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Examples for Context Free Grammars

(Regular Language)

$L_1 = \Sigma^* = (0|1)^* = \{w \mid w \text{ is a binary string}\}$

$$S \rightarrow \epsilon \mid 0S \mid 1S$$

$$L_2 = \{\}$$

$$S \rightarrow S$$

$$L_3 = \{\epsilon\}$$

$$S \rightarrow \epsilon$$

$L_4 = 0^* = \{w \mid w \text{ is arbitrary length string of } 0's\}$

$$S \rightarrow \epsilon \mid 0S$$

$$L_5 = 00^*11^*$$

A: arbitrary length of 0s
 B: arbitrary length of 1s

$$S \rightarrow OA1B$$

$$A \rightarrow OA \mid \epsilon$$

$$B \rightarrow 1B \mid \epsilon$$

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$$L_6 = 0^* | 0^* 1$$

$$S \rightarrow A1A1$$

A: arbitrary length string of 0s $A \rightarrow 0A | \epsilon$

$$L_7 = \Sigma^* 010 \Sigma^*$$

$$S \rightarrow A010A$$

= {w | w has substr'g 010}

A: any binary string

$$A \rightarrow \epsilon | 0A | 1A$$

$$L_8 = [(01)^*]^\ast$$

= {w | w has even length}

grammar(1): $S \rightarrow \epsilon | AAS$

(ambiguous)

$$A \rightarrow 0 | 1$$

grammar(2): $S \rightarrow 00S | 01S | 10S | 11S | \epsilon$

(unambiguous)

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unambiguous

$$L_{S_0} = \{01001\} (01 \cup 00)^* 1 \quad S \rightarrow A1$$

$$S \rightarrow A1$$

$$A \rightarrow 01 \mid 00 \mid AA$$

is another solution which is ambiguous

$$L_{S_0} = \{w \mid w \text{ has equal # of } a's \text{ and } b's\} \quad S \rightarrow \epsilon \mid aM \mid bN$$

S: strings with equal # of a's and b's

$$M \rightarrow b \mid aMM \mid bS$$

M: strings with one more b

N: strings with one more a

$$N \rightarrow a \mid bNN \mid aS$$

(unambiguous)

$$L_H = \{w \mid w \text{ has even # of } a's\}$$

$$S \rightarrow bAS \mid aA \mid \epsilon$$

S: strings with even # of a's

$$A \rightarrow bA \mid aS \mid a$$

A: strings with odd # of a's

(unambiguous)

$$L_{12} = \{w \mid w \text{ has at least two } 1's\}$$

$$S \rightarrow A1A1A$$

$$= (0U1)^* 1 (0U1)^* 1 (0U1)^*$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

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$L_{13} = \{w \mid w \text{ ends with } 01\}$ $S \rightarrow A01$

$$= (0U1)^* 01$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

$L_{14} = \{w \mid w \text{ has length 5 and its third symbol is } 0\}$

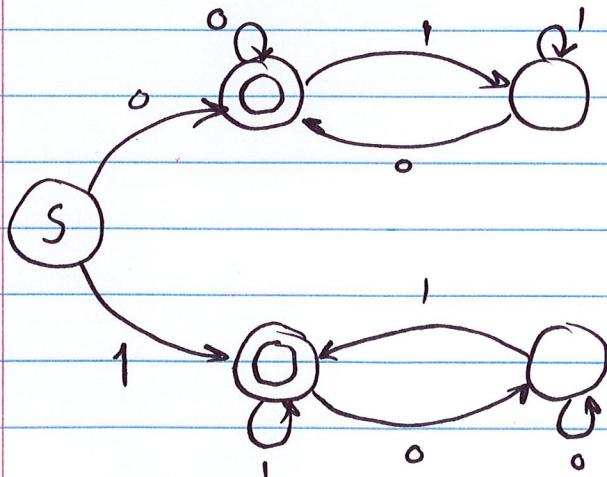
$$= (0U1).(0U1).0.(0U1).(0U1)$$

$$T \$ \rightarrow BB0BB$$

$$B \rightarrow 0 \mid 1$$

$L_{15} = \{w \mid w \text{ starts and ends with the same symbol}\}$

$$(1.(0U1)^*.1) \cup (0.(0U1)^*.0) \cup 0 \cup 1$$



$$F \$ \rightarrow 0A0 \mid 1A1 \mid 0 \mid 1$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

