Rebuttal of Easy-to-Hard Generalization:
Scalable Alignment Beyond Human Supervision

Anonymous Authors[1]

1. Rebuttal Revisions

Below we show revisions that will be incorporated into the paper. We include the relevant subsection title, and highlight text that contains modifications in red. For space considerations we do not show all text from the main paper.
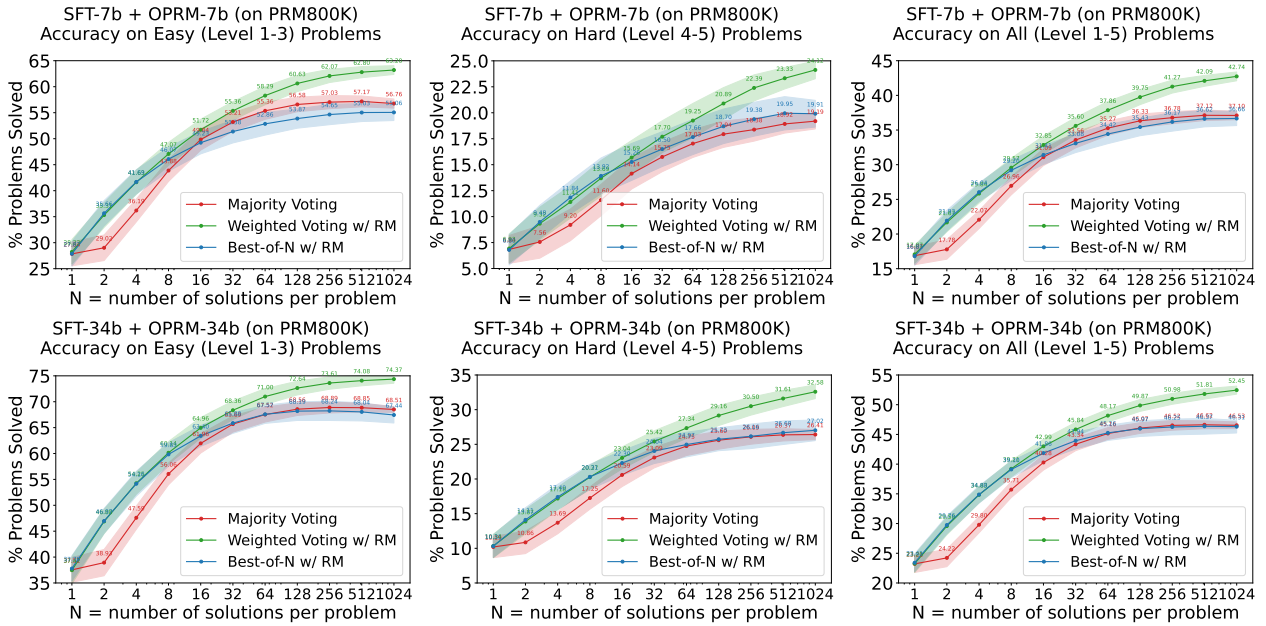

Figure 1. Easy-to-hard generalization of 7b (upper) and 34b (lower) evaluators. Both SFTs and RMs are trained on the easy data. We found that RMs trained on easy tasks can significantly improve the re-ranking (i.e., weighted voting) performance on hard tasks.

2. Revision: Methodology

2.1. Evaluators

Similarly, we consider the following evaluator models that can be trained either on the easy tasks only, or on the full dataset. Notably, unlike final-answer rewards, reward models trained on easy tasks can be applied to evaluate solutions to hard problems.

Final-Answer Reward is a symbolic reward that provides a binary reward based on the accuracy of the model's final answer.

[1] Anonymous Institution, Anonymous City, Anonymous Region, Anonymous Country. Correspondence to: Anonymous Author <anon.email@domain.com>.

The matching is performed after normalization[1].

Outcome Reward Model (ORM) is trained on the Final-Answer rewards. Following Cobbe et al. (2021); Uesato et al. (2022); Lightman et al. (2023), we train the reward head to predict on every token whether the solution is correct, in a similar sense to a value model (Yu et al., 2023a). At inference time, we use the ORM's prediction at the final token as the reward of the solution.

Process Reward Model (PRM) is trained to predict whether each step (delimited by newlines) in the chain-of-thought reasoning path is correct. The labels are usually labeled by humans (Uesato et al., 2022; Lightman et al., 2023) or estimated with rollouts (Silver et al., 2016; Wang et al., 2023).

Outcome & Process Reward Model (OPRM) Building on the distinct advantages of ORMs and PRMs, we introduce the *Outcome & Process Reward Model (OPRM)*, which harnesses the complementary strengths of both. OPRM is trained on the mixed data of ORMs and PRMs. Specifically, OPRM is designed to evaluate the correctness of each intermediate reasoning step, akin to PRMs, while also assessing the overall solution's accuracy at the final answer stage, mirroring the functionality of ORMs.
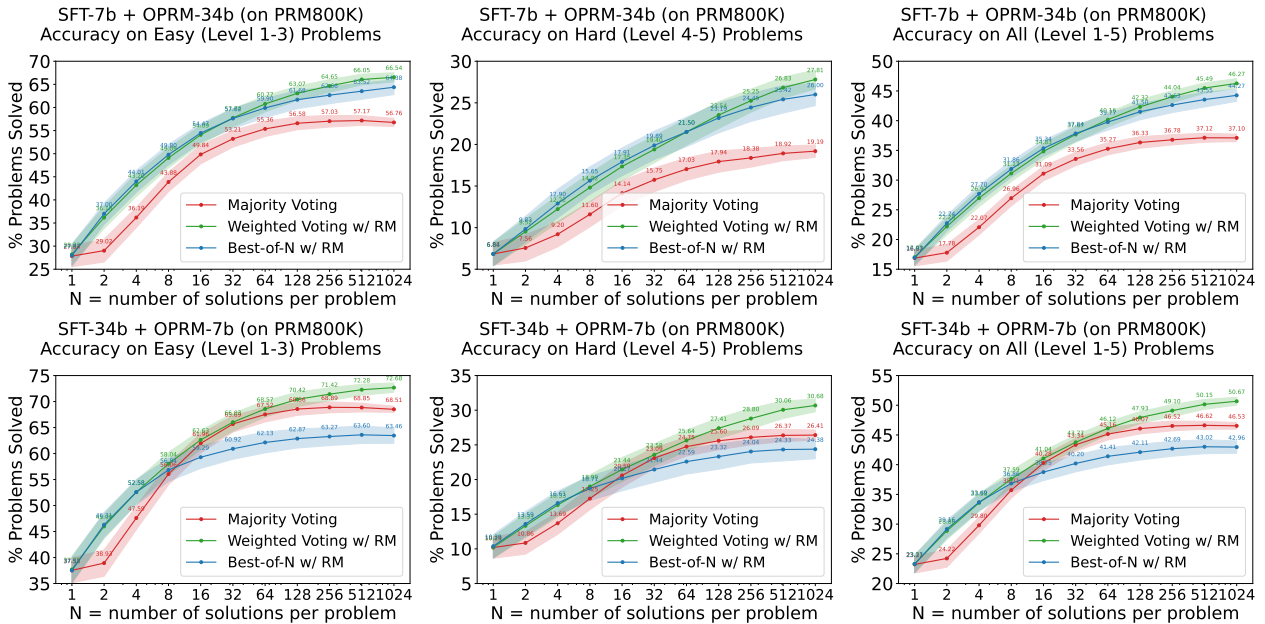

Figure 2. Easy-to-hard generalization of evaluators applied to generators of different sizes. We evaluated 7b generator + 34b evaluator (upper) and 34b generator + 7b evaluator (lower). Both SFTs and RMs are trained on the easy data.

3. Revision: Main Results

3.1. Easy-to-Hard Generalization of Evaluators

The primary metric we use to assess the effectiveness of our process reward model is not the average accuracy of verifying each step in a solution but rather the overall performance achieved through re-ranking methods (See discussion in Sec. **??**). We first use re-ranking to evaluate the easy-to-hard generalization performance of evaluators.

