Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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invariants

$\mathbf{X}_t$

$\Rightarrow$

price at horizon

$$P = g(\mathbf{X}) \qquad (3.101)$$

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invariants

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$$P_{T+1} = \text{?}$$

stocks, FX

invariants

$$\mathbf{X}_t$$

price at horizon

$$P = g(\mathbf{X}) \quad (3.101)$$

---

compounded return

$$X_t \equiv \ln\left(\frac{P_t}{P_{t-1}}\right)$$

$$P_{T+1} = P_T e^{X_{T+1}}$$

stocks, FX

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stocks, FX

$$Z_{T+1}^{(E)} = \textbf{?}$$

bonds

time of maturity

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stocks, FX

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change in yield to maturity

$$X_t^{(v)} \equiv Y_t^{(v)} - Y_{t-1}^{(v)}$$

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$$X_t^{(v)} \equiv Y_t^{(v)} - Y_{t-1}^{(v)}$$

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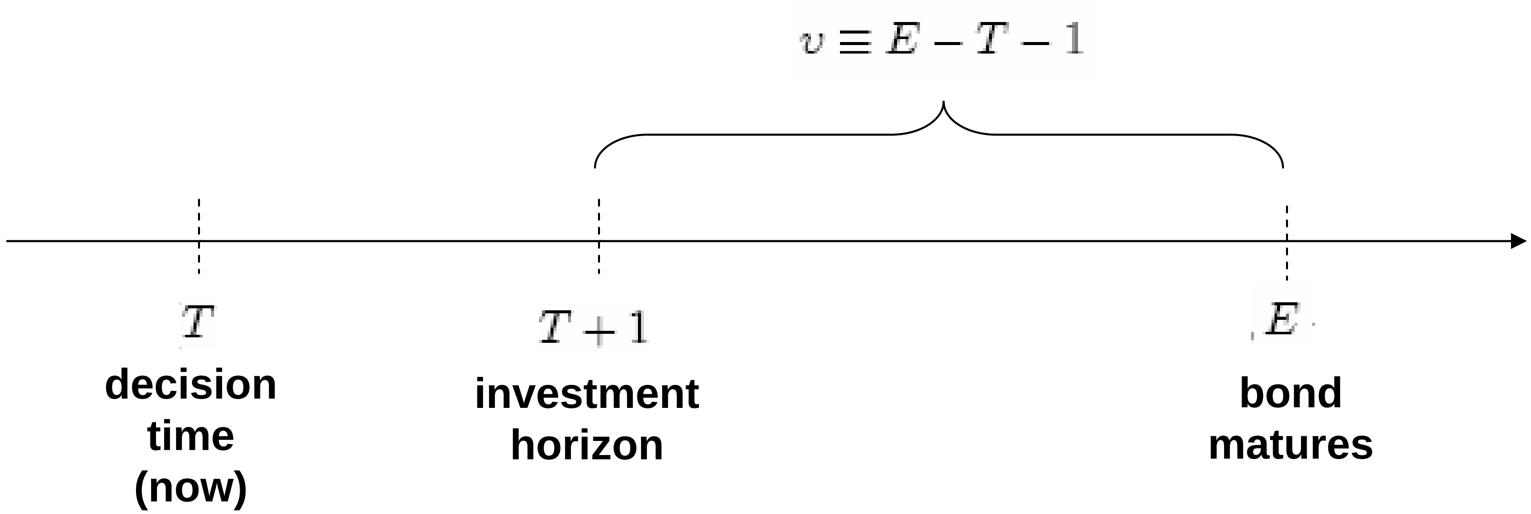
bonds



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invariants

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bonds

$$C_{T+1}(K, E) = \ ?$$

derivatives

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bonds

$$C_{T+1}(K,E) = C_{BS}\left(P_{T+1}, \sigma_{T+1}(K,E); K, E\right)$$

derivatives

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invariants $\mathbf{X}_t$ ⇒ **price at horizon** $P = g(\mathbf{X})$ (3.101)

