Triangle meshes

Georgia Tech College of Computing
Interactive & Intelligent Computing

- Topology and terminology
- Data-structure and generation
- Operators and traversals
  - Holes
  - Shells
  - Handles
  - Solids
  - Rings

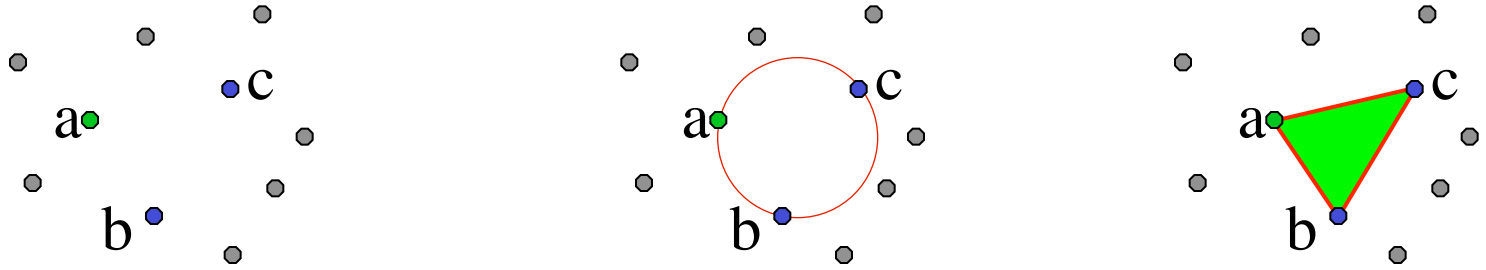
Updated November 9, 2012

Lecture Objectives

- Learn how to **triangulate** an unstructured **point could** in 3D
- Learn the terminology: **Incidence**, **adjacency**, orientation,…
- Learn how to represent a simple triangle mesh using a **Corner Table** (VOG) data structure
- Learn how to **build** a Corner Table from a Face/Vertex index file
- Learn how to **implement** and **use** the **corner operators** for traversing the mesh
- Learn how to design **algorithms** that traverse the mesh to estimate surface **normals** at vertices and to identify the **shells**
- Learn the formula for computing the **genus** of each shell
- Understand the **topological** limitations of the Corner Table and how to use it for representing meshes with **holes**

3D point cloud ball-rolling triangulation

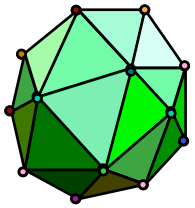
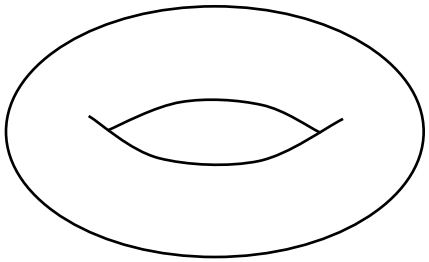
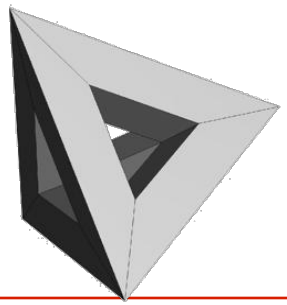
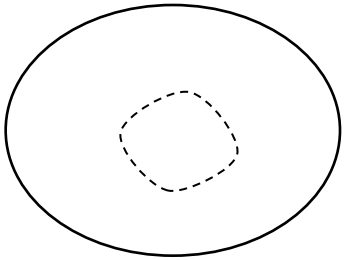
- Pick a radius r (statistics of average distance to nearest point)
- For each ordered triplet of points a, b, and c if there is a point o such that sphere(o,r) touches a, b, and c and contains no other point, then create the **oriented** triangle (a,b,c), so that they appear counterclockwise as seen from o.



- Each triangle has a neighbor across each edge (roll the ball)
- Two triangles with the same vertices have opposite orientations

Interpreting the shells in 3D

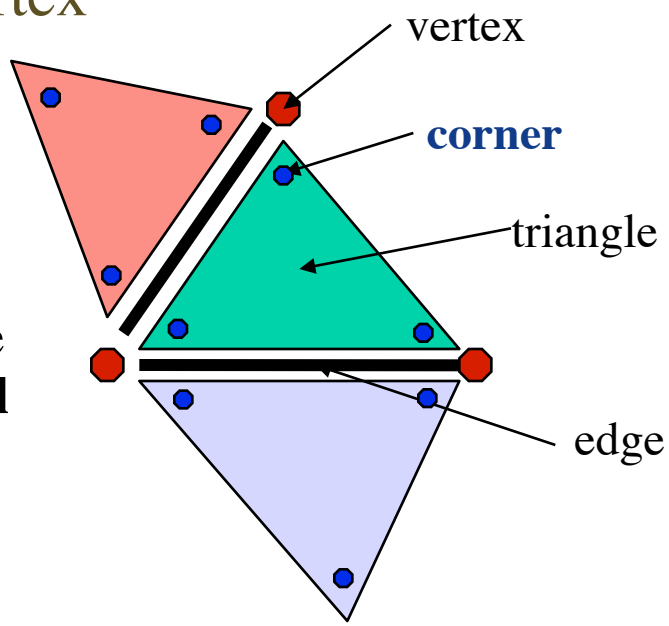
- Form **shells** of triangles by identifying triangle-triangle adjacency.
- **Cracks** are pairs of triangles **in a shells** that share the same three vertices (different order / orientation). The two triangles of a crack may be adjacent to one another (interior walls around a door frame).
- The shells divide space into **regions** (topologically open 3-cells). Each region is bounded by one shell or more.
- Each region may have **interior walls** (cracks bounding the region on both sides), **cavities** (holes), and **handles** (higher genus).

Corners, incidence and **adjacency**

Triangle/vertex **incidence**: identifies corners

- **Corner**: association of a triangle with a vertex (vertex-use)
  - A triangle has 3 corners
  - On average, 6 corners share a vertex



Triangle/triangle **adjacency:**

Identifies **neighboring** triangles

Neighboring triangles share a common edge

Adjacency may be computed from the incidence

Adjacency is convenient to **accelerate traversal**

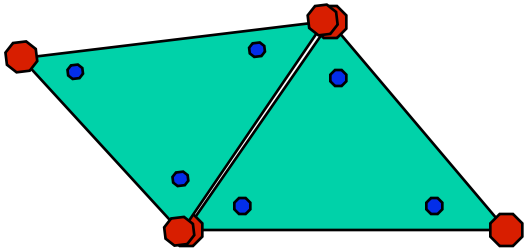
- Walk from one triangle to an adjacent one
- Estimate surface normals at vertices

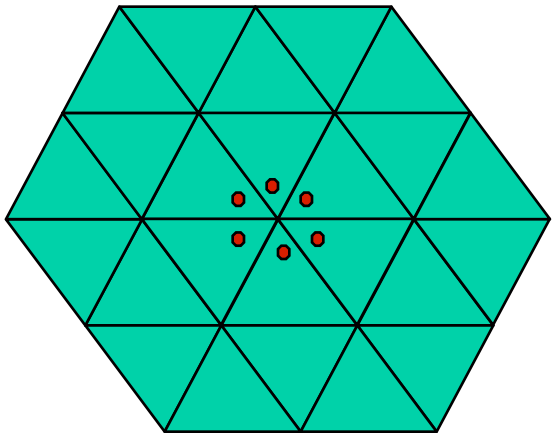
We will use the **Corner Table** to represent incidence and adjacency

Representation as **independent** triangles

- **For each triangle:**
  - **Store the location of its 3 vertices**

	vertex 1			vertex 2			vertex 3		
Triangle 1	x	y	z	x	y	z	x	y	z
Triangle 2	x	y	z	x	y	z	x	y	z
Triangle 3	x	y	z	x	y	z	x	y	z





- **This is a poor choice of representation**
  - **Each vertex is repeated 6 times (on average)**
  - **Expensive to identify an adjacent triangle**
    - **Not suited for traversing a mesh**

Representing **vertices + incidence**

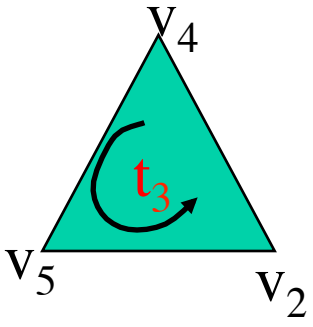
- Samples: Location of vertices + attributes (color, mass)
- Triangle/vertex incidence: indices of the 3 vertices of each triangle
  - Eliminates vertex repetition
  - But still does not support a direct access to neighboring triangles (adjacency)

Samples (vertices):

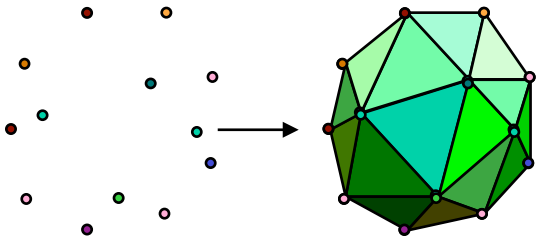
vertex 1	x	y	z	c
vertex 2	x	y	z	c
vertex 3	x	y	z	c
. . .	. . .			

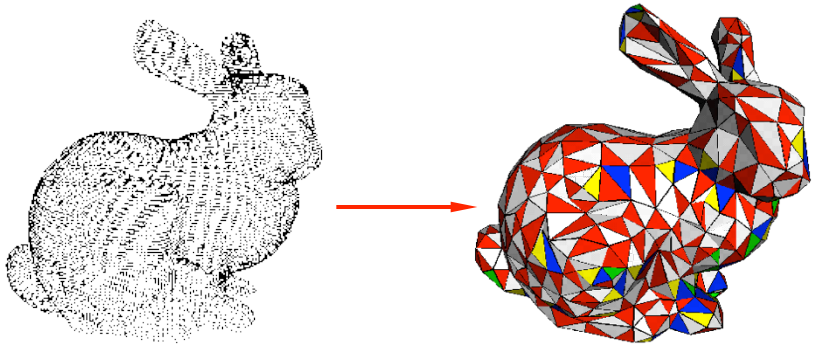
Triangle/vertex incidence:

Triangle 1	1	2	3
Triangle 2	3	2	4
Triangle 3	4	5	2
Triangle 4	7	5	6
Triangle 5	6	5	8
Triangle 6	8	5	1
. . .	. . .		



