Decision making under uncertainty

Outline

1. Components of decision making
2. Criteria for decision making
3. Utility theory
4. Decision tree: Example 1
5. Posterior probabilities using Bayes' rule
6. Decision tree: Example 2
7. Decision tree: Example 3
8. The Monty Hall problem

1. Components of decision making

A decision problem may be represented by a model in terms of the following elements:

- **The decision maker** who is responsible for making the decision.
- **Alternative courses of action**: given that the alternatives are specified, the decision involves a choice among the alternative course action.
- **Events**: the scenarios or states of the environment not under control of the decision maker that may occur.
- **Consequences**: measures of the net benefit, or payoff received by the decision maker. They can be conveniently summarised in a payoff or decision matrix.

Components of decision making

Alternative course of action

- Given alternatives are specified, the decision involves a choice among the alternatives course of action.
- When the opportunity to acquire information is available, the decision maker's problem is to choose a best information source and a best overall strategy.
- A strategy is a set of <u>decision rules</u> indicating which action should be taken contingent on a specific observation received from the chosen information source.

Components of decision making

Events

- The events are defined to be mutually exclusive and collectively exhaustive.
- Uncertainty of an event is measured in terms of the probability assigned to this event.
- A characteristic of decision analysis is that the probabilities of events can be:
  - subjective (reflecting the decision maker's state of knowledge or beliefs) or
  - objective (theoretically or empirically determined) or
  - both.

2. Criteria for decision making

1. **Optimistic (Maximax) :** maximize the maximum possible profit.

2. **Pessimistic (Maximin):** maximize the minimum possible profit.

3. **Minimax-regret criterion:** minimize the regret for not having chosen the best alternative.

Regret =

(profit from the best decision) - (profit from the nonoptimal decision)

4. **The expected value criterion:** choose the action that yields the largest expected rewards.

Example.

John sells newspapers at a bus interchange. He pays the company 20 cents and sells the paper for 25 cents. Newspapers that are unsold at the end of the day are worthless. He knows that he can sell between 6 and 10 papers a day.

Payoff matrix:

Papers Ordered	Papers demanded				
	6	7	8	9	10
6	**30**	30	30	30	30
7	10	**35**	35	35	35
8	-10	15	**40**	40	40
9	-30	-5	20	**45**	45
10	-50	-25	0	25	**50**

decision

Criteria for decision making

Maximax criterion.

Papers Ordered	Papers demanded				
	6	7	8	9	10
6	**30**	30	30	30	30
7	10	**35**	35	35	35
8	-10	15	**40**	40	40
9	-30	-5	20	**45**	45
10	-50	-25	0	25	**50**

Papers ordered	States with best outcome	Best outcome
6	6,7,8,9,10	30 cents
7	7,8,9,10	35 cents
8	8,9,10	40 cents
9	9,10	45 cents
10	10	**50 cents**

Decision:
Order 10 papers for a potential profit of 50 cents.

Criteria for decision making

Maximin criterion.

Papers Ordered	Papers demanded				
	6	7	8	9	10
6	**30**	30	30	30	30
7	10	**35**	35	35	35
8	-10	15	**40**	40	40
9	-30	-5	20	**45**	45
10	-50	-25	0	25	**50**

Papers ordered	Worst state	Reward in worst state
6	6,7,8,9,10	**30 cents**
7	6	10 cents
8	6	-10 cents
9	6	-30 cents
10	6	-50 cents

Decision:
Order 6 papers and earn a profit of at least 30 cents.

Criteria for decision making

Minimax regret criterion.

Papers Ordered	Papers demanded				
	6	7	8	9	10
6	**30**	30	30	30	30
7	10	**35**	35	35	35
8	-10	15	**40**	40	40
9	-30	-5	20	**45**	45
10	-50	-25	0	25	**50**

Regret matrix:

Papers Ordered	Papers demanded					Max regret
	6	7	8	9	10	
6	**30-30 = 0**	35-30 = 5	40-30 = 10	45-30=15	50-30 = 20	**20**
7	30-10 = 20	**35-35 = 0**	40-35 = 5	45-35=10	50-35 = 15	**20**
8	30-(-10) = 40	35-15 = 20	**40-40 = 0**	45-40=5	50-40 =10	40
9	30-(-30) = 60	35-(-5) = 40	20	**45-45 = 0**	50-45 = 5	60
10	30-(-50) = 80	35-(-25) = 60	40-0 = 40	45-25= 20	**50-50 = 0**	80

Decision:
Order 6 or 7 papers to minimize the max regret.

Criteria for decision making

The expected value criterion.

Papers Ordered	Papers demanded				
	6	7	8	9	10
6	**30**	30	30	30	30
7	10	**35**	35	35	35
8	-10	15	**40**	40	40
9	-30	-5	20	**45**	45
10	-50	-25	0	25	**50**

Expected value:

Papers ordered	Expected reward
6	0.2×(30+30+30+30+30) = 30
7	0.2×(10+35+35+35+35) = 30
8	0.2×(-10+15+40+40+40) = 25
9	0.2×(-30-5+20+45+45) = 15
10	0.2×(-50-25+0+25+50) = 0

Decision:
Order 6 or 7 papers to maximize the expected reward
Note: Equal probability of demands is assumed

3. Utility theory

- **Utility function** is a formula or a method for converting any profit of a decision maker to an associated utility.
- Suppose you are asked to choose between two lotteries L_1 and L_2 .
- With certainty, Lottery L_1 yields \$10,000.
- Lottery L_2 consists of tossing a coin. Head, \$30,000 and tail \$0.
- L_1 yields an expected reward of \$10,000 and L_2 yields an expected reward of $0.5\times(30000 + 0) = \$15,000$.
- Although L_2 has larger expected value than L_1, most people prefer L_1 to L_2.