---

compounded return

$$X_t \equiv \ln\left(\frac{P_t}{P_{t-1}}\right)$$

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stocks, FX

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derivatives

$$C_{T+1}(K,E) = C_{BS}\left(P_{T+1}, \sigma_{T+1}\cancel{(K,E)}; K, E\right)$$

invariant coordinates

$$(t, m, v) \mapsto \sigma_t(m, v)$$

moneyness

time to expiry

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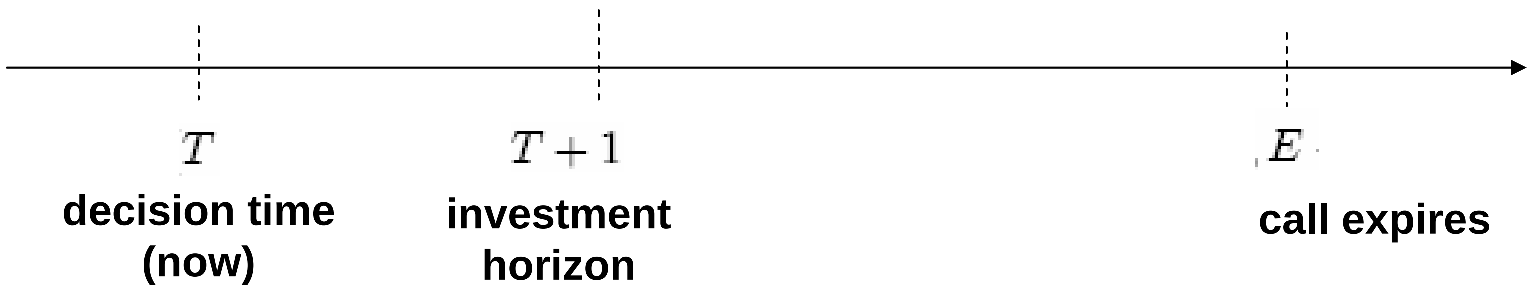

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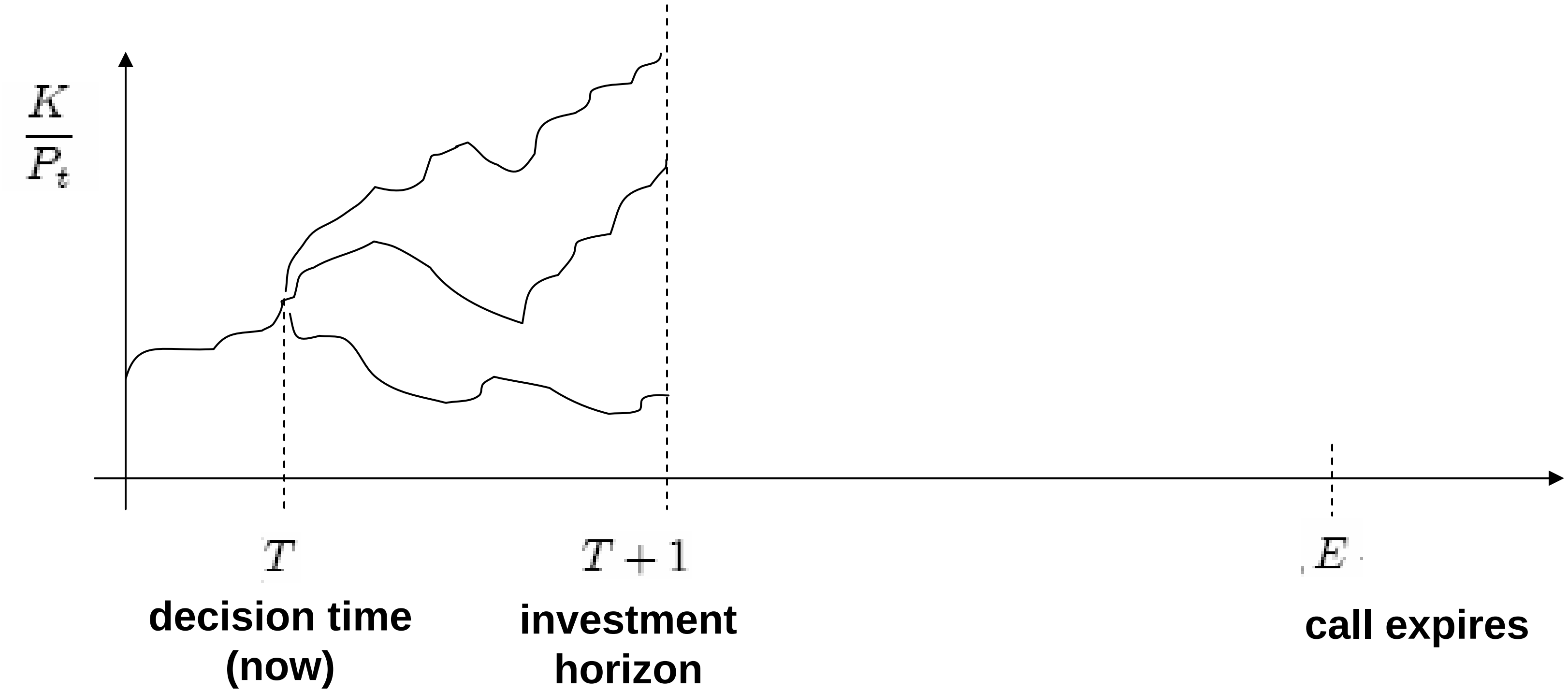

