Attilio Meucci

www.symmys.com

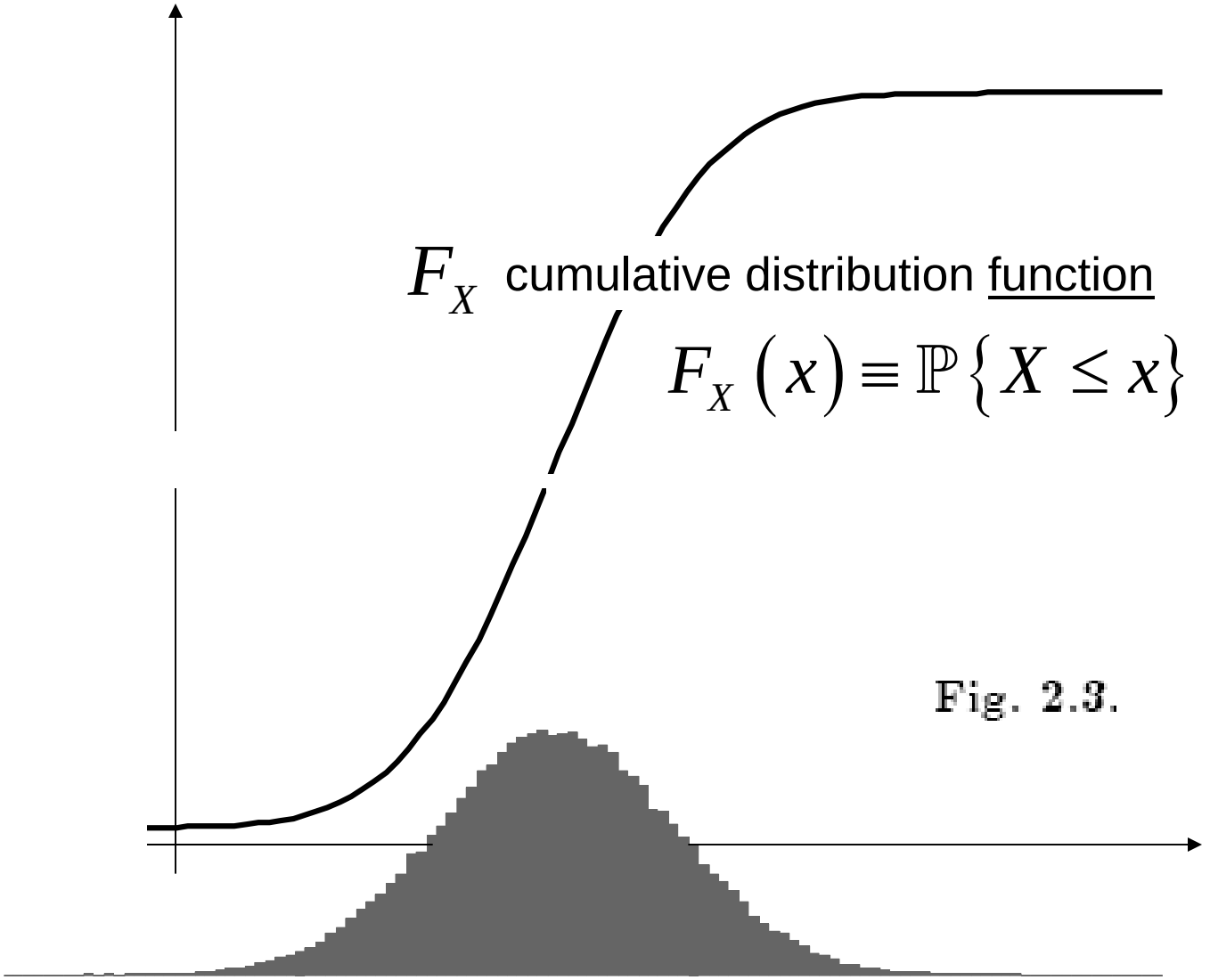
Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

Fig. 2.3.

X generic random variable with generic distribution



F_X cumulative distribution function

$$F_X\left(x\right) \equiv \mathbb{P}\left\{X \leq x\right\}$$

Fig. 2.3.

