COMP 157 : THE DESIGN AND ANALYSIS OF ALGORITHMS

MIDTERM : CHAPTER 1 – CHAPTER 5

REQIRED TIME : 120 MINUTES

NAME :

GRADE : (/ 70)

1. (5 points) Define if each of the following is false or true.
   a. $\frac{n(n+1)}{2} \in O(n^3)$
   b. $\frac{n(n+1)}{2} \in \Theta(n^3)$
   c. $\frac{n(n+1)}{2} \in \Omega(n)$

2. (10 points) Find the order of growth of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. (Use the simplest $g(n)$ possible in your answers.) Prove your answers using $\lim_{n\to\infty}$
   a. $5n^7 + 14n^4 \log n + 3$
   b. $\sum_{i=0}^{n}(i+n)$

3. (15 points) Consider the following algorithm:

ALGORITHM *Count (A[0..n - 1])*
$//Input: Array\ A[0..n-1]\ of\ orderable\ values$
$//Output: Array\ C[0..n-1]$
$\boldsymbol{for}\ i \leftarrow 0\ \boldsymbol{to}\ n-1\ \boldsymbol{do}$
$\boldsymbol{for}\ j \leftarrow 1\ \boldsymbol{to}\ n-1\ \boldsymbol{do}$
$\boldsymbol{if}\ A[i] < A[j]$
$C[j] \leftarrow C[j] + 1$
$\boldsymbol{return}\ \boldsymbol{C[0..n-1]}$

a. (2 points) What does this algorithm compute?
b. (2 points) Considering space efficiency, is this "in space" or "out of space"? Why?
c. (2 points) What is the input size?
d. (2 points) What is the basic operation?
e. (5 points) How many time the basic operation executed (aka what is $C(n)$)?
f. (2 points) What is the efficiency class of this algorithm? Indicate the class $\Theta(g(n))$ the function belongs to. (Use the simplest $g(n)$ possible in your answers.)

4. (15 points) Consider the following recursive algorithm:

$\boldsymbol{ALGORITHM}\ calculate(n)$
$\therefore\therefore\ Input: A\ positive\ integer\ n$
$\boldsymbol{if}\ n = 1\ \boldsymbol{return}\ 1$
$\boldsymbol{else\ return}\ Q(n-1) + 2 * n$

a. (2 points) What does this algorithm compute?
b. (6 points) Set up a recurrence relation for the algorithm's basic operation count and solve it (aka what are $T(1)$ and $T(n)$).
c. (2 points) What is the efficiency class of this algorithm? Indicate the class $\Theta(g(n))$ the function belongs to. (Use the simplest $g(n)$ possible in your answers.)
d. (5 points) Suggest an improvement, or a better algorithm and indicate its efficiency class.

5. (15 points) Consider the following recursive algorithm.

ALGORITHM $Riddle(A[0..n-1])$

$\therefore\therefore$ ***Input***: $An\ array\ A[0..n-1]\ of\ real\ numbers$

if $n = 1$ ***return*** $A[0]$

else

$temp \leftarrow Riddle(A[0..n-2])$

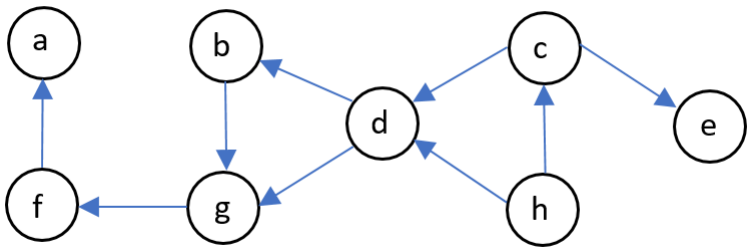
if $temp \leq A[n-1]$ ***return*** $temp$

else return $A[n-1]$

a. What does this algorithm compute?

b. Set up a recurrence relation for the algorithm's basic operation and solve it.

6. (10 points) Apply the DFS algorithm to solve the topological sorting problem for the following digraph and provide the order of (push to stack, pop from stack, topological sort)



7. (10 points) Apply Russian Peasant Multiplication on the bellow numbers:

n	m	addition
86	35	
answer		

8. (5 points) Suppose you are playing as the first player in a 1 pile Nim game.
   There are 48 coins available and, in each move, you are eligible to take at least 1 and at most 4 coins.

   a. How many coins would you take in your first move as the first player?
   b. If your opponent is an expert and will play as the first player, do you have a chance in winning if you start the game?

9. (10 points) The state wants to organize a series of soccer games between different cities, but is not sure if it would be supported. They want to start with a trial game with minimum cost. They need to select the first two cities that will go against each other. The game will be played in one of those cities. Help them select the most beneficial two cities. Propose an algorithm and then analyze it's growth class.