Attilio Meucci

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Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{ \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \qquad (8.76)$$

$$\begin{aligned} \mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = {} & \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1} + \boldsymbol{\mu}) \\ & - \frac{1}{2\zeta} \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T) \boldsymbol{\Sigma} \operatorname{diag}(\mathbf{p}_T) \boldsymbol{\alpha} \end{aligned} \qquad (8.25)$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = [\operatorname{diag}(\mathbf{p}_T)]^{-1} \boldsymbol{\Sigma}^{-1} \left(\zeta \boldsymbol{\mu} + \frac{w_T - \zeta \mathbf{1}' \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}}{\mathbf{1}' \boldsymbol{\Sigma}^{-1} \mathbf{1}} \mathbf{1} \right) \qquad (8.77)$$

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$$\alpha(\theta) \equiv \underset{\alpha \in \mathcal{C}_\theta}{\operatorname{argmax}} \{\mathcal{S}_\theta(\alpha)\} \qquad (8.76)$$

$$\begin{aligned}\mathrm{CE}_{\mu,\Sigma}(\alpha) &= \alpha' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1}+\mu) \qquad (8.25)\\ &\quad -\frac{1}{2\zeta}\alpha' \operatorname{diag}(\mathbf{p}_T)\,\Sigma \operatorname{diag}(\mathbf{p}_T)\,\alpha\end{aligned}$$

$$\alpha(\mu,\Sigma) = [\operatorname{diag}(\mathbf{p}_T)]^{-1}\,\Sigma^{-1}\left(\zeta\mu + \frac{w_T - \zeta\mathbf{1}'\Sigma^{-1}\mu}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\mathbf{1}\right) \quad (8.77)$$

$$\mathrm{OC}_\theta(\alpha) \equiv \overline{\mathcal{S}}(\theta) - \mathcal{S}_\theta(\alpha)$$

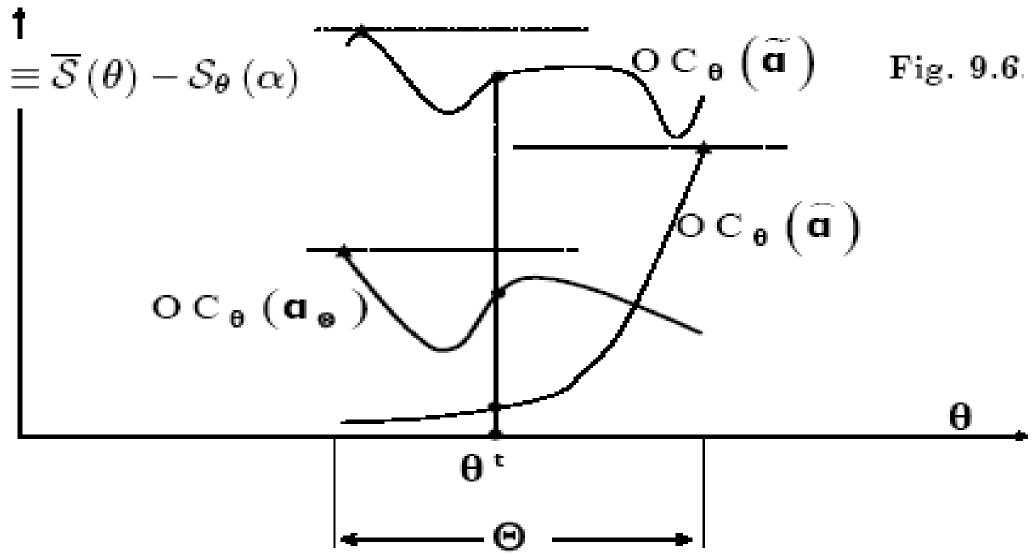

Fig. 9.6

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$$\mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) - \frac{1}{2\zeta}\boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)\,\boldsymbol{\Sigma}\operatorname{diag}(\mathbf{p}_T)\,\boldsymbol{\alpha} \quad (8.25)$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha}\in\mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}}\left\{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\right\} \quad (8.76)$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu},\boldsymbol{\Sigma}) = [\operatorname{diag}(\mathbf{p}_T)]^{-1}\boldsymbol{\Sigma}^{-1}\left(\zeta\boldsymbol{\mu} + \frac{w_T - \zeta\mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}\mathbf{1}\right) \quad (8.77)$$

$$\boldsymbol{\alpha}_{\boldsymbol{\Theta}} \equiv \underset{\boldsymbol{\alpha}\in\mathcal{C}_{\boldsymbol{\Theta}}}{\operatorname{argmin}}\left\{\max_{\boldsymbol{\theta}\in\boldsymbol{\Theta}}\left\{\overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\right\}\right\} \quad (9.104)$$

$$(9.103) \quad \boldsymbol{\alpha}\in\mathcal{C}_{\boldsymbol{\Theta}} \equiv \{\boldsymbol{\alpha}\in\mathcal{C}_{\boldsymbol{\theta}} \text{ for all } \boldsymbol{\theta}\in\boldsymbol{\Theta}\}$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\alpha) \equiv \overline{\mathcal{S}}(\theta) - \mathcal{S}_{\boldsymbol{\theta}}(\alpha)$$

