# BENCHMARK ALLOCATION Risk and Asset Allocation - Springer - symmys.com

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www.symmys.com

Formulas and figures in this presentation refer to the book Risk and Asset Allocation, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

total-return allocation 
$$\begin{cases} \Psi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} & \text{(6.169)} \\ \\ \mathcal{S}\left(\alpha\right) \approx \widetilde{\mathcal{H}}\left(\mathbb{E}\left\{\Psi_{\alpha}\right\}, \operatorname{Var}\left\{\Psi_{\alpha}\right\}\right) & \text{(6.173)} \end{cases}$$

total-return allocation 
$$\begin{cases} \Psi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} & \text{(6.169)} \\ \mathcal{S}\left(\alpha\right) \approx \widetilde{\mathcal{H}}\left(\mathbb{E}\left\{\Psi_{\alpha}\right\}, \operatorname{Var}\left\{\Psi_{\alpha}\right\}\right) & \text{(6.173)} \end{cases} \qquad \widetilde{\alpha}\left(v\right) = \underset{\substack{\alpha' \mathbf{p}_{T} = w \\ \operatorname{Var}\left\{\Psi_{\alpha}\right\} = v}}{\operatorname{argmax}} \mathbb{E}\left\{\Psi_{\alpha}\right\} & \text{(6.174)} \end{cases}$$

total-return allocation 
$$\begin{cases} \Psi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} & \text{(6.169)} \\ \\ \mathcal{S}\left(\alpha\right) \approx \widetilde{\mathcal{H}}\left(\mathbf{E}\left\{\Psi_{\alpha}\right\}, \operatorname{Var}\left\{\Psi_{\alpha}\right\}\right) & \text{(6.173)} \end{cases}$$

$$\widetilde{\boldsymbol{\alpha}}\left(\boldsymbol{v}\right) = \underset{\boldsymbol{\alpha}'\mathbf{p_{\scriptscriptstyle T}} = \boldsymbol{w}}{\operatorname{argmax}} \operatorname{E}\left\{\boldsymbol{\varPsi_{\alpha}}\right\} \quad (6.174)$$
 
$$\operatorname{Var}\left\{\boldsymbol{\varPsi_{\alpha}}\right\} = \boldsymbol{v}$$

$$\widetilde{\alpha} = \alpha_{MV} + \left[e - \mathbb{E}\left\{\Psi_{\alpha_{MV}}\right\}\right] \frac{\alpha_{SR} - \alpha_{MV}}{\mathbb{E}\left\{\Psi_{\alpha_{SR}}\right\} - \mathbb{E}\left\{\Psi_{\alpha_{MV}}\right\}}$$

$$(6.177) \ w \operatorname{Cov}\left\{\mathbf{P}_{T+\sigma}\right\}^{-1} \mathbb{E}\left\{\mathbf{P}_{T+\sigma}\right\}$$

$$\alpha_{SR}^{(6.177)} = \frac{w \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \operatorname{E} \left\{ \mathbf{P}_{T+\tau} \right\}}{\mathbf{p}_{T}' \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \operatorname{E} \left\{ \mathbf{P}_{T+\tau} \right\}}$$
$$\alpha_{MV}^{(6.176)} = \frac{w \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \mathbf{p}_{T}}{\mathbf{p}_{T}' \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \mathbf{p}_{T}}$$

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$$\widetilde{\boldsymbol{\alpha}}\left(\boldsymbol{v}\right) = \underset{\begin{array}{c} \boldsymbol{\alpha}' \mathbf{p_{\scriptscriptstyle T}} = \boldsymbol{w} \\ \mathrm{Var}\{\boldsymbol{\varPsi_{\scriptscriptstyle \alpha}}\} = \boldsymbol{v} \end{array}}{\operatorname{Var}\{\boldsymbol{\Psi_{\scriptscriptstyle \alpha}}\} = \boldsymbol{v}} \tag{6.174}$$

$$\widetilde{\alpha} = \alpha_{MV} + \left[e - \operatorname{E}\left\{\Psi_{\alpha_{MV}}\right\}\right] \frac{\alpha_{SR} - \alpha_{MV}}{\operatorname{E}\left\{\Psi_{\alpha_{SR}}\right\} - \operatorname{E}\left\{\Psi_{\alpha_{MV}}\right\}}$$

