1: Sets

(a) Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.

- $\{1, 3, 5, 7, ...\}$
- $\{n|\ n = 2m,\ \text{for some } m \in \mathcal{N}\}$
- $\{n|\ n = 2m,\ \text{for some } m \in \mathcal{N} \text{ and } n = 3k \text{ for some } k \in \mathcal{N}\}$
- $\{n|n = n + 1, n \in \mathcal{Z}\}$

(b) Write the following sets using set notation

- The set containing the numbers 1, 10, and 100
- The set containing all integers that are greater than 5
- The set containing all natural numbers that are less than 5

(c) Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$.

- Is $A \subseteq B$?
- Is $B \subseteq A$?
- What is $A \cup B$?
- What is $A \cap B$?
- What is $A \times B$?
- $\mathcal{P}(B)$? (powerset of B)

2: Functions

Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{6, 7, 8, 9, 10\}$. The unary function $f : X \to Y$ and the binary function $g : X \times Y \to Y$ are described in the following tables:

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

- What is the value of $f(2)$?
- What are the range and domain of f?
- What is the value of $g(2, 10)$?
- What are the range and domain of g?
- What is the value of $g(4, f(4))$?

3: Proof by Contradiction

Prove the following Theorem: *There is no largest even number*

4: Graphs

Consider the undirected graph $G = (V, E)$ where V, the set of nodes, is $\{1, 2, 3, 4\}$ and E, the set of edges, is $\big\{\{1, 2\}, \{2, 3\}, \{1, 3\}, \{2, 4\}, \{1, 4\}\big\}$.

Draw the graph G. What are the degrees of each node? Indicate a path from node 3 to node 4 on your drawing of G.

5: Languages and alphabets

Consider the Language described by the following regular expression over the binary alphabet i.e. $\Sigma = \{0, 1\}$.

$$(0 \cup 1)^* 01 (0 \cup 1)^*$$

- Give 2 strings that belong to the language.
- Give 2 strings that do not belong to the language.
- How would you describe this language in english.