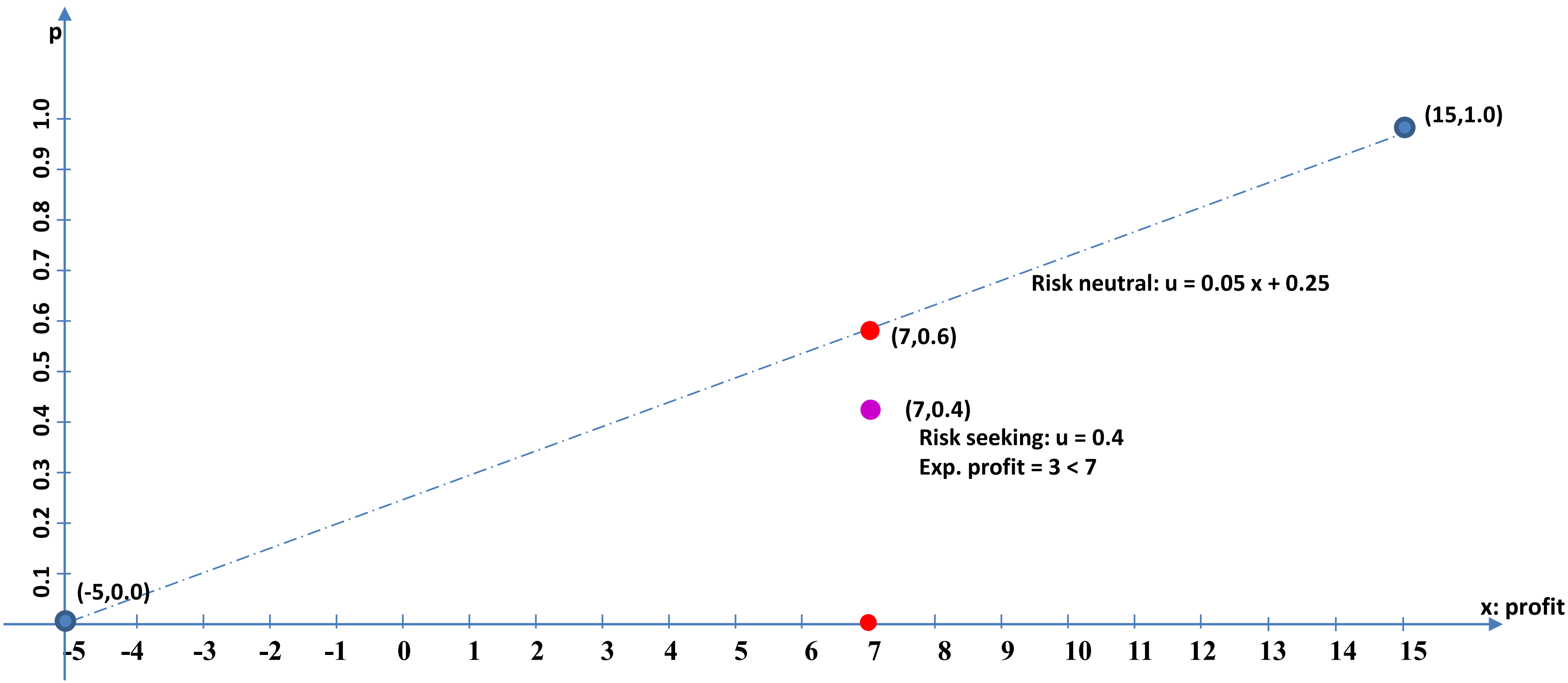
Converting profit into utilities

- Suppose that there are 9 possible profit levels. In decreasing order:

  15, 7, 5, 4, 3, 2, -1, -1, and -5 millions.
- We assign a utility of 100 to the highest profit and 0 to the lowest profit.
- Consider the profit of 7 millions.
  - Would you prefer a lottery that offers a potential winning of 15 with probability of 0.05 and winning of -5 with probability of 0.95 or a guaranteed profit of 7.
  - Would you prefer a lottery that offers a potential winning of 15 with probability of 0.95 and winning of -5 with probability of 0.05 or a guaranteed profit?
  - The goal is to determine a probability p in the lottery for winning 15 and (1-p) probability for winning -5 so that you will be indifferent to playing the lottery or taking the guaranteed profit of 7.

Converting profit into utilities

- Suppose p = 0.40, that is, you are indifferent between playing a lottery that offers 15 with p = 0.40, and -5 with probability 1 - p = 0.60 or taking a guaranteed profit of 7.
- With p = 0.40, the expected profit is $0.40 \times 15 + 0.6 \times (-5) = 3$.



