Final Exam

1.

$$\begin{array}{c} R1 \\ R2 \\ R3 \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 1 & 8 \\ 3 & 5 & 2 & 12 \end{bmatrix} \quad \begin{array}{r} \\ R_2 - 2 \cdot R_1 \\ R3 - 3 \cdot R_1 \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 6 \\ 0 & 2 & -1 & 9 \end{bmatrix}$$

$$R3 - 2R_2 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 6 \\ 0 & 0 & 1 & -3 \end{bmatrix} \quad R_2 + R_3 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

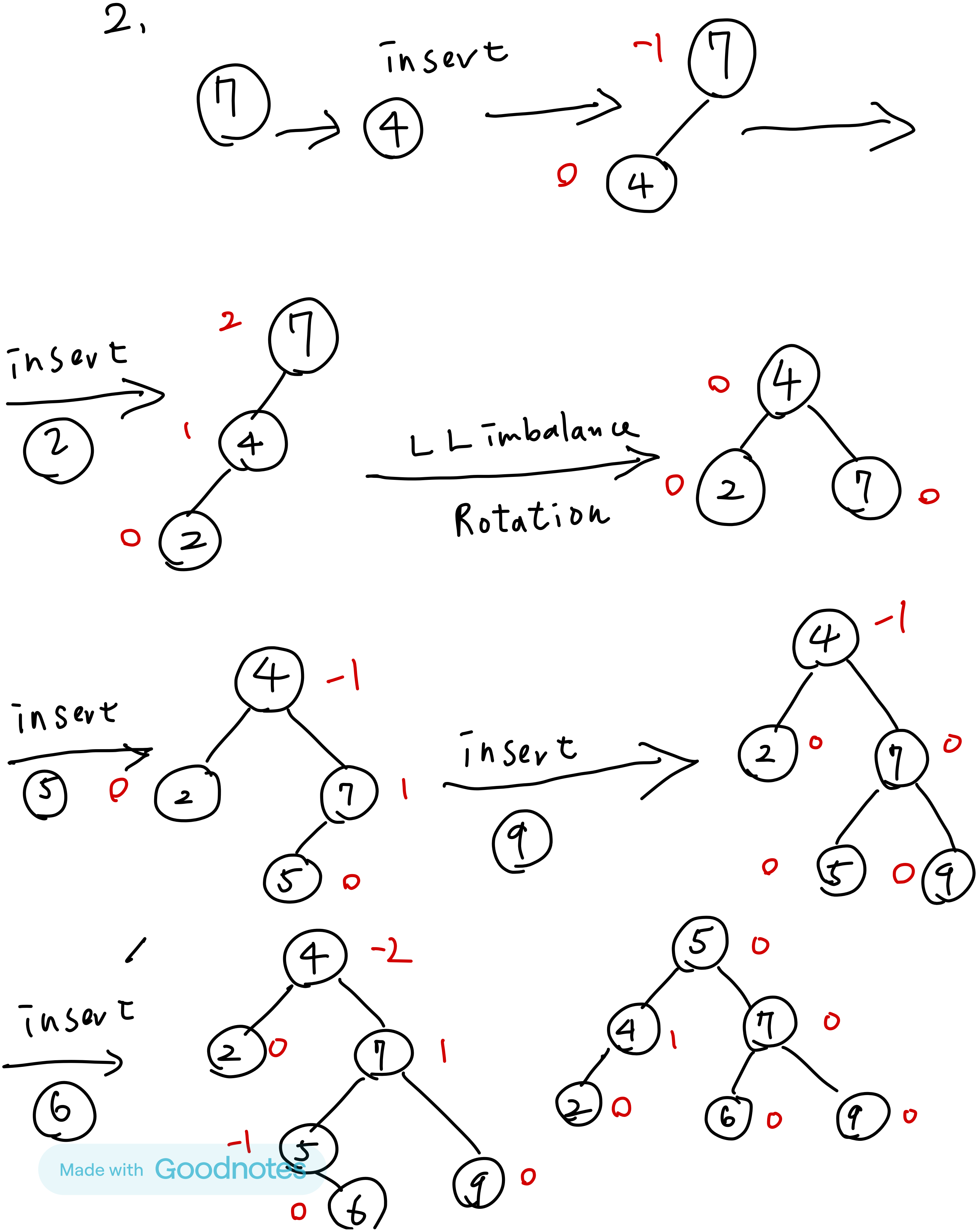
$$R_1 - R_2 - R_3 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

$x_1 = 1$

$x_2 = 3$

$x_3 = -3$

2.



