#### Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book Risk and Asset Allocation, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

invariants

 $\mathbf{X}_t$ 

price at horizon

$$P = g\left(\mathbf{X}\right) \quad (3.101)$$

invariants  $\mathbf{X}_t$  price at horizon  $P = g\left(\mathbf{X}\right)$  (3.101)

$$P_{T+1} = ?$$

stocks, FX

invariants

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compounded return

$$X_t \equiv \ln\left(\frac{P_t}{P_{t-1}}\right)$$

$$P_{T+1} = P_T e^{X_{T+1}}.$$

stocks, FX

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stocks, FX

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$$Z_{T+1}^{(E)} = ?$$

bonds

time of maturity

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stocks, FX

change in yield to maturity

$$X_t^{(\upsilon)} \equiv Y_t^{(\upsilon)} - Y_{t-1}^{(\upsilon)}$$
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bonds

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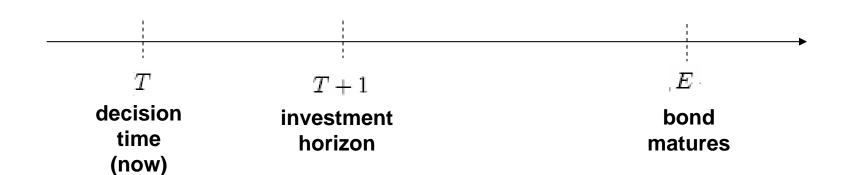
$$P_{T+1} = P_T e^{X_{T+1}}$$

stocks, FX

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bonds



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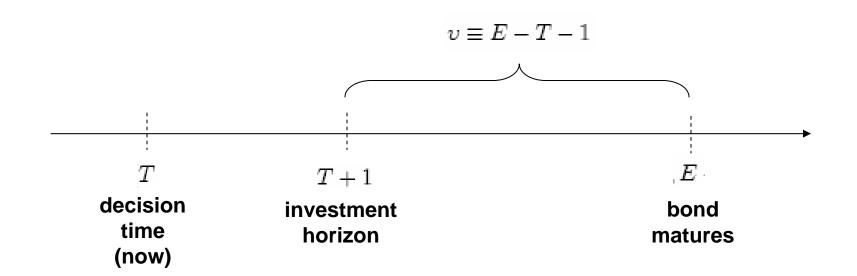
stocks, FX

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$$X_t^{(v)} \equiv Y_t^{(v)} - Y_{t-1}^{(v)}$$

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$$C_{T+1}\left(K,E\right)=C_{BS}\left(P_{T+1},\sigma_{T+1}\left(K,E\right);K,E\right) \qquad \text{derivatives}$$

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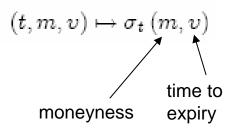
bonds

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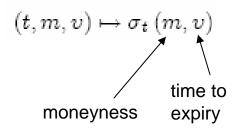
$$C_{T+1}\left(K,E\right)=C_{BS}\left(P_{T+1},\underbrace{\sigma_{T+1}\left(K,E\right)};K,E\right) \qquad \text{derivatives}$$

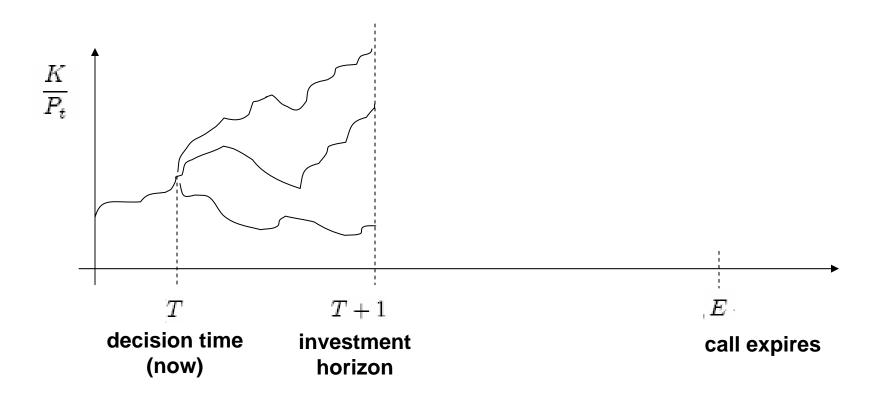
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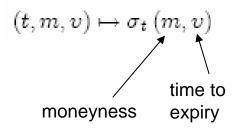
#### invariant coordinates