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$$EOP(\alpha) \equiv E\{\Phi_{\alpha}\} \quad (6.178)$$

$$TE(\alpha) \equiv Sd\{\Phi_{\alpha}\}$$
 (6.179)

$$\Phi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} - \gamma \widetilde{\boldsymbol{\beta}}' \mathbf{P}_{T+\tau}$$

$$\gamma \equiv \frac{\alpha' \mathbf{p}_{T}}{\widetilde{\boldsymbol{\beta}}' \mathbf{p}_{T}}$$

$$(6.170)$$

$$\mathcal{S}(\alpha) \approx \widetilde{\mathcal{K}} \left( \text{EOP}(\alpha), \text{TE}^{2}(\alpha) \right)$$

$$(6.181)$$

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total-return allocation

$$\Psi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} \qquad (6.169)$$

$$\mathcal{S}(\alpha) \approx \widetilde{\mathcal{H}}(\mathbb{E}\{\Psi_{\alpha}\}, \operatorname{Var}\{\Psi_{\alpha}\}) \quad (6.173)$$

$$\widetilde{\boldsymbol{\alpha}}\left(\boldsymbol{v}\right) = \underset{\begin{subarray}{c} \boldsymbol{\alpha}' \mathbf{p_{T}} = \boldsymbol{w} \\ \mathrm{Var}\{\boldsymbol{\varPsi_{\alpha}}\} = \boldsymbol{v} \end{subarray}}{\operatorname{Argmax}} \mathbf{E}\left\{\boldsymbol{\varPsi_{\alpha}}\right\} \quad (6.174)$$

$$\widetilde{\alpha} = \alpha_{MV} + \left[e - \mathbb{E}\left\{\Psi_{\alpha_{MV}}\right\}\right] \frac{\alpha_{SR} - \alpha_{MV}}{\mathbb{E}\left\{\Psi_{\alpha_{SR}}\right\} - \mathbb{E}\left\{\Psi_{\alpha_{MV}}\right\}}$$

$$\alpha_{SR}^{(6.177)} = \frac{w \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \operatorname{E} \left\{ \mathbf{P}_{T+\tau} \right\}}{\mathbf{p}_{T}' \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \operatorname{E} \left\{ \mathbf{P}_{T+\tau} \right\}}$$
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EOP 
$$(\alpha) \equiv \mathbb{E} \{ \Phi_{\alpha} \}$$
 (6.178)

$$TE(\alpha) \equiv Sd\{\Phi_{\alpha}\}$$
 (6.179)

$$\widehat{\boldsymbol{\alpha}}(u) = \underset{\boldsymbol{\alpha}' \mathbf{p}_{T} = w}{\operatorname{argmax}} \operatorname{EOP}(\boldsymbol{\alpha}) \quad (6.182)$$

$$\operatorname{TE}^{2}(\alpha) = u$$

$$\Phi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} - \gamma \widetilde{\boldsymbol{\beta}}' \mathbf{P}_{T+\tau}$$

$$\gamma \equiv \frac{\alpha' \mathbf{p}_{T}}{\widetilde{\boldsymbol{\beta}}' \mathbf{p}_{T}}$$

$$\mathcal{S}(\alpha) \approx \widetilde{\mathcal{K}} \left( \text{EOP}(\alpha), \text{TE}^{2}(\alpha) \right)$$

$$(6.171)$$

$$(6.181)$$

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total-return allocation

$$\Psi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} \quad (6.169)$$

$$S(\alpha) \approx \widetilde{\mathcal{H}} \left( \mathbb{E} \left\{ \Psi_{\alpha} \right\}, \operatorname{Var} \left\{ \Psi_{\alpha} \right\} \right) \quad (6.173)$$

$$\widetilde{\boldsymbol{\alpha}}\left(\boldsymbol{v}\right) = \underset{\begin{array}{c} \boldsymbol{\alpha}' \mathbf{p}_{T} = \boldsymbol{w} \\ \mathrm{Var}\{\boldsymbol{\Psi}_{\boldsymbol{\alpha}}\} = \boldsymbol{v} \end{array}}{\operatorname{Argmax}} \mathbf{E}\left\{\boldsymbol{\varPsi}_{\boldsymbol{\alpha}}\right\} \quad (6.174)$$

$$\widetilde{\alpha} = \alpha_{MV} + \left[e - \operatorname{E}\left\{\Psi_{\alpha_{MV}}\right\}\right] \frac{\alpha_{SR} - \alpha_{MV}}{\operatorname{E}\left\{\Psi_{\alpha_{SR}}\right\} - \operatorname{E}\left\{\Psi_{\alpha_{MV}}\right\}}$$

$$\alpha_{SR}^{(6.177)} = \frac{w \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \operatorname{E} \left\{ \mathbf{P}_{T+\tau} \right\}}{\mathbf{p}_{T}' \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \operatorname{E} \left\{ \mathbf{P}_{T+\tau} \right\}}$$
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 $EOP(\alpha) \equiv E\{\Phi_{\alpha}\}$ 

$$TE(\alpha) \equiv Sd\{\Phi_{\alpha}\}$$
 (6.179)