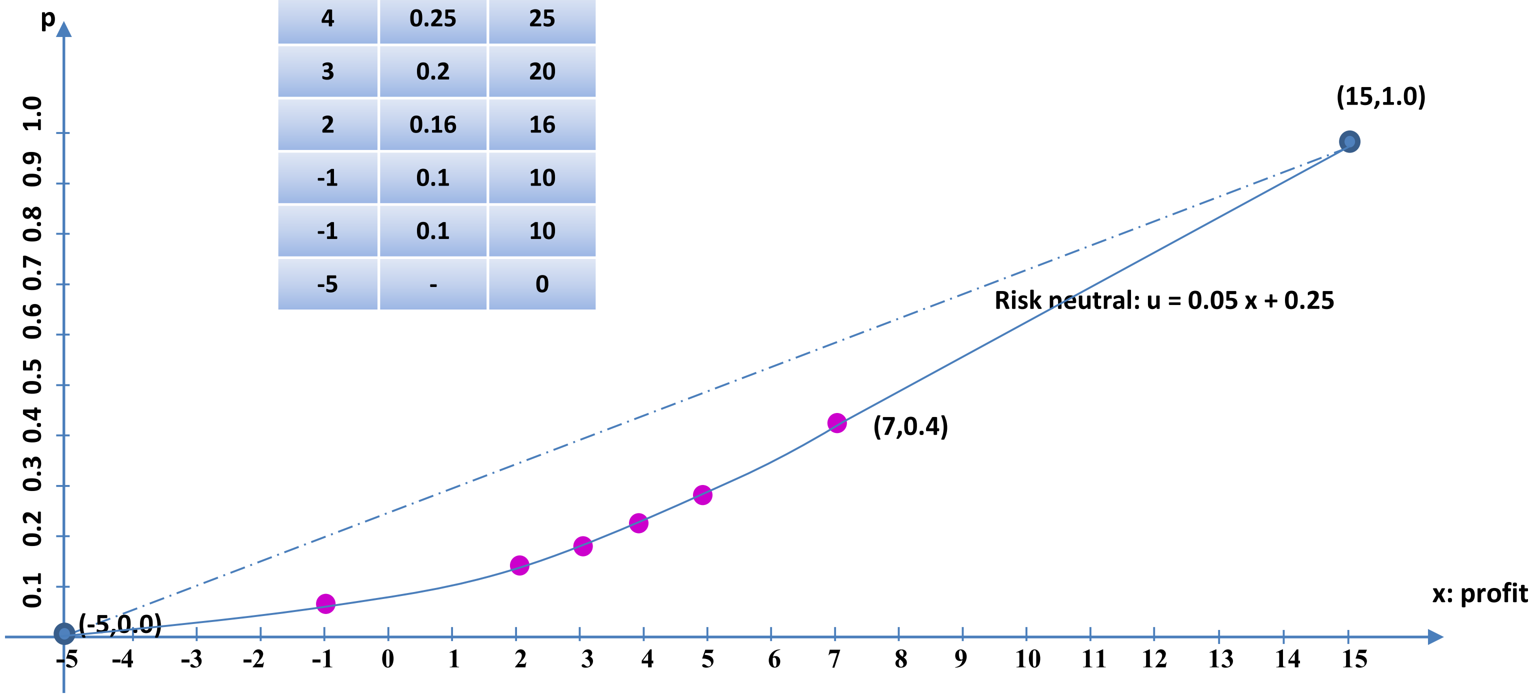
Converting profit into utilities

- Suppose $p = 0.40$, that is, you are indifferent between playing a lottery that offers 15 with $p = 0.40$, or -5 with probability $1 - p = 0.60$ or taking a guaranteed profit of 7.
- With $p = 0.40$, the expected profit is $0.40 \times 15 + 0.6 \times (-5) = 3$.
- You are willing to pay 7 to play a lottery whose expected profit is only 3. This is because you value the opportunity of making 15 relatively more.
- Hence for profit = 7, $p = 0.40$ and utility = $p \times 100 = 40$.
- We must now repeat the interview process to find the utilities for each of the remaining profit.
  - A risk-seeking decision maker has a utility function that indicates preference for taking risks (Convex utility function).
  - Risk-averse decision maker (Concave utility function).
  - Risk-neutral decision maker (Linear utility function).

Converting profit into utilities

Utility table and risk-seeking utility function:

profit	p	Utility (%)
15	-	100
7	0.4	40
5	0.3	30
4	0.25	25
3	0.2	20
2	0.16	16
-1	0.1	10
-1	0.1	10
-5	-	0



Computing utility function: an example.

If you are risk-averse and have assigned the following two endpoints on your utility function:

$U(-30) = 0$

$U(70) = 1$

what is a lower bound on U(30)?

Answer:

$p(70) + (1-p)(-30) \geq 30$

$100\,p \geq 60$ or $p \geq 0.6$

Computing utility function: a second example.

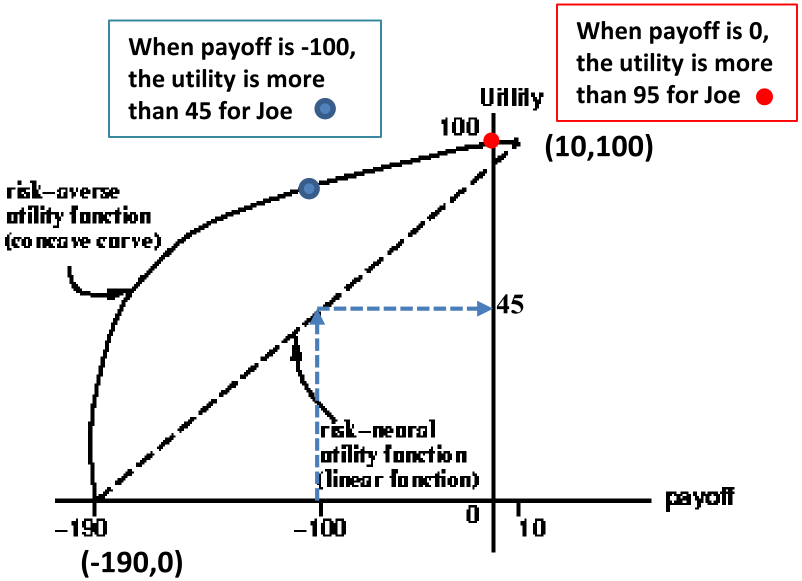
If Joe is risk-averse , which of the following lotteries he prefers?

L_1: with probability .10 Joe loses \$100

with probability .90 Joe wins \$0

L_2: with probability .10 Joe loses \$190

with probability .90 Joe wins \$10



Answer:

Straight line equation:

$$y = 0.5x + 95$$

L_1: $0.10\,U(-100) + 0.90\,U(0) > 0.10\,(45) + 0.90\,(95) = 90$

L_2: $0.10\,U(-190) + 0.90\,U(10) = 0.10(0) + 0.90(100) = 90$

L_1 is preferred, its utility is strictly greater than 90.

Utility function: a paradox.

Suppose we are offered a choice between 2 lotteries:

- L_1: with probability 1, we receive \$1 million.
- L_2:
  - with probability .10, we receive \$5 million.
  - with probability .89, we receive \$1 million.
  - with probability .01, we receive \$0.

Which lottery do we prefer?

Answer: **L_1**

$$U(1M) > 0.10\ U(5M) + 0.89\ U(1M) + 0.01\ U(0M)$$

or

$$\underline{0.11\ U(1M) \geq 0.10\ U(5M) + 0.01\ U(0M)}$$

Utility function: a paradox - continued.

Suppose we are offered a choice between 2 lotteries:

- L3:
  - with probability .11, we receive $1 million.
  - with probability .89, we receive $0.
- L4:
  - with probability .10, we receive $5 million.
  - with probability .90, we receive $0.

Which lottery do we prefer?

Answer: $\mathbf{L_4}$

$$0.11\ U(1M) + 0.89\ U(0M) < 0.10\ U(5M) + 0.90\ U(0M)$$

or

$$\underline{0.11\ U(1M) \leq 0.10\ U(5M) + 0.01\ U(0M)}$$

compare with previous slide!

Utility theory

Utility function: a paradox.

Suppose we are offered a choice between 2 lotteries:

- L_1: with probability 1, we receive \$1 million.
- L_2:
  - with probability .10, we receive \$5 million.
  - with probability .89, we receive \$1 million.
  - with probability .01, we receive \$0.

Which lottery do we prefer?

