Attilio Meucci

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Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

$$\text{classical estimation: } i_T \mapsto \widehat{\boldsymbol{\theta}} \tag{7.2}$$

$$\text{Bayesian estimation: } i_T, e_C \mapsto f_{\mathrm{po}}(\boldsymbol{\theta}) \tag{7.3}$$

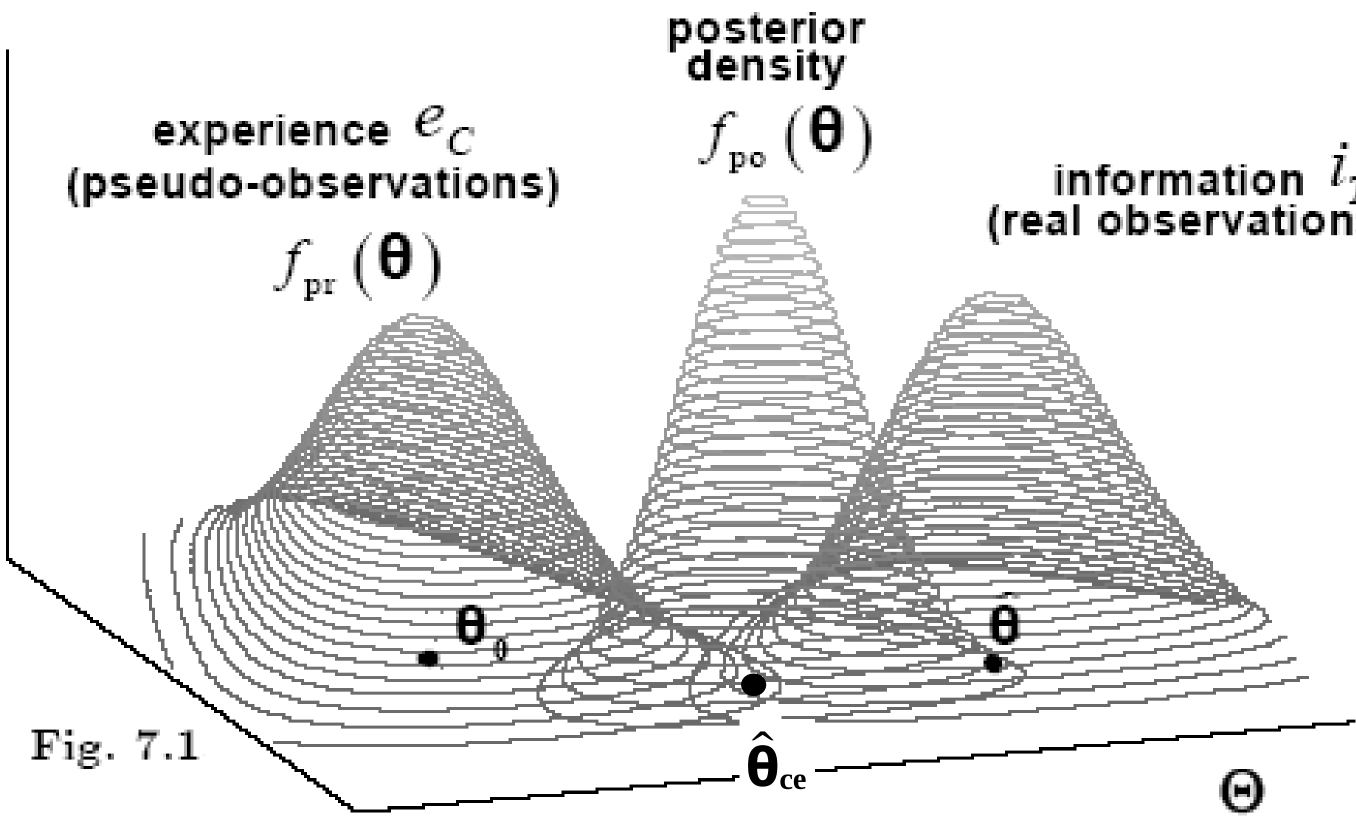

Fig. 7.1

$$f_{\mathrm{po}}(\boldsymbol{\theta}) \begin{array}{l} \nearrow \ \boldsymbol{\theta}_0 \quad (C \to \infty) \\ \\ \searrow \ \widehat{\boldsymbol{\theta}} \quad (T \to \infty). \end{array} \tag{7.4}$$

