MEAN-VARIANCE PITFALLS – SUFFICIENT CONDITIONS FOR MV

***Risk and Asset Allocation* - Springer – symmys.com**

Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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market

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}' \mathbf{M}. \quad (6.123)$$

preferences

$$\mathcal{S}(\boldsymbol{\alpha}) = \mathcal{H}\left(\mathrm{E}\left\{\Psi_{\boldsymbol{\alpha}}\right\}, \mathrm{CM}_2\left\{\Psi_{\boldsymbol{\alpha}}\right\}, \mathrm{CM}_3\left\{\Psi_{\boldsymbol{\alpha}}\right\}, \ldots\right) \quad (6.122)$$

$$\mathcal{S}(\boldsymbol{\alpha}) \approx \widetilde{\mathcal{H}}\left(\mathrm{E}\left\{\Psi_{\boldsymbol{\alpha}}\right\}, \mathrm{Var}\left\{\Psi_{\boldsymbol{\alpha}}\right\}\right) \quad (6.124)$$

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two-step:

step 1

$$\boldsymbol{\alpha}(v) \equiv \underset{\substack{\boldsymbol{\alpha} \in \mathcal{C} \\ \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\} = v}}{\operatorname{argmax}} \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (6.68)$$

step 2

$$\boldsymbol{\alpha}^* \equiv \boldsymbol{\alpha}(v^*) \equiv \underset{v \geq 0}{\operatorname{argmax}}\, \mathcal{S}(\boldsymbol{\alpha}(v)) \quad (6.69)$$

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$$\mathcal{S}(\boldsymbol{\alpha}) \approx \widetilde{\mathcal{H}}\left(\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\}, \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\}\right) \quad (6.124)$$

$$u(\psi) = \psi - \frac{1}{2\zeta}\psi^2 \quad (6.125)$$

preferences

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$$u(\psi) = \psi - \frac{1}{2\zeta}\psi^2 \quad (6.125)$$

preferences

$$\mathbf{M} \sim \mathrm{El}(\boldsymbol{\mu}, \boldsymbol{\Sigma}, g_N) \quad (6.126)$$

market

MEAN-VARIANCE PITFALLS – ONE-STEP VS. TWO-STEP

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two-step:

step 1

$$\boldsymbol{\alpha}(v) \equiv \underset{\substack{\boldsymbol{\alpha} \in \mathcal{C} \\ \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\}=v}}{\operatorname{argmax}} \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \qquad (6.68)$$

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MEAN-VARIANCE PITFALLS – ONE-STEP VS. TWO-STEP

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$$\boldsymbol{\alpha}^* \equiv \boldsymbol{\alpha}(v^*) \equiv \underset{v \geq 0}{\operatorname{argmax}} \, \mathcal{S}(\boldsymbol{\alpha}(v)) \qquad (6.69)$$

$$\Updownarrow$$

two-step:

step 1
$$\boldsymbol{\alpha}(\lambda) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}}{\operatorname{argmax}} \{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \lambda \, \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\}\} \qquad (6.129)$$

step 2
$$\lambda^* \equiv \underset{\lambda \in \mathbb{R}}{\operatorname{argmax}} \, \mathcal{S}(\boldsymbol{\alpha}(\lambda)) \qquad (6.130)$$

MEAN-VARIANCE PITFALLS – ONE-STEP VS. TWO-STEP

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two-step:

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one-step: pseudo-index of satisfaction $$\mathcal{S}^*(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \lambda^* \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\} \quad (6.137)$$

MEAN-VARIANCE PITFALLS – ONE-STEP VS. TWO-STEP

two-step:

step 1
$$\boldsymbol{\alpha}(v) \equiv \underset{\substack{\boldsymbol{\alpha}\in\mathcal{C} \\ \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\}=v}}{\operatorname{argmax}} \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (6.68)$$

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$$\boldsymbol{\alpha}^* \equiv \boldsymbol{\alpha}(v^*) \equiv \underset{v\geq 0}{\operatorname{argmax}}\, \mathcal{S}(\boldsymbol{\alpha}(v)) \quad (6.69)$$

$$\Updownarrow$$

two-step:

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one-step: pseudo-index of satisfaction $\mathcal{S}^*(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \lambda^* \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\}$ (6.137)

same investor displays different risk aversion coefficients λ^* when facing different markets