3. happy

a	b	c	d	e	f	g	h	i	j
3	5	5	5	5	5	5	4	5	5
k	l	m	n	o	p	q	r	s	t
5	5	5	5	5	1	5	5	5	5
u	v	w	x	y	z	-			
5	5	5	5	5	5	5			

4. $h(k)$ is the primary function for this hash table. If there is a collision, use the second function.

0	1	2	3	4	5	6	7	8	9
		63	23		25	46		38	79

$h(23) = 23 \bmod 10 = 3$

$h(25) = 25 \bmod 10 = 5$

$h(46) = 46 \bmod 10 = 6$

$h(79) = 79 \bmod 10 = 9$

$h(38) = 38 \bmod 10 = 8$

$h(63) = 63 \bmod 10 = 3$. but 3 is taken

There is a collision.

use $h(k,i) = (h(k) + i \times s(k)) \bmod 10$

i start frm 1

$h(63,1) = (h(63) + 1 \times 3) \bmod 10 = 6$

6 is taken by 46

$i = 3$ $h(63,3) = (h(63) + 3 \times S(63)) \bmod 10$

$= 12 \bmod 10 = 2$ not taken

5. We have to start from (1, 1) and end at (5, 5). draw the table to keep track of the Maximum coins. $F(i, j)$ maximum number of coins collectable up to cell (i, j)

$$f(n) = \max\{F(i-1, j), F(1, j-1)\} + c_{i,j}$$

$$F(0, j) = 0, \quad F(i, 0) = 0$$

0	0	0	1	1
0	0	0	2	2
1	1	1	2	3
1	1	2	2	3
1	2	2	2	3

$$F(1, 1) = \max(F(0, 1), F(1, 0)) + c_{i,j} = 0$$

6. We need to construct a dynamic programming table to calculate the average comparison costs

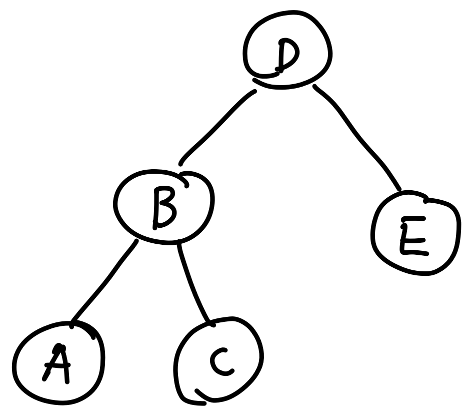
$$C(i,j) = \min\{ C(i,k-1) + C(k+1,j) + \sum_{s=i}^{j} ,$$

$$C(i,i) = P_i$$

$$C(i,i-1) = 0 \qquad C(1,n) = \text{goal}$$

	0	1	2	3	4	5
1	0	0.15	0.45	0.7	1.3	1.9
2		0	0.25	0.5	1.05	1.55
3			0	0.15	0.6	1.05
4				0	0.3	0.75
5					0	0.2
6						0

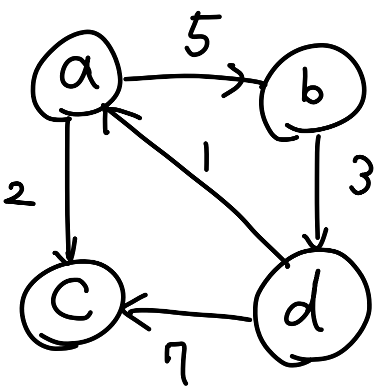
Main table



	0	1	2	3	4	5
1		1	2	2	4	4
2			2	2	4	4
3				3	4	4
4					4	4
5						5
6						

root table

7.