X generic random variable with generic distribution

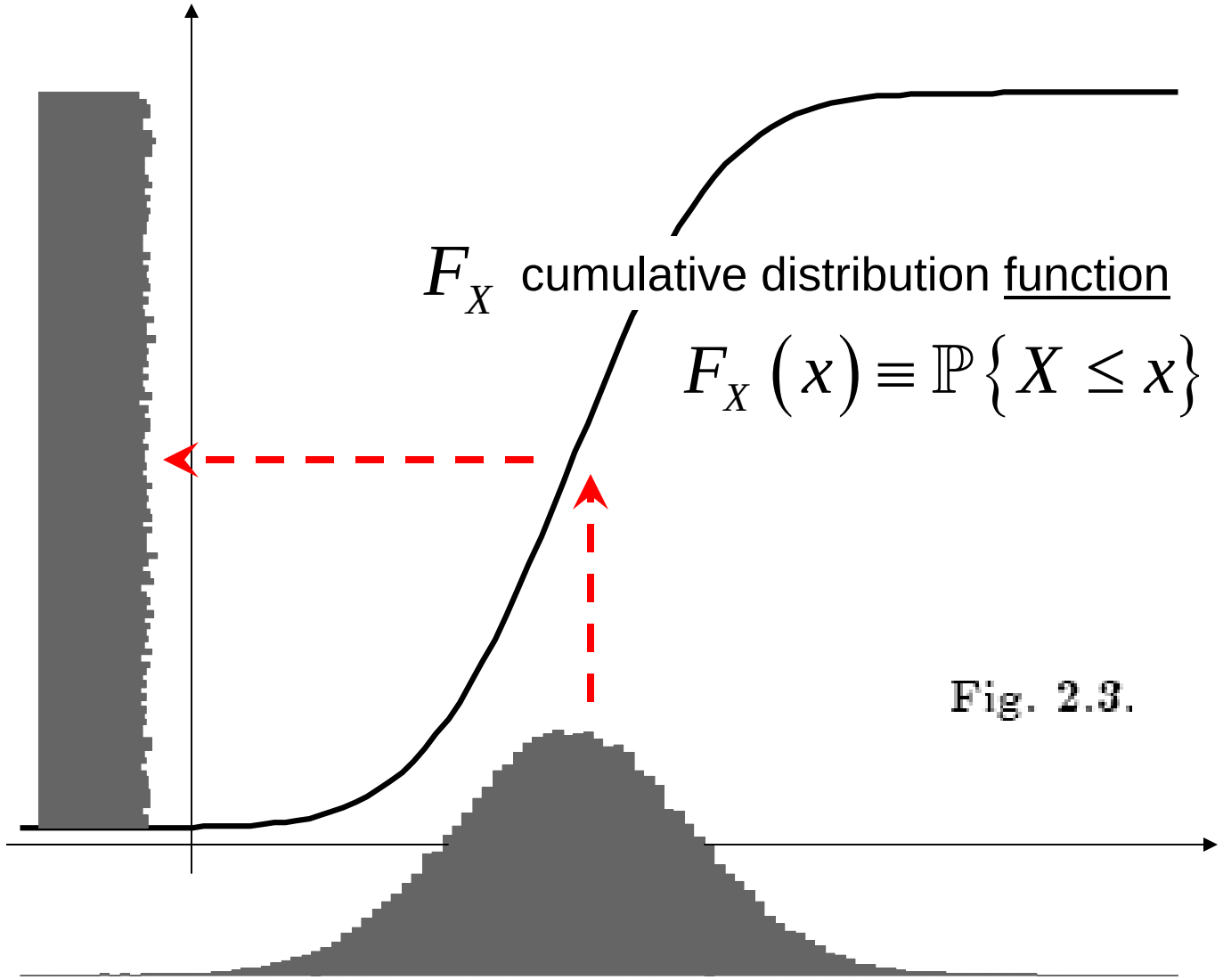


Fig. 2.3.

