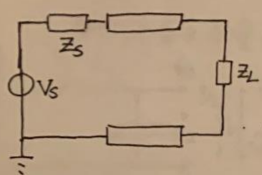


$Z_S = 10\Omega$

$Z_L = 75\Omega$

$Z_0 = 50\Omega$

$V_S(t) = u(t)$

1. 為什麼是這樣的波形？

由于信源内阻R_S、负载电阻R_L均不匹配。信源发射的信号到达负载时，由于不匹配会产生一个反射电压；反射电压回到电源后由于不匹配又会再次产生回到负载的反射信号。如此往复导致了负载电压的振荡特性。

2. 传输线稳定后的反射电压

先考查第一个延时有：$V_0^+ = V_0 \cdot \frac{Z_0}{Z_S+Z_0}$

由 $\begin{cases} \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \\ \Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \end{cases}$

于是 $t_d = 1ns$ 时，负载电压 $V_L(1) = (1+\Gamma_L)V_0^+ = \frac{2Z_L}{Z_L+Z_0} \cdot \frac{Z_0}{Z_S+Z_0} V_0$

而 $V_L(1) = \frac{2 \cdot 75}{75+50} \cdot \frac{50}{10+50} V_0 = V_0 = 1V$

当来自负载的反射电压 $V_{ref}(1) = \Gamma_L \cdot V_0^+$ 回到信源时，由于信源不匹配，立刻就有再次反射回负载的信号：

$V_{ref}(2) = \Gamma_L \cdot V_0^+ \cdot \Gamma_S = V_0^+(\Gamma_L\Gamma_S)^1 = -0.1111V$

负载获得的信号：$V_L(3) = V_{ref}(2)(1+\Gamma_L) = -0.133V$

于是此时传输线负载电压

$V = V_L(1) + V_L(3) = 0.8667V$

类似地：$V_L(5) = (1+\Gamma_L)\Gamma_L\Gamma_S \cdot V_{ref}(2)$

$= (1+\Gamma_L)\Gamma_L\Gamma_S \cdot (\Gamma_S\Gamma_L)V_0^+ = V_0^+(1+\Gamma_L)(\Gamma_S\Gamma_L)^2$

故：$V_{stable} = \{V_0^+(1+\Gamma_L)[(\Gamma_S\Gamma_L)^0 + (\Gamma_S\Gamma_L)^1 + \cdots]$

$= V_0^+(1+\Gamma_L) \cdot \frac{1}{1-\Gamma_S\Gamma_L} = \boxed{0.8824V}$

故有传输线时的稳定电压为：

$V_{stable} = 0.8824V$

3. 无传输线时电压

若不存在传输线，则一个t_d后信号即稳定，有：

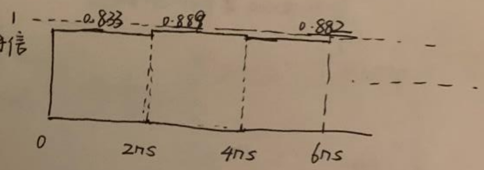
$V = V_0 \cdot \frac{Z_L}{Z_L+Z_S} = 0.8824V$

即两者相等。

理解：由于传输线是无损传输线，因而其双端不匹配并不影响系统的稳态解。但不匹配会放大其瞬态解，因而无传输线时 1ns 即到达稳态，有传输线时则需要很长时间。

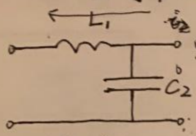
4. 不匹配则数字信号不稳定，影响到晶体管的判定阈值问题。这就必需提高晶体管摆幅以维持高的完整精度，就会影响数字电路的速度。

5. 输入端口波形：



显然，输入端口收敛速度强于输出端口。

二. 考查网络Z参量与ABCD矩阵：



$$\begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix} = \begin{bmatrix} Z_{11}, & Z_{12} \\ Z_{21}, & Z_{22} \end{bmatrix} \begin{bmatrix} \tilde{I}_1 \\ \tilde{I}_2 \end{bmatrix}$$

立即得到：

$$\begin{cases} Z_{11} = j\omega L_1 + \dfrac{1}{j\omega C_2} \\ Z_{22} = \dfrac{1}{j\omega C_2} \\ Z_{21} = \dfrac{1}{j\omega C_2} \\ Z_{12} = \dfrac{1}{j\omega C_2} \end{cases}$$

故有 $Z = \begin{bmatrix} j\omega L_1 + \dfrac{1}{j\omega C_2}, & \dfrac{1}{j\omega C_2} \\ \dfrac{1}{j\omega C_2}, & \dfrac{1}{j\omega C_2} \end{bmatrix}$

由：$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -i_2 \end{bmatrix}$.

