

INDEPENDENCE - *Risk and Asset Allocation* - Springer – *symmys.com*

Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from **www.symmys.com**

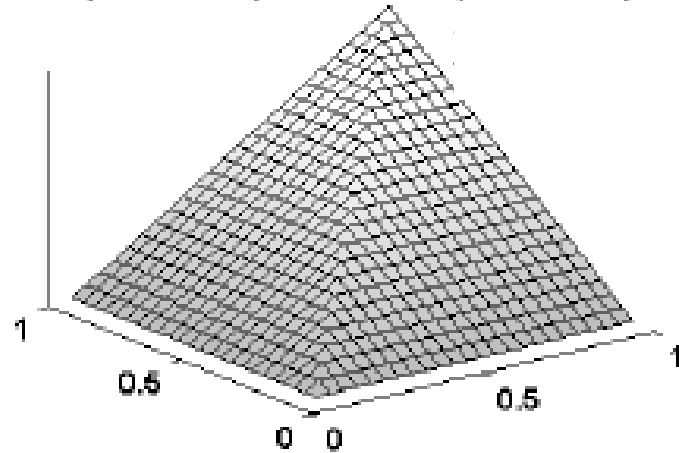
INDEPENDENCE - *Risk and Asset Allocation* - Springer – *symmys.com*

$$f_{\mathbf{X}_B | \mathbf{x}_A}(\mathbf{x}_B) = \frac{f_{\mathbf{X}}(\mathbf{x}_A, \mathbf{x}_B)}{f_{\mathbf{X}_A}(\mathbf{x}_A)} \quad (2.40)$$

$$(\mathbf{X}_A, \mathbf{X}_B) \text{ independent} \Rightarrow f_{\mathbf{X}_B | \mathbf{x}_A}(\mathbf{x}_B) = f_{\mathbf{X}_B}(\mathbf{x}_B) \quad (2.45)$$

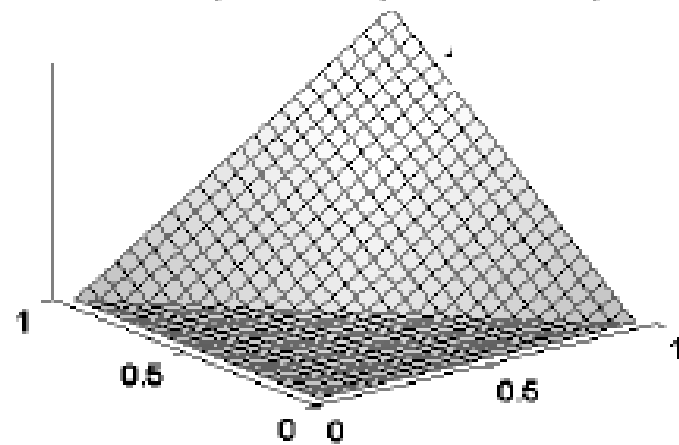
$$(\mathbf{X}_A, \mathbf{X}_B) \text{ independent} \Leftrightarrow f_{\mathbf{X}}(\mathbf{x}_A, \mathbf{x}_B) = f_{\mathbf{X}_A}(\mathbf{x}_A) f_{\mathbf{X}_B}(\mathbf{x}_B) \quad (2.44)$$

$$T(u_m, u_n) \equiv \min(u_m, u_n) \quad (2.106)$$



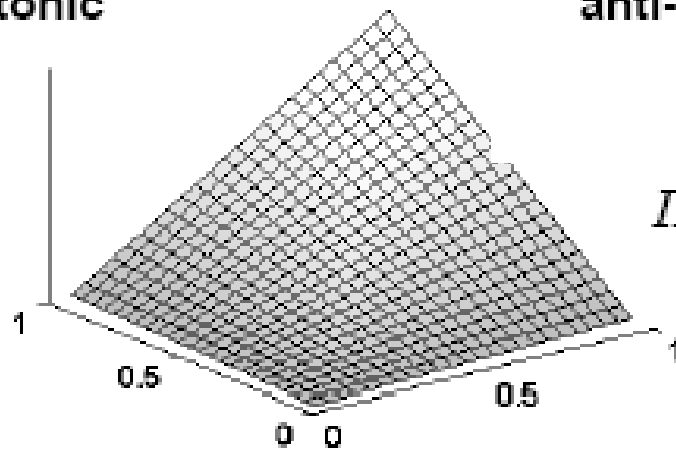
co-monotonic

$$B(u_m, u_n) \equiv \max(u_m + u_n - 1, 0) \quad (2.105)$$



anti-monotonic

$$\Pi(u_m, u_n) \equiv u_m u_n \quad (2.102)$$



independent

Fig. 2.10. Cumulative distribution function of special bivariate copulas

INDEPENDENCE - *Risk and Asset Allocation* - Springer – *symmys.com*

$$(X_m, X_n) \text{ independent} \Leftrightarrow \text{Dep} \{X_m, X_n\} \equiv 0.$$

