FACTORS ON DEMAND

Optimized Flexible Factors for Risk Estimation and Attribution

Attilio Meucci

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EXECUTIVE SUMMARY

TRADITIONAL MULTI-PURPOSE FACTOR MODELS

FACTORS ON DEMAND – THEORY

FACTORS ON DEMAND – APPLICATIONS

REFERENCES

Risk Estimation

- Identify Risk Factors
  to impose structure on estimate of large multivariate market distribution
- Compute overall portfolio risk (Standard Deviation and Tail Risk) from market distribution
- **Goal**: maximize predictive power

Risk Attribution

- Define Attribution Factors
- Allocate overall portfolio risk obtained from Risk Estimation to Attribution Factors
- **Goal**: maximize interpretability and practicality for hedging/trading

Risk Estimation

- Identify Risk Factors $\boxed{F_k}$
  to impose structure on estimate of large multivariate market distribution
- Compute overall portfolio risk (Standard Deviation and Tail Risk) from market distribution
- **Goal**: maximize predictive power

Risk Attribution

- Define Attribution Factors $\boxed{\widetilde{F}_k}$
- Allocate overall portfolio risk obtained from Risk Estimation to Attribution Factors
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Traditional Factor Models: same or similar factors for Risk Estimation and Attribution

- Suboptimal choice of “systematic” factors
  - Suboptimal statistical properties for risk estimation
  - Risk attribution factors are not most practical for hedging/interpretation
  - Not portfolio-specific estimation/attribution
- Inflexible choice of loadings (“betas”)
  - Rigid bottom-up aggregation (beta of portfolio is sum of beta of securities)
  - Rigid maximization target (R-square)
  - Rigid unconstrained maximization (CAPM beta)
- Incorrect modeling of non-linear products/derivatives

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- **Goal**: maximize predictive power

Risk Attribution

- Define Attribution Factors $\boxed{Z_k}$
- Allocate overall portfolio risk obtained from Risk Estimation to Attribution Factors
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Factors On Demand: different factors for Risk Estimation and Risk Attribution

- Flexible choice of factors: “dominant”, instead of “systematic”
  - Ideal statistical properties for risk estimation
  - Ideal hedging/interpretation properties for risk attribution
  - Portfolio-specific estimation/attribution
- Flexible choice of loadings (“betas”)
  - Flexible top-down aggregation
  - Flexible maximization target (R-square, CVaR, etc.)
  - Flexible constrained maximization (best pool, long-only, etc.)
- Consistent across non-linear products/derivatives (full conditional distribution of Z_k)

Risk Estimation

1. Stocks return estimation

$$R_n = \sum_k b_{n,k} F_k + U_n$$

R_n = security return
$b_{n,k}$ = loading
F_k = systematic factor
U_n = idiosyncratic shock

Risk Estimation Rationales

- Estimate the joint distribution of security returns, imposing structure with factor model

Traditional Risk Estimation Techniques

- Regression analysis

Risk Estimation

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3. Aggregation

$$R_w = \sum_n w_n R_n$$

4. Portfolio risk estimation

$$Sdev\{R_w\} = w' \left[b \Sigma_F b' + diag\left(\sigma_U^2\right) \right] w$$

$VaR\{R_w\} =$ Normal assumption

Risk Estimation Rationales

- Estimate the joint distribution of security returns, imposing structure with factor model
- Use the portfolio positions *w* to determine aggregated portfolio return distribution
- Define and compute risk: standard deviation, Value at Risk (tail risk), etc.

Traditional Risk Estimation Techniques

- Regression analysis
- Dimension reduction
- Parametric assumptions

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Pricing

$$R_n = g\left(X_1, \ldots, X_S\right)$$

Traditional modeling of non-linear securities

- For non-equity securities such as bonds and derivatives, the returns *R* are not "invariants", i.e. they do not behave identically and independently across time

Example: bond

$$R = \frac{P\left(X_1, X_2\right)}{P_0} - 1$$

P : discount formula
X_1 : govt curve changes
X_2 : spread changes

Example: option

$$R = \frac{BS\left(X_1, X_2\right)}{P_0} - 1$$

BS : Black-Scholes formula
X_1 : log-return of underlying
X_2 : log-return of implied vol.

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2. Pricing

$$R_n \approx \sum_s \boxed{\delta_{n,s}} X_s$$

Traditional modeling of non-linear securities

- For non-equity securities such as bonds and derivatives, the returns *R* are not "invariants", i.e. they do not behave identically and independently across time
- Therefore, estimation cannot be performed on returns, but rather on risk drivers *X*, which are "invariants"
- Then, risk drivers *X* are transformed into returns *R* by "delta" or "duration" coefficients $\boxed{\delta}$

Example: bond

$$\boxed{R \approx \delta_1 X_1 + \delta_2 X_2}$$

$$\begin{cases} \delta_1 & : \text{curve duration} \\ \delta_2 & : \text{spread duration} \\ X_1 & : \text{govt curve changes} \\ X_2 & : \text{spread changes} \end{cases}$$

Example: option

$$\boxed{R \approx \delta_1 X_1 + \delta_2 X_2}$$

$$\begin{cases} \delta_1 & : \text{delta} \\ \delta_2 & : \text{vega} \\ X_1 & : \text{log-return of underlying} \\ X_2 & : \text{log-return of implied vol.} \end{cases}$$

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$$R_n \approx \sum_s \delta_{n,s} X_s$$

3. Aggregation

$$R_w = \sum_n w_n R_n$$

4. Portfolio risk estimation

$$Sdev\left\{R_w\right\} = w'\delta\left[b\Sigma_F b' + diag\left(\sigma_U^2\right)\right]\delta' w$$

$VaR\left\{R_w\right\} =$ Normal assumption

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- For non-equity securities such as bonds and derivatives, the returns *R* are not “invariants”, i.e. they do not behave identically and independently across time
- Therefore, estimation cannot be performed on returns, but rather on risk drivers *X*, which are “invariants”
- Then, risk drivers *X* are transformed into returns *R* by “delta” or “duration” coefficients δ
- The risk computations follow

Risk Estimation

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Risk Attribution

5. Attribution factors

$$F_k$$

6. Security-level attribution

$$R_n = \sum_k b_{n,k} F_k + U_n$$

$$b_{n,k} = \sum_s \delta_{n,s} b_{s,k}$$

Risk Attribution Rationales

- After obtaining aggregate portfolio risk (Sdev, VaR, CVaR, etc.), attribute it to individual factors
- Purpose: see how factors contributed to portfolio risk and make hedging decision

Traditional Risk Attribution Techniques

- Use same factors for attribution as for estimation
- Perform linear operations to define security-level risk attribution

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Risk Attribution

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6. Security-level attribution

$$R_n = \sum_k b_{n,k} F_k + U_n$$

$$b_{n,k} = \sum_s \delta_{n,s} b_{s,k}$$

7. Portfolio risk attribution: bottom up

$$R_w = \sum_k b_{w,k} F_k + U_w$$

$$b_{w,k} = \sum_n w_n b_{n,k}$$

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- After obtaining aggregate portfolio risk (Sdev, VaR, CVaR, etc.), attribute it to individual factors
- Purpose: see how factors contributed to portfolio risk and make hedging decision

Traditional Risk Attribution Techniques

- Use same factors for attribution as for estimation
- Perform linear operations to define security-level risk attribution
- Perform bottom-up aggregation for portfolio-level risk attribution

Risk Estimation

1. Risk drivers estimation

$$X_s = \sum_k \boxed{b_{s,k} F_k} + U_s$$

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$$b_{n,k} = \sum_s \delta_{n,s} b_{s,k}$$

7. Portfolio risk attribution: bottom up

$$R_w = \sum_k b_{w,k} F_k + U_w$$

$$b_w = \mathbb{C}ov\{R_w, F\} / \mathbb{C}ov\{F\}$$
$$= \underset{\beta}{\operatorname{argmin}}\, \mathbb{V}ar\{R_w - \sum_k \beta_k F_k\}$$

Pitfalls

- Same factors used for both estimation and attribution: choice neither optimizes the estimation power nor the interpretability or practicality for hedging

- As an estimation model, *b* and *F* maximize r-square
- As an attribution model, *b* and *F* maximize r-square (CAPM)
- “delta” assumption can be inappropriate
- Bottom-up aggregation not flexible: small exposures better in residual

