# Projectile Motion in Celestial Bodies


Edward Rios[1, *]

[1] *Department of Applied Mathematics and Physics, Valdosta State University*



This article analyzes a numerical model for the launching of a probe in four different celestial bodies: Venus, Earth, Moon and Mars, with the influence of air resistance. Each celestial body has its own air density , except the Moon that does not have an atmosphere. The air density causes the probe to have different initial conditions to land at least 20 km from the lander. The probe's trajectory is determined by using a fourth order Runge-Kutta method, considering different parameters such as the body's gravitational acceleration, atmospheric density, initial velocity, launch angle, and probe mass. The results of the simulations of the probe are being found by the horizontal distance it has traveled, initial velocity, launch angle, projectile mass, and maximum height attained by the probe. The celestial body that takes the most effort to reach 20 km is Venus because of its high air density (65 $kg/m^3$), the mass of the projectile is also high (15,925 $kg$), on the Earth and the Moon is 7,000 $kg$ and on Mars is 5,000 $kg$. The velocity when the probe is being launched is another determinant factor. In Venus, the initial velocity is $700,000$ $m/s$, extremely high initial velocity. On Earth the initial velocity is 504.5 $m/s$, on the Moon is 180 $m/s$ and on Mars is 273.5 $m/s$. Other results as the launching angle and the maximum height of the probe are useful to analyze the probe's trajectory on every body.


## I. INTRODUCTION

Projectile motion is one of the most fundamental topics of physics that studies the motion of objects thrown into the air and how the object is being influenced by different components as the air resistance and the force of gravity acting on it. The background history of projectile motion goes way back to Galileo's earliest work. The first objective of his study on motion was to discover the curve described by a projectile. In 1638 Galileo found that in the context of projectile motion, it's important to note that the horizontal and vertical motions occur autonomously, with each motion operating independently and having no influence on the other.

Projectile motion plays an important role in our daily life, in this case, it is useful for space explorations and scientific research. This fundamental concept also provides a practical application of kinematics. The main goal of this article is to study the projectile motion and its behavior on different celestial bodies with the influence of air resistance. The projectile motion experiment helps to understand the physics principles under varying environmental conditions. Each planet has unique gravitational forces, atmospheric compositions, and air densities, proving that the experiment is not going to have the same characteristics for each body. As Barthelemy (1) discusses, understanding the atmospheres of celestial bodies is crucial for comprehending their influence on projectile motion.The study of projectile motion on different planetary surfaces is crucial for space explorations, it helps design and execute safe landings, enabling the deployment of scientific instruments, rovers, and even human missions. Anderson's research (2009) (2) investigates into the aerodynamics and flight dynamics of spacecraft during atmospheric entry, including the effects of air resistance. Gathering data and information about a planet's properties is one of the other big tools of projectile motion. This also leads to develops newer technology to improve the experiment and test it in farther bodies.

This article addresses several important questions. First, it explores the ideal weight an object must have to travel a distance of 20 kilometers from its starting point. Second, it investigates the required initial velocity to achieve this 20-kilometer journey. Additionally, it examines the significant impact of air density on determining these various parameters. Given the consideration of air resistance, each celestial body presents unique characteristics. The shape of the object being launched also plays a role, as different shapes result in varying drag coefficients, influencing the initial conditions.

The next section of the paper provides a detailed analysis of how to determine the initial conditions for the projectile and their influence In Section II, the method is explained, detailing the theoretical framework for understanding the motion of objects through the air, considering air resistance, and the computer program used for experiments on various celestial bodies. In Section III, visual aids such as figures and tables are used to illustrate the core concepts of the model and the alignment between theoretical predictions and observed data. Additionally, a comparison is made between the outcomes of computer simulations and analytical values.Finally, Sections IV and V summarize the insights gained from the experiments and describe the `python` code used to investigate the motion of objects with air resistance on different celestial bodies.


