Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

MARKET INVARIANTS - EQUITIES

Risk and Asset Allocation **- Springer –** `symmys.com`

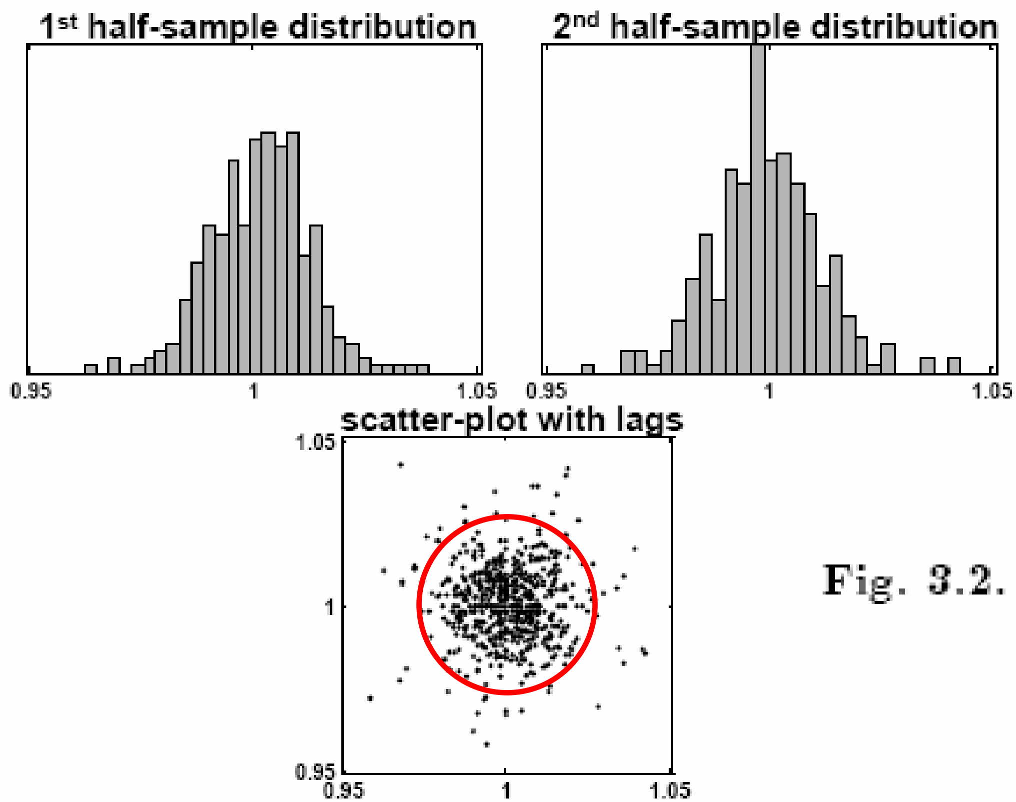

Fig. 3.2.

total returns $$H_{t,\tau} \equiv \frac{P_t}{P_{t-\tau}} \qquad (3.9)$$

MARKET INVARIANTS - EQUITIES

***Risk and Asset Allocation* - Springer – `symmys.com`**

total returns $$H_{t,\tau} \equiv \frac{P_t}{P_{t-\tau}} \qquad (3.9)$$

linear returns

$$L_{t,\tau} \equiv \frac{P_t}{P_{t-\tau}} - 1 \qquad (3.10)$$

$$\Leftrightarrow$$

compounded returns

$$\boxed{C_{t,\tau} \equiv \ln\left(\frac{P_t}{P_{t-\tau}}\right)} \qquad (3.11)$$

