Review Session 6

July 18, 2015

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8.1.4 Mean-variance for derivatives

Consider a market of at-the-money call options on the underlyings whose daily time series are provided in the file DB (30, 91 and 182 are the time to expiry). Assume that the investment horizon is two days.

Fit a normal distribution to the invariants, namely the log-changes in the underlying in the file DB and the log-changes in the respective at-the-money implied volatilities in the file DB.

Project this distribution analytically to the horizon.

Generate simulations for the sources of risk, namely underlying prices and implied volatilities at the horizon.

Price the above simulations through the full Black-Scholes formula, assuming no skewness correction for the implied volatilities and a constant risk-free rate at 4%.

Compute the distribution of the linear returns, as represented by the simulations: the current prices of the options can be obtained similarly to the prices at the horizon by assuming that the current values of underlying and implied volatilities are the last observations in the database.

Compute numerically the mean-variance inputs.

Compute the mean-variance efficient frontier in terms of relative weights, assuming the standard long-only and full investment constraints.

Plot the efficient frontier in the plane of weights and standard deviation.

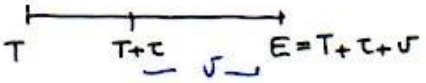
Hint. Compare with Exercise 5.6: here we are making unrealistic dimension reduction assumptions on the dynamics of the implied volatility surface.

0) MARKET : At the money call options

$\tau + \upsilon =$ 30, 91, 182 days

(TIME TO EXPIRY)

TODAY



P1) INVARIANTS $\begin{pmatrix} C_{\tilde{\tau}} \\ \Delta_{\tilde{\tau}} \ln \sigma \end{pmatrix} \sim N(\mu, \Sigma)$

P2) ESTIMATION $\hat{\mu}, \hat{\Sigma}$ (sample mean/cov for simplicity)

P3) PROJECTION

$\tilde{\tau} = 1$ day

$\tau = 2$ days $\qquad \frac{\tau}{\tilde{\tau}} = 2 \qquad \begin{pmatrix} C_{\tau} \\ \Delta_{\tau} \ln \sigma \end{pmatrix} \sim N\left(2\hat{\mu}, 2\hat{\Sigma}\right)$

P4) PRICING

Generate scenarios for $P_{T+\tau}$ (underlying) and $\sigma_{T+\tau}$ (impl vol surface) and price the call options using BS formula

$J = 1 \dots 10000$ (scenarios)

$$P^{(J)}_{T+\tau} = P_T \cdot \exp\left(C_\tau^{(J)}\right)$$

$$\sigma^{(J)}_{T+\tau} = \sigma_T \cdot \exp\left(\Delta_\tau \ln \sigma^{(J)}\right)$$

$$Call^{(J)}_{T+\tau} = BS\left(\underbrace{\frac{P^{(J)}_{T+\tau}}{P_T}}_{\text{STRIKE (at the money)}},\ \upsilon,\ 4\%,\ \underbrace{\sigma^{(J)}_{T+\tau}\left(\frac{P_{T+\tau}}{P_T}, \upsilon\right)}_{\text{n.-lin. interpolation}}\right)$$

P5) AGGREGATION

Linear rets $\quad L^{(J)}_{T\to T+\tau} = \dfrac{CALL^{(J)}_{T+\tau}}{CALL_T} - 1$

$\hookrightarrow BS\left(\frac{P_T}{P_T},\ \upsilon+\tau,\ 4\%,\ \sigma_T(1, \upsilon+\tau)\right)$

$\Psi_w = w' \cdot L_{T\to T+\tau}$

P8) MV optimization

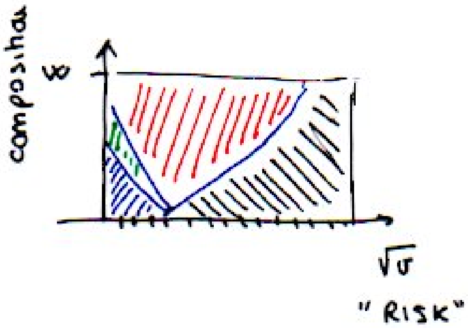
inputs $E[L]$

$Cov[L]$

$$\begin{cases} w(e) = \underset{w}{\operatorname{argmin}} \; w' \, Cov[L] \, w \\ w' E[L] = e \\ w \geq 0 \\ w' 1 = 1 \end{cases} \longrightarrow \text{LIN CONSTRAINTS} \Downarrow \text{DUAL}$$

QUADPROG

Plot Frontier



10.2.1 Robust mean-variance for derivatives

Consider the market of call options in Exercise 8.1.4.

Consider a small ellipsoidal neighborhood of the expected linear returns, as determined by reasonable $\mathbf{T}$, $\mathbf{m}$, and q^2, see (9.118) in Meucci (2005), and assume no uncertainty in the estimation of the covariance of the linear returns, see (9.119) in Meucci (2005), and set up the robust optimization problem (9.117) in the form of SOCP, see comments after (9.130).

Compute the robust mean-variance efficient frontier in terms of relative weights, assuming the standard long-only and full investment constraints. Then plot the efficient frontier in the plane of weights and standard deviation.

