# Attilio Meucci

**www.symmys.com**


Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

# OPTIMAL ALLOCATION – *Risk and Asset Allocation* - Springer – `symmys.com`

MARKET

$\mathbf{p}_T$

$\mathbf{P}_{T+\tau} \sim$ **?**

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\xi}, \boldsymbol{\Phi}) \quad (6.13)$$

$\mathcal{T}(\widetilde{\boldsymbol{\alpha}}, \boldsymbol{\alpha})$

$$\mathcal{T}(\widetilde{\boldsymbol{\alpha}}, \boldsymbol{\alpha}) \equiv \mathbf{k}' \left| \widetilde{\boldsymbol{\alpha}} - \boldsymbol{\alpha} \right| \quad (6.14)$$

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INVESTOR

$$w_T \equiv \mathbf{p}_T' \boldsymbol{\alpha}^{(0)} \quad (6.1)$$

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**primary objective & evaluation**

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}' \mathbf{M} \quad (6.2)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}' \mathbf{P}_{T+\tau} \quad (6.4)$$

$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \quad (5.48)$$

$$\mathrm{CE}(\boldsymbol{\alpha}) = -\zeta \ln \left( \phi_{\Psi_{\boldsymbol{\alpha}}} \left( \frac{i}{\zeta} \right) \right) \quad (6.8)$$

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$$\widetilde{\Psi}_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}' \widetilde{\mathbf{M}}. \quad (6.9)$$

$$\widetilde{\Psi}_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}' (\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (6.11)$$

$$\boldsymbol{\alpha} \mapsto \widetilde{\mathcal{S}}(\boldsymbol{\alpha})$$

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$$\mathcal{C}_1: \ \mathbf{p}_T' \boldsymbol{\alpha} + \mathcal{T}\left(\boldsymbol{\alpha}^{(0)}, \boldsymbol{\alpha}\right) - b \leq 0 \quad (6.23)$$

$$\mathcal{C}_1: \ \mathbf{p}_T' \boldsymbol{\alpha} = w_T \equiv \mathbf{p}_T' \boldsymbol{\alpha}^{(0)} \quad (6.24)$$

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market information

investor's profile

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