MARKET INVARIANTS - BONDS

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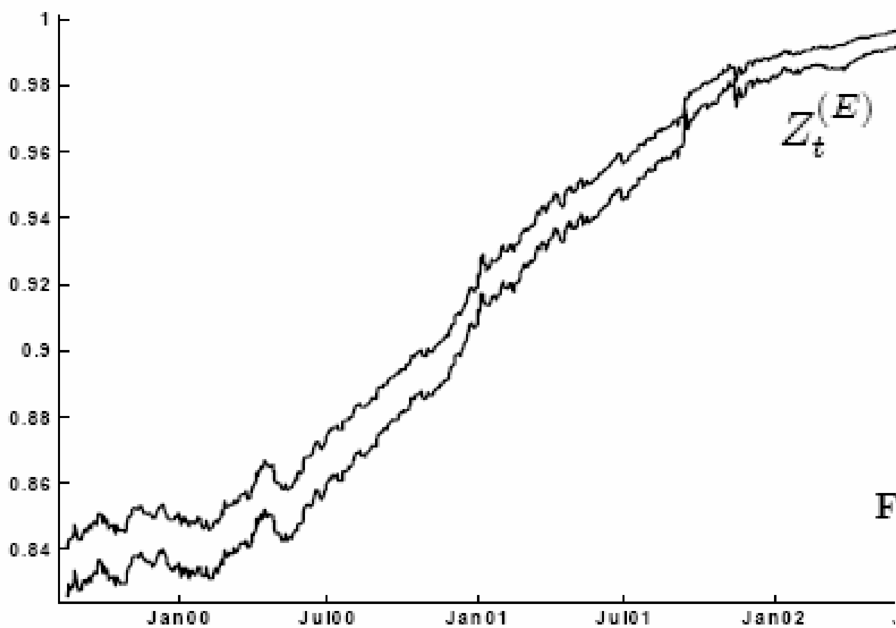


$Z_t^{(E)}$ **price at time t of bond maturing at time E**

Fig. 3.3. Lack of time-homogeneity of bond prices

Fig. 3.4. Time-homogeneity of bond prices with fixed time to maturtity

price at time t of bond with time to maturity υ $Z_t^{(t+\upsilon)}$

MARKET INVARIANTS - BONDS

Risk and Asset Allocation **- Springer –** `symmys.com`

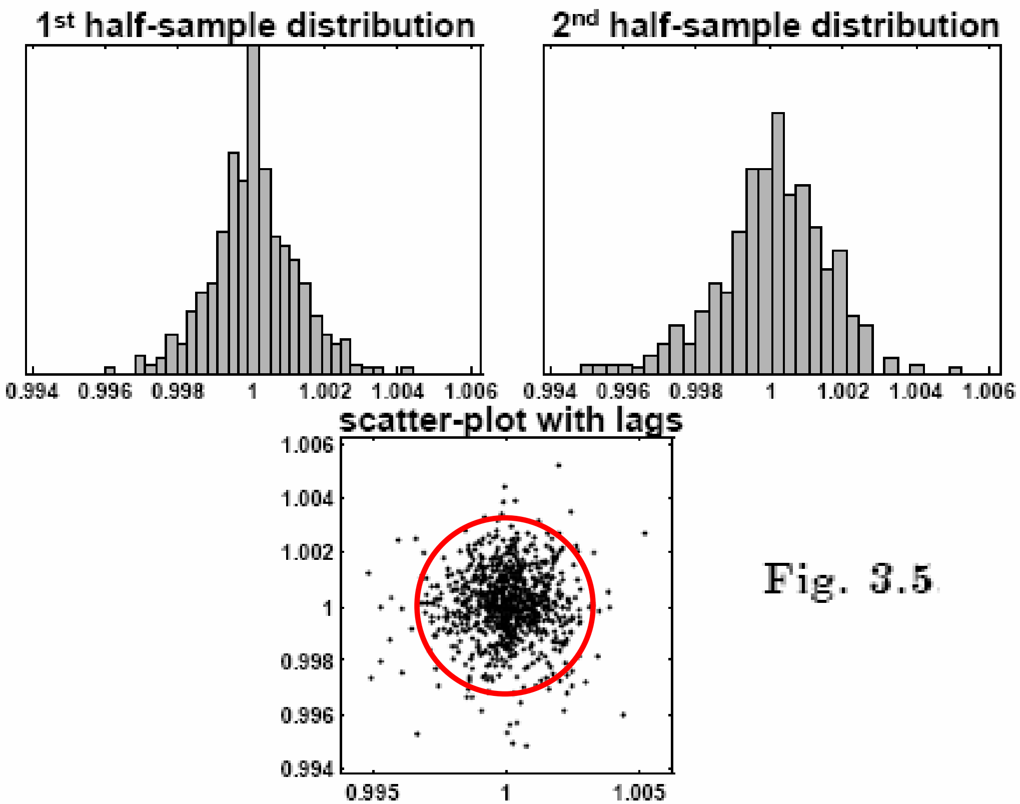


Fig. 3.5

$$R_{t,\tilde{\tau}}^{(v)} \equiv \frac{Z_t^{(t+v)}}{Z_{t-\tilde{\tau}}^{(t+v-\tilde{\tau})}} \quad (3.27)$$