Answer: L_1

$U(1M) > 0.10\ U(5M) + 0.89\ U(1M) + 0.01\ U(0M)$

or

$\underline{0.11\ U(1M) \geq 0.10\ U(5M) + 0.01\ U(0M)}$

Suppose we are offered a choice between 2 lotteries:

- L3:
  - with probability .11, we receive \$1 million.
  - with probability .89, we receive \$0.
- L4:
  - with probability .10, we receive \$5 million.
  - with probability .90, we receive \$0.

Which lottery do we prefer?

Answer: L_4

$0.11\ U(1M) + 0.89\ U(0M) < 0.10\ U(5M) + 0.90\ U(0M)$

or

$\underline{0.11\ U(1M) \leq 0.10\ U(5M) + 0.01\ U(0M)}$

Contradiction!

4. Decision tree: Example 1

A decision tree enables a decision maker to decompose a large complex decision problem into several smaller problems.

Example: Colaco has assets of $150,000 and wants to decide whether to market a new chocolate-flavour soda.

It has 3 alternatives:

- Alternative 1. Test market Chocola locally, then utilise the results of the market study to determine whether or not to market nationally.
- Alternative 2. Immediately market Chocola nationally.
- Alternative 3. Immediately decide not to market Chocola nationally.

With no market study, Colaco believes that Chocola has a 55% chance of success and 45% chance of failure.

If successful, assets will increase by $300,000, otherwise decrease by $100,000.

Decision trees: Example 1

- Alternative 2. Expected return

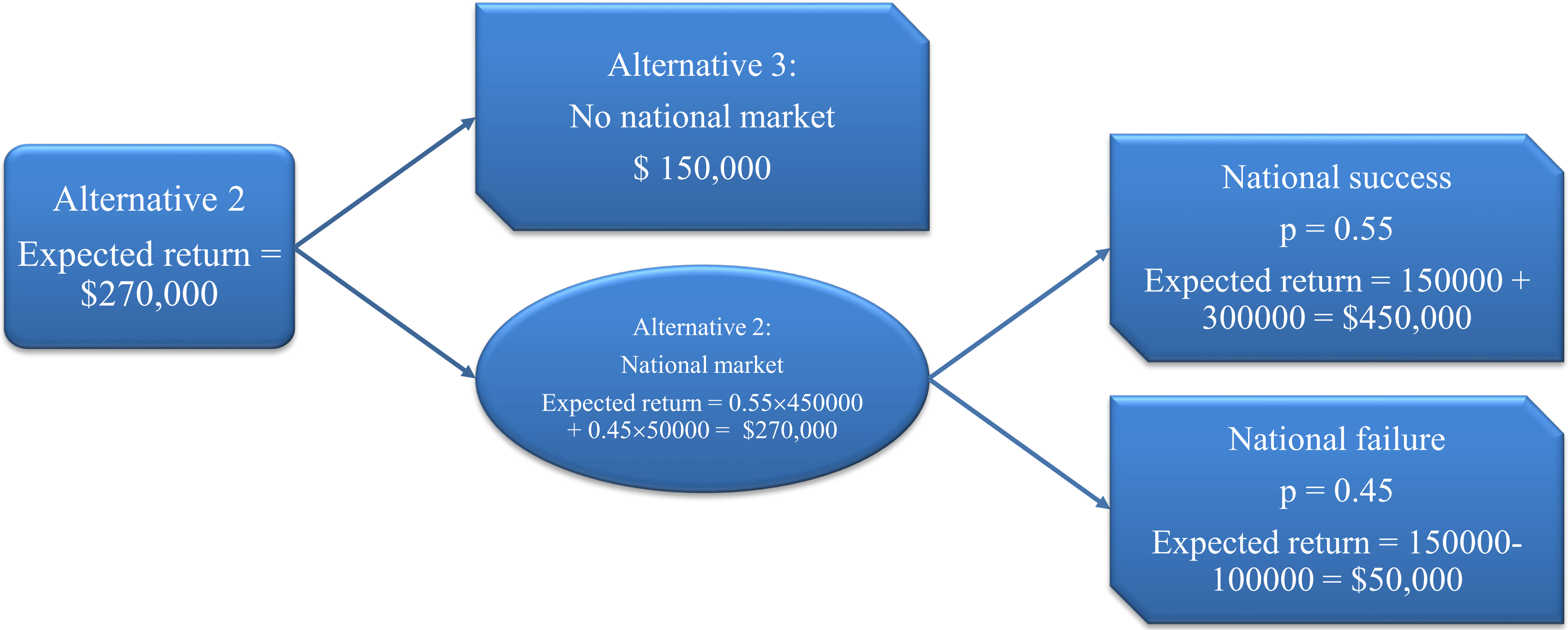
  $= 0.55\times(300000 + 150000) + 0.45\times(150000 - 100000) = \$270{,}000$.

- Alternative 3. Expected return = $150,000.

- If no test marketing (Alternatives 2 and 3):

  Expected return = max($270,000, $150,000) = $270,000.

- Alternative 2 is better than alternative 3, giving an expected return of $270,000.



Decision trees: Example 1

- If Colaco performs market study (cost \$30,000), there is a 60% chance of a favourable result (local success) and 40% of local failure.
  - If local success, 85% chance of national success.
  - If local failure, 10% chance of national success.

➢ Alternative 1.

- Local success and market nationally.

  Expected return =

  $0.85\times(150000 - 30000 + 300000) + 0.15\times(150000 - 30000 - 100000) = 360000$.

- Local success and do not market nationally.

  Expected return = $150000 - 30000 = 120000$.

Decision trees: Example 1

➢ Alternative 1 (continued)

- Local failure and market nationally.

  Expected return =

  $0.10 \times (150000 - 30000 + 300000) + 0.90 \times (150000 - 30000 - 100000) = 60000.$

- Local failure and do not market nationally.

