Iterative Improvement

Iterative Improvement

- Only applicable to optimization problems
- Constructs a solution through a sequence of steps:
- Start with some feasible solution
- Which satisfies the constraints of the problem
- Improve it by repeated application of some simple steps

The Maximum-Flow Problem

- Maximizing the flow of a material
- Through a transportation network

- Pipeline system
- Communication system
- Electrical distribution system

The Maximum-Flow Problem

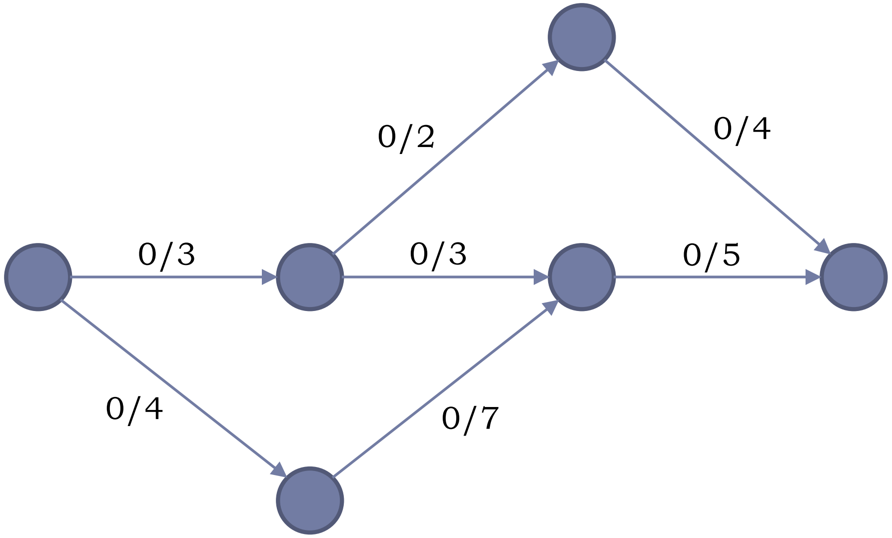
- Flow network is:
- A connected, weighted graph with n nodes and e edges
- source→ exactly one node with no entering edge
- sink→ exactly one node with no leaving edge
- Capacity u_{ij} → capacity of edge from node i to node j
- All the flow starts from source
- All the flow ends up in sink
- Other nodes can be used to direct the flow
- Flow conservation→ flow entering is equal to flow leaving

Ford-Flokerson Method

- Start with 0 flow on all edges
- On each iteration find an augmented path from source to sink
- Each edge on augmented path should have some capacity
- Send as much flow as possible through the path
- Adjust used and remaining capacity on each edge

- In finding augmented path, think of graph as undirected
- In such path we could travel on edges in two directions:
- going in direction → forward edge
- Going in wrong direction→ backward edge
- When adjusting the remaining capacity:
- Forward edge→ use the free capacity
- Backward edge → free the used capacity

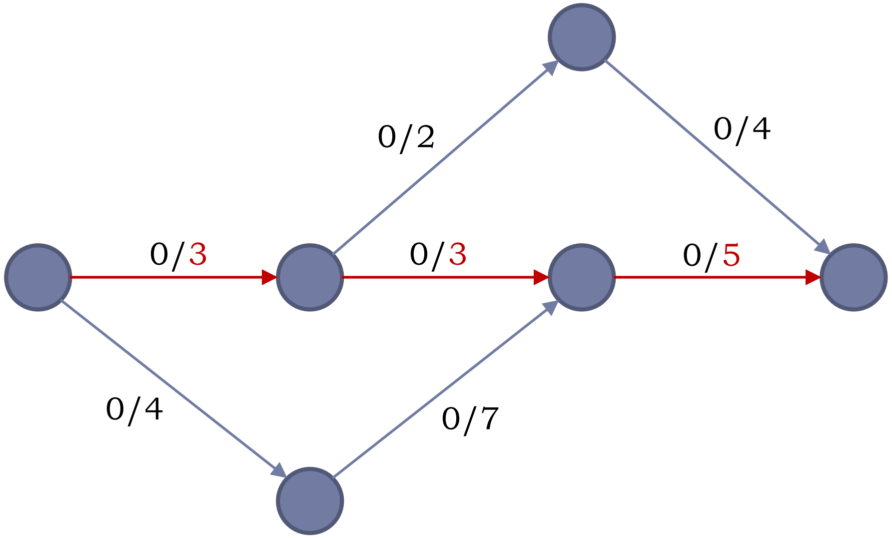
Ford-Flokerson Method



Ford-Flokerson Method

How much can we pass at most?