Risk Estimation

1. Risk drivers estimation

$$X_s = \sum_k b_{s,k} \boxed{F_k} + U_s$$

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Risk Attribution

5. Enhanced attribution factors

$$\boxed{\widetilde{F}_j} = \sum_k a_{j,k} F_k \qquad F_k = \sum_j a^{-1}_{k,j} \widetilde{F}_j$$

6. Security-level attribution

$$R_n = \sum_j \boxed{\widetilde{b}_{n,j} \widetilde{F}_j} + U_n$$

$$\widetilde{b}_{n,j} = \sum_{k,s} \delta_{n,s} b_{s,k} a^{-1}_{k,j}$$

7. Portfolio risk attribution: bottom up

$$R_w = \sum_j \boxed{\widetilde{b}_{w,j} \widetilde{F}_j} + U_w$$

$$\widetilde{b}_{w,j} = \sum_n w_n \widetilde{b}_{n,j}$$

Pitfalls

- Similar factors used for both estimation and attribution: choice neither optimizes the estimation power nor the interpretability or practicality for hedging
- Factors restricted by the “systematic + idiosyncratic” assumption
- As an estimation model, *b* and *F* maximize r-square
- As an attribution model, *b* and *F* maximize r-square (CAPM)
- “delta” assumption can be inappropriate
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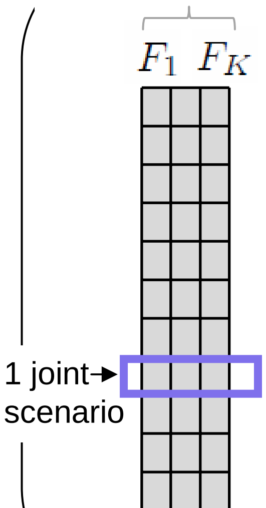
Risk Estimation

1. Risk drivers estimation

Σ_k F_k

F_k = dominant factor

E.g. PCA facts



Risk Estimation

1. Risk drivers estimation

$$X_s = \sum_k b_{s,k} F_k + U_s$$

$$\begin{cases} X_s & = \text{risk driver} \\ b_{n,k} & = \text{loading} \\ F_k & = \text{dominant factor} \\ U_n & = \text{residual} \end{cases}$$

Risk Estimation

1. Risk drivers estimation

$$X_s = \sum_k b_{s,k} F_k + U_s$$

X_s = risk driver
$b_{n,k}$ = loading
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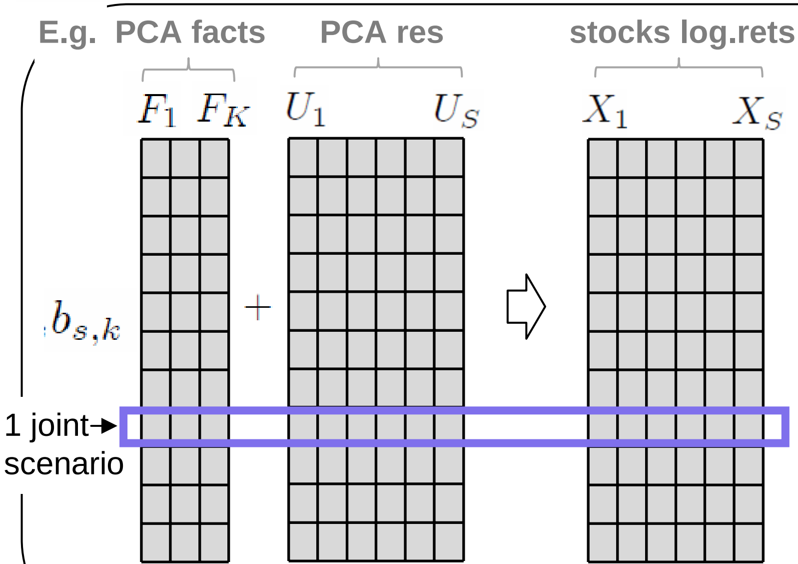

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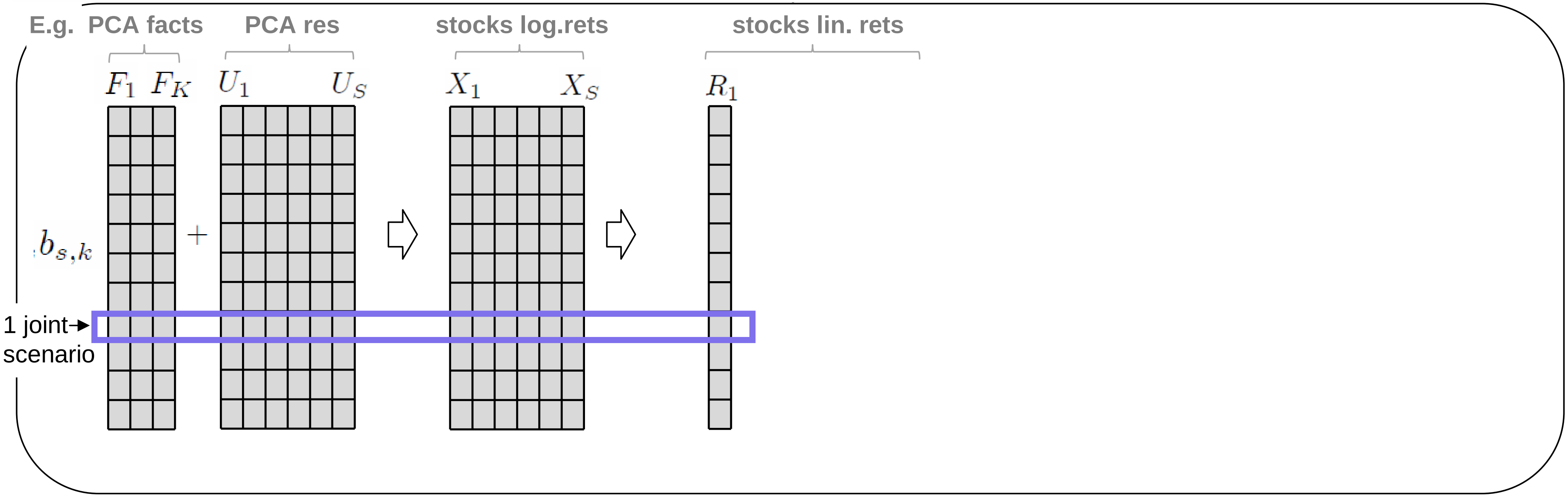
2. Pricing

$$R_n = g_n\left(X_1, \dots, X_S\right)$$



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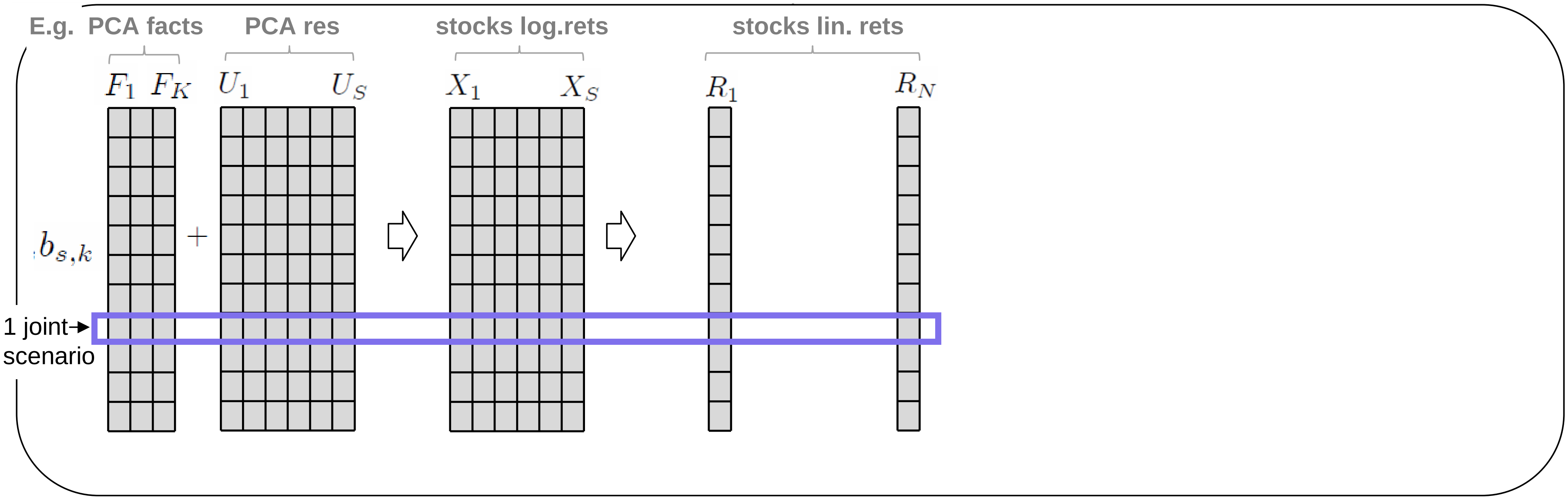