$$D(0) = \begin{array}{cc} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 5 & 2 & \infty \\ \infty & 0 & \infty & 3 \\ \infty & \infty & 0 & \infty \\ 1 & \infty & 7 & 0 \end{bmatrix} \end{array}$$

$$D(1) = \begin{array}{cc} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 5 & 2 & \infty \\ \infty & 0 & \infty & 3 \\ \infty & \infty & 0 & \infty \\ 1 & 6 & 3 & 0 \end{bmatrix} \end{array}$$

$$D(2) = \begin{array}{cc} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 5 & 2 & 8 \\ \infty & 0 & \infty & 3 \\ \infty & \infty & 0 & \infty \\ 1 & 6 & 3 & 0 \end{bmatrix} \end{array}$$

$$D(3) = \begin{array}{cc} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 5 & 2 & 8 \\ \infty & 0 & \infty & 3 \\ \infty & \infty & 0 & \infty \\ 1 & 6 & 3 & 0 \end{bmatrix} \end{array}$$

$$D(4) = \begin{array}{cc} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 5 & 2 & 8 \\ 4 & 0 & 6 & 3 \\ \infty & \infty & 0 & \infty \\ 1 & 6 & 3 & 0 \end{bmatrix} \end{array}$$

8. start with tree consisting of one node through a series of expanding subtrees

$a(-,-)$ $b(a,3)$, $c(a,7)$, $d(a,1)$ small

$e(-\infty)$, $f(-,\infty)$, $g(-,\infty)$

$h(-,\infty)$

find the shortest from node a

$d(a,1)$ $b(a,3)$, $c(a,7)$, $e(d,2)$, $f(d,2)$

small

$g(-\infty)$, $h(-\infty)$

I choose e, because e < f by alphabet

$e(d,2)$ $b(a,3)$, $c(a,7)$, $f(d,2)$, $g(e,1)$

small

$h(-,\infty)$

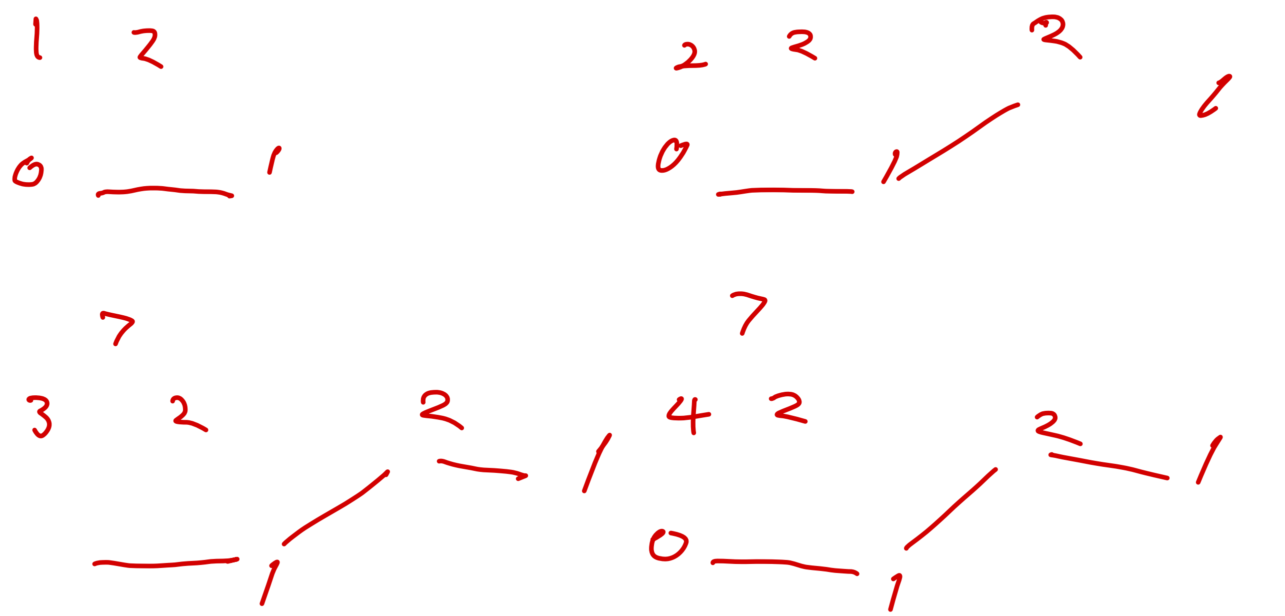
$g(e,1)$ $b(a,3)$, $c(a,7)$, $f(d,2)$, $h(g,5)$