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$L_{16} = \{ w \mid w \text{ has length } 5 \text{ and its third symbol is } 0 \}$
OR w starts and ends with the same symbol

$$L_{16} = L_{14} \cup L_{15}$$

$$S \rightarrow T \mid F$$

$$T \rightarrow BB \circ BB$$

$$B \rightarrow 0 \mid 1$$

$$F \rightarrow OAo \mid 1A1 \mid o \mid 1$$

$$A \rightarrow OA \mid 1A \mid \epsilon$$

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$L_{17} = \{ w \mid w \text{ has } \overset{\text{both}}{\checkmark} \text{ even } \# \text{ of } 0's \text{ and even } \# \text{ of } 1's \}$

S: strings with even 0's and even 1's

~~S, X, Y, Z~~

X: strings with odd 0's and even 1's

Y: strings with even 0's and odd 1's

Z: strings with odd 0's and ~~even~~ ^{odd} 1's

$S \rightarrow 0X \mid 1Y \mid \epsilon$

$X \rightarrow 0S \mid 1Z$

$Y \rightarrow 0Z \mid 1S$

$Z \rightarrow 0Y \mid 1X$

What is the language of this grammar?

$S \rightarrow 0S1S \mid 1S0S \mid \epsilon$

$L_{18} = \{ w \mid w \text{ has length even} \}$

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Examples for Context Free Grammars

(non Regular Languages)

$$L_{16} = \{w \mid w = 0^n 1^n, n \in \mathbb{N}, n \geq 0\}$$

$$S \rightarrow 0 S 1 \mid \epsilon$$

$$L = \{w \mid w = 0^m 1^n, m, n \in \mathbb{N}, m, n \geq 0\} = 0^* 1^*$$

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow 0A \mid \epsilon \\ B &\rightarrow 1B \mid \epsilon \end{aligned}$$

$$L_{17} = \{w \mid w = 0^n 1^n, n \in \mathbb{N}, n \geq 1\}$$

$$S \rightarrow 0 S 1 \mid 01$$

$$L_{18} = \{w \mid w \text{ is a palindrome}\}$$

palindrome is a string that reads the same whether you read it from left-to-right or right-to-left

$$S \rightarrow 0 S 0 \mid 1 S 1 \mid \epsilon$$

$$L_{19} = \{w \mid w \text{ is a balanced parenthesis string}\}$$

$$S \rightarrow (S) \mid SS \mid \epsilon$$

note: for this language $\Sigma = \{(), ()\}$

alphabet ↲

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alphabet

$$\Sigma = \{a, b, c\}$$

$$L_{20} = \{w \mid w = a^i b^{i+k} c^k, i, k \in \mathbb{N} \text{ and } i, k \geq 0\}$$

$$L_{21} = \{w \mid w = a^i b^i, i \in \mathbb{N}, i \geq 0\}$$

$$T \xrightarrow{*} a T b \mid \epsilon$$

$$L_{22} = \{w \mid w = b^k c^k, k \in \mathbb{N} \text{ and } k \geq 0\}$$

$$F \xrightarrow{*} b F c \mid \epsilon$$

Concatenation of two other languages

$$L_{20} = L_{21} \cdot L_{22}$$

$$S \xrightarrow{*} T F$$

$$T \xrightarrow{*} a T b \mid \epsilon$$

$$F \xrightarrow{*} b F c \mid \epsilon$$

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$L_{23} = \{ \omega \mid \text{length of } \omega \text{ is odd and middle symbol is 1} \}$

$S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 1$

another solution (conceptually the same as above):

$S \rightarrow ASA \mid 1$

$A \rightarrow 0 \mid 1$