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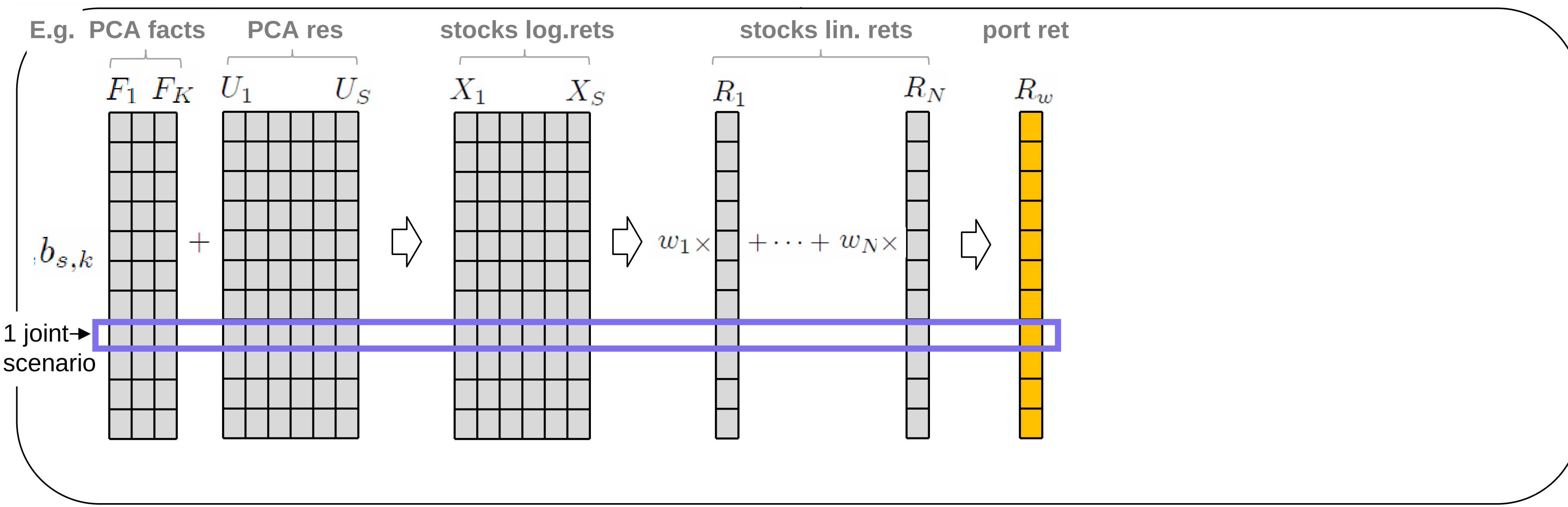

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$Sdev\{R_w\} =$ exact

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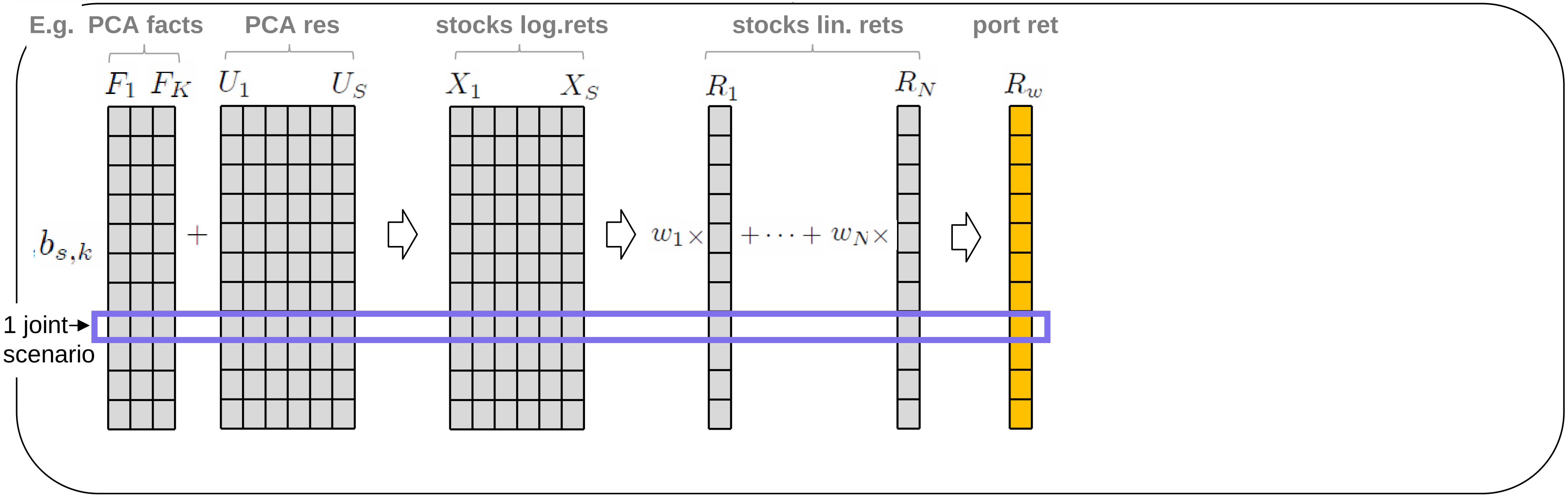

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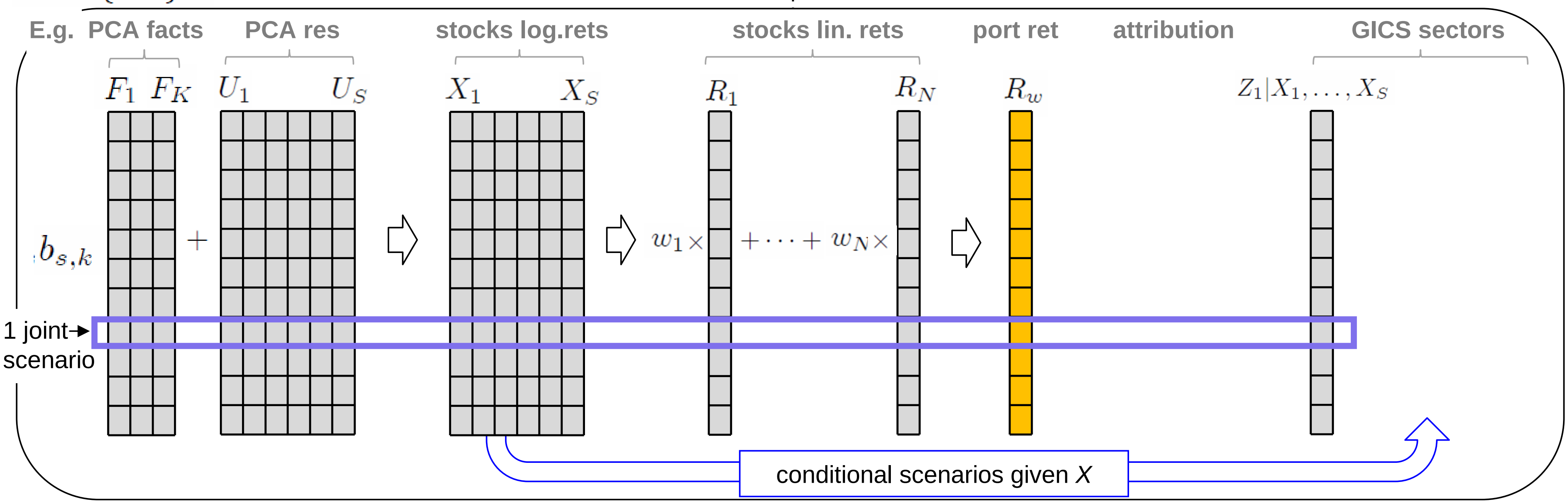
Risk Attribution