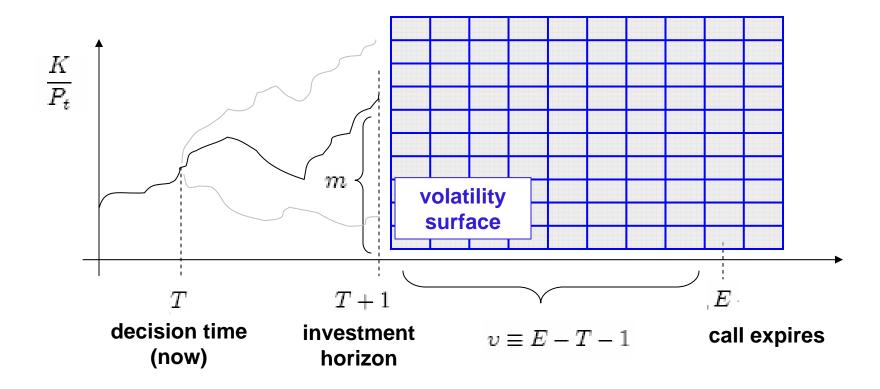


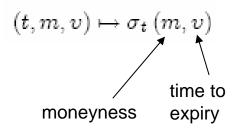


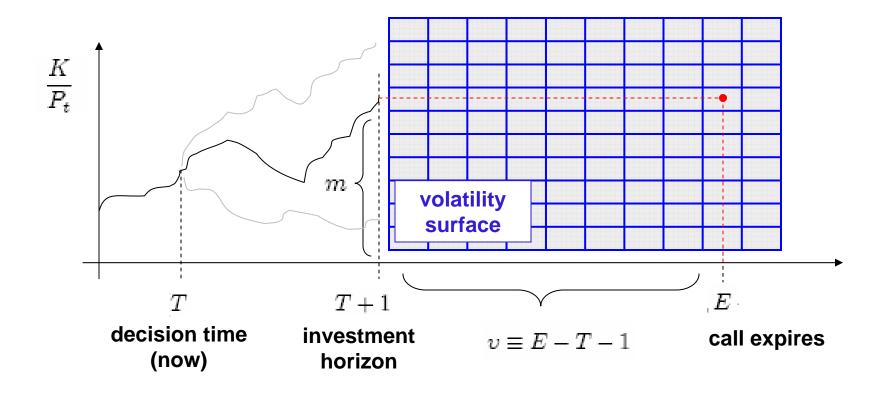












$$(t,m,v)\mapsto \sigma_t\left(m,v
ight)$$
 time to moneyness expiry

$$C_{T+1}\left(K,E\right)=C_{BS}\left(P_{T+1},\sigma_{T+1}\left(\frac{K}{P_{T+1}},E-\left(T+1\right)\right);K,E\right)$$

#### PRICING - Risk and Asset Allocation - Springer - symmys.com volatility vector $\frac{K}{P_t}$ $\Leftrightarrow$ mvolatility surface ETT+1decision time investment call expires $v \equiv E - T - 1$ (now) horizon

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change in log-implied volatility vector

$$X_t \equiv \ln \sigma_t - \ln \sigma_{t-1}$$

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horizon

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(now)

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			Analytical	Scenarios	
Full evaluation	$P=g\left( \mathbf{X}\right)$	(3.101)	only simple securities (e.g. stocks)	~ always, but costly	