  Expected return = $150000 - 30000 = 120000.$

- **Expected return for Alternative 1:**

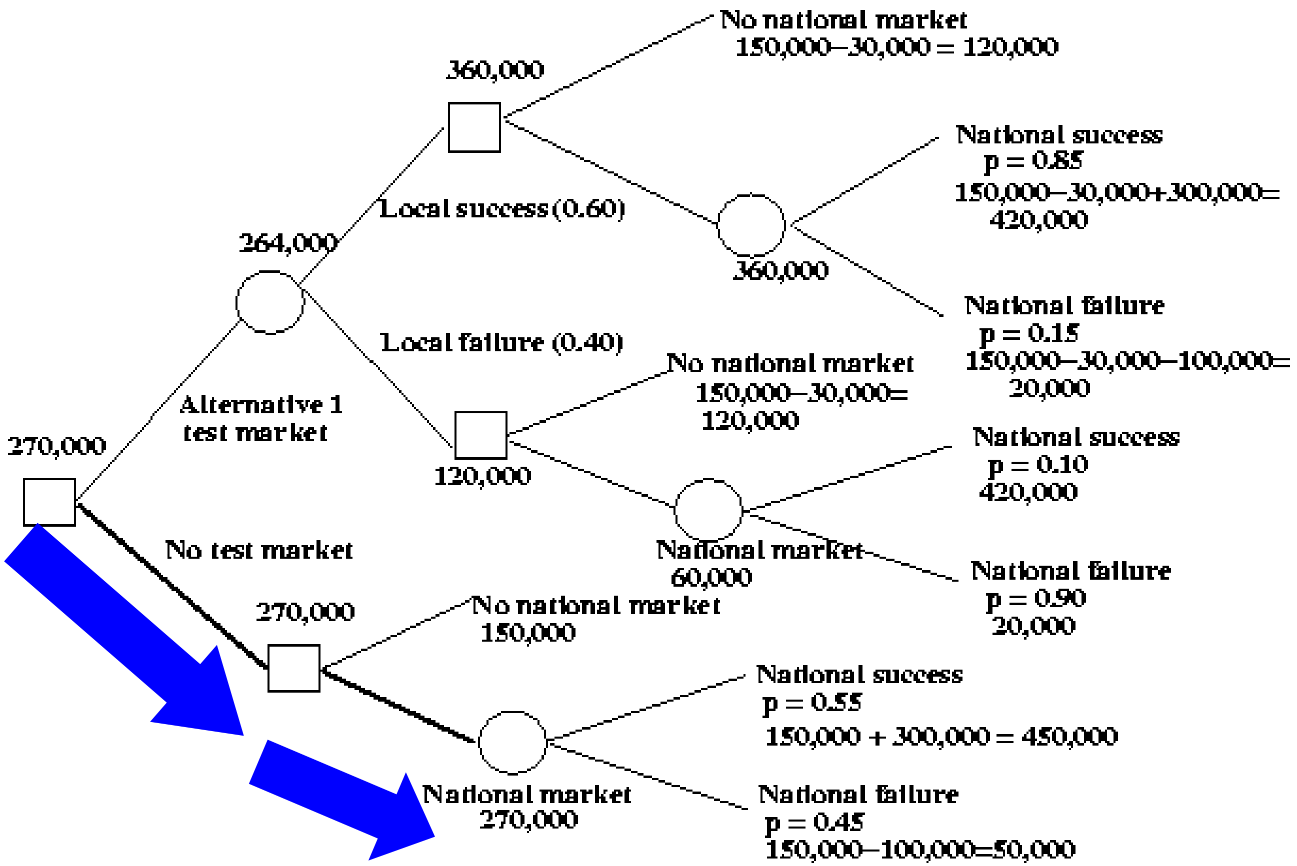
$$0.60 \times (360000) + 0.40 \times (120000) = \$264{,}000$$

Decision trees: Example 1

- Alternative 1. Expected return = $264,000.
- Alternative 2. Expected return = $270,000.
- Alternative 3. Expected return = $150,000.

Best decision: Alternative 2, do not test Chocola locally and then market it nationally

Decision trees: Example 1



Alternative 2 gives the highest expected value

A decision fork (square): a decision has to be made by Colaco.

An event fork (circle): outside forces determine which of the events will take place.

Decision trees: Example 1

Expected Value of Perfect Information (EVPI)

- **Perfect information**: information about the future that, in hindsight, would allow you to have chosen the best alternative.
- For Colaco example, by perfect information we mean that all uncertain events that can affect its final asset position still occur with the given probabilities, but Colaco finds out whether Colaco is a national success or a national failure before making the decision to market nationally.
- The expected value of perfect information = the expected value with perfect information - the expected value with original information

$$EVPI = EVWPI - EVWOI$$

$$= 315000 - 270000 = 45000$$

- How do we compute EVWPI?

Expected Value With Perfect Information (EVWPI)

- Expected value with perfect information:

$$0.55\times[\max(450000, 150000)] + 0.45\times[\max(50000, 150000)] = 315000$$

- Recall probability of national success = 0.55.

If we know it is going to be a national success (perfect information), then we would market nationally and increase the asset by 300000 to 450000.

- Similarly, the probability of failure = 0.45.

If we know it is a national failure, we won't market nationally and Colaco's asset remains 150000.

- Note, with perfect information, we market nationally only if it is going to be a success (national success p = 0.55). Expected gain is $0.55 \times 300000 + 0.45 \times 0 = 165000$.
- Compare this to the gain of 120000 with original information ($165000 - 120000 = 45000$)

Decision trees: Example 1

Expected Value of Sample Information (EVSI)

- If the company acts optimally and the test market study is costless, we have an expected final asset position of $264,000 + $30,000 = $294,000.
- This value is larger than the 'no market test' branch of $270,000.
- Hence, the expected value with sample information (EVWSI) =

  max(294000, 270000) = 294000

- The expected value with original information (EVWOI) = $270,000.
- The expected value of sample information:

  EVSI = EVWSI - EVWOI = $24,000

- This value is **less than** the cost of the market test, hence Colaco **should not** conduct the test market study.

5. Posterior probabilities using Bayes' rule

- In the Colaco's example we have **prior probabilities** of national success $p(NS) = 0.55$ and of national failure $p(NF) = 0.45$.
- **Posterior probabilities** are revised probabilities of outcomes obtained on the basis of indicators from a test marketing procedure.
- The two possible outcomes of the test marketing are LF = local failure and LS = local success.
- The posterior probabilities are

$$p(NS|LS) = 0.85 \qquad p(NS|LF) = 0.10$$

$$p(NF|LS) = 0.15 \qquad p(NF|LF) = 0.90$$

Posterior probabilities using Bayes' rule

- Instead of posterior probabilities, we may be given the **likelihoods**.
- Suppose that 55 products that have been national successes (NS) had previously been test marketed, of these 55, 51 were local successes (LS):

$$P(LS|NS) = 51/55$$

$$P(LF|NS) = 4/55$$

- Suppose that 45 products that have been national failures (NF), 36 of them had been predicted correctly by market tests:

$$P(LS|NF) = 9/45$$

$$P(LF|NF) = 36/45$$

- With the help of Bayes' rule, we can use the prior probabilities and the likelihoods to determine the posterior probabilities.