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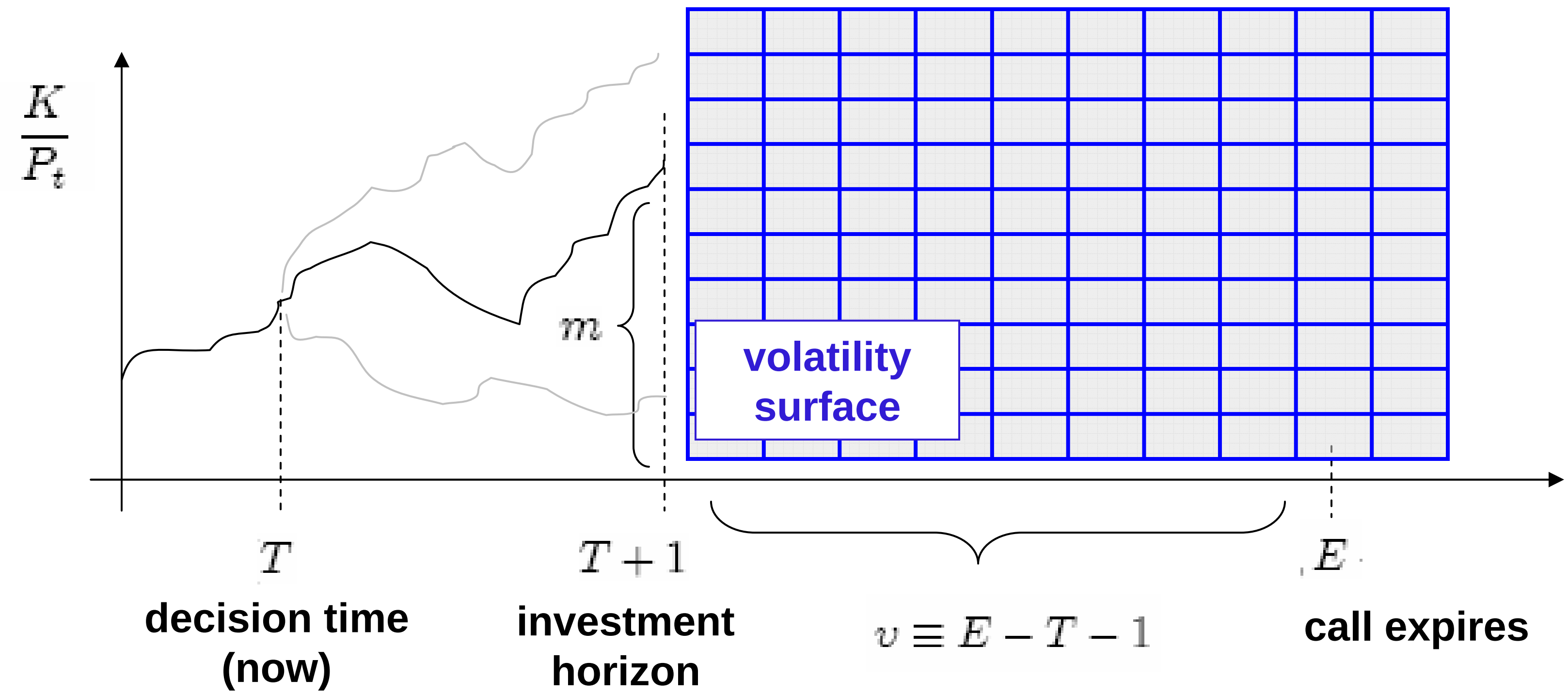

