Stress Analysis on Gravity Separator Bearings

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The pieces of a machine can experience various forces and stresses within their operational lifetime. Modeling these can give better insight to the potential failings of a machine and enable more mindful engineering. We conduct an analysis into the mechanics of a hypothetical spherical roller bearings within a gravity separator machine. Such bearings have been known to fail in a shorter time span than expected. The cause of such failing is open to exploration by examining the forces and possible stress imposed onto the system. We analyze certain modes which were suspected of causing instability in the bearing. This includes motion in the y-direction for the separator platform, the restoring spring force from the planks connected to the platform, torque in the z-direction, and the platform motion in the x-direction. We cannot make a definite conclusion regarding the failure of the bearing in the machine as there are possible avenues which may be uncovered with further analysis.

I. INTRODUCTION

Machinery bearings serve to lower friction and support loads during rotational or linear motion, making them crucial parts of a variety of mechanical systems. They are essential to many different industries, including the automotive, aerospace, manufacturing, and construction sectors. They are utilized in everything from massive industrial gear to little home gadgets. By using higher-quality bearings, machine tool spindles become more dependable and efficient, which boosts output and lessens wear and tear on the equipment. Understanding the types, functions, and importance of bearings is fundamental. One type of bearings that companies use is often custom-designed to accommodate the unique motion requirements of the application. One particular type of bearing is the elliptical bearing, which is found in cross-training apparatus and elliptical trainers. The purpose of these bearings is to support the machine's pedals or arms as they move in an elliptical motion. Typically, they are designed with a specific function in mind to allow for smooth operation and reduced friction. The type of bearing we are going to focus on is the roller bearings, specifically spherical roller bearings for more accurate results of this study.

Spherical roller bearing is made to support moderate axial loads in both directions as well as strong radial loads. They have two rows of symmetrical barrel-shaped rollers that operate on a single spherical outer raceway, enabling the bearing to self-align. Because of its construction, the bearing can withstand shaft deflection and misalignment, which makes it appropriate for situations in which shafts are not precisely aligned or exhibit large deflections([1]). The spherical roller bearing's misalignment tolerance is especially useful in heavy machinery, mining, and construction applications where shafts must withstand severe loads and fluctuating operating conditions. These bearings are appropriate for applications requiring high rotational speeds because of their design, which also permits them to function at high speeds.

Within the complex field of mechanical engineering, bearing stress analysis is an essential component in guaranteeing the durability and performance of machinery, especially that which is subjected to dynamic and oscillatory loads. Situations like the ones the Valdosta State University engineering delegation experienced while visiting Premium Peanuts in Douglas, Georgia, serve as an example of this importance. There, a problem with recurrent bearing failures in peanut sorting equipment such as gravities and shellers was observed. This led to a thorough investigation of the mechanical interactions involved, particularly the effect of structural flexures on bearing performance. The purpose of machinery bearings is to enable smooth rotational motion and support heavy loads. However, structural deformations and misalignments can provide additional dynamic forces, which could cause the bearings to fail prematurely.

The bearing system, which is essential for controlling rotational and load loads while reducing friction between moving parts, facilitating smooth operation, and extending the lifespan of machinery, is a crucial part that frequently determines how well agricultural machinery performs. The "L10" (US) or "B10" (otherwise) life of a bearing is often specified. This refers to the time interval within which 10% of the bearings in that application are expected to fail because of classical fatigue failure (and not any other mode of failure, such as lubrication starvation, incorrect mounting, etc.); alternatively, this is the time interval within which 90% of the bearings will still be in operation. The bearing's theoretical L10/B10 life may not accurately reflect its service life([2]). In the context of peanut sorting machinery, bearings play a pivotal role by supporting the mechanical rotating axes and handling the loads generated during the sorting process.

By reducing friction between moving parts, bearings are made to withstand the heaviest mechanical loads and enable machinery to operate smoothly. However, when bearings fail, it can cause everything from little operational issues to massive machine breakdowns that can stop whole production lines. Although there are many different reasons why bearings fail, material fatigue, unequal load distribution, misalignment, insufficient lubrication, and particle contamination are among the most common causes. All of these factors add to the total stress that the bearing experiences, which ultimately causes it to deteriorate and fail. Bearings are exposed to extreme stress and wear as a result of these circumstances as well as the high-speed and high-load factors associated with the sorting process. As an example, the severe conditions in which peanut sorting equipment functions including high amounts of dust, moisture, and temperature fluctuations can cause failure. Bearing failures highlight a crucial topic in mechanical engineering: bearing stress analysis. These failures frequently result in major downtime and replacement expenses.