BAYESIAN ALLOCATION - predictive distribution

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{ \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \tag{8.76}$$

$$\begin{aligned} \mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = {} & \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) \\ & -\frac{1}{2\zeta}\boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)\,\boldsymbol{\Sigma}\operatorname{diag}(\mathbf{p}_T)\,\boldsymbol{\alpha} \end{aligned} \tag{8.25}$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu},\boldsymbol{\Sigma}) = [\operatorname{diag}(\mathbf{p}_T)]^{-1}\,\boldsymbol{\Sigma}^{-1}\left(\zeta\boldsymbol{\mu} + \frac{w_T - \zeta\mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}\mathbf{1}\right) \tag{8.77}$$

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$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{ \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \qquad (8.76)$$

$$\Uparrow$$

$$\mathcal{S}(\boldsymbol{\alpha}) \equiv u^{-1}\left(\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\}\right) \qquad (9.2)$$

$$\Downarrow$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{ \mathrm{E}\left\{ u\left(\Psi_{\boldsymbol{\alpha}}^{\boldsymbol{\theta}}\right)\right\}\right\} \qquad (9.3)$$

$$\begin{aligned} \mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = {} & \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) \qquad (8.25) \\ & - \frac{1}{2\zeta} \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)\, \boldsymbol{\Sigma} \operatorname{diag}(\mathbf{p}_T)\, \boldsymbol{\alpha} \end{aligned}$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu},\boldsymbol{\Sigma}) = \left[\operatorname{diag}(\mathbf{p}_T)\right]^{-1} \boldsymbol{\Sigma}^{-1} \left(\zeta \boldsymbol{\mu} + \frac{w_T - \zeta \mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}} \mathbf{1} \right) \qquad (8.77)$$

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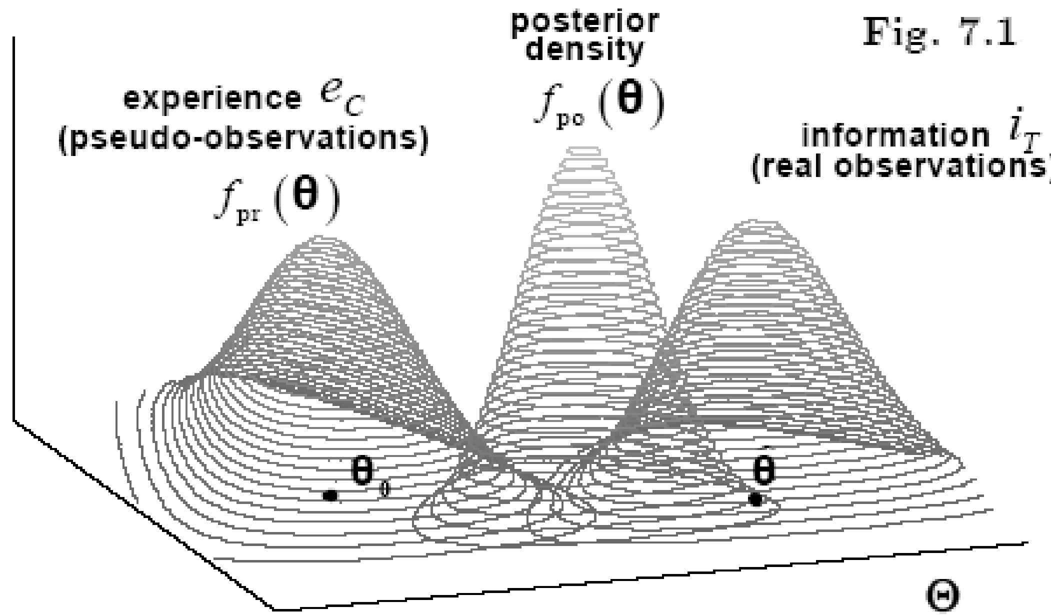
$$\overline{\boldsymbol{\alpha}} \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \int \mathrm{E}\left\{ u\left(\Psi_{\boldsymbol{\alpha}}^{\boldsymbol{\theta}}\right)\right\} f_{\mathrm{po}}(\boldsymbol{\theta})\, d\boldsymbol{\theta} \right\}. \qquad (9.7)$$

$$\begin{aligned} \mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = {} & \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) \\ & -\frac{1}{2\zeta}\boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)\,\boldsymbol{\Sigma} \operatorname{diag}(\mathbf{p}_T)\,\boldsymbol{\alpha} \end{aligned} \qquad (8.25)$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu},\boldsymbol{\Sigma}) = \left[\operatorname{diag}(\mathbf{p}_T)\right]^{-1} \boldsymbol{\Sigma}^{-1} \left(\zeta\boldsymbol{\mu} + \frac{w_T - \zeta \mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}\mathbf{1}\right) \qquad (8.77)$$

