Ever see crazy binary numbers and wonder what they meant? Ever see numbers with letters mixed in and wonder what is going on? You'll find out all of this and more in this article. Hexadecimal doesn't have to be scary.

Introduction: What is a Number System?

You probably already know what a number system is - ever hear of binary numbers or hexadecimal numbers? Simply put, a number system is a way to represent numbers. We are used to using the base-10 number system, which is also called decimal. Other common number systems include base-16 (hexadecimal), base-8 (octal), and base-2 (binary).

Looking at Base-10

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11... You've counted in base-10 all of your life. Quick, what is 7+5? If you answered 12, you are thinking in base-10. Let's take a closer look at what you've been doing all these years without ever thinking about it.

Let's take a quick look at counting. First, you go through all the digits: 0, 1, 2... Once you hit 9, you have no more digits to represent the next number. So, you change it back to 0, and add 1 to the tens digit, giving you 10. The process repeats over and over, and eventually you get to 99, where you can't make any larger numbers with two digits, so you add another, giving you 100.

Although that's all very basic, you shouldn't overlook what is going on. The right-most digit represents the number of ones, the next digit represents the number of tens, the next the number of hundreds, etc.

Let's look at what is going on mathematically, using 2347 as an example.

- As you saw, there are 2 groups of a thousand. Not coincidentally, 1000 = 10*10*10 which can also be written as 10^3.
- There are 3 groups of a hundred. Again, not coincidentally, 100 = 10*10 or 10^2.
- There are 4 groups of ten, and, 10 = 10^1.
- Finally, there are 7 groups of one, and 1 = 10^0. (That may seem strange, but *any* number to the power of 0 equals 1, by definition.)

This is essentially the definition of base-10. To get a value of a number in base-10, we simply follow that pattern. Here are a few more examples:

- $892 = 8*10^2+9*10^1+2*10^0$
- $1147 = 1*10^3+1*10^2+4*10^1+7*10^0$
- $53 = 5*10^1+3*10^0$

Base-8

On to base-8, also called octal. Base-8 means just what is sounds like: the system is based on the number eight (as opposed to ten). Remember how in base-10 we had ten digits? Now, in base-8, we are limited to only eight digits: 0, 1, 2, 3, 4, 5, 6, and 7. There's no such thing as 8 or 9.

Admittedly, this all seems a little silly. We all know what value a base-10 number is because we always use base-10, and it comes naturally to us. As we'll see soon, though, if we understand the patterns in the background of base-10, we can understand other bases better.

Talking about numbers written in multiple bases can be confusing. For example, as we have just seen, 10 in base-8 is not the same as 10 in base-10. So, from this point on, I'll use a standard notation where a subscript denotes the base of numbers if needed. For example, our base-8 version of 10 now looks like 10_8.

Converting From Base-8 to Base-10

Let's look at a wordier example now. John offers to give you 47_8 cookies, and Jane offers to give you 43_{10} cookies. Whose offer do you take? Let's figure out its base-10 value so we can make the best decision!

The four in 47_8 represents the number of groups of eight. This makes sense - we are in base-8. So, in total, we have four groups of eight and seven groups of one. If we add these all up, we get $4*8 + 7*1 = 39_{10}$. So, 47_8 cookies is the exact same as 39_{10} cookies. Jane's offer seems like the best one now!

We'll look at 523_8. There are five groups of 8^2, two groups of 8^1 and three groups of 8^0 (remember, $8^0=1$). If we add these all up,

$$5*8^2 + 2*8^1 + 3*8^0 = 5*64+2*8+3 = 339$$

So we get 339_{10} which is our final answer. Here are a couple more examples:

- $111_8 = 1*8^2+1*8^1+1*8^0 = 64+8+1 = 73_{10}$
- $43_8 = 4*8^1+3*8^0 = 32+3 = 35_{10}$
- $6123_8 = 6*8^3+1*8^2+2*8^1+3*8^0 = 3072+64+16+3 = 3155_{10}$

Converting from Base-10 to Base-8

Converting from base-10 to base-8 is a little trickier, but still straightforward. We basically have to reverse the process from above. Let's start with an example: 150_{10}.

