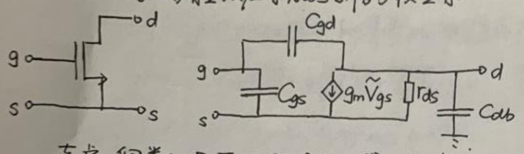
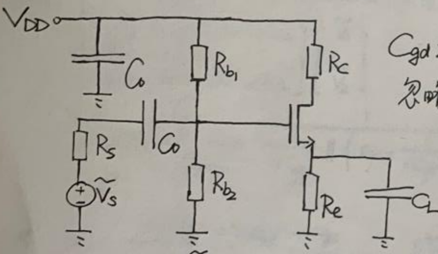
Homework I：

CD组态放大器的抖动现象：

本例的简化版在Razavi《The Design of Analog CMOS Integrated Circuits》6.3节有所叙述。对于NMOS不关心其栅源电阻，忽略背栅效应时MOS小信号模型有：

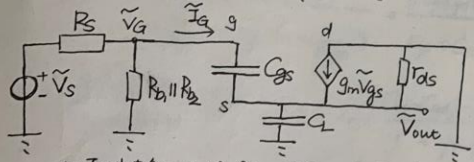


考虑一个类似BJT CE组态放大器的电路：



C_{gd}是大电容，本题中取0

忽略体效应，$C_{db}=0$。故有小信号电路



令源端输入电流 $\tilde{I}_G$，源级电压 $\tilde{V}_G$

有输入阻抗：$Z_{in} = \tilde{V}_G / \tilde{I}_G$，

有：$\tilde{V}_G = \tilde{V}_o + \frac{1}{sC_{gs}}\tilde{I}_G$，考查 $\tilde{V}_o$ 易得：

$$\tilde{V}_o\left(sC_L + g_{ds} + g_m\right) = \tilde{I}_G + g_m\tilde{V}_G$$

$$\Rightarrow \tilde{V}_G = \frac{g_m}{sC_L + g_{ds} + g_m}\tilde{V}_G + \frac{1}{sC_L + g_{ds} + g_m}\tilde{I}_G + \frac{1}{sC_{gs}}\tilde{I}_G$$

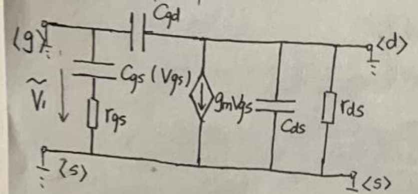
故而有

$$\tilde{Z}_{in} = \frac{sC_L + g_{ds} + g_m}{sC_{gs}\left(sC_L + g_{ds}\right)} + \frac{1}{sC_L + g_{ds}}$$

故而：
$$\tilde{Z}_{in} = \frac{j\omega\left(C_{gs} + C_L\right) + \left(g_m + g_{ds}\right)}{-\omega^2 C_{gs} C_L + j\omega C_{gs} g_{ds}}$$

由于其分母存在一负电阻项，由多数四则运算，其可能进入右半平面，$Re\{\tilde{Z}_{in}\} = R_{in} < 0$。此时从栅级看入放大器呈负阻特性。电路退化为负阻振荡器，可能产生抖动。

最高振荡频率求解



f_{max}: 最大功率增益为1时的频率. 所谓最大功率增益为1,

即: $P = \frac{1}{2}(\tilde{V}_1 \tilde{I}_1^H + \tilde{V}_1^H \tilde{I}_1) + \frac{1}{2}(\tilde{V}_2 \tilde{I}_2^H + \tilde{V}_2^H \tilde{I}_2) \leqslant 0$

考查有源性条件. 即有: $(g_{12}+g_{21})^2 + (b_{12}-b_{21})^2 \geqslant 4 g_{11} g_{22}$.

