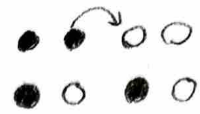
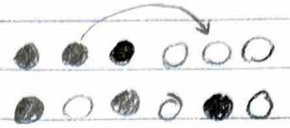
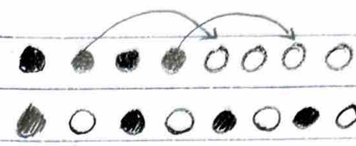
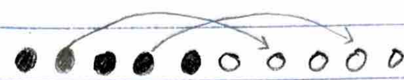
Homework 2

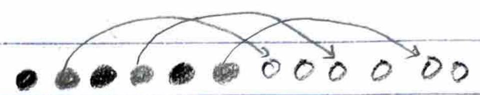
① a. $n \geq 2$

if $n = 2$  move one time

if $n = 3$  move one time

if $n = 4$  move two times

if $n = 5$  move two times

if $n = 6$  move three times

According to the pattan of move times and n, only the position of even number have to be moved. We can find the number of even position by $\frac{n}{2}$

b. There are C^{2n}_{n} scenarioes in random order. we can find the worst case is the filled cup in even position. So it have to move n times.

The all possible move times is $0 - n$.

② a. visited nodes: a, c, f, d, e, b, g, h, j, i

Stack: ~~a~~, ~~c~~, ~~f~~ ~~d~~ ~~e~~. ~~b~~ g. ~~h~~ ~~j~~, ~~i~~

b. visited nodes: a, b, f, g, c, h, d, e

Stack: ~~a~~ ~~b~~, ~~f~~, g ~~c~~ ~~h~~, ~~d~~ ~~e~~

3. Combination (k, S)

if k = 0

  return {∅}

if k > |S|

  return ∅

e is any member of S

Sout := Combination (k, S − {e})

Sin := {C ∪ {e}} C ∈ Combination (k−1, S − {e})}

return Sout ∪ Sin

4. a. the largest number of key comparison for binary search

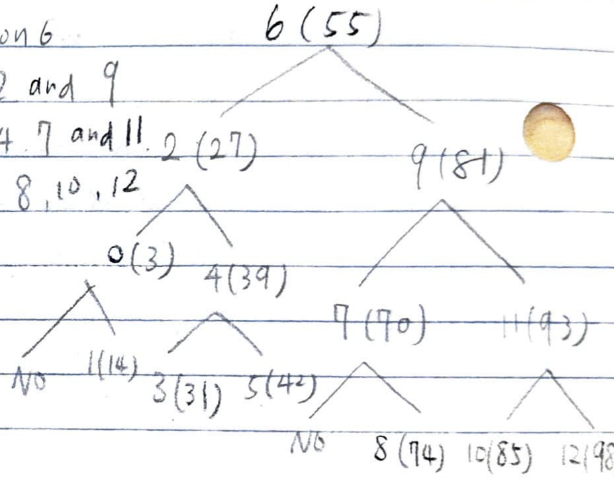
is $\log_2 13 \approx 4$

b. the first comparison is position 6

the second comparison is position 2 and 9

the third comparison is position 0, 4, 7 and 11

the fourth comparison is position 1, 3, 5, 8, 10, 12



C. The position 0 is 3 times of number comparison

The position 1 is 4 times of number comparison

The position 2 is 2 times

The position 3 is 4 times

The position 4 is 3 times

The position 5 is 4 times

The position 6 is 1 time

The position 7 is 3 times

The position 8 is 4 times

The position 9 is 2 times

The position 10 is 4 times

The position 11 is 3 times

The position 12 is 4 time

the average of number comparisons

$$\frac{3+4+2+4+3+4+1+3+4+2+4+3+4}{13} \approx 3.15$$

d. Three comparisons are required to do an unsuccessful search for a key that is less than the value of the key at position 0 and to search for a key that is in between the values of the keys at the position 6 and 7. For the remaining 12 of 14 intervals, there will be comparisons incurred for an unsuccessful key search

Hence, the average number of key comparisons for an unsuccessful is $3 \times (\frac{2}{14}) + 4 \times (\frac{12}{14}) = \frac{54}{14} = 3.86$

5. Sum up the numbers from 0 to n $\sum_{i=0}^{n} i$

and then Sum up the numbers from 0 to n-1 $\sum_{i=0}^{n-1} i$

$$\sum_{i=0}^{n} i - \sum_{i=0}^{n-1} i = \text{missing number}$$

The time complexity is $O(1)$, since it doesn't depend on the size of n. It's a constant operation according to how large n is.

6.

a. Yes, this algorithm can be classified as a variable-size decrease Algorithm. the size of the problem is reduced by a variable amount

b. The worst case in a binary search tree is skewed to right wing or left wing. So the time complexity is $O(n)$

7. In this game, it's better to go first or second player depends on the initial size of chocolate bar (mxn)

① Even - Even: the second player has a winning strategy. Because the first player will spilt the chocolate into two small even-sized chocolate and the second player just mirror their moves. The first player will lose this game.

② odd - odd: the first player has a winning strategy. the first player can spilt the chocolate bar into different size, ensuring the second player is left with spoiled chocolate. The first player will win this game

③ Even - odd: The first player can also win. The first player can spilt chocolate bar into two pieces of unequal sizes. ensuring that second player is left with odd-sied piece containing the spoiled square. the first player will win.