3.1.1. RE-RANKING

We consider two re-ranking strategies: Best-of-n (or rejection sampling) and Weighted Voting. In our easy-to-hard generalization setting, both SFT models and Reward Models (RMs) are trained on easier tasks (levels 1-3), but evaluated on all difficulty levels (1-5). We compare the performance between majority voting (SFT only) and re-ranking (SFT + OPRM) on the PRM800K dataset in Figure 1-2, and the performance of different reward models (PRMs, ORMs, & OPRMs) on the PRM800K dataset in Figure 4-5. Specifically, we use `min` as the reward aggregation function for best-of-n and `prod` for

---

[1] https://github.com/openai/prm800k/blob/main/prm800k/grading/grader.py

Table 1. Comparing reinforcement learning (RL) approaches for easy-to-hard generalization. All methods are of 7b size and evaluated with greedy decoding. † indicates the model is trained with additional final-answer labels on hard tasks (similar to Singh et al. (2023)), which is not strictly a easy-to-hard generalization setup.

| | RL DATA | REWARD | | ACCURACY | | |
		FINAL-ANSWER	PROCESS RM	EASY (LEVEL 1-3)	HARD (LEVEL 4-5)	ALL
(SFT / PRM trained on level 1-3 of PRM800K)						
SFT				28.2	12.2	19.8
REST-EM	EASY	EASY	×	33.2	12.6	22.4
REST-EM	HARD	HARD	×	31.9	8.0	19.4
REST-EM†	ALL	ALL	×	35.7	8.8	21.6
ITERATIVE DPO	EASY	EASY	√	<u>42.0</u>	12.2	26.4
ITERATIVE DPO†	ALL	ALL	√	38.2	11.5	24.2
PPO	EASY	EASY	×	<u>42.0</u>	<u>14.1</u>	<u>27.4</u>
PPO	HARD	HARD	×	34.0	9.2	21.0
PPO†	ALL	ALL	×	<u>42.0</u>	10.7	25.6
PPO	ALL	EASY	√	**45.4**	**14.9**	**29.4**
(SFT / PRM trained on level 1-5 of MetaMath / Math-Shepherd)						
LLEMMA-BASED SFT SoTA (OURS)				51.7	13.7	31.4
PREVIOUS RL SoTA (WANG ET AL., 2023)				-	-	33.0
(SFT / PRM trained on level 1-3 of MetaMath / Math-Shepherd)						
SFT				44.1	14.9	28.8
REST-EM	EASY	EASY	×	50.4	14.5	31.6
ITERATIVE DPO	EASY	EASY	√	**53.8**	**16.0**	**34.0**
ITERATIVE DPO	ALL	EASY	√	<u>49.6</u>	10.7	29.2
ITERATIVE DPO†	ALL	ALL	√	47.9	12.2	29.2
PPO	EASY	EASY	×	<u>50.8</u>	<u>15.3</u>	<u>32.2</u>
PPO†	ALL	ALL	×	<u>50.8</u>	13.4	31.2
PPO	ALL	EASY	√	**53.8**	**16.0**	**34.0**

Table 2. Easy-to-hard generalization of evaluators on coding problems (APPS). Both SFTs and RMs are trained on the easy (Introductory) data. We found that outcome reward models trained on easy tasks can improve the re-ranking (Best-of-N) performance on hard (Interview & Competition) coding problems.

| | SFT / ORM TRAIN DATA | DECODING | AVERAGE ACCURACY (%) | | | | STRICT ACCURACY (%) | | | |
			INTRO.	INTER.	COMP.	ALL	INTRO.	INTER.	COMP.	ALL
CODE LLAMA - 7B	ALL	GREEDY	**31.4**	15.5	**12.2**	**18.0**	17.0	2.3	2.0	5.2
	EASY	GREEDY	26.8	14.1	9.5	15.7	11.0	3.0	0.0	4.0
	EASY	BEST-OF-1	25.4	12.0	0.1	13.5	16.0	2.7	0.0	4.8
	EASY	BEST-OF-4	27.1	13.8	8.1	15.3	14.0	4.0	0.0	5.2
	EASY	BEST-OF-16	29.7	**16.3**	11.3	**18.0**	19.0	5.0	3.0	7.4
CODE LLAMA - 34B	ALL	GREEDY	37.6	19.9	**11.3**	21.7	22.0	5.0	2.0	7.8
	EASY	GREEDY	33.9	19.4	8.5	20.1	21.0	6.0	1.0	8.0
	EASY	BEST-OF-1	28.5	14.5	4.4	15.3	21.0	3.3	0.0	6.2
	EASY	BEST-OF-4	36.3	21.3	10.5	22.1	24.0	8.7	1.0	10.2
	EASY	BEST-OF-16	**45.9**	**25.8**	10.0	**26.6**	30.0	10.7	3.0	13.0

weighted voting[2]. The figures illustrate the performance of different decoding strategies or reward models under the same number of sampled solutions per problem. We have the following findings:

OPRMs outperforms ORMs and PRMs   This confirms our hypothesis that Process Reward Models (PRMs) and Outcome Reward Models (ORMs) capture different aspects of task-solving processes. By integrating the strengths of both PRMs and ORMs, Outcome & Process Reward Models (OPRMs) demonstrate superior performance. However, follow-up experiments conducted on the MetaMath/Math-Shepherd datasets do not demonstrate significant improvements from incorporating

---

[2]See more detailed analysis of reward aggregation functions in Appendix. H.

Table 3. Pass@N scores on coding problems (APPS).

	SFT / ORM TRAIN DATA	DECODING	ACCURACY (%)			
			INTRO.	INTER.	COMP.	ALL
CODE LLAMA - 7B	EASY	PASS @ 1	11.0	1.6	0.0	3.2
	EASY	PASS @ 4	20.0	4.7	2.0	7.2
	EASY	PASS @ 16	39.0	9.3	6.0	14.6
CODE LLAMA - 34B	EASY	PASS @ 1	19.0	5.3	2.0	7.3
	EASY	PASS @ 4	32.0	10.7	3.0	13.4
	EASY	PASS @ 16	44.0	18.3	8.0	21.4

additional ORM training examples. This lack of enhancement may be attributed to the fact that Math-Shepherd is already generated from final-answer rewards. This suggests that there remains a substantial difference between process rewards labeled by humans (e.g., PRM800K) and those generated automatically (e.g., Math-Shepherd).

Weighted voting outshines Best-of-n   This finding diverges from past research where minimal performance differences were observed between weighted voting and Best-of-n (Lightman et al., 2023; Uesato et al., 2022). Our hypothesis is that this discrepancy arises from our specific experiment, which involves training a less powerful base model (Llemma; Azerbayev et al. 2023) on more difficult tasks (MATH; Hendrycks et al. 2021b). This setup might diminish the effectiveness of the reward model, potentially leading to an over-optimization of rewards (Gao et al., 2023). Given these insights, weighted voting is preferred as the primary re-ranking method for further discussions. Nevertheless, Best-of-n still achieves competitive performance to majority voting when producing only one full solution. In Figure 2, we also find that the 34b evaluator can significantly improve the 7b generator, while the 7b evaluator can still improve the performance of the 34b generator.

Greater effectiveness of re-ranking on harder tasks: Weighted voting not only consistently surpasses majority voting but also shows a more pronounced advantage on harder tasks. This observation leads to the conclusion that *evaluators demonstrate better easy-to-hard generalization capabilities in comparison to generators*. This motivates us to explore RL approaches that optimize the generator against the evaluator to further improve the performance of easy-to-hard generation.

3.1.2. REINFORCEMENT LEARNING (RL)

Given the conclusion above, an important question arises: how can evaluators once again assist generators in achieving enhanced easy-to-hard generalization capabilities? We further investigate the enhancement of policy models through RL, utilizing easy-to-hard evaluators as reward models. Similar to re-ranking, SFT and PRM are only trained on easy data. For a fair comparison between PRM800K and MetaMath, we only use vanilla PRMs in the RL training. All the RL methods use the validation accuracy for selecting the best checkpoint[3]. Our comparison spans offline (ReST & DPO) and online (PPO) RL algorithms under various training conditions:

Easy Questions & Easy Final Answers: The SFT model samples from easy questions and receives the corresponding Final-Answer and optional PRM rewards.

All Questions & Easy Final Answers: This assumes access to a range of easy and hard problems for RL training, with rewards for hard tasks solely provided by the easy-to-hard evaluator.

All Questions & All Final Answers: This setting uses all data with the corresponding final answers, which is similar to Singh et al. (2023), but not strictly a easy-to-hard generalization setup.

Based on the results reported in Table 1, we have the following findings:

DPO and PPO excel over ReST: Among the RL algorithms trained on the PRM800K dataset, PPO emerges as the most effective, significantly surpassing both ReST and DPO. On the MetaMATH dataset, PPO and DPO achieve top performance, while ReST shows only marginal improvements over the SFT baseline. The comparative analysis between DPO and PPO across the PRM800K and MetaMATH datasets indicates that while DPO's efficacy is on par with PPO given a high-quality

---

[3]This includes stopping iterations in ReST-EM and iterative DPO, and stopping online steps in PPO.

SFT model as initialization, PPO's effectiveness is less contingent on the quality of the underlying SFT model (Ouyang et al., 2022; Rafailov et al., 2023).

PRM rewards are more beneficial than Final-Answer rewards for hard tasks: Notably, models trained with PRM rewards with human supervision on the easy tasks (achieving a top performance of 34.0) outperform the previous state-of-the-art model trained across all task levels (33.0). This highlights the effectiveness of leveraging easy-to-hard evaluations to improve generator performance across varying task difficulties.