问题退化为求解该MOS的Laplace域y参量:

$$\begin{bmatrix} \tilde{i}_1 \\ \tilde{i}_2 \end{bmatrix} = \begin{bmatrix} g_{11}+jb_{11}, & g_{12}+jb_{12} \\ g_{21}+jb_{21}, & g_{22}+jb_{22} \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix}$$

$$g_{11}+jb_{11} = \left.\frac{\tilde{i}_1}{\tilde{V}_1}\right|_{\tilde{V}_2=0} = \frac{1}{\frac{1}{sC_{gd}} \parallel \left(r_{gs} + \frac{1}{sC_{gs}}\right)}$$

$$g_{11}+jb_{11} = \frac{\frac{1}{sC_{gd}} + \frac{1}{sC_{gs}} + r_{gs}}{\frac{1}{sC_{gd}} \cdot \left(r_{gs} + \frac{1}{sC_{gs}}\right)} = \frac{s^2 C_{gs} C_{gd} r_{gs} + s(C_{gd}+C_{gs})}{r_{gs} \cdot s C_{gs} + 1}$$

$$= \frac{s^2 C_{gs} C_{gd} r_{gs} + s(C_{gs}+C_{gd})}{1 + s C_{gs} r_{gs}} = \frac{j\omega(C_{gs}+C_{gd}) - \omega^2 C_{gs} C_{gd} r_{gs}}{1 + j\omega C_{gs} r_{gs}}$$

$$= \frac{1}{1+\omega^2 C_{gs}^2 r_{gs}^2}\left(-\omega^2 C_{gs} C_{gd} r_{gs} + j\omega(C_{gs}+C_{gd})\right)\left(1 - j\omega C_{gs} r_{gs}\right)$$

$$= \frac{-\omega^2 C_{gs} C_{gd} r_{gs} + \omega^2 C_{gs} r_{gs}(C_{gs}+C_{gd})}{1+\omega^2 C_{gs}^2 r_{gs}^2} + j\left(\frac{\omega^3 C_{gs}^2 C_{gd} r_{gs}^2}{1+\omega^2 C_{gs}^2 r_{gs}^2} + \frac{\omega(C_{gs}+C_{gd})}{1+\omega^2 C_{gs}^2 r_{gs}^2}\right)$$

$$= \frac{\omega^2 C_{gs}^2 r_{gs}}{1+\omega^2 C_{gs}^2 r_{gs}^2} + \frac{j\omega}{1+\omega^2 C_{gs}^2 r_{gs}^2}\left[\omega^2 C_{gs}^2 C_{gd} r_{gs}^2 + C_{gs} + C_{gd}\right]$$

$$\begin{cases} g_{11} = \dfrac{\omega^2 C_{gs}^2 r_{gs}}{1+\omega^2 C_{gs}^2 r_{gs}^2} \\ b_{11} = \dfrac{\omega}{1+\omega^2 C_{gs}^2 r_{gs}^2}\left[\omega^2 C_{gs}^2 C_{gd} r_{gs}^2 + C_{gs} + C_{gd}\right] \end{cases}$$

$$g_{22}+jb_{22} = \left.\frac{\tilde{i}_2}{\tilde{V}_2}\right|_{\tilde{V}_1=0} = g_{ds} + sC_{ds} + sC_{gd}$$

$$\Rightarrow \begin{cases} g_{22} = g_{ds} \\ b_{22} = \omega(C_{ds}+C_{gd}) \end{cases}$$

$$g_{12}+jb_{12} = \left.\frac{\tilde{i}_1}{\tilde{V}_2}\right|_{\tilde{V}_1=0} = \left.-sC_{gd}\right|_{s=j\omega} = -j\omega C_{gd}$$

$$\Rightarrow \begin{cases} g_{12} = 0 \\ b_{12} = -\omega C_{gd} \end{cases}$$

$$g_{21}+jb_{21} = \left.\frac{\tilde{i}_2}{\tilde{V}_1}\right|_{\tilde{V}_2=0} = \left(-\tilde{V}_1 \cdot sC_{gd} + \tilde{V}_1 \frac{\frac{1}{sC_{gs}}}{r_{gs}+\frac{1}{sC_{gs}}} \cdot g_m\right) \Big/ \tilde{V}_1$$

