

BENCHMARK ALLOCATION

Risk and Asset Allocation - Springer – *symmys.com*

Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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total-return
allocation

$$\left\{ \begin{array}{l} \Psi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} \quad (6.169) \\ S(\alpha) \approx \tilde{\mathcal{H}}(\mathbb{E}\{\Psi_{\alpha}\}, \text{Var}\{\Psi_{\alpha}\}) \quad (6.173) \end{array} \right.$$

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total-return allocation

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$$\tilde{\alpha} = \alpha_{MV} + [e - \mathbb{E}\{\Psi_{\alpha_{MV}}\}] \frac{\alpha_{SR} - \alpha_{MV}}{\mathbb{E}\{\Psi_{\alpha_{SR}}\} - \mathbb{E}\{\Psi_{\alpha_{MV}}\}} \quad (6.175)$$

$$\alpha_{SR} \stackrel{(6.177)}{=} \frac{w \text{Cov}\{\mathbf{P}_{T+\tau}\}^{-1} \mathbb{E}\{\mathbf{P}_{T+\tau}\}}{\mathbf{p}_T' \text{Cov}\{\mathbf{P}_{T+\tau}\}^{-1} \mathbb{E}\{\mathbf{P}_{T+\tau}\}}$$

$$\alpha_{MV} \stackrel{(6.176)}{\equiv} \frac{w \text{Cov}\{\mathbf{P}_{T+\tau}\}^{-1} \mathbf{p}_T}{\mathbf{p}_T' \text{Cov}\{\mathbf{P}_{T+\tau}\}^{-1} \mathbf{p}_T}$$

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$$\left\{ \begin{array}{l} \Phi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} - \gamma \tilde{\beta}' \mathbf{P}_{T+\tau} \quad (6.170) \\ \gamma \equiv \frac{\alpha' \mathbf{p}_T}{\tilde{\beta}' \mathbf{p}_T} \quad (6.171) \\ S(\alpha) \approx \tilde{\mathcal{K}}(\text{EOP}(\alpha), \text{TE}^2(\alpha)) \quad (6.181) \end{array} \right.$$

$$\text{EOP}(\alpha) \equiv \mathbb{E}\{\Phi_{\alpha}\} \quad (6.178)$$

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benchmark allocation

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$$\text{TE}(\alpha) \equiv \text{Sd}\{\Phi_{\alpha}\} \quad (6.179)$$

$$\hat{\alpha}(u) = \underset{\substack{\alpha' \mathbf{p}_T = w \\ \text{TE}^2(\alpha) = u}}{\text{argmax}} \text{EOP}(\alpha) \quad (6.182)$$

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benchmark allocation

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$$EOP(\alpha) \equiv E\{\Phi_{\alpha}\} \quad (6.178)$$

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$$\hat{\alpha}(u) = \underset{\substack{\alpha' \mathbf{p}_T = w \\ TE^2(\alpha) = u}}{\text{argmax}} EOP(\alpha) \quad (6.182)$$

$$\hat{\alpha} = \beta + [e - E\{\Psi_{\beta}\}] \frac{\alpha_{SR} - \alpha_{MV}}{E\{\Psi_{\alpha_{SR}}\} - E\{\Psi_{\alpha_{MV}}\}} \quad (6.190)$$

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$$\Psi_{\alpha} \equiv \alpha' P_{T+\tau} \quad (6.169)$$

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$$\tilde{\alpha}(v) = \underset{\substack{\alpha' p_T = w \\ \text{Var}\{\Psi_{\alpha}\} = v}}{\text{argmax}} E\{\Psi_{\alpha}\} \quad (6.174)$$