* erios@valdosta.edu

## II. METHODS

The theoretical model originates from Galileo's experiments. He conducted tests by rolling balls at different speeds and dropping them from various heights. This led to the discovery that when objects are thrown, they follow a curved path resembling a parabola. To implement the theoretical model, the parameters for each celestial bodies are given in the next table :

| **Celestial Body** | **Mass (kg)** | **Air Density** $(kg/m^3)$ |
|---|---|---|
| Earth | $5.97 \times 10^{24}$ | 1.204 |
| Moon | $7.348 \times 10^{22}$ | 0 |
| Mars | $6.39 \times 10^{23}$ | 0.02 |
| Venus | $4.867 \times 10^{24}$ | 65 |

Table I: This table shows the parameters such as the mass, and the air density are shown for each celestial body.

Gravitational Force : This is the gravitational force acting on the probe. Using Newton's law of universal gravitation which relies on the mass and the radius of the celestial body gives a different value for the force of gravity acting on the probe.

$$\text{g(acceleration)} = \frac{G \cdot M}{R^2} \tag{1}$$

The incorporation of air resistance into the motion is what is causing the probe to have different initial conditions. To calculate how much air slows an object down, the drag equation is applied. This equation accounts for the object's shape and its speed. The drag force opposes the object's motion and relies on factors such as its size ($A$), the air's density ($\rho$), its aerodynamic characteristics ($C_0$), and its velocity ($v$).

$$\text{Drag Force} = \frac{1}{2} \cdot \rho \cdot A \cdot C_d \cdot v^2 \tag{2}$$

The drag coefficient as I mentioned before, depends on its shape. For this experiment, I use a streamlined body as a shape for the probe and it contains a drag coefficient ($C_d$) of 0.04. The $v$ is the speed of the object relative to the fluid. It's important to understand that air resistance significantly affects the probe's motion. Therefore, calculating and considering these factors is crucial for accurately simulating its trajectory on different celestial bodies.

To conduct the experiment and get the results, I employ a numerical method and write as a code using the program language `python`. I use the Runge-Kutta method for the numerical integration. The numerical method used for simulating the projectile motion is the fourth- order Runge-Kutta method. This method enables the calculation of the probe's trajectory by considering parameters such as the gravitational acceleration ($g$), air density ($\rho$), initial velocity ($v_0$), launch angle ($\theta$), and probe mass ($m$). The Runge-Kutta methods used to solve differential equations rely on finding the formal (exact) integral of the differential equation.

1. Velocity in the x-direction:

$$k_{1v_x} = \Delta t \cdot \left( -\frac{1}{2} \cdot \rho \cdot A \cdot C_d \cdot v_x^2 + g \right) \tag{3}$$

$$k_{2v_x} = \Delta t \cdot \left( -\frac{1}{2} \cdot \rho \cdot A \cdot C_d \cdot (v_x + \frac{1}{2} k_{1v_x})^2 + g \right) \tag{4}$$

$$k_{3v_x} = \Delta t \cdot \left( -\frac{1}{2} \cdot \rho \cdot A \cdot C_d \cdot (v_x + \frac{1}{2} k_{2v_x})^2 + g \right) \tag{5}$$

$$k_{4v_x} = \Delta t \cdot \left( -\frac{1}{2} \cdot \rho \cdot A \cdot C_d \cdot (v_x + k_{3v_x})^2 + g \right) \tag{6}$$

$$v_x = v_x + \frac{1}{6} \cdot (k_{1v_x} + 2k_{2v_x} + 2k_{3v_x} + k_{4v_x}) \tag{7}$$