5. Attribution factors

$$Z_k | X_1, \ldots, X_S$$



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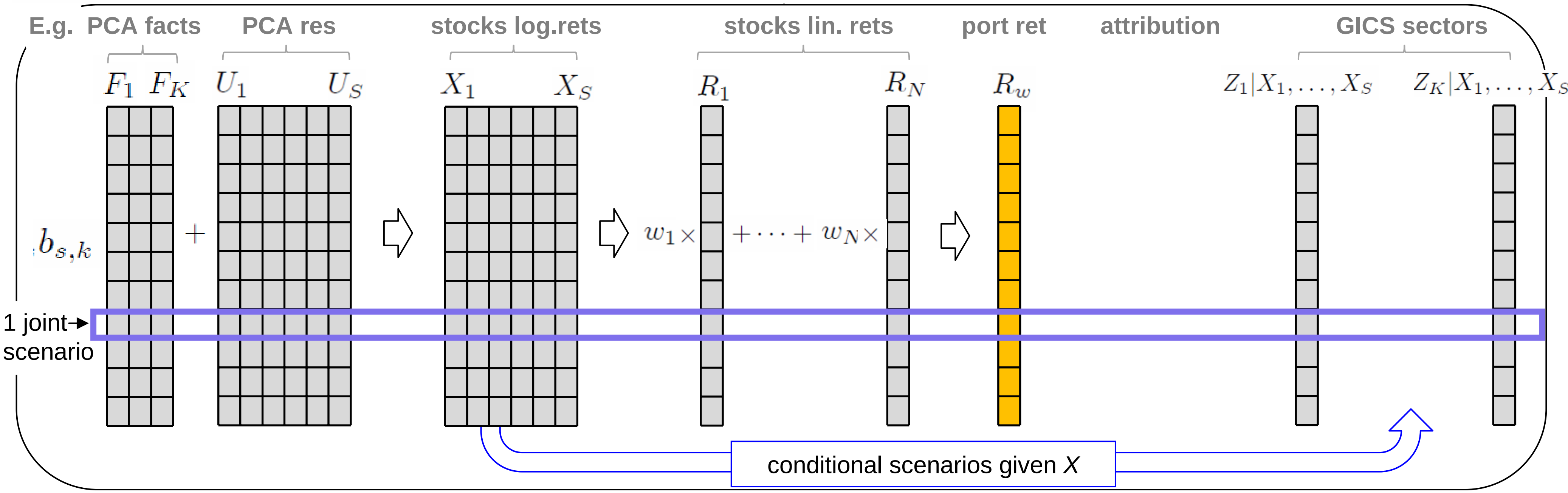

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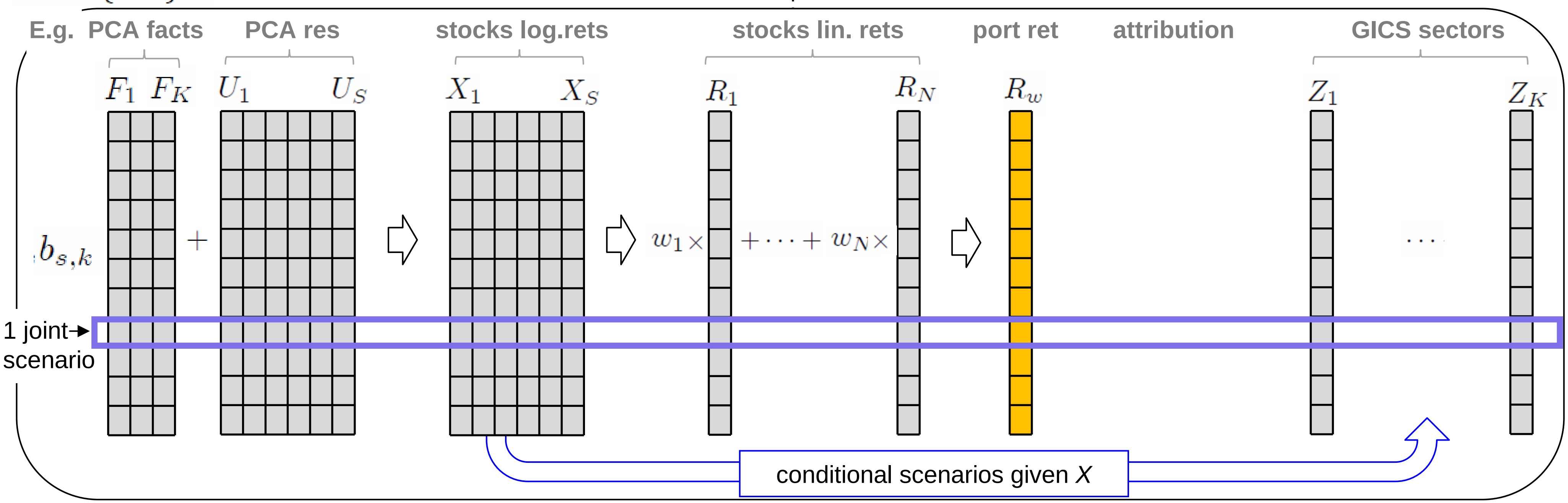

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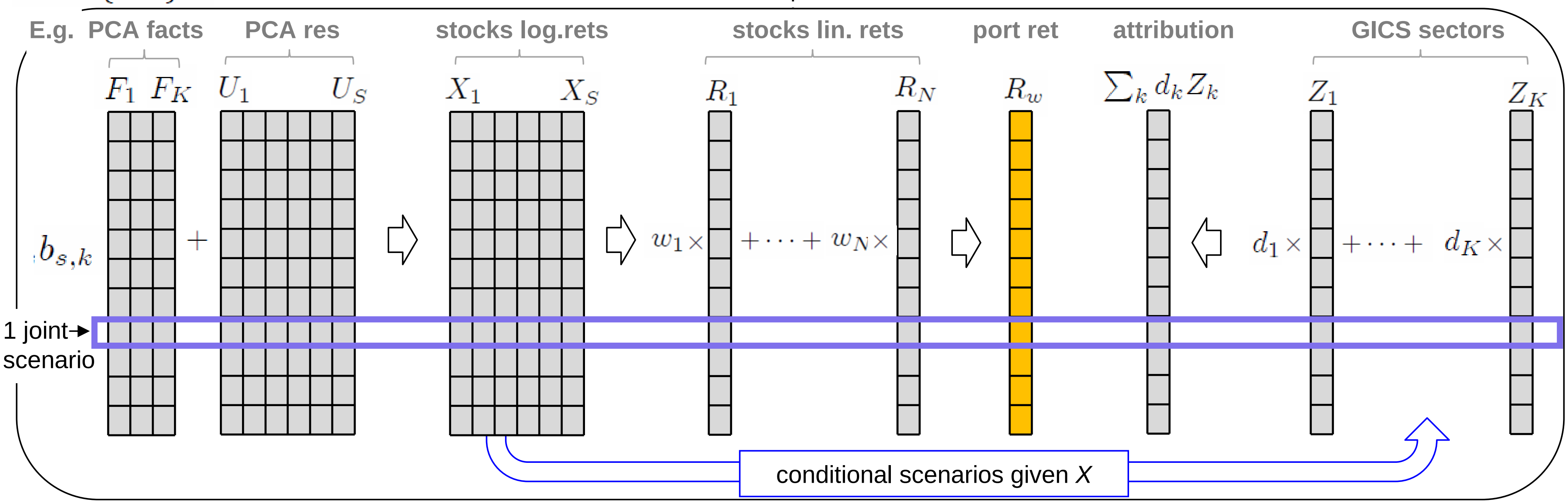