Order of vertex references defines outward direction (triangle orientation)

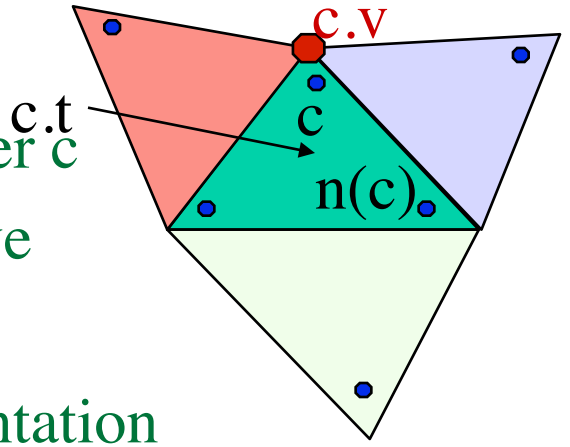


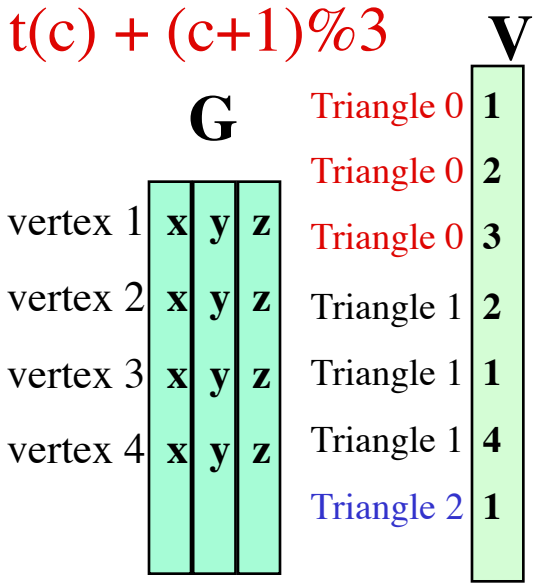


Representing the incidence as the **V table**

- **Integer IDs for** vertices **(0, 1, 2… V-1)** & triangles **(0, 1, 2…T-1)**
- V-table:
  - Identifies the vertex ID v© for each corner c
  - The 3 corners of a triangle are consecutive
    - Triangle number: t(c) = c DIV 3
  - Corners order for a triangle respects orientation
    - Next corner around triangle: n(c) = 3* t(c) + (c+1)%3
    - Previous corner: p(c)=n(n(c))
- Samples stored in geometry table (G):
  - Location of vertex of corner c is

    g(c) = G[V[c]]





Representing **adjacency** with the O table

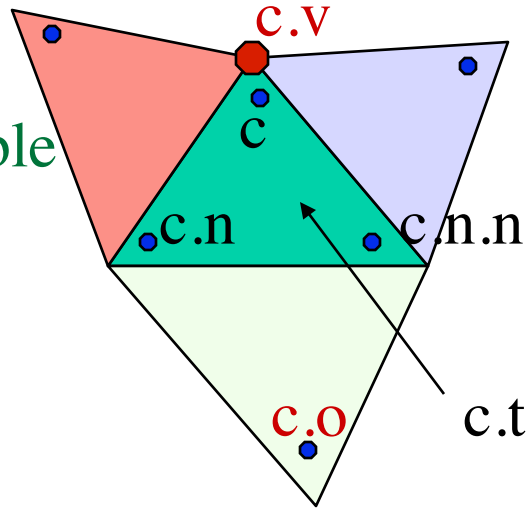
For each corner c store:

c.v : integer reference to an entry in the G table

- Content of V[c] in the V table

c.o : integer id of the opposite corner

- Content of O[c] in the O table

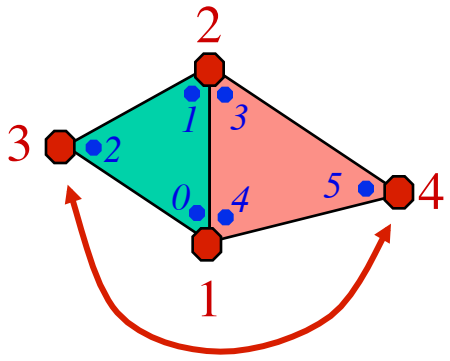


Computing the O table from V

For each corner a do For each corner b do

if (v(n(a))==v(p(b)) && v(p(a))==v(n(b))) { O[a]:=b; O[b]:=a } ;

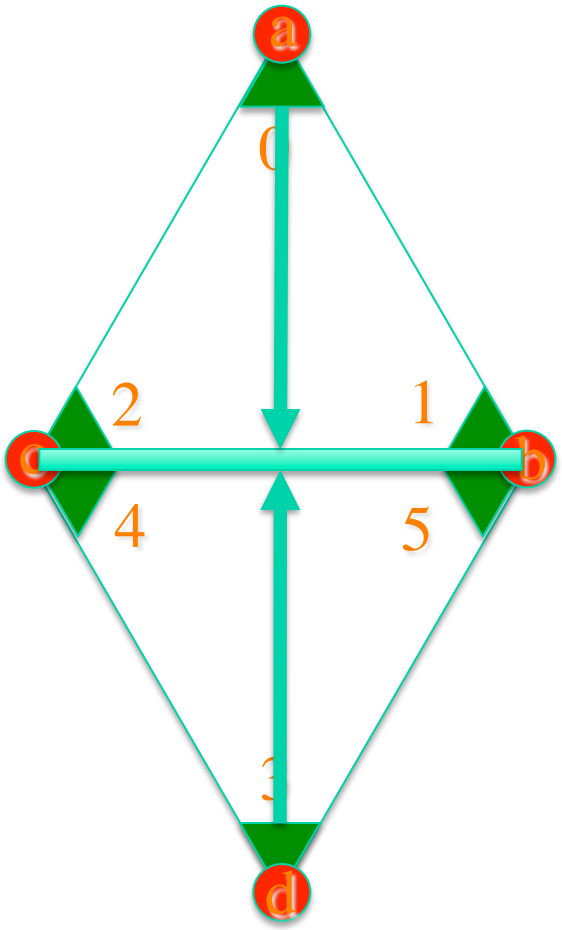
	V	O
Triangle 0 corner 0	1	7
Triangle 0 corner 1	2	8
Triangle 0 corner 2	3	5
Triangle 1 corner 3	2	9
Triangle 1 corner 4	1	6
Triangle 1 corner 5	4	2



	x	y	z
vertex 1			
vertex 2			
vertex 3			
vertex 4			

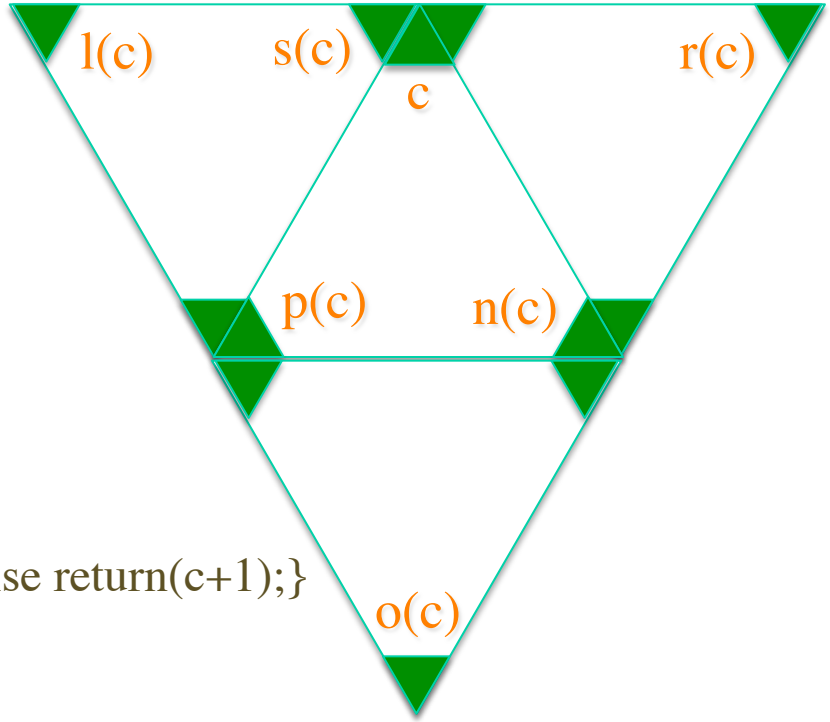
Corner Table for Triangle Meshes (review)

Corner ID	Vertex Table	Opposite Table
0	a	3
1	b	1
2	c	2
3	d	0
4	c	4
5	b	5



- 6 references per triangle
  - 3 for vertex
  - 3 for opposite

Corner Table for Triangle Meshes (review)

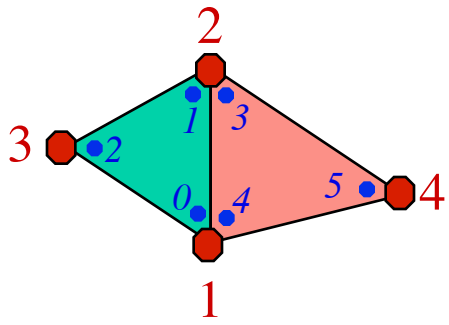


Implementation

```
int v(c) {return V[c];}
pt g(c) {return G[V[c]];}
int o(c) {return O[c];}
int t(c) {return int(c/3);}
int n(c){if ((c%3)==2) return(c–1); else return(c+1);}
int p(c) {return n(n(c));}
int l(c) {return o(p(c));}
int r(c) {return o(n(c));}
int s(c) {return n(l(c));}
```