2. Velocity in the y-direction:

$$k_{1v_y} = \Delta t \cdot \left( -\frac{1}{2} \cdot \rho \cdot A \cdot C_d \cdot v_y^2 - g \right) \tag{8}$$

$$k_{2v_y} = \Delta t \cdot \left( -\frac{1}{2} \cdot \rho \cdot A \cdot C_d \cdot (v_y + \frac{1}{2} k_{1v_y})^2 - g \right) \tag{9}$$

$$k_{3v_y} = \Delta t \cdot \left( -\frac{1}{2} \cdot \rho \cdot A \cdot C_d \cdot (v_y + \frac{1}{2} k_{2v_y})^2 - g \right) \tag{10}$$

$$k_{4v_y} = \Delta t \cdot \left( -\frac{1}{2} \cdot \rho \cdot A \cdot C_d \cdot (v_y + k_{3v_y})^2 - g \right) \tag{11}$$

$$v_y = v_y + \frac{1}{6} \cdot (k_{1v_y} + 2k_{2v_y} + 2k_{3v_y} + k_{4v_y}) \tag{12}$$

3. Position in the x-direction:

$$x = x + \Delta t \cdot v_x \tag{13}$$

4. Position in the y-direction:

$$y = y + \Delta t \cdot v_y \tag{14}$$

Runge-Kutta equations for projectile motion with air resistance are complex and involve multiple steps, including updating velocity and position at each time step. These equations describe how the probe's position and velocity change over time, taking into account gravitational acceleration and the drag force due to air resistance.

Runge-Kutta becomes helpful for projectile motion with air resistance because it's like a smart tool that aids in understanding and predicting how objects move through the air. When an object is launched, it faces two significant challenges: gravity pulling it downward and air slowing it down. Runge-Kutta assists in determining the object's trajectory and velocity. It acts like a specialized calculator that breaks down the problem into small, manageable steps. These steps allow for tracking the object's position and speed at each instant. This method enables accurate predictions of its path.

By using `Python` the motion of the projectile is easier to apply because it can be define as a function that relies on different variables as gravity and air resistance. It works by looking at the object's speed and other characteristics and then predicts how the object's speed and position will change over time. The code used to derive this experiment can be seen in the Appendix V

## III. RESULTS

For this section, I analyze the results obtained once the programs has been run and the probe has reached the 20km line on every celestial body. The horizontal distance covered by the probe is an important parameter to accomplish the goal of the experiment. The numerical simulations revealed that the probe traveled different distances on each celestial body due to variations in air density and gravitational acceleration. These results are summarized in the following table.

| **Celestial Body** | **Distance Traveled (m)** |
|---|---|
| Earth | 20,006.74 |
| Moon | 20,001.37 |
| Mars | 20,012.84 |
| Venus | 20,005.04 |

Table II: Horizontal Distance Traveled by the Probe on Different Celestial Bodies

The probe needs different initial conditions to land 20 km awy from the lander on each celestial body. The results obtained of the parameters of the probe are being shown in the following table :

| **Body** | **Mass (kg)** | **Initial Velocity (m/s)** | **Launch Angle (°)** | **Max. Height (m)** |
|---|---|---|---|---|
| Earth | 7,000 | 504.5 | 45 | 5,606.05 |
| Moon | 7,000 | 180 | 45 | 5,000.44 |
| Mars | 5,000 | 273.5 | 41.5 | 4,438.06 |
| Venus | 15,925 | 700,000 | 41.5 | 14,548.95 |

Table III: This table shows the initial conditions of the probe on each celestial body to reach the goal of 20 km distance.

The mass of the probe is a critical factor because it directly impacts how much force is required to overcome gravity. On planets with stronger gravitational pull, like Earth, a heavier probe is needed to ensure it can reach a specific height and distance. The initial velocity at launch is determined by a balance between gravity and air resistance. It must be set precisely to counteract gravity but not exceed a speed that air resistance would significantly slow it down. For example, on Mars, which has lower air density, a lower initial velocity is sufficient to achieve the experiment's goals. The maximum height achieved by the probe is a result of the initial velocity, the launch angle, and the strength of gravity. The probe needs to reach a certain height to gather specific data or to ensure that it has a safe landing trajectory. This height varies depending on the objectives of the experiment and the conditions of the celestial body.

