Attilio Meucci

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Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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$$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (2.155)$$

$$\begin{cases} f^{\mathrm{N}}_{\boldsymbol{\mu},\Sigma}(\mathbf{x}) = (2\pi)^{-\frac{N}{2}} |\boldsymbol{\Sigma}|^{-\frac{1}{2}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})} & (2.156) \\ \phi^{\mathrm{N}}_{\boldsymbol{\mu},\Sigma}(\boldsymbol{\omega}) = e^{i\boldsymbol{\mu}'\omega - \frac{1}{2}\omega'\boldsymbol{\Sigma}\omega} & (2.157) \end{cases}$$

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$$\begin{cases} \mathrm{E}\{\mathbf{X}\} = \mathrm{Mod}\{\mathbf{X}\} = \boldsymbol{\mu} & (2.158) \\ \mathrm{Cov}\{\mathbf{X}\} = \mathrm{MDis}\{\mathbf{X}\} = \boldsymbol{\Sigma}. & (2.159) \end{cases}$$

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$$(X_m, X_n) \text{ independent} \Leftrightarrow \mathrm{Cov}\{X_n, X_n\} = 0. \quad (2.167)$$

$$\mathbf{a} + \mathbf{B}\mathbf{X} \sim \mathrm{N}(\mathbf{a} + \mathbf{B}\boldsymbol{\mu}, \mathbf{B}\boldsymbol{\Sigma}\mathbf{B}') \quad (2.163)$$

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$$\mathbf{X} \sim \mathrm{N}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \quad (2.155)$$

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$$\mathbf{X} \equiv \begin{pmatrix} \mathbf{X}_A \\ \mathbf{X}_B \end{pmatrix} \quad (2.160)$$

$$\boldsymbol{\mu} \equiv \begin{pmatrix} \boldsymbol{\mu}_A \\ \boldsymbol{\mu}_B \end{pmatrix}, \quad \boldsymbol{\Sigma} \equiv \begin{pmatrix} \boldsymbol{\Sigma}_{AA} & \boldsymbol{\Sigma}_{AB} \\ \boldsymbol{\Sigma}_{BA} & \boldsymbol{\Sigma}_{BB} \end{pmatrix} \quad (2.161)$$

$$\mathbf{X}_B | \mathbf{x}_A \sim \mathrm{N}\left(\boldsymbol{\mu}_B | \mathbf{x}_A, \boldsymbol{\Sigma}_B | \mathbf{x}_A\right) \quad (2.164)$$

$$\boldsymbol{\mu}_B | \mathbf{x}_A \equiv \boldsymbol{\mu}_B + \boldsymbol{\Sigma}_{BA}\boldsymbol{\Sigma}_{AA}^{-1}\left(\mathbf{x}_A - \boldsymbol{\mu}_A\right) \quad (2.165)$$

$$\boldsymbol{\Sigma}_B | \mathbf{x}_A \equiv \boldsymbol{\Sigma}_{BB} - \boldsymbol{\Sigma}_{BA}\boldsymbol{\Sigma}_{AA}^{-1}\boldsymbol{\Sigma}_{AB} \quad (2.166)$$

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$$\mathbf{X} \sim \mathrm{St}\left(\nu, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \quad (2.187)$$

$$\mathrm{St}\left(\infty, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right) = \mathrm{N}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \quad (2.196)$$

$$\mathrm{Ca}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \equiv \mathrm{St}\left(1, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right). \quad (2.208)$$

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$$\mathrm{Ca}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \equiv \mathrm{St}(1, \boldsymbol{\mu}, \boldsymbol{\Sigma}). \quad (2.208)$$

$$f^{\mathrm{St}}_{\nu,\boldsymbol{\mu},\boldsymbol{\Sigma}}(\mathbf{x}) = (\nu\pi)^{-\frac{N}{2}} \frac{\Gamma\left(\frac{\nu+N}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} |\boldsymbol{\Sigma}|^{-\frac{1}{2}} \left(1 + \frac{1}{\nu}(\mathbf{x}-\boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\mathbf{x}-\boldsymbol{\mu})\right)^{-\frac{\nu+N}{2}} \quad (2.188)$$

$$\begin{cases} \mathrm{E}\{\mathbf{X}\} = \mathrm{Mod}\{\mathbf{X}\} = \boldsymbol{\mu}. & (2.190) \\ \mathrm{Cov}\{\mathbf{X}\} = \dfrac{\nu}{\nu-2}\boldsymbol{\Sigma} & (2.191) \end{cases}$$