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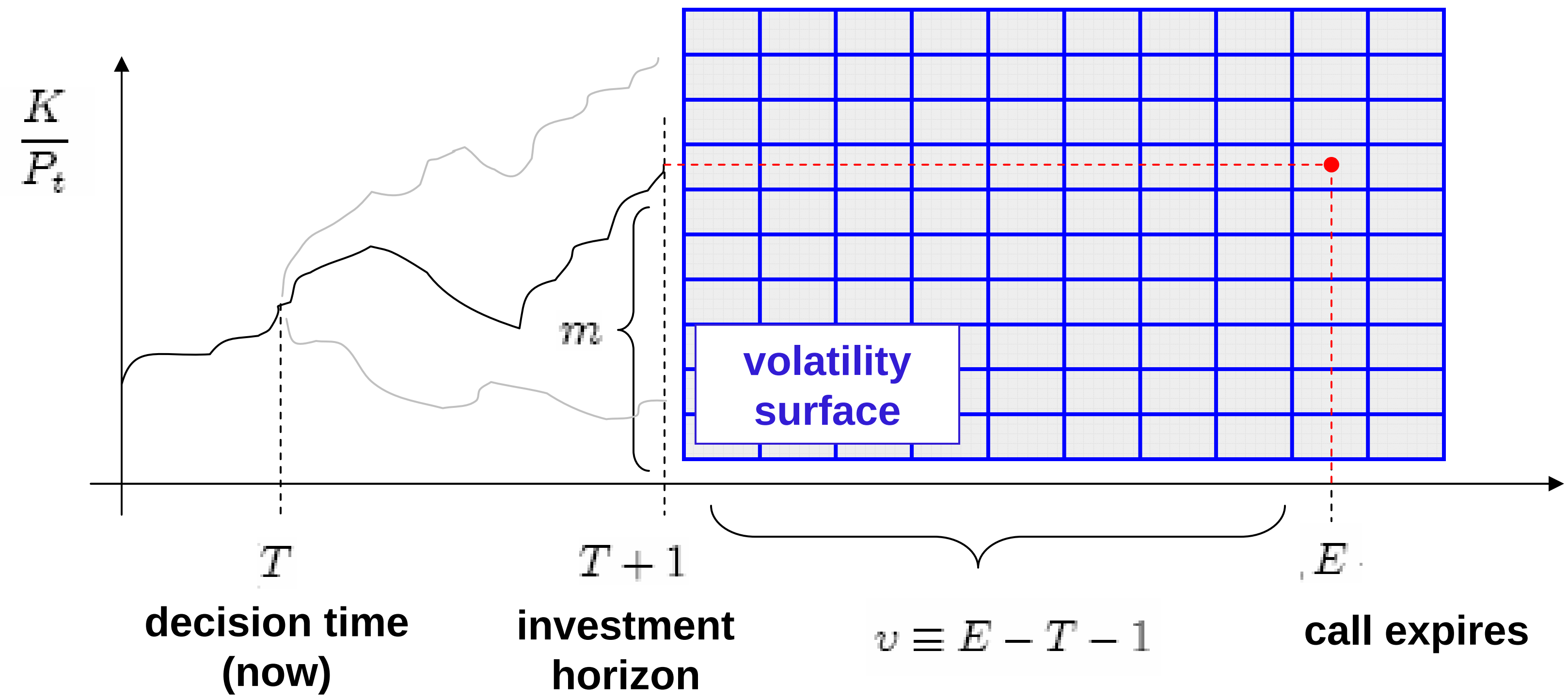
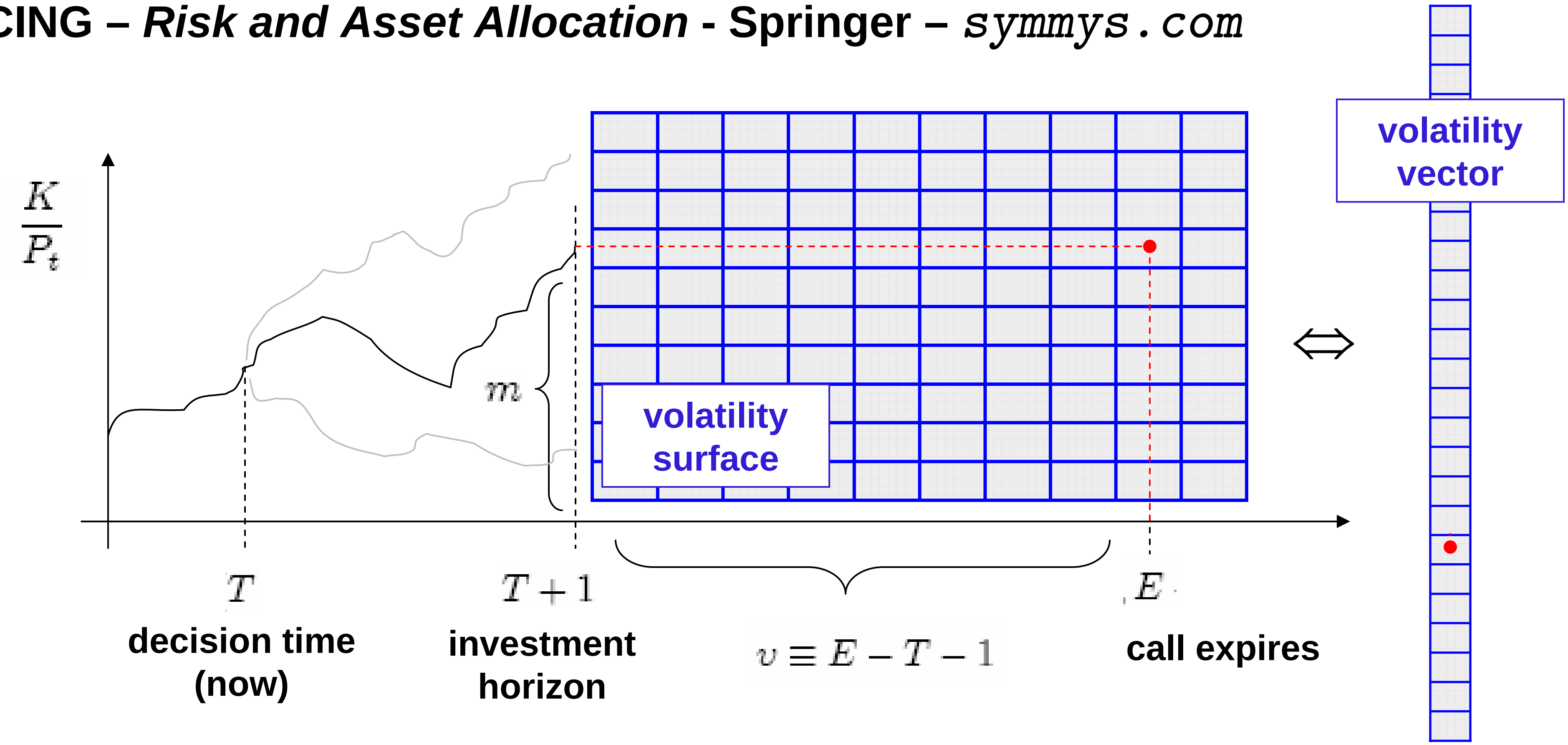