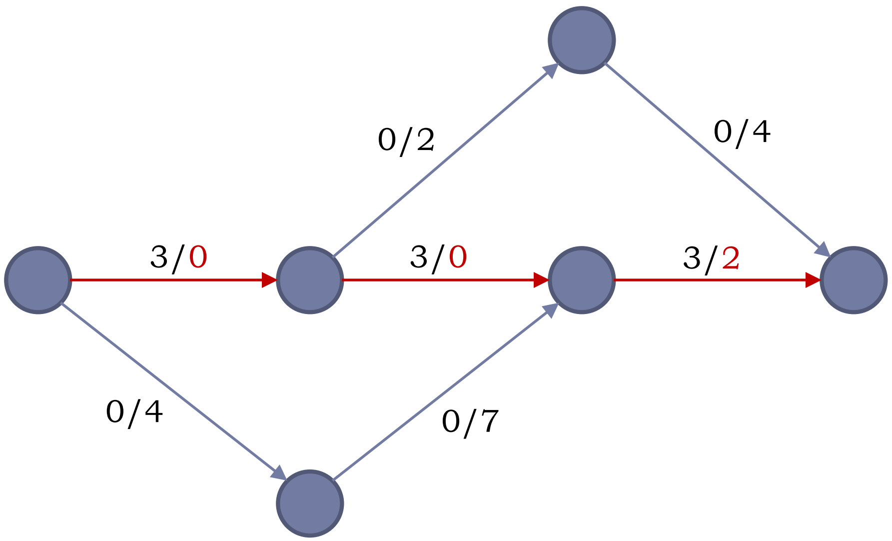
$$min(3,3,5) = 3$$



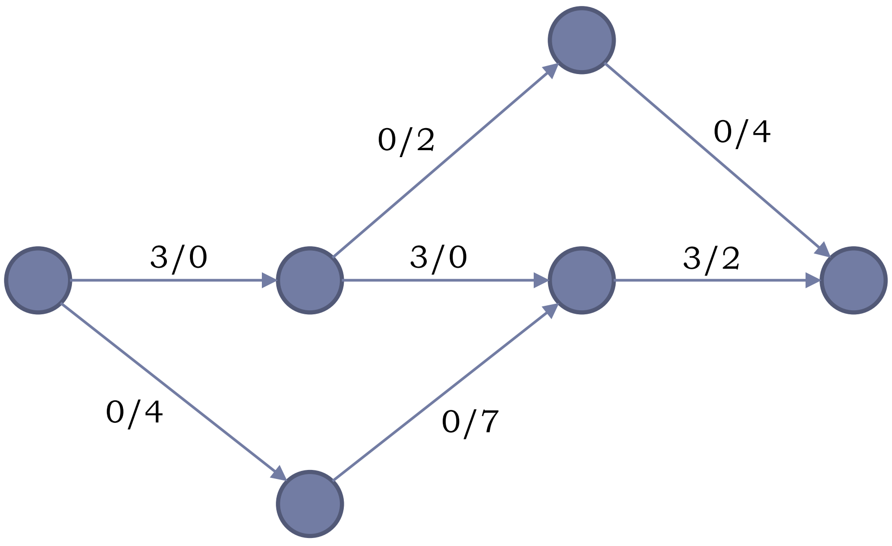
Ford-Flokerson Method

How much can we pass at most?

$$min(3,3,5) = 3$$



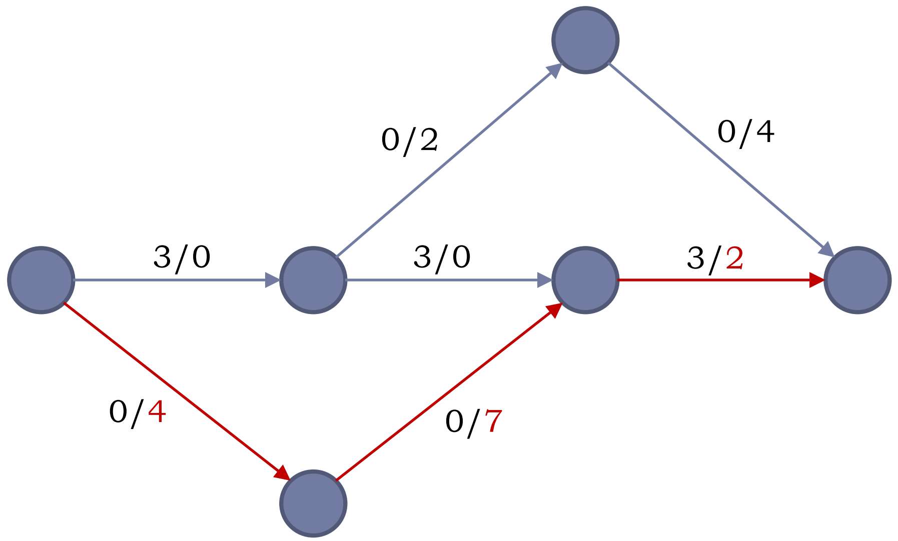
Ford-Flokerson Method



Ford-Flokerson Method

How much can we pass at most?