$$= g_m \cdot \frac{\frac{1}{sC_{gs}}}{r_{gs}+\frac{1}{sC_{gs}}} - s \cdot C_{gd}$$

$$= g_m \cdot \frac{1}{1+sC_{gs} r_{gs}} - sC_{gd} = \frac{g_m}{1+j\omega C_{gs} r_{gs}} - j\omega C_{gd}$$

$$= \frac{g_m}{1+\omega^2 C_{gs}^2 r_{gs}^2}(1 - j\omega C_{gs} r_{gs}) - j\omega C_{gd}$$

$$= \frac{g_m}{1+\omega^2 C_{gs}^2 r_{gs}^2} - j\omega\left(C_{gd} + \frac{g_m C_{gs} r_{gs}}{1+\omega^2 C_{gs}^2 r_{gs}^2}\right)$$

$$\begin{cases} g_{21} = \dfrac{g_m}{1+\omega^2 C_{gs}^2 r_{gs}^2} \\ b_{21} = -\omega\left(C_{gd} + \dfrac{g_m C_{gs} r_{gs}}{1+\omega^2 C_{gs}^2 r_{gs}^2}\right) \end{cases}$$

于是就获得了其y参量:

$$y = \begin{bmatrix} \dfrac{\omega^2 C_{gs}^2 r_{gs}}{1+\omega^2 C_{gs}^2 r_{gs}^2} + j\dfrac{\omega}{1+\omega^2 C_{gs}^2 r_{gs}^2}\left[\omega^2 C_{gs}^2 C_{gd} r_{gs}^2 + C_{gs} + C_{gd}\right], & -j\omega C_{gd} \\ \dfrac{g_m}{1+\omega^2 C_{gs}^2 r_{gs}^2} - j\omega\left(C_{gd} + \dfrac{g_m C_{gs} r_{gs}}{1+\omega^2 C_{gs}^2 r_{gs}^2}\right), & g_{ds} + j\omega(C_{gd}+C_{ds}) \end{bmatrix}$$

由其有源性条件: $(g_{12}+g_{21})^2+(b_{12}-b_{21})^2 \geqslant 4g_{11}g_{22}$.

由: $\begin{cases} g_{12}=0 \\ g_{21} = \dfrac{g_m}{1+\omega^2 C_{gs}^2 r_{gs}^2} \end{cases}$ $\begin{cases} b_{12} = -\omega C_{gd} \\ b_{21} = -\omega\left(C_{gd} + \dfrac{g_m C_{gs} r_{gs}}{1+\omega^2 C_{gs}^2 r_{gs}^2}\right) \end{cases}$

$$\begin{cases} g_{11} = \dfrac{\omega^2 C_{gs} r_{gs}}{1+\omega^2 C_{gs}^2 r_{gs}^2} \\ g_{22} = g_{ds}. \end{cases}$$

即有: $\left(\dfrac{g_m}{1+\omega^2 C_{gs}^2 r_{gs}^2}\right)^2 + \left(\dfrac{\omega g_m C_{gs} r_{gs}}{1+\omega^2 C_{gs}^2 r_{gs}^2}\right)^2 \geqslant 4g_{ds}\cdot\dfrac{\omega^2 C_{gs} r_{gs}}{1+\omega^2 C_{gs}^2 r_{gs}^2}$

即: $g_m^2 + g_m^2 C_{gs}^2 r_{gs}^2 \omega^2 \geqslant 4\omega_m^2 C_{gs}^2 r_{gs} g_{ds}\left(1+\omega^2 C_{gs}^2 r_{gs}^2\right)$.