$$(t, m, v) \mapsto \sigma_t(m, v)$$

moneyness

time to expiry

$$C_{T+1}(K, E) = C_{BS}\left(P_{T+1}, \sigma_{T+1}\left(\frac{K}{P_{T+1}}, E - (T+1)\right); K, E\right)$$



$$C_{T+1}(K,E) = C_{BS}\left(P_{T+1}, \underbrace{\sigma_{T+1}\left(\frac{K}{P_{T+1}}, E-(T+1)\right)}_{=[\boldsymbol{\sigma}_{T+1}]_{\boxed{j}}}; K, E\right)$$

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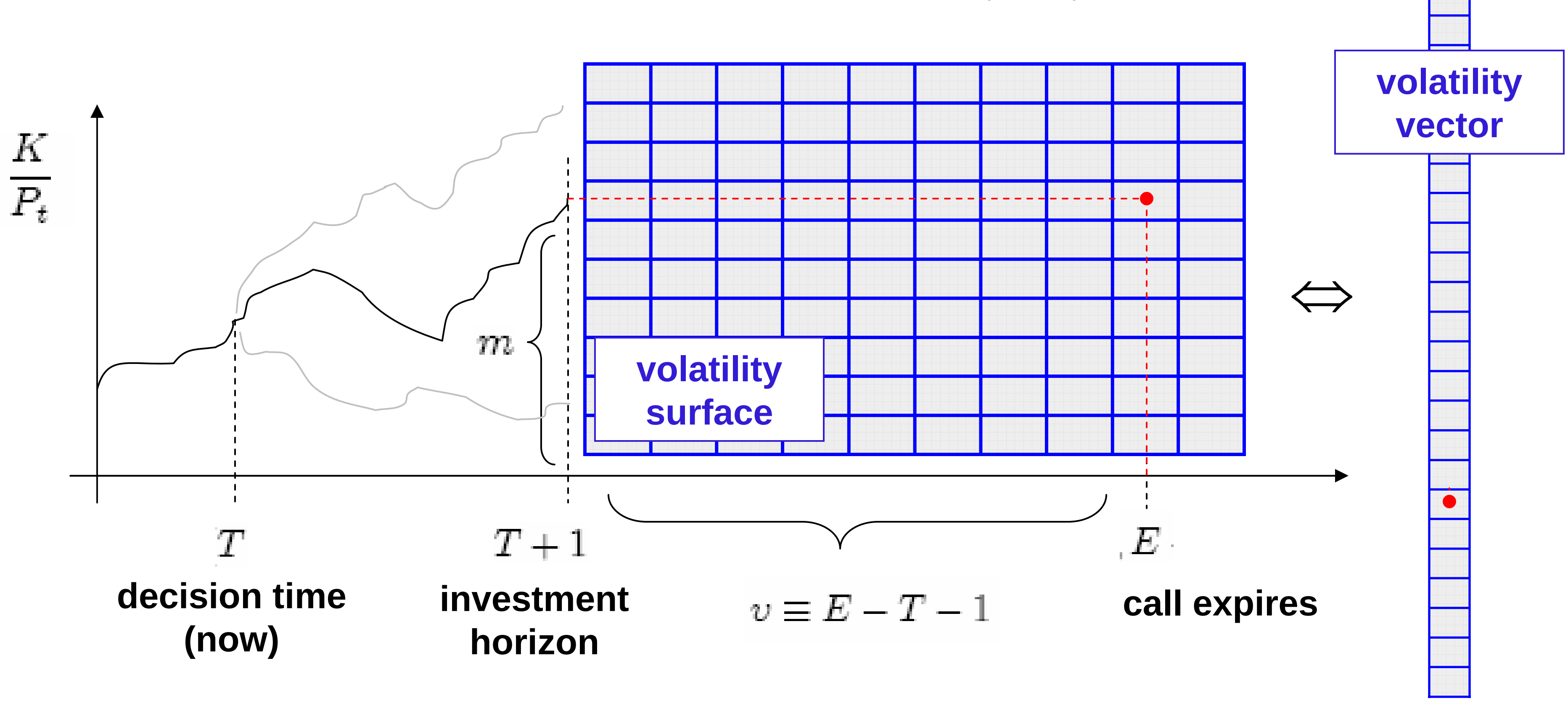

