Attilio Meucci

The Black-Litterman Approach

- "The Black-Litterman Approach: Original Model and Extensions"

  The Encyclopedia of Quantitative Finance, Wiley - 2009, to appear

  available at www.symmys.com > Research > Working Papers

- "Enhancing the Black-Litterman and Related Approaches: Views and Stress-Test on Risk Factors"

  The Journal of Asset Management - 2009, to appear

  available at www.symmys.com > Research > Working Papers

BLACK-LITTERMAN - original model – *symmys.com*

$\mathbf{X} \sim \mathrm{N}(\mu, \Sigma)$ **returns on asset classes/funds**

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$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$\boldsymbol{\mu} \sim \mathrm{N}(\boldsymbol{\pi}, \tau \boldsymbol{\Sigma})$

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no estimation uncertainty $\mathbf{X} \sim \mathrm{N}(\boldsymbol{\pi}, \boldsymbol{\Sigma})$

unconstrained mean-variance $\mathbf{w}_{\lambda} \equiv \underset{\mathbf{w}}{\operatorname{argmax}} \{\mathbf{w}'\boldsymbol{\pi} - \lambda\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}\}$

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$\boldsymbol{\mu} \sim \mathrm{N}(\boldsymbol{\pi}, \tau\boldsymbol{\Sigma})$

$\boldsymbol{\pi} \equiv 2\bar{\lambda}\boldsymbol{\Sigma}\widetilde{\mathbf{w}}.$

equilibrium portfolio

no estimation uncertainty $\mathbf{X} \sim \mathrm{N}(\boldsymbol{\pi}, \boldsymbol{\Sigma})$

unconstrained mean-variance $\mathbf{w}_{\lambda} \equiv \underset{\mathbf{w}}{\operatorname{argmax}} \{\mathbf{w}'\boldsymbol{\pi} - \lambda\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}\}$

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$$\boldsymbol{\mu} \sim \mathrm{N}(\boldsymbol{\pi}, \tau \boldsymbol{\Sigma})$$

$$\boldsymbol{\pi} \equiv 2\overline{\lambda}\boldsymbol{\Sigma}\widetilde{\mathbf{w}}. \qquad \tau \approx \frac{1}{T}. \qquad \widehat{\boldsymbol{\mu}} \equiv \frac{1}{T}\sum_{t=1}^{T}\mathbf{X}_t \sim \mathrm{N}\left(\boldsymbol{\pi}, \frac{\boldsymbol{\Sigma}}{T}\right)$$

equilibrium portfolio

BLACK-LITTERMAN - original model – *symmys.com*

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$$\mathbf{P}\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{v}, \boldsymbol{\Omega})$$

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equilibrium portfolio

$$\mathbf{P}\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{v}, \boldsymbol{\Omega})$$

$$v_k \equiv (\mathbf{P}\boldsymbol{\pi})_k + \eta_k \sqrt{(\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}')_{k,k}},$$

$$\boldsymbol{\Omega} \equiv \frac{1}{c}\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}'$$

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$$\boldsymbol{\pi} \equiv 2\overline{\lambda}\boldsymbol{\Sigma}\widetilde{\mathbf{w}}. \qquad \tau \approx \frac{1}{T}.$$

equilibrium portfolio

$$\boldsymbol{\Omega} \equiv \frac{1}{c}\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}'$$

$$\boldsymbol{\mu}|\mathbf{v}; \boldsymbol{\Omega} \sim \mathrm{N}(\boldsymbol{\mu}_{BL}, \boldsymbol{\Sigma}^{\mu}_{BL})$$

$$\boldsymbol{\mu}_{BL} \equiv \left((\tau\boldsymbol{\Sigma})^{-1} + \mathbf{P}'\boldsymbol{\Omega}^{-1}\mathbf{P}\right)^{-1} \left((\tau\boldsymbol{\Sigma})^{-1}\boldsymbol{\pi} + \mathbf{P}'\boldsymbol{\Omega}^{-1}\mathbf{v}\right)$$

$$\boldsymbol{\Sigma}^{\mu}_{BL} \equiv \left((\tau\boldsymbol{\Sigma})^{-1} + \mathbf{P}'\boldsymbol{\Omega}^{-1}\mathbf{P}\right)^{-1}$$

