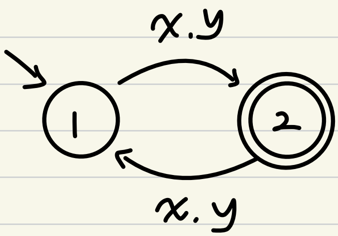
a)

$L1 = \{ w \mid w \text{ has odd length} \}$

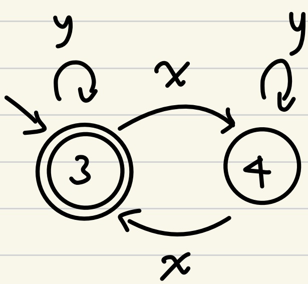
$L2 = \{ w \mid w \text{ has a even number of } x\text{'s} \}$

L1

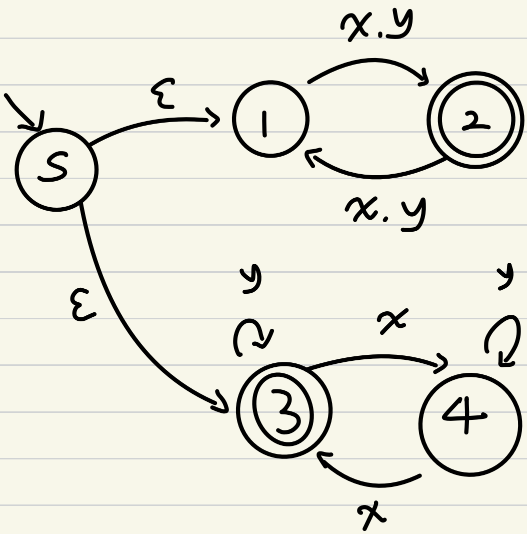


L1 is already DFA

L 2



$L1 \cup L2$

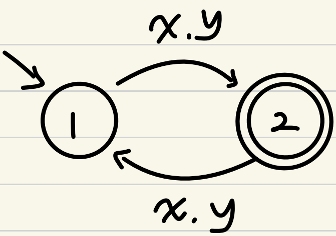


b) $L1 = \{ w \mid w \text{ has odd length} \}$

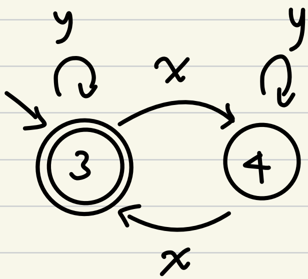
$L2 = \{ w \mid w \text{ has a even number of } x\text{'s} \}$

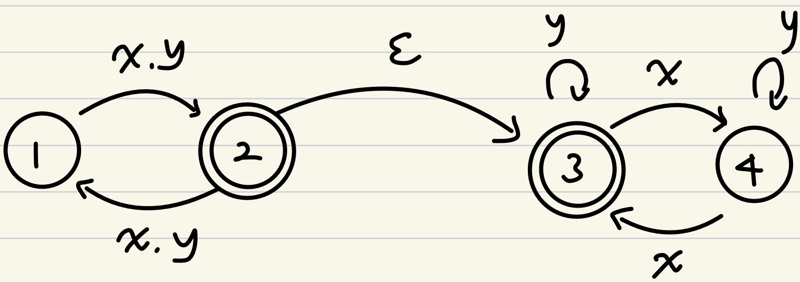
$L1 \cap L2 = L1 \cdot L2 = L6$

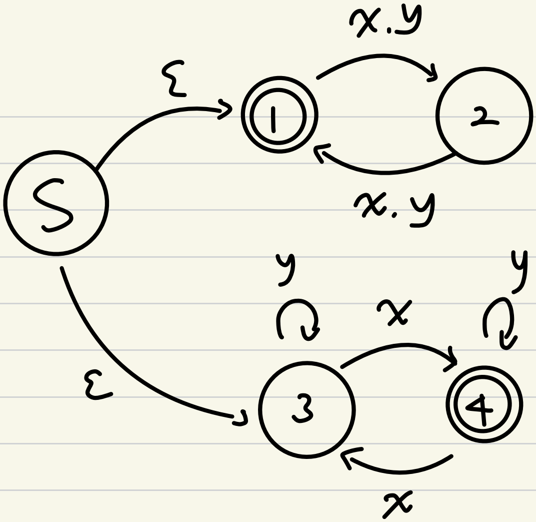
L1



L 2

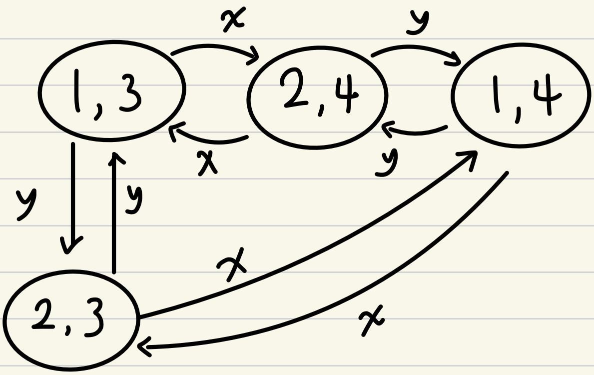






$$\overline{(\overline{L1} \cup \overline{L2})} = L1 \cap L2$$
$$= L6$$

Convert NFA to DFA



	x	y
1,3	24	23
24	13	14
23	14	13
14	23	24

C.

$L1 = \{\varepsilon, 0, 1, 00, 111, 1001, 00000\}$

$Imp(L1) = \{w \in L1 \mid w \text{ has odd }\}$

$= \{\cancel{\varepsilon}, 0, 1, \cancel{00}, 111, \cancel{1001}, 00000\}$

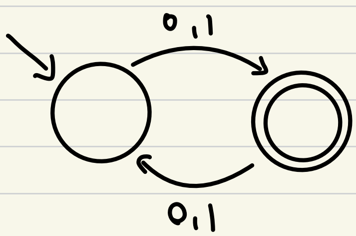
$= \{0, 1, 111, 00000\}$

$K = \{ w \mid w \text{ has odd length} \}$

Because L1 is a binary regular language

Imp(L1) is the language of those strings in L1 that have odd length

So, we can write $Imp(L1) = L1 \cap K$



This DFA can prove that K is a regular language

Hypothesis: L1 is a regular language

K is a regular language

$Imp(L1) = L1 \cap K$

Therefore, $L1 \cap K$ is a regular language

By definition, $Imp(L1) = L1 \cap K$.

So, Imp(L1) is also a regular language