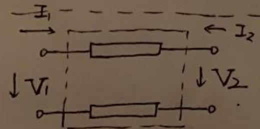
$$A = \frac{V_1}{V_2}\Big|_{i_2=0} = \frac{\frac{1}{j\omega C_2} + j\omega L_1}{\frac{1}{j\omega C_2}} = 1 + (j\omega)^2 L_1 C_2$$

$$-B = \frac{V_1}{i_2}\Big|_{V_2=0} = -j\omega L_1 \Rightarrow B = j\omega L_1$$

$$C = \frac{i_1}{V_2}\Big|_{i_2=0} = j\omega C_2.$$

$$-D = \frac{i_1}{i_2}\Big|_{V_2=0} = -1 \Rightarrow D = 1.$$

于是，LC网络的ABCD参量为：$\begin{bmatrix} 1 + (j\omega)^2 L_1 C_2, & j\omega L_1 \\ j\omega C_2, & 1 \end{bmatrix}$



$$\begin{cases} V(z) = V_0^+\left(\exp(\gamma z) + \Gamma_0 \exp(-\gamma z)\right) \\ I(z) = V_0^+\left(\exp(\gamma z) + \Gamma_0 \exp(-\gamma z)\right). \end{cases}$$

根据《射频电路设计》教材，可得其 $V(z)$，$I(z)$ 关系：

$$\begin{cases} V(z) = 2jV^+ \sin\beta d \\ I(z) = \dfrac{V^+}{Z_0}\cos\beta d \end{cases} \text{（短路时）}$$

$$\begin{cases} V(z) = V^+ \cos\beta d \\ I(z) = j\dfrac{V^+}{Z_0}\sin\beta d \end{cases} \text{（开路时）}.$$

由此先看其Z参量阵：

$Z_{11} = \dfrac{V(l)}{I(l)}$，此时由于端口2开路，需要用开路时的结果：

$$Z_{11} = \frac{V^+\cos\beta l}{j\frac{V^+}{Z_0}\sin\beta l} = -jZ_0\cot\beta l$$

$$Z_{22} = \frac{V(0)}{I(0)} = Z_{11} = -jZ_0\cot\beta l$$

$$Z_{21} = \frac{V(0)}{I(l)}\Big|_{i_2=0} = \frac{V^+}{j\frac{V^+}{Z_0}\sin\beta l} = -jZ_0/\sin\beta l$$

$$Z_{12} = Z_{21} = -jZ_0/\sin\beta l.$$

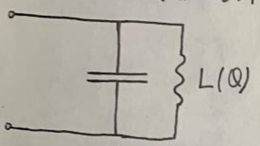
故其Z参量为：

$$Z = \begin{bmatrix} -jZ_0\cot\beta l, & -jZ_0/\sin\beta l \\ -j\dfrac{Z_0}{\sin\beta l}, & -jZ_0\cot\beta l \end{bmatrix}.$$

类似地可得其ABCD参量.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l, & jZ_0\sin\beta l \\ jY_0\sin\beta l, & \cos\beta l \end{bmatrix}.$$

习题三：并联諧振及其带宽



$C = 10pF$
$Q = 200$
$L = 10\mu H$

对于该并联谐振系统，立刻有：（Q值是一个谐振概念，要在谐振频点上考查Q值）

自由振荡频率 $\omega_0 = \frac{1}{\sqrt{LC}} = 10^8 rad/s$

$$f_0 = \frac{1}{2\pi} \cdot \omega_0 = 1.592 \times 10^7 Hz = 15.92 MHz$$

特征阻抗：$Z_0 = \sqrt{\frac{L}{C}} = 1k\Omega$.

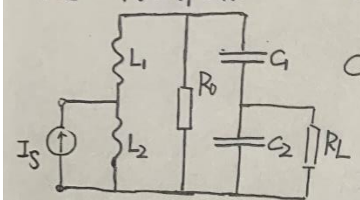
串联等效电阻：$r_S = \frac{Z_0}{Q} = 5\Omega$.

于是等效并臂电阻：$R_P = Q^2 r_S = 200k\Omega$.