Posterior probabilities using Bayes' rule

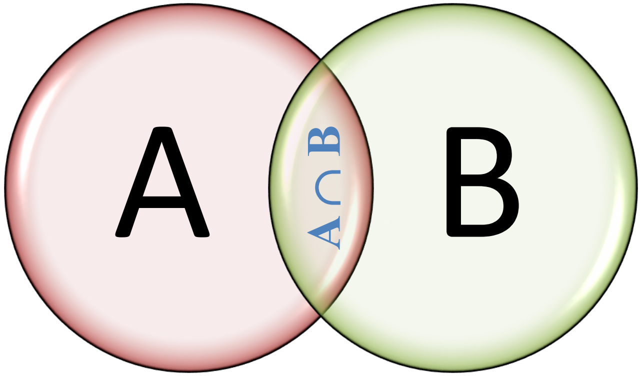
Bayes' formula:

$$P(H_i|A) = \frac{P(H_i)P(A|H_i)}{P(A)}$$
$$= \frac{P(H_i)P(A|H_i)}{\sum_{j=1}^{k} P(H_j)P(A|H_j)}, \quad i = 1, 2, \ldots, k$$

The simple formula for conditional probability

$P(A|B) = P(A \cap B)/P(B)$

is sufficient!



Posterior probabilities using Bayes' rule

We have for the Colaco's example, **joint probabilities**:

$$P(NS \cap LS) = P(NS)\ P(LS|NS) = 0.55 \times (51/55) = 0.51$$

$$P(NS \cap LF) = P(NS)\ P(LF|NS) = 0.55 \times (4/55) = 0.04$$

$$P(NF \cap LS) = P(NF)\ P(LS|NF) = 0.45 \times (9/45) = 0.09$$

$$P(NF \cap LF) = P(NF)\ P(LF|NF) = 0.45 \times (36/45) = 0.36$$

Hence, the **marginal probabilities** are:

$$P(LS) = P(NS \cap LS) + P(NF \cap LS) = 0.51 + 0.09 = 0.60$$

$$P(LF) = P(NS \cap LF) + P(NF \cap LF) = 0.04 + 0.36 = 0.40$$

Note: $P(LS) + P(LF) = 1$

Bayes' rule is applied to obtain the **posterior probabilities**:

$$P(NS|LS) = P(NS \cap LS)/P(LS) = 0.51/0.60 = 0.85$$

$$P(NF|LS) = P(NF \cap LS)/P(LS) = 0.09/0.60 = 0.15$$

$$P(NS|LF) = P(NS \cap LF)/P(LF) = 0.04/0.40 = 0.10$$

$$P(NF|LF) = P(NF \cap LF)/P(LF) = 0.36/0.40 = 0.90$$

Posterior probabilities using Bayes’ rule

Summary:

Prior probability	
NS	0.55
NF	0.45

	Likelihood	
	LS	LF
NS	P(LS|NS) = 51/55	P(LF|NS) = 4/55
NF	P(LS|NF) = 9/45	P(LF|NF) = 36/45

	Joint probability ∩	
	LS	LF
NS	0.55(51/55) = 0.51	0.55(4/55) = 0.04
NF	0.45(9/45) = 0.09	0.45(36/45) = 0.36

Marginal probability	
LS	LF
0.51+0.09 = 0.60	0.04 + 0.36 = 0.40

	Posterior probability	
	NS	NF
LS	P(NS|LS) = 0.85	P(NF|LS) = 0.15
LF	P(NS|LF) = 0.10	P(NF|LF) = 0.90

6. Decision tree: Example 2

Example.

- Ali has 2 drawers.
- One drawer contains 3 gold coins and the other contains one gold and two silver coins.
- You are allowed to choose from one drawer, and you will be paid $500 for each gold coin and $100 for each silver coin in that drawer.
- Before choosing, we may pay Ali $200, and he will draw a randomly selected coin (each of the six coins has an equal chance of being chosen) and tells us whether it is gold or silver.
- For instance, Ali may say that he drew a gold coin from drawer 1.
- Would you pay him $200?
- What is EVSI and EVPI?

Decision tree: Example 2

Answer.

- If we pick GOOD (drawer with 3 gold coins), we gain \$1500. Otherwise, we pick BAD (the other drawer), we gain \$700.
- Without sample information, P(GOOD) = P(BAD) = 0.5. Hence, the expected profit is $0.5\times(1500+700) = 1100$.
- If we had the perfect information, we would always choose GOOD.

  Hence, the EVPI = EVWPI - EVWOI = 1500 - 1100 = 400.
- Suppose now we ask Ali to draw a coin from a drawer (sample information).
- The coin drawn is either GOLD or SILVER.
- We have the conditional probabilities:
  - $P(GOLD|GOOD) = 1$ $P(SILVER|GOOD) = 0$
  - $P(GOLD|BAD) = 1/3$ $P(SILVER|BAD) = 2/3$

Decision tree: Example 2

Answer (continued).

- What are the values of $P(GOOD|GOLD)$, $P(BAD|GOLD)$, $P(GOOD|SILVER)$, $P(BAD|SILVER)$?
- That is, suppose that the coin shown by Ali is GOLD, we would like to know if the drawer contains 3 gold coins, i.e. GOOD. Compute the join probabilities:

$$P(GOOD \cap GOLD) = P(GOOD)\times P(GOLD|GOOD) = 0.5\times 1 = 0.5$$

$$P(BAD \cap GOLD) = P(BAD)\times P(GOLD|BAD) = 0.5\times 1/3 = 1/6$$

$$P(GOOD \cap SILVER) = P(GOOD)\times P(SILVER|GOOD) = 0.5\times 0 = 0$$

$$P(BAD \cap SILVER) = P(BAD)\times P(SILVER|BAD) = 0.5\times 2/3 = 1/3$$

- Compute the marginal probabilities:

$$P(GOLD) = 0.5 + 1/6 = 2/3 \quad P(SILVER) = 0 + 1/3 = 1/3$$

(any easier way to find these probabilities?)

Decision tree: Example 2

Answer (continued).