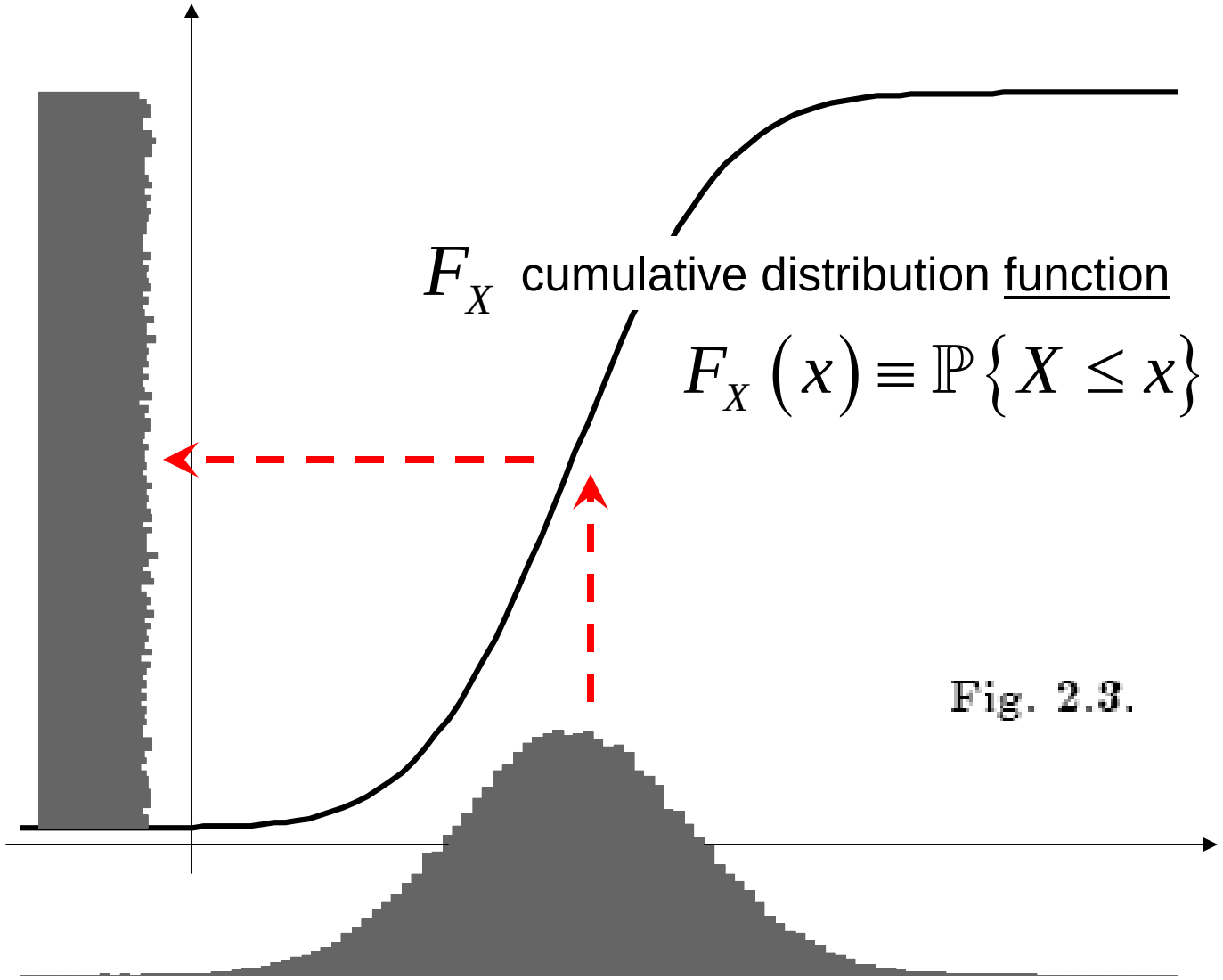
X generic random variable with generic distribution

