MEAN-VARIANCE ANALYTICAL

Risk and Asset Allocation **- Springer –** *symmys.com*

Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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$$\boxed{\boldsymbol{\alpha}(v) \equiv \underset{\substack{\boldsymbol{\alpha}'\mathbf{d}=c \\ \operatorname{Var}\{\Psi_{\boldsymbol{\alpha}}\}=v}}{\operatorname{argmax}} \operatorname{E}\{\Psi_{\boldsymbol{\alpha}}\}} \quad (6.96)$$

$$\begin{aligned} \mathcal{L}(\boldsymbol{\alpha},\lambda,\mu) &\equiv \operatorname{Var}\{\Psi_{\boldsymbol{\alpha}}\} - \lambda(\boldsymbol{\alpha}'\mathbf{d} - c) - \mu(\operatorname{E}\{\Psi_{\boldsymbol{\alpha}}\} - e). \qquad (T6.21) \\ &= \boldsymbol{\alpha}' \operatorname{Cov}\{\mathbf{M}\}\boldsymbol{\alpha} - \lambda(\boldsymbol{\alpha}'\mathbf{d} - c) - \mu(\boldsymbol{\alpha}' \operatorname{E}\{\mathbf{M}\} - e) \end{aligned}$$

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$$\left\{ \begin{aligned} \mathbf{0} &= \frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}} = 2\,\mathrm{Cov}\{\mathbf{M}\}\boldsymbol{\alpha} - \lambda\mathbf{d} - \mu\,\mathrm{E}\{\mathbf{M}\} \quad (T6.22) \\ 0 &= \frac{\partial \mathcal{L}}{\partial \lambda} = \boldsymbol{\alpha}'\mathbf{d} - c \\ & \qquad\qquad (T6.23) \\ 0 &= \frac{\partial \mathcal{L}}{\partial \mu} = \boldsymbol{\alpha}' \mathrm{E}\{\mathbf{M}\} - e \end{aligned} \right.$$

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$$\boldsymbol{\alpha} = \frac{\lambda}{2} \operatorname{Cov}\{\mathbf{M}\}^{-1}\mathbf{d} + \frac{\mu}{2} \operatorname{Cov}\{\mathbf{M}\}^{-1} \operatorname{E}\{\mathbf{M}\} \quad (T6.24)$$

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$$c = \frac{\lambda}{2}A + \frac{\mu}{2}B. \quad (T6.26)$$

$$e = \frac{\lambda}{2}B + \frac{\mu}{2}C \quad (T6.27)$$

$$\begin{aligned} A &\equiv \mathbf{d}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathbf{d} & B &\equiv \mathbf{d}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathrm{E}\{\mathbf{M}\} \\ C &\equiv \mathrm{E}\{\mathbf{M}\}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathrm{E}\{\mathbf{M}\} & D &\equiv AC - B^2 \end{aligned} \quad (T6.25)$$

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$$\boxed{v = \frac{A}{D}e^2 - \frac{2cB}{D}e + \frac{c^2C}{D}} \quad (6.102)$$

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$$\boxed{v = \frac{A}{D}e^2 - \frac{2cB}{D}e + \frac{c^2C}{D}} \quad (6.102)$$

$$\boldsymbol{\alpha} = (1 - \gamma(\boldsymbol{\alpha}))\,\boldsymbol{\alpha}_{MV} + \gamma(\boldsymbol{\alpha})\,\boldsymbol{\alpha}_{SR}. \quad (T6.33)$$

$$\gamma \equiv \frac{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{MV}}\}}{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{SR}}\} - \mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{MV}}\}} \quad (T6.41)$$

$$\boldsymbol{\alpha}_{MV} \overset{(6.99)}{\equiv} \frac{c\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathbf{d}}{\mathbf{d}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathbf{d}} \qquad \boldsymbol{\alpha}_{SR} \overset{(6.100)}{\equiv} \frac{c\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathrm{E}\{\mathbf{M}\}}{\mathbf{d}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathrm{E}\{\mathbf{M}\}}$$