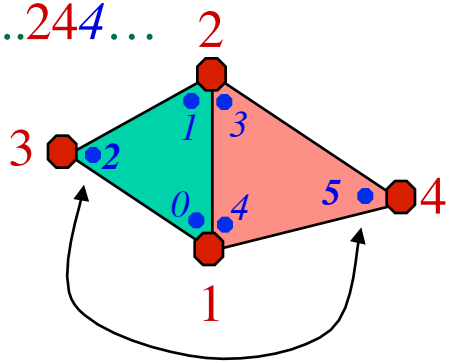
A faster computation of the O table

1. List all of triplets {min(c.n.v, c.p.v), max(c.n.v, c.p.v), *c*}
   - 230, 131, 122, 143, 244, 125, …



	v	o	a
Triangle 1 corner 0	1		a
Triangle 1 corner 1	2		b
Triangle 1 corner 2	3		c
Triangle 2 corner 3	2		c
Triangle 2 corner 4	1		d
Triangle 2 corner 5	4		e

2. Bucket-sort the triplets:
   - 122, 125 ...131... 143 ...230...244 …
3. Pair-up consecutive entries 2k and 2k+1
   - (122, 125)...131... 143...230...244…
4. Their corners are opposite
   - (122,125)...131...143...230...244…

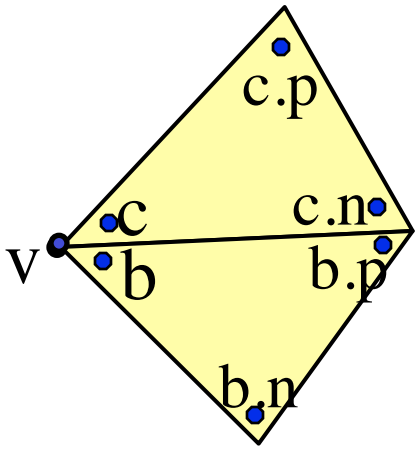


	v	o	a
Triangle 1 corner 0	1		a
Triangle 1 corner 1	2		b
Triangle 1 corner 2	3	5	c
Triangle 2 corner 3	2		c
Triangle 2 corner 4	1		d
Triangle 2 corner 5	4	2	e

Linear cost computation of the O table (new)

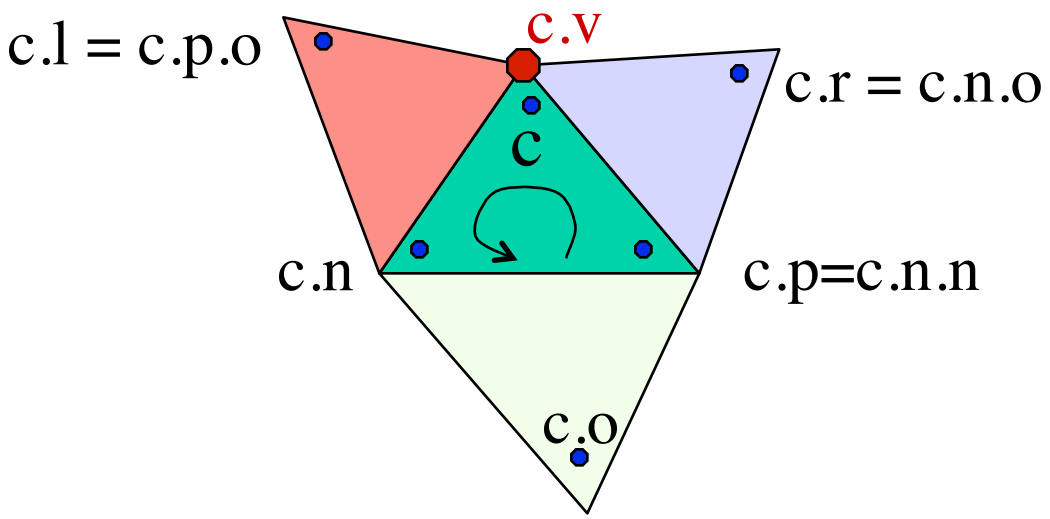
- Compute, valence[v], by incrementing valence[v(c)] for each corner c.
- Compute a running valence sum bin[v] for each vertex v as the sum of the valences of the previous vertices.
- valence[v] consecutive entries in C[], starting at C[bin[v]], are allocated to v.
  - Note that C has a total of $3n_T$ entries divided into n_V bins.
- To fill these entries with corners incident upon v, store with each vertex v the index corner[v] to the first empty entry in its bin and initialize it to bin[v].
- For each corner c, C[corner[v(c)]++]=c ;
- At the end of this process, the bin of vertex v contains the list of corners incident upon v. The integer IDs of the corresponding corners are stored in C between C[bin[v]] and C[bin[v]+valence[v]].
- One can now compute O as follows:

```
for (each vertex v) {
  for (each corner c the bin of v)
    for (each corner b in the bin of v)
      if  (v(n(c))==v(p(b)) ) {O[p(c)]=n(b); O[n(b)]=p(c); }}.
```

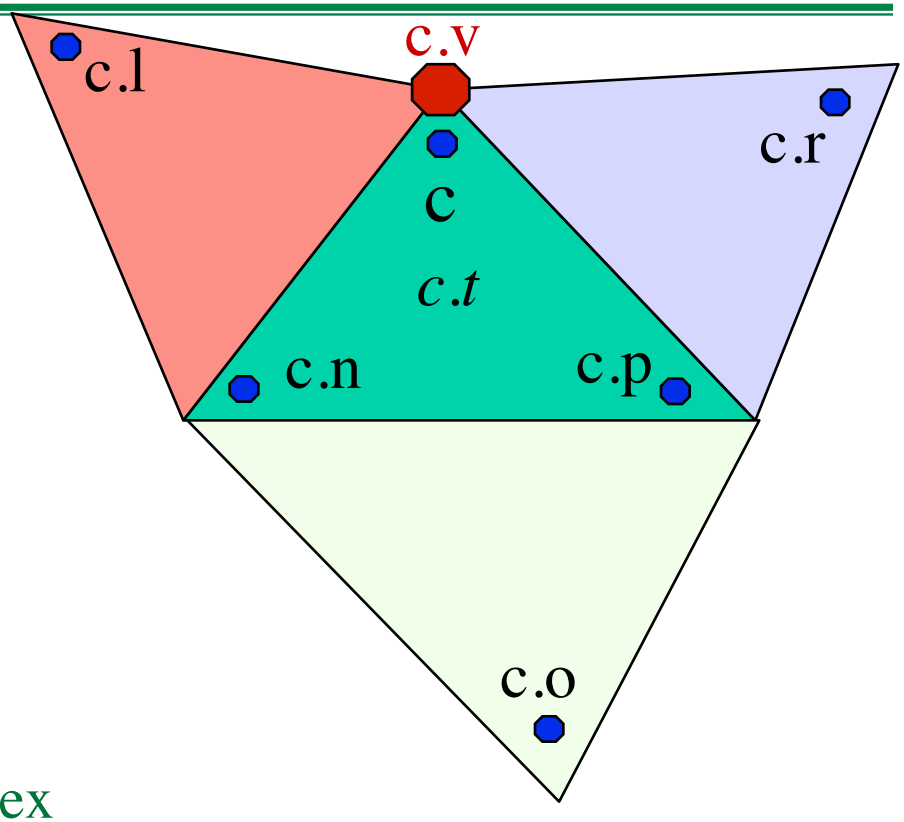


Accessing left and right neighbors

- **Direct access to opposite corners of right and left neighbors**
  - c.r = c.n.o
  - c.l = c.p.o



Summary notation



- **Given corner c**
  - c.v is the integer ID of its vertex
    - On average, 6 corners have the same vertex ID
  - c.v.g (or simply c.g) is the 3D point where c.v is located
    - Must use .g for vector operations. Ex: c.n.g – c.g is vector along edge

Storage (data structure)

```
class Mesh {
  int nv;              // number of vertices
  pt G[nv];            // geometry (vertices)
  int nt;              // number of triangles
  int nc;              // number of corners (3 per triangle)
  int V[nc];           //  corner/vertex incidence
  int O[nc];           // opposite corners

 boolean[] visible = new boolean[nt];   // triangle is not deleted
```

Make a mesh for a w×w grid

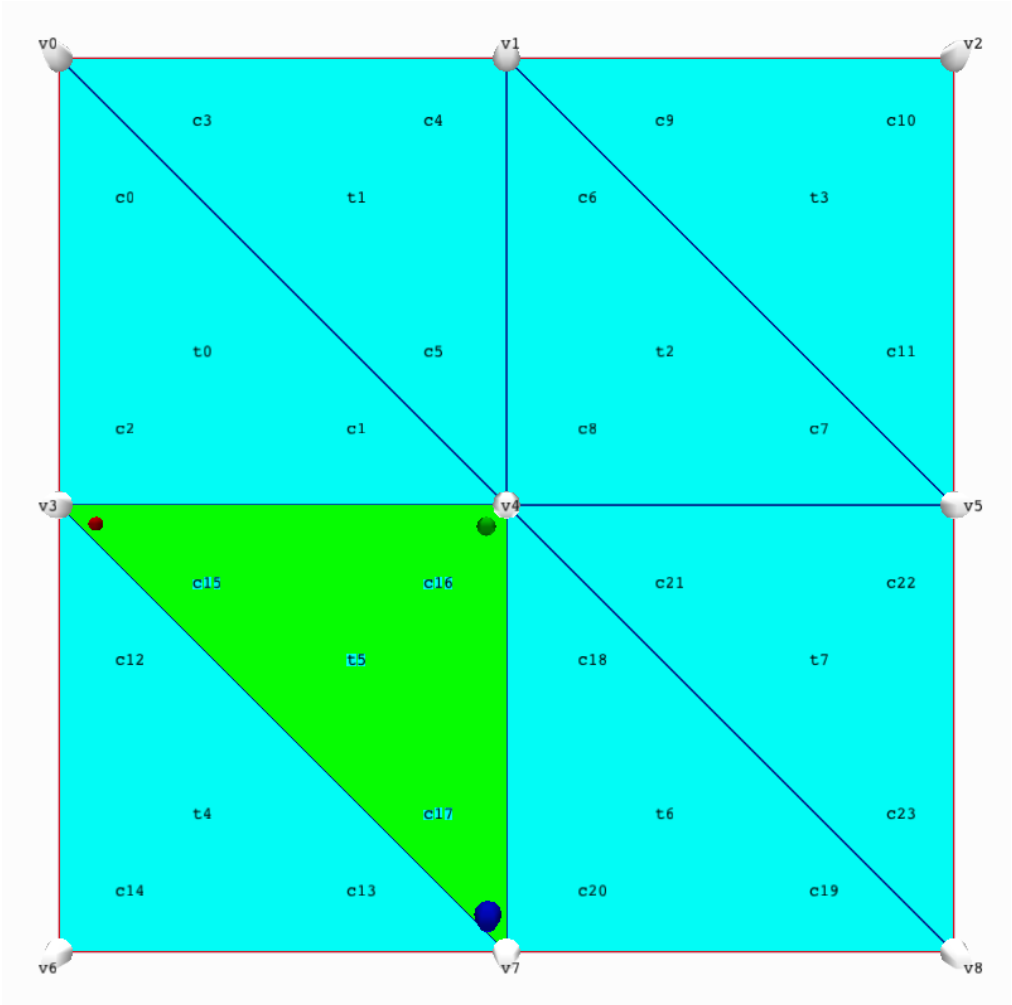
```
void makeGrid (int w) {                    // make a 2D grid of wxw vertices
 for (int i=0; i<w; i++) {for (int j=0; j<w; j++) {
   G[w*i+j].setTo(h*.8*j/(w-1)+h/10,h*.8*i/(w-1)+h/10,0);}}
 for (int i=0; i<w-1; i++) {for (int j=0; j<w-1; j++) {// define the triangles
   V[(i*(w-1)+j)*6]=i*w+j;
   V[(i*(w-1)+j)*6+2]=(i+1)*w+j;
   V[(i*(w-1)+j)*6+1]=(i+1)*w+j+1;
   V[(i*(w-1)+j)*6+3]=i*w+j;
   V[(i*(w-1)+j)*6+5]=(i+1)*w+j+1;
   V[(i*(w-1)+j)*6+4]=i*w+j+1;}; };
 nv = w*w;
 nt = 2*(w-1)*(w-1);
 nc=3*nt;
 }
```