$$C_{T+1}(K,E) = C_{BS}\Bigg(P_{T+1}, \underbrace{\sigma_{T+1}\left(\frac{K}{P_{T+1}}, E-(T+1)\right)}_{=[\boldsymbol{\sigma}_{T+1}]_{\boxed{j}}}; K, E\Bigg)$$

invariants

change in log-implied volatility vector

$$\mathbf{X}_t \equiv \ln \boldsymbol{\sigma}_t - \ln \boldsymbol{\sigma}_{t-1}$$

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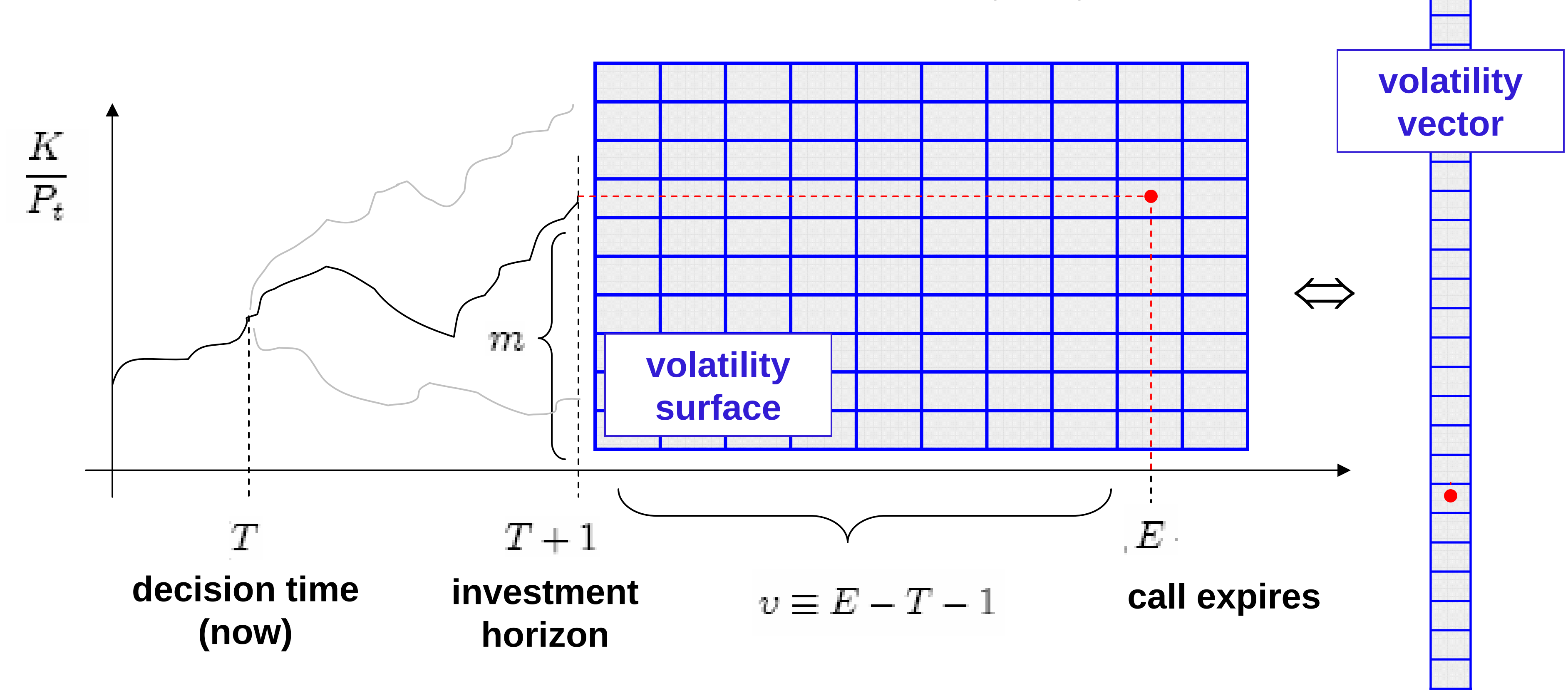