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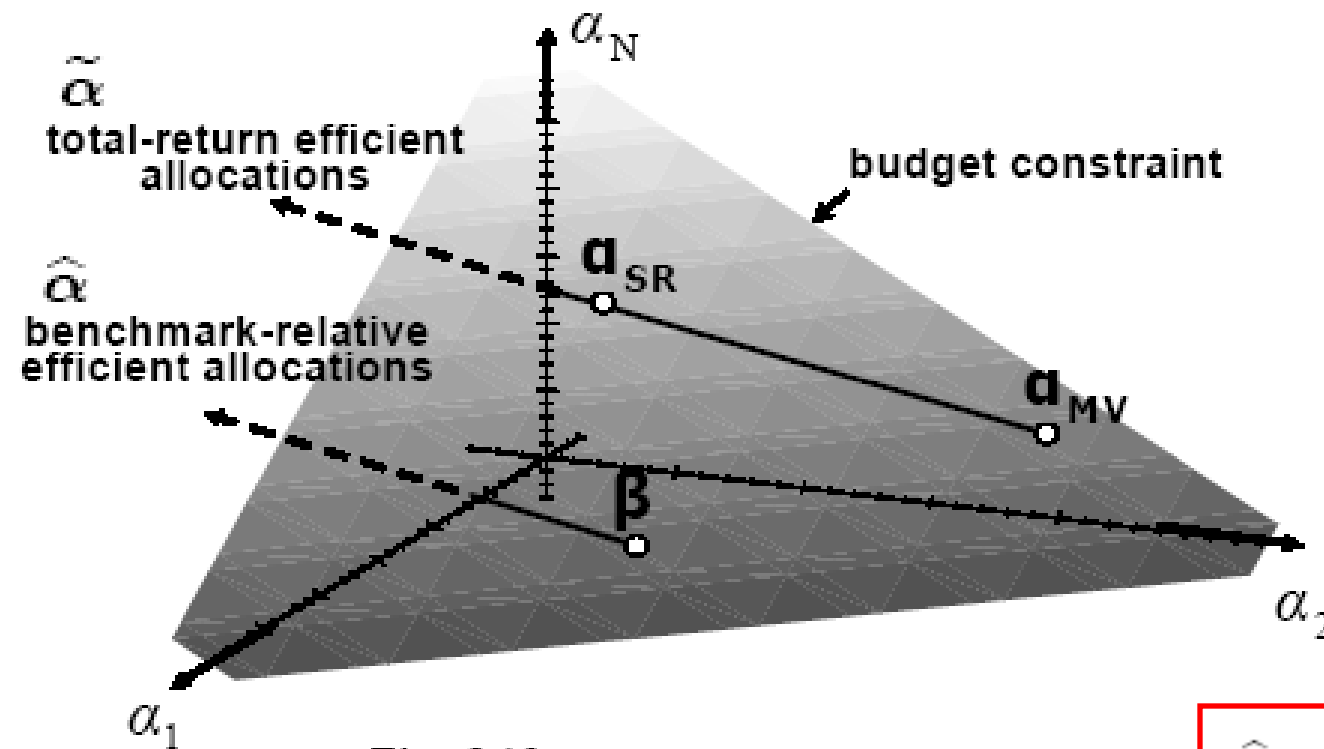


Fig. 6.19

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$$\Psi_{\alpha} \equiv \alpha' P_{T+\tau} \quad (6.169)$$

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$$\tilde{\alpha}(v) = \underset{\substack{\alpha' p_T = w \\ \text{Var}\{\Psi_{\alpha}\} = v}}{\text{argmax}} E\{\Psi_{\alpha}\} \quad (6.174)$$

$$\tilde{\alpha} : v = \frac{A}{D} e^2 - \frac{2wB}{D} e + \frac{w^2 C}{D} \Leftrightarrow \tilde{\alpha} = \alpha_{MV} + [e - E\{\Psi_{\alpha_{MV}}\}] \frac{\alpha_{SR} - \alpha_{MV}}{E\{\Psi_{\alpha_{SR}}\} - E\{\Psi_{\alpha_{MV}}\}} \quad (6.175)$$

