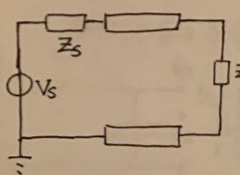
清华大学 TSINGHUA UNIVERSITY **数学作业纸**

班级： 姓名：刘开济 编号：2 科目：通信电路



$Z_S = 10\Omega$
$Z_L = 75\Omega$
$Z_0 = 50\Omega$
$V_S(t) = u(t)$

1. 為什麼是這樣的波形？

由于信源内阻 R_S、负载电阻 R_L 均不匹配。信源发射的信号到达负载时，由于不匹配会产生一个反射电压；反射电压回到电源后由于不匹配又会再次产生回到负载的反射信号。如此往复导致了负载电压的振荡特性。

2. 传输线稳定后的反射电压.

先考查第一个延时有：$V_0^+ = V_0 \cdot \dfrac{Z_0}{Z_S+Z_0}$

由 $\begin{cases} \Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} \\ \Gamma_S = \dfrac{Z_S - Z_0}{Z_S + Z_0} \end{cases}$

于是 $t_d = 1ns$ 时，负载电压 $V_l(1) = (1+\Gamma_L)V_0^+ = \dfrac{2Z_L}{Z_L+Z_0}\dfrac{Z_0}{Z_S+Z_0}V_0$.

而 $V_l(1) = \dfrac{2\cdot 75}{75+50}\cdot\dfrac{50}{10+50}V_0 = V_0 = 1V$.

当来自负载的反射电压 $V_{ref}(1) = \Gamma_L \cdot V_0^+$ 回到信源时，由于信源不匹配，立刻就有再次反射回负载的信号：

$$V_{ref}(2) = \Gamma_L \cdot V_0^+ \cdot \Gamma_S = V_0^+(\Gamma_L\Gamma_S)^1 \doteq -0.1111V$$

负载获得的信号：$V_l(3) = V_{ref}(2)(1+\Gamma_L) = -0.133V$

于是此时传输线负载电压.

$$V = \overset{V_l(1)}{\cancel{V_l(1)}} + V_l(3) = 0.8667V.$$

类似地：$V_l(5) = (1+\Gamma_L)\Gamma_L\cdot\Gamma_S\, V_{ref}(2)$

$$= (1+\Gamma_L)\Gamma_L\Gamma_S\cdot(\Gamma_S\Gamma_L)V_0^+ = \underline{V_0^+(1+\Gamma_L)(\Gamma_S\Gamma_L)^2}.$$

故：$V_{stable} = \{V_0^+(1+\Gamma_L)\left[(\Gamma_S\Gamma_L)^0 + (\Gamma_S\Gamma_L)^1 + \cdots\right]$

$$= V_0^+(1+\Gamma_L)\cdot\frac{1}{1-\Gamma_S\Gamma_L} = \boxed{0.8824V}$$

故有传输线时的稳定电压为：

$$V_{stable} = 0.8824V$$

3: 无传输线时电压

若不存在传输线，则一个 t_d 后信号即稳定，有：

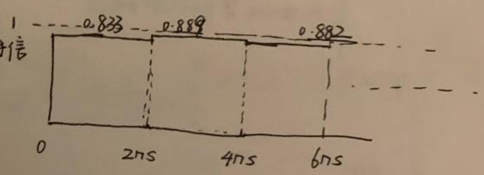
$$V = V_0 \cdot \frac{Z_L}{Z_L+Z_S} = 0.8824V.$$

即两者相等。

理解：由于传输线是无损传输线，因而其双端不匹配并不影响系统的稳态解。但不匹配会放大其瞬态解，因而无传输线时 1ns 即到达稳态，有传输线时则需要很长时间。

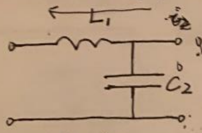
4. 不匹配则数字信号不稳定，影响到晶体管的判定阈值问题。这就必需提高晶体管摆幅以维持高的電壓裕度，就会影响数字电路的速度。

5: 输入端口波形：



显然，输入端口收敛速度强于输出端口.

二. 考查网络Z参量与ABCD矩阵：



$$\begin{bmatrix}\tilde{V}_1\\ \tilde{V}_2\end{bmatrix}=\begin{bmatrix}Z_{11}, & Z_{12}\\ Z_{21}, & Z_{22}\end{bmatrix}\begin{bmatrix}\tilde{I}_1\\ \tilde{I}_2\end{bmatrix}$$

立即得到：

$$\begin{cases}Z_{11}=j\omega L_1+\dfrac{1}{j\omega C_2}\\ Z_{22}=\dfrac{1}{j\omega C_2}\\ Z_{21}=\dfrac{1}{j\omega C_2}\\ Z_{12}=\dfrac{1}{j\omega C_2}\end{cases}$$

故有 $Z=\begin{bmatrix}j\omega L_1+\dfrac{1}{j\omega C_2}, & \dfrac{1}{j\omega C_2}\\ \dfrac{1}{j\omega C_2}, & \dfrac{1}{j\omega C_2}\end{bmatrix}$

由：$\begin{bmatrix}V_1\\ i_1\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}V_2\\ -i_2\end{bmatrix}$.