MEAN-VARIANCE PITFALLS – ONE-STEP VS. TWO-STEP

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two-step:

step 1 $$\boldsymbol{\alpha}(v) \equiv \underset{\substack{\boldsymbol{\alpha}\in\mathcal{C} \\ \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\}=v}}{\operatorname{argmax}} \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (6.68)$$

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two-step:

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one-step: pseudo-index of satisfaction $\mathcal{S}^*(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \lambda^* \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\}$ (6.137)

same investor displays different risk aversion coefficients λ^* when facing different markets

one-step exception:

$$\mathbf{M} \equiv \mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.143)$$

$$u(\psi) \equiv -e^{-\frac{1}{\zeta}\psi}; \quad (5.92)$$

$$\mathrm{CE}(\boldsymbol{\alpha}) = \boldsymbol{\alpha}'\boldsymbol{\mu} - \frac{\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}}{\boxed{2\zeta}} \quad (5.144)$$

MEAN-VARIANCE PITFALLS – DUAL FORMULATIONS

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step 1

two-step

$$\boldsymbol{\alpha}^{*} \equiv \boldsymbol{\alpha}(v^{*}) \equiv \underset{v\geq 0}{\operatorname{argmax}}\, \mathcal{S}(\boldsymbol{\alpha}(v)) \qquad (6.69)$$

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$$\Downarrow \textbf{?}$$

$$\boldsymbol{\alpha}(v) \equiv \underset{\substack{\boldsymbol{\alpha} \in \mathcal{C} \\ \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\} \leq v}}{\operatorname{argmax}} \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \qquad (6.144)$$

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$\Downarrow$?

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$\Updownarrow$?

$$\boldsymbol{\alpha}(e) \equiv \underset{\substack{\boldsymbol{\alpha} \in \mathcal{C} \\ \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \geq e}}{\operatorname{argmin}} \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\} \qquad (6.146)$$

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$$\boldsymbol{\alpha}(v) \equiv \underset{\substack{\boldsymbol{\alpha}\in\mathcal{C} \\ \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\}=v}}{\operatorname{argmax}} \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \qquad (6.143)$$

$\Downarrow$ **?**

$$\boldsymbol{\alpha}(v) \equiv \underset{\substack{\boldsymbol{\alpha}\in\mathcal{C} \\ \mathrm{Var}\{\Psi_{\boldsymbol{\alpha}}\}\leq v}}{\operatorname{argmax}} \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \qquad (6.144)$$

$\cancel{\Updownarrow}$

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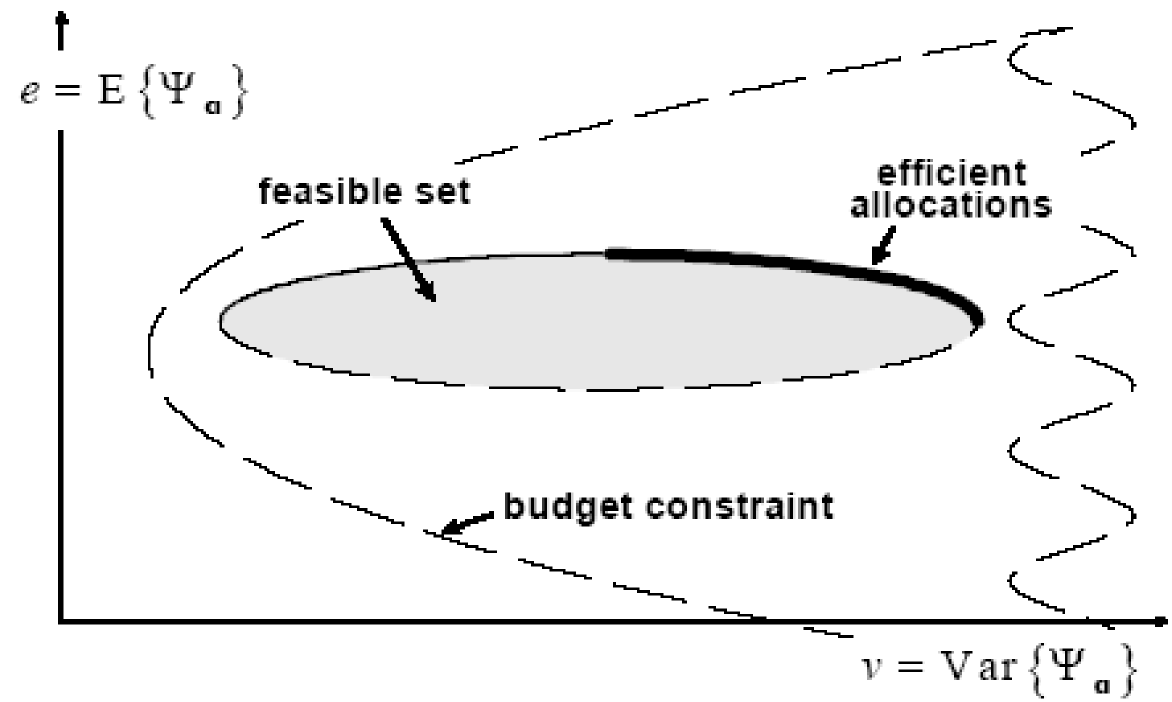
Fig. 6.17