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$$\mathbf{X} \sim \mathrm{St}\left(\nu, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \quad (2.187)$$

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$$\mathbf{X} \equiv \begin{pmatrix} \mathbf{X}_A \\ \mathbf{X}_B \end{pmatrix} \quad (2.192)$$

$$\mathbf{X}_B | \mathbf{x}_A \sim \text{?}$$

$$\mathbf{a} + \mathbf{B}\mathbf{X} \sim \mathrm{St}\left(\nu, \mathbf{a} + \mathbf{B}\boldsymbol{\mu}, \mathbf{B}\boldsymbol{\Sigma}\mathbf{B}'\right) \quad (2.195)$$

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$$\mathbf{X}_t \sim \mathrm{N}(\mathbf{0}, \mathbf{\Sigma}), \quad t = 1, \ldots, \nu \tag{2.221}$$

$$\mathbf{W} \equiv \mathbf{X}_1 \mathbf{X}_1' + \cdots + \mathbf{X}_\nu \mathbf{X}_\nu' \tag{2.222}$$

$$\mathbf{W} \sim \mathrm{W}(\nu, \mathbf{\Sigma}) \tag{2.223}$$

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$$\mathbf{W} \sim \mathrm{W}(\nu, \mathbf{\Sigma}) \tag{2.223}$$

$$f_{\nu,\mathbf{\Sigma}}^{\mathrm{W}}(\mathbf{W}) = \frac{1}{\kappa} |\mathbf{\Sigma}|^{-\frac{\nu}{2}} |\mathbf{W}|^{\frac{\nu - N - 1}{2}} e^{-\frac{1}{2} \operatorname{tr}(\mathbf{\Sigma}^{-1} \mathbf{W})} \tag{2.224}$$

$$\begin{cases} \mathrm{E}\{W_{mn}\} = \nu \Sigma_{mn} & (2.227) \\ \mathrm{Cov}\{W_{mn}, W_{pq}\} = \nu (\Sigma_{mp} \Sigma_{nq} + \Sigma_{mq} \Sigma_{np}) & (2.228) \end{cases}$$