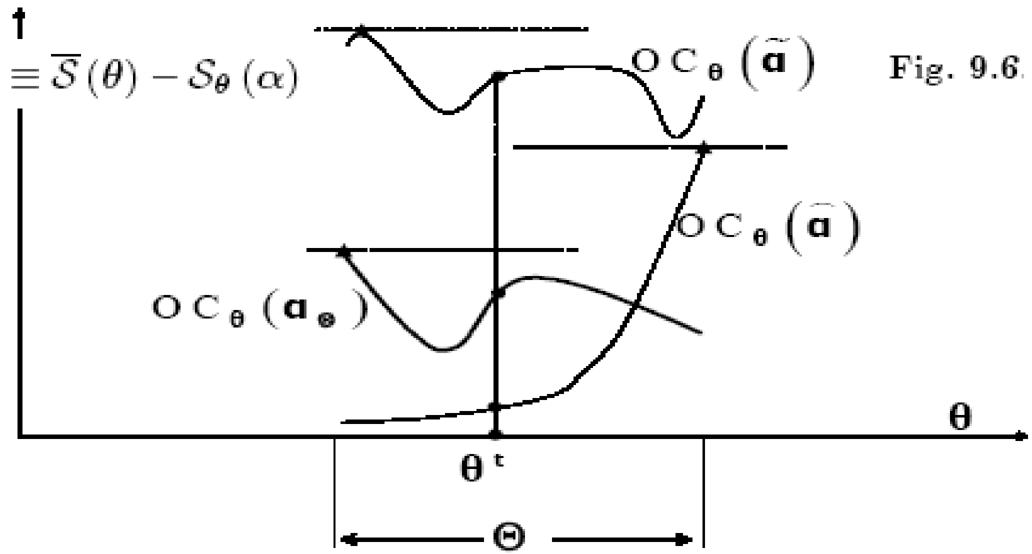

Fig. 9.6

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$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{ \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \qquad (8.76)$$

$$\begin{aligned} \mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = {} & \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) \qquad (8.25) \\ & - \frac{1}{2\zeta} \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)\, \boldsymbol{\Sigma} \operatorname{diag}(\mathbf{p}_T)\, \boldsymbol{\alpha} \end{aligned}$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = [\operatorname{diag}(\mathbf{p}_T)]^{-1} \boldsymbol{\Sigma}^{-1} \left(\zeta \boldsymbol{\mu} + \frac{w_T - \zeta \mathbf{1}' \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}}{\mathbf{1}' \boldsymbol{\Sigma}^{-1} \mathbf{1}} \mathbf{1} \right) \qquad (8.77)$$

$$\boldsymbol{\alpha}_{\boldsymbol{\Theta}} \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\Theta}}}{\operatorname{argmin}} \left\{ \max_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} \left\{ \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \right\} \qquad (9.104)$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\alpha) \equiv \overline{\mathcal{S}}(\theta) - \mathcal{S}_{\boldsymbol{\theta}}(\alpha)$$

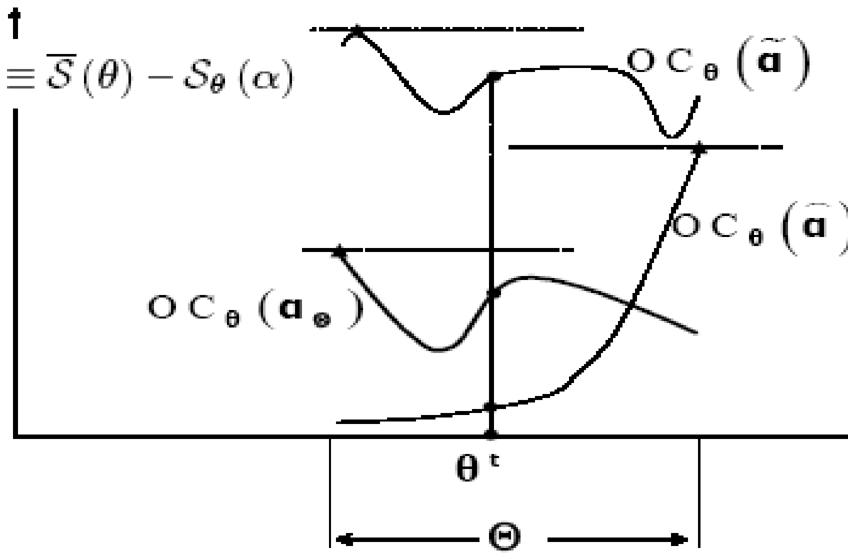

Fig. 9.6

$$i_T \mapsto \widehat{\boldsymbol{\Theta}}[i_T] \qquad (9.106)$$

$$\widehat{\Theta}_{\mu}[i_T] \equiv \left\{ \boldsymbol{\mu} \text{ such that } \mathrm{Ma}^2\left(\boldsymbol{\mu}, \widehat{\boldsymbol{\mu}}[i_T], \boldsymbol{\Sigma}'\right) \leq \frac{Q_{\chi^2_N}(p)}{T} \right\} \qquad (9.108)$$

