PROJECTION TO THE HORIZON

Risk and Asset Allocation* - Springer – *symmys.com

Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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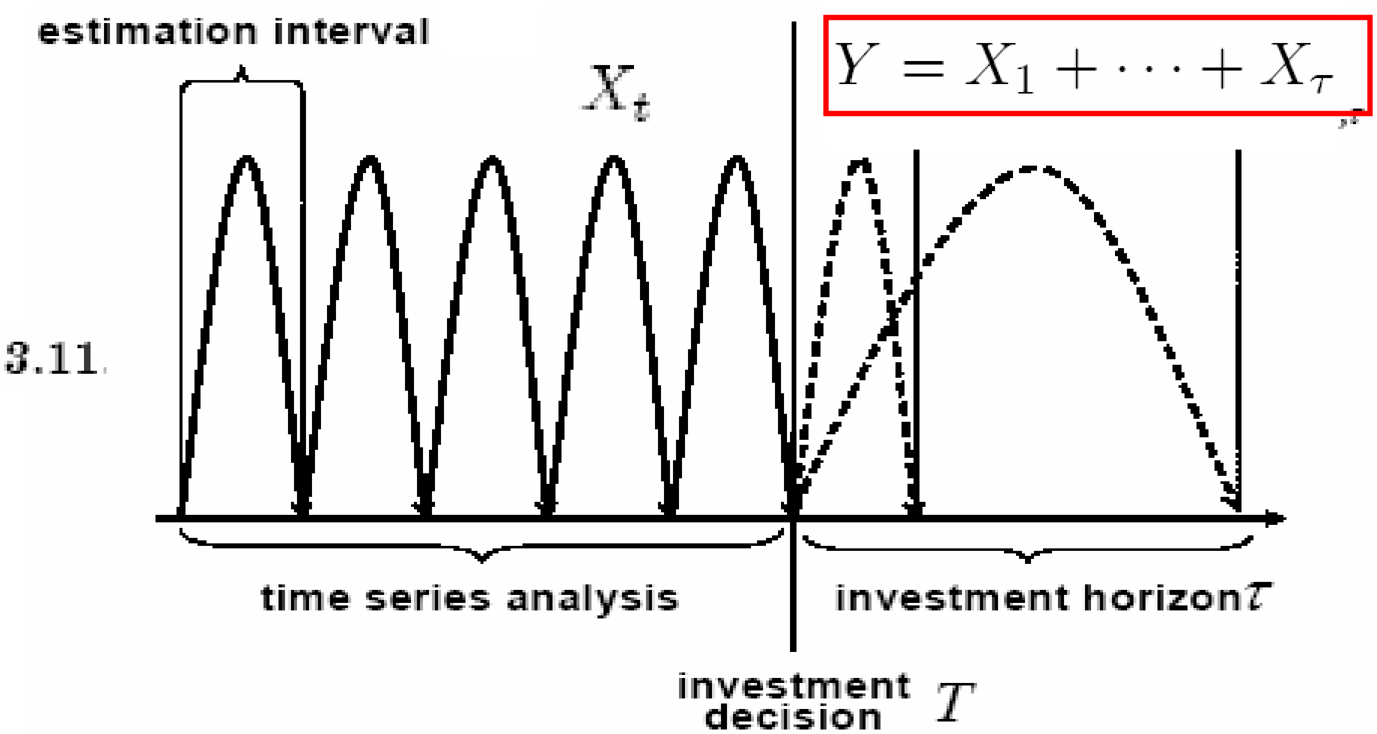
$$Y = X_1 + \cdots + X_\tau \quad (3.60)$$

Fig. 3.11.