		Analytical	Scenarios
Taylor 1	$P = g(\mathbf{m}) + (\mathbf{X} - \mathbf{m})' \left. \partial_{\mathbf{x}} g \right _{\mathbf{x} = \mathbf{m}} $ (3.108)	~ any analytical distribution for <b>X</b>	~ always
Full evaluation	$P=g\left( \mathbf{X} ight)$ (3.101)	only simple securities (e.g. stocks)	~ always, but costly

		Analytical	Scenarios
	$P=g\left(\mathbf{m} ight)+\left(\mathbf{X}-\mathbf{m} ight)'\left.\partial_{\mathbf{x}}g\right _{\mathbf{x}=\mathbf{m}}$ (3.108) <u>e</u> : carry + duration theta + delta + vega	~ any analytical distribution for <b>X</b>	~ always
Full evaluation	on $P=g\left(\mathbf{X} ight)$ (3.101)	only simple securities (e.g. stocks)	~ always, but costly

		Analytical	Scenarios
Taylor 1  fixed-income: canderivatives: the	$P=g\left(\mathbf{m} ight)+\left(\mathbf{X}-\mathbf{m} ight)'\left.\partial_{\mathbf{x}}g\right _{\mathbf{x}=\mathbf{m}}$ (3.108) arry + duration ta + delta + vega	~ any analytical distribution for <b>X</b>	~ always
Taylor 2	$P = g(\mathbf{m}) + (\mathbf{X} - \mathbf{m})' \partial_{\mathbf{x}} g _{\mathbf{x} = \mathbf{m}} $ (3.108) $+ \frac{1}{2} (\mathbf{X} - \mathbf{m})' \partial_{\mathbf{x}\mathbf{x}}^2 g _{\mathbf{x} = \mathbf{m}} (\mathbf{X} - \mathbf{m})$	<b>X</b> normal ( <b>X</b> Student)	~ always
Full evaluation	$P = g\left(\mathbf{X}\right)$ (3.101)	only simple securities (e.g. stocks)	~ always, but costly

		Analytical	Scenarios
Taylor 1  fixed-income: car derivatives: theta	-	~ any analytical distribution for <b>X</b>	~ always
<u>fixed-income</u> : car	$P = g\left(\mathbf{m}\right) + \left(\mathbf{X} - \mathbf{m}\right)' \left.\partial_{\mathbf{x}} g\right _{\mathbf{x} = \mathbf{m}} \qquad (3.108)$ $+ \frac{1}{2} \left(\mathbf{X} - \mathbf{m}\right)' \left.\partial_{\mathbf{x}\mathbf{x}}^2 g\right _{\mathbf{x} = \mathbf{m}} \left(\mathbf{X} - \mathbf{m}\right)$ $+ \mathbf{r}\mathbf{y} + \mathbf{d}\mathbf{u}\mathbf{r}\mathbf{a}\mathbf{t}\mathbf{i}\mathbf{o}\mathbf{n} + \mathbf{c}\mathbf{o}\mathbf{n}\mathbf{v}\mathbf{e}\mathbf{x}\mathbf{i}\mathbf{t}\mathbf{y}$ $+ \mathbf{d}\mathbf{e}\mathbf{l}\mathbf{t}\mathbf{a} + \mathbf{v}\mathbf{e}\mathbf{g}\mathbf{a} + \mathbf{g}\mathbf{a}\mathbf{m}\mathbf{m}\mathbf{a} \left(\mathbf{+}\mathbf{v}\mathbf{a}\mathbf{n}\mathbf{n}\mathbf{a}, \mathbf{v}\mathbf{o}\mathbf{l}\mathbf{g}\mathbf{a}\right)$	<b>X</b> normal ( <b>X</b> Student)	~ always
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	Analytical	Scenarios
Taylor 1 $P = g(\mathbf{m}) + (\mathbf{X} - \mathbf{m})' \left. \partial_{\mathbf{x}} g \right _{\mathbf{x} = \mathbf{m}}$ (3.108) <u>fixed-income</u> : carry + duration <u>derivatives</u> : theta + delta + vega	~ any analytical distribution for <b>X</b>	~ always
Taylor 2 $\begin{aligned} P &= g\left(\mathbf{m}\right) + \left(\mathbf{X} - \mathbf{m}\right)' \left.\partial_{\mathbf{x}} g\right _{\mathbf{x} = \mathbf{m}} \\ &+ \frac{1}{2} \left(\mathbf{X} - \mathbf{m}\right)' \left.\partial_{\mathbf{x} \mathbf{x}}^2 g\right _{\mathbf{x} = \mathbf{m}} \left(\mathbf{X} - \mathbf{m}\right) \\ &\frac{\text{fixed-income: carry + duration + convexity}}{\text{derivatives: theta + delta + vega + gamma (+ vanna, volga)} \end{aligned}$	X normal (X Student)	~ always
Intra/Extrapolation	N/A	~ always, but <b>curse of</b> <b>dimensionality</b>
Full evaluation $P=g\left(\mathbf{X} ight)$ (3.101)	only simple securities (e.g. stocks)	~ always, but costly