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$$\boxed{v = \frac{A}{D}e^2 - \frac{2cB}{D}e + \frac{c^2C}{D}} \quad (6.102)$$

$$\boldsymbol{\alpha} = (1-\gamma(\boldsymbol{\alpha}))\,\boldsymbol{\alpha}_{MV} + \gamma(\boldsymbol{\alpha})\,\boldsymbol{\alpha}_{SR}. \quad (T6.33)$$

$$\gamma \equiv \frac{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{MV}}\}}{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{SR}}\} - \mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{MV}}\}} \quad (T6.41)$$

$$\boldsymbol{\alpha}_{MV} \overset{(6.99)}{\equiv} \frac{c\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathbf{d}}{\mathbf{d}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathbf{d}} \qquad \boldsymbol{\alpha}_{SR} \overset{(6.100)}{\equiv} \frac{c\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathrm{E}\{\mathbf{M}\}}{\mathbf{d}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathrm{E}\{\mathbf{M}\}}$$

$$\boxed{\boldsymbol{\alpha}(e) = \boldsymbol{\alpha}_{MV} + \left[e - \mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{MV}}\}\right]\frac{\boldsymbol{\alpha}_{SR} - \boldsymbol{\alpha}_{MV}}{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{SR}}\} - \mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{MV}}\}}} \quad (6.97)$$

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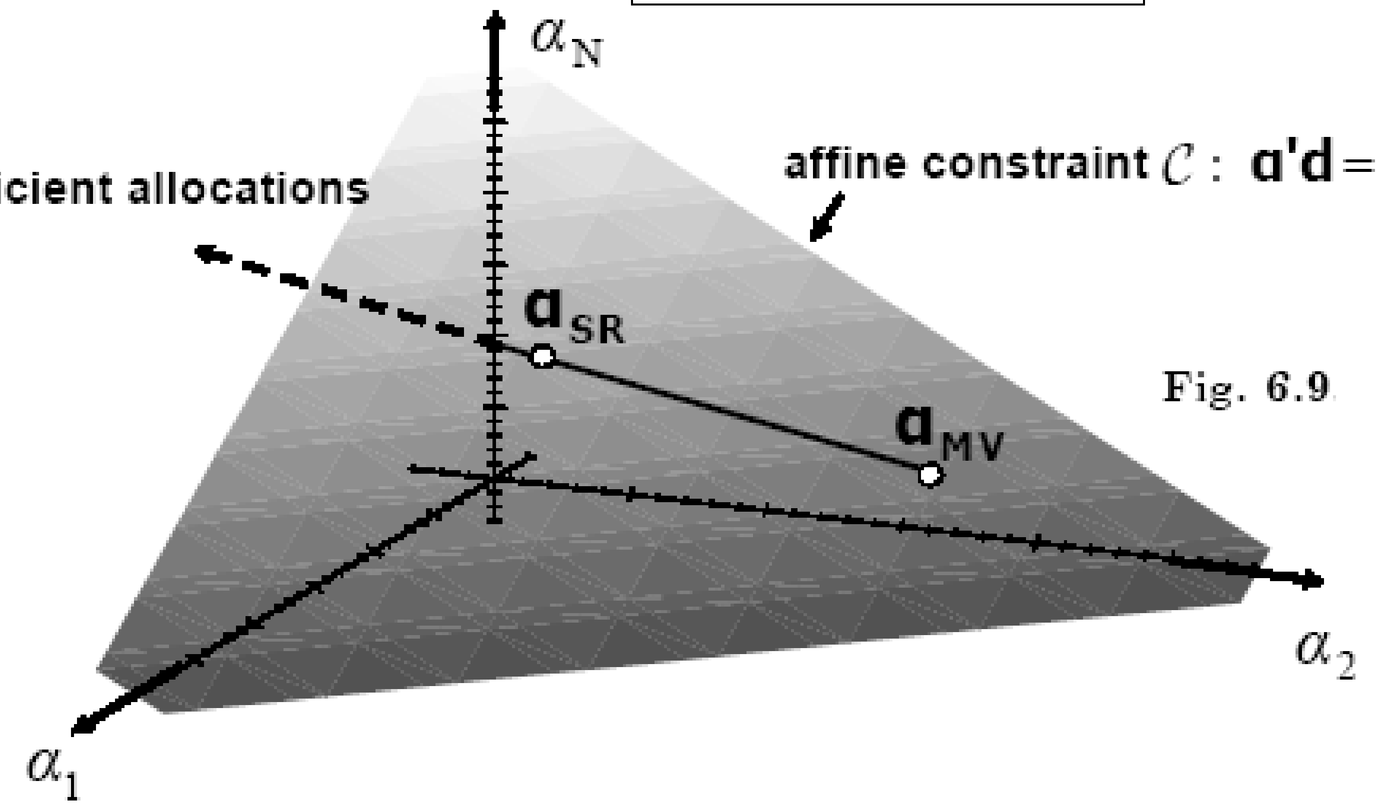

