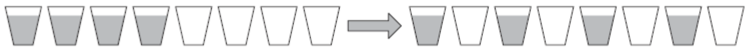
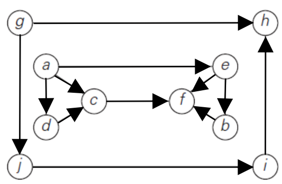
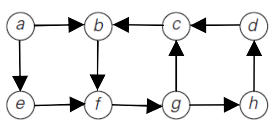
1. (5 points) Alternating glasses



   a. There are $2n$ glasses standing next to each other in a row, the first n of them filled with a soda drink and the remaining n glasses empty. Make the glasses alternate in a filled-empty-filled-empty pattern in the minimum number of glass moves.
   b. Solve the same problem if $2n$ glasses—n with a drink and n empty—are initially in a random order.

2. (5 points) Apply the DFS-based algorithm to solve the topological sorting problem for the following digraphs:

   a. 

   b. 

3. (5 points) Design a decrease-and-conquer algorithm for generating all combinations of k items chosen from n, i.e., all $k - \boldsymbol{element}$ subsets of a given $n - \boldsymbol{element}$ set.

4. (5 points) Binary Search

3	14	27	31	39	42	55	70	74	81	85	93	98

   a. What is the largest number of key comparisons made by binary search in searching for a key in the array?
   b. List all the keys of this array that will require the largest number of key comparisons when searched for by binary search.
   c. Find the average number of key comparisons made by binary search in a successful search in this array. Assume that each key is searched for with the same probability.
   d. Find the average number of key comparisons made by binary search in an unsuccessful search in this array. Assume that searches for keys in each of the 14 intervals formed by the array's elements are equally likely.

5. (5 points) An Array $A[0..n-2]$ contains $n-1$ integers from 1 to n increasing order. (Thus one integer in this range is missing) Design the most efficient algorithm you can to find the missing integer and indicate its time efficiency.
6. (5 points) Outline an algorithm for finding the largest key in a binary search tree.

    a. Would you classify your algorithm as a variable-size-decrease algorithm? Why?
    b. What is the efficiency of your algorithm in the worst case?

7. (5 points) Two players take turn by breaking a $m \times n$ chocolate bar, which has one spoiled 1×1 square. Each break must be a single straight line cutting all the way across the bar along the boundaries between the squares. After each break, the player who broke the bar last eats the piece that does not contain the spoiled square. The player left with the spoiled square loses the game. Is it better to go first or second in this game?