pseudo-returns

MARKET INVARIANTS - BONDS

Risk and Asset Allocation **- Springer –** `symmys.com`

$$R_{t,\widetilde{\tau}}^{(\upsilon)} \equiv \frac{Z_t^{(t+\upsilon)}}{Z_{t-\widetilde{\tau}}^{(t+\upsilon-\widetilde{\tau})}} \qquad (3.27)$$

pseudo-returns

$$\Updownarrow$$

$$X_{t,\widetilde{\tau}}^{(\upsilon)} \equiv -\frac{1}{\upsilon} \ln\left(R_{t,\widetilde{\tau}}^{(\upsilon)}\right) \qquad (3.31)$$

MARKET INVARIANTS - BONDS

Risk and Asset Allocation - Springer – `symmys.com`

$$R_{t,\tilde{\tau}}^{(v)} \equiv \frac{Z_t^{(t+v)}}{Z_{t-\tilde{\tau}}^{(t+v-\tilde{\tau})}} \qquad (3.27)$$

pseudo-returns

$$\Updownarrow$$

$$X_{t,\tilde{\tau}}^{(v)} : -\frac{1}{v} \ln\left(R_{t,\tilde{\tau}}^{(v)}\right) \qquad (3.31)$$

$$\Updownarrow$$

changes in yield to maturity

$$\boxed{X_{t,\tilde{\tau}}^{(v)} \equiv Y_t^{(v)} - Y_{t-\tilde{\tau}}^{(v)}} \qquad (3.31)$$

$$Y_t^{(v)} \equiv -\frac{1}{v} \ln\left(Z_t^{(t+v)}\right) \qquad (3.30)$$

yield to maturity

MARKET INVARIANTS - DERIVATIVES, no implied volatility

$C_t(K, E)$ **price at time t of call with strike K expiring at time E**

MARKET INVARIANTS - DERIVATIVES, no implied volatility

$C_t\left(K,E\right)$ **price at time t of call with strike K expiring at time E**

$$Y_t^{(E)} \equiv \frac{C_t\left(K,E\right)}{C_{t-1}\left(K,E\right)}$$ **total return**

MARKET INVARIANTS - DERIVATIVES, no implied volatility

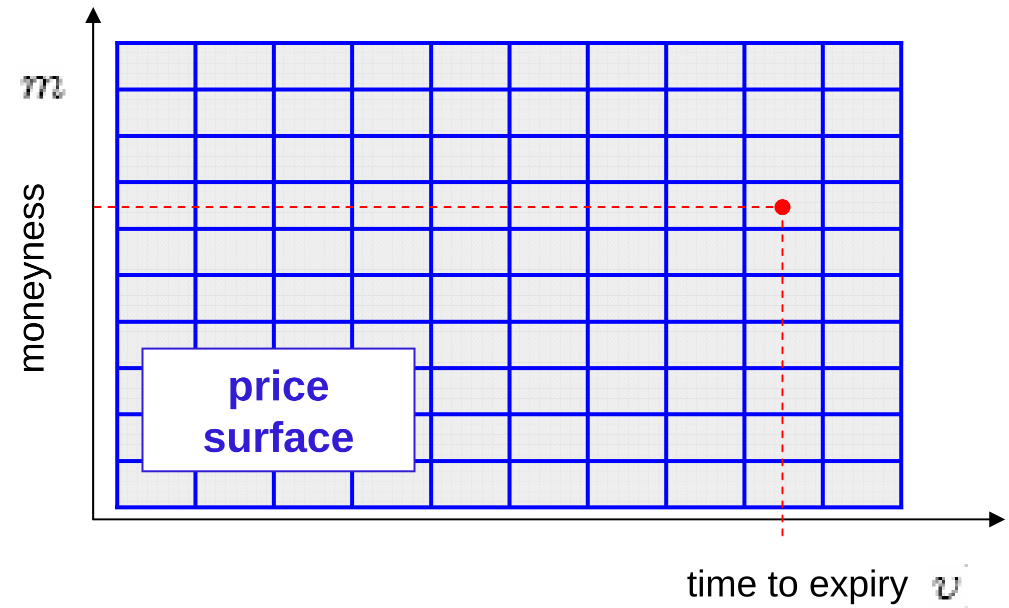
$C_t(K,E)$ **price at time t of call with strike K expiring at time E**

$$Y_t^{(E)} \equiv \frac{C_t(K,E)}{C_{t-1}(K,E)}$$

MARKET INVARIANTS - DERIVATIVES, no implied volatility

$C_t(K,E)$ **price at time t of call with strike K expiring at time E**

$$Y_t^{(E)} \equiv \frac{C_t(K,E)}{C_{t-1}(K,E)}$$



$C_t(mS_t, t+v)$ **price at time t of call with strike** mS_t, **and time to expiry** v