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$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha}\in\mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{ \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \qquad (8.76)$$

$$\begin{aligned} \mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = {} & \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) \\ & -\frac{1}{2\zeta}\boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)\,\boldsymbol{\Sigma}\operatorname{diag}(\mathbf{p}_T)\,\boldsymbol{\alpha} \end{aligned} \qquad (8.25)$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu},\boldsymbol{\Sigma}) = [\operatorname{diag}(\mathbf{p}_T)]^{-1}\,\boldsymbol{\Sigma}^{-1}\left(\zeta\boldsymbol{\mu} + \frac{w_T - \zeta\mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}\mathbf{1}\right) \qquad (8.77)$$

$$\boldsymbol{\alpha}_{\Theta} \equiv \underset{\boldsymbol{\alpha}\in\mathcal{C}_{\Theta}}{\operatorname{argmin}} \left\{ \max_{\boldsymbol{\theta}\in\Theta} \left\{ \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \right\} \qquad (9.104)$$

$$\mathrm{OC}_{\boldsymbol{\theta}}(\alpha) \equiv \overline{\mathcal{S}}(\theta) - \mathcal{S}_{\boldsymbol{\theta}}(\alpha)$$

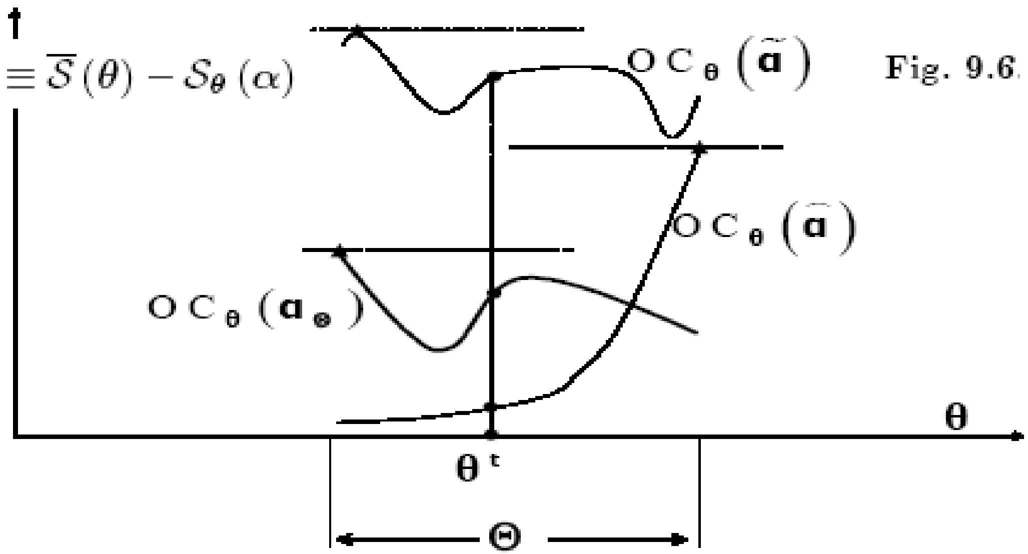

Fig. 9.6

$$i_T \mapsto \widehat{\Theta}[i_T] \qquad (9.106)$$

$$\widehat{\Theta}_{\mu}[i_T] \equiv \left\{ \boldsymbol{\mu} \text{ such that } \mathrm{Ma}^2\left(\boldsymbol{\mu}, \widehat{\boldsymbol{\mu}}[i_T], \boldsymbol{\Sigma}'\right) \leq \frac{Q_{\chi^2_N}(p)}{T} \right\} \qquad (9.108)$$

$$\boldsymbol{\alpha}_{r}[i_T] \equiv \underset{\boldsymbol{\alpha}\in\mathcal{C}_{\widehat{\Theta}[i_T]}}{\operatorname{argmin}} \left\{ \max_{\boldsymbol{\theta}\in\widehat{\Theta}[i_T]} \left\{ \overline{\mathcal{S}}(\boldsymbol{\theta}) - \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \right\} \qquad (9.110)$$

$$\alpha^{(i)} = \underset{\alpha}{\operatorname{argmax}}\, \alpha' \mu \qquad (9.115)$$

$$\text{subject to } \begin{cases} \alpha \in \mathcal{C} \\ \alpha' \Sigma \alpha \leq v^{(i)}. \end{cases}$$

$$\mu \equiv \mathrm{E}\{\mathbf{M}\} \qquad \Sigma \equiv \mathrm{Cov}\{\mathbf{M}\} \qquad (9.116)$$

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$$\boldsymbol{\alpha}_{\mathbf{r}}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \boxed{\widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}}}} \left\{ \boldsymbol{\alpha}' \boldsymbol{\mu} \right\} \right\} \qquad (9.117)$$

$$\text{subject to } \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max_{\boldsymbol{\Sigma} \in \boxed{\widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}}}} \left\{ \boldsymbol{\alpha}' \boldsymbol{\Sigma} \boldsymbol{\alpha} \right\} \leq v^{(i)} \end{cases}.$$

$$\boldsymbol{\alpha}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}}\, \boldsymbol{\alpha}' \boldsymbol{\mu} \qquad (9.115)$$

$$\text{subject to } \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \boldsymbol{\alpha}' \boldsymbol{\Sigma} \boldsymbol{\alpha} \leq v^{(i)} \end{cases}.$$

$$\boldsymbol{\mu} \equiv \mathrm{E}\left\{\mathbf{M}\right\} \qquad \boldsymbol{\Sigma} \equiv \mathrm{Cov}\left\{\mathbf{M}\right\} \qquad (9.116)$$

$$\boldsymbol{\alpha}_{\mathrm{r}}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}}} \{\boldsymbol{\alpha}'\boldsymbol{\mu}\} \right\} \quad (9.117)$$

$$\text{subject to } \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max_{\boldsymbol{\Sigma} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}}} \{\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}\} \leq v^{(i)} \end{cases}$$