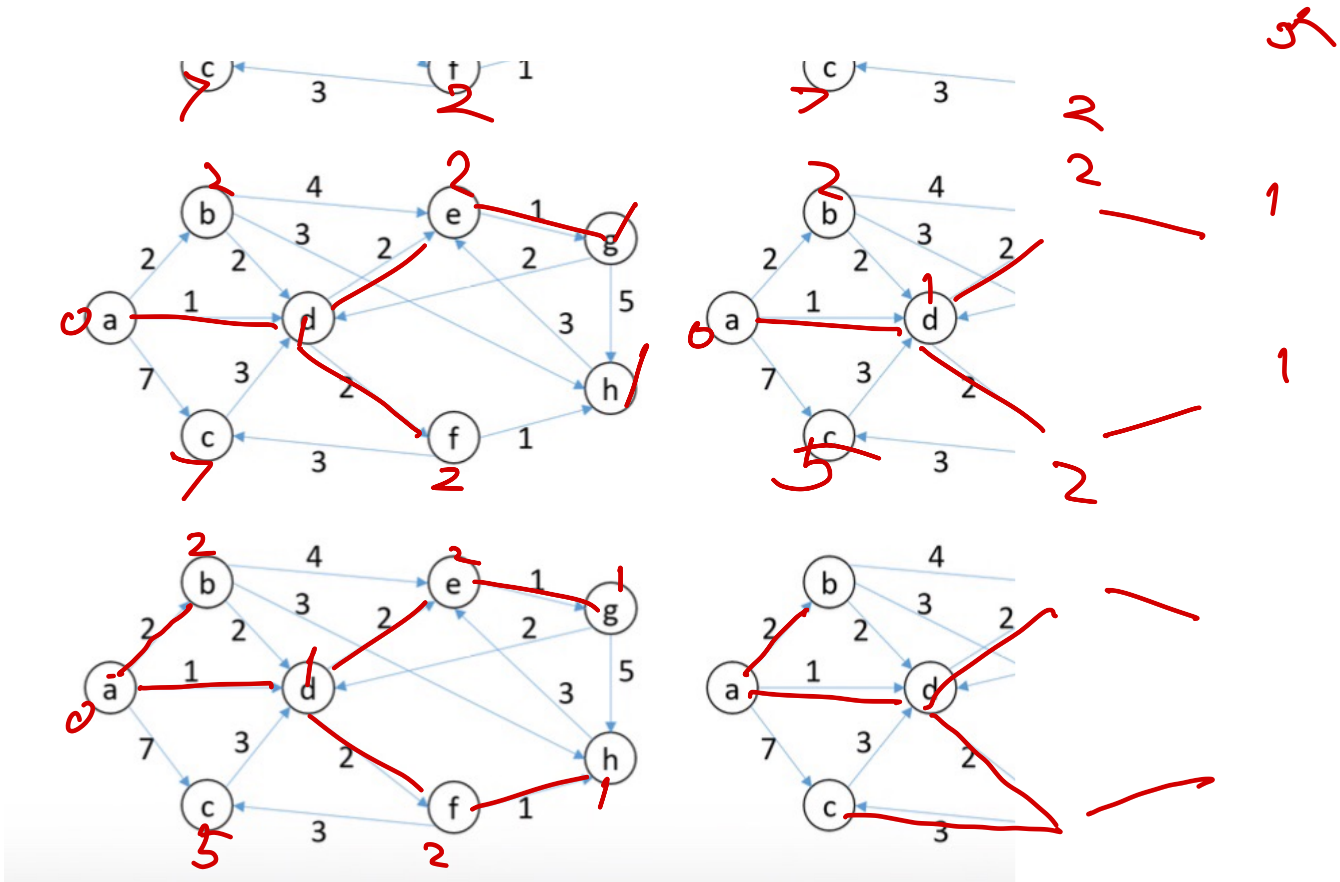
small

$f(d,2)$ $b(a,3)$, $c(f,3)$, $h(f,1)$ small

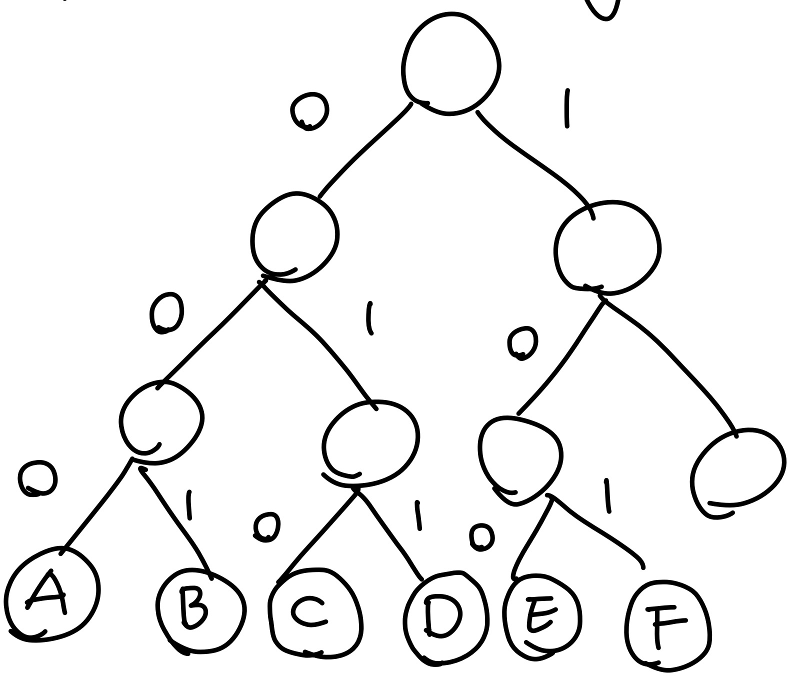
$h(f,1)$ $b(a,3)$, $c(f,3)$ Choose b

$b(a,3) \rightarrow c(f,3)$