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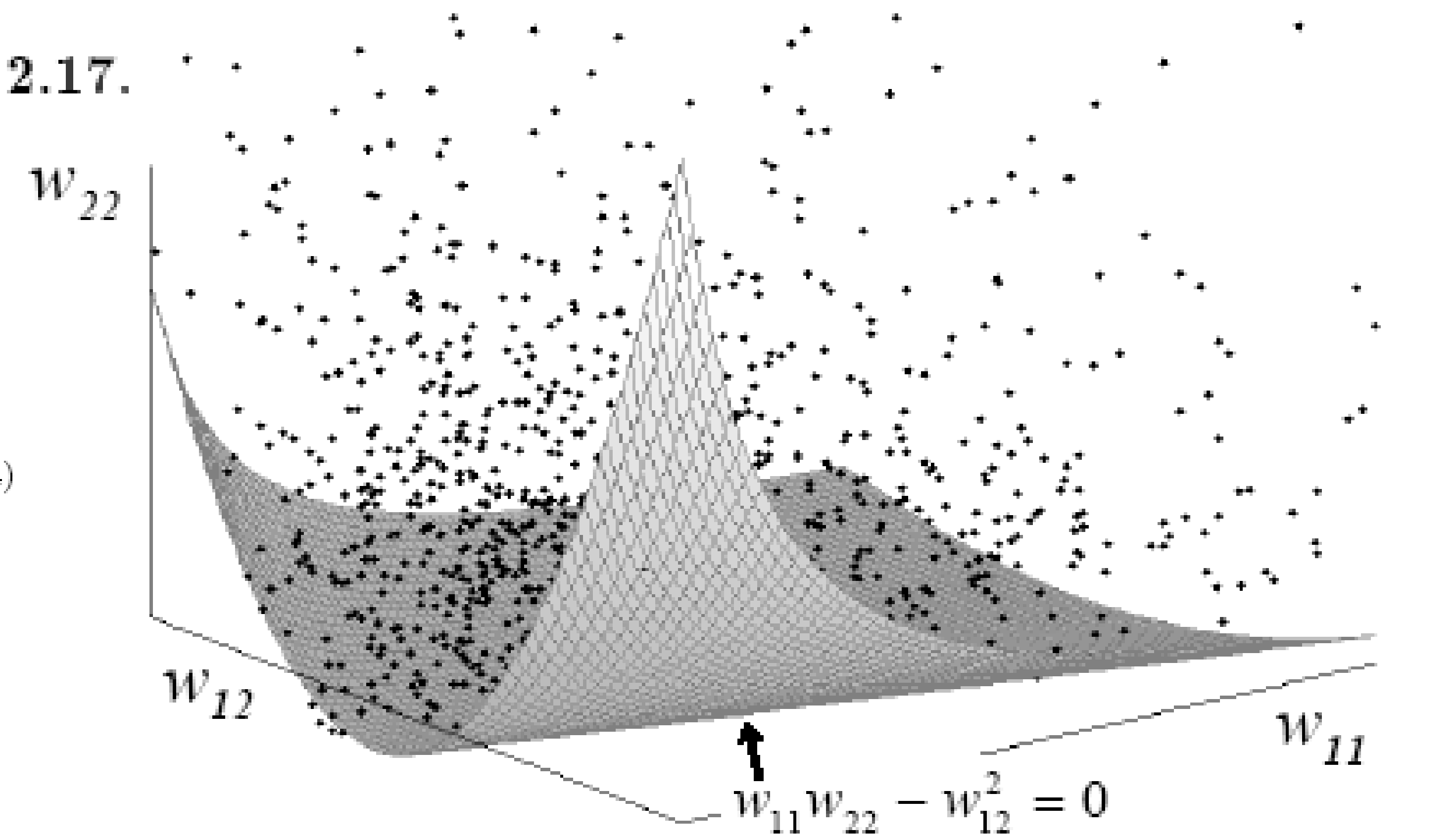
$$\mathbf{a}'\mathbf{W}\mathbf{a} \sim \mathrm{Ga}(\nu, \mathbf{a}'\mathbf{\Sigma}\mathbf{a}) \tag{2.230}$$

Fig. 2.17.



$$\mathbf{W} \equiv \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix} \quad (2.234)$$

$$\lambda_1 \lambda_2 > 0, \qquad \lambda_1 + \lambda_2 > 0. \quad (2.235)$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$|\mathbf{W}| \equiv W_{11}W_{22} - W_{12}^2 \geq 0 \quad (2.236) \qquad \mathrm{tr}(\mathbf{W}) \equiv W_{11} + W_{22} \geq 0, \quad (2.237)$$

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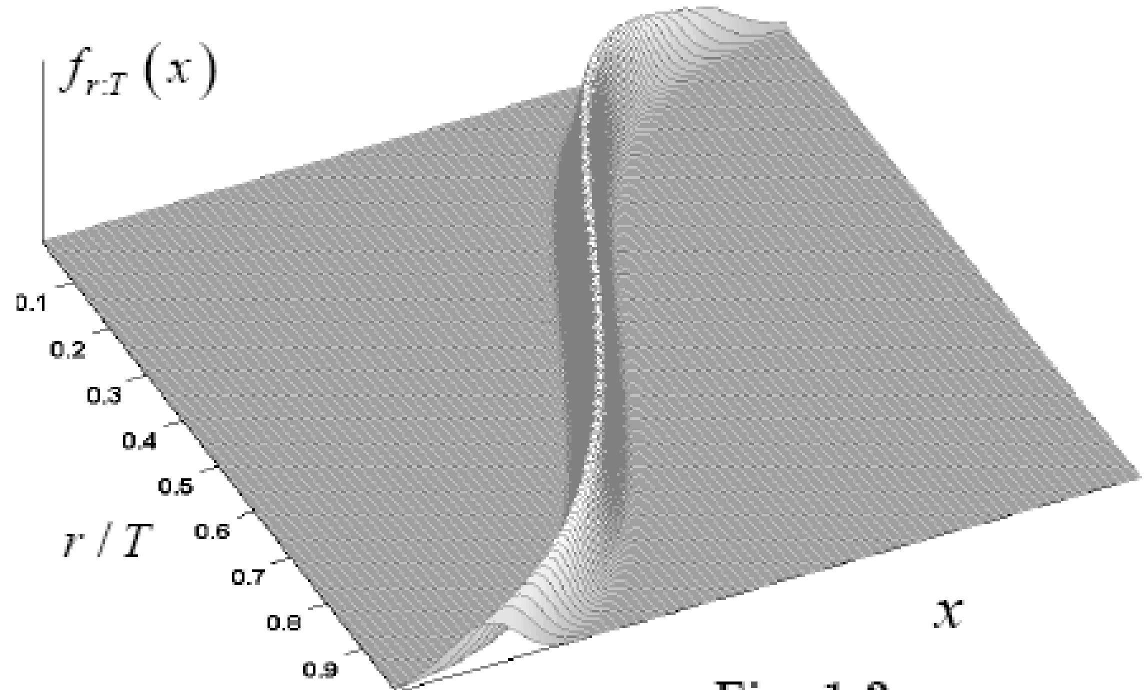

