Context Free Grammars

1) Consider the following Grammar over alphabet: $\Sigma = \{a, +, \times, (,)\}$

$$E \to E + T | T$$
$$T \to T \times F | F$$
$$F \to (E) | a$$

a) Give derivations for the following strings: a, a+a, (a)

b) Give parse trees for the following string:

$a + a \times a$

2) Consider the grammar $S \to aS \mid aSbS \mid \epsilon$

Show that the grammar is ambiguous by finding 2 different parse trees for the string aab

Context Free Grammars

3) Give a grammar for the following languages:

a) $L_1 = \{w \mid w$ contains at least three $1's\}$

b) $L_2 = \{w \mid$ length of w is odd$\}$

c) $L_3 = \{w \mid$ length of w is odd and the middle symbol is $1\}$