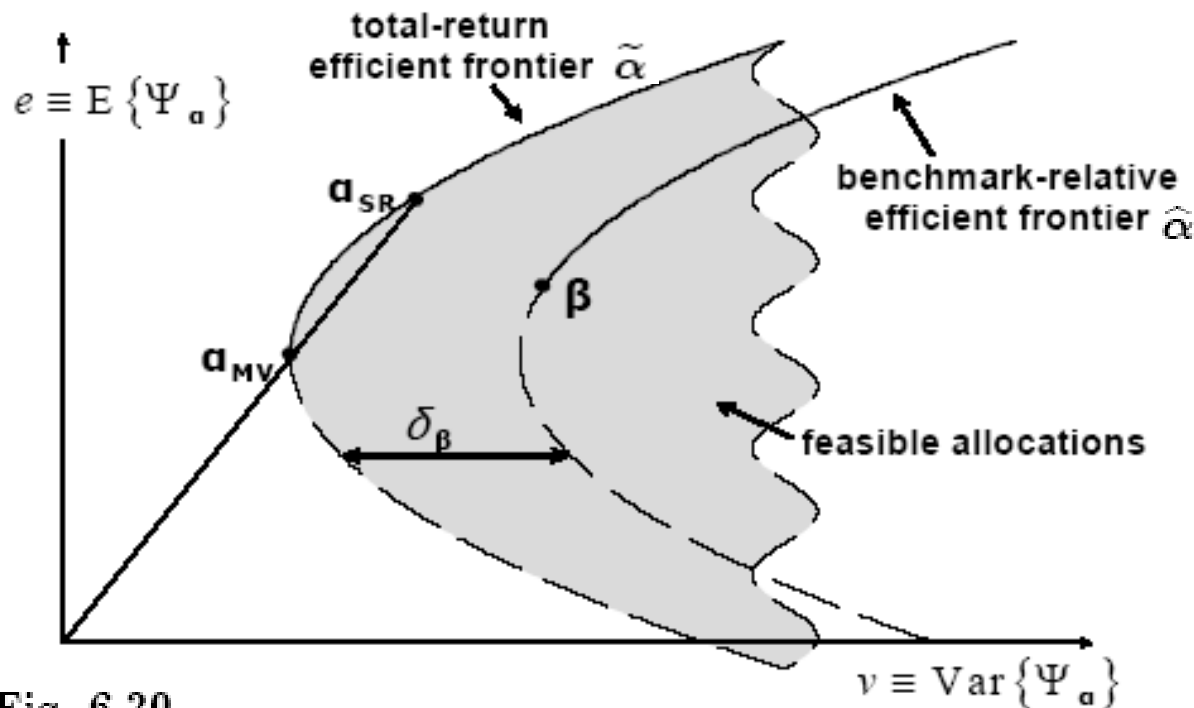


Fig. 6.20.

$$\alpha_{SR} = \frac{(6.177) \quad w \text{Cov}\{P_{T+\tau}\}^{-1} E\{P_{T+\tau}\}}{p_T' \text{Cov}\{P_{T+\tau}\}^{-1} E\{P_{T+\tau}\}}$$

$$\alpha_{MV} \equiv \frac{(6.176) \quad w \text{Cov}\{P_{T+\tau}\}^{-1} p_T}{p_T' \text{Cov}\{P_{T+\tau}\}^{-1} p_T}$$

$$EOP(\alpha) \equiv E\{\Phi_{\alpha}\} \quad (6.178)$$

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$$\hat{\alpha}(u) = \underset{\substack{\alpha' p_T = w \\ TE^2(\alpha) = u}}{\text{argmax}} EOP(\alpha) \quad (6.182)$$

$$\hat{\alpha} : v = \frac{A}{D} e^2 - \frac{2wB}{D} e + \frac{w^2 C}{D} + \delta_{\beta} \Leftrightarrow \hat{\alpha} = \beta + [e - E\{\Psi_{\beta}\}] \frac{\alpha_{SR} - \alpha_{MV}}{E\{\Psi_{\alpha_{SR}}\} - E\{\Psi_{\alpha_{MV}}\}} \quad (6.190)$$

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$$\tilde{\alpha}(v) = \underset{\substack{\alpha' p_T = w \\ \text{Var}\{\Psi_{\alpha}\} = v}}{\text{argmax}} E\{\Psi_{\alpha}\} \quad (6.174)$$

$$\tilde{\alpha} : u = \frac{A}{D} p^2 + \delta \beta \quad \Leftarrow \quad \tilde{\alpha} = \alpha_{MV} + [e - E\{\Psi_{\alpha_{MV}}\}] \frac{\alpha_{SR} - \alpha_{MV}}{E\{\Psi_{\alpha_{SR}}\} - E\{\Psi_{\alpha_{MV}}\}} \quad (6.175)$$