Fig. 7.1



BAYESIAN ALLOCATION - predictive distribution

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$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{ \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \qquad (8.76)$$

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$$\overline{\boldsymbol{\alpha}} \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \int \mathrm{E}\left\{ u\left(\Psi_{\boldsymbol{\alpha}}^{\boldsymbol{\theta}}\right)\right\} f_{\mathrm{po}}(\boldsymbol{\theta})\, d\boldsymbol{\theta} \right\}. \qquad (9.7)$$

$$f_{\mathrm{prd}}\left(\mathbf{m}; i_T, e_C\right) \equiv \int f_{\boldsymbol{\theta}}(\mathbf{m})\, f_{\mathrm{po}}\left(\boldsymbol{\theta}; i_T, e_C\right) d\boldsymbol{\theta}.$$

$$\boldsymbol{\alpha}_{\mathrm{B}}\left[i_T, e_C\right] = \underset{\boldsymbol{\alpha} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \int u\left(\boldsymbol{\alpha}'\mathbf{m}\right) f_{\mathrm{prd}}\left(\mathbf{m}; i_T, e_C\right) d\mathbf{m} \right\}$$

$$\begin{aligned} \mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = {} & \boldsymbol{\alpha}' \operatorname{diag}\left(\mathbf{p}_T\right)(\mathbf{1} + \boldsymbol{\mu}) \qquad (8.25) \\ & -\frac{1}{2\zeta} \boldsymbol{\alpha}' \operatorname{diag}\left(\mathbf{p}_T\right) \boldsymbol{\Sigma} \operatorname{diag}\left(\mathbf{p}_T\right) \boldsymbol{\alpha} \end{aligned}$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \left[\operatorname{diag}\left(\mathbf{p}_T\right)\right]^{-1} \boldsymbol{\Sigma}^{-1} \left(\zeta \boldsymbol{\mu} + \frac{w_T - \zeta \mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}} \mathbf{1} \right) \qquad (8.77)$$

BAYESIAN ALLOCATION - predictive distribution

$$\begin{aligned}\mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) &= \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1}+\boldsymbol{\mu}) \qquad (8.25)\\ &\quad -\frac{1}{2\zeta}\boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)\,\boldsymbol{\Sigma} \operatorname{diag}(\mathbf{p}_T)\,\boldsymbol{\alpha}\end{aligned}$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu},\boldsymbol{\Sigma}) = [\operatorname{diag}(\mathbf{p}_T)]^{-1}\,\boldsymbol{\Sigma}^{-1}\left(\zeta\boldsymbol{\mu} + \frac{w_T - \zeta\mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}\mathbf{1}\right) \qquad (8.77)$$

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha}\in\mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}}\left\{\mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha})\right\} \qquad (8.76)$$

$$\mathcal{S}(\boldsymbol{\alpha}) \equiv u^{-1}\left(\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}\right)\right\}\right) \qquad (9.2)$$

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$$\overline{\boldsymbol{\alpha}} \equiv \underset{\boldsymbol{\alpha}\in\mathcal{C}}{\operatorname{argmax}}\left\{\int \mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}^{\boldsymbol{\theta}}\right)\right\} f_{\mathrm{po}}(\boldsymbol{\theta})\, d\boldsymbol{\theta}\right\}. \qquad (9.7)$$

$$f_{\mathrm{prd}}(\mathbf{m}; i_T, e_C) \equiv \int f_{\boldsymbol{\theta}}(\mathbf{m})\, f_{\mathrm{po}}(\boldsymbol{\theta}; i_T, e_C)\, d\boldsymbol{\theta}.$$

$$\boldsymbol{\alpha}_{\mathrm{B}}[i_T, e_C] = \underset{\boldsymbol{\alpha}\in\mathcal{C}}{\operatorname{argmax}}\left\{\int u(\boldsymbol{\alpha}'\mathbf{m})\, f_{\mathrm{prd}}(\mathbf{m}; i_T, e_C)\, d\mathbf{m}\right\} \equiv \underset{\boldsymbol{\alpha}\in\mathcal{C}}{\operatorname{argmax}}\left\{\mathrm{E}\left\{u\left(\Psi_{\boldsymbol{\alpha}}^{i_T, e_C}\right)\right\}\right\} \qquad (9.9)$$