a
b
c
d
e
f
g
h

9. a. Fixed-length Coding for this question



$\lceil \log_2 6 \rceil = 4$ height,

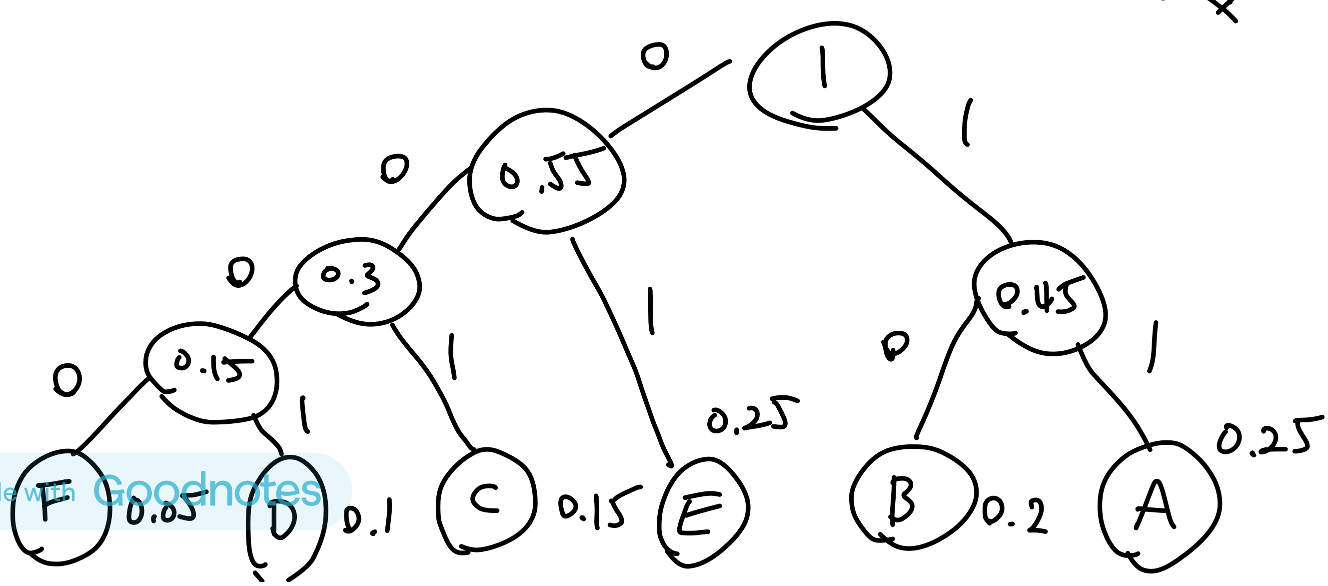
character	code
A	000
B	001
C	010
D	011
E	100
F	101

b.