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$$EOP(\alpha) \equiv E\{\Phi_{\alpha}\} \quad (6.178)$$

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$$\hat{\alpha}(u) = \underset{\substack{\alpha' p_T = w \\ TE^2(\alpha) = u}}{\text{argmax}} EOP(\alpha) \quad (6.182)$$

$$\hat{\alpha} : u = \frac{A}{D} p^2 \quad \Leftarrow \quad \hat{\alpha} = \beta + [e - E\{\Psi_{\beta}\}] \frac{\alpha_{SR} - \alpha_{MV}}{E\{\Psi_{\alpha_{SR}}\} - E\{\Psi_{\alpha_{MV}}\}} \quad (6.190)$$

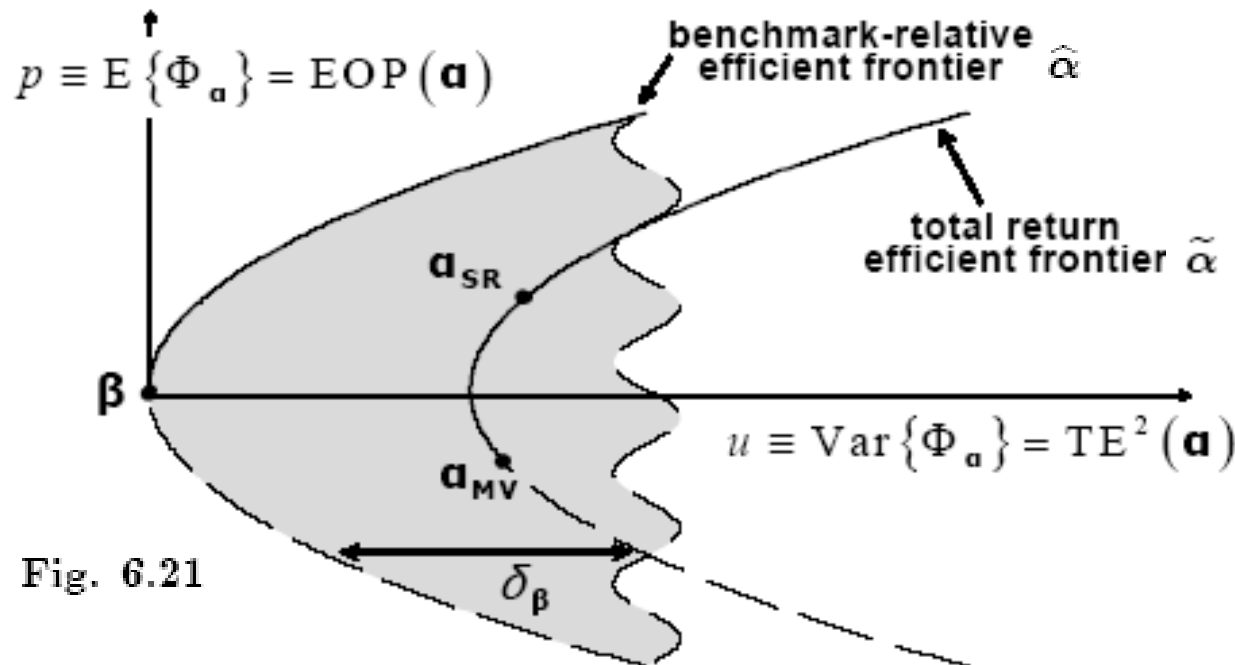


Fig. 6.21

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$$\tilde{\alpha}(v) = \underset{\substack{\alpha' p_T = w \\ \text{Var}\{\Psi_{\alpha}\} = v}}{\text{argmax}} E\{\Psi_{\alpha}\} \quad (6.174)$$

$$(6.200) \quad \tilde{\alpha}: u = \frac{A}{D} p^2 + \delta \beta. \Leftrightarrow \tilde{\alpha} = \alpha_{MV} + [e - E\{\Psi_{\alpha_{MV}}\}] \frac{\alpha_{SR} - \alpha_{MV}}{E\{\Psi_{\alpha_{SR}}\} - E\{\Psi_{\alpha_{MV}}\}} \quad (6.175)$$

$$(6.177) \quad \alpha_{SR} = \frac{w \text{Cov}\{P_{T+\tau}\}^{-1} E\{P_{T+\tau}\}}{p_T' \text{Cov}\{P_{T+\tau}\}^{-1} E\{P_{T+\tau}\}}$$

$$(6.176) \quad \alpha_{MV} \equiv \frac{w \text{Cov}\{P_{T+\tau}\}^{-1} p_T}{p_T' \text{Cov}\{P_{T+\tau}\}^{-1} p_T}$$