MEAN-VARIANCE PITFALLS – RETURNS: ESTIMATION VS OPTIMIZATION

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$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}' \mathbf{P}_{T+\tau} \quad (6.82)$$

$$\boldsymbol{\alpha}(v) \equiv \underset{\substack{\boldsymbol{\alpha} \in \mathcal{C} \\ \boldsymbol{\alpha}' \operatorname{Cov}\{\mathbf{P}\} \boldsymbol{\alpha} = v}}{\operatorname{argmax}} \boldsymbol{\alpha}' \operatorname{E}\{\mathbf{P}\} \quad (6.74)$$

step 1

$$\boldsymbol{\alpha}^{*} \equiv \boldsymbol{\alpha}(v^{*}) \equiv \underset{v \geq 0}{\operatorname{argmax}}\, \mathcal{S}(\boldsymbol{\alpha}(v)) \quad (6.69)$$

step 2

two-step

MEAN-VARIANCE PITFALLS – RETURNS: ESTIMATION VS OPTIMIZATION

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$$\mathbf{w} \equiv \frac{\operatorname{diag}(\mathbf{p}_T)}{\boldsymbol{\alpha}' \mathbf{p}_T} \boldsymbol{\alpha} \quad (6.86)$$

$$L \equiv \frac{P_{T+\tau}}{P_T} - 1 \quad (6.81)$$

MEAN-VARIANCE PITFALLS – RETURNS: ESTIMATION VS OPTIMIZATION

$$\Psi_{\alpha} \equiv \boldsymbol{\alpha}' \mathbf{P}_{T+\tau} \quad (6.82)$$

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$$L^{\Psi_{\alpha}} \equiv \frac{\Psi_{\alpha}}{w_T} - 1 = \mathbf{w}' \mathbf{L} \quad (6.84)(6.87)$$

MEAN-VARIANCE PITFALLS – RETURNS: ESTIMATION VS OPTIMIZATION

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$$\mathbf{w}(v) = \underset{\substack{\mathbf{w} \in \mathcal{C} \\ \mathbf{w}' \operatorname{Cov}\{\mathbf{L}_{T+\tau,\tau}\} \mathbf{w} = v}}{\operatorname{argmax}} \mathbf{w}' \operatorname{E}\{\mathbf{L}_{T+\tau,\tau}\} \quad (6.147)$$

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invariants = returns $\quad \mathbf{X}_{t,\widetilde{\tau}} \equiv \mathbf{L}_{t,\widetilde{\tau}} \quad (6.148)$

estimation interval = investment horizon $\quad \tau \equiv \widetilde{\tau}. \quad (6.149)$

$$\operatorname{E}\{\mathbf{L}_{T+\tau,\tau}\} = \operatorname{E}\{\mathbf{L}_{t,\widetilde{\tau}}\}$$

$$(6.150)$$

$$\operatorname{Cov}\{\mathbf{L}_{T+\tau,\tau}\} = \operatorname{Cov}\{\mathbf{L}_{t,\widetilde{\tau}}\}$$

$$\mathbf{w}(v) = \underset{\substack{\mathbf{w} \in \mathcal{C} \\ \mathbf{w}' \operatorname{Cov}\{\mathbf{L}_{T+\tau,\tau}\} \mathbf{w} = v}}{\operatorname{argmax}} \mathbf{w}' \operatorname{E}\{\mathbf{L}_{T+\tau,\tau}\} \quad (6.147)$$