(6.178)

$$\widehat{\boldsymbol{\alpha}}\left(\boldsymbol{u}\right) = \underset{\boldsymbol{\alpha}'\mathbf{p_{T}} = \boldsymbol{w}}{\operatorname{argmax}} \ \operatorname{EOP}\left(\boldsymbol{\alpha}\right) \quad (6.182)$$

$$\operatorname{TE}^{2}(\boldsymbol{\alpha}) = \boldsymbol{u}$$

benchmark allocation

$$\varphi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} - \gamma \beta' \mathbf{P}_{T+\tau} \qquad (6.170)$$

$$\gamma \equiv \frac{\alpha' \mathbf{p}_{T}}{\widetilde{\beta}' \mathbf{p}_{T}} \qquad (6.171)$$

$$\mathcal{S}(\alpha) \approx \widetilde{\mathcal{K}} \left( \text{EOP}(\alpha), \text{TE}^{2}(\alpha) \right) \qquad (6.181)$$

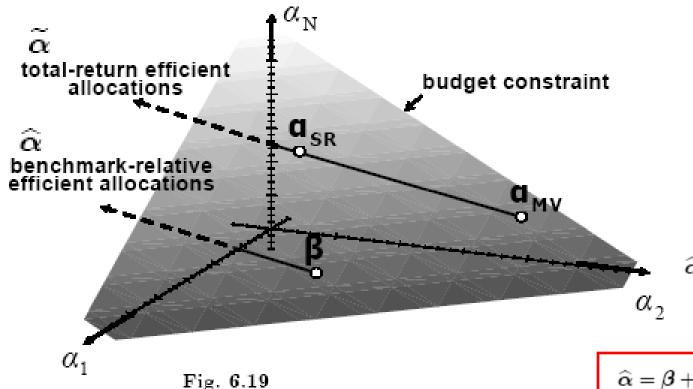
$$\widehat{\alpha} = \beta + [e - E\{\Psi_{\beta}\}] \frac{\alpha_{SR} - \alpha_{MV}}{E\{\Psi_{\alpha_{SR}}\} - E\{\Psi_{\alpha_{MV}}\}}$$
(6.190)

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$$\Psi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} \qquad (6.169) \qquad \qquad \widetilde{\alpha} (v) = \underset{\alpha' \mathbf{p}_{T} = w}{\operatorname{argmax}} \mathbf{E} \{ \Psi_{\alpha} \} \qquad (6.174)$$

$$\Phi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} - \gamma \widetilde{\boldsymbol{\beta}}' \mathbf{P}_{T+\tau} \qquad (6.170)$$

$$\widetilde{\alpha} = \alpha_{MV} + \left[e - \mathbb{E}\left\{\Psi_{\alpha_{MV}}\right\}\right] \frac{\alpha_{SR} - \alpha_{MV}}{\mathbb{E}\left\{\Psi_{\alpha_{SR}}\right\} - \mathbb{E}\left\{\Psi_{\alpha_{MV}}\right\}}$$
(6.175)



$$\alpha_{SR}^{(6.177)} = \frac{w \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \operatorname{E} \left\{ \mathbf{P}_{T+\tau} \right\}}{\mathbf{p}_{T}^{\prime} \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \operatorname{E} \left\{ \mathbf{P}_{T+\tau} \right\}}$$
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 $EOP(\alpha) \equiv E\{\Phi_{\alpha}\}$  (6.178)

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$$\widehat{\boldsymbol{\alpha}}(u) = \underset{\boldsymbol{\alpha}' \mathbf{p}_{T} = w}{\operatorname{argmax}} \operatorname{EOP}(\boldsymbol{\alpha}) \quad (6.182)$$

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$$\Psi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} \quad (6.169) \qquad \qquad \widetilde{\alpha} \left( v \right) = \underset{\mathbf{\alpha}' \mathbf{P}_{T} = w}{\operatorname{argmax}} \mathbf{E} \left\{ \Psi_{\alpha} \right\} \quad (6.174)$$

$$\Phi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} - \gamma \widetilde{\boldsymbol{\beta}}' \mathbf{P}_{T+\tau} \quad (6.170)$$