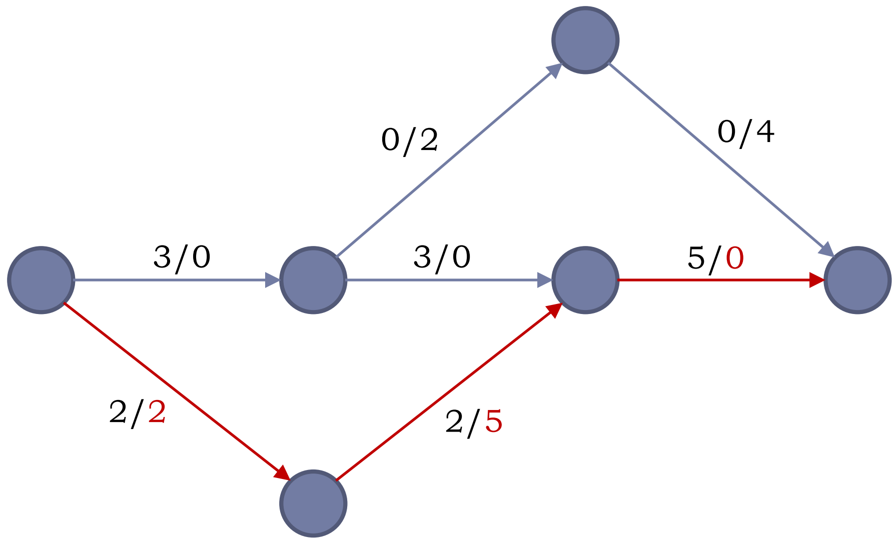
$$min(4,7,2) = 2$$



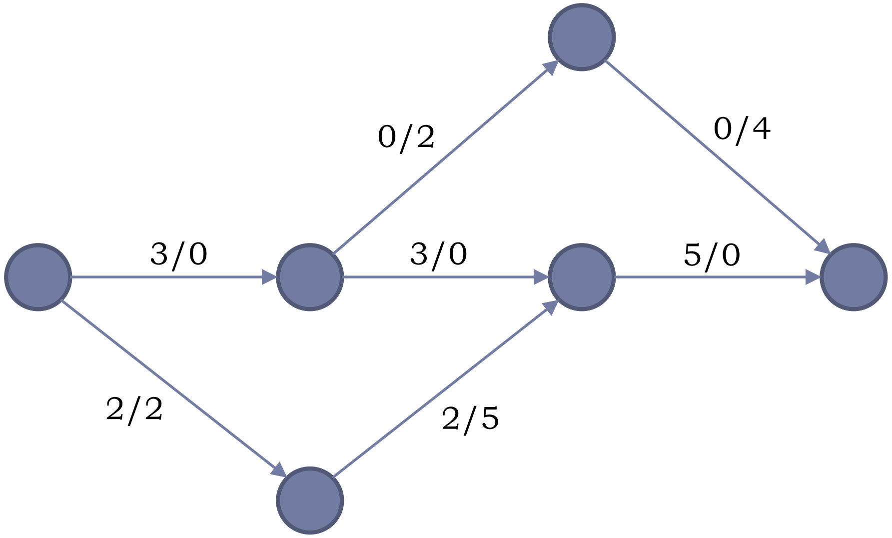
Ford-Flokerson Method

How much can we pass at most?

$$min(4,7,2) = 2$$



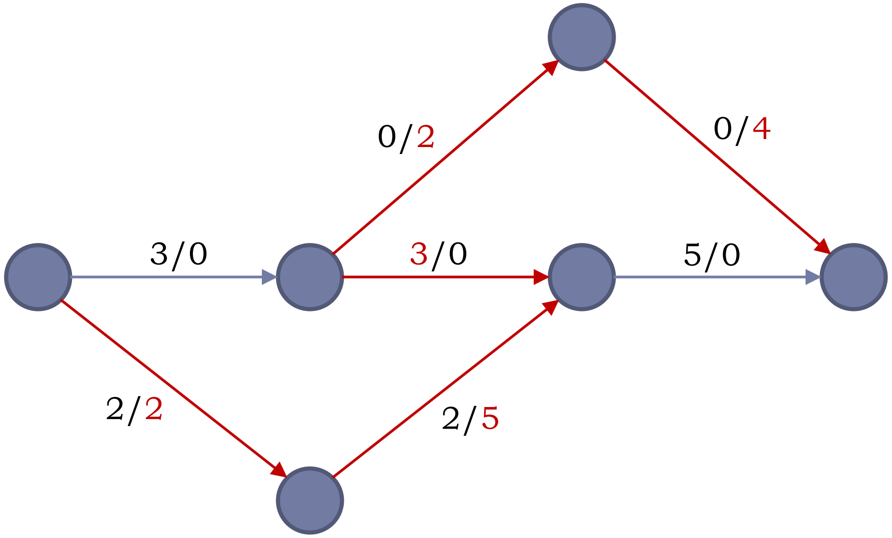
Ford-Flokerson Method



Ford-Flokerson Method

How much can we pass at most?