MEAN-VARIANCE PITFALLS – RETURNS: LINEAR VS. COMPOUNDED

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$$\mathbf{w} \equiv \frac{\operatorname{diag}(\mathbf{p}_T)}{\boldsymbol{\alpha}'\mathbf{p}_T}\boldsymbol{\alpha} \quad (6.86)$$

$$C_{t,\tau} \equiv \ln\left(\frac{P_t}{P_{t-\tau}}\right) \quad (6.152)$$

$$L^{\Psi_{\alpha}} \equiv \frac{\Psi_{\alpha}}{w_T} - 1 = \mathbf{w}'\mathbf{L} \quad (6.84)(6.87)$$

$$\mathbf{w}(v) = \underset{\substack{\mathbf{w}\in\mathcal{C} \\ \mathbf{w}'\,\mathrm{Cov}\{\mathbf{L}_{T+\tau,\tau}\}\mathbf{w}=v}}{\operatorname{argmax}} \mathbf{w}'\,\mathrm{E}\{\mathbf{L}_{T+\tau,\tau}\} \quad (6.147)$$

MEAN-VARIANCE PITFALLS – RETURNS: LINEAR VS. COMPOUNDED

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$$\widetilde{\mathbf{w}}(v) \equiv \underset{\substack{\mathbf{w} \in \mathcal{C} \\ \mathbf{w}' \operatorname{Cov}\{\mathbf{C}_{T+\tau,\tau}\} \mathbf{w} = v}}{\operatorname{argmax}} \mathbf{w}' \operatorname{E}\{\mathbf{C}_{T+\tau,\tau}\} \quad (6.164)$$

MEAN-VARIANCE PITFALLS – RETURNS: LINEAR VS. COMPOUNDED

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$$\boxed{C^{\Psi_{\alpha}}_{T+\tau,\tau} \neq \mathbf{w}' \mathbf{C}_{T+\tau,\tau}} \quad (6.166)$$

$$\widetilde{\mathbf{w}}(v) \equiv \underset{\substack{\mathbf{w} \in \mathcal{C} \\ \mathbf{w}' \operatorname{Cov}\{\mathbf{C}_{T+\tau,\tau}\} \mathbf{w} = v}}{\operatorname{argmax}} \mathbf{w}' \operatorname{E}\{\mathbf{C}_{T+\tau,\tau}\} \quad (6.164)$$

MEAN-VARIANCE PITFALLS – RETURNS: LINEAR VS. COMPOUNDED

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MEAN-VARIANCE PITFALLS – RETURNS: LINEAR VS. COMPOUNDED - HORIZON

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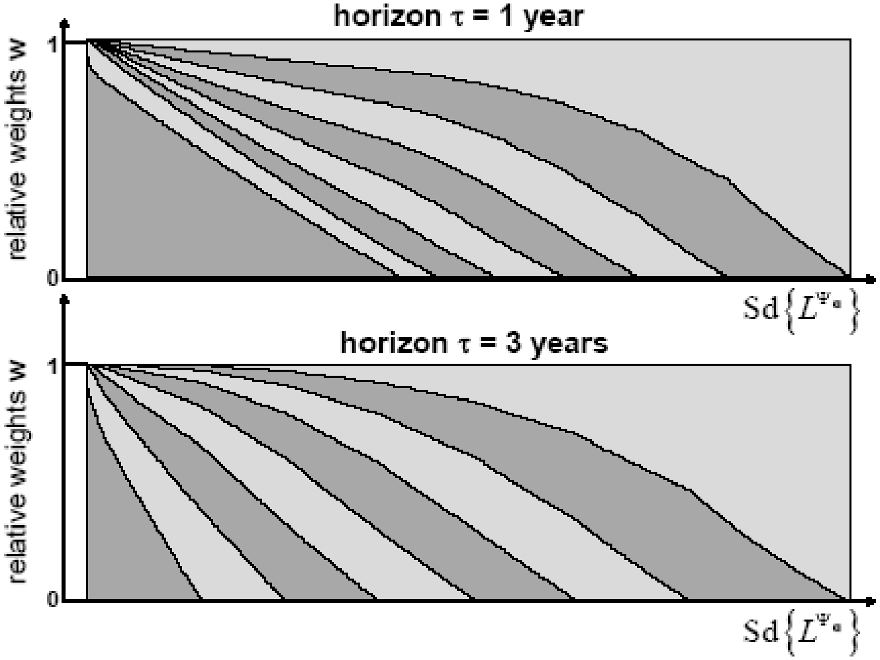