Fig. 6.9

$$\boldsymbol{\alpha}_{MV} \overset{(6.99)}{\equiv} \frac{c\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathbf{d}}{\mathbf{d}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathbf{d}} \qquad \boldsymbol{\alpha}_{SR} \overset{(6.100)}{\equiv} \frac{c\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathrm{E}\{\mathbf{M}\}}{\mathbf{d}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathrm{E}\{\mathbf{M}\}}$$

$$v = \frac{A}{D}e^2 - \frac{2cB}{D}e + \frac{c^2C}{D} \quad (6.102)$$

$$\boxed{\boldsymbol{\alpha}(e) = \boldsymbol{\alpha}_{MV} + [e - \mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{MV}}\}] \frac{\boldsymbol{\alpha}_{SR} - \boldsymbol{\alpha}_{MV}}{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{SR}}\} - \mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{MV}}\}}} \quad (6.97)$$

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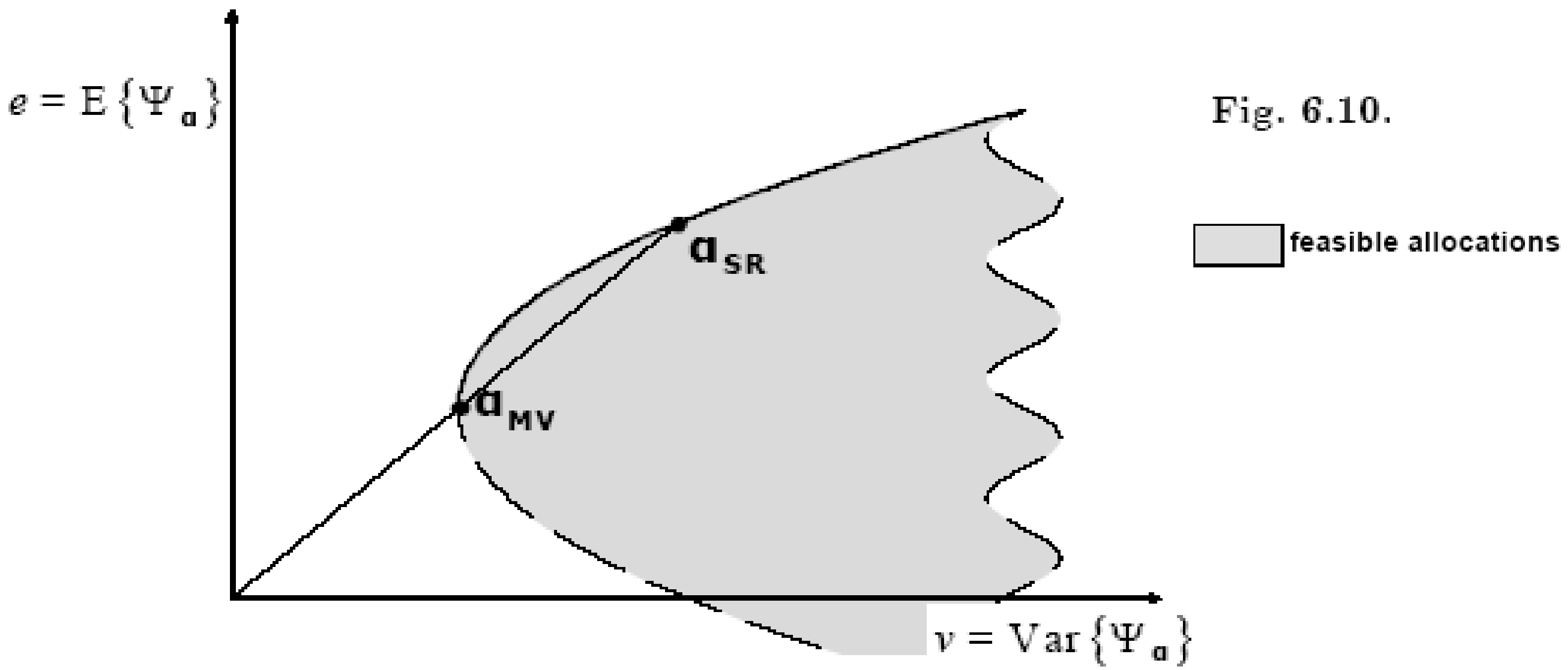

Fig. 6.10.

$$\alpha_{MV} \overset{(6.99)}{\equiv} \frac{c\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathbf{d}}{\mathbf{d}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathbf{d}} \qquad \alpha_{SR} \overset{(6.100)}{\equiv} \frac{c\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathrm{E}\{\mathbf{M}\}}{\mathbf{d}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathrm{E}\{\mathbf{M}\}}$$

$$\boxed{v = \frac{A}{D}e^2 - \frac{2cB}{D}e + \frac{c^2 C}{D}} \quad (6.102)$$

$$\alpha(e) = \alpha_{MV} + \left[e - \mathrm{E}\{\Psi_{\alpha_{MV}}\}\right] \frac{\alpha_{SR} - \alpha_{MV}}{\mathrm{E}\{\Psi_{\alpha_{SR}}\} - \mathrm{E}\{\Psi_{\alpha_{MV}}\}} \quad (6.97)$$