$$\Rightarrow 4C_{gs}^4 r_{gs}^3 g_{ds}(\omega^2)^2 + \left[4C_{gs}^2 r_{gs} g_{ds} - g_m^2 C_{gs}^2 r_{gs}^2\right]\omega^2 - g_m^2 \leqslant 0$$

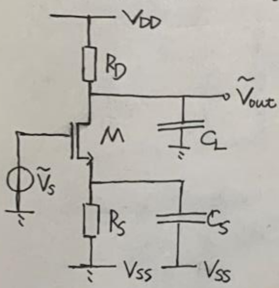
$$(\omega^2)^2 + \frac{4C_{gs}^2 r_{gs} g_{ds} - g_m^2 C_{gs}^2 r_{gs}^2}{4C_{gs}^4 r_{gs}^3 g_{ds}}\omega^2 - \frac{g_m^2}{4C_{gs}^4 r_{gs}^3 g_{ds}} \leqslant 0.$$

$$\omega_{max}^2 = \frac{1}{2}\left[\frac{g_m^2 r_{gs} - 4g_{ds}}{4C_{gs}^2 r_{gs}^2 g_{ds}} + \sqrt{\left(\frac{4g_{ds} - g_m^2 r_{gs}}{4g_{ds} C_{gs}^2 r_{gs}^2}\right)^2 + \frac{g_m^2}{C_{gs}^4 r_{gs}^3 g_{ds}}}\right].$$

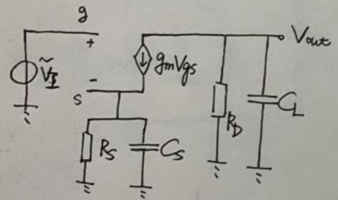
$$\Rightarrow f_{max} = \frac{1}{2\pi}\sqrt{\frac{1}{2}\left[\frac{g_m^2 r_{gs} - 4g_{ds}}{4C_{gs}^2 r_{gs}^2 g_{ds}} + \sqrt{\left[\frac{4g_{ds} - g_m^2 r_{gs}}{4g_{ds} C_{gs}^2 r_{gs}^2}\right]^2 + \frac{g_m^2}{C_{gs}^4 r_{gs}^3 g_{ds}}}\right]}.$$

Homework I：Band Expansion：

零點補償極點法： $g_m R_D = 9$, $\omega_0 = \frac{1}{R_D C_L}$



其小信号电路是直观的：



显然有：$g_m(\tilde{V}_I - \tilde{V}_S)(R_S \| \frac{1}{sC_S}) = \tilde{V}_S$

$$\tilde{V}_{out} = -g_m(\tilde{V}_I - \tilde{V}_S)(R_D \| \frac{1}{sC_L})$$

故有：$\tilde{V}_S = \frac{g_m (R_S \| \frac{1}{sC_S})}{1 + g_m (R_S \| \frac{1}{sC_S})} \tilde{V}_I$

$$\tilde{V}_{out} = -g_m \cdot \frac{R_D}{1+sC_L R_D} \cdot \frac{1}{1+g_m(R_S \| \frac{1}{sC_S})} \tilde{V}_I$$

$$\Rightarrow \tilde{H}(s) = \frac{\tilde{V}_{out}}{\tilde{V}_I} = -g_m R_D \frac{1}{1+sC_L R_D} \cdot \frac{1}{1+g_m \cdot \frac{1}{\frac{1}{R_S}+sC_S}}$$

$$= -g_m R_D \frac{1}{1+sC_L R_D} \cdot \frac{1}{1+g_m \frac{R_S}{1+sC_S R_S}}$$

$$= -g_m R_D \frac{1}{1+sC_L R_D} \cdot \frac{1+sC_S R_S}{1+g_m R_S + sC_S R_S}$$

$$= -g_m R_D \cdot \frac{1}{1+g_m R_S} \cdot \frac{1}{1+sC_L R_D} \cdot \frac{1+sC_S R_S}{1+s\cdot\frac{C_S R_S}{1+g_m R_S}}$$

$$\tilde{H}(s) = -\frac{g_m R_D}{1+g_m R_S} \cdot \frac{1}{1+sC_L R_D} \cdot \frac{1+sC_S R_S}{1+\frac{sC_S R_S}{1+g_m R_S}}$$