3.2. Easy-to-Hard Generalization on the Coding Domain

We conduct further experiments in the coding domain with the APPS dataset (Hendrycks et al., 2021a). Following Lightman et al. (2023), we sub-sampled 500 questions from the original test set of APPS as our test set. Specifically, we sub-sampled 100 Introductory questions, 300 Interview questions, and 100 Competition questions, following the original distribution in the test set.

In Table 2, we compare the performance of SFT-trained Code Llama (Roziere et al., 2023) (7b & 34b) with greedy decoding and best-of-N approach. In the latter, an Outcome Reward Model (ORM) of the same model size is trained to select the best coding one from N sampled solutions.

We found that while the reward model is only trained on the outcome supervision of easy (Introductory) data, it significantly improves the model performance on hard (Interview & Competition) data. These findings extend the premise of easy-to-hard generalization beyond the confines of mathematical reasoning, suggesting its applicability across diverse domains.

ReST-EM Following Singh et al. (2023), we sample 32 solutions for each question from the language model using top-K sampling with K=40. We also used a cut-off threshold of 10 for the maximum number of solutions per problem (Zelikman et al., 2022; Singh et al., 2023). We performed iterative ReST training for two epochs, and observed performance degeneration starting from the third epoch. For PRM800K, we used a temperature of 1.0, while for MetaMath, we used a temperature of 1.2. The rest training hyper-parameters are the same as in SFT training.

Iterative DPO We sample 8 solutions for each question from the language model using top-K sampling with K=20 and temperature of 1.0. We use the process reward model to assign a score between 0 and 1 to each solution, and use final-answer reward to assign an additional 0/1 score to each solution. A preference training pair is constructed only when the score difference between positive and negative solutions is greater than 1.0. We used a cut-off threshold of 3 for the maximum number of preference pairs per problem.

For all DPO training (Rafailov et al., 2023), we used a learning rate of 2×10^{-6}, a batch size of 64, and a DPO training epoch of 1. We set $\beta = 0.1$ for all DPO experiments, and performed at most 5 DPO iterations (i.e., sampling new solutions and performing one DPO epoch).

PPO We follow Dubois et al. (2023) on the implementation of the PPO algorithm, which is a variant of (Ouyang et al., 2022)[4]. Specifically, we normalize the advantage across the entire batch of rollouts obtained for each PPO step and initialize the value model from the reward model.

We clipped the gradient by its Euclidean norm at a limit of 1. Our training spanned 500 PPO steps on the RL data (MATH questions except MATH500 and our 500 validation questions). For generalized advantage estimation (GAE; Schulman et al. (2015)), both λ and γ were set at 1.

For PRM800K, we used a batch size of 512 for each PPO step. This comprised 8 epochs of gradient steps, each having 64 rollouts. We applied a peak learning rate of 2×10^{-5} with cosine decay. We opted for a constant KL regularizer coefficient of 0.01, and a sampling temperature of 0.7.

For MetaMath/Math-Shepherd, we used a batch size of 512 for each PPO step. This comprised 2 epochs of gradient steps, each having 256 rollouts. We applied a peak learning rate of 5×10^{-6} with cosine decay. We opted for a constant KL regularizer coefficient of 0.002, and a sampling temperature of 1.2.

---

[4]https://github.com/openai/lm-human-preferences

C. Qualitative Comparison

QUESTION	The value of y varies inversely as $\sqrt{x}$ and when $x = 24$, $y = 15$. What is x when $y = 3$?
METAMATH-SFT	If y varies inversely as $\sqrt{x}$, then $y = \frac{k}{\sqrt{x}}$ for some constant k. Plugging in the given values, we have $15 = \frac{k}{\sqrt{24}}$, or $k = \frac{15\sqrt{24}}{24} = \frac{15\sqrt{6}}{4}$. Now we can find x when $y = 3$ by plugging in these values into the equation $y = \frac{k}{\sqrt{x}}$. We have $3 = \frac{\frac{15\sqrt{6}}{4}}{\sqrt{x}}$, or $3 = \frac{15\sqrt{6}}{4\sqrt{x}}$. Cross-multiplying, we get $3 \cdot 4\sqrt{x} = 15\sqrt{6}$, or $12\sqrt{x} = 15\sqrt{6}$. Dividing both sides by 12, we find $\sqrt{x} = \frac{15\sqrt{6}}{12\sqrt{6}} = \frac{5}{2}$. Squaring both sides, we get $x = \left(\frac{5}{2}\right)^2 = \boxed{6.25}$.
METAMATH-PPO	If y varies inversely as $\sqrt{x}$, then we can write the equation as $y = \frac{k}{\sqrt{x}}$ for some constant k. We are given that when $x = 24$, $y = 15$, so we can substitute these values into the equation to solve for k: $15 = \frac{k}{\sqrt{24}}$ Multiplying both sides by $\sqrt{24}$ gives $15\sqrt{24} = k$ Simplifying the left side gives $15 \cdot 2\sqrt{6} = k$, which further simplifies to $30\sqrt{6} = k$ Now we can use this value of k in the equation to find x when $y = 3$: $3 = \frac{30\sqrt{6}}{\sqrt{x}}$ Multiplying both sides by $\sqrt{x}$ gives $3\sqrt{x} = 30\sqrt{6}$ Dividing both sides by 3 gives $\sqrt{x} = 10\sqrt{6}$ Squaring both sides gives $x = (10\sqrt{6})^2 = 100(6) = \boxed{600}$

Table 5. Qualitative comparison between MetaMath-7b-SFT and MetaMath-7b-PPO, as well as the first error detected in MetaMath-SFT greedy decoding by Math-Shepherd. Examples are cherry-picked for better illustration.
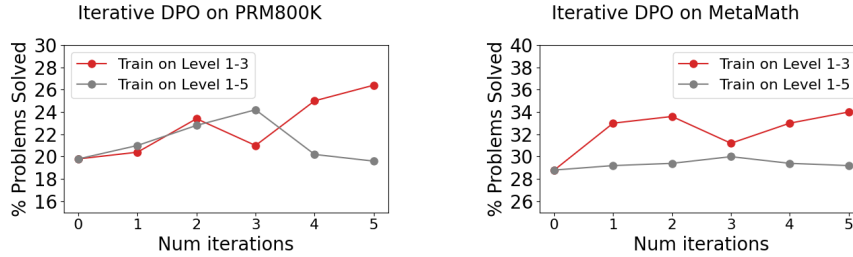
D. Performance of DPO Iterations

Figure 3. Test performance as a function of DPO iterations.

