COMP 157 : THE DESIGN AND ANALYSIS OF ALGORITHMS

MIDTERM : CHAPTER 6 – CHAPTER 10

REQIRED TIME : 180 MINUTES

NAME :

GRADE : (/ 100)

1. (10 points) Solve the following system by Gaussian Elimination:
   Provide the middle matrixes and all your work

   $x_1 + x_2 + 3x_3 = 1$
   $2x_1 + 3x_2 + x_3 = 8$
   $3x_1 + 5x_2 + 2x_3 = 12$

   $x_1 =$
   $x_2 =$
   $x_3 =$

2. (15 points) Construct an AVL tree by inserting their elements **successively,** starting with the empty tree. Redraw the tree after each insertion or rotation. In case of rotation, indicate what rotation you have used.

   7, 4, 2, 5, 9, 6

3. (5 points) Apply Harpools' string matching on the following text for pattern "**happy**" Define the number of jumps required for all the characters.

c	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	-
#																											

4. (10 points) Construct the Closed Hash Table with double hashing collision management for the following records and hash functions:

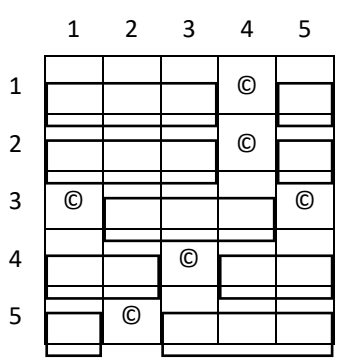
   $23, 25, 46, 79, 38, 63$

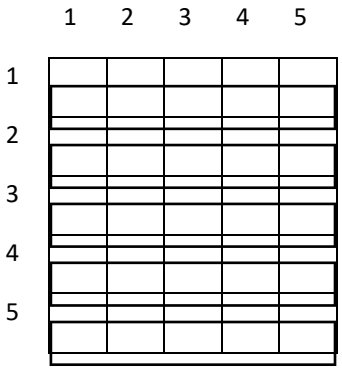
   $h(K) = K \ mod \ 10$

   $s(k) = k \ mod \ 6$

0	1	2	3	4	5	6	7	8	9

5. (5 points) Coin-Collecting Problem:
   Starting from upper-left and reaching bottom-right.
   Available moves are one cell to right, or one cell to down.
   On the way, collect the maximum number of coins.
   a. Write down the recursive solution.
   b. Apply it to the given problem.
   c. Mark the path on the board





6. (15 points) Create the optimal binary search tree for the following keys:
   a. Fill in the average comparison table and root table
   b. Draw the resulting tree

number	**1**	**2**	**3**	**4**	**5**
character	A	B	C	D	E
Probability of search	0.1	0.25	0.15	0.3	0.20

6.

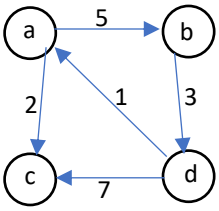
Average comparisons

	0	1	2	3	4	5
1						
2						
3						
4						
5						
6						

Root

	0	1	2	3	4	5
1						
2						
3						
4						
5						
6						

7. (10 points) Considering Floyd's algorithm:
   a. Define its use-case.
   b. Apply the algorithm to the following graph.



