1: Context Free Grammars

(a) Consider the following grammar:

$$\begin{array}{lcl} S & \rightarrow & A1B \\ A & \rightarrow & 1A \mid \epsilon \\ B & \rightarrow & 0B \mid 1B \mid \epsilon \end{array}$$

- Give a leftmost derivation for string 1100
- Give a parse tree for string 1010
- The language of this grammar is a regular language, can you find an equivalent regular expression for this language?

(b) In each case below, indicate what language is generated by the context-free grammar. The alphabet is $\Sigma = \{0, 1\}$

- $S \rightarrow 0S \mid 1S \mid \epsilon$
- $S \rightarrow SS \mid 1S \mid 0$
- $S \rightarrow S0S \mid 1S$

In the following questions the alphabet is $\Sigma = \{0, 1\}$:

(c) Write a grammar for the language $\{0^i 1^j \mid i, j \geq 0\}$.

(d) Write a grammar for the language $\{0^i 1^j \mid i > j\}$.

(e) Write a grammar for the language $\{0^n 1^m 0^m 1^n \mid n, m \geq 0\}$.

(f) Write a grammar for the language $\{u1v \mid u \in \Sigma^*, v \in \Sigma^* \text{ and } |u| = |v|\}$.

(g) Let B be the collection of strings that contain at least one 1 in their second half. In other words, $B = \{uv \mid u \in \Sigma^*, v \in \Sigma^* 1 \Sigma^* \text{ and } |u| \geq |v|\}$.
Give a CFG that generates B. (HINT: Start with the grammar that you used for question f and modify it).

(h) Given context-free grammars for languages L_1 and L_2 with start variables S_1 and S_2 respectively. Give the grammar for $L_1 \cup L_2$, the grammar for $L_1 \circ L_2$, (the concatenation), and the grammar for L_1^*.