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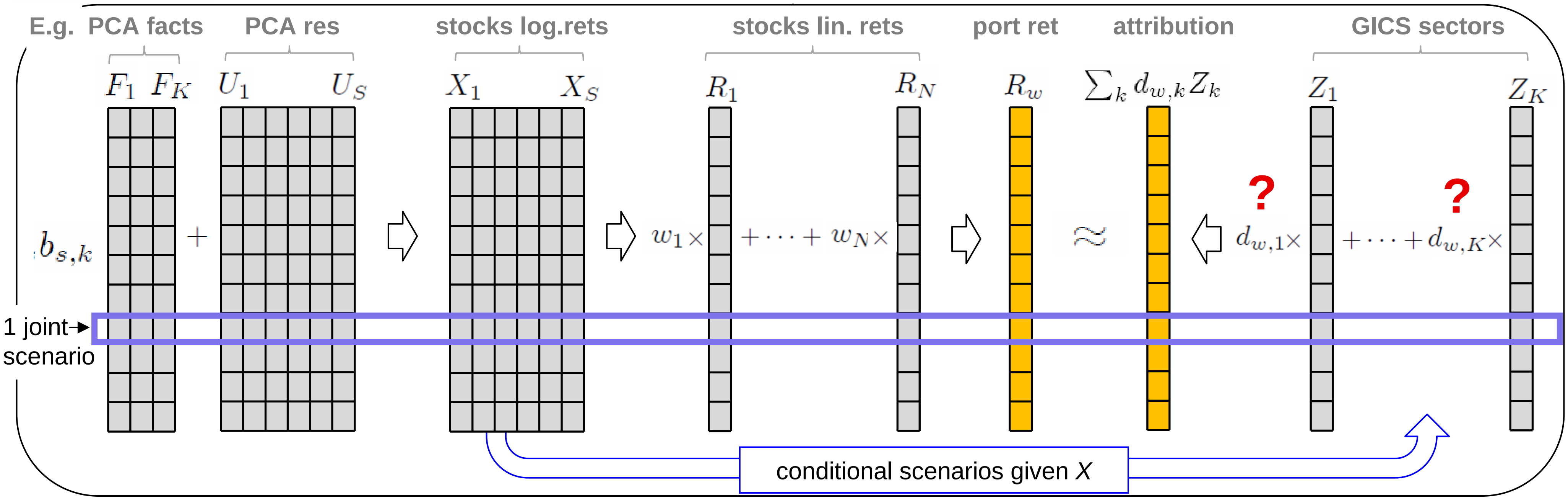
6. Portfolio risk attribution: top down

$$R_w = \sum_k d_{w,k} Z_k + \eta_w$$

- R_w = portfolio return
- $d_{w,k}$ = attribution loading
- Z_k = attribution factor
- η_w = residual



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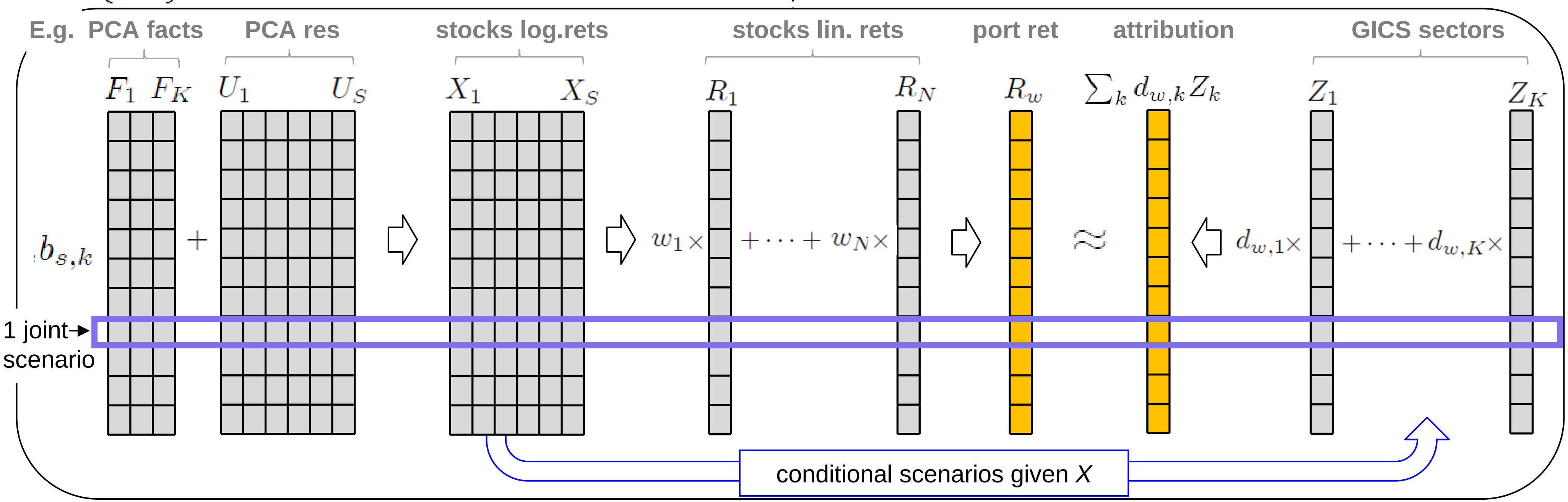
$$\begin{cases} R_w & = \text{portfolio return} \\ d_{w,k} & = \text{attribution loading} \\ Z_k & = \text{attribution factor} \\ \eta_w & = \text{residual} \end{cases}$$

$$d_w = \operatorname{argmin} \mathbb{E}\{(R_w - \sum_k d_k Z_k)^2\}$$



Risk Estimation

1. Risk drivers estimation

$$X_s = \sum_k b_{s,k} F_k + U_s$$

X_s = risk driver
$b_{n,k}$ = loading
F_k = dominant factor
U_n = residual

2. Pricing

$$R_n = g_n(X_1, \ldots, X_S)$$

3. Aggregation

$$R_w = \sum_n w_n R_n$$

4. Portfolio risk estimation

$Sdev\{R_w\} =$ exact

$VaR\{R_w\} =$ exact

Risk Attribution

5. Attribution factors

$$Z_k | X_1, \ldots, X_S$$

6. Portfolio risk attribution: top down

$$R_w = \sum_k d_{w,k} Z_k + \eta_w$$

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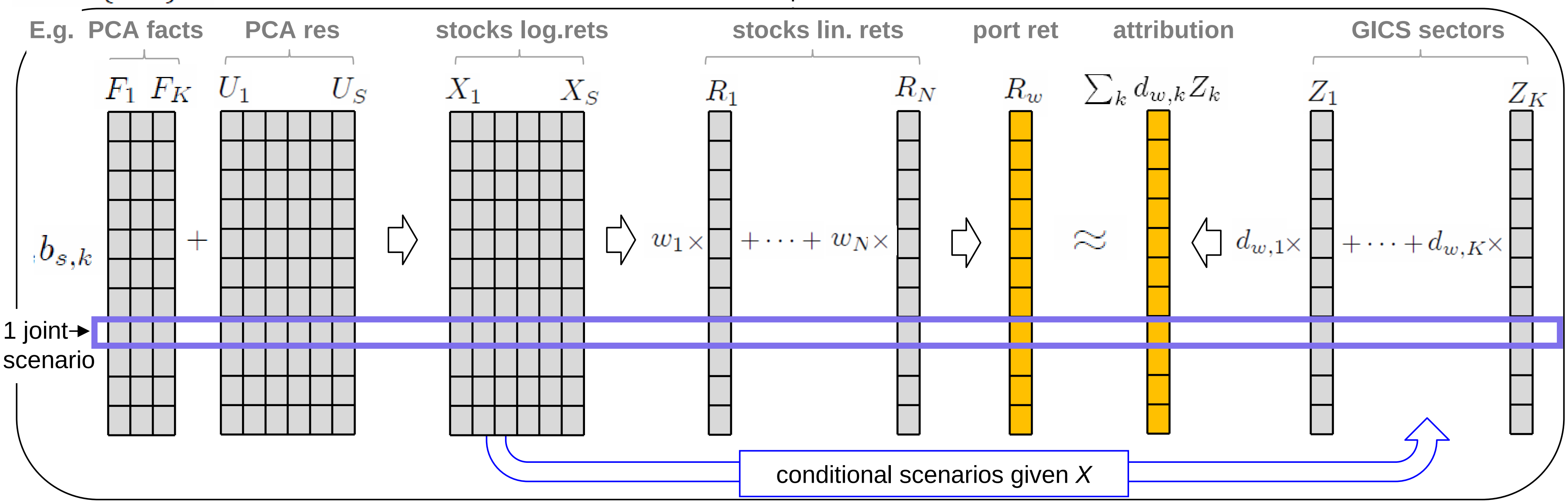