BLACK-LITTERMAN - original model – *symmys.com*

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds** $\mathbf{X} \stackrel{d}{=} \boldsymbol{\mu} + \mathbf{Z}$, $\mathbf{Z} \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma})$

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$$\boldsymbol{\mu} \sim \mathrm{N}(\boldsymbol{\pi}, \tau\boldsymbol{\Sigma})$$

$$\boldsymbol{\pi} \equiv 2\overline{\lambda}\boldsymbol{\Sigma}\widetilde{\mathbf{w}}. \qquad \tau \approx \frac{1}{T}.$$

equilibrium portfolio

$$\mathbf{P}\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{v}, \boldsymbol{\Omega}).$$

$$v_k \equiv (\mathbf{P}\boldsymbol{\pi})_k + \eta_k \sqrt{(\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}')_{k,k}},$$

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$$\boldsymbol{\mu}|\mathbf{v}; \boldsymbol{\Omega} \sim \mathrm{N}(\boldsymbol{\mu}_{BL}, \boldsymbol{\Sigma}^{\mu}_{BL})$$

$$\boldsymbol{\mu}_{BL} \equiv \left((\tau\boldsymbol{\Sigma})^{-1} + \mathbf{P}'\boldsymbol{\Omega}^{-1}\mathbf{P}\right)^{-1} \left((\tau\boldsymbol{\Sigma})^{-1}\boldsymbol{\pi} + \mathbf{P}'\boldsymbol{\Omega}^{-1}\mathbf{v}\right)$$

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$$\mathbf{X}|\mathbf{v}; \boldsymbol{\Omega} \sim \mathrm{N}(\boldsymbol{\mu}_{BL}, \boldsymbol{\Sigma}_{BL})$$

$$\boldsymbol{\Sigma}_{BL} \equiv \boldsymbol{\Sigma} + \boldsymbol{\Sigma}^{\mu}_{BL}.$$

BLACK-LITTERMAN - original model – *symmys.com*

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

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$\mathbf{P}\boldsymbol{\mu} \sim \mathrm{N}(\mathbf{v}, \boldsymbol{\Omega})$

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$$\boldsymbol{\Sigma}^{\mu}_{BL} \equiv \left((\tau\boldsymbol{\Sigma})^{-1} + \mathbf{P}'\boldsymbol{\Omega}^{-1}\mathbf{P}\right)^{-1}$$

$\mathbf{X}|\mathbf{v};\boldsymbol{\Omega} \sim \mathrm{N}(\boldsymbol{\mu}_{BL}, \boldsymbol{\Sigma}_{BL})$

$$\begin{aligned} \boldsymbol{\mu}_{BL} &= \boldsymbol{\pi} + \tau\boldsymbol{\Sigma}\mathbf{P}'(\tau\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}' + \boldsymbol{\Omega})^{-1}(\mathbf{v} - \mathbf{P}\boldsymbol{\pi}) \\ \boldsymbol{\Sigma}_{BL} &= (1+\tau)\boldsymbol{\Sigma} - \tau^2\boldsymbol{\Sigma}\mathbf{P}'(\tau\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}' + \boldsymbol{\Omega})^{-1}\mathbf{P}\boldsymbol{\Sigma}. \end{aligned}$$

BLACK-LITTERMAN - market model – *symmys.com*

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$$\boldsymbol{\pi} \equiv 2\overline{\lambda}\boldsymbol{\Sigma}\widetilde{\mathbf{w}}.$$

equilibrium portfolio

$$v_k \equiv (\mathbf{P}\boldsymbol{\pi})_k + \eta_k \sqrt{(\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}')_{k,k}},$$

$$\boldsymbol{\Omega} \equiv \frac{1}{c}\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}'$$

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$$v_k \equiv (\mathbf{P}\boldsymbol{\pi})_k + \eta_k \sqrt{(\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}')_{k,k}},$$

$$\boldsymbol{\Omega} \equiv \frac{1}{c}\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}'$$

$$\mathbf{X}|\mathbf{v};\boldsymbol{\Omega} \sim \mathrm{N}(\boldsymbol{\mu}^m_{BL}, \boldsymbol{\Sigma}^m_{BL})$$

$$\begin{aligned} \boldsymbol{\mu}^m_{BL} &\equiv \boldsymbol{\pi} + \boldsymbol{\Sigma}\mathbf{P}'\left(\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}' + \boldsymbol{\Omega}\right)^{-1}(\mathbf{v} - \mathbf{P}\boldsymbol{\pi}) \\ \boldsymbol{\Sigma}^m_{BL} &\equiv \boldsymbol{\Sigma} - \boldsymbol{\Sigma}\mathbf{P}'\left(\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}' + \boldsymbol{\Omega}\right)^{-1}\mathbf{P}\boldsymbol{\Sigma}. \end{aligned}$$