- Goldfarb and Iyengar (2003)

$$\widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}} \equiv \{\boldsymbol{\mu} \text{ such that } \underline{\boldsymbol{\mu}} \leq \boldsymbol{\mu} \leq \overline{\boldsymbol{\mu}}\} \quad (9.125)$$

$$\widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}} \equiv \left\{\mathbf{BGB}' + \operatorname{diag}(\mathbf{d})\right\} \quad (9.126)$$

$$\mathbf{G} \text{ known} \quad \underline{\mathbf{d}} \leq \mathbf{d} \leq \overline{\mathbf{d}} \quad \mathbf{b}_{(n)} \in \mathcal{E}_n, n = 1, \ldots, N. \quad (9.127)$$

$$\boldsymbol{\alpha}_{\mathrm{r}}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \widehat{\Theta}_{\mu}} \{\boldsymbol{\alpha}'\boldsymbol{\mu}\} \right\} \quad (9.117)$$

$$\text{subject to } \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max_{\boldsymbol{\Sigma} \in \widehat{\Theta}_{\boldsymbol{\Sigma}}} \{\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}\} \leq v^{(i)}. \end{cases}$$

- Goldfarb and Iyengar (2003)

$$\widehat{\Theta}_{\mu} \equiv \{\boldsymbol{\mu} \text{ such that } \underline{\boldsymbol{\mu}} \leq \boldsymbol{\mu} \leq \overline{\boldsymbol{\mu}}\} \quad (9.125)$$

$$\widehat{\Theta}_{\boldsymbol{\Sigma}} \equiv \{\mathbf{BGB}' + \operatorname{diag}(\mathbf{d})\} \quad (9.126)$$

$$\mathbf{G} \text{ known} \quad \underline{\mathbf{d}} \leq \mathbf{d} \leq \overline{\mathbf{d}} \quad \mathbf{b}_{(n)} \in \mathcal{E}_n, n = 1, \ldots, N. \quad (9.127)$$

- Halldorsson and Tutuncu (2003)

$$\widehat{\Theta}_{\mu} \equiv \{\boldsymbol{\mu} \text{ such that } \underline{\boldsymbol{\mu}} \leq \boldsymbol{\mu} \leq \overline{\boldsymbol{\mu}}\} \quad (9.128)$$

$$\widehat{\Theta}_{\boldsymbol{\Sigma}} \equiv \{\boldsymbol{\Sigma} \succeq \mathbf{0} \text{ such that } \underline{\boldsymbol{\Sigma}} \leq \boldsymbol{\Sigma} \leq \overline{\boldsymbol{\Sigma}}\} \quad (9.129)$$

$$\boldsymbol{\alpha}_r^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \widehat{\Theta}_{\mu}} \{\boldsymbol{\alpha}'\boldsymbol{\mu}\} \right\} \quad (9.117)$$

$$\text{subject to } \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max_{\boldsymbol{\Sigma} \in \widehat{\Theta}_{\Sigma}} \{\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}\} \leq v^{(i)} \end{cases}$$

- Goldfarb and Iyengar (2003)

$$\widehat{\Theta}_{\mu} \equiv \left\{\boldsymbol{\mu} \text{ such that } \underline{\boldsymbol{\mu}} \leq \boldsymbol{\mu} \leq \overline{\boldsymbol{\mu}}\right\} \quad (9.125)$$

$$\widehat{\Theta}_{\Sigma} \equiv \left\{\mathbf{BGB}' + \operatorname{diag}(\mathbf{d})\right\} \quad (9.126)$$

$$\mathbf{G} \text{ known} \quad \underline{\mathbf{d}} \leq \mathbf{d} \leq \overline{\mathbf{d}} \quad \mathbf{b}_{(n)} \in \mathcal{E}_n, n = 1, \ldots, N. \quad (9.127)$$

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$$\widehat{\Theta}_{\mu} \equiv \left\{\boldsymbol{\mu} \text{ such that } \underline{\boldsymbol{\mu}} \leq \boldsymbol{\mu} \leq \overline{\boldsymbol{\mu}}\right\} \quad (9.128)$$

$$\widehat{\Theta}_{\Sigma} \equiv \left\{\boldsymbol{\Sigma} \succeq \mathbf{0} \text{ such that } \underline{\boldsymbol{\Sigma}} \leq \boldsymbol{\Sigma} \leq \overline{\boldsymbol{\Sigma}}\right\} \quad (9.129)$$

- Meucci (2005)

$$\widehat{\Theta}_{\mu} \equiv \left\{\boldsymbol{\mu} : \ (\boldsymbol{\mu} - \widehat{\boldsymbol{\mu}}_{\mathrm{ce}})' \, \mathbf{S}_{\mu}^{-1} (\boldsymbol{\mu} - \widehat{\boldsymbol{\mu}}_{\mathrm{ce}}) \leq q_{\mu}^2 \right\} \quad (9.149)$$

