EXPECTATION-COVARIANCE INTERPRETATION

Risk and Asset Allocation – Springer – `symmys.com`

Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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$$\mathrm{E}\{\mathbf{X}\} \equiv \left(\mathrm{E}\{X_1\}, \ldots, \mathrm{E}\{X_N\}\right)' \quad (2.54)$$

$$\mathrm{Cov}\{\mathbf{X}\} \equiv \mathrm{E}\left\{(\mathbf{X} - \mathrm{E}\{\mathbf{X}\})(\mathbf{X} - \mathrm{E}\{\mathbf{X}\})'\right\} \quad (2.67)$$

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$$= \mathbf{E}\boldsymbol{\Lambda}\mathbf{E}' \quad (2.76)$$

vector

orthogonal eigenvectors (columns)

positive eigenvalues (diagonal)

algebra

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$$\mathcal{E}_{\mathrm{E},\mathrm{Cov}} \equiv \left\{ \mathrm{x} \text{ such that } (\mathrm{x}-\mathrm{E})' \operatorname{Cov}^{-1} (\mathrm{x}-\mathrm{E}) \leq 1 \right\} \quad (2.75)$$

$$\Updownarrow$$

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algebra

vector	orthogonal eigenvectors (columns)	positive eigenvalues (diagonal)
$\Updownarrow$	$\Updownarrow$	$\Updownarrow$
center of ellipsoid	directions of principal axes	length of principal axes

geometry

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vector	orthogonal eigenvectors (columns)	positive eigenvalues (diagonal)	algebra
$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	
center of ellipsoid	directions of principal axes	length of principal axes	geometry
$\Updownarrow$	$\Updownarrow$	$\Updownarrow$	
expected values	uncorrelated directions	volatility in uncorrelated directions	statistics

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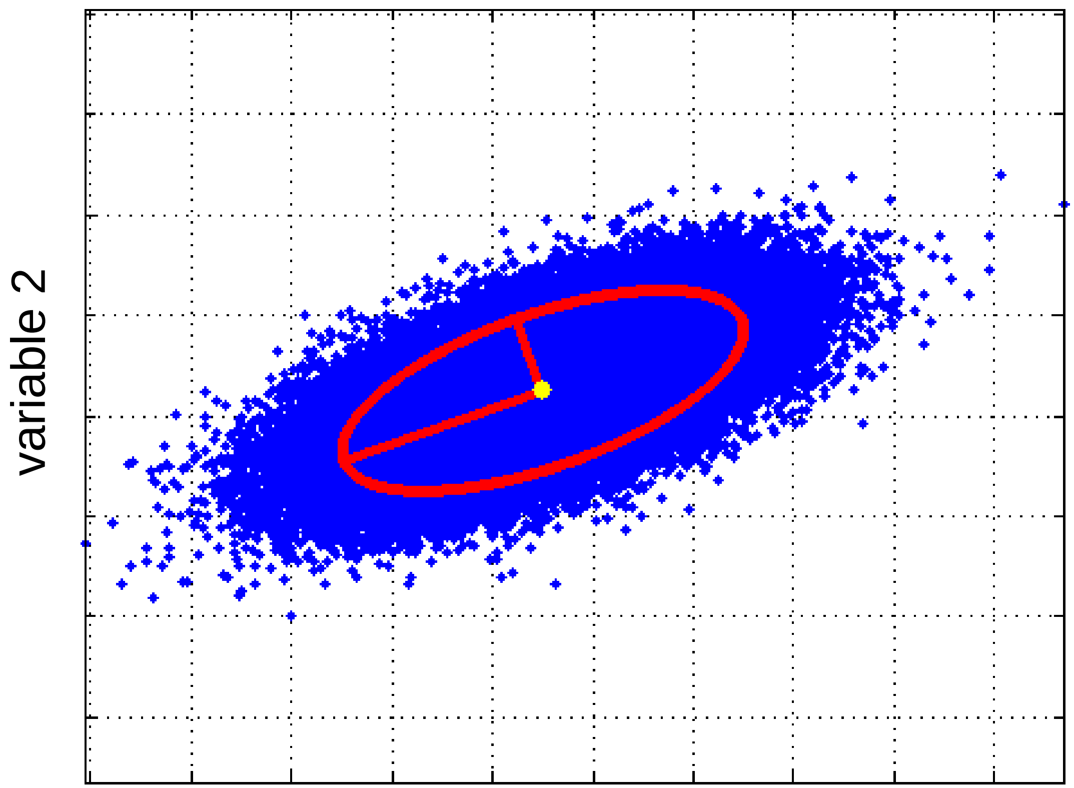