Fig. 1.2

$$f_{X_{r:T}}(x) = \frac{T!}{(r-1)!\,(T-r)!} F_X^{r-1}(x)\,(1 - F_X(x))^{T-r} f_X(x) \qquad (2.248)$$

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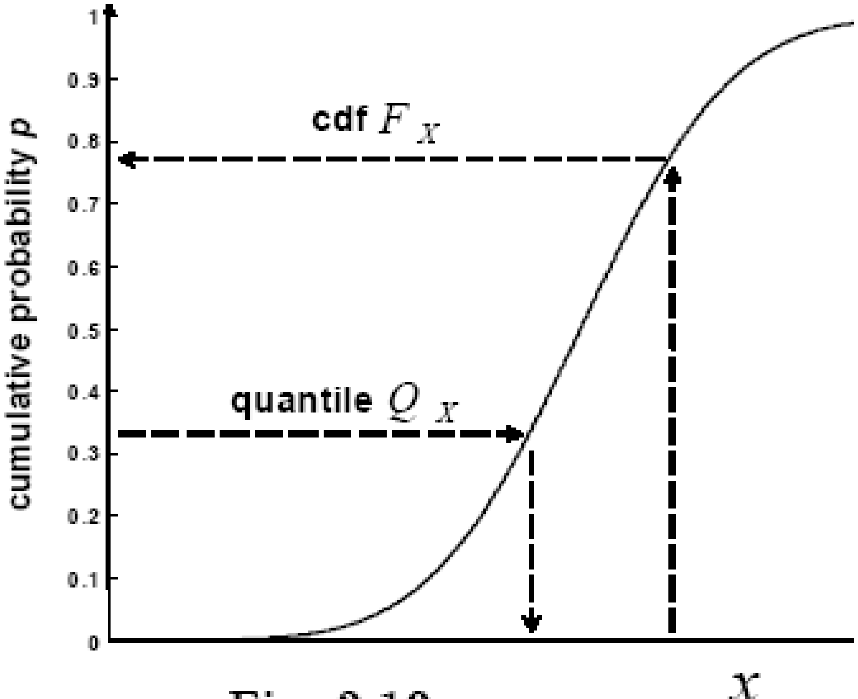

Fig. 2.19.