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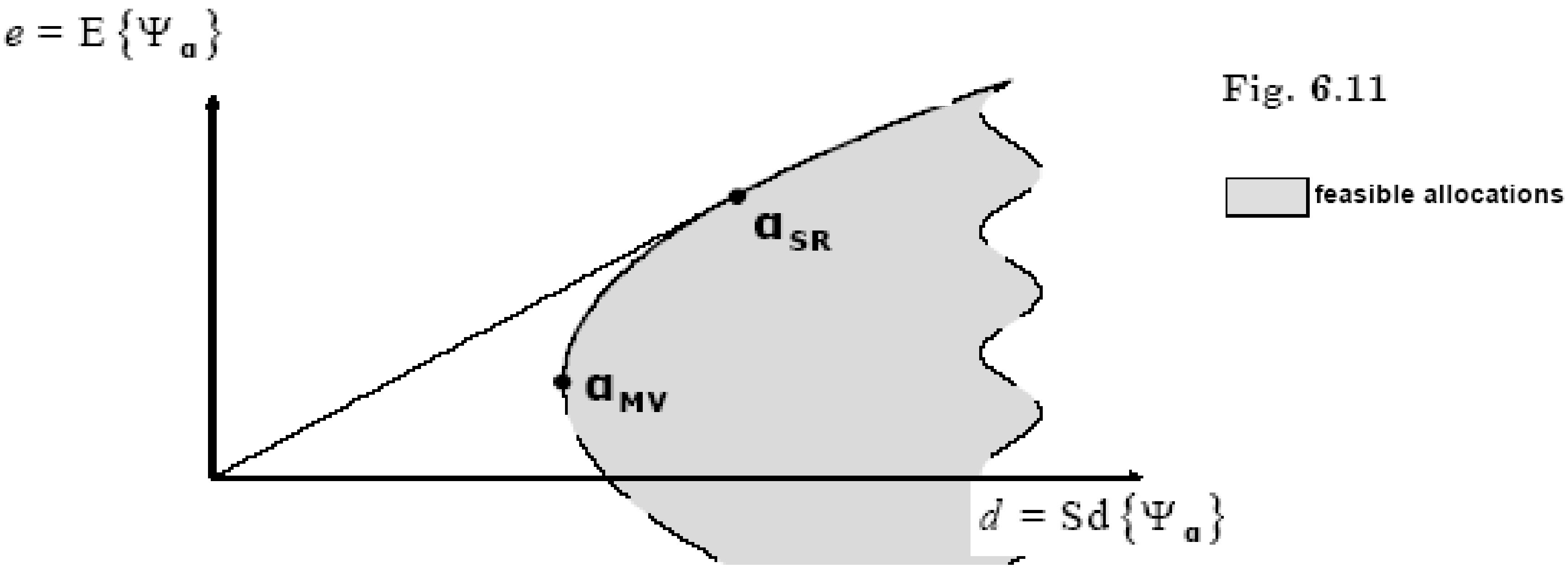

Fig. 6.11

$$\boxed{d^2 = \frac{A}{D}e^2 - \frac{2cB}{D}e + \frac{c^2C}{D}} \quad (6.106)$$

$$\Updownarrow$$

$$v = \frac{A}{D}e^2 - \frac{2cB}{D}e + \frac{c^2C}{D} \quad (6.102)$$

$$\boldsymbol{\alpha}_{MV} \overset{(6.99)}{\equiv} \frac{c\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathbf{d}}{\mathbf{d}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathbf{d}} \qquad \boldsymbol{\alpha}_{SR} \overset{(6.100)}{\equiv} \frac{c\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathrm{E}\{\mathbf{M}\}}{\mathbf{d}'\,\mathrm{Cov}\{\mathbf{M}\}^{-1}\mathrm{E}\{\mathbf{M}\}}$$

$$\boldsymbol{\alpha}(e) = \boldsymbol{\alpha}_{MV} + [e - \mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{MV}}\}] \frac{\boldsymbol{\alpha}_{SR} - \boldsymbol{\alpha}_{MV}}{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{SR}}\} - \mathrm{E}\{\Psi_{\boldsymbol{\alpha}_{MV}}\}} \quad (6.97)$$