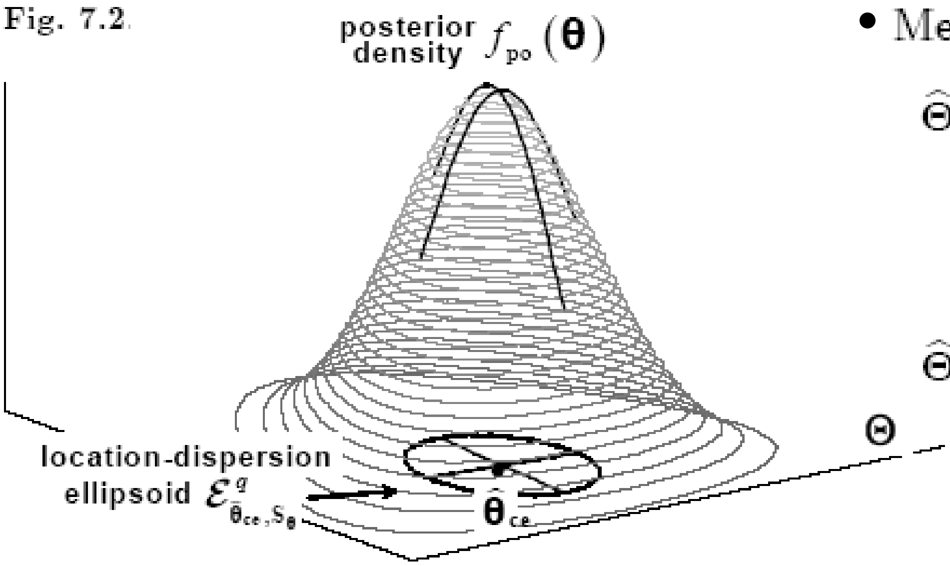
scatter matrix for $\boldsymbol{\mu}$

normal-inverse-Wishart posterior classical-equivalent

$$\widehat{\Theta}_{\Sigma} \equiv \left\{\boldsymbol{\Sigma} : \ \operatorname{vech}\left[\boldsymbol{\Sigma} - \widehat{\boldsymbol{\Sigma}}_{\mathrm{ce}}\right]' \mathbf{S}_{\Sigma}^{-1} \operatorname{vech}\left[\boldsymbol{\Sigma} - \widehat{\boldsymbol{\Sigma}}_{\mathrm{ce}}\right] \leq q_{\Sigma}^2 \right\} \quad (9.152)$$

scatter matrix for $\operatorname{vech}[\boldsymbol{\Sigma}]$

Fig. 7.2.



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$$\boldsymbol{\alpha}_{\mathrm{r}}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \widehat{\Theta}_{\boldsymbol{\mu}}} \left\{ \boldsymbol{\alpha}' \boldsymbol{\mu} \right\} \right\} \quad (9.117)$$

$$\text{subject to } \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max_{\boldsymbol{\Sigma} \in \widehat{\Theta}_{\boldsymbol{\Sigma}}} \left\{ \boldsymbol{\alpha}' \boldsymbol{\Sigma} \boldsymbol{\alpha} \right\} \leq v^{(i)} \end{cases}.$$

$$\widehat{\Theta}_{\boldsymbol{\mu}} \equiv \left\{ \boldsymbol{\mu} : (\boldsymbol{\mu} - \mathbf{m})' \mathbf{T}^{-1} (\boldsymbol{\mu} - \mathbf{m}) \leq q^2 \right\}. \quad (T9.72)$$

$$\widehat{\Theta}_{\boldsymbol{\Sigma}} \equiv \widehat{\boldsymbol{\Sigma}}. \quad (9.119)$$

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$$\boldsymbol{\alpha}_{\mathrm{r}}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}}} \{\boldsymbol{\alpha}' \boldsymbol{\mu}\} \right\} \quad (9.117)$$

$$\text{subject to } \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max\limits_{\boldsymbol{\Sigma} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}}} \{\boldsymbol{\alpha}' \boldsymbol{\Sigma} \boldsymbol{\alpha}\} \leq v^{(i)} \end{cases}.$$

$$\widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}} \equiv \left\{ \boldsymbol{\mu} : (\boldsymbol{\mu} - \mathbf{m})' \, \mathbf{T}^{-1} (\boldsymbol{\mu} - \mathbf{m}) \leq q^2 \right\}. \quad (T9.72)$$

$$\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{m}, \mathbf{T}) \quad (9.123)$$

$$q^2 \equiv Q_{\chi^2_N}(p) \quad (9.120)$$

$$\mathbb{P}\left\{ \boldsymbol{\mu} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}} \right\} = p \quad (9.124)$$

$$\widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}} \equiv \widehat{\boldsymbol{\Sigma}}. \quad (9.119)$$

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$$\boldsymbol{\alpha}_{\mathrm{r}}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}}} \{\boldsymbol{\alpha}'\boldsymbol{\mu}\} \right\} \quad (9.117)$$

$$\text{subject to } \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max\limits_{\boldsymbol{\Sigma} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}}} \{\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}\} \leq v^{(i)} \end{cases}$$

$$\widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}} \equiv \left\{ \boldsymbol{\mu} : (\boldsymbol{\mu} - \mathbf{m})' \, \mathbf{T}^{-1} (\boldsymbol{\mu} - \mathbf{m}) \leq q^2 \right\}. \quad (T9.72)$$

$$\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{m}, \mathbf{T}) \quad (9.123) \qquad q^2 \equiv Q_{\chi^2_N}(p) \quad (9.120) \qquad \mathbb{P}\left\{ \boldsymbol{\mu} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}} \right\} = p \quad (9.124)$$

$$\widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}} \equiv \widehat{\boldsymbol{\Sigma}}. \quad (9.119)$$