PROJECTION TO THE HORIZON – Simple statistics

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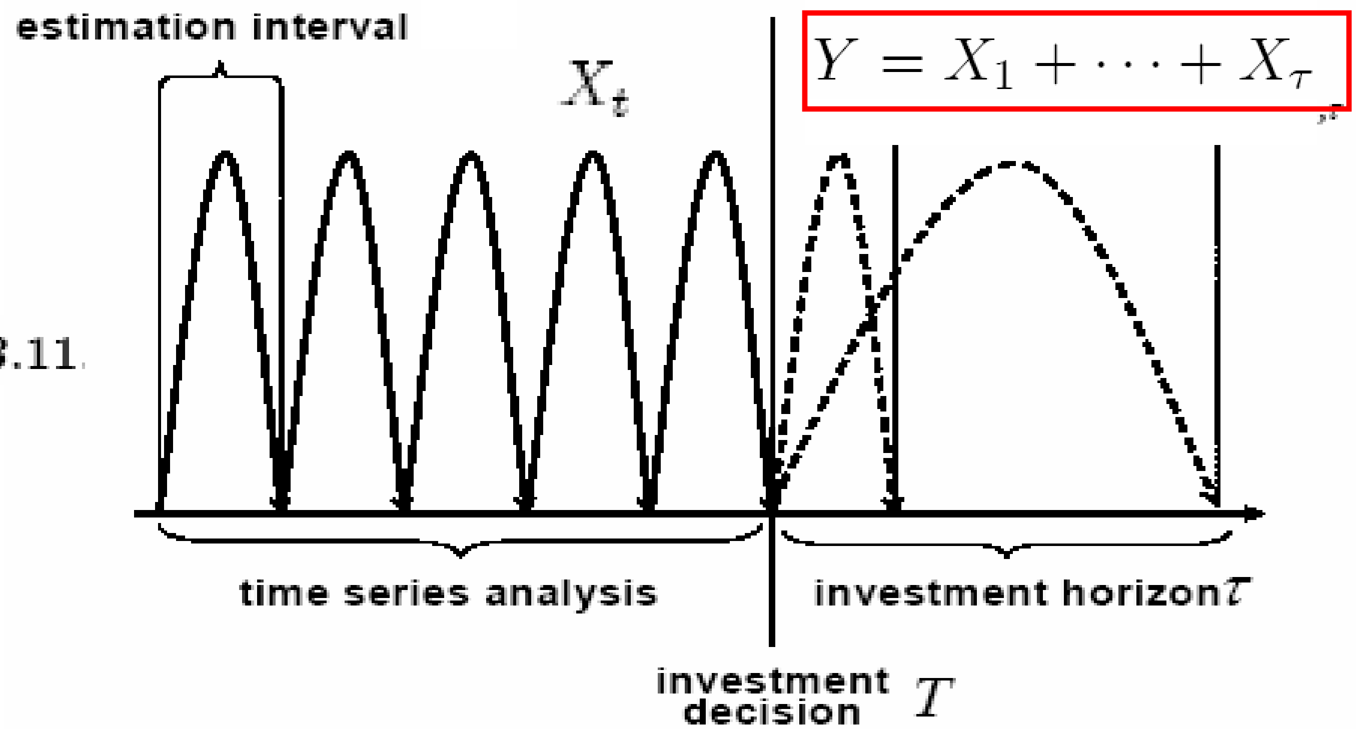
Fig. 3.11



$$\boxed{Y = X_1 + \cdots + X_\tau} \quad (3.60)$$

$$\mu_X \equiv \mathrm{E}\{X_t\} \qquad \Rightarrow \qquad \mu_Y = \tau \mu_X$$

$$\sigma_X \equiv \sqrt{\mathrm{E}\{(X_t - \mu_X)^2\}} \qquad \Rightarrow \qquad \sigma_Y = \sqrt{\tau}\,\sigma_X$$

“square-root rule” (3.76)

PROJECTION TO THE HORIZON – All statistics

***Quant Nugget 4* -** http://ssrn.com/abstract=1635484

$$X_t$$

$$Y = X_1 + \cdots + X_\tau$$

Summary statistics

$$\mu_X, \sigma_X, sk_X, ku_X, \ldots, \gamma_X^{(n)}$$

$$\mu_Y, \sigma_Y, sk_Y, ku_Y, \ldots, \gamma_Y^{(n)}$$

$$sk_X \equiv \mathrm{E}\{(X_t - \mu_X)^3\}/\sigma_X^3$$

$$ku_X \equiv \mathrm{E}\{(X_t - \mu_X)^4\}/\sigma_X^4$$

$$\gamma_X^{(n)} \equiv \mathrm{E}\{(X_t - \mu_X)^n\}/\sigma_X^n,$$

PROJECTION TO THE HORIZON – All statistics

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$$\mu_Y, \sigma_Y, sk_Y, ku_Y, \ldots, \gamma_Y^{(n)}$$