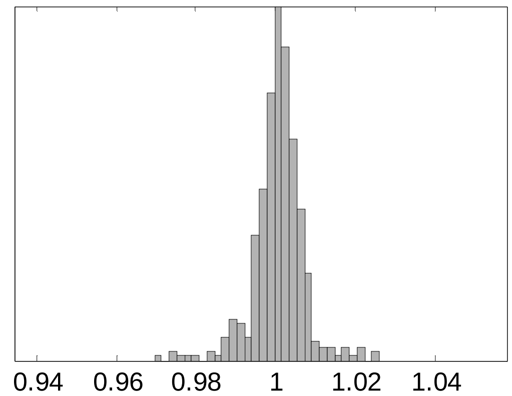
"moneyness"

MARKET INVARIANTS - DERIVATIVES, no implied volatility

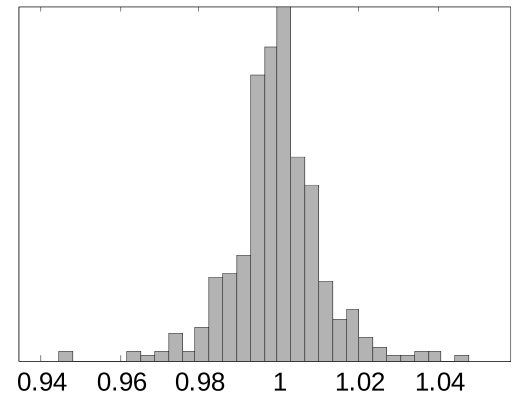
$C_t(K,E)$ **price at time t of call with strike K expiring at time E**

$$Y_t^{(E)} \equiv \frac{C_t(K,E)}{C_{t-1}(K,E)}$$

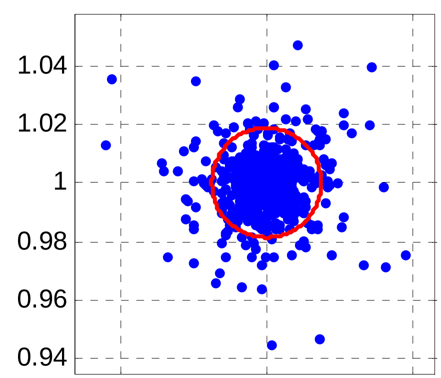
1st half-sample distribution



2nd half-sample distribution



$C_t(mS_t, t+v)$ **price at time t of call with strike** mS_t **and time to expiry** v



scatter-plot with lags

$$\boxed{Z_t^{(m,v)} \equiv \frac{C_t(mS_t, t+v)}{C_{t-1}(mS_{t-1}, t-1+v)},}$$

pseudo-return

MARKET INVARIANTS - DERIVATIVES, implied volatility

Invariants:
compounded returns

$$\epsilon_t \equiv \ln S_t - \ln S_{t-1}$$

MARKET INVARIANTS - DERIVATIVES, implied volatility

Invariants:
compounded returns

$$\epsilon_t \equiv \ln S_t - \ln S_{t-1} \sim \mathrm{N}\left(\mu, \sigma^2\right)$$ **Black-Scholes assumption**

MARKET INVARIANTS - DERIVATIVES, implied volatility

Invariants:
compounded returns

$$\epsilon_t \equiv \ln S_t - \ln S_{t-1} \quad \sim \mathrm{N}\left(\mu, \sigma^2\right)$$