Bearing stresses are the tensions created when two elastic bodies are forced together([3]). Stress analysis is necessary because of the complex relationships between the forces acting on bearings and the serious effects of their failure. Evaluation of the forces, both static and dynamic, exerted on a bearing during its operational lifecycle, as well as the bearing's response to these loads under various circumstances, are all included in stress analysis. This analytical procedure provides information regarding possible mechanisms of failure and required design enhancements.

This research aims to analyze the stress analysis of machinery bearings and determine whether the plank's bending and subsequent force exerted when recoiling creates interference with the bearing's rotation, such as resonance. We need to use a comprehensive analytical framework that integrates data collecting, theoretical modeling, and computational simulations to evaluate these dynamics successfully.

II. METHODS

In order to understand the problem and start equations such as the forces exerted on the bearing, the position of the bearing and so on, we need to make a simple diagram 1 of the system to analyze it better.

This problem can be evaluated from different perspectives by having different values or different

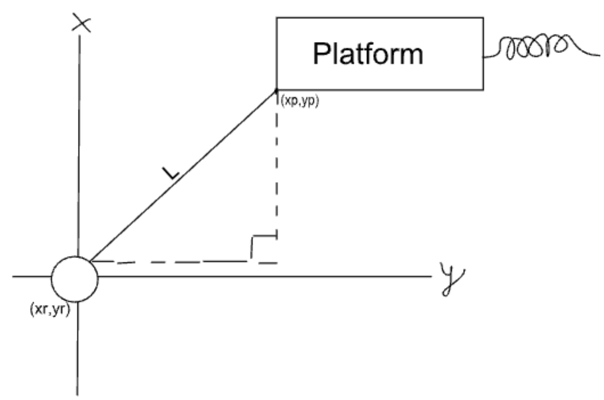

Figure 1. This diagram shows the rod being connected from the spherical bearing to the platform of the machinery. Next to the platform we can see the spring force exerted from the planks.

shapes for the material such as the bearing in this case. By analyzing the machine where the bearing is connected to, we take a look at the flexible planks, and by looking at their motion, we have determined that they are oscillating, therefore they can be treated as a spring force. The spring force is trying to push the top edge of the rod to the left, and then the rod itself makes a force acting to the right, which means that there are two forces acting on the top end of the rod, which for convenience in this paper we call it "x_p". The bottom end of the rod is connected to the bearing, therefore we must count the bearing to be part of the bottom end of the rod. The expression for the bottom end of the rod are determined as

$$x_r = acos(wt + \phi_0), \tag{1}$$

where a is the radius of the bearing, ϕ_0 is the angle when the flexible planks are closed to be vertical. Similar expression for y_r

$$y_r = asin(wt + \phi_0). \tag{2}$$

We call the rod length to be "L" and "y_p" which is the y-coordinate position of the top of the rod, and both of them are arbitrary constants. By doing some trigonometry, the expression for x_p can be found as

$$x_p = acos(wt + \phi_0) + \sqrt{L^2 - (y_p - asin(wt + \phi_0))^2}, \tag{3}$$

We can see how equation 3 is similar to equation 1, where the only difference is that now we are adding this extra part from the length of the rod. Another important thing to notice is that both equations are depending on time, and this is because we want to keep track on how this position x_p differs with time. Now, we can determine the initial position (x_p) by setting the time equal to 0, just as shown in the next formula:

$$x_p(t = 0) = acos(\phi_0) + \sqrt{L^2 - (y_p - asin(\phi_0))^2}. \tag{4}$$

The next step is to find out what is the force of the planks acting on the rod. We can determine this force with equation 3 and 4. We have mentioned that the planks force are acting as a spring force which the formula is just, $F = -kx$, where k is the spring constant and x is the position. The planks force equation is

$$F_{plank} = -k(x_p - xp_0). \tag{5}$$

Now we need to find out what is the net force being exerted from the rod in the x-direction. In order to find out the net force, we need to add the force from the rod and the planks:

$$F_{net} = F_{rodx} + F_{plank}, \tag{6}$$

where F_{net} is Newton's second law ($F = ma$) or the same as $F = m\ddot{x}$. Rewrite F_{net}:

$$m\ddot{x} = F_{rodx} + F_{plank}, \tag{7}$$

now by moving terms to the other side of the equation we can solve for F_{rodx}:

$$F_{rodx} = m\ddot{x_p} + k(x_p - xp_0), \tag{8}$$

where m is the mass of the bearing. Notice that to solve for F_{rodx}, first we must need to take the first and second derivative of equation 3. By just looking at equation3, we can assume that the second derivative is going to be a little messy, and therefore is going to take a lot of time and human errors can happen while we are taking the derivative of such long expressions. For that reason, we use `Mathematica` to compute difficult expressions and to obtain the most accurate results. The code used to solve this expressions can be seen in Appendix A.