sort the character by ascending Probability

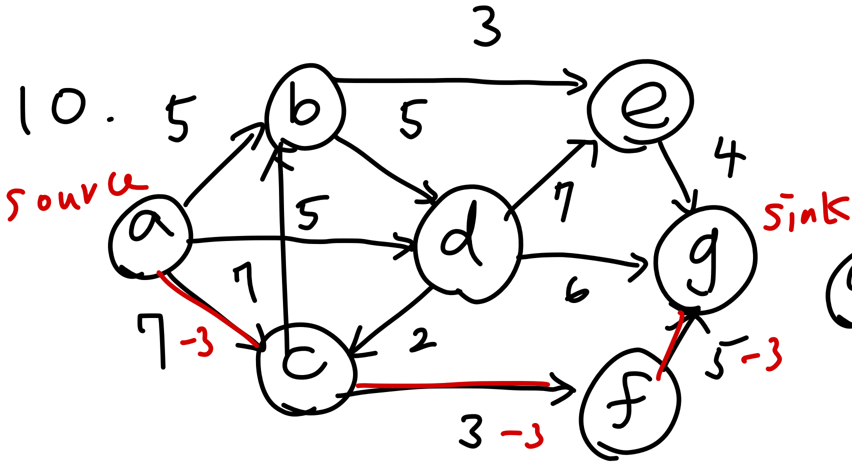
	F	D	C	B	A	E
	0.05	0.1	0.15	0.2	0.25	0.25
code	0000	0001	001	10	11	01

$$\frac{4+4+3+2+2+2}{6} = 2.8$$

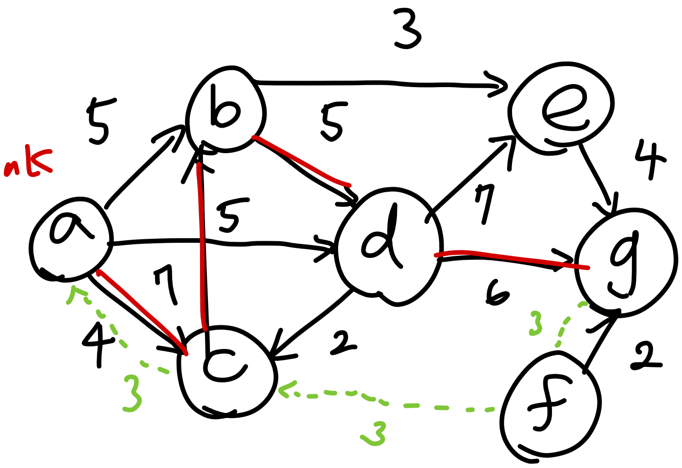


c. Compression rate $= \frac{3-2.8}{3} = 0.06667$

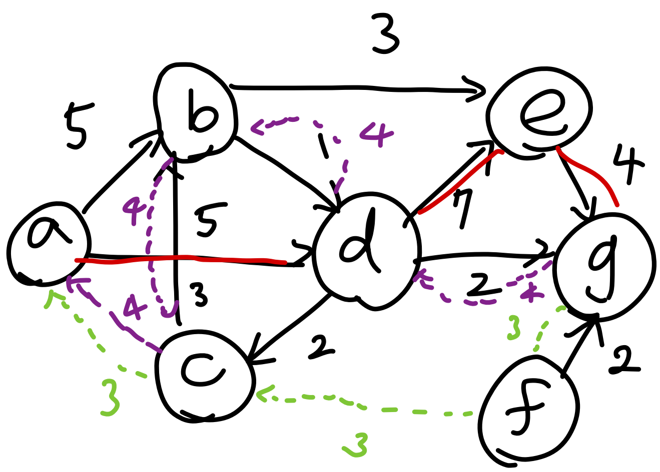
10.