dependence

dependence

$$(X_m, X_n) \text{ independent} \Leftrightarrow \text{Dep} \{X_m, X_n\} \equiv 0.$$

$$\text{SW} \{X_m, X_n\} \equiv k_p \left(\int_{\mathbb{Q}} |F_{U_m, U_n}(u_m, u_n) - \Pi(u_m, u_n)|^p du_m du_n \right)^{\frac{1}{p}} \tag{2.103}$$

$$k_p \equiv \frac{1}{\|B - \Pi\|_p} = \frac{1}{\|T - \Pi\|_p} \tag{2.107}$$

dependence

$$(X_m, X_n) \text{ independent} \Leftrightarrow \text{Dep} \{X_m, X_n\} \equiv 0.$$

$$\text{SW} \{X_m, X_n\} \equiv k_p \left(\int_{\mathbb{Q}} |F_{U_m, U_n}(u_m, u_n) - \Pi(u_m, u_n)|^p du_m du_n \right)^{\frac{1}{p}} \tag{2.103}$$

$$k_p \equiv \frac{1}{\|B - \Pi\|_p} = \frac{1}{\|T - \Pi\|_p} \tag{2.107}$$

concordance

$$\tau \{X_m, X_n\} \equiv 4 \int_{\mathbb{Q}} \left(F_{U_m, U_n}(u_m, u_n) - \frac{1}{4} \right) f_{U_m, U_n}(u_m, u_n) du_m du_n \tag{2.128}$$

INDEPENDENCE - *Risk and Asset Allocation* - Springer – *symmys.com*

$$(X_m, X_n) \text{ independent} \Leftrightarrow \text{Dep} \{X_m, X_n\} \equiv 0.$$

$$\text{SW} \{X_m, X_n\} \equiv k_p \left(\int_{\mathbb{Q}} |F_{U_m, U_n}(u_m, u_n) - \Pi(u_m, u_n)|^p du_m du_n \right)^{\frac{1}{p}} \quad (2.103)$$

$$k_p \equiv \frac{1}{\|B - \Pi\|_p} = \frac{1}{\|T - \Pi\|_p} \quad (2.107)$$

$$(X_m, X_n) \text{ independent} \Rightarrow \text{Con} \{X_m, X_n\} = 0.$$

$$\tau \{X_m, X_n\} \equiv 4 \int_{\mathbb{Q}} \left(F_{U_m, U_n}(u_m, u_n) - \frac{1}{4} \right) f_{U_m, U_n}(u_m, u_n) du_m du_n \quad (2.128)$$

dependence

concordance

INDEPENDENCE - *Risk and Asset Allocation* - Springer – *symmys.com*

$$(X_m, X_n) \text{ independent} \Leftrightarrow \text{Dep} \{X_m, X_n\} \equiv 0.$$

$$\text{SW} \{X_m, X_n\} \equiv k_p \left(\int_{\mathbb{Q}} |F_{U_m, U_n}(u_m, u_n) - \Pi(u_m, u_n)|^p du_m du_n \right)^{\frac{1}{p}} \quad (2.103)$$

$$k_p \equiv \frac{1}{\|B - \Pi\|_p} = \frac{1}{\|T - \Pi\|_p} \quad (2.107)$$

$$(X_m, X_n) \text{ independent} \Rightarrow \text{Con} \{X_m, X_n\} = 0.$$

$$\rho \{X_m, X_n\} \equiv \frac{\text{Cov} \{U_m, U_n\}}{\text{Sd} \{U_m\} \text{Sd} \{U_n\}} \quad (2.130)$$

$$\tau \{X_m, X_n\} \equiv 4 \int_{\mathbb{Q}} \left(F_{U_m, U_n}(u_m, u_n) - \frac{1}{4} \right) f_{U_m, U_n}(u_m, u_n) du_m du_n \quad (2.128)$$

dependence

concordance