Corner operators

```
int t (int c) {return int(c/3);};          // triangle of corner
int n (int c) {return 3*t(c)+(c+1)%3;};   // next corner in the same t(c)
int p (int c) {return n(n(c));};  // previous corner in the same t(c)
int v (int c) {return V[c] ;};   // id of the vertex of c
pt g (int c) {return G[v(c)];};  // point of the vertex v(c) of corner c
boolean b (int c) {return O[c]==c;};       // if faces a border (has no opposite)
int o (int c) {return O[c];}; // opposite (or self)
int l (int c) {return o(n(c));}; // left neighbor or next if b(n(c))
int r (int c) {return o(p(c));}; // right neighbor or next if b(p(c))
int s (int c) {return n(l(c));}; // swings around v(c) or around a border loop
```

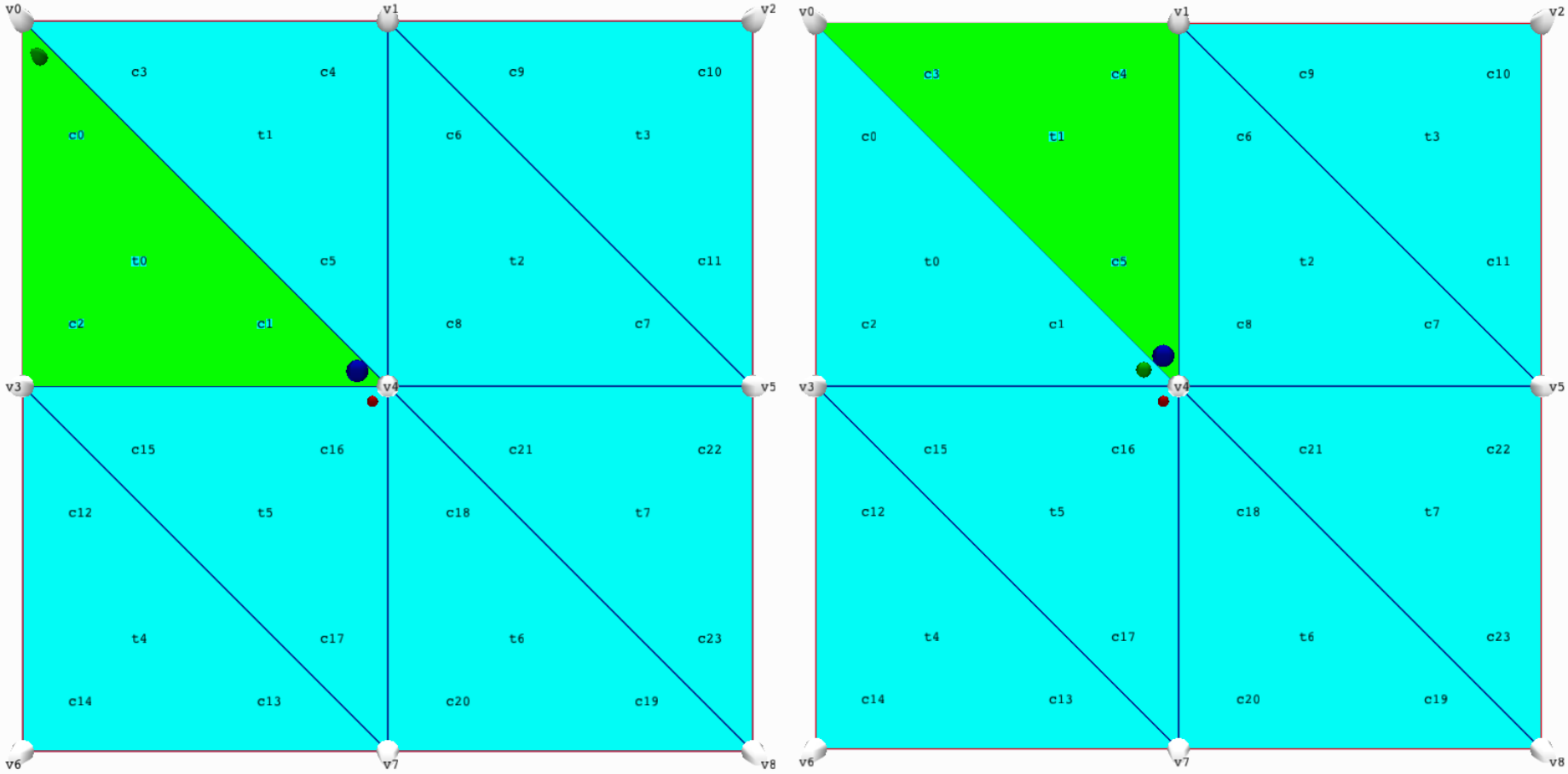
p(c), n(c) to walk around a triangle

Current (blue), previously current (green), previously (red)



s(c) to swing around a vertex

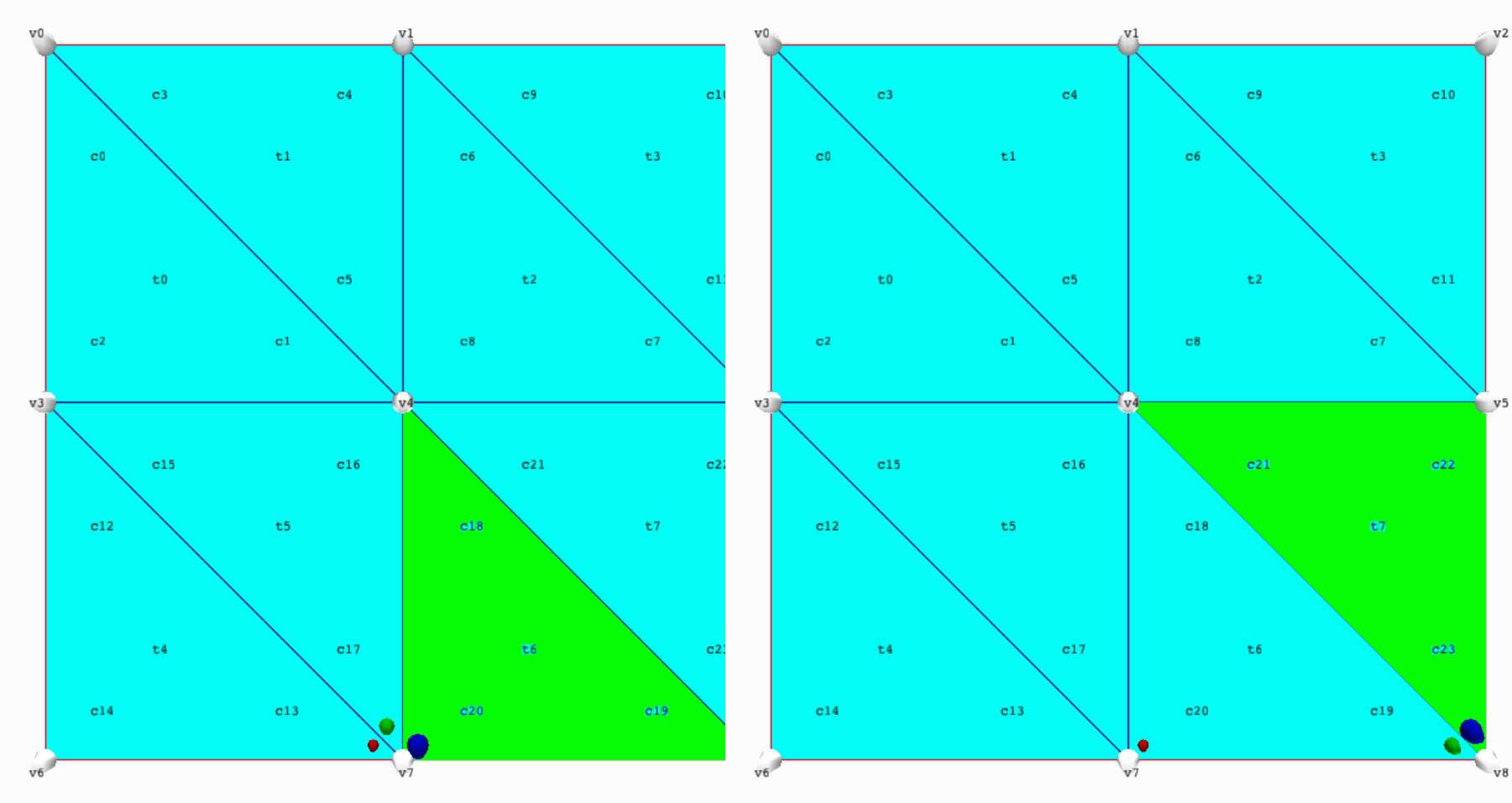
int s (int c) {return n(l(c));}; while (…) c=s(c);



s(c) to walk around a border loop

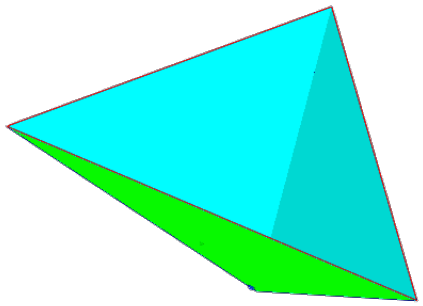
int s (int c) {return n(l(c));};

while (…) c=s(c);