location dispersion ellipsoid

$$(\boldsymbol{r}-\boldsymbol{m})'\boldsymbol{S}^{-1}(\boldsymbol{r}-\boldsymbol{m}) \equiv \text{constant}$$

$$\boldsymbol{m} \equiv \mathrm{E}\{\boldsymbol{X}\}$$

$$\boldsymbol{S} \equiv \mathrm{Cov}\{\boldsymbol{X}\} \equiv \boldsymbol{E}\boldsymbol{\Lambda}\boldsymbol{E}'$$

algebra		statistics		geometry
orthogonal eigenvectors	$\Leftrightarrow$	uncorrelated directions	$\Leftrightarrow$	direction of principal axes
square root of eigenvalues	$\Leftrightarrow$	volatility in uncorr. dir.	$\Leftrightarrow$	length of principal axes

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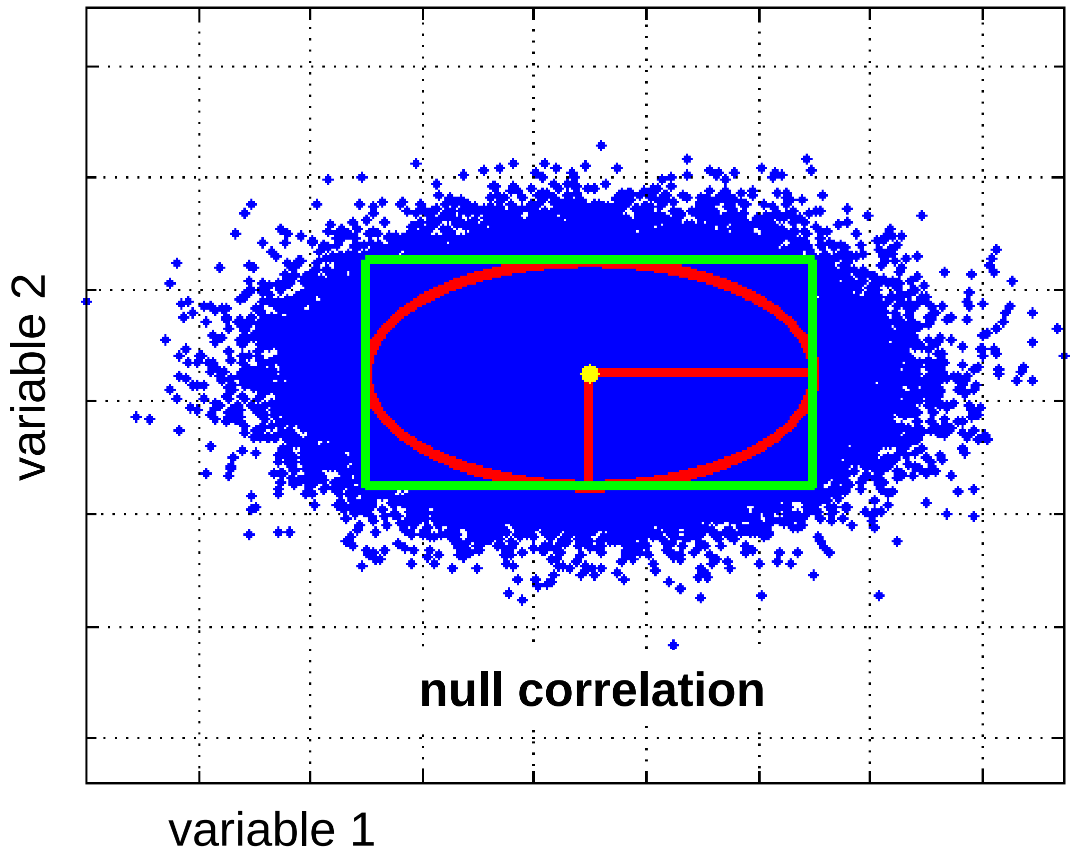

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algebra		statistics		geometry
orthogonal eigenvectors	⇔	uncorrelated directions	⇔	directions of principal axes
square root of eigenvalues	⇔	volatility in uncorr. dir.	⇔	length of principal axes