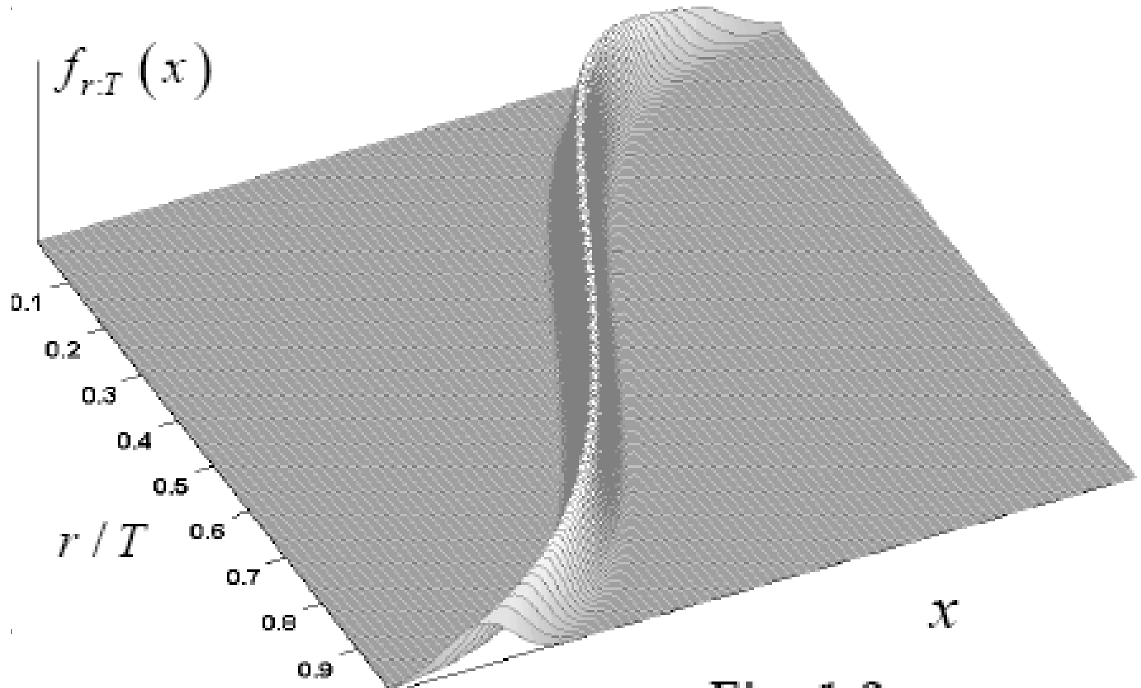

Fig. 1.2

$$f_{X_{r:T}}(x) = \frac{T!}{(r-1)!\,(T-r)!} F_X^{r-1}(x)\,(1 - F_X(x))^{T-r} f_X(x) \qquad (2.248)$$

$$\mathrm{E}\{X_{r:T}\} = \int_{\mathbb{R}} Q_X(u)\,\widetilde{\delta}_{r,T}(u)\,du \qquad (2.250)$$

$$\widetilde{\delta}_{r,T} \overset{T\to\infty}{\longrightarrow} \delta^{(r/T)}$$

$$\boxed{\mathrm{E}\{X_{r:T}\} \approx Q_X\left(\frac{r}{T}\right)} \qquad (2.253)$$

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$$\mathbf{X} \sim \mathrm{El}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}, g_N\right). \quad (2.268)$$

$$\boxed{f_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\mathbf{x}) = \left|\boldsymbol{\Sigma}\right|^{-\frac{1}{2}} g_N\left(\mathrm{Ma}^2\left(\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right)\right)} \quad (2.261)$$

$$\mathrm{Ma}\left(\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \equiv \sqrt{\left(\mathbf{x} - \boldsymbol{\mu}\right)' \boldsymbol{\Sigma}^{-1} \left(\mathbf{x} - \boldsymbol{\mu}\right)}.$$

$$\int_0^{\infty} v^{\frac{N}{2}-1} g_N(v)\, dv < \infty; \quad (2.262)$$

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$$\int_0^\infty v^{\frac{N}{2}-1} g_N\left(v\right) dv < \infty; \quad (2.262)$$

$$\boxed{\mathbf{X} \equiv \boldsymbol{\mu} + \mathbf{A}\mathbf{Y}.} \quad (2.258)$$

$$\mathbf{A}\mathbf{A}' = \boldsymbol{\Sigma}. \qquad \mathbf{Y} \stackrel{d}{=} \boldsymbol{\Gamma}\mathbf{Y}.$$

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$$\boxed{\mathbf{X} \equiv \boldsymbol{\mu} + R\mathbf{A}\mathbf{U}.} \quad (2.259)$$

$$\mathbf{A}\mathbf{A}' = \boldsymbol{\Sigma}. \qquad R \equiv \left\|\mathbf{A}^{-1}\left(\mathbf{X} - \boldsymbol{\mu}\right)\right\| \qquad \mathbf{U} \equiv \frac{\mathbf{A}^{-1}\left(\mathbf{X} - \boldsymbol{\mu}\right)}{\left\|\mathbf{A}^{-1}\left(\mathbf{X} - \boldsymbol{\mu}\right)\right\|}$$

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$$\mathbf{X} \sim \mathrm{El}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}, g_N\right) \qquad (2.268)$$

$$\mathbf{a} + \mathbf{B}\mathbf{X} \sim \mathrm{El}\left(\mathbf{a} + \mathbf{B}\boldsymbol{\mu}, \mathbf{B}\boldsymbol{\Sigma}\mathbf{B}', g_K\right) \qquad (2.270)$$