Draw borders

```
void showBorder() {
   for (int i=0; i<nc; i++)  if (b(i)) drawEdge(i);  };
```



Compute O (simple, but slow)

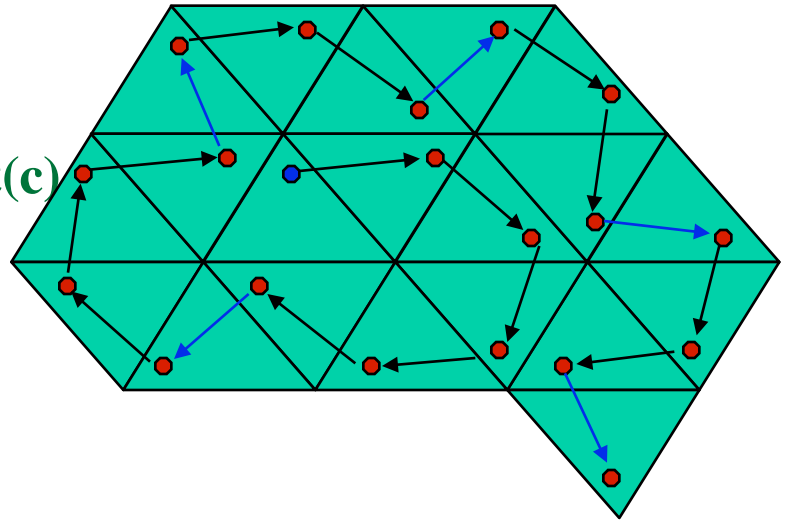
```
void computeOsimple() { // sets O from V, assumes orientation
  for (int i=0; i<3*nt; i++) {O[i]=-1;};  // init to -1: no opposite
  for (int i=0; i<nc; i++) {            // for each corner i,
    for (int j=i+1; j<nc; j++) {       // for each other corner j,
      if( (v(n(i))==v(p(j))) && (v(p(i))==v(n(j))) ) { // if match
      O[i]=j; O[j]=i;};};};              // make i and j opposite
 }}}}
```

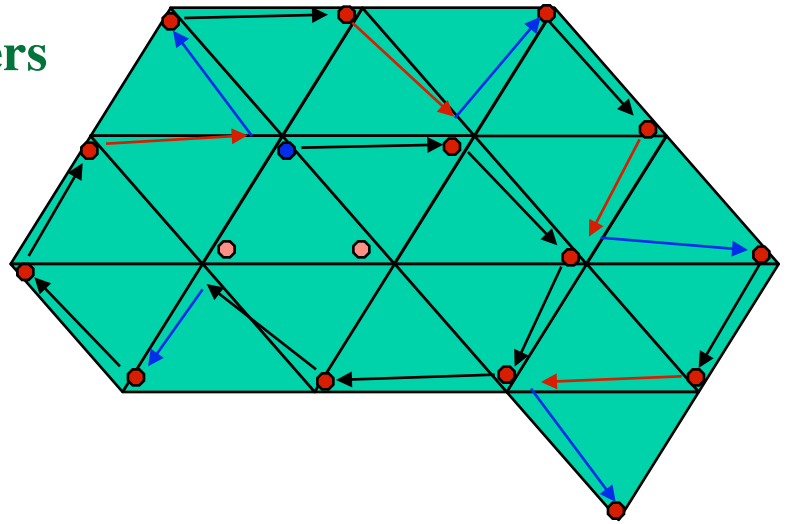
Compute O (fast)

```
void computeO() {
  int val[] = new int [nv]; for (int v=0; v<nv; v++) val[v]=0;  // count of incident corners
  for (int c=0; c<nc; c++) val[v(c)]++;
  int fic[] = new int [nv]; int rfic=0; // head of incident corners list for each vertex
  for (int v=0; v<nv; v++) {fic[v]=rfic; rfic+=val[v];};
  for (int v=0; v<nv; v++) val[v]=0;   // clear valences to track count of incident corners
  int [] C = new int [nc]; // vor each vertex: the list of val[v] incident corners starts at C[fic[v]]
  for (int c=0; c<nc; c++) C[fic[v(c)]+val[v(c)]++]=c;
  for (int c=0; c<nc; c++) O[c]=-1;    // init O table to -1
  for (int v=0; v<nv; v++)            // for each vertex...
    for (int a=fic[v]; a<fic[v]+val[v]-1; a++)
      for (int b=a+1; b<fic[v]+val[v]; b++)  { // for each pair (C[a],C[b[]) of its incident corners
      if (v(n(C[a]))==v(p(C[b]))) {                    // if C[a] follows C[b] around v,
        O[p(C[a])]=n(C[b]); O[n(C[b])]=p(C[a]); };   // then p(C[a]) and n(C[b]) are opposite
      if (v(n(C[b]))==v(p(C[a]))) {                    // if C[b] follows C[a] around v,
        O[p(C[b])]=n(C[a]); O[n(C[a])]=p(C[b]); }}} // then p(C[b]) and n(C[a]) are opposite
```

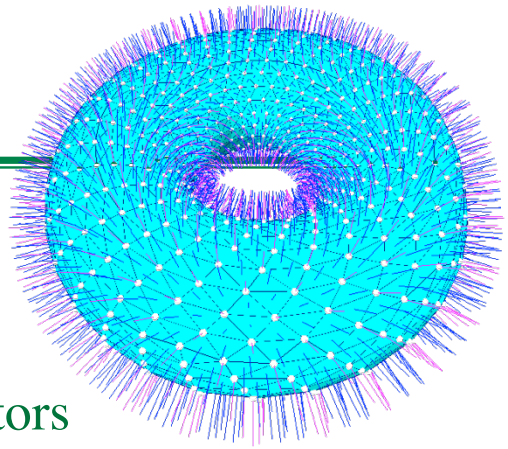
Using adjacency table for T-mesh traversal

- Visit T-mesh
  - **Mark triangles as you visit them**
  - **Start with any corner c and call Visit(c)**
  - **Visit(c)**
    - mark c.t;
    - IF NOT marked(c.r.t) THEN visit(c.r);
    - IF NOT marked(c.l.t) THEN visit(c.l);
- Label vertices **(for example as 1, 2, 3 …)**
  - **Label vertices with consecutive integers**
  - **Label(c.n.v); Label(c.n.n.v); Visit(c);**
  - **Visit(c)**
    - IF NOT labeled(c.v) THEN Label(c.v);
    - mark c.t;
    - IF NOT marked(c.r.t) THEN visit(c.r);
    - IF NOT marked(c.l.t) THEN visit(c.l);

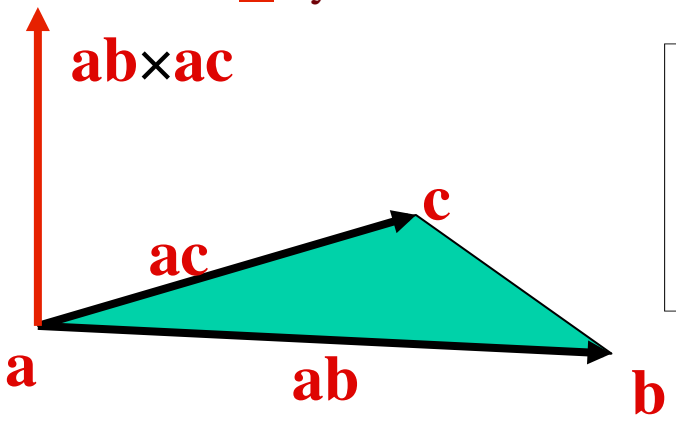


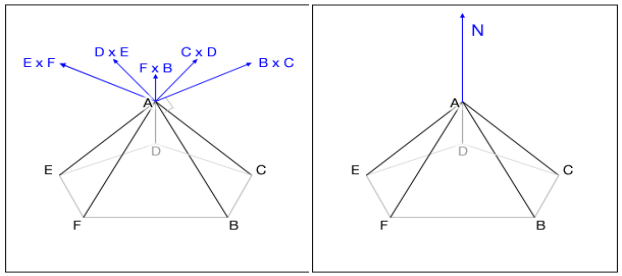


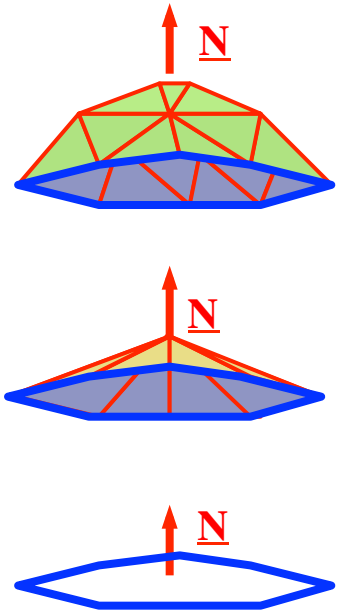
Estimating a vertex normal



- At vertex a having b, c, d, e, f as neighbors
- N=ab×ac+ac×ad+ad×ae+ae×af+af×ab
  - The notation U×V is the cross product of the two vectors
  - The notation ac is the vector between a and c. In other words: ac=c–a
  - Note that N is independent of the position of vertex a
    - (b–a)×(c–a)+(b–a)×(c–a)+… = b×c+a×a–b×a–a×c + c×d+a×a–c×a–a×d +… –a×b +…
    - a×a = 0 , –a×c and –c×a cancel out, same for all other cross-products containing a
    - We are left with = b×c + c×d + … which does not depend on a
- Then divide N by its norm to make it a unit vector