Black-Scholes assumption

$$C_{BS}\left(t, S, \sigma; K, E\right) \equiv S\Phi\left(d_1\right) - Ke^{-r(E-t)}\Phi\left(d_2\right)$$

theory > price:

$$C_t\left(K, E\right) = C_{BS}\left(t, S_t, \sigma; K, E\right)$$

$$d_1 \equiv \frac{-\ln\left(\frac{K}{S}\right) + \left(E-t\right)\left(r + \frac{\sigma^2}{2}\right)}{\sqrt{\sigma^2\left(E-t\right)}}$$

$$d_2 \equiv \frac{-\ln\left(\frac{K}{S}\right) + \left(E-t\right)\left(r - \frac{\sigma^2}{2}\right)}{\sqrt{\sigma^2\left(E-t\right)}}.$$

MARKET INVARIANTS - DERIVATIVES, implied volatility

Invariants:
compounded returns

$$\epsilon_t \equiv \ln S_t - \ln S_{t-1} \quad \sim \mathrm{N}\left(\cancel{\mu}, \sigma^2\right)$$

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$$C_{BS}\left(t, S, \sigma; K, E\right) \equiv S\Phi\left(d_1\right) - Ke^{\boxed{-r}\left(E-t\right)}\Phi\left(d_2\right)$$

theory > price: $\mathbb{Q}$

$$C_t\left(K, E\right) = C_{BS}\left(t, S_t, \sigma; K, E\right)$$

$$d_1 \equiv \frac{-\ln\left(\frac{K}{S}\right) + \left(E-t\right)\left(\boxed{r} + \frac{\sigma^2}{2}\right)}{\sqrt{\sigma^2\left(E-t\right)}}$$

$$d_2 \equiv \frac{-\ln\left(\frac{K}{S}\right) + \left(E-t\right)\left(\boxed{r} - \frac{\sigma^2}{2}\right)}{\sqrt{\sigma^2\left(E-t\right)}}$$

MARKET INVARIANTS - DERIVATIVES, implied volatility

Invariants:
compounded returns

$\epsilon_t \equiv \ln S_t - \ln S_{t-1} \quad \sim \mathrm{N}\left(\mu, \sigma^2\right)$ **Black-Scholes assumption**

theory > price:

$C_t\left(K, E\right) = C_{BS}\left(t, S_t, \sigma; K, E\right)$

implied volatility surface

$\left(t, K, E\right) \mapsto \sigma_t\left(K, E\right)$

MARKET INVARIANTS - DERIVATIVES, implied volatility

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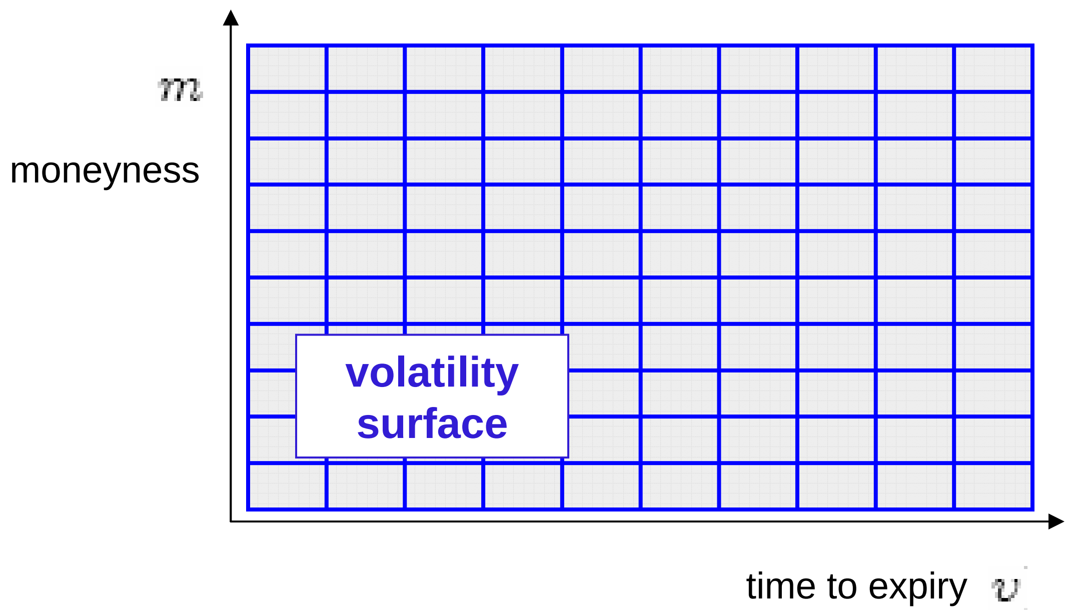
invariant coordinates

$$\left(t, m, v\right) \mapsto \sigma_t\left(m, v\right)$$

moneyness

time to expiry



MARKET INVARIANTS - DERIVATIVES, implied volatility

Invariants:
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$\epsilon_t \equiv \ln S_t - \ln S_{t-1} \quad \sim \mathrm{N}\left(\mu, \sigma^2\right)$ **Black-Scholes assumption**