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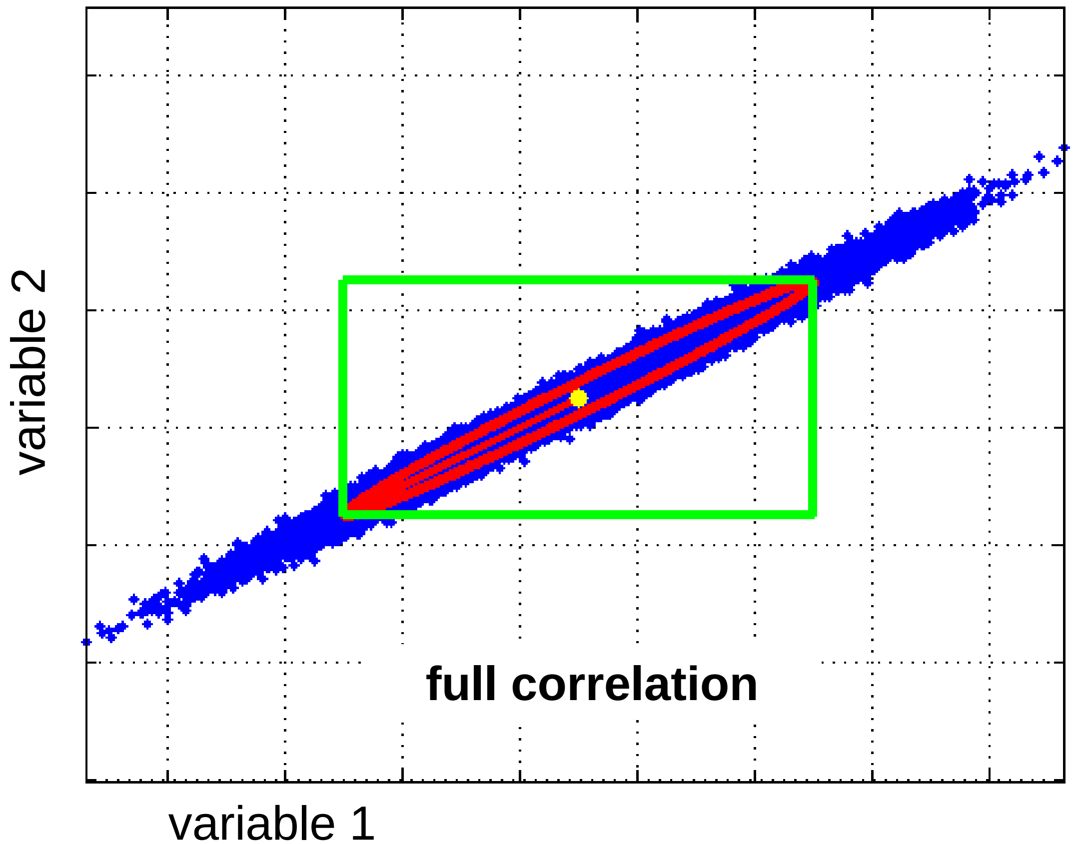

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algebra | **statistics** | **geometry**

orthogonal eigenvectors $\Leftrightarrow$ **uncorrelated directions** $\Leftrightarrow$ **directions of principal axes**

square root of eigenvalues $\Leftrightarrow$ **volatility in uncorr. dir.** $\Leftrightarrow$ **length of principal axes**

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$$\text{Cov}\{\mathbf{X}\} = \mathbf{E}\boldsymbol{\Lambda}\mathbf{E}'. \quad (2.76)$$

$$\begin{cases} \mathbf{Z} \equiv \mathbf{E}'\mathbf{X} = \begin{pmatrix} \left[\mathbf{e}^{(1)}\right]'\mathbf{X} \\ \vdots \\ \left[\mathbf{e}^{(N)}\right]'\mathbf{X} \end{pmatrix} & (2.79) \\ \text{Cov}\{Z_m, Z_n\} = 0. & (2.80) \\ \text{Var}\{Z_n\} = \lambda_n. & (2.81) \end{cases}$$