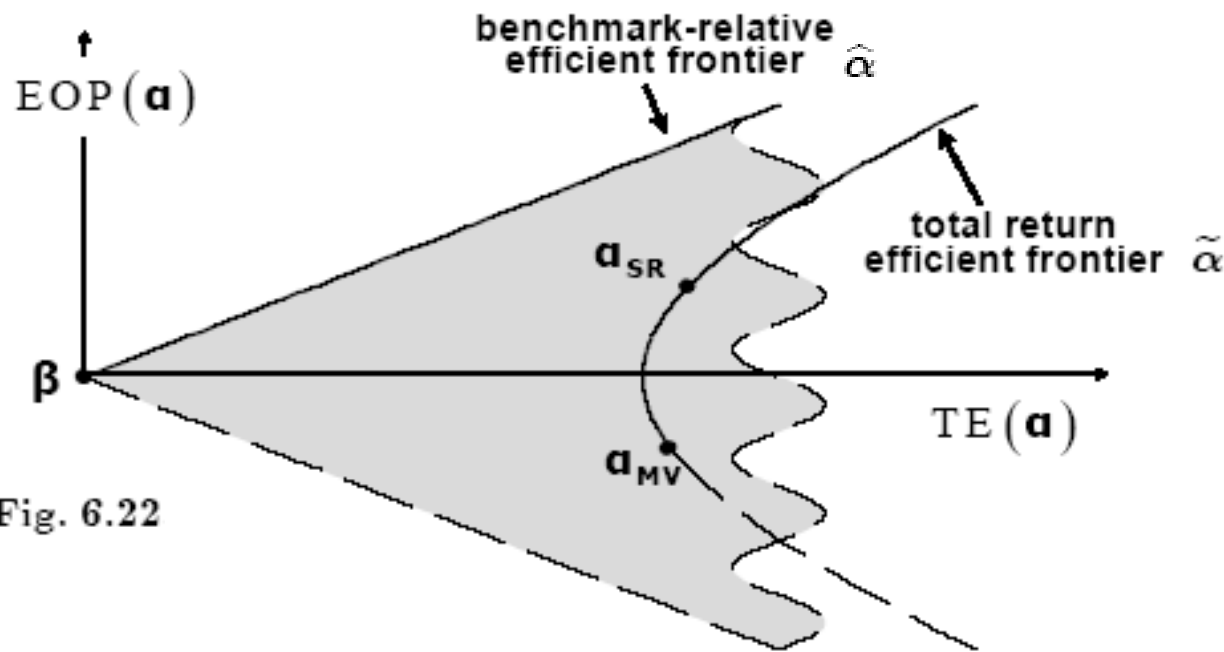


Fig. 6.22

$$EOP(\alpha) \equiv E\{\Phi_{\alpha}\} \quad (6.178)$$

$$TE(\alpha) \equiv Sd\{\Phi_{\alpha}\} \quad (6.179)$$

$$\hat{\alpha}(u) = \underset{\substack{\alpha' p_T = w \\ TE^2(\alpha) = u}}{\text{argmax}} EOP(\alpha) \quad (6.182)$$

$$(6.201) \quad EOP(\hat{\alpha}) = \sqrt{\frac{D}{A}} TE(\hat{\alpha}) \Leftrightarrow (6.199) \quad \hat{\alpha}: u = \frac{A}{D} p^2, \Leftrightarrow \hat{\alpha} = \beta + [e - E\{\Psi_{\beta}\}] \frac{\alpha_{SR} - \alpha_{MV}}{E\{\Psi_{\alpha_{SR}}\} - E\{\Psi_{\alpha_{MV}}\}} \quad (6.190)$$