并联谐振频率：$f_P = \frac{1}{2\pi} \omega_0 \sqrt{1 - \frac{1}{Q^2}} \approx f_0$

于是 $BW_{3dB} = \frac{f_P}{Q} = \cancel{795.8 kHz}\ 79.58 kHz$

习题四：抽头部分接入：



$L_1 = L_2 = 5\mu H$

$Q_0 = 100$

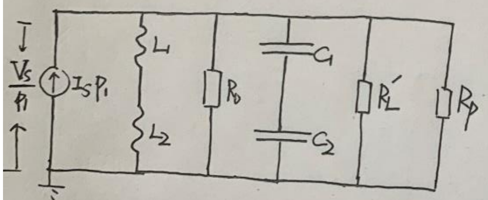
$C_1 = C_2 = 8pF$

$R_0 = 40k\Omega$

$R_L = 10k\Omega$

$$p_1 = \frac{L_2}{L_1+L_2} = \frac{1}{2}, \quad p_2 = \frac{C_1}{C_1+C_2} = \frac{1}{2}$$

故需做两次部分接入等效，有等效电路



无阻尼谐振频率有：$f_m = \frac{1}{2\pi} \frac{1}{\sqrt{(L_1+L_2)(C_1\|C_2)}} \sqrt{1-\frac{1}{Q^2}}$

得：$f_m = 25.163 MHz$.

并联谐振回路特征阻抗：$Z_0 = \sqrt{\frac{(L_1+L_2)}{C_1\|C_2}} = \cancel{790.56}\ 1.581k\ \Omega$

故并联电阻：$R_p = Z_0 \cdot Q = 158.11k\Omega$.

$$R_L' = R_L / p_2^2 = 40k\Omega.$$

而：$\frac{V_s/p_1}{I_s p_1} = \frac{V_s}{I_s} \cdot \frac{1}{p_1^2} = R_0 \| R_p \| R_L'$

$$\Rightarrow Z_{in}\Big|_{\omega=\omega_p} = p_1^2 (R_0 \| R_p \| R_L') = 4.44k\Omega.$$

~~考查其3dB带宽~~

考查电路整体Q值，若不拿掉负载电阻，其有

$$Q_{R_L} = \frac{Y_0}{G} = \frac{\sqrt{\frac{C}{L}}}{G_0+G_p+G_L'} = 11.2288$$

若拿掉负载电阻，则：

$$Q_{no.R_L} = \sqrt{\frac{C_1\|C_2}{L_1+L_2}} \Big/ G_0+G_p = 20.1904$$

故无R_L时带宽有

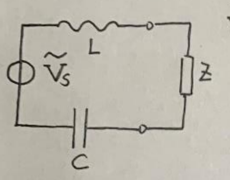
$$BW_{3dB\ no.R_L} = \frac{f_m}{Q_{no\,R_L}} = 1.2463 MHz.$$

有R_L时带宽有：

$$BW_{3dB\ R_L} = \frac{f_m}{Q_{R_L}} = 2.2410 MHz.$$

习题五:阻抗测量: $\tilde{f}_s = 1\,MHz$

$V_{ps} = 0.1V$



一:短接电路.

短接电路时有谐振频率: $f_m = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}$

短接时,有: $1\,MHz = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L \cdot 100pF}}$

$$\Rightarrow L = \frac{1}{(2\pi f_m)^2 C} = 2.533 \times 10^{-4} H = \cancel{2.53}\ 253.30\,\mu H$$

谐振时,电路整体呈阻性,电压电源全部加在L的串联等效电阻 r_s 上,故有

$$Q = \frac{V_{pp} \cdot C}{2 V_{p \cdot s}} = 100 = \frac{\frac{1}{\omega_0 C}}{r_s}$$

于是有: $r_s = \frac{1}{Q \omega_0 C} = 15.916\Omega$.

于是有 $\begin{cases} \text{电感无载品质因数}: Q = 100 \\ \text{电感数值}: L = 253.30\,\mu H \end{cases}$

二:接某一阻抗元件,记其有 $Z_L = R_L + \frac{1}{j\omega C_L}$

之刻有: $C_2 \parallel C_L = C \Rightarrow C_L = 200pF$

$$\frac{V_{pp} \cdot C}{2 \cdot V_{ps}} = Q = \frac{\sqrt{\frac{L}{C_2 \parallel C_L}}}{r_s + R_L}$$

$$\Rightarrow R_L = 47.75\Omega.$$

故 $\begin{cases} \text{阻抗电阻 } R_L = 47.75\Omega \\ \text{阻抗电容}: C_L = 200pF. \end{cases}$