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$$\mathbf{X}|\mathbf{v};\boldsymbol{\Omega} \sim \mathrm{N}(\boldsymbol{\mu}_{BL}^{m}, \boldsymbol{\Sigma}_{BL}^{m})$$

$$\begin{aligned} \boldsymbol{\mu}_{BL}^{m} &\equiv \boldsymbol{\pi} + \boldsymbol{\Sigma}\mathbf{P}'\left(\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}' + \boldsymbol{\Omega}\right)^{-1}(\mathbf{v} - \mathbf{P}\boldsymbol{\pi}) \\ \boldsymbol{\Sigma}_{BL}^{m} &\equiv \boldsymbol{\Sigma} - \boldsymbol{\Sigma}\mathbf{P}'\left(\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}' + \boldsymbol{\Omega}\right)^{-1}\mathbf{P}\boldsymbol{\Sigma}. \end{aligned}$$

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}_{BL}^{m}, \boldsymbol{\Sigma}_{BL}^{m})$
mkt-posterior

$\nearrow$ $\mathbf{X} \sim \mathrm{N}(\boldsymbol{\pi}, \boldsymbol{\Sigma})$ (no confidence: $\boldsymbol{\Omega} \to \infty$)
prior

$\searrow$ $\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}|\mathbf{v}, \boldsymbol{\Sigma}|\mathbf{v})$ (full confidence: $\boldsymbol{\Omega} \to \mathbf{0}$)
conditional

BLACK-LITTERMAN - enhanced model – *symmys.com*

$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\pi}, \boldsymbol{\Sigma})$ **returns on asset classes/funds**

$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$$\boldsymbol{\pi} \equiv 2\overline{\lambda}\boldsymbol{\Sigma}\widetilde{\mathbf{w}}.$$

equilibrium portfolio

mean-variance

$$\mathbf{w}_{\lambda} \equiv \underset{\mathbf{w}}{\operatorname{argmax}} \left\{ \mathbf{w}'\boldsymbol{\pi} - \lambda \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \right\}$$

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$\boldsymbol{\Sigma}$ **estimated by exponential smoothing**

$$\boldsymbol{\pi} \equiv 2\overline{\lambda}\boldsymbol{\Sigma}\widetilde{\mathbf{w}}.$$

equilibrium portfolio

any risk factors:

- implied volatilities

- macro factors not in p&L

- …

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$\mathrm{X} \sim \mathrm{N}(\pi, \Sigma)$ ~~returns on asset classes/funds~~

Σ ~~estimated by exponential smoothing~~

$\pi \equiv 2\overline{\lambda}\Sigma\widetilde{\mathrm{w}}.$

~~equilibrium portfolio~~

any risk factors:

- implied volatilities
- macro factors not in p&L
- ...

any estimation:

- historical
- shrinkage
- Bayesian
- robust
- implied
- ...

BLACK-LITTERMAN - enhanced model – *symmys.com*

$\mathrm{X} \sim \mathrm{N}(\boldsymbol{\pi}, \Sigma)$ returns on asset classes/funds

Σ estimated by exponential smoothing

any risk factors:

- implied volatilities
- macro factors not in p&L
- …

$$\boldsymbol{\pi} \equiv 2\overline{\lambda}\Sigma\widetilde{\mathrm{w}}.$$

equilibrium portfolio

any estimation:

- historical
- shrinkage, robust,…
- implied
- …

mean-variance

$$\mathrm{w}_{\lambda} \equiv \underset{\mathrm{w}}{\operatorname{argmax}} \left\{ \mathrm{w}'\boldsymbol{\pi} - \lambda \mathrm{w}'\Sigma\mathrm{w} \right\}$$

any index of satisfaction

$$\mathrm{w}^{*} \equiv \underset{\mathrm{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \mathcal{S}\left(\mathrm{w}; \boldsymbol{\pi}, \Sigma\right) \right\}$$