By looking at Table III the high values for the probe in Venus is due to the the high air density (65 $kg/m^3$) which makes it pretty hard to make the experiment requiring a really heavy probe with a super initial velocity. That would probably cause more money to make the experiment and more time developing a probe that meets such requirements. In the case of the Moon where there in not an atmosphere the projectile motion experiment would be applied easily and without difficulties. For the case of Earth the acceleration of gravity is the biggest (9.81 m/s) which means that the probe has to have a higher mass to counterbalance the force of gravity. Mars is a similar case as the Moon, where the air density is small and the force of gravity is not a big factor either. Even the experiment of projectile motion on the Moon has been accomplished before by the Astronaut Alan Shepard. "Shepard took a few moments during the Apollo 14 landing to show off his hobby during a live broadcast from the lunar surface on Feb. 6, 1971. He took two shots, with the second ball going "miles and mile," he said on-camera". Springer's book (1982) (3) covers various aspects of lunar missions, including the analysis of projectile motion in lunar landers.

By computing the characteristics of the probe on each celestial bodies using the methods describing before on Section II, with `Python` I plot the trajectory of the probe on each of the bodies.

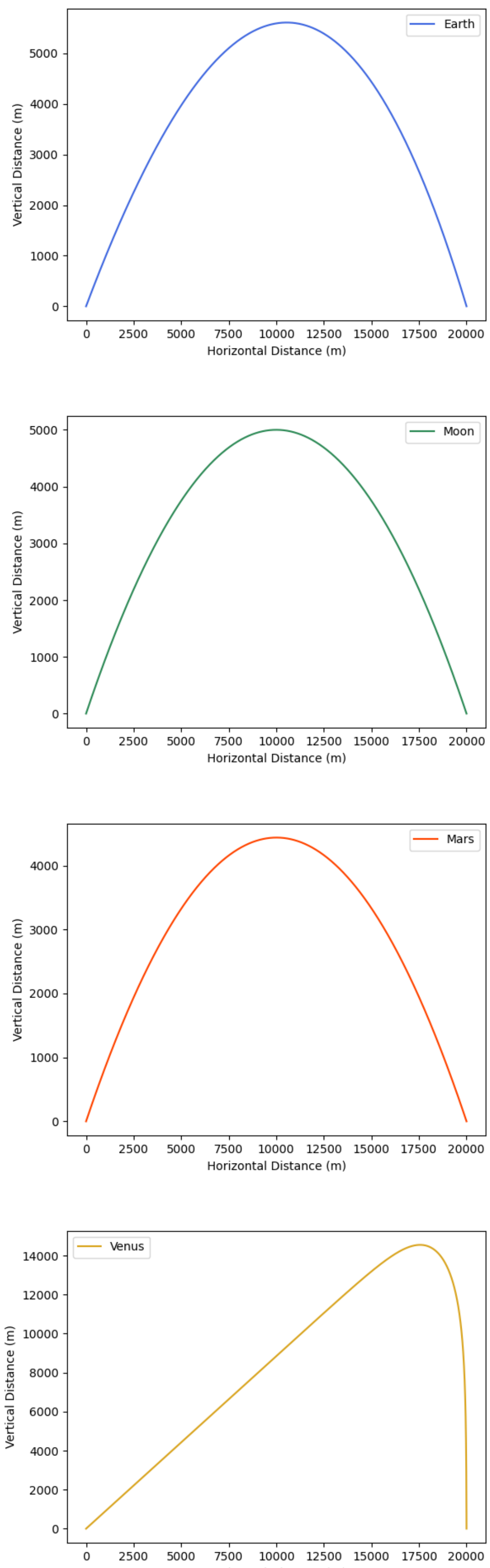


Figure 1: This figure shows the different trajectories of the probe being launched at a certain angle and landing at least 20 km from the lander on each celestial body.

