Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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affine equiv.

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“local” measure	$$\mathrm{Mod}\{X\} \equiv \underset{x\in\mathbb{R}}{\mathrm{argmax}}\{f_X(x)\} \quad (1.30)$$	(square root of) $$\mathrm{MDis}\{X\} \equiv -\left.\frac{1}{\frac{d^2 \ln f_X}{dx^2}}\right\rvert_{x=\mathrm{Mod}\{X\}} \quad (1.38)$$

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"semi-local" measure	$\int_{-\infty}^{\operatorname{Med}\{X\}}f_X(x)\,dx=\frac{1}{2}$ (1.27)	$\operatorname{Ran}\{X\}\equiv Q_X(\overline{p})-Q_X(\underline{p})$ (1.37)

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"global" measure	$\mathrm{E}\{X\} \equiv \int_{-\infty}^{+\infty} x f_X(x)\,dx$ (1.25)	(square root of) $\mathrm{Var}\{X\} \equiv (\mathrm{Sd}\{X\})^2 = \int_{\mathbb{R}} (x-\mathrm{E}\{X\})^2 f_X(x)\,dx.$ (1.43)

raw moments

$$\mathrm{RM}_k^X \equiv \mathrm{E}\left\{X^k\right\} \qquad (1.47)$$

$$\phi_X(\omega) = 1 + (i\omega)\,\mathrm{RM}_1^X + \cdots + \frac{(i\omega)^k}{k!}\,\mathrm{RM}_k^X + \cdots \qquad (T1.32)$$

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central moments

$$\mathrm{CM}_k^X \equiv \mathrm{E}\left\{(X - \mathrm{E}\{X\})^k\right\} \qquad (1.48)$$

$$\mathrm{CM}_k^X = \sum_{j=0}^{k} \frac{k!\,(-1)^{k-j}}{j!\,(k-j)!}\,\mathrm{RM}_j^X \left(\mathrm{RM}_1^X\right)^{k-j} \qquad (T1.38)$$

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notable examples

$$\mathrm{Sk}\{X\} \equiv \frac{\mathrm{CM}_3^X}{(\mathrm{Sd}\{X\})^3} \qquad (1.49)$$

$$\mathrm{Ku}\{X\} \equiv \frac{\mathrm{CM}_4^X}{(\mathrm{Sd}\{X\})^4} \qquad (1.51)$$