- certainty equivalent
- spectral measures
- mean/CVaR
- …

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Black-Scholes formula:
deterministic function of risk into price

$$\boxed{C_{BS}}\left(\boxed{y},\boxed{\sigma};\kappa,T,r\right)\equiv yF\left(d_1\right)-\kappa e^{-rT}F\left(d_2\right)$$

$$d_1\equiv\left(\ln\left(y/\kappa\right)+\left(r+\sigma^2/2\right)T\right)/\sigma\sqrt{T},\qquad d_2\equiv d_1-\sigma\sqrt{T};$$

BLACK-LITTERMAN - enhanced model – *symmys.com*

$$C_{BS}(y,\sigma;\kappa,T,r) \equiv yF(d_1) - \kappa e^{-rT}F(d_2)$$

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$$h(y,\sigma;\kappa,T) \equiv \sigma + a\frac{\ln(y/\kappa)}{\sqrt{T}} + b\left(\frac{\ln(y/\kappa)}{\sqrt{T}}\right)^2$$

empirical smirk and smile

BLACK-LITTERMAN - enhanced model – *symmys.com*

call option price at horizon $P_{t+\tau} = C_{BS}\left(y_t e^{X_y}, h\left(y_t e^{X_y}, \sigma_t + X_\sigma, \kappa, T-\tau\right); \kappa, T-\tau, r\right)$

$$X_y \equiv \ln\left(y_{t+\tau}/y_t\right)$$

$$X_\sigma \equiv \sigma_{t+\tau} - \sigma_t$$

$$C_{BS}\left(y,\sigma;\kappa,T,r\right) \equiv yF\left(d_1\right) - \kappa e^{-rT} F\left(d_2\right)$$

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$$\mathbf{X} \equiv \left(X^M, X^M_{1m}, X^M_{2m}, X^M_{6m}, \ldots, X^G_{6m}, \underbrace{X_{2y}, X_{10y}}\right)'$$

1-month, 2-month and 6-month calls
Microsoft (M), Yahoo (Y) and Google (G)

curve change (growth/inflation)
not directly in pricing

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risk factors are approximately <u>normal</u>

BLACK-LITTERMAN - enhanced model – *symmys.com*

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$$\Pi_{\mathbf{w}} \equiv \sum_{i=1}^{I} w_i\left(C_{BS,i}\left(\mathbf{X}, \mathcal{I}_t\right) - C_{i,t}\right)$$

profit and loss is highly <u>non-linear</u>, highly <u>non-normal</u>

BLACK-LITTERMAN - enhanced model – `symmys.com`

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$$\Pi_{\mathbf{w}} \equiv \sum_{i=1}^{I} w_i\left(C_{BS,i}\left(\mathbf{X}, \mathcal{I}_t\right) - C_{i,t}\right)$$

Mean-Expected Shortfall optimization

$$\mathbf{w}_\lambda \equiv \underset{\underline{\mathbf{b}} \leq \mathbf{B}\mathbf{w} \leq \overline{\mathbf{b}}}{\operatorname{argmax}} \left\{\mathrm{E}\left\{\Pi_{\mathbf{w}}\right\} + \lambda Q_{1-\gamma}\left\{\Pi_{\mathbf{w}}\right\}\right\}$$

- **long-short delta-neutral**
- **no cash upfront**
- **limit on leverage**

BLACK-LITTERMAN - enhanced model – *symmys.com*

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$$\mathbf{X} \equiv \left(X^M, X^M_{1m}, X^M_{2m}, X^M_{6m}, \ldots, , X^G_{6m}, X_{2y}, X_{10y}\right)' \sim \mathrm{N}\left(\boldsymbol{\pi}, \boldsymbol{\Sigma}\right)$$

$$\Pi_{\mathbf{w}} \equiv \sum_{i=1}^{I} w_i \left(C_{BS,i}\left(\mathbf{X}, \mathcal{I}_t\right) - C_{i,t}\right)$$

simulations

$$\mathcal{P}_{j,i} \equiv C_{BS,i}\left(\mathcal{X}_{j,\cdot}, \mathcal{I}_t\right) - C_{i,t},$$