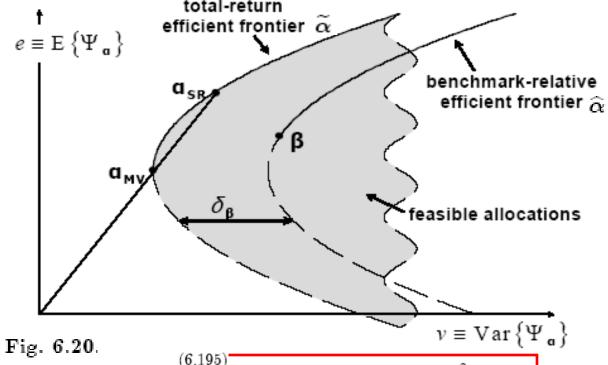
$$\begin{array}{c} (6.193) \\ \widetilde{\alpha} : \quad v = \frac{A}{D} e^{2} - \frac{2wB}{D} e + \frac{w^{2}C}{D} \\ \end{array}$$

$$\begin{array}{c} \widetilde{\alpha} = \alpha_{MV} + \left[ e - \mathbf{E} \left\{ \Psi_{\alpha_{MV}} \right\} \right] \frac{\alpha_{SR} - \alpha_{MV}}{\mathbf{E} \left\{ \Psi_{\alpha_{SR}} \right\} - \mathbf{E} \left\{ \Psi_{\alpha_{MV}} \right\} \\ \end{array}$$

$$\begin{array}{c} (6.175) \\ \mathbf{E} \left\{ \Psi_{\alpha_{SR}} \right\} - \mathbf{E} \left\{ \Psi_{\alpha_{MV}} \right\} \\ \end{array}$$

$$\begin{array}{c} (6.177) \\ \alpha_{SR} = \frac{w \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \mathbf{E} \left\{ \mathbf{P}_{T+\tau} \right\}}{\mathbf{p}_{T}' \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \mathbf{E} \left\{ \mathbf{P}_{T+\tau} \right\}} \\ \end{array}$$

$$\begin{array}{c} (6.176) \\ \alpha_{MV} \equiv \frac{w \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \mathbf{P}_{T}}{\mathbf{P}_{T}} \\ \end{array}$$



$$\alpha_{SR}^{(6.177)} = \frac{w \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \operatorname{E} \left\{ \mathbf{P}_{T+\tau} \right\}}{\mathbf{p}_{T}' \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \operatorname{E} \left\{ \mathbf{P}_{T+\tau} \right\}}$$
$$\alpha_{MV}^{(6.176)} = \frac{w \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \mathbf{p}_{T}}{\mathbf{p}_{T}' \operatorname{Cov} \left\{ \mathbf{P}_{T+\tau} \right\}^{-1} \mathbf{p}_{T}}$$

 $EOP(\alpha) \equiv E\{\Phi_{\alpha}\}$ (6.178)

$$TE(\alpha) \equiv Sd\{\Phi_{\alpha}\}$$
 (6.179)

$$\widehat{\boldsymbol{\alpha}}(u) = \underset{\boldsymbol{\alpha}' \mathbf{p}_{\mathcal{T}} = u}{\operatorname{argmax}} \operatorname{EOP}(\boldsymbol{\alpha})$$
 (6.182)  
 $\mathbf{TE}^{2}(\boldsymbol{\alpha}) = u$ 

$$\widehat{\boldsymbol{\alpha}}: \quad v = \frac{A}{D}e^2 - \frac{2wB}{D}e + \frac{w^2C}{D} + \delta_{\boldsymbol{\beta}} \quad \longleftarrow \quad \widehat{\boldsymbol{\alpha}} = \boldsymbol{\beta} + \left[e - \operatorname{E}\left\{\Psi_{\boldsymbol{\beta}}\right\}\right] \frac{\boldsymbol{\alpha}_{SR} - \boldsymbol{\alpha}_{MV}}{\operatorname{E}\left\{\Psi_{\boldsymbol{\alpha}_{SR}}\right\} - \operatorname{E}\left\{\Psi_{\boldsymbol{\alpha}_{MV}}\right\}}$$
(6.190)