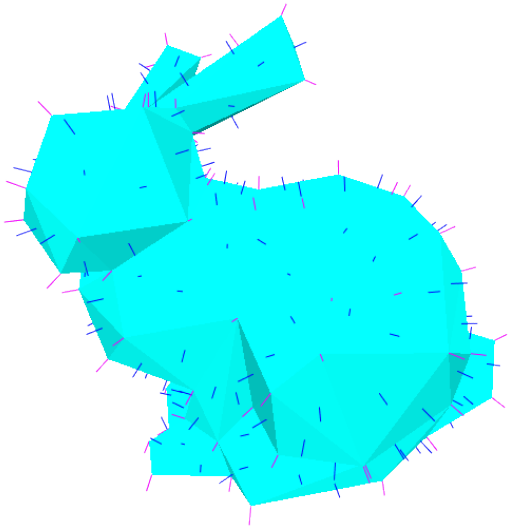






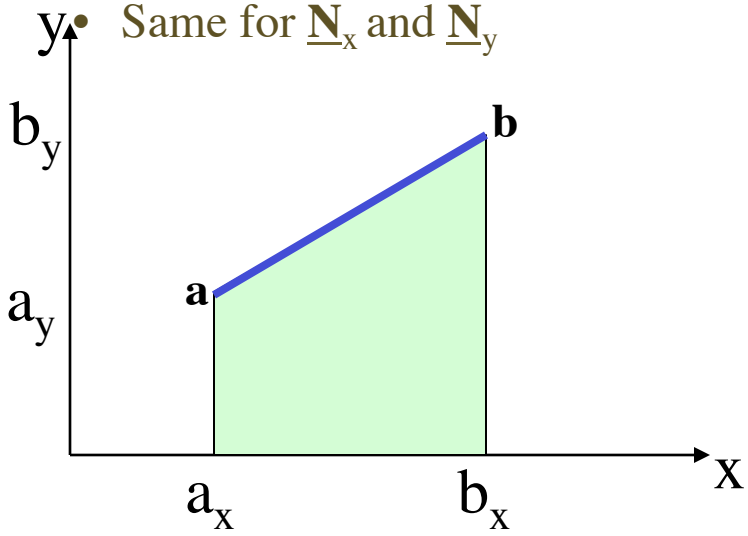
Compute normals and valence

```
vec triNormal(int t) { return C(V(g(3*t),g(3*t+1)),V(g(3*t),g(3*t+2))); };

void computeTriNormals() {
      for (int i=0; i<nt; i++) Nt[i].setToVec(triNormal(i));};


void computeVertexNormals() {
  for (int i=0; i<nv; i++) {Nv[i].setTo(0,0,0);};
  for (int i=0; i<nc; i++) {Nv[v(i)].add(Nt[t(i)]);};
  for (int i=0; i<nv; i++) {Nv[i].makeUnit();};
  };
```

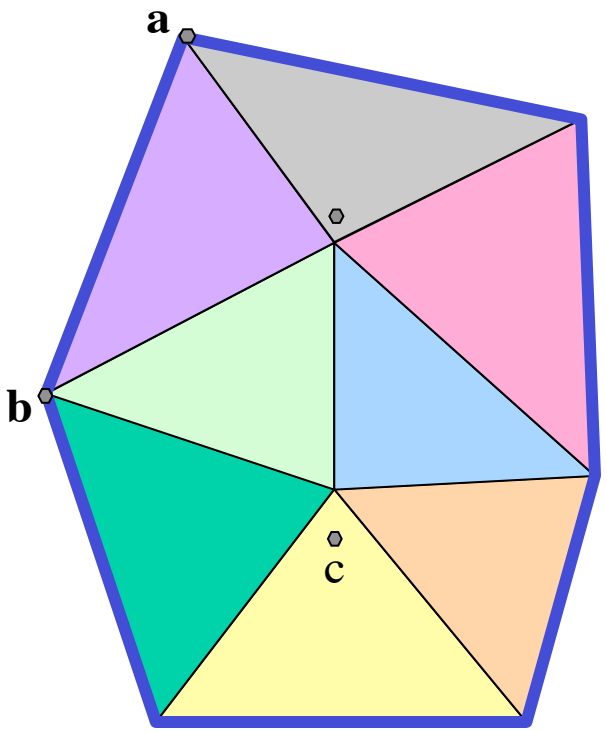


Faster computation of the normal N to a patch

- N may be computed from the projections of the border edges (a,b) onto the 3 principal planes:
- Compute signed areas of "shadows" of the border loop on the YZ, ZX, and XY planes
  - $\mathbf{N}_z$:= the sum of signed areas of 2D trapozoids under the projection of (**a**,**b**), for each border edge (**a**,**b**).
  - Same for $\mathbf{N}_x$ and $\mathbf{N}_y$



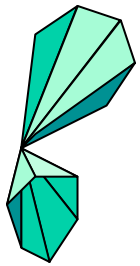
$$\text{area}(\mathbf{a},\mathbf{b}) := (a_y + b_y)(b_x - a_x)$$



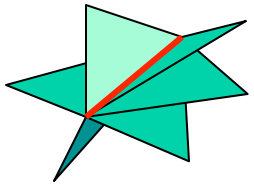
Assume T-mesh is an **orientable manifold**

- A set of triangles forms a **manifold** mesh when:
  - The 3 corners of a triangle refer to different vertices (not zero area)
  - Each edge bounds exactly 2 triangles
  - The star of each vertex v forms a single cone (connected if we remove v)
    - Star = union of edges and triangles incident upon the vertex

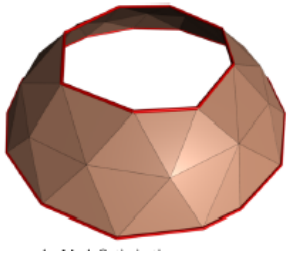
Non-manifold vertex



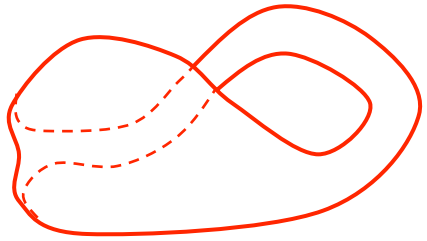
Non-manifold edge



border edges



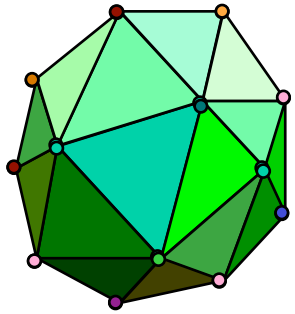
- A manifold triangles mesh is **orientable** when:
  - Triangle can be oriented consistently
- **Pseudo-manifold**
  - Start with a Manifold T-mesh
  - Displacing the geometry (vertices)
  - Different vertices may have the same location
  - Mesh may self-intersect

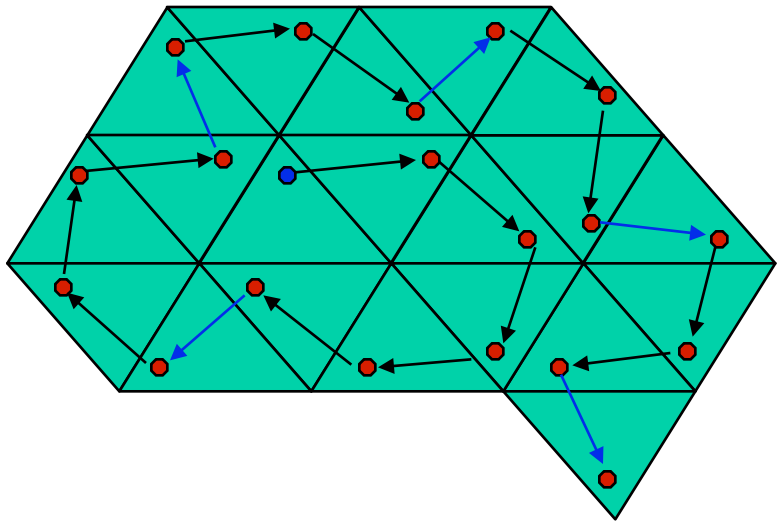


Klein bottle

Shells: connected portions of T-meshes

- **All triangles of a** shell **form a** connected **set**
  - Two adjacent triangles are connected (through their common edge)
  - Connectivity is a transitive relation (can identify a mesh by invading it)
- To identify a new shell
  - Pick a new "color" (ID) and a virgin triangle
  - Use the swirl(c) procedure to reach all of the triangle of the shell and paint them



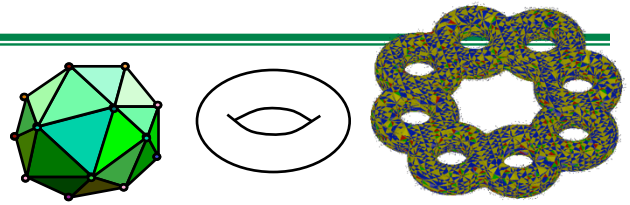


Swirl

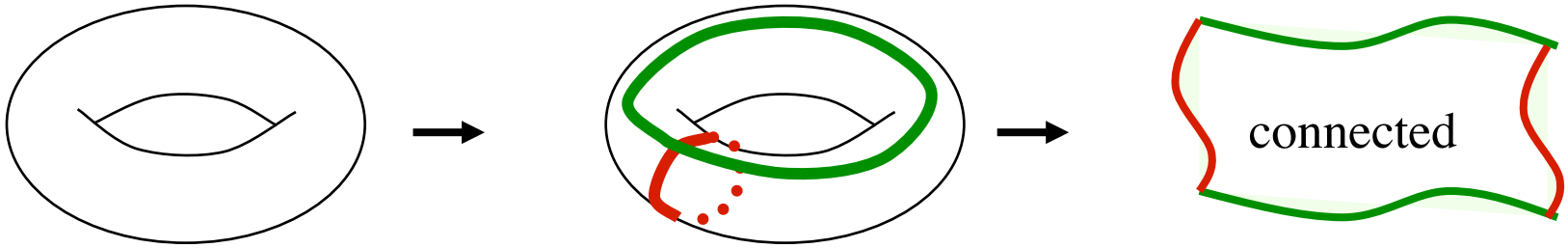
We compute the number k of shells and identify a corner, firstCorner[s], for each shell s.

```
k=0; // shell count
for (each triangle t) shell[t]=0; // shell number
for (each corner c)
        if (shell[t(c)]==0) {firstCorner[k++]=c; swirl(c,k)},

void swirl(c,k) {
   if (shell[t(c)]==0) {shell[t(c)]=k; swirl(c.l,k); swirl(c.r,k);}}.
```