D^0

	1	2	3	4
1				
2				
3				
4				

D^1

	1	2	3	4
1				
2				
3				
4				

D^2

	1	2	3	4
1				
2				
3				
4				

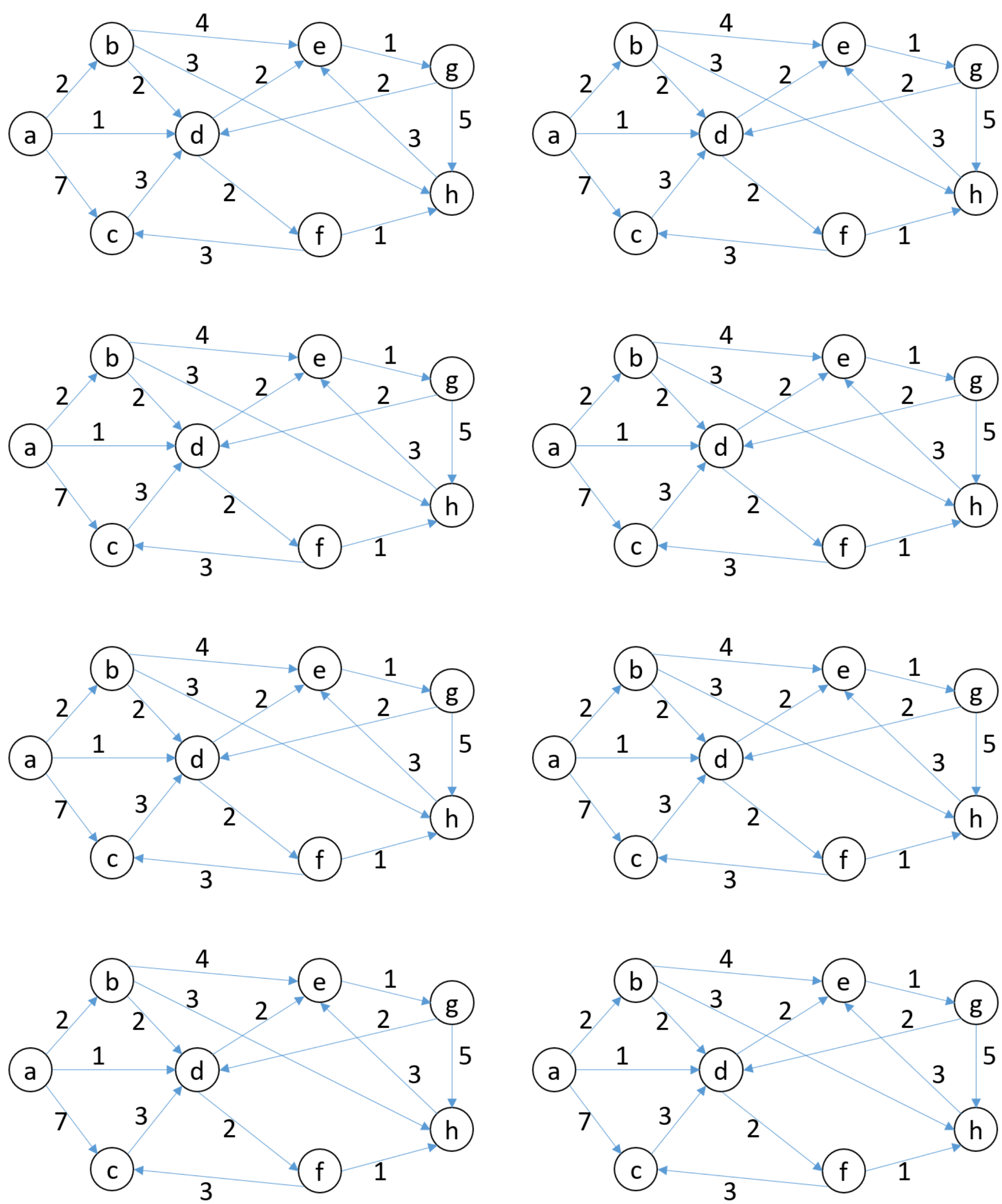
D^3

	1	2	3	4
1				
2				
3				
4				

D^4

	1	2	3	4
1				
2				
3				
4				

8. (10 points) Considering Prim’s algorithm:
   a. Define its use-case
   b. Apply the algorithm to the following graph:
      Define the process of edge selecting step by step



9. (10 points) Apply Huffman codding on the following characters:

Character	**A**	**B**	**C**	**D**	**E**	**F**
Probability	0.25	0.20	0.15	0.10	0.25	0.05
Code						

a. Fixed-length codding:

b. Huffman average code length:

c. Compression rate:

10. (10 points) Find the maximum flow in the following network, using augmented paths:
You might not need all the graphs. Use as many as needed.