Once the force of the rod (F_{rodx}) has been found, then we can find the total force applied in the x-direction. We must have to account the force of the rod and the difference of x-positions from the bottom end of rod and the top end. This total force can be expressed as

$$F_{tot} = \frac{F_{rodx}}{(x_p - x_r)/L}, \tag{9}$$

we can see how the total force (F_{tot}) is involving equations 1, 3 and 8, which means that the output expression is not going to be easy to compute by hand, that is why we recommend to use `Mathematica` to solve this expression, and others. `Mathematica` has a command where we can fully simplify expressions, which makes the equation easier to understand.

All we have done so far is solving for the total force 9 in the x-direction, which means that we must also need to consider the force exerted in the y-direction. To account for the force in the y-direction, we also need to take in account the forces we found in equations 8 and 9. The absolute magnitude of the force in the y-direction can be determined as

$$|F_y| = \sqrt{F_{tot}^2 - F_{rodx}^2} = F_{tot} sin(\alpha), \tag{10}$$

which can also be expressed in terms of y_p and y_r:

$$|F_y| = F_{tot}(\frac{y_p - y_r}{L}). \tag{11}$$

We must take a look at this expression for F_y, and we find that the sign for the force in the y-direction changes for certain values of θ. When $F_y > 0, 0 \leq \theta < 180°$, i.e. $y \geq 0$. When $F_y < 0, 180° \leq \theta \leq 360°$, i.e. $y < 0$. We found this force in the y-direction, in order to find the torque of the rod to see what is the force that cause the bearing to rotate about an axis. The torque of the rod is found by the following expression :

$$\tau_{\vec{rod}} = \vec{r} \times \vec{F_{rodx}} + \vec{r} \times \vec{F_y}, \tag{12}$$

by doing the hand-rule, we determine that it is going in the z-direction, and torque can be simplified and found as

$$\tau_{\vec{rod}} = F_{rodx} y_r z + (-F_y)(-xz) = (F_{rodx} y_r + F_y x_r) z \tag{13}$$

For this project we have encountered different ways to approach the problem. We also have tried a method where the equations depend on the angle θ instead of time. We obtained the results after plotting the forces over time, and look into those graphs to see if they show any interesting behavior.

III. RESULTS

The purpose of this was to determine the presence of resonance in this system through analytical means, and through the use of `Mathematica` we were able to obtain some graphs that simulate the torque and force on the rod.

$$w = 5,\ L = 1,\ a = 0.02,\ k = 100,$$

$$y_p = 0.2,\ m = 70,\ \phi = 21(\pi/180)$$

Applying different numbers for the variables in the equations allowed some room for us to experiment with our results. Some of the things we had to account for consisted of the velocity of the planks (w), the radius of the bearings (a), the plank constant (k), the mass in kg (m), and the angle at which the planks are vertical (ϕ). Other things are the length of the rod (L) and the initial position of the rod when vertical (y_p) that were discussed in Section II Methods.

In figure 2 we have a visual of how the torque acts over time. The curve peaks may indicate that the maximum value in either the positive or negative z-direction is not set. This may warrant further investigation on the effects of torque on the bearing stability. Looking over this we are not able to see any discernible outliers that would make for any interesting types of behavior. In our experiments with this, we attempted to change the various parameters listed above that are used within the equations. This involved changing things in an attempt to make the torque either all positive or all negative. However, this is not possible while still trying to keep all of the measurements realistic.

Moving on to the visualization of the rod within this system, we can tell that force is exerted in a sinusoidal pattern. Throughout the graph there are no outlying patterns that would give us reason to suspect unwanted resonance. Things we would expect to see in this case would be spikes at some of the peaks, however, the flow of force over time remained constant.

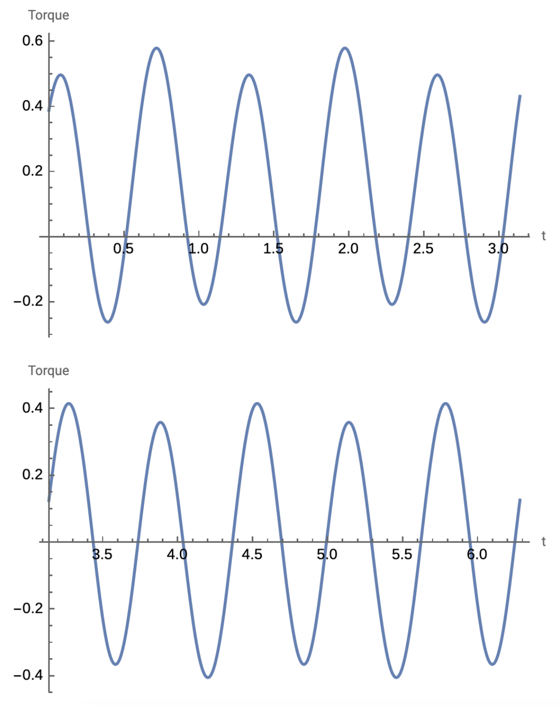

Figure 2. This figure shows a plot of torque over time where the two results are separated due to the force in the y-direction having a changed sign in the second plot. There are slight variations in the peaks of the curves which indicates how strong the torque vector is in the z-direction.