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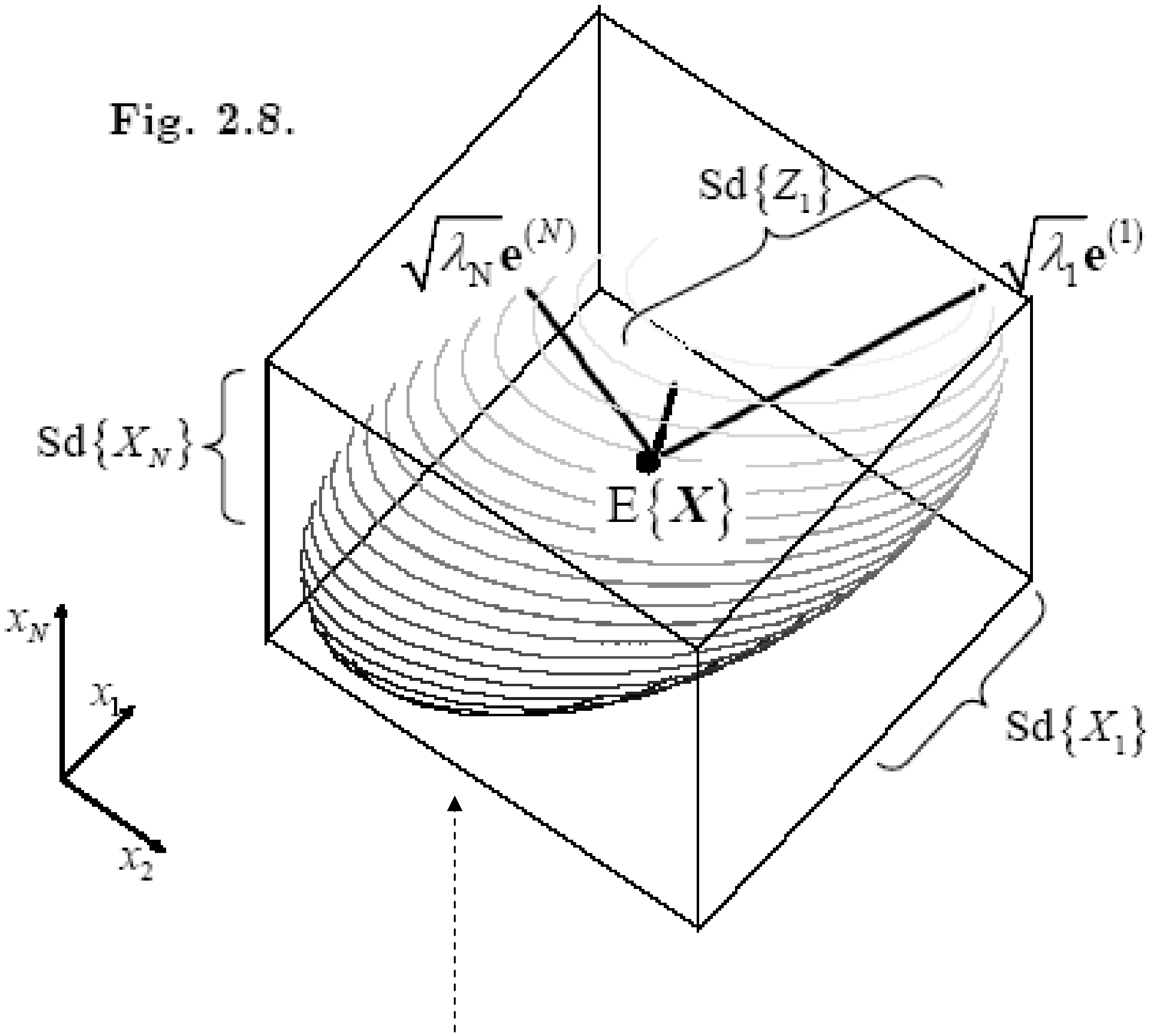
Fig. 2.8.



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$$\mathrm{Cov}\{Z_m, Z_n\} = 0. \quad (2.80)$$

$$\mathrm{Var}\{Z_n\} = \lambda_n. \quad (2.81)$$

$$\mathrm{E}\{X_n\} - \mathrm{Sd}\{X_n\} \leq x_n \leq \mathrm{E}\{X_n\} + \mathrm{Sd}\{X_n\} \quad (2.86)$$