E. Re-ranking Performance Analysis of PRMs, ORMs & OPRMs

We compare the re-ranking performance of Process Reward Models (PRMs), Outcome Reward Models (ORMs), and our proposed Outcome & Process Reward Models (OPRMs). Figure 4 shows the results on 7b models and Figure 5 is on 34b models. We find that in our setting of Llemma (Azerbayev et al., 2023) + MATH (Hendrycks et al., 2021b), PRMs and ORMs perform similarly, with PRMs slightly outperforming ORMs on hard tasks. But the OPRMs that trained on the mixed data of PRMs and ORMs significantly outperforms both of them.
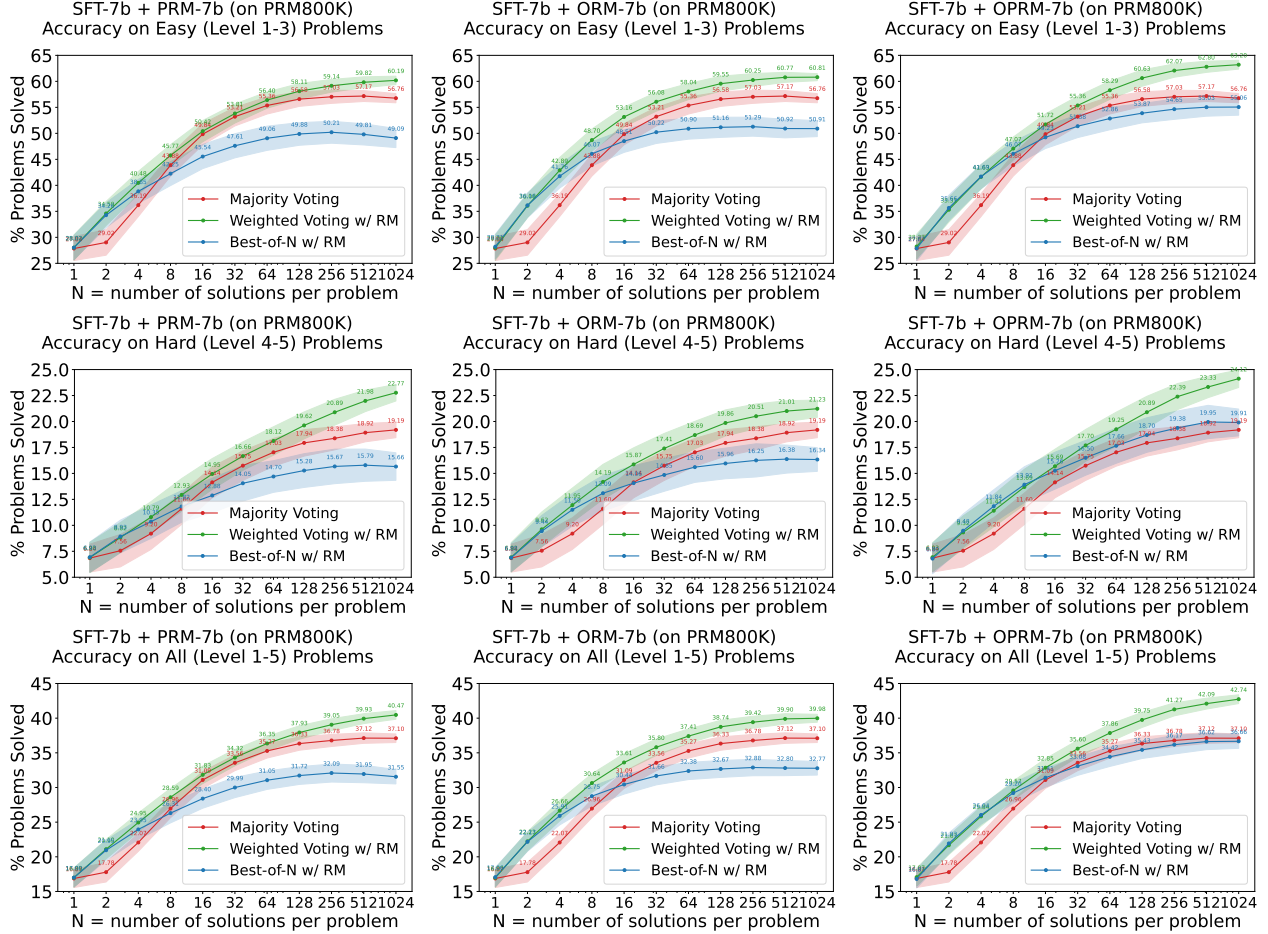

Figure 4. Comparing process reward models (PRMs, left), outcome reward models (ORMs, middle), and outcome & process reward models (OPRMs, right) on 7b models trained on the PRM800K dataset. Both SFTs and RMs are trained on the easy data.
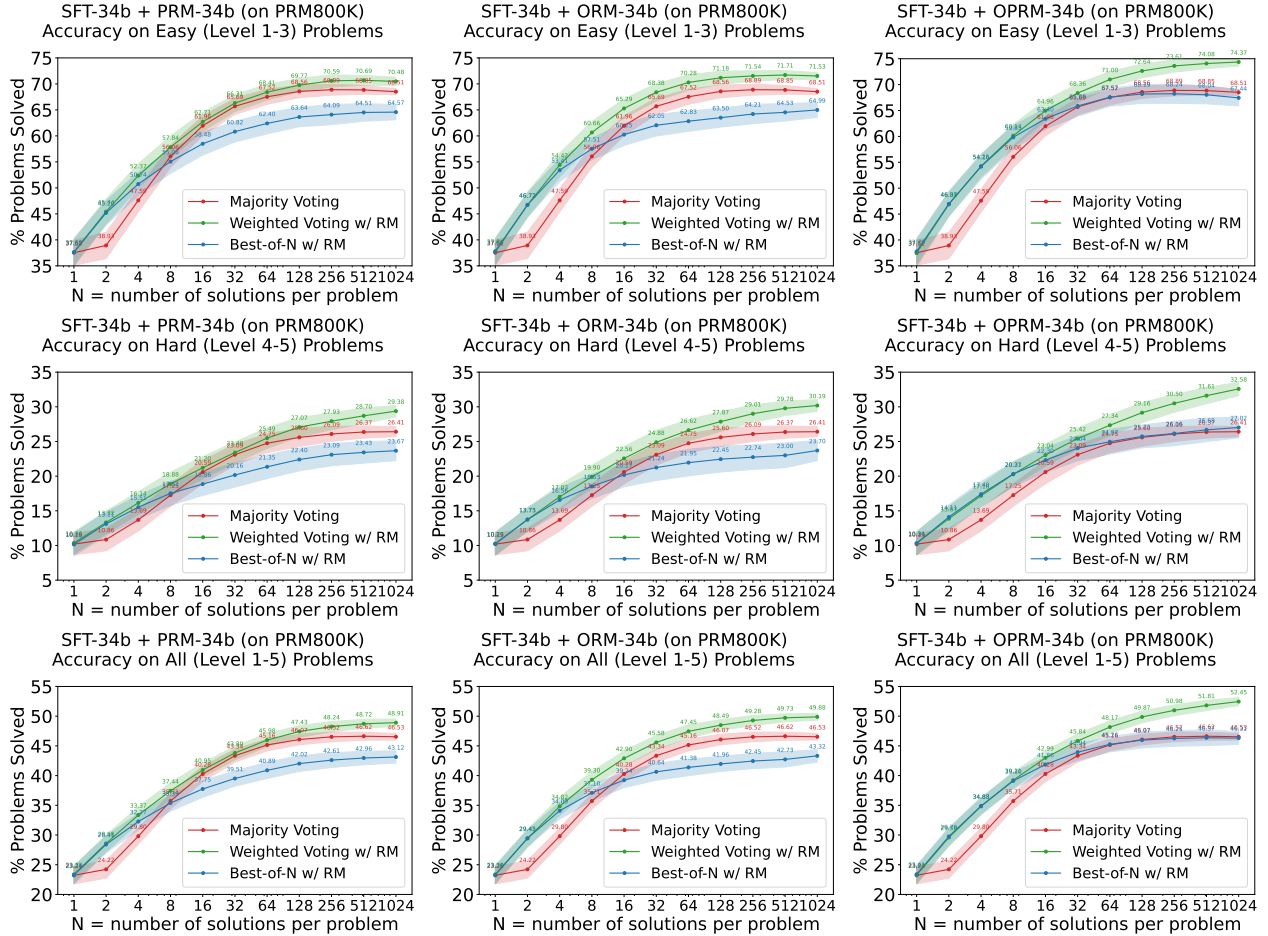
Figure 5. Comparing process reward models (PRMs, left), outcome reward models (ORMs, middle), and outcome & process reward models (OPRMs, right) on 34b models trained on the PRM800K dataset. Both SFTs and RMs are trained on the easy data.

F. Re-ranking Results on MetaMath

Similar to Sec. 3.1.1, we assess the effectiveness of process reward models on the MetaMath/Math-Shepherd dataset (Yu et al., 2023b; Wang et al., 2023). From Figure 6, we can see that PRMs are also more effective on harder tasks when trained on MetaMath/Math-Shepherd.
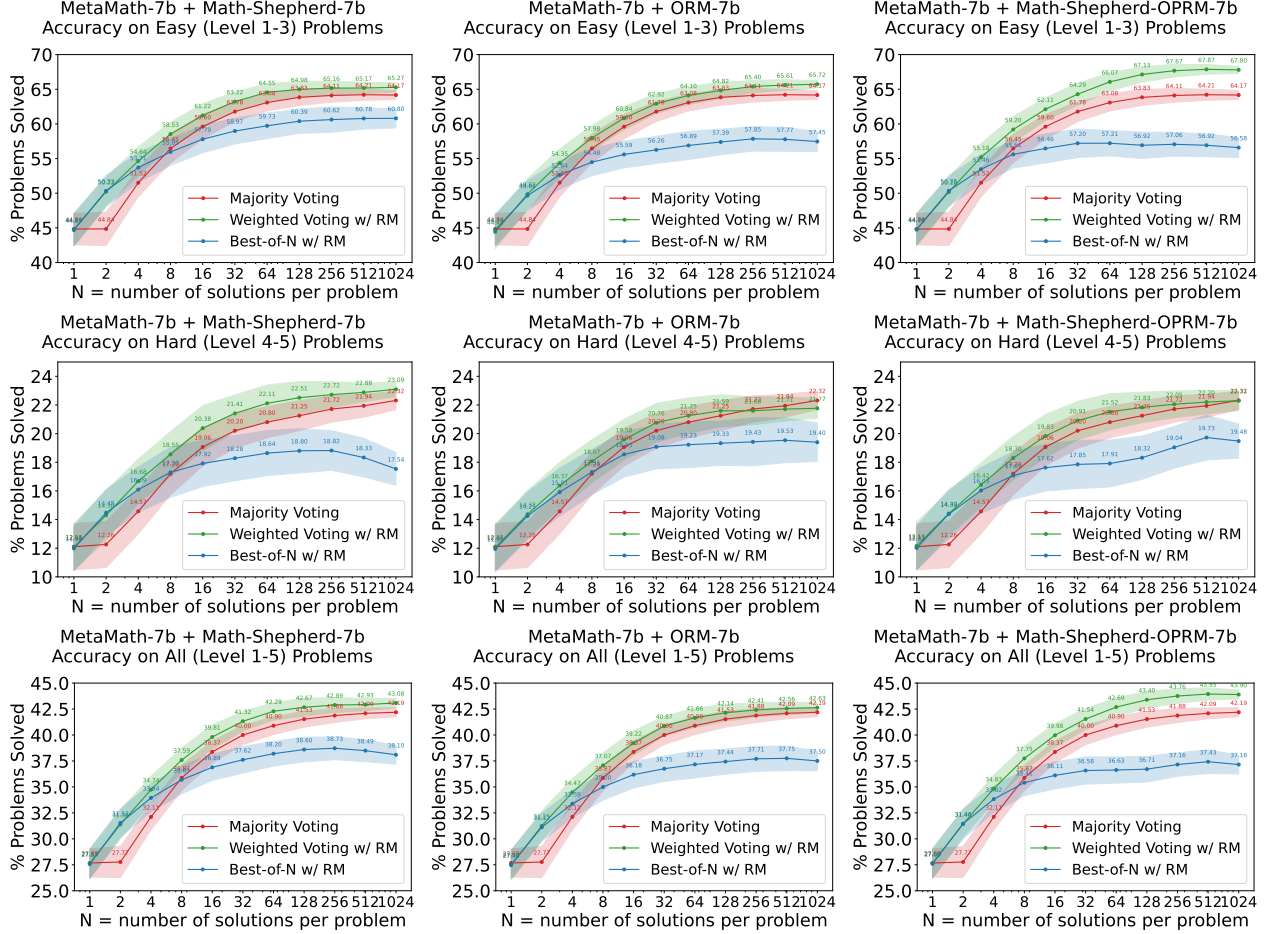