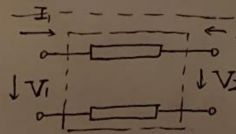
$$A=\frac{V_1}{V_2}\bigg|_{i_2=0}=\frac{\frac{1}{j\omega C_2}+j\omega L_1}{\frac{1}{j\omega C_2}}=1+(j\omega)^2L_1C_2$$

$$-B=\frac{V_1}{i_2}\bigg|_{V_2=0}=-j\omega L_1\Rightarrow B=j\omega L_1$$

$$C=\frac{i_1}{V_2}\bigg|_{i_2=0}=j\omega C_2.$$

$$-D=\frac{i_1}{i_2}\bigg|_{V_2=0}=-1\Rightarrow D=1.$$

于是 LC网络的ABCD参量为：$\begin{bmatrix}1+(j\omega)^2L_1C_2, & j\omega L_1\\ j\omega C_2, & 1\end{bmatrix}$



$$\begin{cases}V(z)=V_0^+\left(\exp(\gamma z)+\Gamma_0\exp(-\gamma z)\right)\\ I(z)=V_0^+\left(\exp(\gamma z)+\Gamma_0\exp(-\gamma z)\right).\end{cases}$$

根据《射频电路设计》教材，可得其 $V(z)$，$I(z)$ 关系：

$$\begin{cases}V(z)=2jV^+\sin\beta d\\ I(z)=\dfrac{V^+}{Z_0}\cos\beta d\end{cases}\quad(\text{短路时})$$

$$\begin{cases}V(z)=V^+\cos\beta d\\ I(z)=j\dfrac{V^+}{Z_0}\sin\beta d\end{cases}\quad(\text{开路时}).$$

由此先看其Z参量阵：

$Z_{11}=\dfrac{V(l)}{I(l)}$，此时由于端口2开路，需要用开路时的结果：

$$Z_{11}=\frac{V^+\cos\beta l}{j\frac{V^+}{Z_0}\sin\beta l}=-jZ_0\cot\beta l$$

$$Z_{22}=\frac{V(0)}{I(0)}=Z_{11}=-jZ_0\cot\beta l$$

$$Z_{21}=\frac{V(0)}{I(l)}\bigg|_{i_2=0}=\frac{V^+}{j\frac{V^+}{Z_0}\sin\beta l}=-jZ_0/\sin\beta l$$

$$Z_{12}=Z_{21}=-jZ_0/\sin\beta l.$$

故其Z参量为：

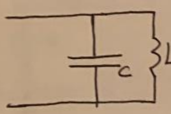
$$Z=\begin{bmatrix}-jZ_0\cot\beta l, & -jZ_0/\sin\beta l\\ -j\dfrac{Z_0}{\sin\beta l}, & -jZ\cot\beta l\end{bmatrix}.$$

类似地可得其ABCD参量.

$$\begin{bmatrix}A & B\\ C & D\end{bmatrix}=\begin{bmatrix}\cos\beta l, & jZ_0\sin\beta l\\ jY_0\sin\beta l, & \cos\beta l\end{bmatrix}.$$

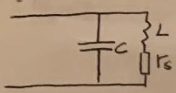
3. 并联諧振带宽分析：

$C = 10pF$



$Q(L) = 200$　　$L = 10\mu H$

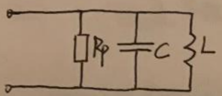
$\Delta f \xrightarrow{?} 40\Delta f$



首先立刻得到：

$$\begin{cases} Z_0 = \sqrt{\frac{L}{C}} = 1000\,\Omega = 1k\Omega \\ r_s = \frac{Z_0}{Q} = 5\Omega \end{cases}$$

于是有等效 $R_p = Q^2 r_s = 200\,k\Omega$



$$\omega_p = \omega_0 \sqrt{1 - \frac{1}{Q^2}}$$

$$Q_p = Q = 200$$

对并联谐振有传递函数：

$$H(s) = A_0 \frac{\frac{1}{Q_p}\left(\frac{s}{\omega_p}\right)}{\left(\frac{s}{\omega_p}\right)^2 + \left(\frac{s}{\omega_p}\right)\frac{1}{Q_p} + 1}$$