Analyzing the figure **??** I can see that the trajectory for Earth , Moon and Mars are pretty similar having a noticeable parabola. In the other hand, the shape of the trajectory on Venus is totally different. The shape of Venus' trajectory is different from the other celestial bodies because Venus has a very thick and heavy atmosphere made up of gases. This dense atmosphere creates a lot of resistance or "air drag" when the probe is flying through it, slowing it down more than the other planets with thinner atmospheres. As a result, the probe needs to be launched with extremely high initial speed and weight to overcome this resistance and reach the same distance. To make the probe travel 20 kilometers away, Venus requires a powerful launch compared to other celestial bodies.

## IV. CONCLUSION

Summarizing the experiment, this study explores into how objects, like rockets or probes, move in the atmospheres of various celestial bodies, including planets like Venus and Mars, the Moon, and Earth. A computer program (`Python`) helped determine what conditions were necessary for these objects to travel at least 20 kilometers from their starting points on each celestial body. It became clear that the planets posed unique challenges. For instance, Venus, with its dense atmosphere, required rockets to be extremely fast and heavy to break through the thick air. In contrast, the Moon, which lacks an atmosphere, offered a simpler path for rocket travel.

In this project I found that depending on the characteristics of the celestial body, the probe requires different conditions to reach the goal and the air resistance changes completely the motion of the probe. The air density of each of the celestial bodies , except the Moon, play a big role on contrasting the motion of the probe and would require higher initial conditions. Future research could investigate the influence of rocket shapes on their performance in space. Additionally, there's an opportunity to develop advanced computational tools for enhanced space exploration. Sharipov and Kharisov's study (2017) (4) looks into the aerodynamic characteristics of spacecraft during atmospheric entry and the implications of air resistance. Better comprehension of how objects move in space supports our ambitions to explore other planets and expand our understanding of the universe. Talking about future expeditions and the development of new techlogies to test new experiments in celestial bodies, there has been already expedition as the one on Mars 2020 Rover mission (5) which contributes to a significant advancement in the field of planetary exploration.