Fig. 6.18.

$$\mathbf{w}(v) = \underset{\substack{\mathbf{w}\in\mathcal{C} \\ \mathbf{w}'\,\mathrm{Cov}\{\mathbf{L}_{T+\tau,\tau}\}\mathbf{w}=v}}{\operatorname{argmax}} \mathbf{w}'\,\mathrm{E}\{\mathbf{L}_{T+\tau,\tau}\} \quad (6.147)$$

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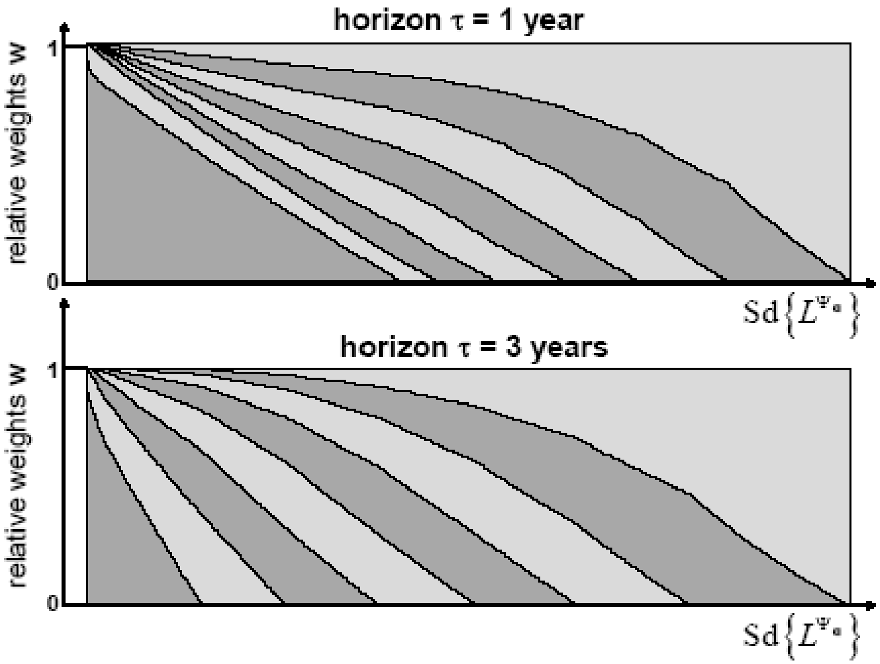

Fig. 6.18.

$$\mathbf{w}(v) = \underset{\substack{\mathbf{w}\in\mathcal{C} \\ \mathbf{w}' \operatorname{Cov}\{\mathbf{L}_{T+\tau,\tau}\}\mathbf{w}=v}}{\operatorname{argmax}} \mathbf{w}' \operatorname{E}\{\mathbf{L}_{T+\tau,\tau}\} \quad (6.147)$$

$$\widetilde{\mathbf{w}}(v) \equiv \underset{\substack{\mathbf{w}\in\mathcal{C} \\ \mathbf{w}' \operatorname{Cov}\{\mathbf{C}_{T+\tau,\tau}\}\mathbf{w}=v}}{\operatorname{argmax}} \mathbf{w}' \operatorname{E}\{\mathbf{C}_{T+\tau,\tau}\} \quad (6.164)$$