We first find the largest power of 8 that is smaller than our number. Here, this is 8^2 or 64 (8^3 is 512). We count how many groups of 64 we can take from 150. This is 2, so the first digit in our base-8 number is 2. We have now accounted for 128 out of 150, so we have 22 left over.

The largest power of 8 that is smaller than 22 is 8^1 (that is, 8). How many groups of 8 can we take from 22? Two groups again, and thus our second digit is 2.

Finally, we are left with 6, and can obviously take 6 groups of one from this, our final digit. We end up with 226_8.

In fact, we can make this process a touch clearer with math. Here are the steps:

1. $150/8^2 = 2$ remainder 22
2. $22/8^1 = 2$ remainder 6
3. $6/8^0 = 6$

Our final answer is then all of our non-remainder digits, or 226. Notice that we still start by dividing by the highest power of 8 that is less that our number.

Base-16

Base-16 is also called hexadecimal. It's commonly used in computer programming, so it's very important to understand. Let's start with counting in hexadecimal to make sure we can apply what we've learned about other bases so far.

Since we are working with base-16, we have 16 digits. So, we have 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ... and yikes! We've run out of digits, but we still need six more. Perhaps we could use something like a circled 10?

The truth is, we could, but this would be a pain to type. Instead, we simply use letters of the alphabet, starting with A and continuing to F. Here's a table with all the digits of base-16:

Base 16 Digit	Value
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
A	10
B	11
C	12
D	13
E	14
F	15

Other than these extra digits, hexadecimal is just like any other base. For example, let's convert $3D_{16}$ to base-10. Following our previous rules, we have: $3D_{16} = 3*16^1 + 13*16^0 = 48 + 13 = 61$. So $3D_{16}$ is equal to 61_{10}. Notice how we use D's value of 13 in our calculation.

We can convert from base-10 to base-16 similar to the way we did with base-8. Let's convert 696_{10} to base-16. First, we find the largest power of 16 that is less than 696_{10}. This is 16^2, or 296. Then:

1. $696/16^2 = 2$ remainder 184
2. $184/16^1 = 11$ remainder 8
3. $8/16^1 = 8$ remainder 0

We have to replace 11 with its digit representation B, and we get $2B8_{16}$.

Binary! (Base-2)

On to the famous base-2, also called binary. While everyone knows binary is made up of 0s and 1s, it is important to understand that it is no different mathematically than any other base.

There's an old joke that goes like this:
"There are only 10 types of people in the world: those who understand binary and those who don't."

Can you figure out what it means?

Let's try a few conversions with base-2. First, we'll convert 101100_2 to base-10. We have: $101100 = 1*2^5 + 1*2^3 + 1*2^2 = 32 + 8 + 4 = 44_{10}$.

Now let's convert 65 to binary. 2^6 is the highest power of 2 less than 65, so:

1. $65/2^6 = 1$ remainder 1
2. $1/2^5 = 0$ remainder 1
3. $1/2^4 = 0$ remainder 1
4. $1/2^3 = 0$ remainder 1
5. $1/2^2 = 0$ remainder 1
6. $1/2^1 = 0$ remainder 1
7. $1/2^0 = 1$ remainder 0

And thus we get our binary number, 1000001.

Tricks and Tips

Normally, when converting between two bases that aren't base-10, you would do something like this:

1. Convert number to base-10

2. Convert result to desired base

However, there's a trick that will let you convert between binary and hexadecimal quickly. First, take any binary number and divide its digits into groups of four. So, say we have the number 1011101_2. Divided up we have 0101 1101. Notice how we can just add extra zeroes to the front of the first group to make even groups of 4. We now find the value for each group as if it was its own separate number, which gives us 5 and 13. Finally, we simply use the corresponding hexadecimal digits to write out base-16 number, $5D_{16}$.

We can go the other direction also, by converting each hexadecimal digit into four binary digits. Try converting $B7_{16}$ to binary. You should get 10110111_2.

This trick works because 16 is a power of 2. What this means is that we use similar trick for base-8, which is also a power of 2:

Understanding different number systems is extremely useful in many computer-related fields. Binary and hexadecimal are very common, and I encourage you to become very familiar with them. Thanks for reading - I hope you've learned a lot from this article! If you have any questions, please ask them to either Daryl or me.