BAYESIAN ALLOCATION - certainty equivalent

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$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{ \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \quad (8.76)$$

$$\begin{aligned} \mathrm{CE}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\alpha}) = & \ \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T)(\mathbf{1} + \boldsymbol{\mu}) \\ & - \frac{1}{2\zeta} \boldsymbol{\alpha}' \operatorname{diag}(\mathbf{p}_T) \boldsymbol{\Sigma} \operatorname{diag}(\mathbf{p}_T) \boldsymbol{\alpha} \end{aligned} \quad (8.25)$$

$$\boldsymbol{\alpha}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = [\operatorname{diag}(\mathbf{p}_T)]^{-1} \boldsymbol{\Sigma}^{-1} \left(\zeta \boldsymbol{\mu} + \frac{w_T - \zeta \mathbf{1}' \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}}{\mathbf{1}' \boldsymbol{\Sigma}^{-1} \mathbf{1}} \mathbf{1} \right) \quad (8.77)$$

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$$\boldsymbol{\alpha}(\boldsymbol{\theta}) \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\boldsymbol{\theta}}}{\operatorname{argmax}} \left\{ \mathcal{S}_{\boldsymbol{\theta}}(\boldsymbol{\alpha}) \right\} \qquad (8.76)$$

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$$\boldsymbol{\alpha}(\boldsymbol{\mu},\boldsymbol{\Sigma}) = [\operatorname{diag}(\mathbf{p}_T)]^{-1} \boldsymbol{\Sigma}^{-1} \left(\zeta\boldsymbol{\mu} + \frac{w_T - \zeta \mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}} \mathbf{1} \right) \qquad (8.77)$$

$$\boldsymbol{\alpha}_{\mathsf{ce}}[i_T, e_C] \equiv \underset{\boldsymbol{\alpha} \in \mathcal{C}_{\widehat{\boldsymbol{\theta}}_{\mathsf{ce}}[i_T, e_C]}}{\operatorname{argmax}} \left\{ \mathcal{S}_{\widehat{\boldsymbol{\theta}}_{\mathsf{ce}}[i_T, e_C]}(\boldsymbol{\alpha}) \right\} \qquad (9.13)$$

$$\boldsymbol{\alpha}_{\mathsf{ce}} \equiv [\operatorname{diag}(\mathbf{p}_T)]^{-1} \widehat{\boldsymbol{\Sigma}}_{\mathsf{ce}}^{-1} \left(\zeta\widehat{\boldsymbol{\mu}}_{\mathsf{ce}} + \frac{w_T - \zeta \mathbf{1}'\widehat{\boldsymbol{\Sigma}}_{\mathsf{ce}}^{-1}\widehat{\boldsymbol{\mu}}_{\mathsf{ce}}}{\mathbf{1}'\widehat{\boldsymbol{\Sigma}}_{\mathsf{ce}}^{-1}\mathbf{1}} \mathbf{1} \right) \qquad (9.21)$$

$$\widehat{\boldsymbol{\mu}}_{\mathsf{ce}}(i_T, e_C) = \frac{T_0\boldsymbol{\mu}_0 + T\widehat{\boldsymbol{\mu}}}{T_0 + T} \qquad (9.19)$$

$$\begin{aligned} \widehat{\boldsymbol{\Sigma}}_{\mathsf{ce}}(i_T, e_C) = {} & \frac{1}{\nu_0 + T + N + 1} \left[\nu_0\boldsymbol{\Sigma}_0 + T\widehat{\boldsymbol{\Sigma}} \right. \\ & \left. + \frac{(\boldsymbol{\mu}_0 - \widehat{\boldsymbol{\mu}})(\boldsymbol{\mu}_0 - \widehat{\boldsymbol{\mu}})'}{\frac{1}{T} + \frac{1}{T_0}} \right]. \end{aligned} \qquad (9.20)$$