Figure 6. Comparing process reward models (PRMs, left, trained on Meth-Shepherd), outcome reward models (ORMs, middle), and outcome & process reward models (OPRMs, right) on 7b models trained on the MetaMath dataset. Both SFTs and RMs are trained on the easy data.
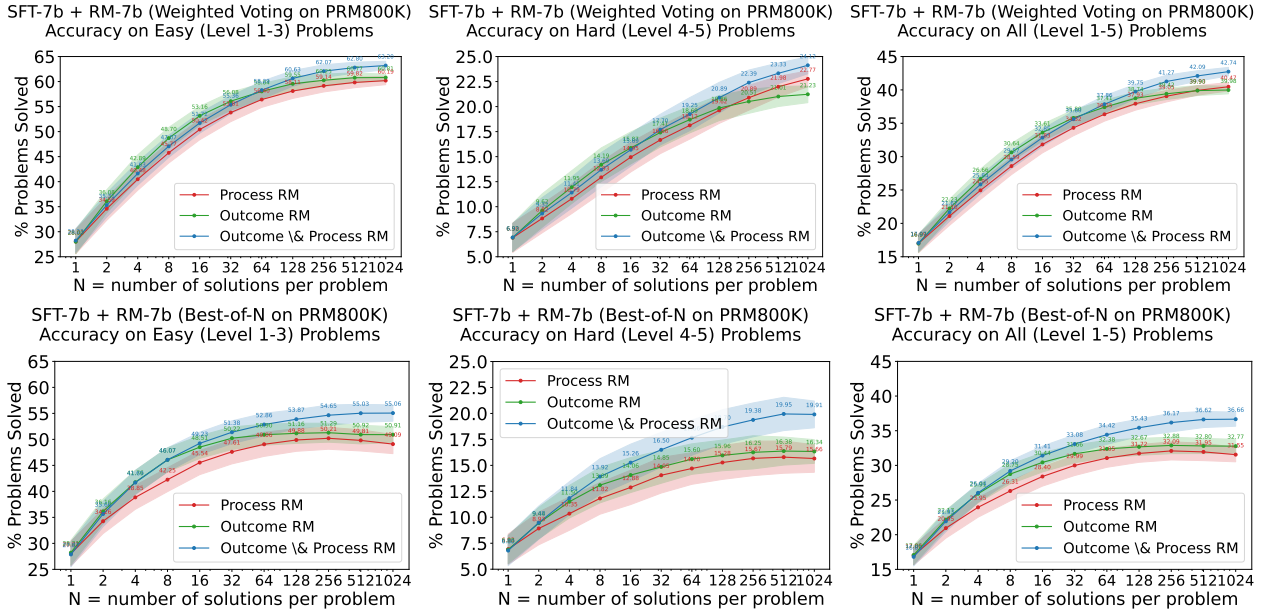
G. More Comparisons

Figure 7. Comparing different reward models with Weighted Voting (upper) and Best-of-N (lower) on 7b models trained on the PRM800K dataset. Both SFTs and RMs are trained on the easy data.
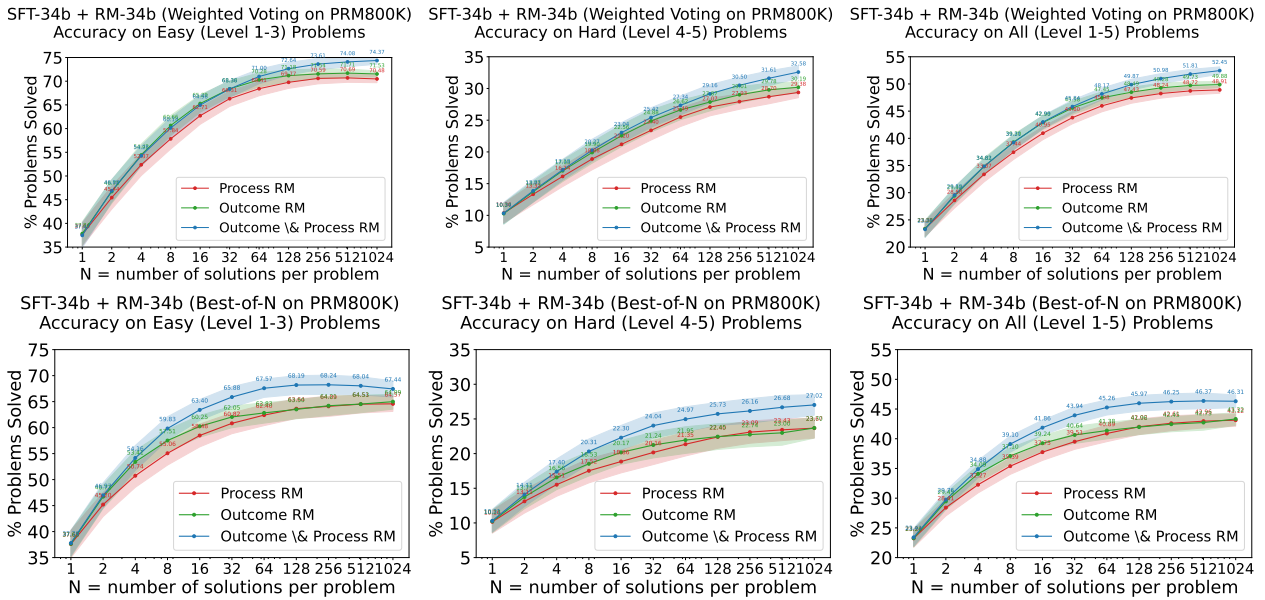

Figure 8. Comparing different reward models with Weighted Voting (upper) and Best-of-N (lower) on 34b models trained on the PRM800K dataset. Both SFTs and RMs are trained on the easy data.