- $$\widehat{\boldsymbol{\mu}}[I_T] \sim \mathrm{N}\left(\boldsymbol{\mu}, \frac{\boldsymbol{\Sigma}}{T} \right) \quad (4.102)$$

$$\mathbf{m} \equiv \widehat{\boldsymbol{\mu}}[i_T] = \frac{1}{T} \sum_{t=1}^{T} \mathbf{x}_t \quad \mathbf{T} \equiv \frac{1}{T} \widehat{\boldsymbol{\Sigma}}$$

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$$\boldsymbol{\alpha}_{\mathrm{r}}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \widehat{\Theta}_{\boldsymbol{\mu}}} \left\{ \boldsymbol{\alpha}' \boldsymbol{\mu} \right\} \right\} \quad (9.117)$$

$$\text{subject to} \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max\limits_{\boldsymbol{\Sigma} \in \widehat{\Theta}_{\boldsymbol{\Sigma}}} \left\{ \boldsymbol{\alpha}' \boldsymbol{\Sigma} \boldsymbol{\alpha} \right\} \leq v^{(i)}. \end{cases}$$

$$\widehat{\Theta}_{\boldsymbol{\mu}} \equiv \left\{ \boldsymbol{\mu} : (\boldsymbol{\mu} - \mathbf{m})' \mathbf{T}^{-1} (\boldsymbol{\mu} - \mathbf{m}) \leq q^2 \right\}. \quad (T9.72)$$

$$\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{m}, \mathbf{T}) \quad (9.123) \qquad q^2 \equiv Q_{\chi^2_N}(p) \quad (9.120) \qquad \mathbb{P}\left\{ \boldsymbol{\mu} \in \widehat{\Theta}_{\boldsymbol{\mu}} \right\} = p \quad (9.124)$$

$$\widehat{\Theta}_{\boldsymbol{\Sigma}} \equiv \widehat{\boldsymbol{\Sigma}}. \quad (9.119)$$

- $$\widehat{\boldsymbol{\mu}}[I_T] \sim \mathrm{N}\left(\boldsymbol{\mu}, \frac{\boldsymbol{\Sigma}}{T} \right) \quad (4.102)$$

  $$\mathbf{m} \equiv \widehat{\boldsymbol{\mu}}[i_T] = \frac{1}{T} \sum_{t=1}^{T} \mathbf{x}_t \quad \mathbf{T} \equiv \frac{1}{T} \widehat{\boldsymbol{\Sigma}}$$

- Ceria and Stubbs (2004)

  $$\mathbf{m} \equiv \widehat{\boldsymbol{\mu}}[i_T], \ \mathbf{T} \text{ exogenous}, \quad (9.121)$$

- De Santis and Foresi (2002)

  $$\mathbf{m} \equiv \boldsymbol{\mu}_{\mathrm{BL}}, \quad \mathbf{T} \equiv \boldsymbol{\Sigma}_{\mathrm{BL}} \quad (9.122)$$

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$$\boldsymbol{\alpha}_{\mathrm{r}}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}}} \{\boldsymbol{\alpha}'\boldsymbol{\mu}\} \right\} \quad (9.117)$$

$$\text{subject to } \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max_{\boldsymbol{\Sigma} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}}} \{\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}\} \leq v^{(i)} \end{cases}.$$

$$\widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}} \equiv \left\{ \boldsymbol{\mu} : (\boldsymbol{\mu} - \mathbf{m})' \mathbf{T}^{-1} (\boldsymbol{\mu} - \mathbf{m}) \leq q^2 \right\}. \quad (T9.72)$$

$$\widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}} \equiv \widehat{\boldsymbol{\Sigma}}. \quad (9.119)$$

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$$\boldsymbol{\alpha}_{\mathrm{r}}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \widehat{\Theta}_{\boldsymbol{\mu}}} \left\{ \boldsymbol{\alpha}' \boldsymbol{\mu} \right\} \right\} \quad (9.117)$$

$$\text{subject to} \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max_{\boldsymbol{\Sigma} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}}} \left\{ \boldsymbol{\alpha}' \boldsymbol{\Sigma} \boldsymbol{\alpha} \right\} \leq v^{(i)} \end{cases}.$$

$$\widehat{\Theta}_{\boldsymbol{\mu}} \equiv \left\{ \boldsymbol{\mu} : (\boldsymbol{\mu} - \mathbf{m})' \mathbf{T}^{-1} (\boldsymbol{\mu} - \mathbf{m}) \leq q^2 \right\}. \quad (T9.72)$$

$$\widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}} \equiv \widehat{\boldsymbol{\Sigma}}. \quad (9.119)$$

$$\widehat{\Theta}_{\boldsymbol{\mu}} \equiv \left\{ \mathbf{m} + q \mathbf{E} \boldsymbol{\Lambda}^{1/2} \mathbf{u} : \mathbf{u}' \mathbf{u} \leq 1 \right\} \quad (T9.77)$$

$$\mathbf{T} \equiv \mathbf{E} \boldsymbol{\Lambda}^{1/2} \boldsymbol{\Lambda}^{1/2} \mathbf{E}'. \quad (T9.73)$$

$$\mathbf{u} \equiv \frac{1}{q} \boldsymbol{\Lambda}^{-1/2} \mathbf{E}' (\boldsymbol{\mu} - \mathbf{m}) \quad (T9.75)$$