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call option price at horizon $$P_{t+\tau} = C_{BS}\left(y_t e^{X_y}, h\left(y_t e^{X_y}, \sigma_t + X_\sigma, \kappa, T-\tau\right); \kappa, T-\tau, r\right)$$

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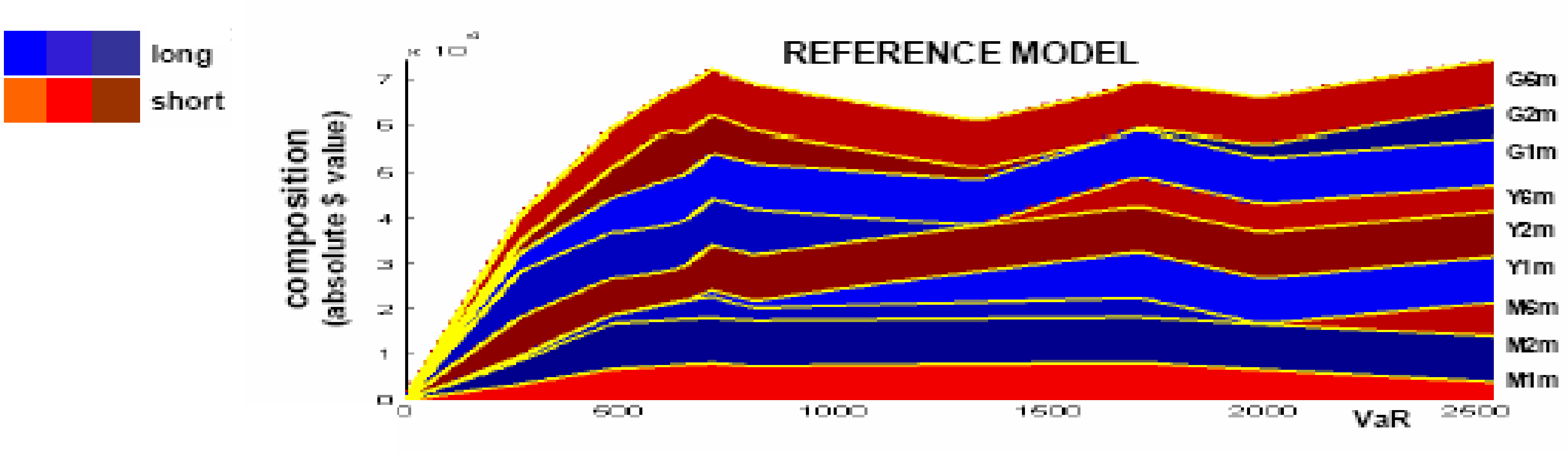
simulations

$$\mathcal{P}_{j,i} \equiv C_{BS,i}\left(\mathcal{X}_{j,\cdot}, \mathcal{I}_t\right) - C_{i,t},$$

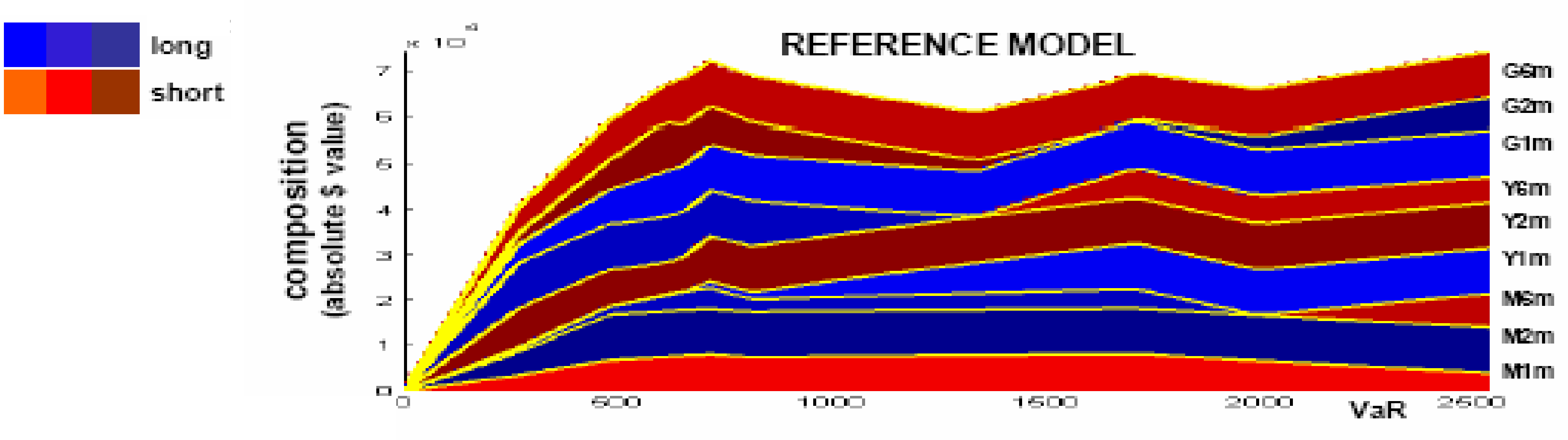
$$\mathbf{w}_\lambda \equiv \underset{\underline{\mathbf{b}} \leq \mathbf{B}\mathbf{w} \leq \overline{\mathbf{b}}}{\operatorname{argmax}} \left\{\mathrm{E}\left\{\Pi_{\mathbf{w}}\right\} + \lambda Q_{1-\gamma}\left\{\Pi_{\mathbf{w}}\right\}\right\} \iff$$