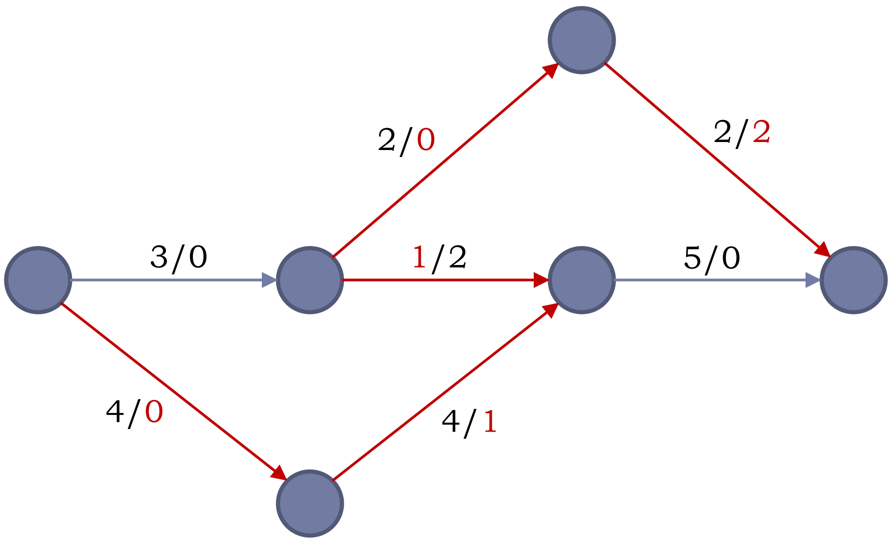
$$min(2,5,3,2,4) = 2$$



Ford-Flokerson Method

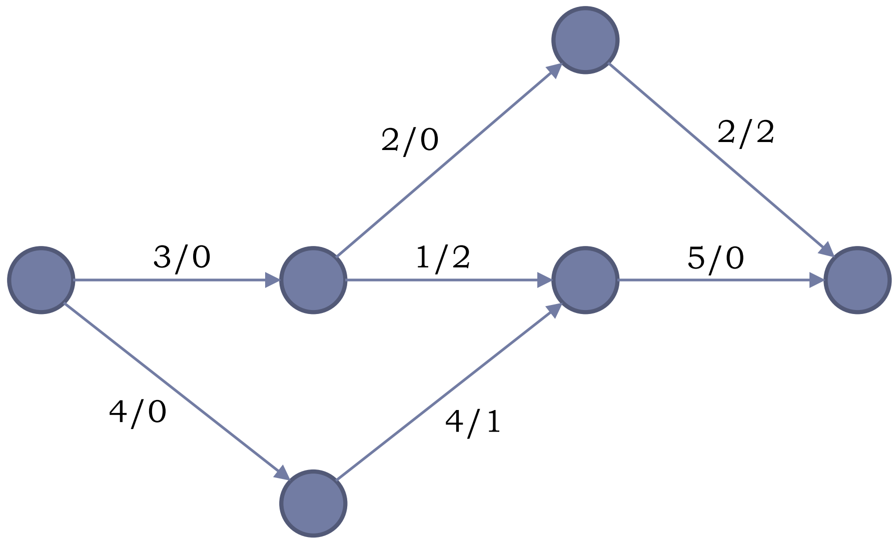
How much can we pass at most?

$min(2,5,3,2,4) = 2$



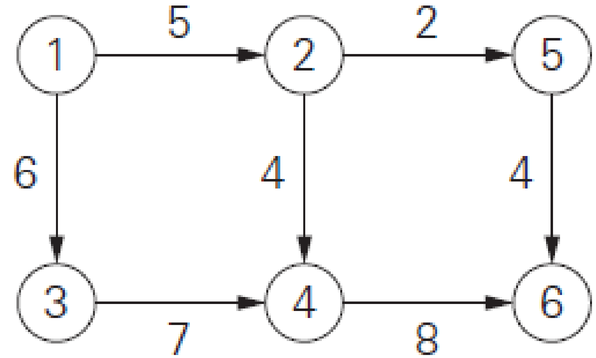
Ford-Flokerson Method

Network flow = 7



Ford-Flokerson Method

a.



b.