MEAN-VARIANCE PITFALLS – RETURNS: LINEAR VS. COMPOUNDED - HORIZON

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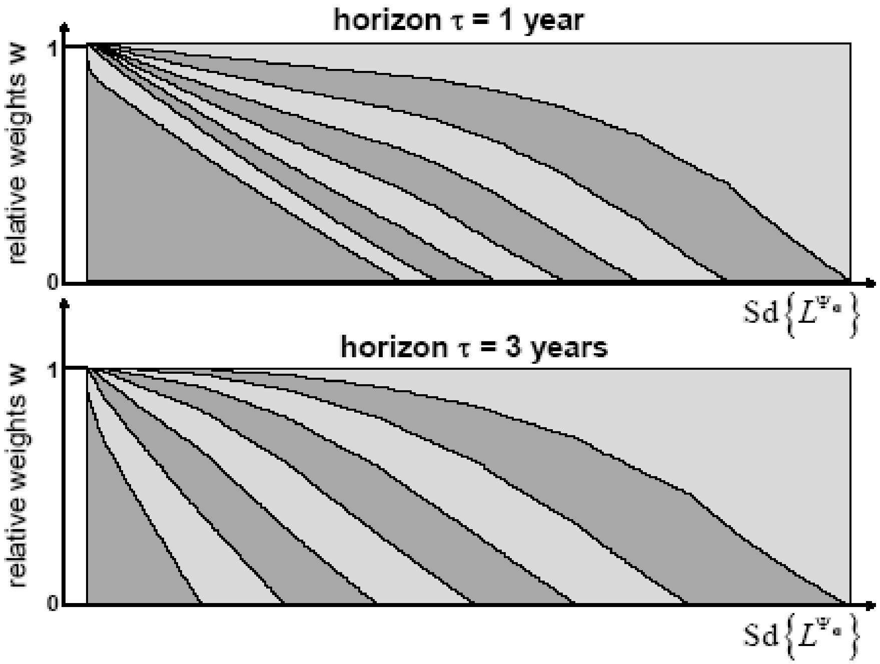

Fig. 6.18.

invariants = returns

$$\mathbf{X}_{t,\tilde{\tau}} \equiv \mathbf{L}_{t,\tilde{\tau}} \quad (6.148)$$

$$\mathrm{E}\left\{\mathbf{C}_{T+\tau,\tau}\right\} = \frac{\tau}{\tilde{\tau}}\,\mathrm{E}\left\{\mathbf{C}_{t,\tilde{\tau}}\right\}$$

(6.165)

$$\mathrm{Cov}\left\{\mathbf{C}_{T+\tau,\tau}\right\} = \frac{\tau}{\tilde{\tau}}\,\mathrm{Cov}\left\{\mathbf{C}_{t,\tilde{\tau}}\right\}$$

$$\underset{\substack{\mathbf{w}\in\mathcal{C} \\ \mathbf{w}'\,\mathrm{Cov}\{\mathbf{C}_{t,\tilde{\tau}}\}\,\mathbf{w}=v}}{\operatorname{argmax}} \mathbf{w}'\,\mathrm{E}\left\{\mathbf{C}_{t,\tilde{\tau}}\right\}$$

$$\Updownarrow$$

$$\widetilde{\mathbf{w}}(v) \equiv \underset{\substack{\mathbf{w}\in\mathcal{C} \\ \mathbf{w}'\,\mathrm{Cov}\{\mathbf{C}_{T+\tau,\tau}\}\mathbf{w}=v}}{\operatorname{argmax}} \mathbf{w}'\,\mathrm{E}\left\{\mathbf{C}_{T+\tau,\tau}\right\} \quad (6.164)$$

$$\mathbf{w}(v) = \underset{\substack{\mathbf{w}\in\mathcal{C} \\ \mathbf{w}'\,\mathrm{Cov}\{\mathbf{L}_{T+\tau,\tau}\}\mathbf{w}=v}}{\operatorname{argmax}} \mathbf{w}'\,\mathrm{E}\left\{\mathbf{L}_{T+\tau,\tau}\right\} \quad (6.147)$$