Fig. 6.21

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$$\frac{\Psi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau}}{\Phi_{\alpha} \equiv \alpha' \mathbf{P}_{T+\tau} - \gamma \widetilde{\boldsymbol{\beta}}' \mathbf{P}_{T+\tau}} (6.169) \qquad \widetilde{\boldsymbol{\alpha}} (v) = \underset{\mathbf{\alpha}' \mathbf{p}_{T} \equiv w}{\operatorname{argmax}} \mathbf{E} \{ \Psi_{\alpha} \} (6.174) \\ \underset{\mathbf{\alpha}' = \pi}{\text{Var}} \{ \Psi_{\alpha} \} = v$$

$$\frac{(6.200)}{\widetilde{\boldsymbol{\alpha}} : u = \frac{A}{D} p^{2} + \delta \boldsymbol{\beta}} \longleftarrow \widetilde{\boldsymbol{\alpha}} = \alpha_{MV} + [e - \mathbf{E} \{ \Psi_{\alpha_{MV}} \}] \frac{\alpha_{SR} - \alpha_{MV}}{\mathbf{E} \{ \Psi_{\alpha_{SR}} \} - \mathbf{E} \{ \Psi_{\alpha_{MV}} \}}$$

$$p \equiv \mathbf{E} \{ \Phi_{\alpha} \} = \mathbf{EOP} (\mathbf{\alpha}) \qquad \underset{\mathbf{efficient frontier}}{\text{benchmark-relative efficient frontier}} \widetilde{\boldsymbol{\alpha}} \qquad \underset{\mathbf{\alpha}'' = \pi}{\text{constance}} \frac{w \operatorname{Cov} \{ \mathbf{P}_{T+\tau} \}^{-1} \mathbf{E} \{ \mathbf{P}_{T+\tau} \}}{\mathbf{p}_{T}' \operatorname{Cov} \{ \mathbf{P}_{T+\tau} \}^{-1} \mathbf{P}_{T}}$$

$$\alpha_{MV} \equiv \frac{(6.176)}{\mathbf{p}_{T}' \operatorname{Cov} \{ \mathbf{P}_{T+\tau} \}^{-1} \mathbf{P}_{T}}{\mathbf{p}_{T}' \operatorname{Cov} \{ \mathbf{P}_{T+\tau} \}^{-1} \mathbf{P}_{T}}$$

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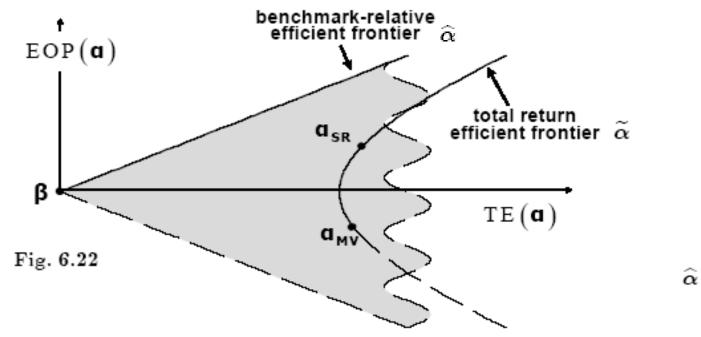
$$\widehat{\boldsymbol{\alpha}}: \quad u = \frac{A}{D}p^2, \quad \longleftarrow \quad \widehat{\boldsymbol{\alpha}} = \boldsymbol{\beta} + \left[e - \operatorname{E}\left\{\boldsymbol{\varPsi}_{\boldsymbol{\beta}}\right\}\right] \frac{\boldsymbol{\alpha}_{SR} - \boldsymbol{\alpha}_{MV}}{\operatorname{E}\left\{\boldsymbol{\varPsi}_{\boldsymbol{\alpha}_{SR}}\right\} - \operatorname{E}\left\{\boldsymbol{\varPsi}_{\boldsymbol{\alpha}_{MV}}\right\}}$$
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 $\alpha' \mathbf{p}_T = w$  $TE^{2}(\alpha)=u$ 

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$$\overset{(6.200)}{\tilde{\alpha}}: \quad u = \frac{A}{D}p^2 + \delta \beta. \quad \longleftarrow \quad \tilde{\alpha} = \alpha_{MV} + \left[e - \operatorname{E}\left\{\varPsi_{\alpha_{MV}}\right\}\right] \frac{\alpha_{SR} - \alpha_{MV}}{\operatorname{E}\left\{\varPsi_{\alpha_{SR}}\right\} - \operatorname{E}\left\{\varPsi_{\alpha_{MV}}\right\}}$$



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$$EOP(\alpha) \equiv E\{\Phi_{\alpha}\}$$
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$$\widehat{\boldsymbol{\alpha}}(u) = \underset{\boldsymbol{\alpha}' \mathbf{p}_{T} = w}{\operatorname{argmax}} \operatorname{EOP}(\boldsymbol{\alpha}) \quad (6.182)$$

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$$(6.201) \longrightarrow (6.199) \longrightarrow (6.1$$