Cumulants

$$\kappa_X^{(n)} \equiv \left. \frac{d^n \ln \left(\mathrm{E} \left\{ e^{zX} \right\} \right)}{dz^n} \right|_{z=0}$$

$$\kappa_Y^{(n)} = \tau \kappa_X^{(n)}$$

PROJECTION TO THE HORIZON – All statistics

***Quant Nugget 4* - http://ssrn.com/abstract=1635484**

	X_t		$Y = X_1 + \cdots + X_\tau$
Summary statistics	$\boxed{\mu_X, \sigma_X, sk_X, ku_X, \ldots, \gamma_X^{(n)}}$		$\boxed{\mu_Y, \sigma_Y, sk_Y, ku_Y, \ldots, \gamma_Y^{(n)}}$
	$\downarrow$		$\uparrow$
Central moments	$\mu_X^{(1)}, \ldots, \mu_X^{(n)}$	($\mu_X^{(n)} \equiv \mathrm{E}\{(X - \mu_X)^n\}$)	$\mu_Y^{(1)}, \ldots, \mu_Y^{(n)}$
	$\downarrow$		$\uparrow$
Non-central moments	$\widetilde{\mu}_X^{(1)}, \ldots, \widetilde{\mu}_X^{(n)}$	($\widetilde{\mu}_X^{(n)} \equiv \mathrm{E}\{X^n\}$)	$\widetilde{\mu}_Y^{(1)}, \ldots, \widetilde{\mu}_Y^{(n)}$
	$\downarrow$		$\uparrow$
Cumulants	$\kappa_X^{(1)}, \ldots, \kappa_X^{(n)}$	$\longrightarrow$	$\kappa_Y^{(1)}, \ldots, \kappa_Y^{(n)}$

	τ	μ_Y	σ_Y	sk_Y	ku_Y	$\gamma_Y^{(5)}$	$\gamma_Y^{(6)}$
Quarter P&L (shifted lognormal)	**1**	0.32	0.55	1.32	6.26	25.53	145.80
	2	0.65	0.78	0.93	4.63	13.70	64.29
	3	0.97	0.95	0.76	4.09	10.00	44.28
Annual P&L (unknown distribution)	**4**	1.29	1.10	0.66	3.82	8.15	35.62
	5	1.62	1.23	0.59	3.65	7.01	30.85
	6	1.94	1.35	0.54	3.54	6.23	27.85
	7	2.26	1.46	0.50	3.47	5.66	25.80
Long-horizon P&L (normal approx.)	**100**	32.31	5.51	0.13	3.03	1.33	15.67

PROJECTION TO THE HORIZON – All distribution

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pdf f_X f_Y

X_t

$Y = X_1 + \cdots + X_\tau$

PROJECTION TO THE HORIZON – All distribution

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$$\begin{tikzcd}[column sep=huge]
\textbf{pdf} & f_X \arrow[d, "\text{FFT}"] & & f_Y \\
\textbf{char. f.} & \phi_X \arrow[rr] & & \phi_Y = \left(\phi_X\right)^{\tau} \ (3.64) \arrow[u, "\text{IFFT}"']
\end{tikzcd}$$

$$\phi_{\mathbf{X}} = \mathcal{F}\left[f_{\mathbf{X}}\right] \quad (2.14)$$

$$X_t \qquad\qquad Y = X_1 + \cdots + X_\tau$$

PROJECTION TO THE HORIZON – All distribution

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pdf	f_X		f_Y
	$\downarrow$ **FFT**	$\phi_{\mathbf{X}} = \mathcal{F}[f_{\mathbf{X}}]$ (2.14)	$\uparrow$ **IFFT**
char. f.	ϕ_X	$\longrightarrow$	$\phi_Y = (\phi_X)^\tau$ (3.64)