于是有 3dB 带宽：$BW_{3dB} = \frac{1}{Q}\frac{f}{f_p}\sqrt{f_1 f_2}$

其中 f_1、f_2 是方程：$\frac{f}{f_p} - \frac{f_p}{f} = \pm\frac{1}{Q}$ 的两根。

可得：

$$\begin{cases} f_1 = 159.6\,MHz \\ f_2 = 158.8\,MHz \end{cases}$$

于是其 3dB 带宽是：$BW_{3dB} = \frac{1}{Q}\sqrt{f_1 f_2} = 7.958 \times 10^5\,Hz$

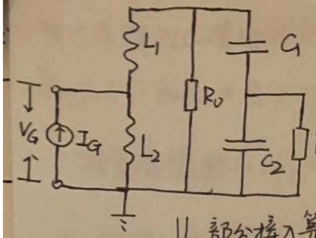
$\approx 795.8\,kHz$

现在若欲扩频，策略：

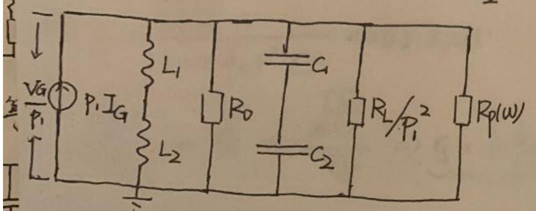
①：减少 Q 值（但这样会降低增益）

②：增大 f_p

习题四.部分接入:



⇓ 部分接入等效: $\begin{cases} p_1 = \dfrac{L_2}{L_1+L_2} \\ p_2 = \dfrac{C_1}{C_1+C_2} \end{cases}$



①: 无阻尼谐振电阻频率 f_m.

$$f_m = \frac{1}{2\pi}\cdot\frac{1}{\sqrt{(L_1+L_2)(C_1\|C_2)}}\cdot\sqrt{1-\frac{1}{Q^2}} = 2.5163\times10^7 Hz$$

故: $f_m = 25.16\ MHz$

②: 谐振时输入电阻 R_{in}

并联谐振时,电路呈阻性 $R_p(\omega_m) = Q^2\cdot\frac{Z_0}{Q} = Q\sqrt{\frac{L_1+L_2}{C_1\|C_2}}$

有 $R_p(\omega_p) = 1.581\times10^5\Omega = 158.1k\Omega$

故有谐振电阻

$$\frac{V_G}{I_G}\cdot\frac{1}{p_1^2} = R_0 \| R_p(\omega_m) \| \frac{R_L}{p_1^2}$$

故而 $R_{in} = p_1^2\ R_0 \| R_p(\omega_m) \| \frac{R_L}{p_1^2} = 4.44\ k\Omega$

③ BW_{3dB}

考查电路整体Q值:

$$Q\Big|_{R_L=40k\Omega} = \left(R_0\|R_p\|\frac{R_L}{p_1^2}\right)\sqrt{\frac{C_1\|C_2}{L_1+L_2}} = 11.2288$$

若拿掉负载电阻R_L,则有

$$Q\Big|_{R_L=\infty} = 20.1904$$

拿不拿掉R_L对谐振频率没有影响,故有

$$BW_{3dB}\Big|_{R_L=10k\Omega} = 2.241\ MHz$$

$$BW_{3dB}\Big|_{R_L=0} = 1.2463\ MHz$$

也就是说拿掉R_L后峰更尖了,选频特性更好

习题五：阻抗测量：

对于串联RLC谐振回路应有：

$$Z_P(j\omega) = j\omega L + r_s + \frac{1}{j\omega C} = r_s + j\left(\omega L - \frac{1}{\omega C}\right)$$

故有谐振频率：$f_0 = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}$

①：短接时有：$f_0 = f_s = 1\,MHz$，$C = 100pF$.

有. $L = \frac{1}{(2\pi f_0)^2 \cdot C} = 253.3\mu H$

$$\frac{V_{pp \cdot C}}{2 \cdot V_p} = Q = \frac{\sqrt{\frac{L}{C}}}{r_s} \Rightarrow r_s = 15.92\Omega$$

②：接入一负载，记 $Z_L = R_L + \frac{1}{j\omega C_L}$. $C = 200pF$

则有. $f_s = f_0 = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L(C \parallel C_L)}}$

$$\Rightarrow 200pF \parallel C_L = 100pF \Rightarrow C_L = 200pF$$

$$\frac{V_{pp.C}}{2 \cdot V_p} = 25 = \frac{\sqrt{\frac{L}{C \parallel C_L}}}{r_s + R_L}$$

$$\Rightarrow R_L = 44.75\Omega.$$ QED.