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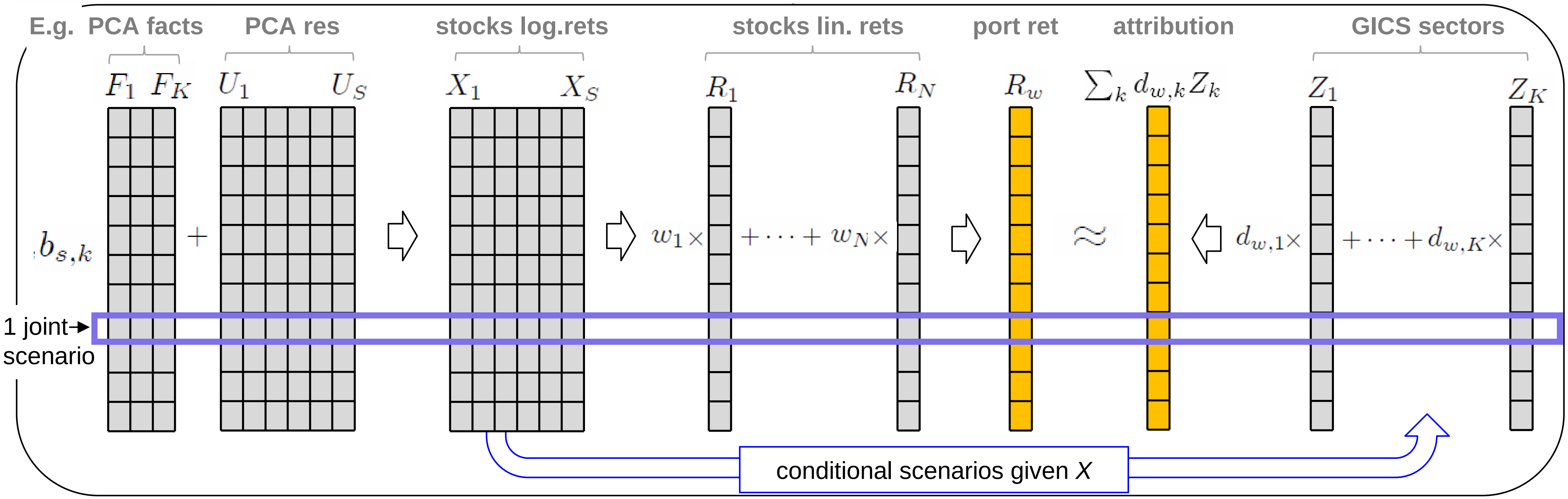

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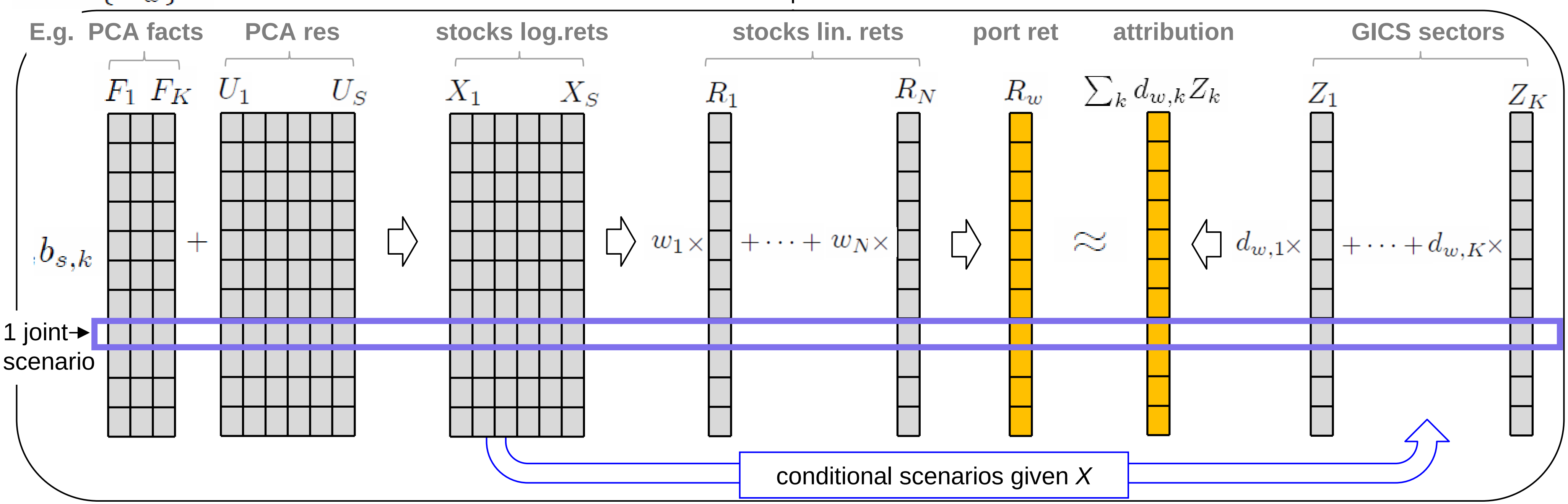

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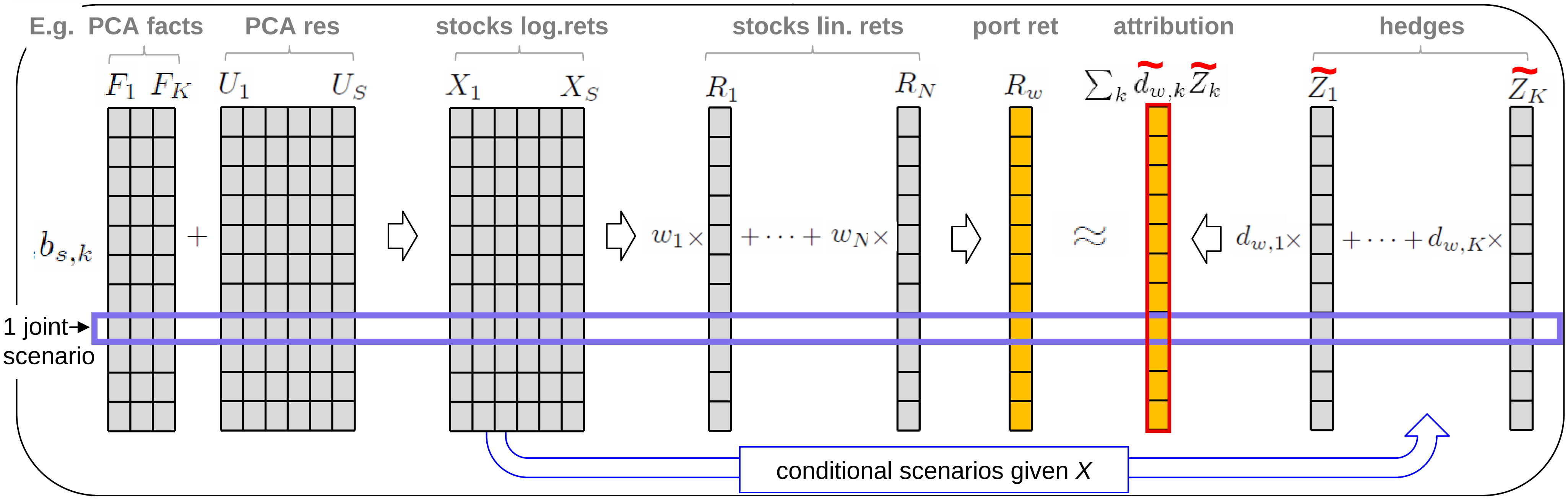

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Factors on Demand - Features

1. Estimation factors *F* and loadings *b* are chosen to optimize the explanation power

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Factors on Demand – Frequently Asked Questions

Q: Why not run a regression of portfolio returns *R* vs. attribution factors *Z*?

A: *R* and *Z* are not necessarily "invariants"

Q: Why abandon "systematic + idiosyncratic" model?

