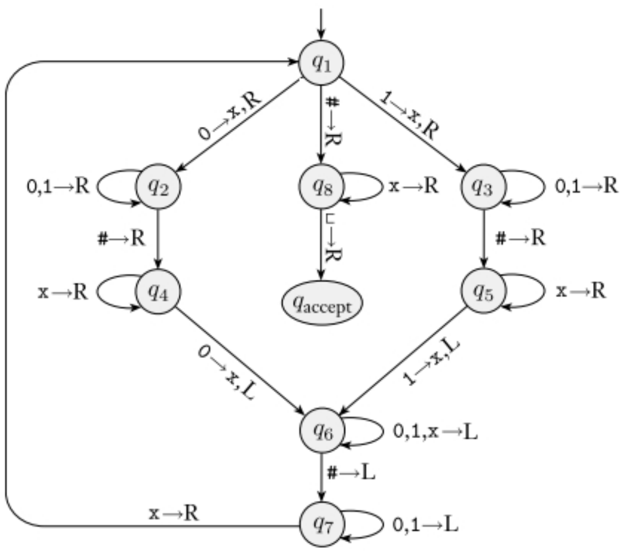
1: Turing Machines Basics

(8 pts) Give the sequence of configurations that the following machine enters when started on input string

- 1#1
- 1##1



2: Turing Machines Design

(1) (7pts) Give the state diagram for a Turing machine that decides the following language $L_1 = \{w \mid w \text{ contains twice as many 0's than 1's}\}$

(2) (3pts) How would modify your machine from the previous question to decide the following language $L_2 = \overline{L_1} = \{w \mid w \text{ does not contain twice as many 0's than 1's}\}$

(3)(10pts) Give the state diagram for a Turing machine for a Turing machine that recognizes the following languages:

- $L_3 = \{0^a1^b | a < b\}$
- $L_4 = \{0^a1^b | a \leq b\}$

(4)(10pts) A nondeterministic Turing machine M with start state q_0 and accepting state q_f has the following transition function:

$\delta(q,a)$	0	1	B
q_0	$\{(q_1, 0, R)\}$	$\{(q_1, 0, R)\}$	$\{(q_1, 0, R)\}$
q_1	$\{(q_1, 1, R), (q_2, 0, L)\}$	$\{(q_1, 1, R), (q_2, 1, L)\}$	$\{(q_1, 1, R), (q_2, B, L)\}$
q_2	$\{(q_f, 0, R)\}$	$\{(q_2, 1, L)\}$	$\{\}$
q_f	$\{\}$	$\{\}$	$\{\}$

Simulate all sequences of 5 moves, starting from initial configuration $q_0 1010$.

(5)(6pts) Show that the set of decidable languages is closed under:

- Complementation (i.e if L is decidable then so is $\overline{L}$)
- Concatenation (it's easier if you use Nondeterministic Turing machines here)

(6)(6pts) Show that the set of Turing-recognizable languages is closed under:

- Intersection
- Star (it's easier if you use Nondeterministic Turing machines here)