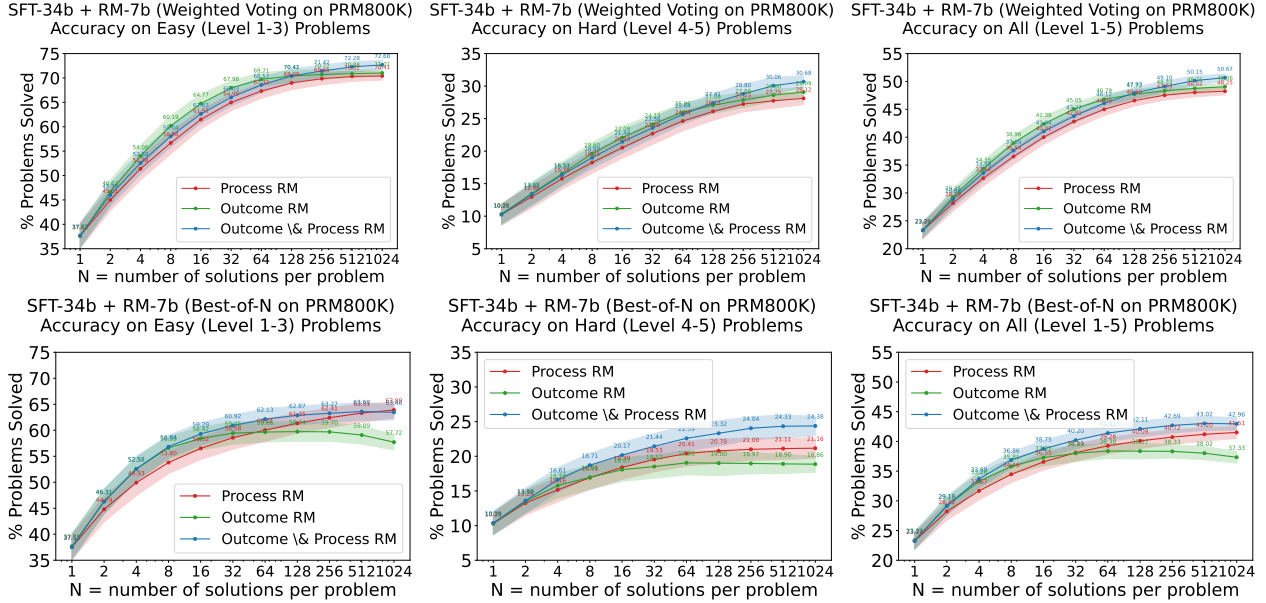*Figure 9.* Comparing different reward models with Weighted Voting (upper) and Best-of-N (lower) on 34b SFT model and 7b reward model trained on the PRM800K dataset. Both SFTs and RMs are trained on the easy data.
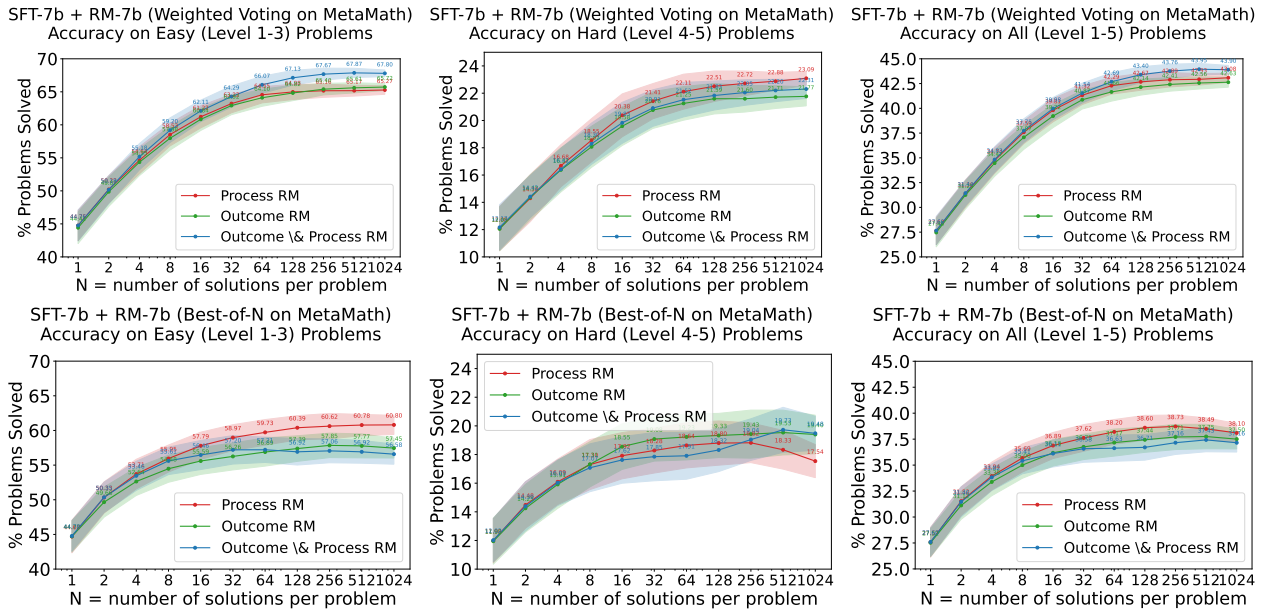

Figure 10. Comparing different reward models with Weighted Voting (upper) and Best-of-N (lower) on 7b models trained on the MetaMath dataset. Both SFTs and RMs are trained on the easy data.

H. Analysis of Aggregation Functions in PRMs & OPRMs

We explored different methods to consolidate step-wise prediction scores into a single score value, a process we describe as employing an aggregation function, during the use of the evaluator. Lightman et al. (2023) report comparable performance when using `min` (minimum) and `prod` (product) as the aggregation function to reduce multiple scores into a single solution-level score. Note that when training PRMs on PRM800K (Lightman et al., 2023), we have already considered neutral steps to be positive as training labels.

Following Wang et al. (2024), given $\{p_1, p_2, \ldots, p_n\}$ as a list of predicted correctness probability of each step (including the final answer), we considered the following aggregation functions:

$$\mathtt{min} = \min\{p_1, p_2, \ldots, p_n\} \tag{1}$$

$$\mathtt{max} = \max\{p_1, p_2, \ldots, p_n\} \tag{2}$$

$$\mathtt{prod} = \prod_i p_i \tag{3}$$

$$\mathtt{mean} = \frac{\sum_i p_i}{n} \tag{4}$$

$$\mathtt{mean\_logit} = \sigma\left(\frac{\sum_i \log \frac{p_i}{1-p_i}}{n}\right) \tag{5}$$

$$\mathtt{mean\_odd} = \mathrm{ReLU}\left(\frac{\sum_i \frac{p_i}{1-p_i}}{n}\right) \tag{6}$$

$$\mathtt{last} = p_n \tag{7}$$

In Figure 11-13, we perform analysis of aggregation functions on PRM800K and Math-Shepherd (from MetaMath) datasets with weighted voting and best-of-n decoding and PRMs or OPRMs. On PRM800K, we find `prod` works well in weighted voting and `min` works well in best-of-n. On MetaMath, we find `min` works well in both weighted voting and best-of-n. So we adopt these strategies in our main experiments, where the RL uses the same strategy as best-of-n.

One interesting finding is that for reward models trained on the human annotated process reward (e.g., PRM800K), the `last` strategy does not perform very well, but `last` works much better on OPRMs and pseudo PRMs (e.g., Math-Shepherd). This could partially explain why OPRMs does not further improve the performance on the Math-Shepherd dataset.
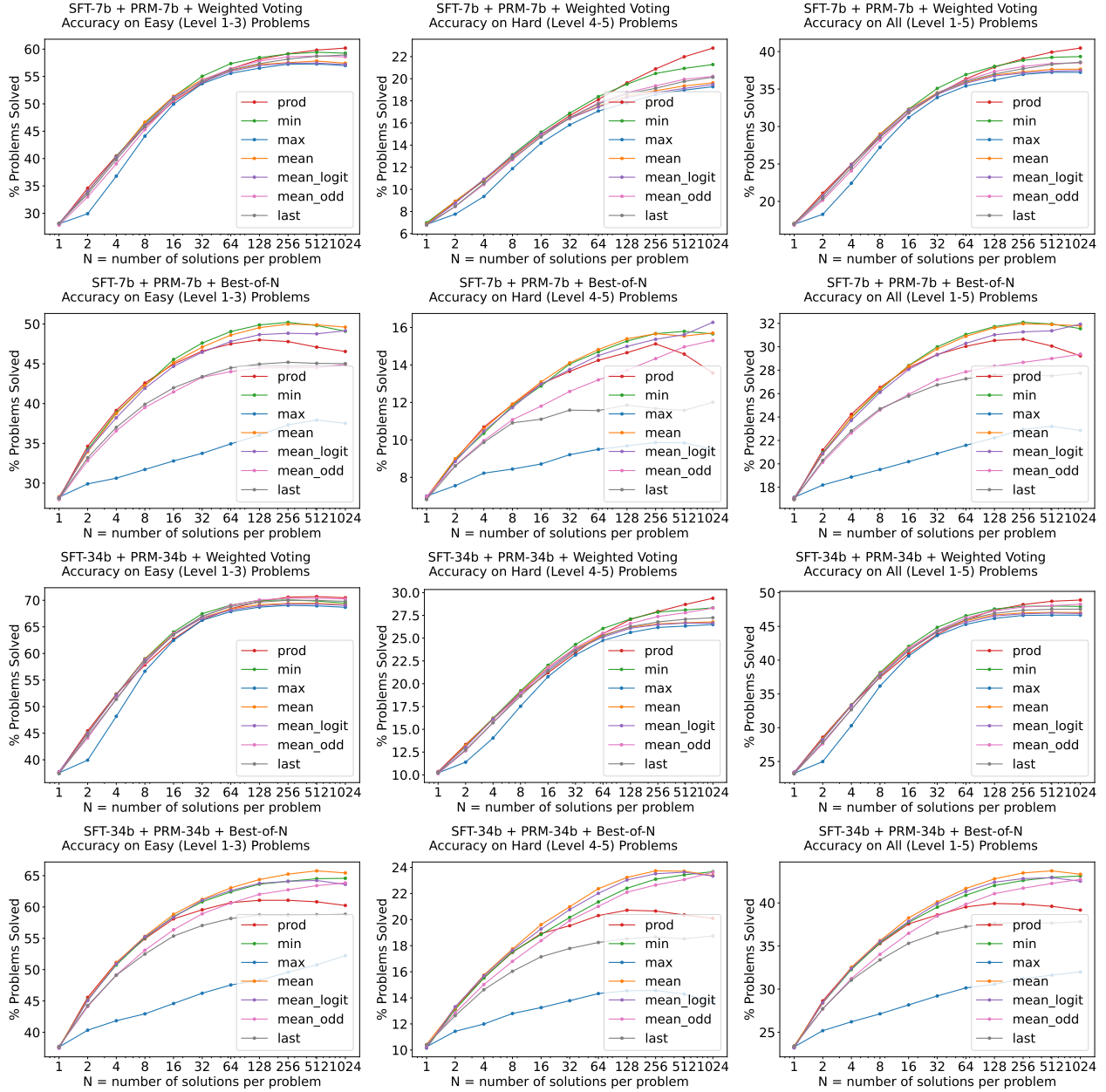
Figure 11. Analysis of aggregation functions in process reward models (PRMs) on the PRM800K dataset with Weighted Voting and Best-of-N. Both SFTs and RMs are trained on the easy data.