	Analytical	Scenarios
Taylor 1 $P = g(\mathbf{m}) + (\mathbf{X} - \mathbf{m})' \left. \partial_{\mathbf{x}} g \right _{\mathbf{x} = \mathbf{m}}$ (3.108) <u>fixed-income</u> : carry + duration <u>derivatives</u> : theta + delta + vega	~ any analytical distribution for <b>X</b>	~ always
Taylor 2 $\begin{aligned} P &= g\left(\mathbf{m}\right) + \left(\mathbf{X} - \mathbf{m}\right)' \left. \partial_{\mathbf{x}} g \right _{\mathbf{x} = \mathbf{m}} \\ &+ \frac{1}{2} \left(\mathbf{X} - \mathbf{m}\right)' \left. \partial_{\mathbf{x} \mathbf{x}}^2 g \right _{\mathbf{x} = \mathbf{m}} \left(\mathbf{X} - \mathbf{m}\right) \\ &\frac{\text{fixed-income: carry + duration + convexity}}{\text{derivatives: theta + delta + vega + gamma (+ vanna, volga)} \end{aligned}$	X normal (X Student)	~ always
Intra/Extrapolation + Taylor (stress matrices)	N/A	~ always
Intra/Extrapolation	N/A	~ always, but curse of dimensionality
Full evaluation $P=g\left(\mathbf{X} ight)$ (3.101)	only simple securities (e.g. stocks)	~ always, but costly

	Analytical	( <mark>Historical</mark> ) Scenarios
Taylor 1 $P = g(\mathbf{m}) + (\mathbf{X} - \mathbf{m})' \left. \partial_{\mathbf{x}} g \right _{\mathbf{x} = \mathbf{m}}$ (3.108) <u>fixed-income</u> : carry + duration <u>derivatives</u> : theta + delta + vega	~ any analytical distribution for <b>X</b>	~ always
Taylor 2 $\begin{aligned} P &= g\left(\mathbf{m}\right) + \left(\mathbf{X} - \mathbf{m}\right)' \left.\partial_{\mathbf{x}} g\right _{\mathbf{x} = \mathbf{m}} \\ &+ \frac{1}{2} \left(\mathbf{X} - \mathbf{m}\right)' \left.\partial_{\mathbf{x} \mathbf{x}}^2 g\right _{\mathbf{x} = \mathbf{m}} \left(\mathbf{X} - \mathbf{m}\right) \\ &\frac{\text{fixed-income}}{\text{derivatives}} \text{: theta + delta + vega + gamma (+ vanna, volga)} \end{aligned} $	<b>X</b> normal ( <b>X</b> Student)	~ always
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