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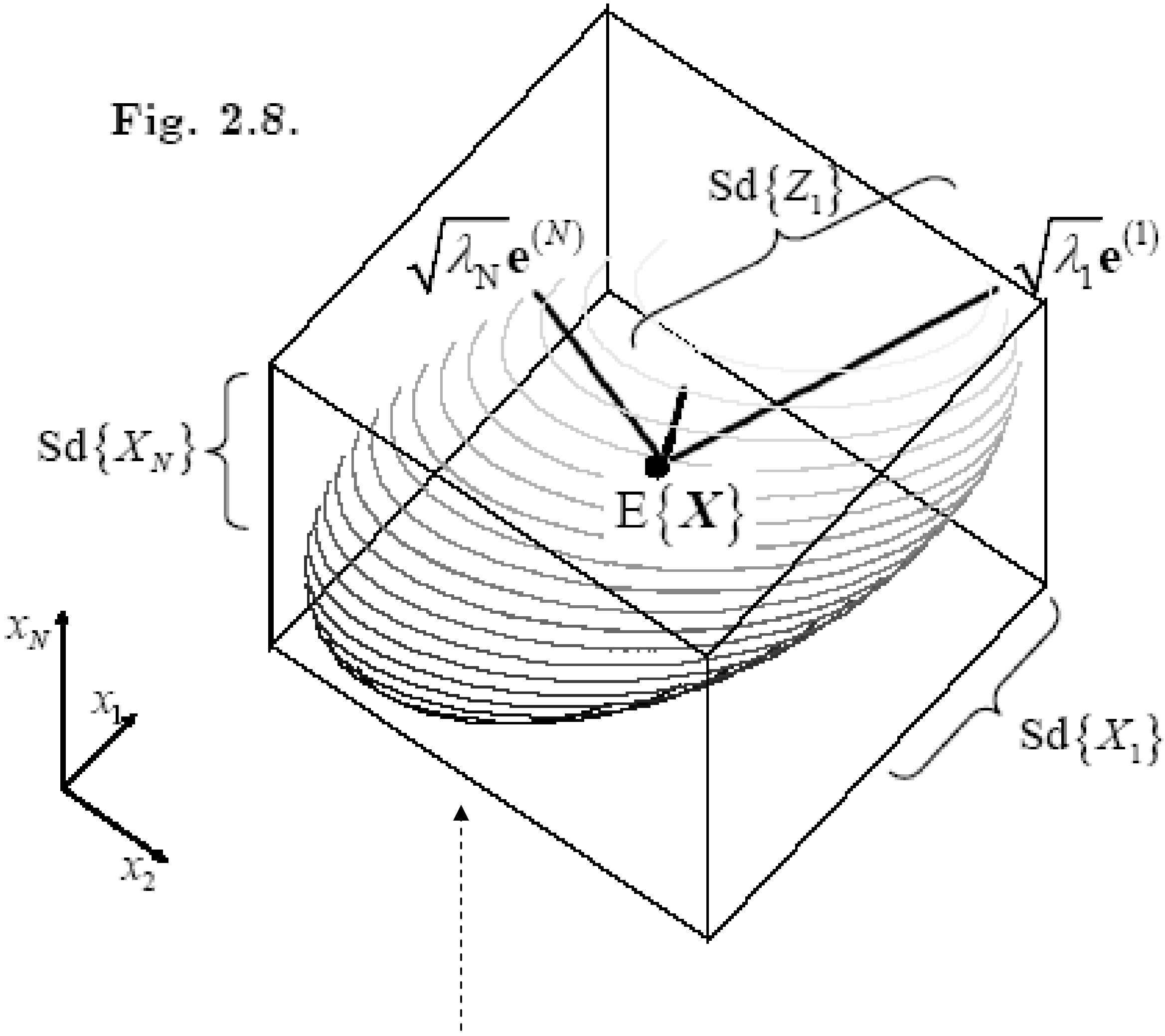

$$\mathrm{E}\{X_n\} - \mathrm{Sd}\{X_n\} \leq x_n \leq \mathrm{E}\{X_n\} + \mathrm{Sd}\{X_n\} \quad (2.86)$$

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$$\left\{\begin{array}{l} \mathbf{Z} \equiv \mathbf{E}'\mathbf{X} = \begin{pmatrix} \left[\mathbf{e}^{(1)}\right]'\mathbf{X} \\ \vdots \\ \left[\mathbf{e}^{(N)}\right]'\mathbf{X} \end{pmatrix} \quad (2.79) \\ \\ \mathrm{Cov}\{Z_m, Z_n\} = 0. \quad (2.80) \\ \\ \mathrm{Var}\{Z_n\} = \lambda_n. \quad (2.81) \end{array}\right.$$

$$\left\{\begin{array}{l} \lambda_1 = \max\limits_{\|\mathbf{e}\|=1} \{\mathrm{Var}\{\mathbf{e}'\mathbf{X}\}\} \quad (2.82) \\ \\ \mathbf{e}^{(1)} = \underset{\|\mathbf{e}\|=1}{\mathrm{argmax}} \{\mathrm{Var}\{\mathbf{e}'\mathbf{X}\}\} \quad (2.83) \end{array}\right.$$