- Compute the revised/posterior probabilities:

  $P(GOOD|GOLD) = P(GOOD \cap GOLD)/P(GOLD) = 0.5/(2/3) = 0.75$

  $P(BAD|GOLD) = P(BAD \cap GOLD)/P(GOLD) = (1/6)/(2/3) = 0.25 = 1 - 0.75$

  $P(GOOD|SILVER) = P(GOOD \cap SILVER)/P(SILVER) = 0$

  $P(BAD|SILVER) = P(BAD \cap SILVER)/P(SILVER) = 1$

- If a silver coin is drawn, then we choose the other drawer and the expected gain is 1500.
- If a gold coin is drawn, then we pick that drawer and the expected gain is

  $P(GOOD|GOLD)\times1500 + P(BAD|GOLD)\times700 = 1300.$

- Hence, the expected profit when a coin is drawn by Ali:

  $P(GOLD)\times1300 + P(SILVER)\times1500 =$

  $(2/3)\times1300 + (1/3)\times1500 = 4100/3$

  Expected value of sample information EVSI:

  $EVSI = EVWSI - EVWOI = 4100/3 - 1100 = 800/3$

  Since EVSI > $200, we would ask Ali to draw a coin.

7. Decision tree: Example 3

Fruit computer company manufactures memory chips in lots of ten chips.

From past experience, it knows that:

80% of all lots contain 10% defective chips (good batch)

20% of all lots contain 50% defective chips (bad batch)

If a good batch is sent to the next stage of production, processing costs of $1000 are incurred.

If a bad batch is sent to the next stage of production, processing costs of $4000 are incurred.

The company also has the alternative of reworking a batch at a cost of $1000.

A reworked batch is sure to be a good batch.

Alternatively, at a cost of $100 the company can test one chip from each batch to determine whether the batch is defective.

How can the company minimize the total cost per batch?

Compute EVSI and EVPI.

7. Decision tree: Example 3

80% of all lots contain 10% defective chips (good batch): $P(G) = 0.80$

20% of all lots contain 50% defective chips (bad batch): $P(B) = 0.20$

When a chip is tested for a batch, the possible outcomes are:

D = defective chip is observed

ND = non-defective chip is observed

We have the following likelihoods:

$P(D|G) = 0.10$ $P(ND|G) = 0.90$ $P(D|B) = 0.50$ $P(ND|B) = 0.50$

Prior probability	
G	0.80
B	0.20

	Likelihood	
	D	ND
G	P(D|G) = 0.10	P(ND|G) = 0.90
B	P(D|B) = 0.50	P(ND|B) = 0.50

7. Decision tree: Example 3

Joint probabilities:

$P(D \cap G) = P(G)\ P(D|G) = 0.80 \times 0.10 = 0.08$

$P(D \cap B) = P(B)\ P(D|B) = 0.20 \times 0.50 = 0.10$

$P(ND \cap G) = P(G)\ P(ND|G) = 0.80 \times 0.90 = 0.72$

$P(ND \cap B) = P(B)\ P(ND|B) = 0.20 \times 0.50 = 0.10$

Marginal probabilities of the outcome from the test:

$P(D) = P(D \cap G) + P(D \cap B) = 0.18$

$P(ND) = P(ND \cap G) + P(ND \cap B) = 0.82$

Posterior (revised) probabilities:

$P(G|D) = P(G \cap D)/P(D) = 0.08/0.18 = 4/9 \approx 0.44$

$P(B|D) = P(B \cap D)/P(D) = 0.10/0.18 = 5/9 \approx 0.56$

$P(G|ND) = P(G \cap ND)/P(ND) = 0.72/0.82 = 36/41 \approx 0.88$

$P(B|ND) = P(B \cap ND)/P(ND) = 0.10/0.82 = 5/41 \approx 0.12$

Prior probability	
G	0.80
B	0.20

	Likelihood	
	D	ND
G	P(D|G) = 0.10	P(ND|G) = 0.90
B	P(D|B) = 0.50	P(ND|B) = 0.50

	Joint probability ∩	
	D	ND
G	0.08	0.72
B	0.10	0.10

Marginal probability	
D	ND
0.18	0.82

7. Decision tree: Example 3

	Posterior probability	
	G	B
D	P(G|D) = 4/9	P(B|D) = 5/9
ND	P(G|ND) = 36/41	P(B|ND) = 5/41

- If we decide to test a chip (cost is \$100):
- Chip is defective (probability = 0.18):

  Send batch on for pre-processing:

  E1 = (4/9)(1000) + (5/9)(4000) = 2666.67

  Rework the batch:

  E2 = 1000 + 1000 = 2000

  = cost of reworking the batch + cost of processing a good batch

  It is better to rework the batch.

- Chip is not defective (probability = 0.82):

  Send batch on for pre-processing:

  E3 = (36/41)(1000) + (5/41)(4000) = 1365.85

  Rework the batch:

  E4 = 1000 + 1000 = 2000

  It is better to send the batch on for pre-processing.

7. Decision tree: Example 3

- If we decide to test a chip (cost is \$100):

  $E5 = 0.18(2000) + 0.82(1365.85) = 1480$

- If we send the batch on for processing:

  $E6 = 0.80(1000) + 0.20(4000) = 1600$

- If we rework the batch

  $E7 = 1000 + 1000 = 2000$

  (cost of reworking is 1000 and cost of processing good batch is 1000)

- Comparing E5, E6, E7, the best decision:

  Test a chip from the batch is the decision with minimum expected cost of

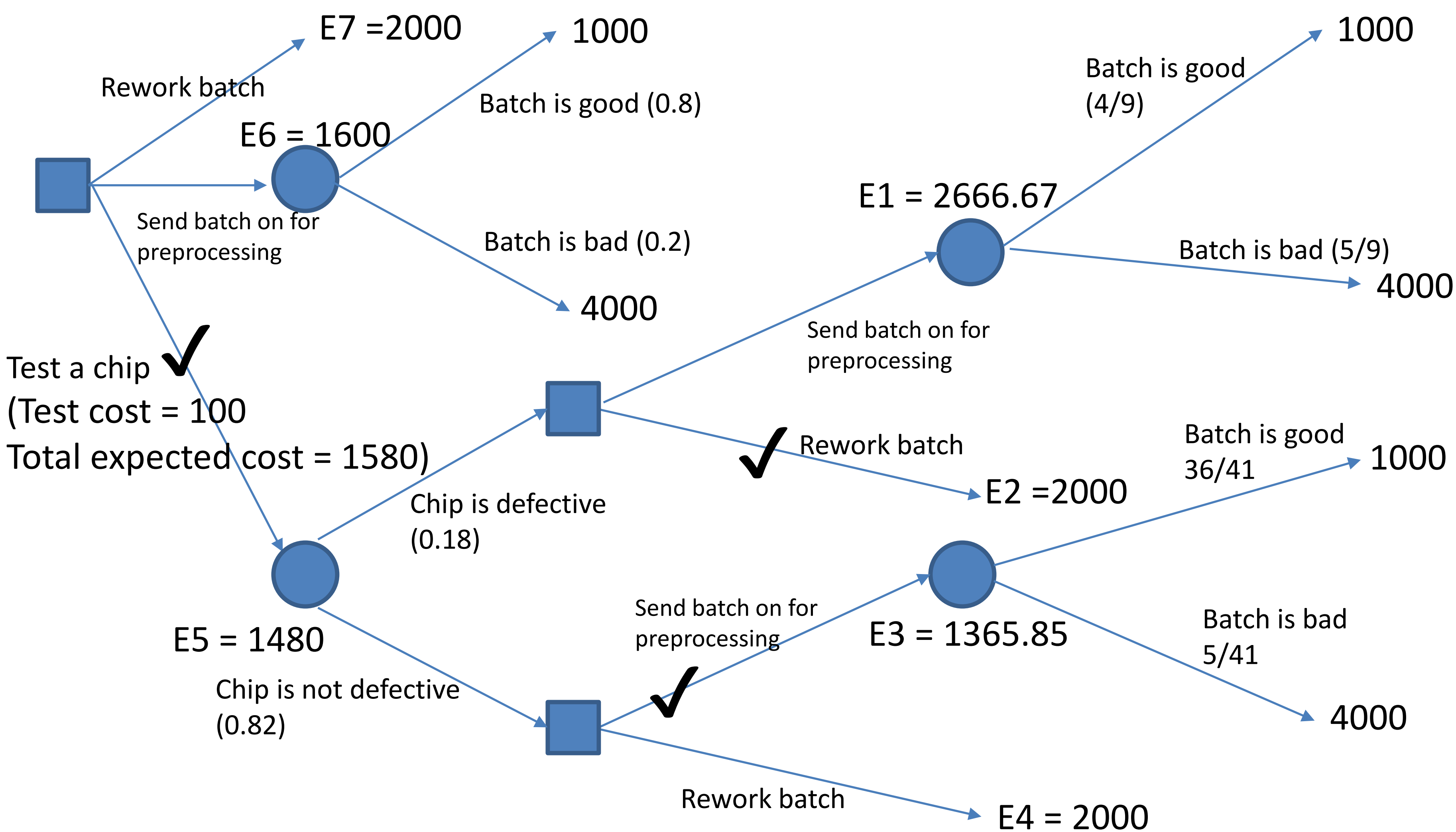
  \$1480 + \$100 = \$1580.

Marginal probability	
D	ND
0.18	0.82

Prior probability	
G	0.80
B	0.20

7. Decision tree: Example 3

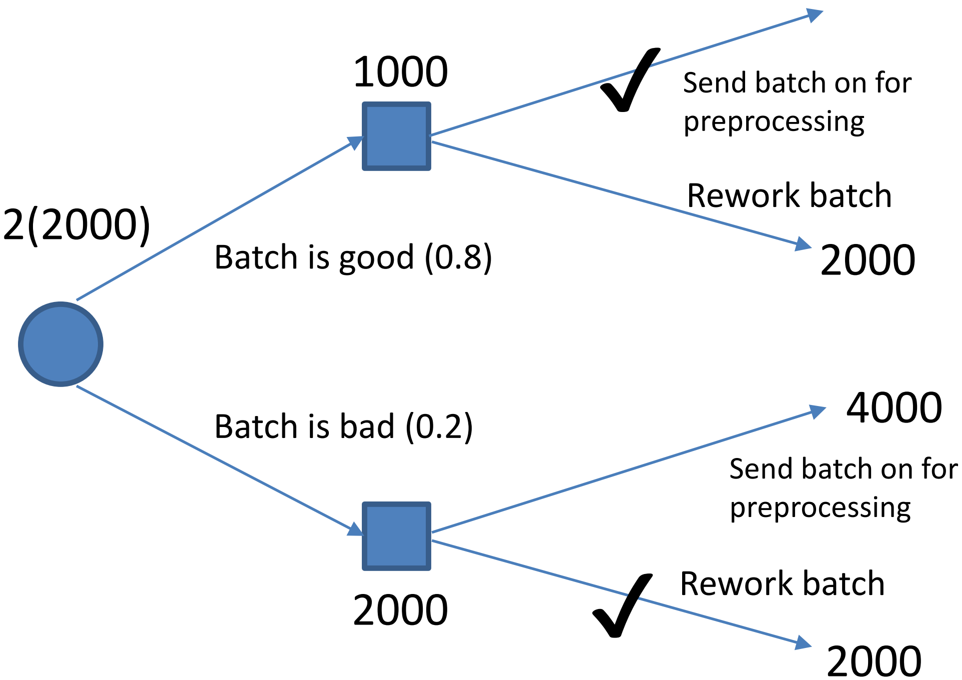
Test a chip from the batch is the decision with minimum expected cost of $1580.



7. Decision tree: Example 3

- Expected value (cost) with original information (EVWOI): min(E6, E7) = $1600
- Expected value (cost) with sample information (EVWSI): $1480
- Expected value of sample information (EVSI) = = $1600 - $1480 = $120
- Expected value (cost) with perfect information: (EVWPI) $1200
- Expected value of perfect information (EVPI) = $1600 - $1200 = $400

EVWPI =
0.8(1000) + 0.2(2000)
= 1200



Note: EVSI = 120 is greater than $100 cost of test. Hence, we will do the test.

6. The Monty Hall problem

Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch to door number 2?

Solution using Bayes' rule. Also here.

Solution using simulation

Reference.

Winston, Wayne L, Operations Research : Applications and algorithms, Australia ;
Belmont, CA : Thomson Brooks/Cole, c2004, Chapter 13. T57.6 Win 2004