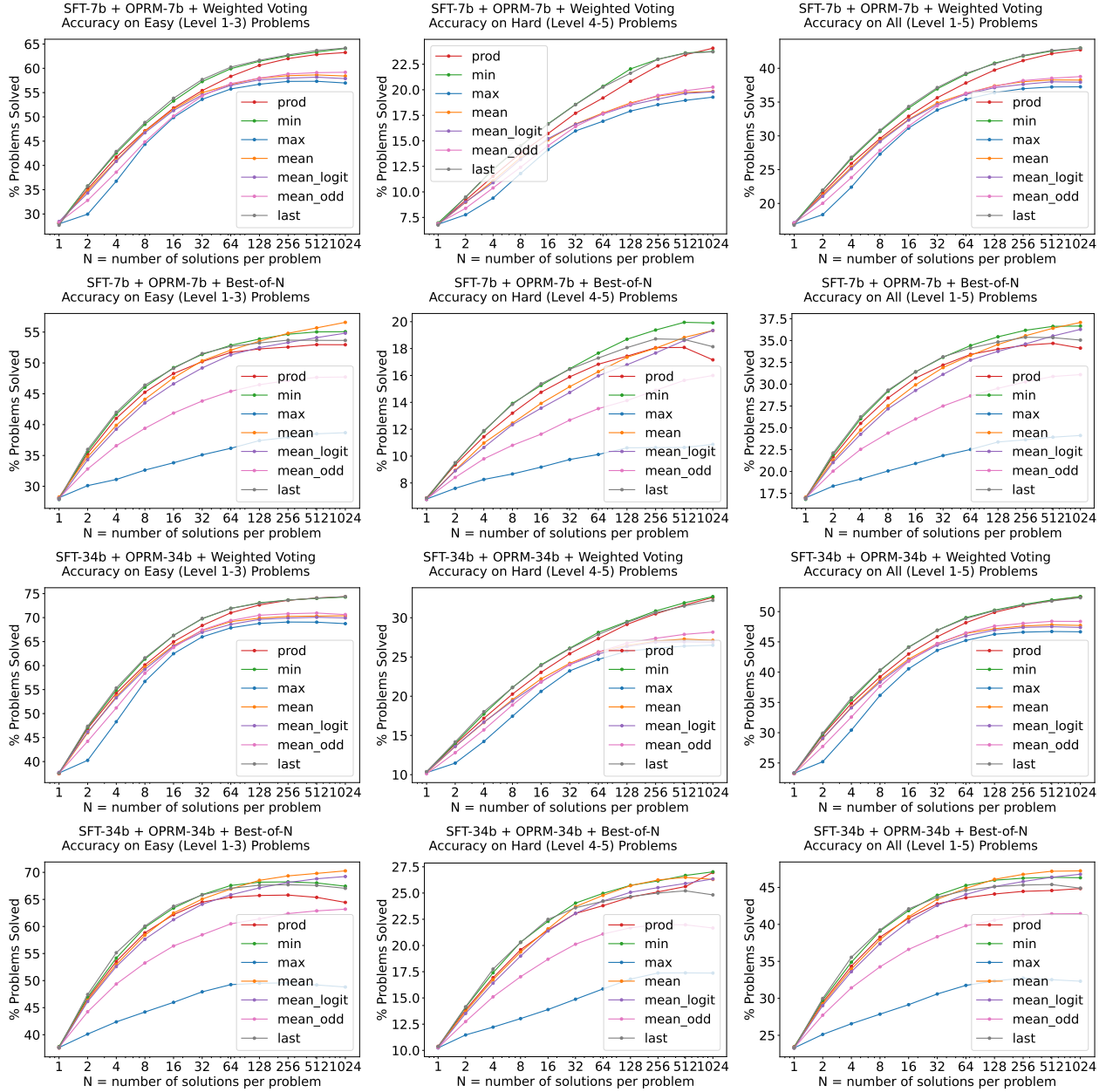*Figure 12.* Analysis of aggregation functions in outcome & process reward models (OPRMs) on the PRM800K dataset with Weighted Voting and Best-of-N. Both SFTs and RMs are trained on the easy data.
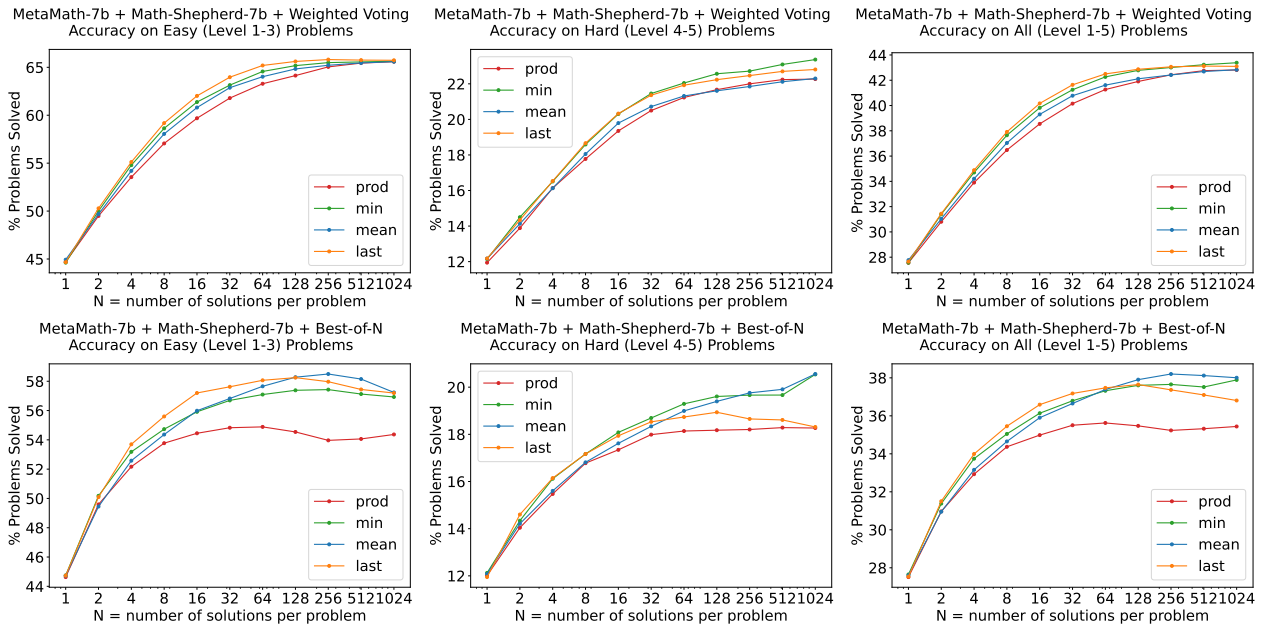
Figure 13. Analysis of aggregation functions in psuedo process reward models (PRMs) on the Math-Shepherd (from MetaMath) dataset with Weighted Voting and Best-of-N. Both SFTs and RMs are trained on the easy data.

References

Anthony, T., Tian, Z., and Barber, D. Thinking fast and slow with deep learning and tree search. *Advances in neural information processing systems*, 30, 2017.

Azar, M. G., Rowland, M., Piot, B., Guo, D., Calandriello, D., Valko, M., and Munos, R. A general theoretical paradigm to understand learning from human preferences. *arXiv preprint arXiv:2310.12036*, 2023.