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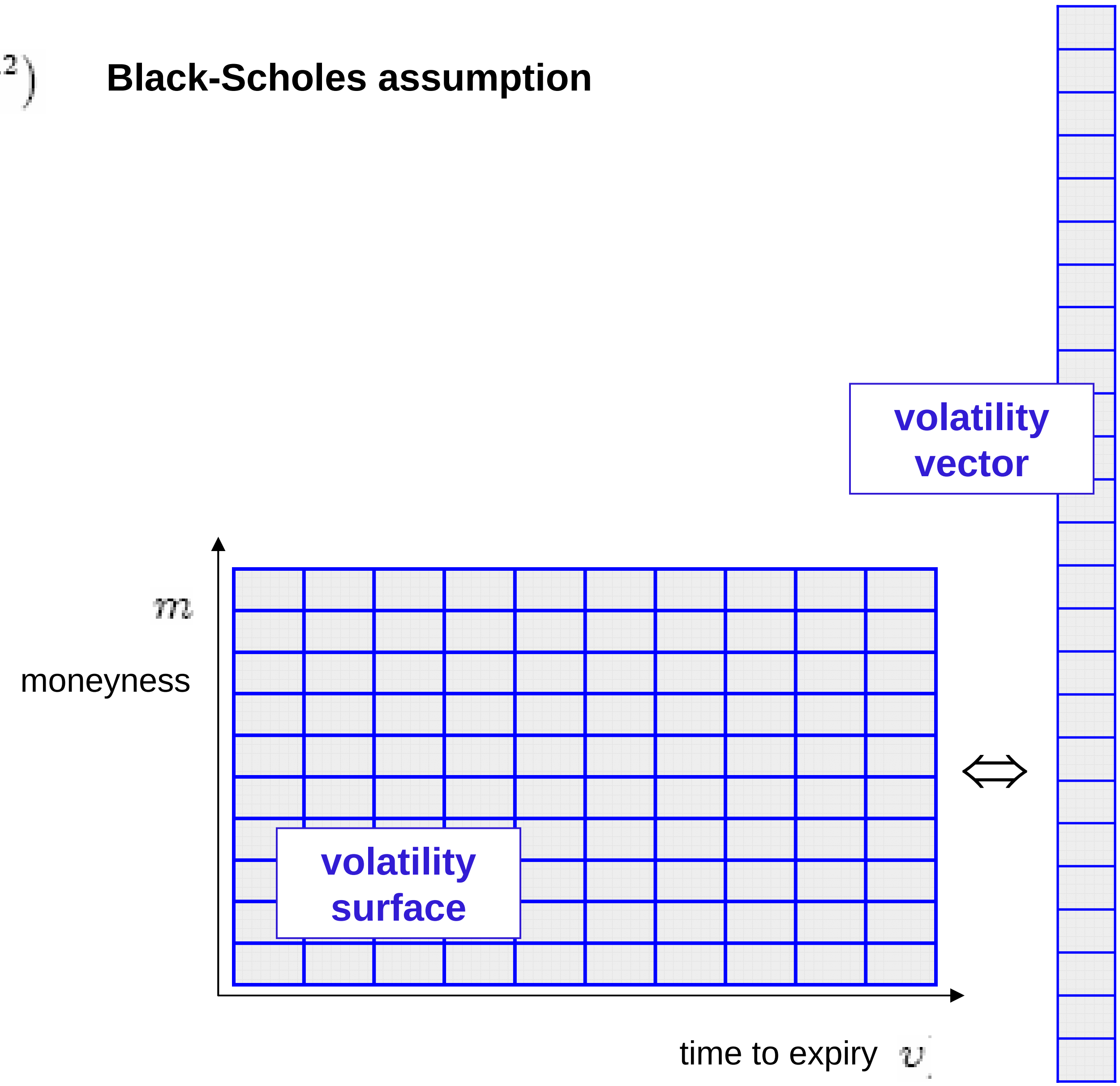
invariant coordinates

$$(t, m, v) \mapsto \sigma_t(m, v) \quad \Leftrightarrow \quad \boldsymbol{\sigma}_t$$