grade

$$U \equiv F_X\left(X\right) \quad (2.25)$$

$$U \sim \mathrm{U}\left([0,1]\right) \quad (2.26)$$

uniform random variable



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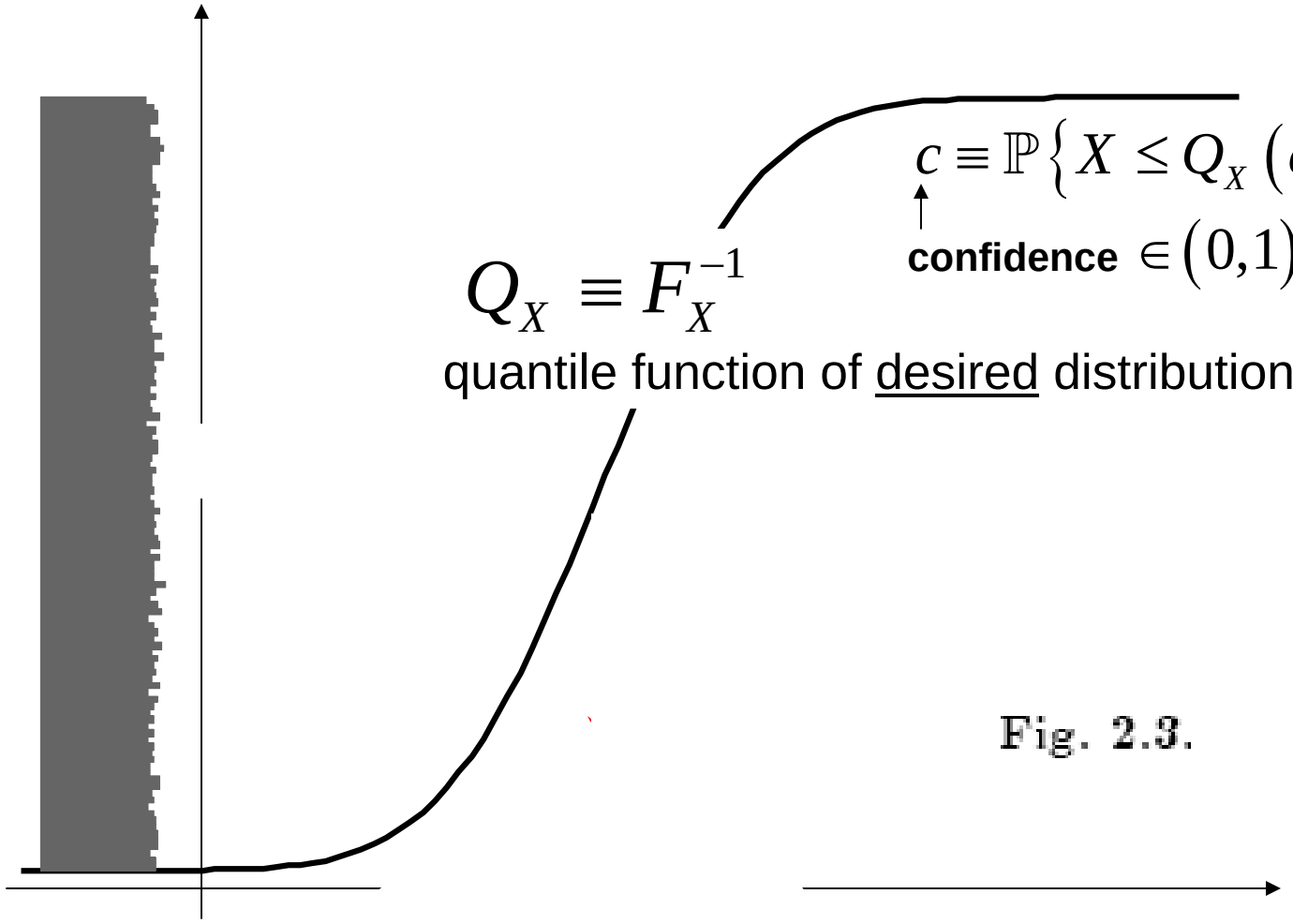

$$c \equiv \mathbb{P}\left\{X \leq Q_X(c)\right\}$$

confidence $\in (0,1)$

$$Q_X \equiv F_X^{-1}$$

quantile function of desired distribution

Fig. 2.3.