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$$\boldsymbol{\alpha}_{\mathrm{r}}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}}} \{\boldsymbol{\alpha}' \boldsymbol{\mu}\} \right\} \quad (9.117)$$

$$\text{subject to} \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max_{\boldsymbol{\Sigma} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}}} \{\boldsymbol{\alpha}' \boldsymbol{\Sigma} \boldsymbol{\alpha}\} \leq v^{(i)} \end{cases}$$

$$\widehat{\Theta}_{\boldsymbol{\mu}} \equiv \left\{ \boldsymbol{\mu} : (\boldsymbol{\mu} - \mathbf{m})' \mathbf{T}^{-1} (\boldsymbol{\mu} - \mathbf{m}) \leq q^2 \right\} . \quad (T9.72)$$

$$\widehat{\boldsymbol{\Theta}}_{\boldsymbol{\Sigma}} \equiv \widehat{\boldsymbol{\Sigma}}. \quad (9.119)$$

$$\begin{aligned} \min_{\boldsymbol{\mu} \in \widehat{\boldsymbol{\Theta}}_{\boldsymbol{\mu}}} \{\boldsymbol{\alpha}' \boldsymbol{\mu}\} &= \min_{\mathbf{u}'\mathbf{u} \leq 1} \left\{ \boldsymbol{\alpha}' \left(\mathbf{m} + q \mathbf{E} \boldsymbol{\Lambda}^{1/2} \mathbf{u} \right) \right\} \quad (T9.78) \\ &= \boldsymbol{\alpha}' \mathbf{m} + q \underbrace{\min_{\mathbf{u}'\mathbf{u} \leq 1} \left\{ \boldsymbol{\alpha}' \mathbf{E} \boldsymbol{\Lambda}^{1/2} \mathbf{u} \right\}}_{= -\left\| \boldsymbol{\Lambda}^{1/2} \mathbf{E}' \boldsymbol{\alpha} \right\|} \quad (T9.80) \end{aligned}$$

$$\widehat{\Theta}_{\mu} \equiv \left\{ \mathbf{m} + q \mathbf{E} \boldsymbol{\Lambda}^{1/2} \mathbf{u} : \mathbf{u}'\mathbf{u} \leq 1 \right\} \quad (T9.77)$$

$$\mathbf{T} \equiv \mathbf{E} \boldsymbol{\Lambda}^{1/2} \boldsymbol{\Lambda}^{1/2} \mathbf{E}'. \quad (T9.73)$$

$$\mathbf{u} \equiv \frac{1}{q} \boldsymbol{\Lambda}^{-1/2} \mathbf{E}' (\boldsymbol{\mu} - \mathbf{m}) \quad (T9.75)$$

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$$\boxed{\boldsymbol{\alpha}_{\mathrm{r}}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \widehat{\Theta}_{\boldsymbol{\mu}}} \{\boldsymbol{\alpha}'\boldsymbol{\mu}\} \right\}} \quad (9.117)$$

$$\text{subject to } \boxed{\begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max_{\boldsymbol{\Sigma} \in \widehat{\Theta}_{\boldsymbol{\Sigma}}} \{\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}\} \leq v^{(i)}. \end{cases}}$$

$$\widehat{\Theta}_{\boldsymbol{\mu}} \equiv \left\{ \boldsymbol{\mu} : (\boldsymbol{\mu} - \mathbf{m})' \mathbf{T}^{-1} (\boldsymbol{\mu} - \mathbf{m}) \leq q^2 \right\}. \quad (T9.72)$$

$$\widehat{\Theta}_{\boldsymbol{\Sigma}} \equiv \widehat{\boldsymbol{\Sigma}}. \quad (9.119)$$

$$\begin{aligned} \min_{\boldsymbol{\mu} \in \widehat{\Theta}_{\boldsymbol{\mu}}} \{\boldsymbol{\alpha}'\boldsymbol{\mu}\} &= \min_{\mathbf{u}'\mathbf{u} \leq 1} \left\{ \boldsymbol{\alpha}' \left(\mathbf{m} + q\mathbf{E}\boldsymbol{\Lambda}^{1/2}\mathbf{u} \right) \right\} \quad (T9.78) \\ &= \boldsymbol{\alpha}'\mathbf{m} + q \underbrace{\min_{\mathbf{u}'\mathbf{u} \leq 1} \left\{ \boldsymbol{\alpha}'\mathbf{E}\boldsymbol{\Lambda}^{1/2}\mathbf{u} \right\}}_{= -\left\| \boldsymbol{\Lambda}^{1/2}\mathbf{E}'\boldsymbol{\alpha} \right\| \; (T9.80)} \end{aligned}$$

$$\widehat{\Theta}_{\boldsymbol{\mu}} \equiv \left\{ \mathbf{m} + q\mathbf{E}\boldsymbol{\Lambda}^{1/2}\mathbf{u} : \mathbf{u}'\mathbf{u} \leq 1 \right\} \quad (T9.77)$$

$$\mathbf{T} \equiv \mathbf{E}\boldsymbol{\Lambda}^{1/2}\boldsymbol{\Lambda}^{1/2}\mathbf{E}'. \quad (T9.73)$$