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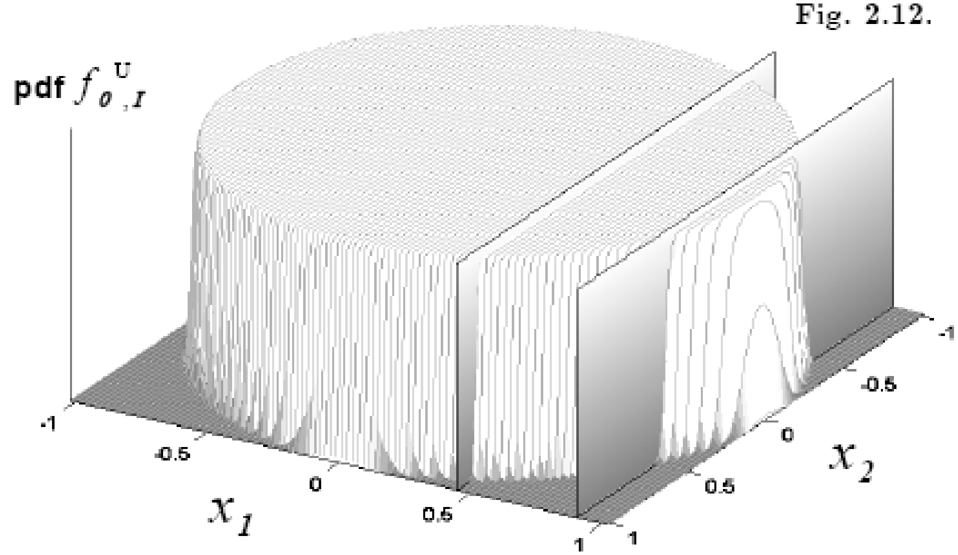
$$\mathbf{X} \sim \mathrm{U}\left(\mathcal{E}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}\right) \qquad (2.144)$$

Fig. 2.12.



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$$\mathbf{X} \sim \mathrm{U}\left(\mathcal{E}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}\right) \qquad (2.144)$$

$$f^{\mathrm{U}}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\mathbf{x}) = \frac{\Gamma\left(\frac{N}{2}+1\right)}{\pi^{\frac{N}{2}}\left|\boldsymbol{\Sigma}\right|^{\frac{1}{2}}} \mathbb{I}_{\mathcal{E}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}}(\mathbf{x}) \qquad (2.145)$$

$$\begin{cases} \mathrm{E}\{\mathbf{X}\} = \boldsymbol{\mu} & (2.148) \\ \mathrm{Cov}\{\mathbf{X}\} = \dfrac{1}{N+2}\boldsymbol{\Sigma}. & (2.149) \end{cases}$$

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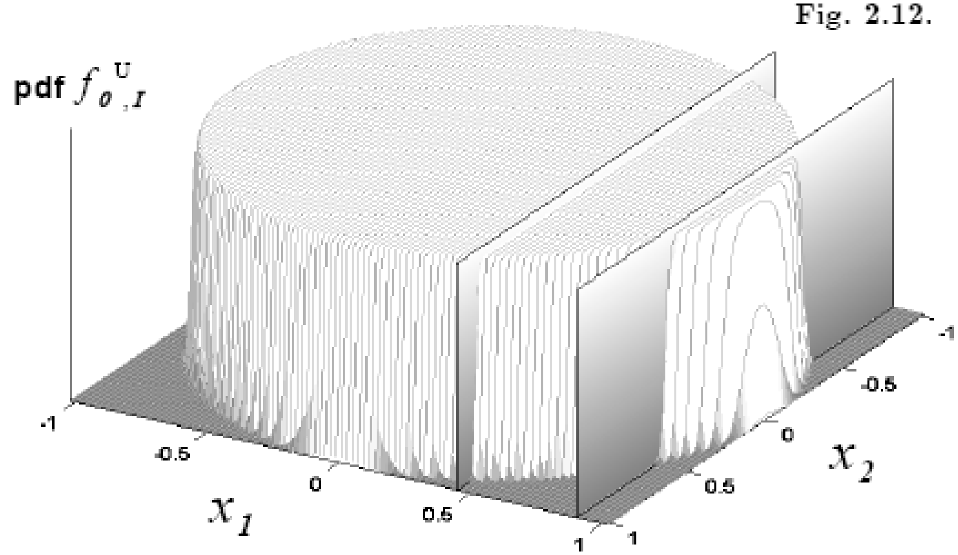