Azerbayev, Z., Schoelkopf, H., Paster, K., Santos, M. D., McAleer, S., Jiang, A. Q., Deng, J., Biderman, S., and Welleck, S. Llemma: An open language model for mathematics. *arXiv preprint arXiv:2310.10631*, 2023.

Bai, Y., Jones, A., Ndousse, K., Askell, A., Chen, A., DasSarma, N., Drain, D., Fort, S., Ganguli, D., Henighan, T., et al. Training a helpful and harmless assistant with reinforcement learning from human feedback. *arXiv preprint arXiv:2204.05862*, 2022a.

Bai, Y., Kadavath, S., Kundu, S., Askell, A., Kernion, J., Jones, A., Chen, A., Goldie, A., Mirhoseini, A., McKinnon, C., Chen, C., Olsson, C., Olah, C., Hernandez, D., Drain, D., Ganguli, D., Li, D., Tran-Johnson, E., Perez, E., Kerr, J., Mueller, J., Ladish, J., Landau, J., Ndousse, K., Lukosuite, K., Lovitt, L., Sellitto, M., Elhage, N., Schiefer, N., Mercado, N., DasSarma, N., Lasenby, R., Larson, R., Ringer, S., Johnston, S., Kravec, S., Showk, S. E., Fort, S., Lanham, T., Telleen-Lawton, T., Conerly, T., Henighan, T., Hume, T., Bowman, S. R., Hatfield-Dodds, Z., Mann, B., Amodei, D., Joseph, N., McCandlish, S., Brown, T., and Kaplan, J. Constitutional ai: Harmlessness from ai feedback, 2022b.

Cobbe, K., Kosaraju, V., Bavarian, M., Chen, M., Jun, H., Kaiser, L., Plappert, M., Tworek, J., Hilton, J., Nakano, R., Hesse, C., and Schulman, J. Training verifiers to solve math word problems. *arXiv preprint arXiv:2110.14168*, 2021.

Dubois, Y., Li, X., Taori, R., Zhang, T., Gulrajani, I., Ba, J., Guestrin, C., Liang, P., and Hashimoto, T. B. Alpacafarm: A simulation framework for methods that learn from human feedback. *arXiv preprint arXiv:2305.14387*, 2023.

Gao, L., Schulman, J., and Hilton, J. Scaling laws for reward model overoptimization. In *International Conference on Machine Learning*, pp. 10835–10866. PMLR, 2023.

Gulcehre, C., Paine, T. L., Srinivasan, S., Konyushkova, K., Weerts, L., Sharma, A., Siddhant, A., Ahern, A., Wang, M., Gu, C., et al. Reinforced self-training (rest) for language modeling. *arXiv preprint arXiv:2308.08998*, 2023.

Hendrycks, D., Basart, S., Kadavath, S., Mazeika, M., Arora, A., Guo, E., Burns, C., Puranik, S., He, H., Song, D., et al. Measuring coding challenge competence with apps. *arXiv preprint arXiv:2105.09938*, 2021a.

Hendrycks, D., Burns, C., Kadavath, S., Arora, A., Basart, S., Tang, E., Song, D., and Steinhardt, J. Measuring mathematical problem solving with the math dataset. *arXiv preprint arXiv:2103.03874*, 2021b.

Lightman, H., Kosaraju, V., Burda, Y., Edwards, H., Baker, B., Lee, T., Leike, J., Schulman, J., Sutskever, I., and Cobbe, K. Let's verify step by step. *arXiv preprint arXiv:2305.20050*, 2023.

Munos, R., Valko, M., Calandriello, D., Azar, M. G., Rowland, M., Guo, Z. D., Tang, Y., Geist, M., Mesnard, T., Michi, A., et al. Nash learning from human feedback. *arXiv preprint arXiv:2312.00886*, 2023.

Ouyang, L., Wu, J., Jiang, X., Almeida, D., Wainwright, C. L., Mishkin, P., Zhang, C., Agarwal, S., Slama, K., Ray, A., et al. Training language models to follow instructions with human feedback. *arXiv preprint arXiv:2203.02155*, 2022.

Rafailov, R., Sharma, A., Mitchell, E., Ermon, S., Manning, C. D., and Finn, C. Direct preference optimization: Your language model is secretly a reward model. *arXiv preprint arXiv:2305.18290*, 2023.

Roziere, B., Gehring, J., Gloeckle, F., Sootla, S., Gat, I., Tan, X. E., Adi, Y., Liu, J., Remez, T., Rapin, J., et al. Code llama: Open foundation models for code. *arXiv preprint arXiv:2308.12950*, 2023.

Schulman, J., Moritz, P., Levine, S., Jordan, M., and Abbeel, P. High-dimensional continuous control using generalized advantage estimation. *arXiv preprint arXiv:1506.02438*, 2015.

Schulman, J., Wolski, F., Dhariwal, P., Radford, A., and Klimov, O. Proximal policy optimization algorithms. *arXiv preprint arXiv:1707.06347*, 2017.

Silver, D., Huang, A., Maddison, C. J., Guez, A., Sifre, L., Van Den Driessche, G., Schrittwieser, J., Antonoglou, I., Panneershelvam, V., Lanctot, M., et al. Mastering the game of Go with deep neural networks and tree search. *Nature*, 529 (7587):484–489, 2016.

Singh, A., Co-Reyes, J. D., Agarwal, R., Anand, A., Patil, P., Liu, P. J., Harrison, J., Lee, J., Xu, K., Parisi, A., et al. Beyond human data: Scaling self-training for problem-solving with language models. *arXiv preprint arXiv:2312.06585*, 2023.

Stiennon, N., Ouyang, L., Wu, J., Ziegler, D., Lowe, R., Voss, C., Radford, A., Amodei, D., and Christiano, P. F. Learning to summarize with human feedback. *Advances in Neural Information Processing Systems*, 33:3008–3021, 2020.

Sun, Z., Shen, Y., Zhang, H., Zhou, Q., Chen, Z., Cox, D., Yang, Y., and Gan, C. Salmon: Self-alignment with principle-following reward models. *arXiv preprint arXiv:2310.05910*, 2023.

Touvron, H., Martin, L., Stone, K., Albert, P., Almahairi, A., Babaei, Y., Bashlykov, N., Batra, S., Bhargava, P., Bhosale, S., et al. Llama 2: Open foundation and fine-tuned chat models. *arXiv preprint arXiv:2307.09288*, 2023.

Uesato, J., Kushman, N., Kumar, R., Song, F., Siegel, N., Wang, L., Creswell, A., Irving, G., and Higgins, I. Solving math word problems with process- and outcome-based feedback. *arXiv preprint arXiv:2211.14275*, 2022.

Wang, P., Li, L., Shao, Z., Xu, R., Dai, D., Li, Y., Chen, D., Wu, Y., and Sui, Z. Math-shepherd: Verify and reinforce llms step-by-step without human annotations. *CoRR, abs/2312.08935*, 2023.

Wang, Z., Li, Y., Wu, Y., Luo, L., Hou, L., Yu, H., and Shang, J. Multi-step problem solving through a verifier: An empirical analysis on model-induced process supervision. *arXiv preprint arXiv:2402.02658*, 2024.

Xu, J., Lee, A., Sukhbaatar, S., and Weston, J. Some things are more cringe than others: Preference optimization with the pairwise cringe loss. *arXiv preprint arXiv:2312.16682*, 2023.

Yu, F., Gao, A., and Wang, B. Outcome-supervised verifiers for planning in mathematical reasoning. *arXiv preprint arXiv:2311.09724*, 2023a.

Yu, L., Jiang, W., Shi, H., Yu, J., Liu, Z., Zhang, Y., Kwok, J. T., Li, Z., Weller, A., and Liu, W. Metamath: Bootstrap your own mathematical questions for large language models. *arXiv preprint arXiv:2309.12284*, 2023b.

Yuan, Z., Yuan, H., Li, C., Dong, G., Tan, C., and Zhou, C. Scaling relationship on learning mathematical reasoning with large language models. *arXiv preprint arXiv:2308.01825*, 2023.

Zelikman, E., Wu, Y., Mu, J., and Goodman, N. Star: Bootstrapping reasoning with reasoning. *Advances in Neural Information Processing Systems*, 35:15476–15488, 2022.

Zhao, Y., Khalman, M., Joshi, R., Narayan, S., Saleh, M., and Liu, P. J. Calibrating sequence likelihood improves conditional language generation. In *The Eleventh International Conference on Learning Representations*, 2022.

Zhao, Y., Joshi, R., Liu, T., Khalman, M., Saleh, M., and Liu, P. J. Slic-hf: Sequence likelihood calibration with human feedback. *arXiv preprint arXiv:2305.10425*, 2023.