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invariants

change in log-implied volatility vector

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invariants

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invariants

$\mathbf{X}_t$

price at horizon

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compounded return

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		Analytical	Scenarios
Full evaluation	$$P = g(\mathbf{X}) \qquad (3.101)$$	only simple securities (e.g. stocks)	~ always, but costly

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		Analytical	Scenarios
Taylor 1	$$P = g(\mathbf{m}) + (\mathbf{X} - \mathbf{m})' \, \partial_{\mathbf{x}} g|_{\mathbf{x}=\mathbf{m}} \quad (3.108)$$	~ any analytical distribution for **X**	~ always
Full evaluation	$$P = g(\mathbf{X}) \quad (3.101)$$	only simple securities (e.g. stocks)	~ always, but costly

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	Analytical	Scenarios
Taylor 1 $$P = g(\mathbf{m}) + (\mathbf{X} - \mathbf{m})' \, \partial_{\mathbf{x}} g\big|_{\mathbf{x}=\mathbf{m}} \quad (3.108)$$ **<u>fixed-income</u>: carry + duration** **<u>derivatives</u>: theta + delta + vega**	~ any analytical distribution for **X**	~ always
Full evaluation $$P = g(\mathbf{X}) \quad (3.101)$$	only simple securities (e.g. stocks)	~ always, but costly

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		Analytical	Scenarios
Taylor 1 **fixed-income: carry + duration** **derivatives: theta + delta + vega**	$P = g(\mathbf{m}) + (\mathbf{X} - \mathbf{m})' \, \partial_{\mathbf{x}} g\big|_{\mathbf{x}=\mathbf{m}}$ (3.108)	~ any analytical distribution for **X**	~ always
Taylor 2	$P = g(\mathbf{m}) + (\mathbf{X} - \mathbf{m})' \, \partial_{\mathbf{x}} g\big|_{\mathbf{x}=\mathbf{m}} + \frac{1}{2} (\mathbf{X} - \mathbf{m})' \, \partial^2_{\mathbf{xx}} g\big|_{\mathbf{x}=\mathbf{m}} (\mathbf{X} - \mathbf{m})$ (3.108)	**X** normal (**X** Student)	~ always
Full evaluation	$P = g(\mathbf{X})$ (3.101)	only simple securities (e.g. stocks)	~ always, but costly