$$U \sim \mathrm{U}([0,1]) \quad (2.26)$$

uniform random variable

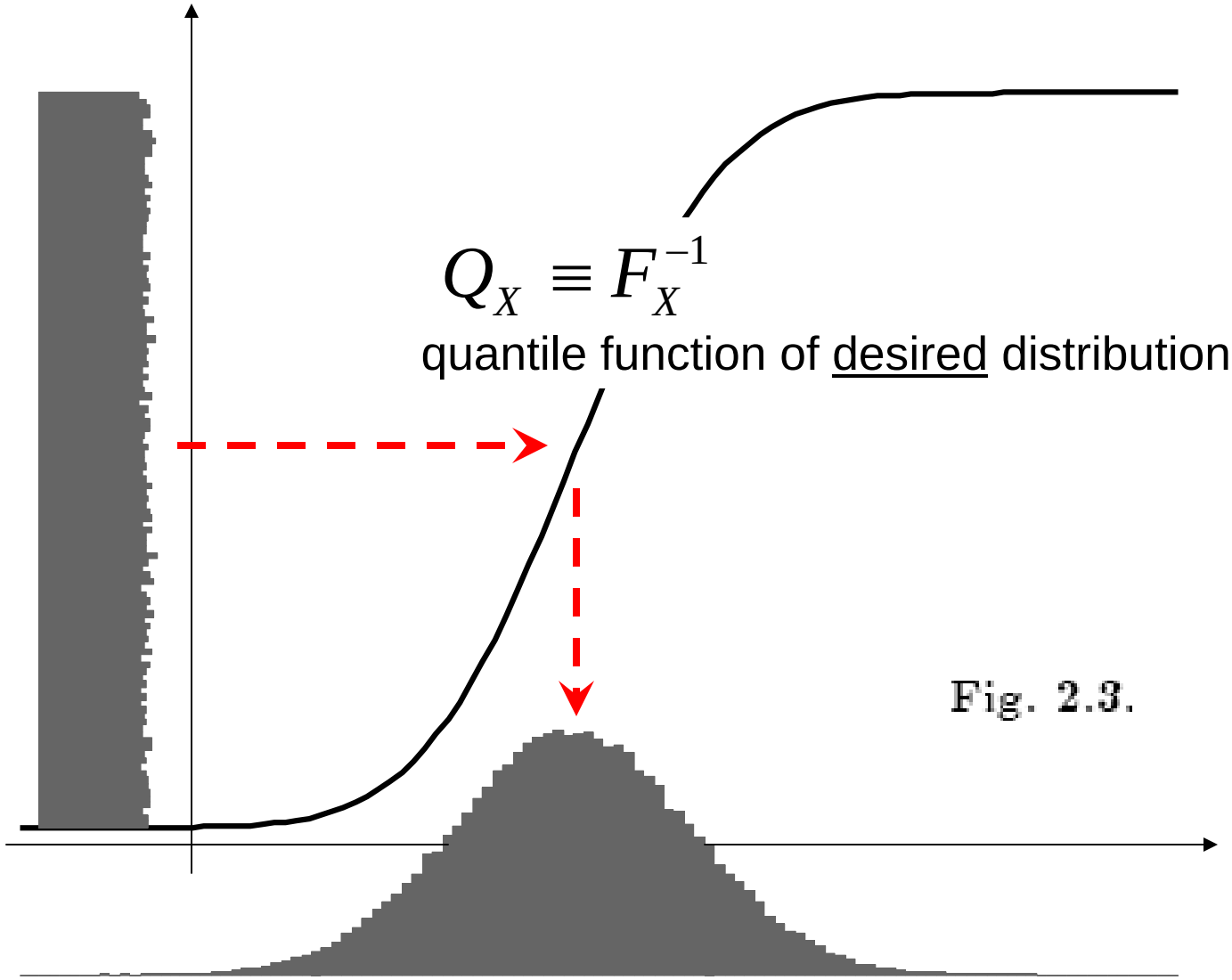

Fig. 2.3.

$X \equiv F_X^{-1}(U)$ random variable with desired distribution (2.27)

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$$\begin{pmatrix} X_1 \\ \vdots \\ X_N \end{pmatrix}$$ multivariate random variable with generic distribution

$$\begin{pmatrix} F_{X_1} \\ \vdots \\ F_{X_N} \end{pmatrix}$$ respective cumulative distribution functions

uniform distribution

$$\begin{pmatrix} U_1 \\ \vdots \\ U_N \end{pmatrix} \equiv \begin{pmatrix} F_{X_1}\left(X_1\right) \\ \vdots \\ F_{X_N}\left(X_N\right) \end{pmatrix} \qquad (2.28)$$

uniform distribution

copula : distribution of the grades

$$\boxed{\textbf{multivariate} = \text{"1-dim"}\ (\textbf{marginals}) + \text{"joint"}(\textbf{copula})} \tag{2.19}$$

the copula, a standardized distribution which summarizes the purely "joint" component of the distribution of $\mathbf{X}$.

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the copula, a standardized distribution which summarizes the purely "joint" component of the distribution of $\mathbf{X}$.

$$(\mathbf{X}, \mathbf{Y})\ \text{co-monotonic} \Leftrightarrow \text{copula of } \mathbf{X} = \text{copula of } \mathbf{Y} \qquad (2.38)$$

e.g.: **X** = prices (e.g. lognormal)

Y = compounded returns (e.g. normal)

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pdf

$$f_{\mathbf{U}}(u_1,\ldots,u_N)=\frac{f_{\mathbf{X}}\left(Q_{X_1}(u_1),\ldots,Q_{X_N}(u_N)\right)}{f_{X_1}\left(Q_{X_1}(u_1)\right)\cdots f_{X_N}\left(Q_{X_N}(u_N)\right)} \tag{2.30}$$

$$[0,1]^N\equiv[0,1]\times\cdots\times[0,1] \tag{2.29}$$

cdf

$$F_{\mathbf{U}}(u_1,\ldots,u_N)=F_{\mathbf{X}}\left(Q_{X_1}(u_1),\ldots,Q_{X_N}(u_N)\right) \tag{2.31}$$