若欲零极点补偿，应有：$C_L R_D = C_S R_S$.

$$\boxed{\text{即：} \omega_0 = \frac{1}{C_S R_S}}$$

此时该放大器传输函数变为：

$$\tilde{H}(s) = -\frac{9}{1+g_m R_S} \cdot \frac{1}{1+s\frac{C_S R_S}{1+g_m R_S}}$$

$$\Rightarrow \omega_0' = \frac{1+g_m R_S}{C_S R_S} \gg \frac{1}{C_S R_S} = \frac{1}{C_L R_D} = \omega_0.$$

此时就完成了Band Expansion的工作。

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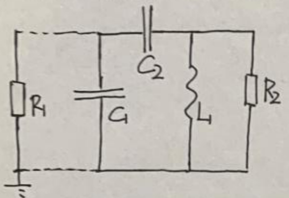
Homework II：A 類功放匹配网络



$f_C = 50\,\text{MHz}$

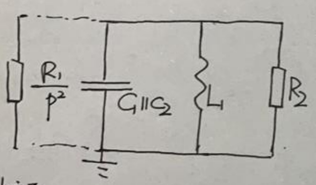
$Q_L = 10$

$R_2 = 200\Omega$

功放最佳匹配负载是易解的：

$$R_{L,opt} = \frac{(V_{CC} - V_{CE,sat})^2}{2 \cdot P_{out}} = 132.25\,\Omega$$

在谐振频点处可作部分接入分析：



$$p = \frac{C_2}{C_1 + C_2}$$

此时应有：

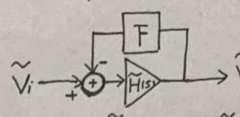
$$\begin{cases} 2\pi f_C = \dfrac{1}{\sqrt{(C_1 \| C_2) L_1}} \\ Q_L = \dfrac{\sqrt{\dfrac{C_1 \| C_2}{L_1}}}{\dfrac{1}{R_2}} \\ R_2 \left(\dfrac{C_2}{C_1 + C_2}\right)^2 = R_1 \end{cases}$$

联合理解有：

$$\begin{cases} C_1 = 0.002 \times 10^{-7} F = 0.2\,nF \\ C_2 = 0.0085 \times 10^{-7} = 0.85\,nF \\ L_1 = 0.6366 \times 10^{-7} = 63.66\,nH \end{cases}$$

Homework III

二阶低通加负反馈：



$$\tilde{H}_{n.F}(s) = \frac{\tilde{V}_o(s)}{\tilde{V}_i(s)} = \frac{\tilde{H}(s)}{1 + F\tilde{H}(s)} = \frac{\dfrac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}}{1 + F \cdot \dfrac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}}$$

$$= \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + (1+F)\omega_0^2}$$

$$= \frac{1}{1+F} \cdot \frac{(\sqrt{1+F}\,\omega_0)^2}{s^2 + \frac{\sqrt{1+F}\,\omega_0}{\sqrt{1+F}\,Q}s + (\sqrt{1+F}\,\omega_0)^2}$$

若期望幅度最大平坦，则应有：

$$\sqrt{1+F} \cdot Q = \frac{1}{\sqrt{2}} \Rightarrow F = \frac{1}{2Q^2} - 1.$$

若期望群延时最大平坦，则应有：

$$\sqrt{1+F}\,Q = \frac{1}{\sqrt{3}} \Rightarrow F = \frac{1}{3Q^2} - 1.$$

由于Q值很低，不管是幅度最大平坦还是群延时最大平坦，都需要很大的反馈增益。