A: *U* is where managers look for "alpha" factors -> $\Sigma_X \neq b \Sigma_F b' + diag\left(\sigma_U^2\right)$

A: otherwise we cannot merge irrelevant "systematic" factors with "idiosyncratic" residual to obtain more efficient attribution/hedging

A: in powerful estimation approaches (PCA,RMT) residual *U* is never idiosyncratic

A: that model is not a consequence of APT/CAPM

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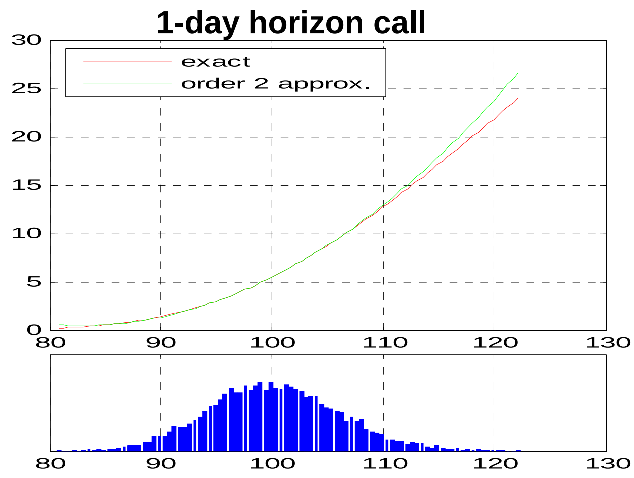
$$d_w = \underset{d \in \mathcal{C}}{\operatorname{argmax}}\, \mathcal{T}\left(R_w, \sum_k d_k Z_k\right)$$

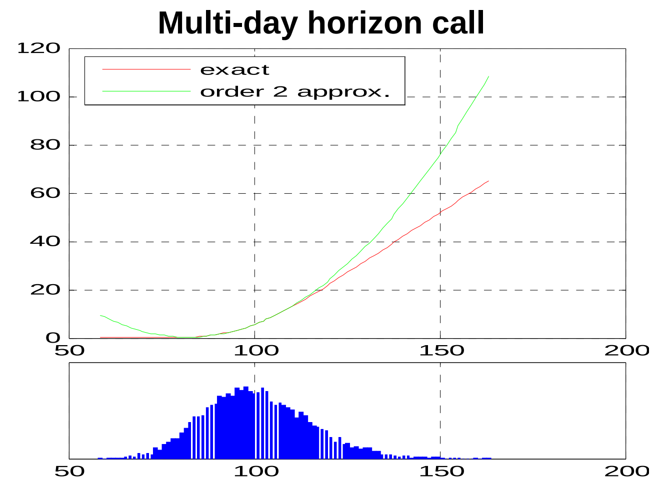
R_w = portfolio return
$d_{w,k}$ = attribution loading
Z_k = attribution factor
η_w = residual

Factors on Demand – Frequently Asked Questions

Q: Why should we not use delta approximation?

A: Risk of derivatives or non linear instruments at multi-day horizon is distorted





Risk Estimation

1. Risk drivers estimation

$$X_s = \sum_k b_{s,k} F_k + U_s$$

X_s = risk driver
$b_{n,k}$ = loading
F_k = dominant factor
U_n = residual

2. Pricing

$$R_n = g_n (X_1, \ldots, X_S)$$

3. Aggregation

$$R_w = \sum_n w_n R_n$$

4. Portfolio risk estimation

$Sdev \{R_w\} =$ exact

$VaR \{R_w\} =$ exact

Risk Attribution

5. Attribution factors

$$Z_k | X_1, \ldots, X_S$$

7. Security-level attribution

$$R_n = \sum_k d_{n,k} Z_k + \eta_n$$

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Factors on Demand – Frequently Asked Questions

Q: Do we have to generate conditional scenarios for *Z*?

A: Not always: if using historical scenarios, use historical (drivers for) *Z*

Q: Does FOD recommend specific estimation/attribution factors/techniques?

A: No, FOD proposes a flexible, modular methodology that hosts all techniques

Q: Does FOD dismiss traditional multi-purpose factor models

A: No, all traditional model are special cases of FOD

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6. Portfolio risk attribution: top down

$$R_w = \sum_k d_{w,k} Z_k + \eta_w$$

$$d_w = \underset{d \in C}{\operatorname{argmax}} \, T\left(R_w, \sum_k d_k Z_k\right)$$

R_w = portfolio return
$d_{w,k}$ = attribution loading
Z_k = attribution factor
η_w = residual

b, *F* : high statistical power

- Principal Component Analysis and Random Matrix Theory can be applied
- Factors and loadings are determined to minimize estimation error although they might be difficult to interpret.

Z : high interpretability/tradability

Attribution factors examples

- GICS Sectors: Material, Technology, Financials
- Macro: S&P500, 10 year yield, Gold price, MSCI EM Index, Russell 2000

FOD Application #1: Optimize Factor Choice for Risk and Portfolio Mgmt.

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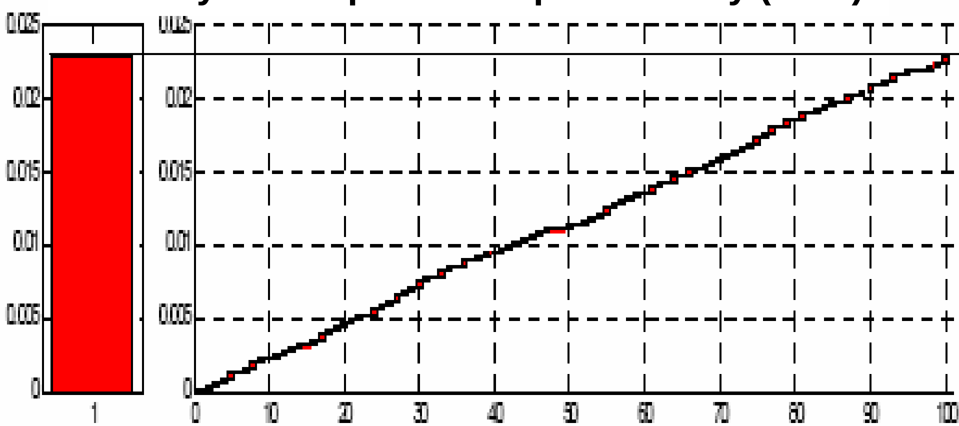
Z_k = attribution factor
η_w = residual

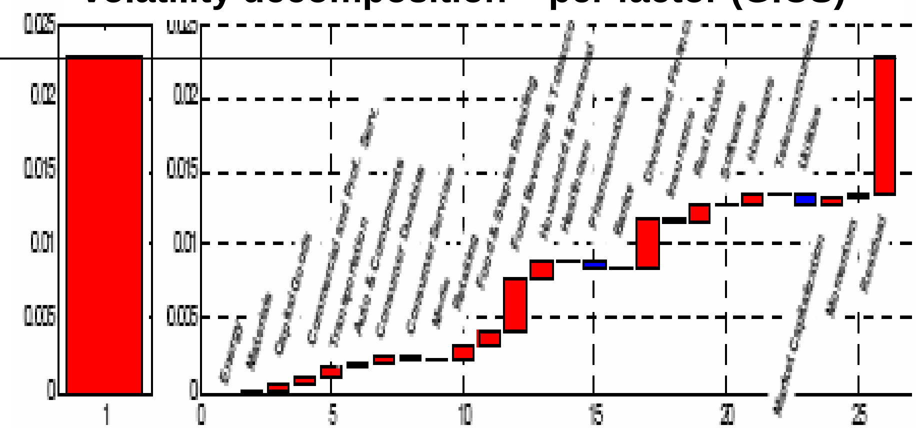
***b*, *F* : high statistical power**

volatility decomposition – per industry (RMT)



***Z* : high interpretability/tradability**

volatility decomposition – per factor (GICS)



FOD Application #2: Custom attribution factors on the fly

Risk Estimation

1. Risk drivers estimation

$$\boxed{X_s} = \sum_k b_{s,k} F_k + U_s$$

$$\begin{cases} X_s & = \text{risk driver} \\ b_{n,k} & = \text{loading} \\ F_k & = \text{dominant factor} \\ U_n & = \text{residual} \end{cases}$$

2. Pricing

$$R_n = g_n(X_1, \ldots, X_S)$$

3. Aggregation

$$R_w = \sum_n w_n R_n$$

4. Portfolio risk estimation

$$Sdev\{R_w\} = \text{exact}$$

$$VaR\{R_w\} = \text{exact}$$

Risk Attribution

5. Attribution factors

$$\boxed{Z_k} | X_1, \ldots, X_S$$

7. Security-level attribution

$$R_n = \sum_k d_{n,k} Z_k + \eta_n$$

6. Portfolio risk attribution: top down

$$R_w = \sum_k d_{w,k} Z_k + \eta_w$$

$$d_w = \underset{d \in \mathcal{C}}{\operatorname{argmax}}\, \mathcal{T}\left(R_w, \sum_k d_k Z_k\right)$$

$$\begin{cases} R_w & = \text{portfolio return} \\ d_{w,k} & = \text{attribution loading} \\ Z_k & = \text{attribution factor} \\ \eta_w & = \text{residual} \end{cases}$$

