow \mathrm{Cor}\left\{X_m, X_n\right\} = \mathrm{Cor}\left\{Y_m, Y_n\right\} & (2.139) \end{cases}$$

$$\operatorname{Cor}\{X_m, X_n\} \equiv [\operatorname{Cor}\{\mathbf{X}\}]_{mn} \equiv \frac{\operatorname{Cov}\{X_m, X_n\}}{\operatorname{Sd}\{X_m\}\operatorname{Sd}\{X_n\}} \qquad (2.133)$$

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$$(X_m, X_n) \text{ independent} \Rightarrow \operatorname{Cor}\{X_m, X_n\} = 0. \qquad (2.136)$$

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but…

$$Y_1 \equiv e^{X_1}, \quad Y_2 \equiv e^{X_2}. \qquad (2.143)$$

The correlation between these variables is bounded within an interval smaller than $[-1, 1]$.