Genus (number of handles) in a shell



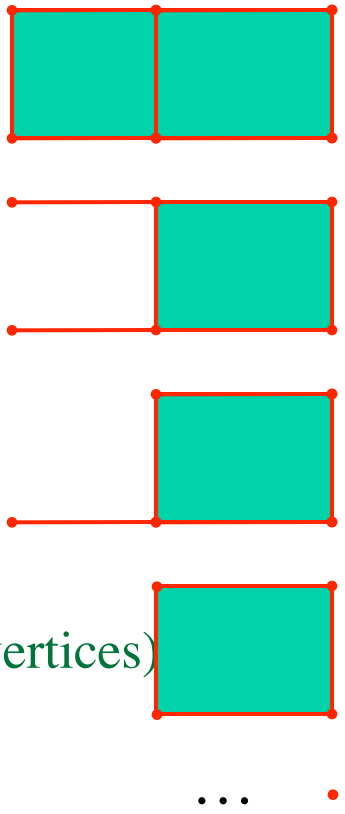
- **Handles** correspond to through-holes
  - A sphere has zero handles, a torus has one
- The number H of handles is called the **genus** of the shell
  - A handle cannot be identified as a particular set of triangles
- A T-mesh has k handles if and only if you can remove at most 2k edge-loops without disconnecting the mesh



- Genus of a shell may be computed using: H = T/4 – V/2 + 1
  - Remember as **T = 2V – 4 + 4H**
- In a **zero-genus mesh**, **T = 2V – 4** (the **Euler-Poincare** formula)

Proof of Euler's formula: F–E+V =2

- Formula attributed to Descartes (1639)
- Euler published first proof (1751)
- Used in Combinatorial Topology founded by Poincaré (c.1900)
- Cauchy's proof (1811)
  - Take a zero-genus polyhedron
    - Faces may have arbitrary number of edges
  - Assume it has F faces, E edges, V vertices
  - Remove one initial face
  - Repeatedly apply one or the other destructor
    - Remove one edge and one face
    - Remove one edge and one vertex
  - All preserve connectivity and F–E+V
  - All keep closed cells (faces with borders, edges with end-vertices)
  - You end up with a single vertex
  - Counting the initial face: F–E+V=2

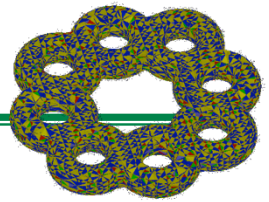


Applying V–E+F=2 to triangle meshes

- Euler formula for zero genus manifold polyhedron: F–E+V=2
- Triangle mesh: each face has 3 edges
  - We will use T to represent the number of faces: T–E+V=2
  - Let's count edge-uses U
  - Each triangle uses 3 edges: U = 3T
  - Each edge is used twice: U = 2E
  - Therefore: E = 3T / 2
- Substitution: T–3T/2+V=2
- Multiply by 2: 2T–3T+2V=4
- Hence: T = 2V – 4
  - There are roughly twice as many triangles as vertices!
  - Check on a tetrahedron: T=4 and V=4

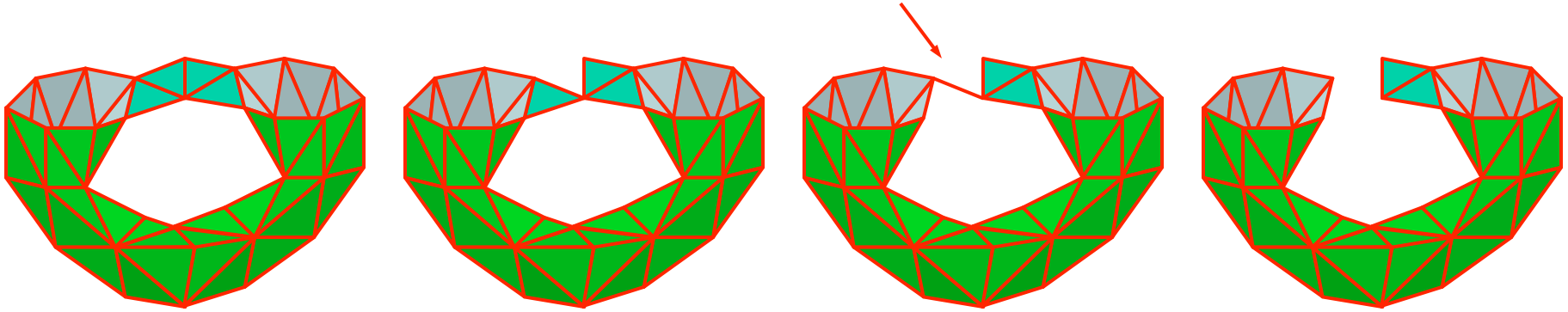
Note that 3T/2=E=T+V–2

Justifying T = 2V – 4 + 4H

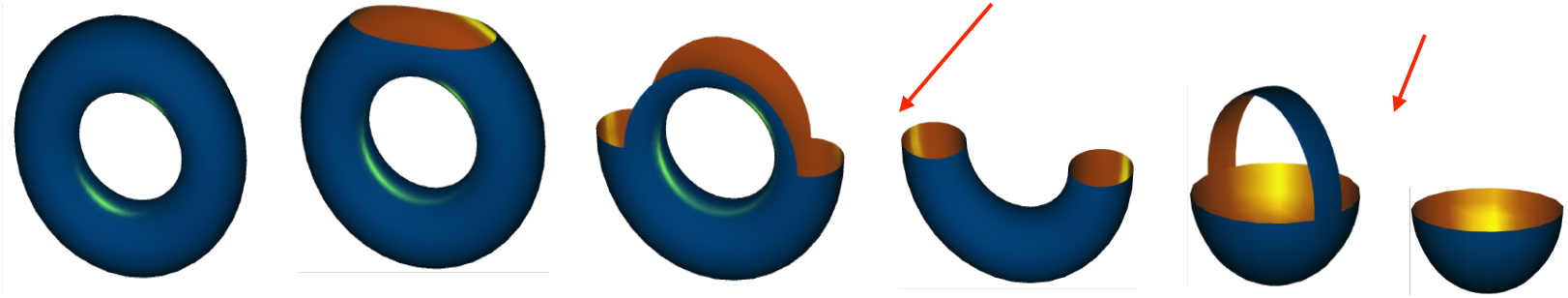


Apply Cauchy's proof to a triangle mesh with 1 handle

We must delete a **bridge** edge without deleting a vertex or a triangle



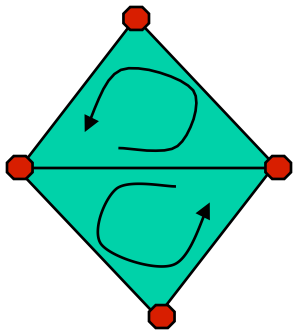
We have 2 bridge edges per handle

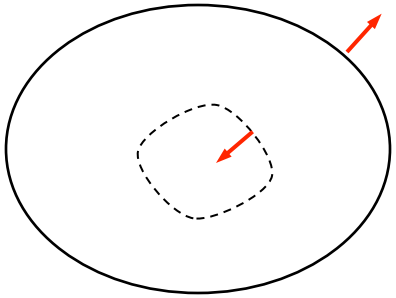


Now 3T/2=E=T+V–2+2H. Hence: **T = 2V – 4 + 4H**

Solids and cavities

- A **solid** (here restricted to be a connected manifold polyhedron) may be represented by its boundary, which may be composed of one or more **manifold shells**
  - One shell defines the external boundary
  - The other shells define the boundaries of internal cavities (holes)
- All the shells of a solid can be consistently oriented
  - It you were a bug on the outward side of an oriented triangle you would have to turn counterclockwise (with respect to your up vector normal to the triangle) to look at the vertices in the order in which their IDs are stored in the V table
  - The outward side of each triangle must be adjacent to the exterior of the solid.





Point-in-solid test

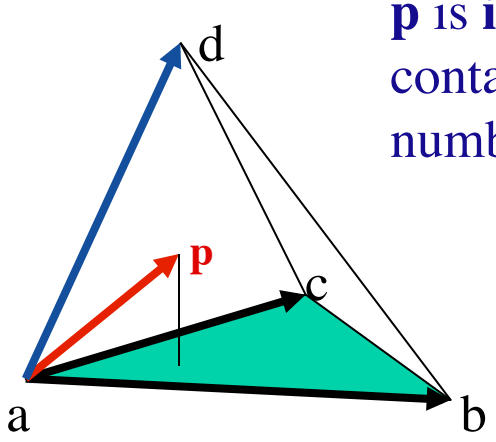
A point p lies inside a solid S bounded by triangles T_i when p lies inside an odd number of tetrahedra, each defined by an arbitrary point o and the 3 vertices of a different triangle T_i.