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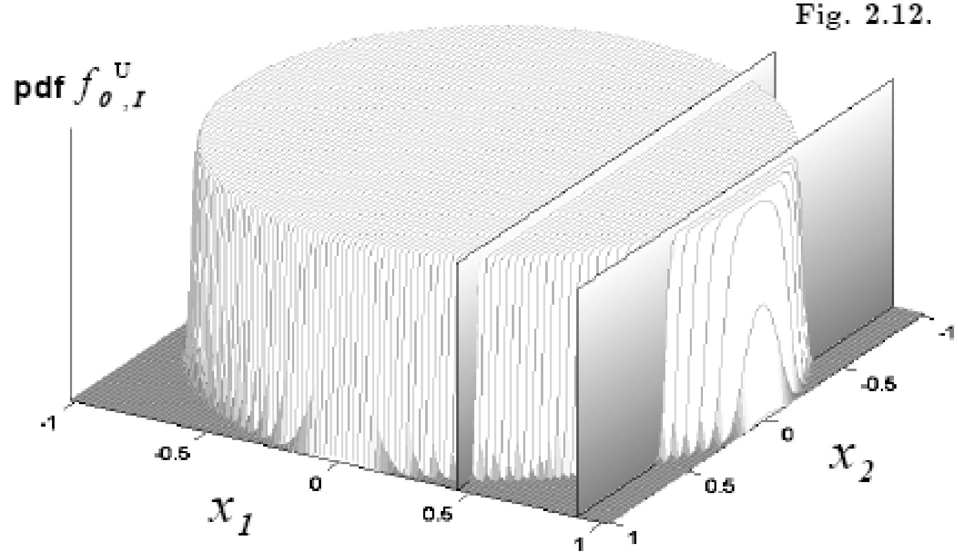

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$$f_{X_1}(x_1) \equiv \int_{-\sqrt{1-x_1^2}}^{+\sqrt{1-x_1^2}} \frac{1}{\pi} dx_2 = \frac{2}{\pi}\sqrt{1-x_1^2}. \qquad (2.151)$$

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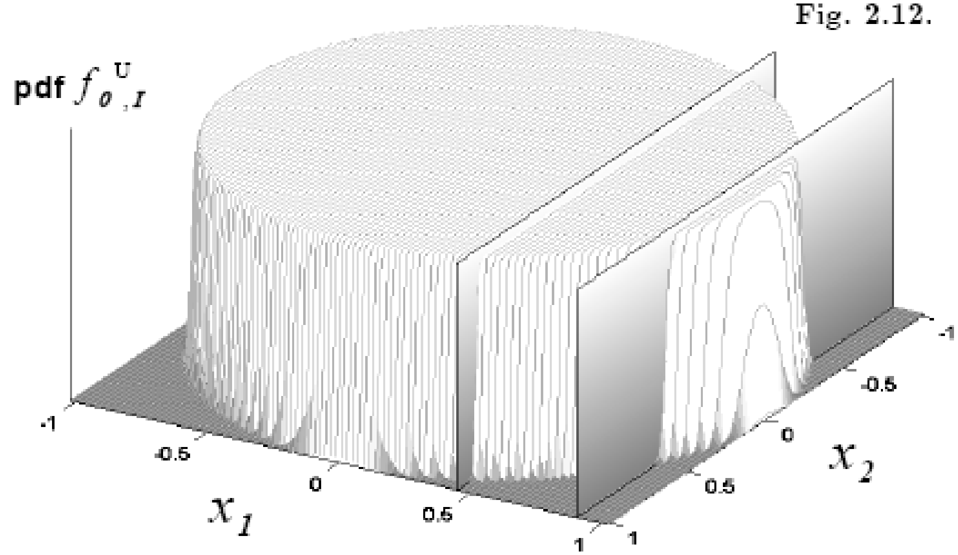

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$$f_{X_2|x_1}(x_2) = \frac{f_{X_1,X_2}(x_1,x_2)}{f_{X_1}(x_1)} = \frac{1}{2\sqrt{1-x_1^2}}. \qquad (2.153)$$

$$\mathrm{Cor}\{X_1, X_2\} = 0 \qquad (2.154)$$