Hint. Use the CVX package, located at www.stanford.edu/~boyd/cvx/

Same market as 8.1.4

- uncertainty set for μ : $\Theta = \{ \mu : (\mu - m)' T^{-1} (\mu - m) \leq q^2 \}$
- no uncertainty for Σ

symmetric pos def

$q^2 = Q_{\chi^2_N}(p)$

confidence $\in (0,1)$

ROBUST OPTIMIZATION PROBLEM :

$$\underset{\alpha \in \mathcal{C}}{\operatorname{Max}} \left\{ \underset{\mu \in \Theta}{\operatorname{Min}} \; \alpha' \mu \right\}$$

↳ worst case scenario

Consider the minimization problem

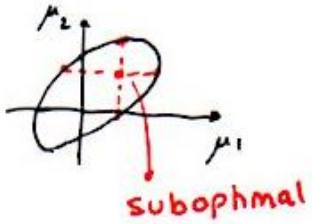
$$\min_{\mu \in \Theta} \alpha' \mu$$

→ The optimizer is on the boundary of Θ ⇒ constraint:

$$(\mu^* - m)' T^{-1} (\mu^* - m) = q^2$$

FIXED

$\alpha_1 \underline{\mu_1} + \alpha_2 \underline{\mu_2}$



Lagrangian:

$$L(\mu, \lambda) = \alpha'\mu - \lambda\left((\mu - m)' T^{-1} (\mu - m) - q^2 \right)$$

1^{st} order criterion

$$\begin{cases} \dfrac{\partial L}{\partial \mu} = \alpha - 2\lambda T^{-1}(\mu - m) \overset{!}{=} 0 & \text{①} \\[2ex] \dfrac{\partial L}{\partial \lambda} = -(\mu - m)' T^{-1} (\mu - m) + q^2 \overset{!}{=} 0 & \text{②} \end{cases}$$

From ① : $\mu - m = \dfrac{T\alpha}{2\lambda}$

↓ plug into ②

$$-\frac{1}{4\lambda^2} \alpha' \cancel{T}\cancel{T^{-1}} T \alpha + q^2 = 0 \quad \rightarrow \quad \lambda = \frac{1}{2q}\sqrt{\alpha' T \alpha}$$

$$\mu^* = m + \frac{T\alpha}{2\lambda} = m + \frac{q}{\sqrt{\alpha' T \alpha}} T\alpha$$

- The MV problem: $\max_{\alpha \in \mathcal{C}} \left\{ \min_{\mu \in \Theta} \alpha' \mu \right\}$

becomes $\max_{\alpha \in \mathcal{C}} \alpha' \mu^* = \alpha' m + \frac{q}{\sqrt{\alpha' T \alpha}} \cdot \alpha' T \alpha$

$\longrightarrow \max_{\alpha \in \mathcal{C}} \left(\alpha' m + \sqrt{\alpha' T \alpha} \cdot q \right)$

SOCP (www. 9.6)

10.3.2 Beyond Black-Litterman

Consider the market prior

$$X \stackrel{d}{=} BZ_{-1} + (1 - B) Z_1 \tag{559}$$

where

$$Z_{-1} \sim \mathrm{N}(-1, 1), \quad Z_1 \sim \mathrm{N}(1, 1); \tag{560}$$

B is Bernoulli with $\mathbb{P}\{B = 1\} \equiv 1/2$; and all the variables are independent.

Compute and plot the posterior market distribution that is the most consistent with the view

$$\widetilde{\mathbb{E}}\{X\} \equiv 0.5. \tag{561}$$

Hint. Use the package "Entropy Pooling: Fully Flexible Views and Stress-testing" available at www.mathworks.com/matlabcentral/fileexchange/21307.

→ PRIOR:

$X \stackrel{d}{=} BZ_{-1} + (1-B) Z_1$

$E[X] = 0.5 \cdot (-1) + 0.5 \cdot (1) = 0$

$$\left.\begin{array}{l} Z_{-1} \sim N(-1, 1) \\ Z_1 \sim N(1, 1) \\ B \sim \text{Bernoulli}(0.5) \end{array}\right\} \text{indep.}$$

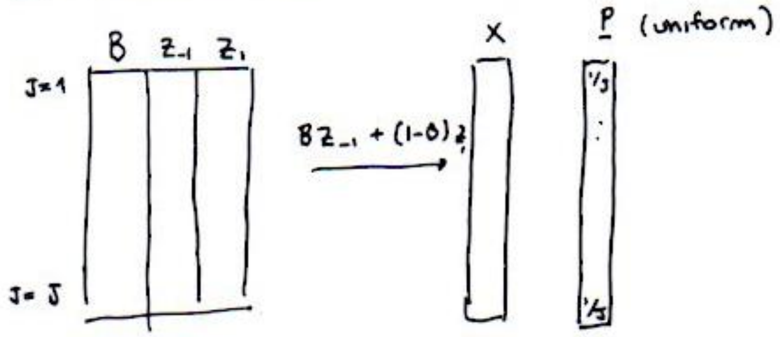
→ view: $E[X] = 0.5$

POSTERIOR MKT DISTRIBUTION CONSISTENT WITH THE VIEW AND "AS CLOSE AS POSSIBLE TO THE PRIOR"

↓

Entropy pooling with Flexible probabilities

① GENERATE SCENARIOS FOR X



② find $\bar{p} = \{p_1, \dots, p_J\}$ to match the view $\sum_{j=1}^{J} x_j p_j = \frac{1}{2}$

→ minimize relative entropy between uniform and $\bar{p}$

$$\begin{cases} \min\limits_{\bar{p}} \sum_{j=1}^{J} p_j \left(\ln p_j - \ln \frac{1}{J} \right) \\ \text{s.t. } p_j \geq 0, \quad \sum p_j = 1, \quad \sum x_j p_j = 0.5 \end{cases}$$

→ Entropy Prog (linear constraints)