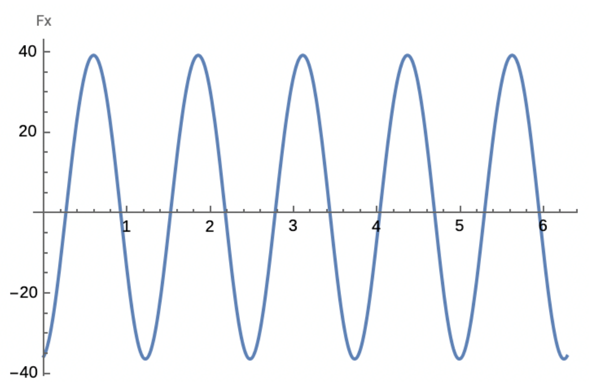

Figure 3. This shows the force exerted on the rod over time visualized as a sinusoidal wave.

IV. CONCLUSION

The forces and stresses of machine components are worth investigating to diagnose and redesign possibly compromised systems. This may hold true for machines and systems that experience rotational resonance as any subtle forces could be overlooked during the design process. The spherical roller bearings on the gravity separator machines were subject to recurrent failure. The investigation within this work focused on the flexing of the system composed of the planks, the separator platform, the bearing, and the rod connecting the bearing to the platform.

The platform moves radially in the positive and negative x-directions. The bearing and rod system rotational motion as described by equation 1 facilitates the motion of the other components in this analysis. The planks exert a spring force onto the platform as they retract back into an equilibrium position.

Our first analysis was focused on the interactions within the rod-bearing system to see if there were any instabilities in the y-direction motion which could be modeled. We found that the movement is in resonance as previously expected. The total force over time of the system was sinusoidal. We then examined the platform-planks system and the opposing forces associated within. We determined that the forces were not obviously contributing to any type of motion indicative of being an instability in the total system. This determination may benefit from further analysis. The following investigation into the platform's possible motion in the the y-direction led us to equation 11. The motion in the y-direction depends on the angular motion of the bearing-rod system in which $F_y < 0, 180° \leq \theta \leq 360°$ results in $y < 0$ and $F_y > 0, 0 \leq \theta < 180°$ leads to $y \geq 0$. We wanted to find an appropriate model describing the torque which led to motion of the bearing along the z-axis. Equation 12 describes the torque changing with time as the center of the bearing revolves a center. Torque in the positive z-direction correlates with positive x-values on this circle in a Cartesian coordinate system and vice versa with negative x-values. We believe a more detailed analysis into the mechanics of the gravity separator roller bearings would give more insight into the reasons structural failure occurs at a higher rate than expected.

There is no empirically supported conclusion to be drawn with our current analysis methods for whether the forces and mechanics explored were stress inducing to the point of structural failure. However, we have not holistically explored all modes for which the bearings could be unstable. This enables opportunity for future work in analyzing with more terms for the mechanics to accurately describe the system and the possible reasons for bearing failure. This gives opportunity for future work to analyze additional mechanical factors to more accurately describe the system and identify potential causes of bearing failure.

Appendix A: CODE

```
Mathematica Code:

f[t_] := a Cos[w t + \[Phi]]+Sqrt[L^2-(
    yp-a Sin[w t+\[Phi]])^2]
(*Solve for the first and second
    derivative for xp*)
f'[t] (*First derivative*)
f''[t] (*Second derivative*)
delta[t_] = f[t]-f[0]
Fplanks[t_]:= -k *delta[t]
Fx[t_]:=m  f''[t]+Fplanks[t]
Fx[t]//FullSimplify
xr[t_]:= a Cos[w t+\[Phi]]
yr [t_]:= a Sin[w t +\[Phi]]
Ftot= Fx[t]/((f[t]-xr[t])/L)
Fy = Ftot*((yp-yr[t])/L)
torque =Fx[t]*yr[t]+Fy*xr[t]
(*Give values for the variables*)
{w=5,L = 1,a = 0.02,k = 100,yp = 0.2,m
    = 70,\[Phi]=21(Pi/180)}
Plot[-Fx[t]*yr[t]-Fy*xr[t], {t, 0, Pi},
     Axes -> True, AxesLabel -> {t,
    Torque}](*Plotting the torque over
    time*)
Plot[-Fx[t]*yr[t]+Fy*xr[t], {t, Pi,2 Pi
    }, Axes -> True, AxesLabel -> {t,
    Torque}]
```

REFERENCES

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[3] E. Library, Bearing stresses | engineering library (2019).