- Remember that point-in-tetrahedron test may be implemented as:

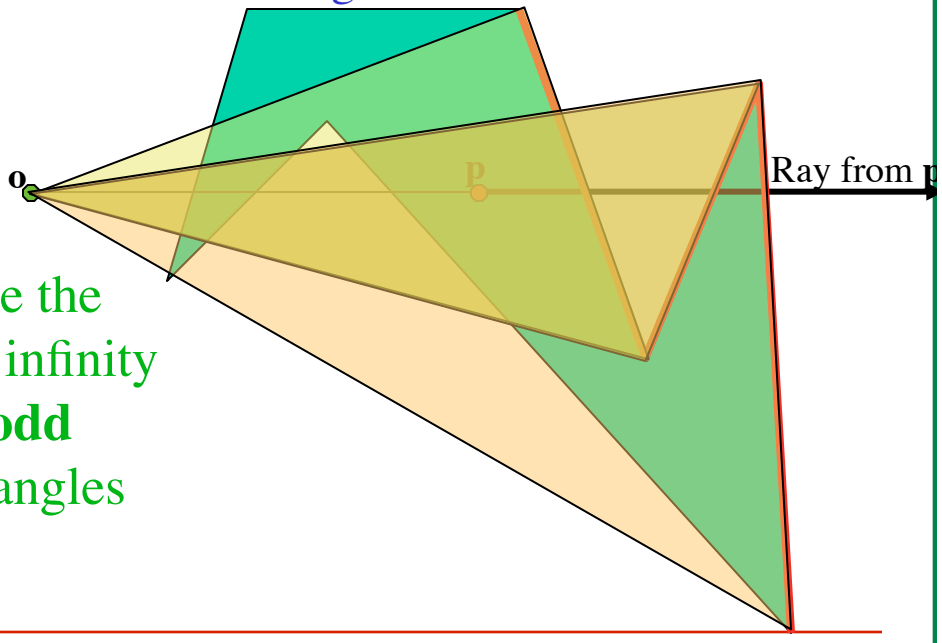
PinT(a,b,c,d,p) := *same*(*s*(a,b,c,d),*s*(p,b,c,d),*s*(a,p,c,d),*s*(a,b,p,d),*s*(a,b,c,p))

where s(a,b,c,d) returns $(\underline{ab} \times \underline{ac}) \bullet \underline{ad} > 0$

- The test does not assume proper orientation of the triangles!



p is **in** because it is contained in an **odd** number of tetrahedra



p is in because the ray from **p** to infinity intersects an **odd** number of triangles

Rationale

Consider an oriented ray R from o through p.

Assume for simplicity that it does not hit any vertex or edge of the mesh.

A point at infinity along the ray is OUT because we assume that the solid is finite.

Assume that the portion of the ray after p intersects the triangle mesh k times.

If we walk from infinity towards p, each time we cross a triangle, we toggle classification.

So, p is IN if and only if k is odd.

Let T denote the set of triangles hit be the portion of the ray that is beyond p.

Let H denote the set of tetrahedra that contain p and have as vertices the point o and the 3 vertices of a triangle in the mesh.

To each triangle of T corresponds a unique tetrahedron of H.

So, the cardinality of T equals the cardinality of H.

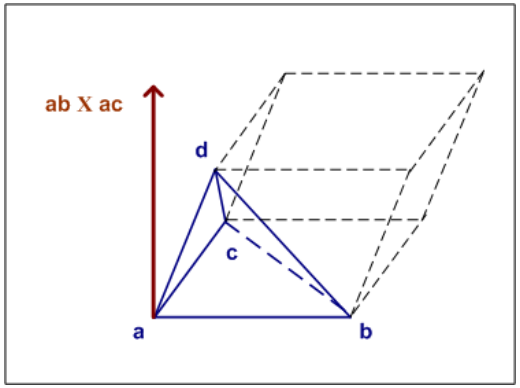
Hence we can use the parity of the cardinality of H.

Volume of a solid

- Given a solid S, bounded by consistently oriented triangles T_1, T_2, … T_n, let H_i denote the tetrahedron having as vertices an arbitrary origin o and the three vertices (b_i,c_i,d_i) of T_i.

- The volume of S is one sixth of the sum of $v(o,b_i,c_i,d_i)$, for all i.
  - $v(o, b_i, c_i, d_i)$ has been defined as $(\underline{ob_i} \times \underline{oc_i}) \bullet \underline{od_i}$
  - Note that it is independent on the choice of o
  - Note that it requires that the triangles be consistently oriented
  - The formula also works for triangulated boundaries of non-manifold solids, provided that the orientation is consistent with the outward orientation of the faces.

- Applications:
  - Physically plausible simulation
  - Product design and optimization
  - Volume preserving 3D morphs and simplification



Examples of questions for tests

- Define incidence, adjacency, corner, shell, solid, genus
- Difference between handle (through-hole) and hole (cavity)
- Explain the content of a corner table
- Provide the implementation of the corner operators: c.v, c.o, c.t, c.n, c.s, c.r
- How can we identify the corner opposite to c without the O table?
- Explain how to build a Corner Table from a list of triangles?
- How to identify the shells of a mesh represented by a corner table?
- How to compute the genus (number fo handles) of each shell?
- Can we represent solid by its bounding triangles (not-oriented)
- How to test whether a vertex lies inside a solid
- How to compute the volume of a solid

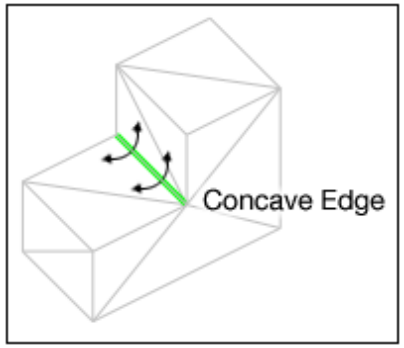
Questions to think about

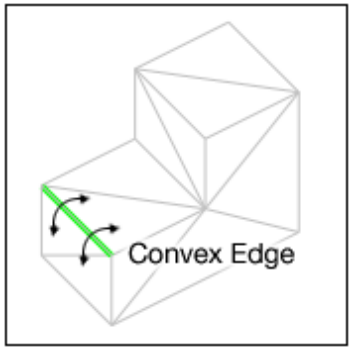
- How to pick the proper outward orientation for a triangle
- How to consistently orient the triangles of a shell
- How to test whether a point P is inside a shell S
- How to identify the shells that bound a solid
- How to identify the solids (and their bounding shells) from a corner table that represents all the triangles
- How to orient the shells bounding a solid
- How to identify the non-manifold vertices of a shell
- How to test whether a shell is free from self-intersections
- How to test whether two shells intersect one another
- What if the triangles do not form a water-tight shell

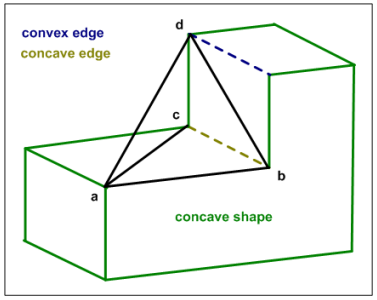
Practice question 1

Concavity test for an edge of a triangle mesh

- Assume that the triangle mesh is properly oriented and represented by a corner table.
- Each edge (a,b) of a triangle mesh may be identified by the opposite corner c of one of its incident triangles, so that c.n.v==a && c.p.v==b or vice versa. Let edge(c) denote this edge.
- Assuming that triangles are oriented counterclockwise, we say that edge(c) is concave when the vertices of c.t appear counter-clockwise from c.o.v.
- An edge that is not concave is either convex or flat.
- Write a very simple test for checking whether edge(c) is concave, convex, or flat. Use only vector notations and cross and dot products. Use the corner table operators. Present it as a function convex(c) that returns 1 if the edge is convex, 0 if it is flat, and -1 if it is concave.
- Make sure that your function can identify flat edges even in the presence of numerical round-off errors.
- Provide a drawing with the vertices and corners marked properly.







Question 1: Solution

- $\underline{N}:=(c.n.v.g-c.v.g)\times(c.p.v.g-c.v.g)$;
  - Outward normal to c.t
- $\underline{n}:=\underline{N}.u$;
  - Make a unit vector
- $d:=(c.o.v.g-c.v.g)\bullet\underline{n}$;
- If $|d|<e$ then the edge is flat.
- if $d>e$ then the edge is concave.
- If $d<e$ then the edge is convex.
  - Pick e carefully. Depends on model size, desired threshold for identifying flat edges.

Practice Question 2

Smooth normals

- Assume that the triangle mesh is properly oriented and represented by a corner table.
- Given a corner c, write the code for estimating the surface normal to the vertex c.v from two layers of neighbors.
- Use only corner table operators and operations on vectors and points.
- Let (c.v.x, c.v.y, c.v.z) denote the 3 coordinates of vertex c.v
- The first layer of neighbors are those vertices connected to c.v by an edge.
- The second layer of neighbors are those connected by an edge to the first layer.
- Use the 3 shadow area method for computing the normal to a patch

Question 2: Solution

We visit all neighbors of a.v and turn around each, collecting the triangle normals.
We mark visited triangles to avoid double counting.
This solution does not use the 3 shadow areas.

```
Input: corner a
N:=0;                                  # Normal is initially null
b:=a;                                  # b will travel ccw around a.v
DO {c:=b.n;                            # c will travel around ccw around b.n.v
      DO {
            IF (c.t.m==0) {            # triangle already processed
                  N+= (c.n.v.g–c.v.g)×(c.p.v,g–c.v.g); # add cross-product
                  c.t.b:=1 };          # mark triangle
            c:=c.r.n;                  # next corner around b.n.v
            } WHILE (c != b.n);        # finished turning around b.n.v
      b:= b.r.n                        # next b
      } WHILE (b != a);                # stop turning around a.v
n:=N.u;                                # make unit vector
```

Practice question 3

Shell containment

- Assume that you have two manifold shells A and B that do not intersect.
- Assume that each shell is properly oriented (so that its triangles appear counterclockwise when seen from the outside) and is represented by a corner table.
- Provide the pseudo-code for detecting whether A lies inside B, B lies inside A, or neither.
- Now, assume that A lies inside B. Provide the pseudo-code for testing whether a point p lies inside the solid S bounded by these two shells.

Question 3: Solution

- A is inside B if and only if the first vertex, v_o, of A is inside B.
  - A shell is connected. The shells do not intersect. So, A is either entirely inside B or entirely outside of it. Therefore we can use any vertex of A for the test.
- To test whether v_o is inside B, we pick a point x and make tetrahedra that each join x to a triangle of B. v_o is inside B if it lies in an odd number of these tetrahedra. The point-in-tetrahedron test is performed by comparing the signs of five mixed-products (see slides).
- To test whether B is inside A, just swap A and B above.

- Assuming that A lies inside B, the solid S is B–A, which is a simple CSG expression. Point p is in S if it is in B and out of A.
  - We already discussed how to test whether a point is inside a shell.
- An alternative solution is to treat A and B as a single set of triangles and count the parity of the tetrahedra they form with an arbitrary point x and that contain p. It has to be odd for p to be in S.