linear programming

BLACK-LITTERMAN - enhanced model – *symmys.com*



BLACK-LITTERMAN - enhanced model – `symmys.com`

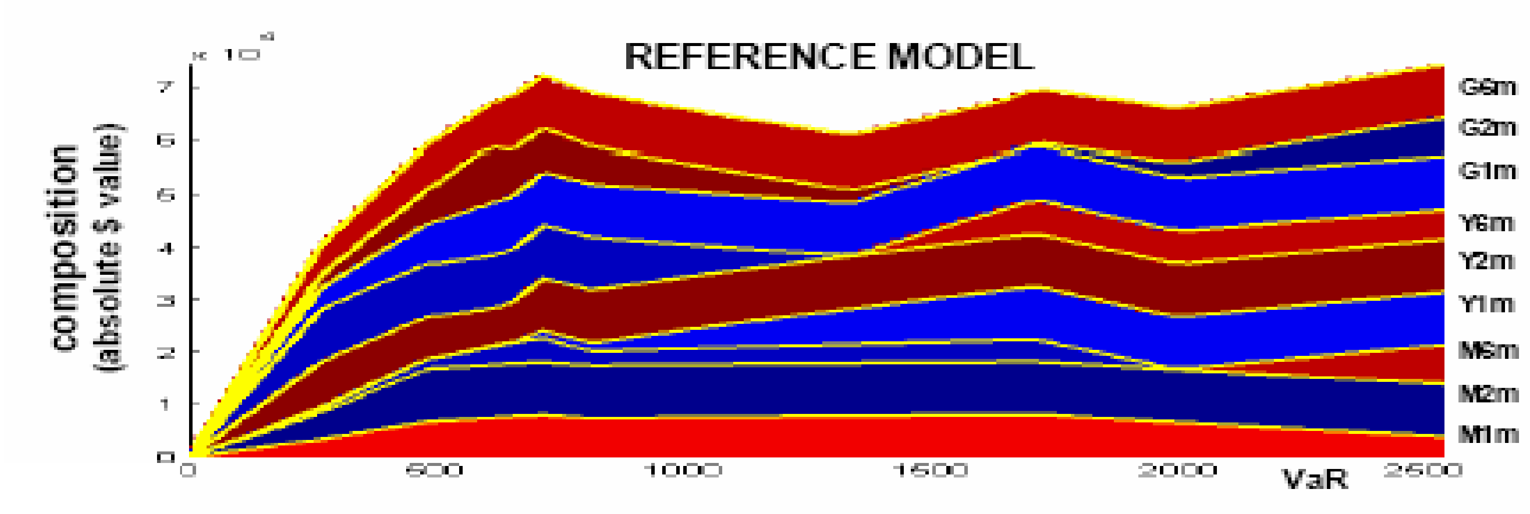


$\mathrm{X} \sim \mathrm{N}(\pi, \Sigma)$ > **Black-Litterman >** **bullish M 1m / bearish Y 1m**

BLACK-LITTERMAN - enhanced model – *symmys.com*

long

short



$X \sim N(\pi, \Sigma)$ > **Black-Litterman >** **bullish M 1m / bearish Y 1m**

long

short