# V. APPENDIX

## Appendix A: Python Code for Projectile Motion

Listing 1: Your Python Code

```python
import numpy as np
# Import the NumPy library
import matplotlib.pyplot as plt
# Import the matplotlib library for
plotting
from scipy.constants import G
# Import the gravitational constant

# Define constants
x_f = 2e4  # Minimum probe distance (m)
r = 1.2  # Radius of probe (m)
Cd = 0.04  # Coefficient of drag
A = np.pi * r**2  # Cross-sectional
area of the probe
N = 100000  # Number of steps to take
tau = 200.  # Total time for simulation
(in seconds)
h = tau / float(N - 1)  # Time step
times = np.arange(0, tau + h, h)
# Array of time points

def accel(M, R):
    g = G * M / R**2  # Calculate
    gravitational acceleration
    return g

def rk4(y, t, h, derivs, grav, rho, m):
    # Runge-Kutta 4th order integration
    for motion
    k1 = h * derivs(y, t, grav, rho, m)
    k2 = h * derivs(y + 0.5 * k1, t +
    0.5 * h, grav, rho, m)
    k3 = h * derivs(y + 0.5 * k2, t +
    0.5 * h, grav, rho, m)
    k4 = h * derivs(y + k3, t + h,
    grav, rho, m)
    y_next = y + (k1 + 2 * k2 + 2 *
    k3 + k4) / 6  # Update the state
    return y_next

# Define characteristics for Earth,
Moon, Mars, and Venus
celestial_bodies = [
    {"name": "Earth", "M": 5.97e24,
    "R": 6.38e6, "rho": 1.204, "v_i"
    : 504.5, "theta": 45, "color": '
␣␣␣␣royalblue', "mass": 7000},
    {"name": "Moon", "M": 7.348e22,
    "R": 1.74e6, "rho": 0, "v_i": 180,
    "theta": 45, "color": 'seagreen',
    "mass": 7000},
    {"name": "Mars", "M": 6.39e23,
    "R": 3.4e6, "rho": 0.02, "v_i":
    273.5, "theta": 41.5, "color":
    'orangered', "mass": 5000},
    {"name": "Venus", "M": 4.867e24,
    "R": 6.05e6, "rho": 65, "v_i":
    700000, "theta": 41.5, "color":
    'goldenrod', "mass": 15925}
]

def motion(z, t, grav, rho, m):
    z_p = np.zeros(4)
    v_mag = np.sqrt(z[1]**2 + z[3]**2)
    # Calculate the velocity magnitude
    z_p[0] = z[1]
    z_p[1] = -0.5 * A * Cd * rho *
    v_mag * z[1] / m  # Calculate
    horizontal acceleration
    z_p[2] = z[3]
    z_p[3] = -grav - 0.5 * A * Cd *
    rho * v_mag * z[3] / m  #
    Calculate vertical acceleration
    return z_p

# Initialize a list to store the results
results = []

# Loop through the celestial bodies
and store the results
for body in celestial_bodies:
    xmax = 0
    grav = accel(body["M"], body["R"])
    # Calculate gravitational
    acceleration
    rho = body["rho"]
    v_i = body["v_i"]  # Initial velocity
    theta = body["theta"]  # Launch angle
    color = body["color"]
    mass = body["mass"]

    x_0, y_0 = 0, 0
    v_x0, v_y0 = v_i *
    np.cos(np.radians(theta)), v_i *
    np.sin(np.radians(theta))
    # Initial horizontal and vertical
    velocities

    states_rk4 = np.zeros((N, 4))
    states_rk4[0, :] = [x_0, v_x0,
    y_0, v_y0]
    # Set initial state

    for j in range(0, N - 1):
        states_rk4[j + 1, :] =
        rk4(states_rk4[j, :], times[j],
        h, motion, grav, rho, mass)
        # Run RK4 integration
        if states_rk4[j + 1, 2] <= 0:
```

```
        # If the probe hits the ground
            states_rk4 = states_rk4[:j,
            :]  # Remove extraneous
            rows
            xmax = states_rk4[-1, 0]
            # Record the horizontal
            distance at impact
            break

    results.append({"name":
    body["name"], "trajectory":
    states_rk4, "initial_velocity":
    v_i, "color": color, "theta":
    theta, "mass": mass})

# Plot the trajectories and save figures
for result in results:
    trajectory = result["trajectory"]
    name = result["name"]
    initial_velocity =
    result["initial_velocity"]
    color = result["color"]
    theta = result["theta"]
    mass = result["mass"]

    plt.figure()
    plt.plot(trajectory[:, 0],
    trajectory[:, 2], color=color,
    label=name)  # Add label for legend
    plt.xlabel("Horizontal Distance (m)")
    plt.ylabel("Vertical Distance (m)")
    plt.legend()  # Display the legend
    plt.savefig(f"{name}_trajectory.png")
    # Save the figure
    plt.show()

    distance = trajectory[-1, 0]
    # Calculate the horizontal distance
    at impact
    max_height = max(trajectory[:, 2])
    # Find the maximum height

    print(f"{name}:")
    print(f"Distance: {distance:.2f}
    meters")
    print(f"Initial Velocity:
    {initial_velocity} m/s")
    print(f"Theta Angle: {theta}
    degrees")
    print(f"Projectile Mass:
    {mass} kg")
    print(f"Max Height:
    {max_height:.2f} meters")
    print("\n")

# Display the figures
plt.show()
```


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[5] Ker Than. Nasa's mars 2020 rover: Will perseverance pay off? *National Geographic*, 2020. URL `https://www.nationalgeographic.com/science/2020/06/nasas-mars-2020-rover-will-perseverance-pay-off/`.