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		Analytical	Scenarios
Taylor 1 **fixed-income: carry + duration** **derivatives: theta + delta + vega**	$$P = g(\mathbf{m}) + (\mathbf{X}-\mathbf{m})' \, \partial_{\mathbf{x}} g\big|_{\mathbf{x}=\mathbf{m}} \qquad (3.108)$$	~ any analytical distribution for **X**	~ always
Taylor 2 **fixed-income: carry + duration + convexity** **derivatives: theta + delta + vega + gamma (+ vanna, volga…)**	$$P = g(\mathbf{m}) + (\mathbf{X}-\mathbf{m})' \, \partial_{\mathbf{x}} g\big|_{\mathbf{x}=\mathbf{m}} + \frac{1}{2}(\mathbf{X}-\mathbf{m})' \, \partial^2_{\mathbf{xx}} g\big|_{\mathbf{x}=\mathbf{m}} (\mathbf{X}-\mathbf{m}) \qquad (3.108)$$	**X** normal (**X** Student)	~ always
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	Analytical	Scenarios
Taylor 1 $$P = g(\mathbf{m}) + (\mathbf{X}-\mathbf{m})' \left.\partial_{\mathbf{x}} g\right\|_{\mathbf{x}=\mathbf{m}} \quad (3.108)$$ **fixed-income**: carry + duration **derivatives**: theta + delta + vega	~ any analytical distribution for **X**	~ always
Taylor 2 $$P = g(\mathbf{m}) + (\mathbf{X}-\mathbf{m})' \left.\partial_{\mathbf{x}} g\right\|_{\mathbf{x}=\mathbf{m}} + \frac{1}{2}(\mathbf{X}-\mathbf{m})' \left.\partial^2_{\mathbf{xx}} g\right\|_{\mathbf{x}=\mathbf{m}} (\mathbf{X}-\mathbf{m}) \quad (3.108)$$ **fixed-income**: carry + duration + convexity **derivatives**: theta + delta + vega + gamma (+ vanna, volga…)	**X** normal (**X** Student)	~ always
Intra/Extrapolation	**N/A**	~ always, but **curse of dimensionality**
Full evaluation $$P = g(\mathbf{X}) \quad (3.101)$$	only simple securities (e.g. stocks)	~ always, but costly

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	Analytical	Scenarios
Taylor 1 $$P = g(\mathbf{m}) + (\mathbf{X}-\mathbf{m})' \, \partial_{\mathbf{x}} g\big|_{\mathbf{x}=\mathbf{m}} \quad (3.108)$$ **<u>fixed-income</u>: carry + duration** **<u>derivatives</u>: theta + delta + vega**	~ any analytical distribution for **X**	~ always
Taylor 2 $$P = g(\mathbf{m}) + (\mathbf{X}-\mathbf{m})' \, \partial_{\mathbf{x}} g\big|_{\mathbf{x}=\mathbf{m}} + \frac{1}{2}(\mathbf{X}-\mathbf{m})' \, \partial^2_{\mathbf{xx}} g\big|_{\mathbf{x}=\mathbf{m}} (\mathbf{X}-\mathbf{m}) \quad (3.108)$$ **<u>fixed-income</u>: carry + duration + convexity** **<u>derivatives</u>: theta + delta + vega + gamma (+ vanna, volga…)**	**X** normal (**X** Student)	~ always
Intra/Extrapolation + Taylor (stress matrices)	**N/A**	~ always
Intra/Extrapolation	**N/A**	~ always, but curse of dimensionality
Full evaluation $$P = g(\mathbf{X}) \quad (3.101)$$	only simple securities (e.g. stocks)	~ always, but costly

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	Analytical	(Historical) Scenarios
Taylor 1 $$P = g(\mathbf{m}) + (\mathbf{X}-\mathbf{m})' \, \partial_{\mathbf{x}} g\big|_{\mathbf{x}=\mathbf{m}} \quad (3.108)$$ **<u>fixed-income</u>: carry + duration** **<u>derivatives</u>: theta + delta + vega**	~ any analytical distribution for **X**	~ always
Taylor 2 $$P = g(\mathbf{m}) + (\mathbf{X}-\mathbf{m})' \, \partial_{\mathbf{x}} g\big|_{\mathbf{x}=\mathbf{m}} + \frac{1}{2}(\mathbf{X}-\mathbf{m})' \, \partial^2_{\mathbf{xx}} g\big|_{\mathbf{x}=\mathbf{m}} (\mathbf{X}-\mathbf{m}) \quad (3.108)$$ **<u>fixed-income</u>: carry + duration + convexity** **<u>derivatives</u>: theta + delta + vega + gamma (+ vanna, volga…)**	**X** normal (**X** Student)	~ always
Intra/Extrapolation + Taylor (stress matrices)	**N/A**	~ always
Intra/Extrapolation	**N/A**	~ always, but curse of dimensionality
Full evaluation $$P = g(\mathbf{X}) \quad (3.101)$$	only simple securities (e.g. stocks)	~ always, but costly