$$\mathbf{u} \equiv \frac{1}{q}\boldsymbol{\Lambda}^{-1/2}\mathbf{E}' (\boldsymbol{\mu} - \mathbf{m}) \quad (T9.75)$$

$$\boxed{\left(\boldsymbol{\alpha}_{\mathrm{r}}^{(i)}, z_{\mathrm{r}}^{(i)} \right) = \underset{\boldsymbol{\alpha}, z}{\operatorname{argmax}} \{\boldsymbol{\alpha}'\mathbf{m} - z\}} \quad (T9.83)$$

$$(T9.83) \quad \boxed{\begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ q \left\| \boldsymbol{\Lambda}^{1/2}\mathbf{E}'\boldsymbol{\alpha} \right\| \leq z \\ \boldsymbol{\alpha}'\widehat{\boldsymbol{\Sigma}}\boldsymbol{\alpha} \leq v_i. \end{cases}}$$

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$$\boxed{\boldsymbol{\alpha}_{\mathrm{r}}^{(i)} = \underset{\boldsymbol{\alpha}}{\operatorname{argmax}} \left\{ \min_{\boldsymbol{\mu} \in \widehat{\Theta}_{\boldsymbol{\mu}}} \{\boldsymbol{\alpha}' \boldsymbol{\mu}\} \right\}} \quad (9.117)$$

$$\widehat{\Theta}_{\boldsymbol{\mu}} \equiv \left\{ \boldsymbol{\mu} : (\boldsymbol{\mu} - \mathbf{m})' \mathbf{T}^{-1} (\boldsymbol{\mu} - \mathbf{m}) \leq q^2 \right\}. \quad (T9.72)$$

$$\text{subject to } \boxed{\begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ \max\limits_{\boldsymbol{\Sigma} \in \widehat{\Theta}_{\boldsymbol{\Sigma}}} \{\boldsymbol{\alpha}' \boldsymbol{\Sigma} \boldsymbol{\alpha}\} \leq v^{(i)}. \end{cases}}$$

$$\widehat{\Theta}_{\boldsymbol{\Sigma}} \equiv \widehat{\boldsymbol{\Sigma}}. \quad (9.119)$$

$$\begin{aligned} \min_{\boldsymbol{\mu} \in \widehat{\Theta}_{\boldsymbol{\mu}}} \{\boldsymbol{\alpha}' \boldsymbol{\mu}\} &= \min_{\mathbf{u}'\mathbf{u} \leq 1} \left\{ \boldsymbol{\alpha}' \left(\mathbf{m} + q \mathbf{E} \boldsymbol{\Lambda}^{1/2} \mathbf{u} \right) \right\} \quad (T9.78) \\ &= \boldsymbol{\alpha}' \mathbf{m} + q \underbrace{\min_{\mathbf{u}'\mathbf{u} \leq 1} \left\{ \boldsymbol{\alpha}' \mathbf{E} \boldsymbol{\Lambda}^{1/2} \mathbf{u} \right\}} \\ &= - \left\| \boldsymbol{\Lambda}^{1/2} \mathbf{E}' \boldsymbol{\alpha} \right\| \quad (T9.80) \end{aligned}$$

$$\widehat{\Theta}_{\boldsymbol{\mu}} \equiv \left\{ \mathbf{m} + q \mathbf{E} \boldsymbol{\Lambda}^{1/2} \mathbf{u} : \mathbf{u}'\mathbf{u} \leq 1 \right\} \quad (T9.77)$$

$$\mathbf{T} \equiv \mathbf{E} \boldsymbol{\Lambda}^{1/2} \boldsymbol{\Lambda}^{1/2} \mathbf{E}'. \quad (T9.73)$$

$$\mathbf{u} \equiv \frac{1}{q} \boldsymbol{\Lambda}^{-1/2} \mathbf{E}' (\boldsymbol{\mu} - \mathbf{m}) \quad (T9.75)$$

$$\boxed{\left(\boldsymbol{\alpha}_{\mathrm{r}}^{(i)}, z_{\mathrm{r}}^{(i)} \right) = \underset{\boldsymbol{\alpha}, z}{\operatorname{argmax}} \{\boldsymbol{\alpha}' \mathbf{m} - z\}} \quad (T9.83)$$

$$(T9.83) \quad \begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ q \left\| \boldsymbol{\Lambda}^{1/2} \mathbf{E}' \boldsymbol{\alpha} \right\| \leq z \\ \boldsymbol{\alpha}' \widehat{\boldsymbol{\Sigma}} \boldsymbol{\alpha} \leq v_i. \end{cases}$$

$$\widehat{\boldsymbol{\Sigma}} \equiv \mathbf{F} \boldsymbol{\Gamma}^{1/2} \boldsymbol{\Gamma}^{1/2} \mathbf{F}' \quad (T9.86)$$

$$\boxed{\begin{cases} \boldsymbol{\alpha} \in \mathcal{C} \\ q \left\| \boldsymbol{\Lambda}^{1/2} \mathbf{E}' \boldsymbol{\alpha} \right\| \leq z \\ \left\| \boldsymbol{\Gamma}^{1/2} \mathbf{F}' \boldsymbol{\alpha} \right\| \leq \sqrt{v_i}. \end{cases}}$$