MARKET INVARIANTS - DERIVATIVES, implied volatility

Invariants:
compounded returns

$$\epsilon_t \equiv \ln S_t - \ln S_{t-1} \quad \sim \mathrm{N}\left(\mu, \sigma^2\right)$$

Black-Scholes assumption

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invariant coordinates

$$\left(t, m, v\right) \mapsto \sigma_t\left(m, v\right) \quad \Leftrightarrow \quad \boldsymbol{\sigma}_t$$

invariants

$$\eta_t \equiv \ln \boldsymbol{\sigma}_t - \ln \boldsymbol{\sigma}_{t-1}$$

MARKET INVARIANTS - DERIVATIVES, implied volatility

Invariants:
compounded returns

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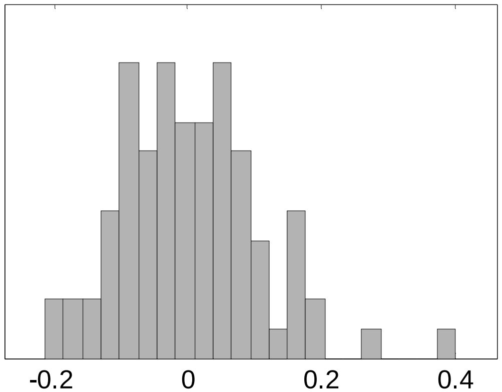
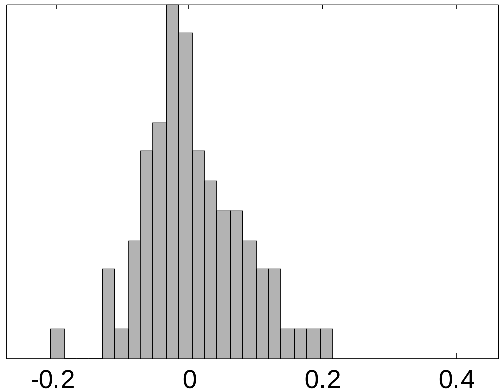
invariants

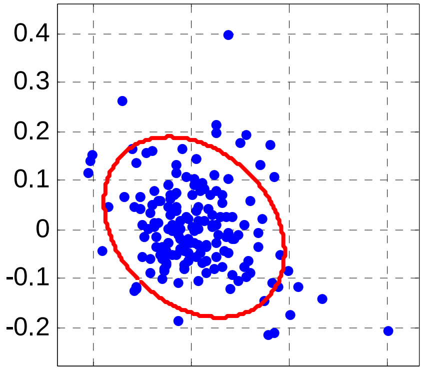
$$\eta_t \equiv \ln \boldsymbol{\sigma}_t - \ln \boldsymbol{\sigma}_{t-1}$$

1st half-sample distribution



2nd half-sample distribution





scatter-plot with lags

MARKET INVARIANTS - DERIVATIVES, implied volatility

Invariants:
compounded returns

$$\epsilon_t \equiv \ln S_t - \ln S_{t-1} \quad \sim \mathrm{N}\left(\mu, \sigma^2\right)$$

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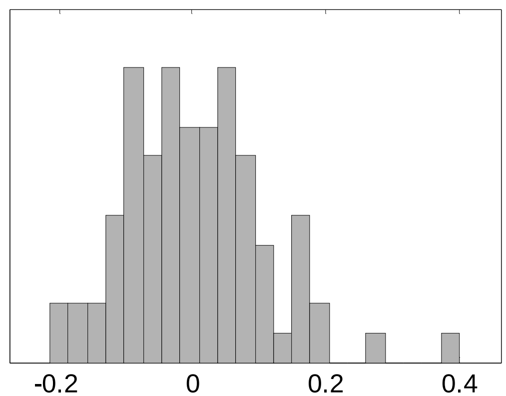
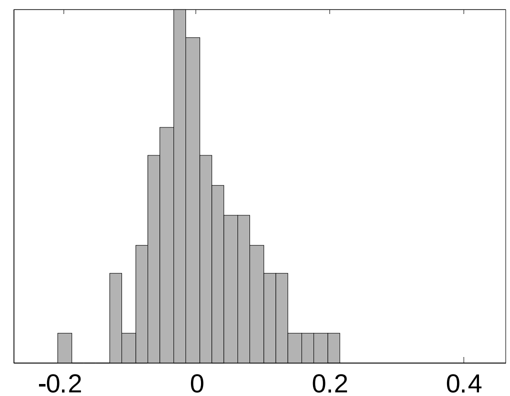
invariants