X : historical

- No factor modes for *X*, pure historical realization of risk drivers
- *R* is not the time series of the returns
- Explicitly no idiosyncratic term

Z : g(*X*)

- Attribution factors are deterministic functions of risk drivers
- For instance, *Z* can be user-supplied definitions of value/momentum factors
- FOD then allows to compare in real time the attribution to different, user-supplied factor models *Z* and $\tilde{Z}$
- All models share the *same* risk statistics

Risk Estimation

1. Risk drivers estimation

$$X_s = \sum_k \boxed{b_{s,k} F_k} + U_s$$

X_s = risk driver
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U_n = residual

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$$R_w = \sum_k d_{w,k} Z_k + \eta_w$$

$$d_w = \underset{d \in C}{\text{argmax}}\, \mathcal{T}\left(R_w, \sum_k d_k Z_k\right)$$

R_w = portfolio return
$d_{w,k}$ = attribution loading
Z_k = attribution factor
η_w = residual

b, *F*: regional equity factor model

• Regional factors *F* constructed by cross-sectional regression on given loadings *b*

e.g. US Model: US sector factors

$$\mathbf{R}^{(\alpha)} \equiv \mathbf{B}^{(\alpha)}\mathbf{F}^{(\alpha)} + \mathbf{U}^{(\alpha)}$$

e.g. UK Model: UK financial, UK utilities,…)

$$\mathbf{R}^{(\omega)} \equiv \mathbf{B}^{(\omega)}\mathbf{F}^{(\omega)} + \mathbf{U}^{(\omega)}.$$

Z : global equity factors

• Global factors *Z* are deterministic, linear functions (aggregations) of the regional factors

$$\mathbf{Z} \equiv \mathbf{A} \begin{pmatrix} \mathbf{F}^{(\alpha)} \\ \vdots \\ \mathbf{F}^{(\omega)} \end{pmatrix}$$

e.g. global financial, global utilities,…

FOD Application #4:
New attribution target: minimize CVAR for hedging

Risk Estimation

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Risk Attribution

5. Attribution factors

$$\boxed{Z_k} X_1, \ldots, X_S$$

7. Security-level attribution

$$R_n = \sum_k d_{n,k} Z_k + \eta_n$$

6. Portfolio risk attribution: top down

$$R_w = \sum_k d_{w,k} Z_k + \eta_w$$

$$d_w = \underset{d \in \mathcal{C}}{\text{argmin}} \boxed{CVaR} \{R_w - \sum_k d_k Z_k\}$$

R_w = portfolio return
$d_{w,k}$ = attribution loading
Z_k = attribution factor
η_w = residual

Z : returns of hedging instruments; *d*: attribution target as CVaR

- For hedging, the attribution factors must be the linear returns *Z=P(t+1)/P(t)-1* of tradables
- *Linear* attribution (6) is important for hedging: only portfolios, i.e. linear combinations, are traded
- Profits and losses of hedged p&l η play a *non-symmetrical* role: non-linear pricing (2) properly induces asymmetries on *R*; downside target CVaR in (6) accounts for asymmetries in η
- Thus FOD hedging (full-pricing/CVaR) and Black-Scholes hedging (delta/r-square) are different

Example: units of underlying to hedge call options

	100 days	150 days	200 days	250 days	300 days
FOD	5.8	5.3	5.0	4.9	4.8
BS	5.7	5.4	5.2	5.1	5.0

FOD Application #5: Best Pool on Demand / flexible constraints

Risk Estimation

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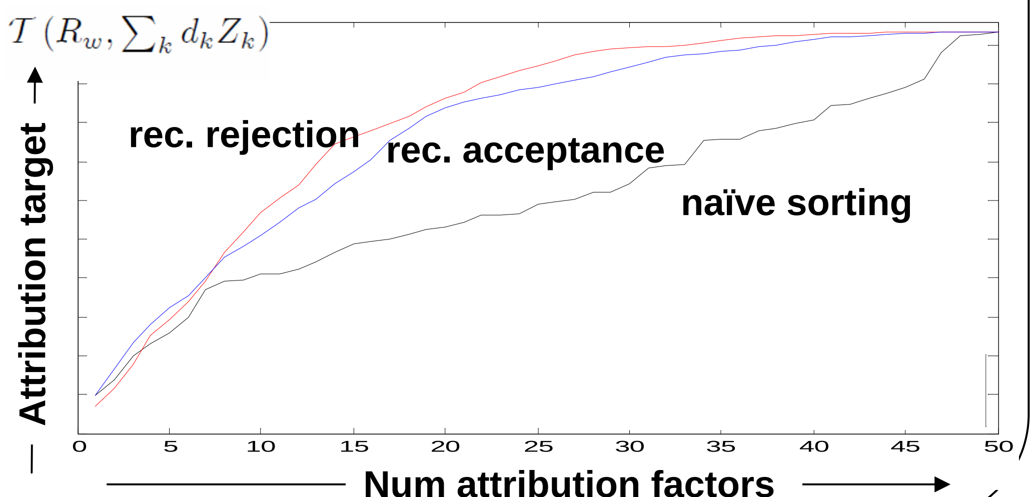
$$d_w = \underset{\boxed{d \in \mathcal{C}}}{\operatorname{argmax}} \mathcal{T}\left(R_w, \sum_k d_k Z_k\right)$$

R_w = portfolio return
$d_{w,k}$ = attribution loading
Z_k = attribution factor
η_w = residual

d: constraint "few relevant out of many" in top-down attribution

- For hedging, traders prefer to put on fewer hedges. Therefore the selection of the best few trades should be optimized
- For factor modeling, it does not make sense to include minimally represented factors in analysis. Better to add them to residual
- Other constraints can be added (e.g. long only, sum-to-one, etc.)



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R_w = portfolio return
$d_{w,k}$ = attribution loading
Z_k = attribution factor
η_w = residual

Z : returns of sub-portfolios; portfolios: past holdings

• The attribution of the current holdings to the past holdings allows the portfolio manager to evaluate the turnover (half-life) of their positions

$$\begin{cases} Z_1 = w'_{t-1} R \\ \vdots \\ Z_K = w'_{t-K} R \end{cases}$$

Risk Estimation

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$$R_n = \sum_k d_{n,k} Z_k + \eta_n$$

6. Portfolio risk attribution: top down

$$R_w = \sum_k d_{w,k} Z_k + \eta_w$$

$$d_w = \underset{\boxed{d \in C}}{\operatorname{argmin}} \boxed{Rsq}\{R_w, \sum_k d_k Z_k\}$$

R_w = portfolio return
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Z_k = attribution factor
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Z : returns of sub-portfolios; portfolios: past holdings

$$\begin{cases} Z_1 = w'_{t-1} R \\ \vdots \\ Z_K = w'_{t-K} R \end{cases}$$

- The attribution of the current holdings to the past holdings allows portfolio managers to evaluate the turnover (half-life) of their positions

- If the attribution target in (6) is set as the r-square and the attribution optimization is unconstrained we obtain the analytical solution in Grinold (2006)

$$d_w = (W' \Sigma_R W)^{-1} W' \Sigma_R w_t$$

- FOD allows portfolio managers to customize their analysis, with arbitrary targets and constraints

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6. Portfolio risk attribution: top down

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$$d_w = \operatorname{argmax}_{\boxed{d'1 = 1, d \geq 0}} \mathcal{T}\left(R_w, \sum_k d_k Z_k\right)$$

R_w = portfolio return
$d_{w,k}$ = attribution loading
Z_k = attribution factor
η_w = residual

Z : style factors; constraints: long-only, sum-to-one

- Traditional style analysis a-la-Sharpe runs a constrained regression of portfolio returns *Rp(t)* on style factors *Z(t)*
- In traditional style analysis the past returns are affected by the past allocation decisions *Rp(t-k)= w(t-k) x R(t-k)* includes a component due to rebalancing w(t-k)
- FOD allows to perform point-in-time style analysis based only the current exposures *w(t)*

EXECUTIVE SUMMARY

TRADITIONAL MULTI-PURPOSE FACTOR MODELS

FACTORS ON DEMAND – THEORY

FACTORS ON DEMAND – APPLICATIONS

REFERENCES

- Article

Attilio Meucci - "Factors on Demand"
Risk, July 2010, p 84-89
available at http://ssrn.com/abstract=1565134

- MATLAB examples

MATLAB Central Files Exchange (see above article)

- This presentation

www.symmys.com > Teaching > Talks