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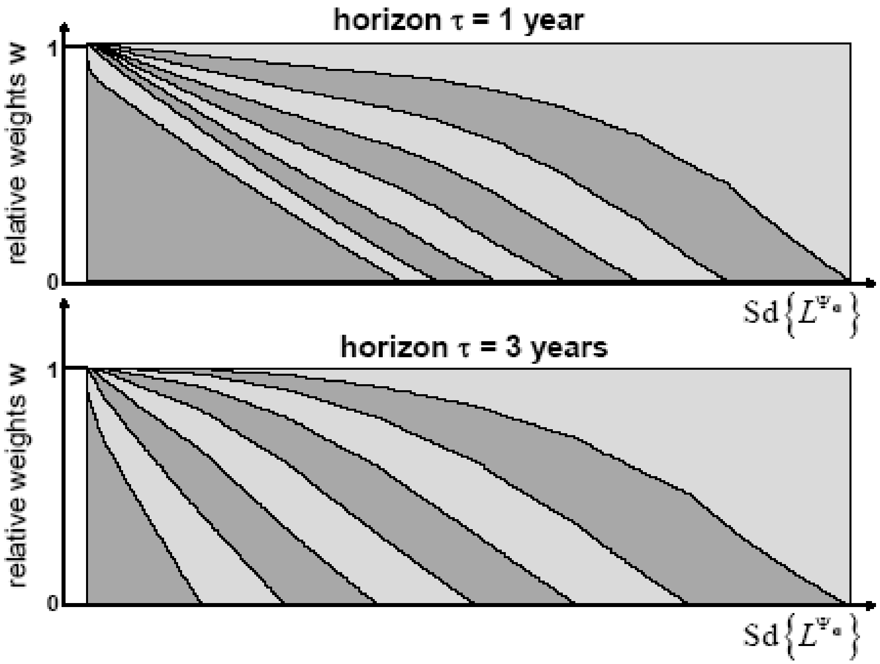
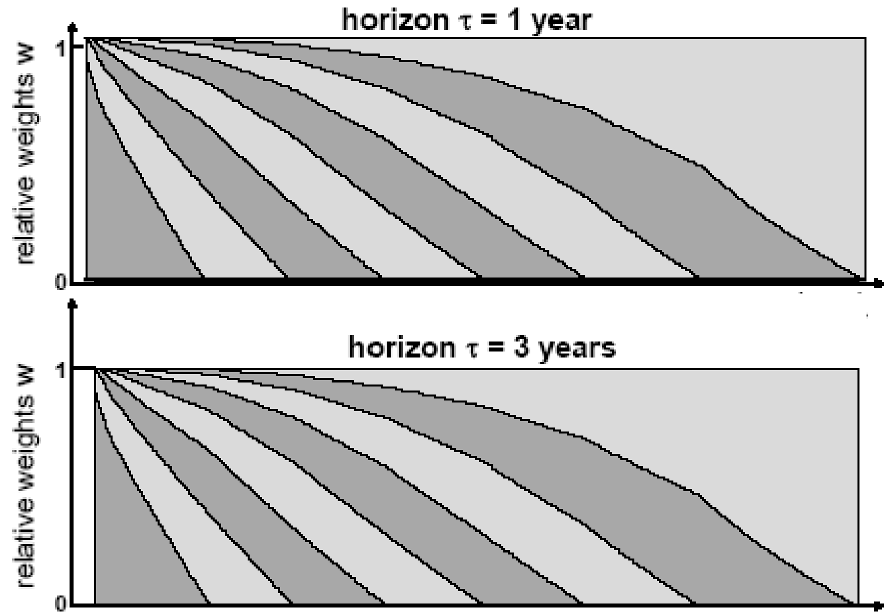


Fig. 6.18.



$$\underset{\substack{\mathbf{w}\in\mathcal{C} \\ \mathbf{w}'\,\mathrm{Cov}\{\mathbf{C}_{t,\tilde{\tau}}\}\,\mathbf{w}=v}}{\operatorname{argmax}} \mathbf{w}'\,\mathrm{E}\{\mathbf{C}_{t,\tilde{\tau}}\}$$

$$\Updownarrow$$

$$\mathbf{w}(v) = \underset{\substack{\mathbf{w}\in\mathcal{C} \\ \mathbf{w}'\,\mathrm{Cov}\{\mathbf{L}_{T+\tau,\tau}\}\mathbf{w}=v}}{\operatorname{argmax}} \mathbf{w}'\,\mathrm{E}\{\mathbf{L}_{T+\tau,\tau}\} \quad (6.147)$$

$$\widetilde{\mathbf{w}}(v) \equiv \underset{\substack{\mathbf{w}\in\mathcal{C} \\ \mathbf{w}'\,\mathrm{Cov}\{\mathbf{C}_{T+\tau,\tau}\}\mathbf{w}=v}}{\operatorname{argmax}} \mathbf{w}'\,\mathrm{E}\{\mathbf{C}_{T+\tau,\tau}\} \quad (6.164)$$