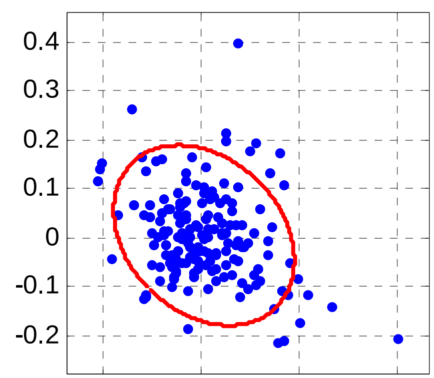
$$\eta_t \equiv \ln \sigma_t - \ln \sigma_{t-1}$$

1st half-sample distribution



2nd half-sample distribution





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MARKET INVARIANTS - DERIVATIVES, implied volatility

Invariants:
compounded returns

$$\epsilon_t \equiv \ln S_t - \ln S_{t-1} \quad \sim \mathrm{N}\left(\mu, \sigma^2\right)$$

theory > price:

$$C_t\left(K, E\right) = C_{BS}\left(t, S_t, \sigma; K, E\right)$$

implied volatility surface

$$\left(t, K, E\right) \mapsto \sigma_t\left(K, E\right)$$

invariant coordinates

$$\left(t, m, v\right) \mapsto \sigma_t\left(m, v\right)$$

invariants

$$\eta_t \equiv \ln \boldsymbol{\sigma}_t - \ln \boldsymbol{\sigma}_{t-1}$$

invariants

$$\eta_t = \ln \boldsymbol{\sigma}_t - \widehat{\Psi} \ln \boldsymbol{\sigma}_{t-1}$$

MARKET INVARIANTS - DERIVATIVES, implied volatility

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theory > price:

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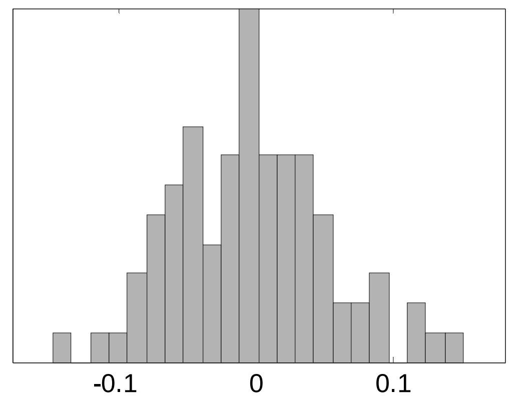
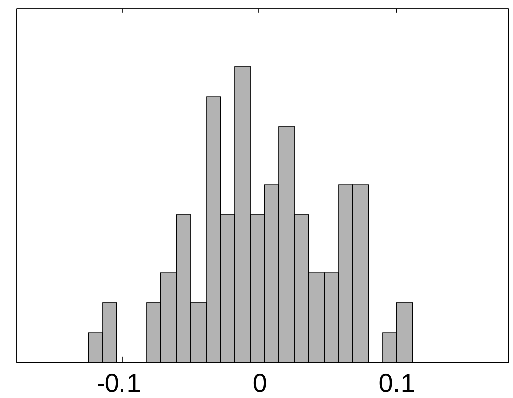
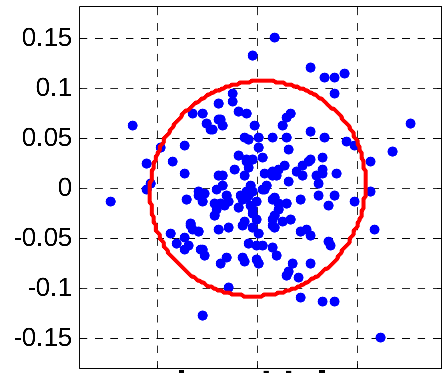
1st half-sample distribution



2nd half-sample distribution





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invariants

$$\eta_t = \ln \boldsymbol{\sigma}_t - \widehat{\Psi} \ln \boldsymbol{\sigma}_{t-1}$$