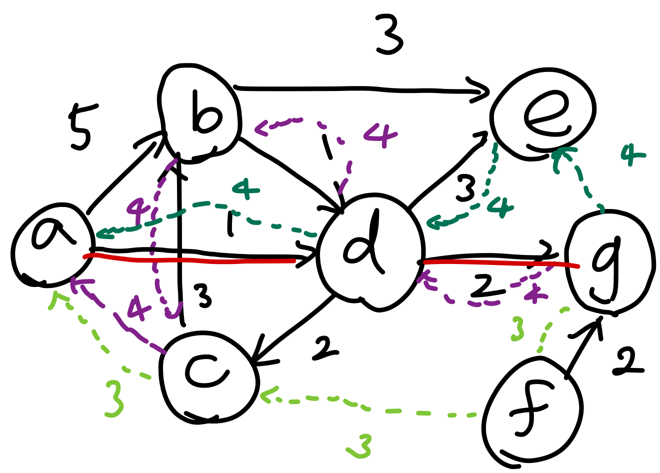
min(7,3,5) = 3



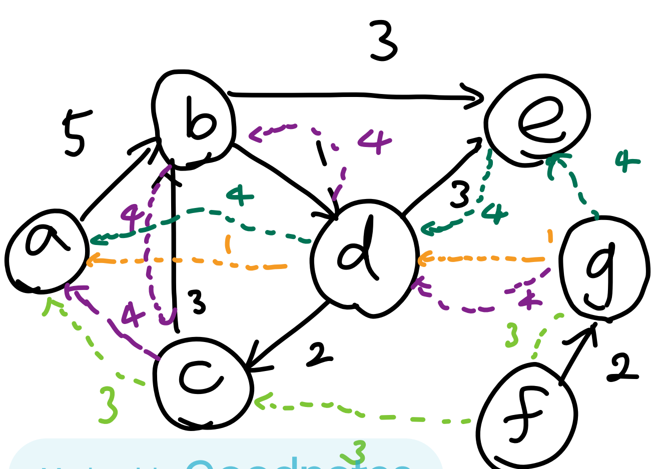
min(4,7,5,6) = 4



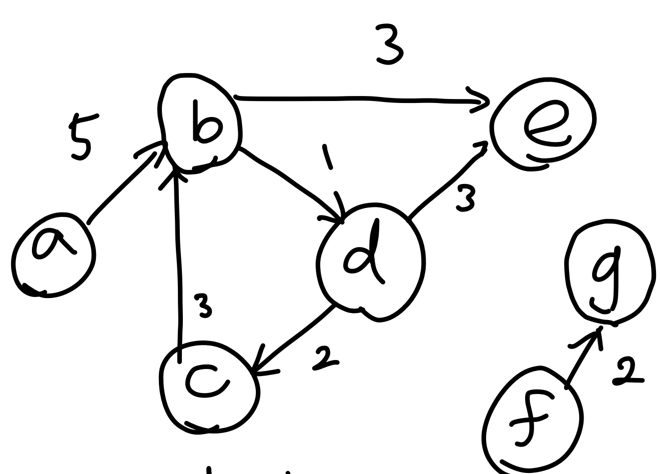
min(5,7,4) = 4



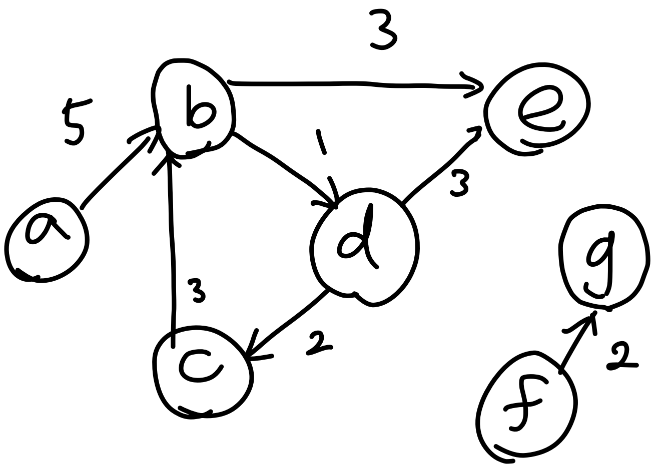
min(1,2) = 1

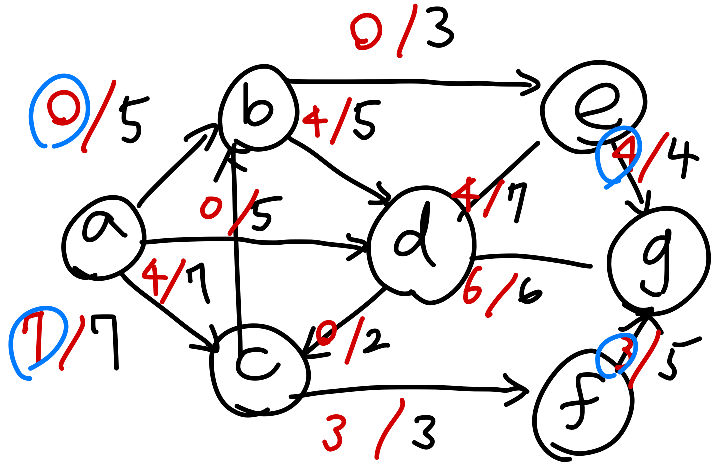


stop here



residual capacity





Network flow = 7

0+7 or 4+3

=7 =7